How Dark Sector Equations of State Govern Interaction Signatures

Peng-Ju Wu,[1,*] Ming Zhang,[2] and Shang-Jie Jin[3,4,5]

[1] *School of Physics, Ningxia University, Yinchuan 750021, China*
[2] *School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China*
[3] *Liaoning Key Laboratory of Cosmology and Astrophysics, College of Sciences, Northeastern University, Shenyang 110819, China*
[4] *Department of Physics, University of Western Australia, Perth WA 6009, Australia*
[5] *Research Center for the Early Universe, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan*

Using late-Universe observations, we demonstrate that freeing dark energy and dark matter equations of state (EoS) dramatically alters the inferred strength and direction of their interactions. When dark sector EoS are fixed to $w_{\rm de} = -1$ and $w_{\rm dm} = 0$, the data consistently favor an energy transfer from dark energy to dark matter across various interaction forms. This apparent evidence, however, proves highly sensitive to the EoS assumptions: treating $w_{\rm de}$ as a free parameter substantially weakens the evidence for interaction, with its value converging to the quintessence regime ($w_{\rm de} > -1$). In contrast, freeing $w_{\rm dm}$ maintains a preference for interaction, revealing a correlation where positive $w_{\rm dm}$ is associated with energy transfer from dark energy to dark matter, and negative $w_{\rm dm}$ with energy transfer from dark matter to dark energy. These findings caution against the simplistic assumption of ΛCDM EoS values when attempting to detect a possible interaction. Despite these fundamental degeneracies, model comparison using the Akaike and Deviance information criteria shows that all of the tested interacting dark energy scenarios receive substantial support over the ΛCDM model.

I. INTRODUCTION

The ΛCDM paradigm, which synthesizes a cosmological constant (Λ) with cold dark matter and standard baryonic physics, provides an economical fit to a wide range of astronomical observations, from the cosmic microwave background (CMB) anisotropies to the large-scale clustering of galaxies [1, 2]. Yet the origin of either dark sector remains unknown. In ΛCDM, dark energy is treated as a perfect fluid with equation of state (EoS) parameter $w_{\rm de} = -1$ at all epochs, while dark matter is assumed to be collisionless and non-relativistic with $w_{\rm dm} = 0$. These two components are postulated to evolve independently. However, given their unknown nature, the possibility of a direct interaction between them remains a theoretically compelling extension, prompting investigations into models beyond the minimal standard paradigm [3–12].

The phenomenological foundation of interacting dark energy (IDE) models is characterized by a set of coupled conservation equations. In the background, this interaction is typically encoded through a source term Q in the continuity equations:

$$\dot{\rho}_{\rm de} + 3H(1 + w_{\rm de})\rho_{\rm de} = +Q,$$
$$\dot{\rho}_{\rm dm} + 3H(1 + w_{\rm dm})\rho_{\rm dm} = -Q, \quad (1)$$

where $\rho_{\rm de}$ and $\rho_{\rm dm}$ represent the energy densities of dark energy and dark matter, respectively, and the dot denotes the derivative with respect to cosmic time. The source term Q is usually taken as $Q = \beta H \rho_{\rm de}$ [13, 14], where H is the Hubble parameter and β is a dimensionless coupling parameter; $\beta > 0$ means that dark matter decays into dark energy, $\beta < 0$ indicates that dark energy decays into dark matter, and $\beta = 0$ denotes that there is no interaction between them. The canonical IDE paradigm remains the ΛCDM values for the dark sector EoS. Under this minimal extension, analyses of observational data have revealed scenarios where a non-zero interaction term ($\beta \neq 0$) is statistically favored, suggesting the possible presence of an interaction between dark energy and dark matter [15–26].

Dark energy may not be a mere cosmological constant and dark matter may not be strictly cold. With recent DESI releases, compelling evidence has emerged suggesting a statistically significant deviation of the dark energy EoS from −1 [27–33], alongside tantalizing hints of a non-zero EoS for dark matter [34–39]. Generally, a more negative dark energy EoS and an energy influx from dark matter to dark energy both enhance the observed cosmic acceleration; conversely, a positive dark matter EoS and an energy outflow from dark energy to dark matter produce analogous imprints on the expansion history. This physical mimicry inherently induces degeneracies exist between the interaction coupling parameter and both dark sector EoS [8, 40–42]. A critical question thus arises: how would relaxing the assumptions on dark energy and dark matter EoS influence the direction and strength of the energy transfer in interactions under current observations? Given its importance, this paper undertakes an investigation of this issue.

Motivated by the distinct preferences between different observational datasets—for instance, where CMB data alone favor a coupling parameter $\beta < 0$, while combined CMB+DESI analysis prefers $\beta > 0$ (see Ref. [19]

* Corresponding author; wupengju@nxu.edu.cn

for details)—this work utilize late-universe observations exclusively. This choice is further justified by the documented discrepancies in the measurements of key cosmological parameters like H_0, S_8, and Ω_K [43–47], which often arise when comparing early and late-universe probes (though we note that different voices exist in the literature). By relying solely on late-universe observations, our analysis is designed to provide a complementary and self-consistent perspective on dark sector interactions, that is less susceptible to early-universe measurements.

II. METHOD AND DATA

Previous studies have demonstrated that the strength and direction of the dark sector interaction are closely tied to its specific coupling form. To explore this dependence and to ensure the generality of our investigation, we consider three representative forms of the interaction term Q, where the energy transfer is proportional to the dark energy density, the dark matter density, and the sum of both dark sector densities, respectively [13, 14]. Specifically, we adopt the following three forms: $Q = \beta H \rho_{\rm de}$, $Q = \beta H \rho_{\rm dm}$, and $Q = \beta H(\rho_{\rm dm} + \rho_{\rm de})$.

For the coupling form $Q = \beta H \rho_{\rm de}$, solutions to continuity equations are:

$$\begin{aligned}\rho_{\rm de}(a) &= \rho_{\rm de,0} a^{-3(1+w_{\rm de})+\beta}, \\ \rho_{\rm dm}(a) &= \rho_{\rm dm,0} a^{-3(1+w_{\rm dm})} \\ &\quad - \frac{\beta \rho_{\rm de,0}}{\Delta}\left[a^{-3(1+w_{\rm de})+\beta} - a^{-3(1+w_{\rm dm})}\right], \end{aligned} \tag{2}$$

where $\Delta = 3(w_{\rm dm} - w_{\rm de}) + \beta$. For the case $\Delta = 0$:

$$\rho_{\rm dm}(a) = \rho_{\rm dm,0} a^{-3(1+w_{\rm dm})} - \beta \rho_{\rm de,0} a^{-3(1+w_{\rm dm})} \ln a. \tag{3}$$

For the coupling form $Q = \beta H \rho_{\rm dm}$, solutions to continuity equations are:

$$\begin{aligned}\rho_{\rm dm}(a) &= \rho_{\rm dm,0} a^{-3(1+w_{\rm dm})-\beta}, \\ \rho_{\rm de}(a) &= \rho_{\rm de,0} a^{-3(1+w_{\rm de})} \\ &\quad + \frac{\beta \rho_{\rm dm,0}}{\Delta}\left[a^{-3(1+w_{\rm dm})-\beta} - a^{-3(1+w_{\rm de})}\right], \end{aligned} \tag{4}$$

where $\Delta = 3(w_{\rm de} - w_{\rm dm}) - \beta$. For the case $\Delta = 0$:

$$\rho_{\rm de}(a) = \rho_{\rm de,0} a^{-3(1+w_{\rm de})} + \beta \rho_{\rm dm,0} a^{-3(1+w_{\rm de})} \ln a. \tag{5}$$

For $Q = \beta H(\rho_{\rm dm}+\rho_{\rm de})$, the coupled system of differential equations is solved via eigenvalue decomposition. The continuity equations can be expressed in matrix form as:

$$\frac{\mathrm{d}}{\mathrm{d}\ln a}\begin{pmatrix}\rho_{\rm dm} \\ \rho_{\rm de}\end{pmatrix} = -\mathbf{A}\begin{pmatrix}\rho_{\rm dm} \\ \rho_{\rm de}\end{pmatrix}. \tag{6}$$

where the interaction matrix $\mathbf{A}$ is defined as:

$$\mathbf{A} = \begin{pmatrix} 3(1+w_{\rm dm}) + \beta & \beta \\ -\beta & 3(1+w_{\rm de}) - \beta \end{pmatrix}. \tag{7}$$

The general solution for the dark sector energy densities is obtained by diagonalizing the matrix $\mathbf{A}$:

$$\begin{pmatrix}\rho_{\rm dm} \\ \rho_{\rm de}\end{pmatrix} = C_1 \mathbf{v}_1 a^{-\lambda_1} + C_2 \mathbf{v}_2 a^{-\lambda_2}, \tag{8}$$

where λ_1 and λ_2 are the eigenvalues of matrix $\mathbf{A}$, $\mathbf{v}_1$ and $\mathbf{v}_2$ are the corresponding eigenvectors, C_1 and C_2 are constants determined by the initial conditions $\rho_{\rm dm}(1) = \rho_{\rm dm,0}$ and $\rho_{\rm de}(1) = \rho_{\rm de,0}$.

We constrain these IDE models using the late-time cosmological observations. The dataset comprises:

- **Baryon Acoustic Oscillations** from DESI DR2, including measurements of the transverse, radial, and volume-averaged distance scales in the redshift range $0.295 < z < 2.33$ [28].
- **Type Ia Supernovae** from DESY5, comprising 1829 distance moduli over $0.025 < z < 1.3$ [48].
- **Cosmic Chronometers**, including 32 Hubble parameter measurements covering $0.07 < z < 1.965$, as compiled by Ref. [49].
- **Strong Lensing Time Delays**, incorporating 7 time-delay distance and 5 angular diameter distance data from lenses at $0.295 < z < 0.745$ and sources at $0.654 < z < 2.375$, following Ref. [31].
- **Gamma Ray Bursts**, consisting of 193 distance moduli calibrated via the Amati relation, spanning $0.034 < z < 8.1$ [50].

These data may provide robust constraints through their complementary redshift coverage and sensitivity to different cosmological parameters. To derive constraints that are fully independent of the early-universe observations, we treat the sound horizon at the drag epoch $r_{\rm d}$ as a free parameter in the baryon acoustic oscillation analysis, rather than anchoring it to a value inferred from the CMB data. Additionally, we self-consistently marginalize over the absolute magnitude M_B of the type Ia supernovae to account for the degeneracy with H_0.

It should be pointed out that despite substantial improvements in the quality and quantity of late-time observations, their constraining power remains limited for extensively extended parameter spaces. For this reason, we adopt a targeted strategy to mitigate parameter degeneracies. Rather than simultaneously freeing the EoS for both dark sectors, we examine the following three distinct scenarios:

- Fixing dark energy and dark matter EoS to their ΛCDM values ($w_{\rm de} = -1$ and $w_{\rm dm} = 0$).
- Freeing dark energy EoS while maintaining the cold dark matter assumption ($w_{\rm dm} = 0$).
- Freeing dark matter EoS while maintaining the cosmological constant assumption ($w_{\rm de} = -1$).

This approach allows us to probe possible deviations from the standard model in a controlled manner, providing clearer insights into how interactions manifest alongside non-standard dark sector properties.

III. RESULTS AND DISCUSSIONS

We adopt the Markov Chain Monte Carlo (MCMC) method to infer the probability distributions of cosmological parameters. We conduct parameter sampling with `Cobaya` package [51]. The chain convergence is assessed via the Gelman-Rubin potential scale reduction factor ($R-1<0.01$ for all parameters). We fit the standard ΛCDM model, which serves as our baseline for comparison, two models with free dark sector EoS, and a series of IDE models to the data combination. The 1σ (68.3% confidence level) errors for the marginalized parameter constraints are summarized in Table I. To elucidate the constraints on the interaction coupling parameter β and the relationships between and the EoS parameters for dark energy $w_{\rm de}$ and dark matter $w_{\rm dm}$, the left panel of Fig. 1 presents their marginalized constraints. Each point represents the best-fit value for a specific IDE scenario, with horizontal and vertical error bars indicating the 1σ uncertainties, respectively.

Our analysis begins by examining the non-interacting case ($Q=0$). When dark matter and dark energy EoS adhere to their standard values ($w_{\rm dm}=0$, $w_{\rm de}=-1$), our constraints yield $H_0=73.57^{+0.87}_{-0.87}$ km/s/Mpc and $\Omega_{\rm m}=0.3047^{+0.0075}_{-0.0075}$. The H_0 constraint is consistent with the SH0ES result $H_0=73.04^{+1.04}_{-1.04}$ km/s/Mpc. When allowing the dark energy EoS to deviate from -1 while maintaining $w_{\rm dm}=0$, we observe $w_{\rm de}=-0.883^{+0.041}_{-0.037}$, which departs from the cosmological constant Λ at a significance of 3.2σ. Conversely, when allowing the dark matter EoS to deviate from 0 while fixing $w_{\rm de}=-1$, we obtain $w_{\rm dm}=-0.059^{+0.026}_{-0.022}$, which departs from the cold dark matter case at a significance of 2.3σ. These results therefore make it natural to explore the interplay between the dark sector interaction (governed by Q and β) and the dark sector EoS. As an essential preliminary, we first establish how the introduction of interaction itself modifies the cosmological constraints. To this end, we examine the interacting scenarios under the canonical assumption of $w_{\rm dm}=0$ and $w_{\rm de}=-1$.

Under the assumption of cold dark matter and a cosmological constant, the constraints on IDE models reveal consistent patterns. For all three interaction forms considered, the coupling parameter β is constrained to be negative, indicating a preference for energy transfer from dark energy to dark matter at 3.2σ, 2.4σ, and 2.7σ confidence levels, respectively. This interaction systematically affects the matter density parameter, with $\Omega_{\rm m}$ elevated to $0.378^{+0.024}_{-0.024}$, $0.347^{+0.018}_{-0.018}$, and $0.359^{+0.020}_{-0.020}$ for the three coupling forms respectively, compared to the ΛCDM value of $0.3047^{+0.0075}_{-0.0075}$. This systematic increase in $\Omega_{\rm m}$ is consistent with the physical picture of energy transfer from dark energy to dark matter, as the transferred energy effectively increases the present-day dark matter density. Meanwhile, the Hubble parameter H_0 shows a modest reduction relative to the non-interacting case but remains statistically consistent within uncertainties.

Allowing the dark energy EoS parameter $w_{\rm de}$ to deviate from its cosmological constant value, we observe systematic shifts in the constrained parameter spaces across all interaction forms. For $Q=\beta H\rho_{\rm de}$, freeing $w_{\rm de}$ causes the interaction parameter to move from significantly negative ($\beta=-0.35^{+0.11}_{-0.11}$) to near-zero ($\beta=-0.02^{+0.75}_{-0.38}$), while simultaneously driving $\Omega_{\rm m}$ to lower values ($0.378\to 0.275$) and constraining $w_{\rm de}=-0.89^{+0.25}_{-0.13}$. For $Q=\beta H\rho_{\rm dm}$, the effect is even more pronounced: β changes sign from negative ($-0.180^{+0.074}_{-0.067}$) to positive ($0.31^{+0.27}_{-0.35}$), $\Omega_{\rm m}$ decreases substantially ($0.347\to 0.239$), and $w_{\rm de}$ converges to $-0.792^{+0.110}_{-0.063}$. The $Q=\beta H(\rho_{\rm dm}+\rho_{\rm de})$ form shows similar behavior with β shifting from $-0.120^{+0.044}_{-0.044}$ to $0.22^{+0.27}_{-0.18}$ and $w_{\rm de}$ converges to $-0.758^{+0.170}_{-0.062}$. In summary, allowing for dynamical dark energy ($w_{\rm de}>-1$) resolves the apparent need for interaction. The data prefer a universe where the late-time acceleration is driven by dark energy with properties slightly different from a cosmological constant (quintessence-like behavior), rather than by energy transfer between the dark sectors. This preference is evidenced by the fact that the deviations of β from zero are constrained at 0.03σ, 0.9σ, and 1.2σ confidence levels for the three interaction coupling forms, respectively, whereas the significance of $w_{\rm de}$ departing from -1 reaches substantially higher values of 0.8σ, 3.3σ, and 3.9σ, respectively.

The dark matter EoS, when treated as a free parameter, systematically alters the inferred strength and direction of the dark sector interactions. This influence is clearly demonstrated across different coupling forms. For $Q=\beta H\rho_{\rm de}$, we find $w_{\rm dm}=0.071^{+0.088}_{-0.100}$ correlating with $\beta=-0.53^{+0.16}_{-0.39}$, reinforcing the preference for energy transfer from dark energy to dark matter. In stark contrast, for $Q=\beta H\rho_{\rm dm}$, the constraint $w_{\rm dm}=-0.20^{+0.20}_{-0.49}$ necessitates a sign reversal in the interaction, yielding $\beta=0.42^{+1.50}_{-0.54}$ (i.e., dark matter decays into dark energy). For the symmetric coupling $Q=\beta H(\rho_{\rm dm}+\rho_{\rm de})$, the constraint $w_{\rm dm}=0.23^{+0.24}_{-0.24}$ correlates with $\beta=-0.39^{+0.16}_{-0.37}$, maintaining the same anti-correlation pattern observed in the asymmetric case. Across different coupling forms, we observe a consistent pattern: a positive $w_{\rm dm}$ correlates with energy transfer from dark energy to dark matter ($\beta<0$), while a negative $w_{\rm dm}$ pairs with energy transfer in the opposite direction ($\beta>0$). This compensatory relationship demonstrates that the cosmological effects of dark matter pressure can be equivalently described by an appropriate interaction between the dark sectors. In contrast to the case with a free $w_{\rm de}$, when $w_{\rm dm}$ is freed, the data continue to favor an interaction. The parameter β deviates from zero at 3.3σ, 0.8σ, and 2.4σ confidence levels for the three interaction forms, respectively. In comparison, the deviation of $w_{\rm dm}$ from zero is substan-

TABLE I. Cosmological parameter constraints for the ΛCDM model, its extensions with a free dark energy or dark matter EoS, and interacting dark energy models using late-time observational data. All models are constrained with a combination of DESI DR2 baryon acoustic oscillations, DESY5 supernova, cosmic chronometers, strong lensing time delays, and gamma-ray bursts. Here H_0 is in units of km/s/Mpc.

Interaction Form	**Setup**	H_0	$\Omega_{\rm m}$	β	**EoS Parameter**
$Q = 0$	$w_{\rm dm} = 0$; $w_{\rm de} = -1$	$73.57^{+0.87}_{-0.87}$	$0.3047^{+0.0075}_{-0.0075}$	0	–
$Q = 0$	$w_{\rm dm} = 0$; Free $w_{\rm de}$	$72.40^{+0.88}_{-0.88}$	$0.2961^{+0.0090}_{-0.0090}$	0	$w_{\rm de} = -0.883^{+0.041}_{-0.037}$
$Q = 0$	Free $w_{\rm dm}$; $w_{\rm de} = -1$	$72.80^{+0.90}_{-0.90}$	$0.365^{+0.024}_{-0.027}$	0	$w_{\rm dm} = -0.059^{+0.026}_{-0.022}$
$Q = \beta H \rho_{\rm de}$	$w_{\rm dm} = 0$; $w_{\rm de} = -1$	$72.51^{+0.91}_{-0.91}$	$0.378^{+0.024}_{-0.024}$	$-0.35^{+0.11}_{-0.11}$	–
$Q = \beta H \rho_{\rm de}$	$w_{\rm dm} = 0$; Free $w_{\rm de}$	$72.50^{+0.91}_{-0.91}$	$0.275^{+0.210}_{-0.092}$	$-0.02^{+0.75}_{-0.38}$	$w_{\rm de} = -0.89^{+0.25}_{-0.13}$
$Q = \beta H \rho_{\rm de}$	Free $w_{\rm dm}$; $w_{\rm de} = -1$	$72.40^{+0.92}_{-0.92}$	$0.372^{+0.021}_{-0.024}$	$-0.53^{+0.16}_{-0.39}$	$w_{\rm dm} = 0.071^{+0.088}_{-0.100}$
$Q = \beta H \rho_{\rm dm}$	$w_{\rm dm} = 0$; $w_{\rm de} = -1$	$72.78^{+0.91}_{-0.91}$	$0.347^{+0.018}_{-0.018}$	$-0.180^{+0.074}_{-0.067}$	–
$Q = \beta H \rho_{\rm dm}$	$w_{\rm dm} = 0$; Free $w_{\rm de}$	$72.33^{+0.91}_{-0.91}$	$0.239^{+0.043}_{-0.066}$	$0.31^{+0.27}_{-0.35}$	$w_{\rm de} = -0.792^{+0.110}_{-0.063}$
$Q = \beta H \rho_{\rm dm}$	Free $w_{\rm dm}$; $w_{\rm de} = -1$	$72.81^{+0.89}_{-0.89}$	$0.51^{+0.11}_{-0.27}$	$0.42^{+1.50}_{-0.54}$	$w_{\rm dm} = -0.20^{+0.20}_{-0.49}$
$Q = \beta H(\rho_{\rm dm} + \rho_{\rm de})$	$w_{\rm dm} = 0$; $w_{\rm de} = -1$	$72.68^{+0.90}_{-0.90}$	$0.359^{+0.020}_{-0.020}$	$-0.120^{+0.044}_{-0.044}$	–
$Q = \beta H(\rho_{\rm dm} + \rho_{\rm de})$	$w_{\rm dm} = 0$; Free $w_{\rm de}$	$72.42^{+0.90}_{-0.90}$	$0.181^{+0.061}_{-0.170}$	$0.22^{+0.27}_{-0.18}$	$w_{\rm de} = -0.758^{+0.170}_{-0.062}$
$Q = \beta H(\rho_{\rm dm} + \rho_{\rm de})$	Free $w_{\rm dm}$; $w_{\rm de} = -1$	$72.44^{+0.91}_{-0.91}$	$0.333^{+0.021}_{-0.039}$	$-0.39^{+0.16}_{-0.37}$	$w_{\rm dm} = 0.23^{+0.24}_{-0.24}$

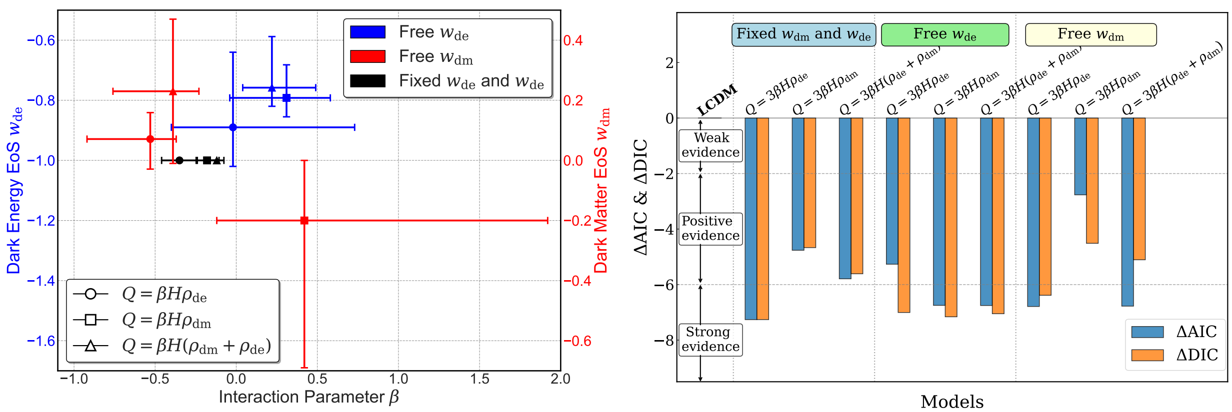

FIG. 1. Analysis of interacting dark energy scenarios: observational constraints and statistical evidence. Left: Marginalized joint constraints (68% confidence level) on the dark energy EoS parameter $w_{\rm de}$, dark matter EoS parameter $w_{\rm dm}$, and the interaction coupling parameter β. Results are shown for three interaction models: $Q = \beta H \rho_{\rm de}$ (circles), $Q = \beta H \rho_{\rm dm}$ (squares), and $Q = \beta H(\rho_{\rm dm} + \rho_{\rm de})$ (triangles). For each model, we present the constraints under three different parameter assumptions: freeing $w_{\rm de}$ (blue), freeing $w_{\rm dm}$ (red), and fixing both EoS to their standard ΛCDM values (black). The error bars on the data points represent the 1σ uncertainties. Right: Graphical representation of model comparison results. In this work, we choose flat ΛCDM model as a reference to calculate the ΔAIC and ΔDIC values. Generally, a negative ΔAIC/DIC value within the range $[-2, 0)$ indicates weak evidence in favor of the model, a value within $[-6, -2)$ indicates positive evidence in favor, a value within $[-10, -6)$ indicates strong evidence in favor, and a value less than -10 indicates very strong evidence in favor. The order of models from left to right is arranged according to the interaction form.

tially less significant, at 0.7σ, 1.0σ, and 0.96σ confidence levels, respectively.

Our parameter constraints on Hubble constant demonstrate consistency with local measurements, with H_0 values ranging between 72.33 and 72.81 km/s/Mpc across all IDE scenarios, aligning with SH0ES collaboration's measurement of 73.04 ± 1.04 km/s/Mpc. The tight H_0 uncertainties ($\pm 0.90 - 0.92$ km/s/Mpc across models) indicate that our dataset strongly constrains the current expansion rate regardless of interaction mechanism. Freeing the dark energy EoS consistently yields a lower $\Omega_{\rm m}$ across all interaction models, indicating a robust degeneracy where a more negative $w_{\rm de}$ compensates for a reduced matter density. In contrast, freeing the dark matter EoS shows no universal trend: $\Omega_{\rm m}$ may increase or decrease depending on the specific interaction form, highlighting the model-dependent nature of $w_{\rm dm}$ and its complex role in balancing the cosmological energy budget.

Having established the parameter constraints for IDE scenarios, we turn to a quantitative model comparison to assess their relative statistical support from the observational data. To this end, we employ the Akaike Information Criterion (AIC) and Deviance Information Criterion (DIC), which provide robust measures of model performance while penalizing excessive complexity. These are respectively defined as:

$$\begin{aligned} {\rm AIC} &= \chi^2_{\rm min} + 2k, \\ {\rm DIC} &= \chi^2_{\rm min} + 2k_{\rm eff}, \end{aligned} \tag{9}$$

where k represents the number of free parameters in the model, and $k_{\rm eff} = \langle\chi^2\rangle - \chi^2_{\rm min}$ denotes the number of effectively constrained parameters, with the angular brackets indicating the average over the posterior distribution. Models yielding smaller AIC/DIC values are considered to be more favored by the observational data. We are not concerned with the absolute values of AIC/DIC, but the relative ones between different models, i.e., ΔAIC/DIC. Here, we adopt the ΛCDM model as baseline for comparison. Therefore, negative ΔAIC/DIC values denote superior fit to data relative to ΛCDM, whereas positive values indicate inferior fit.

Generally, a ΔAIC/DIC value within the range $[-2, 0)$ indicates weak evidence in favor of the model, a value within $[-6, -2)$ denotes positive evidence in favor, a value within $[-10, -6)$ denotes strong evidence in favor, and a value less than -10 indicates very strong evidence in favor. Conversely, the interpretation for positive values is symmetric, indicating equivalent levels of evidence against the model. Our statistical analysis reveals that all IDE models considered in this work receive observational support relative to the flat ΛCDM cosmology, with evidence strengths ranging from positive to strong. Specifically, we find that there are four IDE models are strongly favored by both AIC and DIC criteria, three models show positive evidence according to both metrics, and two models exhibit mixed evidence with one criterion indicating positive support and the other indicating strong support, as systematically summarized in the right panel of Fig. 1. This consistent pattern across complementary statistical measures reinforces the viability of interacting dark sector scenarios in explaining current cosmological observations.

IV. CONCLUSION

This work provides an investigation of IDE scenarios using late-universe observations, examining how freeing dark energy and dark matter EoS affect the strength and direction of dark sector interactions.

Our analysis reveals the following key findings. When the dark sector EoS are fixed to their ΛCDM values ($w_{\rm de} = -1$, $w_{\rm dm} = 0$), all interaction forms consistently yield negative coupling parameters β, indicating a preference for energy transfer from dark energy to dark matter. However, this apparent evidence is critically dependent on the EoS assumptions. We identify significant degeneracies between the interaction coupling parameter and dark sector EoS. When $w_{\rm de}$ is allowed to deviate from -1, the previously strong evidence for interaction substantially weakens across all coupling forms. This systematic pattern demonstrates that the observational signatures previously attributed to dark sector interaction can be equally well explained by a dynamical dark energy with $w_{\rm de} > -1$. In contrast, when $w_{\rm dm}$ is freed, the data continue to favor an interaction between dark sectors. We find a fundamental degeneracy between the interaction coupling parameter and the dark matter EoS. Positive $w_{\rm dm}$ correlates with energy transfer from dark energy to dark matter ($\beta < 0$), while negative $w_{\rm dm}$ associates with energy flow in the opposite direction ($\beta > 0$). Our analysis also find that the interaction form influences the resulting constraints on the coupling parameter. In addition, model comparison using both the AIC and DIC criteria shows that all IDE models considered receive substantial statistical support over the standard ΛCDM cosmology, with evidence levels ranging from positive to strong.

Despite these advances, our analysis has several limitations. Notably, we have not explored scenarios where $w_{\rm de}$ and $w_{\rm dm}$ are simultaneously freed, nor have we considered the time-evolving EoS. These choices are primarily due to the limited constraining power of current late-universe datasets, which would yield uninformatively large uncertainties if applied to such extended parameter spaces. Moving forward, we plan to pursue several promising avenues. First, we will investigate the inclusion of CMB data, which could help break degeneracies through its complementary sensitivity to early-universe physics and thereby impose tighter constraints on the interplay between interactions and the properties of dark matter and dark energy. While promising, this approach requires careful consideration of the potential tensions between CMB and late-time observations regarding the interaction direction, as presented in Ref. [19]. Second, we will examine how synergies of emerging late-universe probes [52], such as 21 cm intensity mapping, fast radio bursts, gravitational wave standard siren, and strong gravitational lensing, can improve constraints on these more complex interacting scenarios.

ACKNOWLEDGMENTS

We thank Jian-Hua Yang, Guo-Hong Du and Tian-Nuo Li for fruitful discussions. This work was supported by the Research Starting Funds for Imported Talents, Ningxia University (Grant No. 030700002562). Shang-Jie Jin is grateful for the support from the China Scholarship Council, and JST ASPIRE Program of Japan (Grant No. JPMJAP2320).

[1] N. Aghanim *et al.* (Planck), "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. **641**, A6 (2020), [Erratum: Astron.Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].

[2] Shadab Alam *et al.* (eBOSS), "Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory," Phys. Rev. D **103**, 083533 (2021), arXiv:2007.08991 [astro-ph.CO].

[3] Luca Amendola, "Coupled quintessence," Phys. Rev. D **62**, 043511 (2000), arXiv:astro-ph/9908023.

[4] Glennys R. Farrar and P. James E. Peebles, "Interacting dark matter and dark energy," Astrophys. J. **604**, 1–11 (2004), arXiv:astro-ph/0307316.

[5] Jussi Valiviita, Elisabetta Majerotto, and Roy Maartens, "Instability in interacting dark energy and dark matter fluids," JCAP **07**, 020 (2008), arXiv:0804.0232 [astro-ph].

[6] Yun-He Li, Jing-Fei Zhang, and Xin Zhang, "Parametrized Post-Friedmann Framework for Interacting Dark Energy," Phys. Rev. D **90**, 063005 (2014), arXiv:1404.5220 [astro-ph.CO].

[7] Yun-He Li and Xin Zhang, "Large-scale stable interacting dark energy model: Cosmological perturbations and observational constraints," Phys. Rev. D **89**, 083009 (2014), arXiv:1312.6328 [astro-ph.CO].

[8] B. Wang, E. Abdalla, F. Atrio-Barandela, and D. Pavon, "Dark Matter and Dark Energy Interactions: Theoretical Challenges, Cosmological Implications and Observational Signatures," Rept. Prog. Phys. **79**, 096901 (2016), arXiv:1603.08299 [astro-ph.CO].

[9] B. Wang, E. Abdalla, F. Atrio-Barandela, and D. Pavón, "Further understanding the interaction between dark energy and dark matter: current status and future directions," Rept. Prog. Phys. **87**, 036901 (2024), arXiv:2402.00819 [astro-ph.CO].

[10] Marcel van der Westhuizen, Amare Abebe, and Eleonora Di Valentino, "I. Linear interacting dark energy: Analytical solutions and theoretical pathologies," Phys. Dark Univ. **50**, 102119 (2025), arXiv:2509.04495 [gr-qc].

[11] Marcel van der Westhuizen, Amare Abebe, and Eleonora Di Valentino, "II. Non-linear interacting dark energy: Analytical solutions and theoretical pathologies," Phys. Dark Univ. **50**, 102120 (2025), arXiv:2509.04494 [gr-qc].

[12] Marcel van der Westhuizen, Amare Abebe, and Eleonora Di Valentino, "III. Interacting Dark Energy: Summary of models, Pathologies, and Constraints," Phys. Dark Univ. **50**, 102121 (2025), arXiv:2509.04496 [gr-qc].

[13] Andrew P. Billyard and Alan A. Coley, "Interactions in scalar field cosmology," Phys. Rev. D **61**, 083503 (2000), arXiv:astro-ph/9908224.

[14] Luca Amendola, "Scaling solutions in general nonminimal coupling theories," Phys. Rev. D **60**, 043501 (1999), arXiv:astro-ph/9904120.

[15] Orfeu Bertolami, F. Gil Pedro, and M. Le Delliou, "Dark Energy-Dark Matter Interaction and the Violation of the Equivalence Principle from the Abell Cluster A586," Phys. Lett. B **654**, 165–169 (2007), arXiv:astro-ph/0703462.

[16] Valentina Salvatelli, Najla Said, Marco Bruni, Alessandro Melchiorri, and David Wands, "Indications of a late-time interaction in the dark sector," Phys. Rev. Lett. **113**, 181301 (2014), arXiv:1406.7297 [astro-ph.CO].

[17] Elisa G. M. Ferreira, Jerome Quintin, Andre A. Costa, E. Abdalla, and Bin Wang, "Evidence for interacting dark energy from BOSS," Phys. Rev. D **95**, 043520 (2017), arXiv:1412.2777 [astro-ph.CO].

[18] Weiqiang Yang, Supriya Pan, Eleonora Di Valentino, Rafael C. Nunes, Sunny Vagnozzi, and David F. Mota, "Tale of stable interacting dark energy, observational signatures, and the H_0 tension," JCAP **09**, 019 (2018), arXiv:1805.08252 [astro-ph.CO].

[19] Tian-Nuo Li, Peng-Ju Wu, Guo-Hong Du, Shang-Jie Jin, Hai-Li Li, Jing-Fei Zhang, and Xin Zhang, "Constraints on Interacting Dark Energy Models from the DESI Baryon Acoustic Oscillation and DES Supernovae Data," Astrophys. J. **976**, 1 (2024), arXiv:2407.14934 [astro-ph.CO].

[20] Tian-Nuo Li, Guo-Hong Du, Yun-He Li, Peng-Ju Wu, Shang-Jie Jin, Jing-Fei Zhang, and Xin Zhang, "Probing the sign-changeable interaction between dark energy and dark matter with DESI baryon acoustic oscillations and DES supernovae data," Sci. China Phys. Mech. Astron. **69**, 210413 (2026), arXiv:2501.07361 [astro-ph.CO].

[21] Tian-Nuo Li, Guo-Hong Du, Yun-He Li, Yichao Li, Jia-Le Ling, Jing-Fei Zhang, and Xin Zhang, "Updated constraints on interacting dark energy: A comprehensive analysis using multiple CMB probes, DESI DR2, and supernovae observations," (2025), arXiv:2510.11363 [astro-ph.CO].

[22] Supriya Pan, Sivasish Paul, Emmanuel N. Saridakis, and Weiqiang Yang, "Interacting dark energy after DESI DR2: a challenge for ΛCDM paradigm?" (2025), arXiv:2504.00994 [astro-ph.CO].

[23] Yun-He Li, Jing-Fei Zhang, and Xin Zhang, "Testing models of vacuum energy interacting with cold dark matter," Phys. Rev. D **93**, 023002 (2016), arXiv:1506.06349 [astro-ph.CO].

[24] William J. Wolf, Carlos García-García, Theodore Anton, and Pedro G. Ferreira, "Assessing Cosmological Evidence for Nonminimal Coupling," Phys. Rev. Lett. **135**, 081001 (2025), arXiv:2504.07679 [astro-ph.CO].

[25] Changyu You, Dan Wang, and Tao Yang, "Dynamical dark energy implies a coupled dark sector: Insights from DESI DR2 via a data-driven approach," Phys. Rev. D **112**, 043503 (2025), arXiv:2504.00985 [astro-ph.CO].

[26] William J. Wolf, Pedro G. Ferreira, and Carlos García-García, "Matching current observational constraints with nonminimally coupled dark energy," Phys. Rev. D **111**, L041303 (2025), arXiv:2409.17019 [astro-ph.CO].

[27] A. G. Adame *et al.* (DESI), "DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations," JCAP **02**, 021 (2025), arXiv:2404.03002 [astro-ph.CO].

[28] M. Abdul Karim *et al.* (DESI), "DESI DR2 results. II. Measurements of baryon acoustic oscillations and cosmological constraints," Phys. Rev. D **112**, 083515 (2025), arXiv:2503.14738 [astro-ph.CO].

[29] Mateus Scherer, Miguel A. Sabogal, Rafael C. Nunes, and Antonio De Felice, "Challenging the ΛCDM model: 5σ evidence for a dynamical dark energy late-time transition," Phys. Rev. D **112**, 043513 (2025), arXiv:2504.20664 [astro-ph.CO].

[30] Miguel A. Sabogal and Rafael C. Nunes, "Robust evidence for dynamical dark energy from DESI galaxy-CMB lensing cross-correlation and geometric probes," JCAP **09**, 084 (2025), arXiv:2505.24465 [astro-ph.CO].

[31] Peng-Ju Wu, "Comparison of dark energy models using late-universe observations," Phys. Rev. D **112**, 043527 (2025), arXiv:2504.09054 [astro-ph.CO].

[32] Eoin Ó. Colgáin, Saeed Pourojaghi, M. M. Sheikh-Jabbari, and Lu Yin, "How much has DESI dark energy evolved since DR1?" (2025), arXiv:2504.04417 [astro-ph.CO].

[33] Yifu Cai, Xin Ren, Taotao Qiu, Mingzhe Li, and Xinmin Zhang, "The Quintom theory of dark energy after DESI DR2," (2025), arXiv:2505.24732 [astro-ph.CO].

[34] Tian-Nuo Li, Peng-Ju Wu, Guo-Hong Du, Yan-Hong Yao, Jing-Fei Zhang, and Xin Zhang, "Exploring non-cold dark matter in the scenario of dynamical dark energy with DESI DR2 data," Phys. Dark Univ. **50**, 102068 (2025), arXiv:2507.07798 [astro-ph.CO].

[35] Utkarsh Kumar, Abhijith Ajith, and Amresh Verma, "Evidence for non-cold dark matter from DESI DR2 measurements," (2025), arXiv:2504.14419 [astro-ph.CO].

[36] Weiqiang Yang, Supriya Pan, Eleonora Di Valentino, Olga Mena, David F. Mota, and Subenoy Chakraborty, "Probing the cold nature of dark matter," Phys. Rev. D **111**, 103509 (2025), arXiv:2504.11973 [astro-ph.CO].

[37] Xingang Chen and Abraham Loeb, "Evolving dark energy or dark matter with an evolving equation-of-state?" JCAP **07**, 059 (2025), arXiv:2505.02645 [astro-ph.CO].

[38] Deng Wang, "Evidence for Dynamical Dark Matter," (2025), arXiv:2504.21481 [astro-ph.CO].

[39] Dorian Araya, Cristian Castillo, Genly Leon, Juan Magaña, Angie Barr Domínguez, and Miguel A. García-Aspeitia, "w_{dm}-w_{de} cosmological model with new data samples of cosmological observations," (2025), arXiv:2506.09281 [astro-ph.CO].

[40] Timothy Clemson, Kazuya Koyama, Gong-Bo Zhao, Roy Maartens, and Jussi Valiviita, "Interacting Dark Energy – constraints and degeneracies," Phys. Rev. D **85**, 043007 (2012), arXiv:1109.6234 [astro-ph.CO].

[41] Supriya Pan, Weiqiang Yang, Eleonora Di Valentino, David F. Mota, and Joseph Silk, "IWDM: the fate of an interacting non-cold dark matter — vacuum scenario," JCAP **07**, 064 (2023), arXiv:2211.11047 [astro-ph.CO].

[42] Rahul Shah, Purba Mukherjee, and Supratik Pal, "Interacting Dark Sectors in light of DESI DR2," Mon. Not. Roy. Astron. Soc. **542**, 2936 (2025), arXiv:2503.21652 [astro-ph.CO].

[43] L. Verde, T. Treu, and A. G. Riess, "Tensions between the Early and the Late Universe," Nature Astron. **3**, 891 (2019), arXiv:1907.10625 [astro-ph.CO].

[44] Adam G. Riess *et al.*, "A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s^{-1} Mpc^{-1} Uncertainty from the Hubble Space Telescope and the SH0ES Team," Astrophys. J. Lett. **934**, L7 (2022), arXiv:2112.04510 [astro-ph.CO].

[45] Eleonora Di Valentino, Alessandro Melchiorri, and Joseph Silk, "Planck evidence for a closed Universe and a possible crisis for cosmology," Nature Astron. **4**, 196–203 (2019), arXiv:1911.02087 [astro-ph.CO].

[46] Will Handley, "Curvature tension: evidence for a closed universe," Phys. Rev. D **103**, L041301 (2021), arXiv:1908.09139 [astro-ph.CO].

[47] Peng-Ju Wu and Xin Zhang, "Measuring cosmic curvature with non-CMB observations," Phys. Rev. D **112**, 063514 (2025), arXiv:2411.06356 [astro-ph.CO].

[48] T. M. C. Abbott *et al.* (DES), "The Dark Energy Survey: Cosmology Results with ∼1500 New High-redshift Type Ia Supernovae Using the Full 5 yr Data Set," Astrophys. J. Lett. **973**, L14 (2024), arXiv:2401.02929 [astro-ph.CO].

[49] Michele Moresco *et al.*, "Unveiling the Universe with emerging cosmological probes," Living Rev. Rel. **25**, 6 (2022), arXiv:2201.07241 [astro-ph.CO].

[50] Lorenzo Amati, Rocco D'Agostino, Orlando Luongo, Marco Muccino, and Maria Tantalo, "Addressing the circularity problem in the $E_{\rm p}-E_{\rm iso}$ correlation of gamma-ray bursts," Mon. Not. Roy. Astron. Soc. **486**, L46–L51 (2019), arXiv:1811.08934 [astro-ph.HE].

[51] Jesus Torrado and Antony Lewis, "Cobaya: Code for Bayesian Analysis of hierarchical physical models," JCAP **05**, 057 (2021), arXiv:2005.05290 [astro-ph.IM].

[52] Peng-Ju Wu, Yue Shao, Shang-Jie Jin, and Xin Zhang, "A path to precision cosmology: synergy between four promising late-universe cosmological probes," JCAP **06**, 052 (2023), arXiv:2202.09726 [astro-ph.CO].