Minimal Flavor Protection for TeV-scale New Physics

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We determine how much TeV-scale new physics can deviate from flavor universality, $U(3)^5$, while respecting stringent bounds on flavor-changing neutral currents. The minimal continuous subgroup that *must* be approximately preserved is identified as $SU(2)_q \times U(1)_X$. With only a few symmetry-breaking spurions of $\mathcal{O}(10^{-2})$, all observed fermion hierarchies may be reproduced, offering a new perspective on the SM flavor puzzle. Remarkably, this framework provides structural flavor protection for generic TeV-scale new physics within the SMEFT, enlarging the space of collider-accessible scenarios beyond MFV and $U(2)^5$ and allowing for richer patterns of flavor violation.

1. Introduction

Mass hierarchies are among the most profound structural questions in particle physics and call for an explanation beyond the Standard Model (BSM). A striking example is the peculiar hierarchical pattern of fermion masses and mixings, the *SM flavor puzzle* [1, 2], which points to the existence of an underlying structure behind the observed Yukawa interactions of the Higgs field. Being dimensionless, the Yukawa couplings do not single out a characteristic scale at which this structure must emerge. In contrast, the electroweak hierarchy problem, arising from the sensitivity of the Higgs mass to ultraviolet (UV) dynamics, suggests that new physics should appear not far above the TeV scale, the present energy frontier of collider experiments.

The hypothesis of TeV-scale, collider-accessible new physics with sizable couplings to SM fields is well motivated and actively pursued in ongoing searches at the Large Hadron Collider (LHC), but it gives rise to a new puzzle, the *new physics flavor puzzle* [2–8]. Stringent indirect bounds from flavor-changing neutral currents (FCNC) and CP-violating observables imply that such new physics cannot have a generic flavor structure, but must exhibit special protection mechanisms to evade current limits from low-energy precision experiments.

Without Yukawa interactions, the SM has a large global flavor symmetry, $U(3)^5$ [9], corresponding to independent unitary rotations of the three generations of each fermion gauge representation. The Yukawa couplings explicitly break this symmetry, leaving only baryon number and individual lepton flavors, $U(1)_B \times U(1)^3$, as exact symmetries. Owing to the hierarchical structure, one can identify approximate flavor symmetries of increasing quality: the largest breaking, driven by $y_t \sim \mathcal{O}(1)$, reduces the original symmetry down to $U(2)^2 \times U(1) \times U(3)^3$, followed by a smaller breaking of $\mathcal{O}(0.01)$, etc.

Mechanisms that protect TeV-scale new physics from excessive flavor violation ensure that the approximate flavor symmetries of the SM are also respected in the new physics sector. This predicts BSM selection rules that mirror those of the SM, keeping flavor and CP observables under control. The classic paradigm is *Minimal Flavor Violation* (MFV) [10, 11], where the three SM Yukawa matrices serve as spurions that encode all sources of $U(3)^5$ and CP violation, including those in the BSM sector. Among the variations and alternatives [12–15], the most extensively studied framework is the minimally-broken $U(2)^5$ flavor symmetry [16–18]. It distinguishes the third family from the first two, reproduces the SM Yukawa structure with four spurions, and simultaneously provides flavor protection for new physics. Within the Standard Model Effective Field Theory (SMEFT) [19–21], used as a proxy for generic short-distance new physics, both flavor symmetries and their breakings can be implemented order by order in the spurion expansion [22, 23]. The resulting power counting provides structural suppression of dangerous FCNC.

The relationship between the two paradigms yields valuable insights. An $\mathcal{O}(1)$ breaking of $U(3)^5$ down to a smaller $U(2)^5$ symmetry allows for TeV-scale new physics which exhibits richer flavor phenomenology, particularly in the b and τ sectors [18, 24]. In addition, the $U(2)^5$ framework provides a setting to (partially) address the SM flavor puzzle, since, unlike MFV, the Yukawas themselves originate from small spurions.

While MFV guarantees sufficient flavor protection, it is not strictly necessary and is thus overly restrictive: it excludes a broad class of potentially interesting flavor phenomena that could still be compatible with TeV-scale dynamics. Following the success of $U(2)^5$ as a first step in this direction, we ask *how much* TeV-scale new physics can *further* break the full $U(3)^5$ flavor symmetry at $\mathcal{O}(1)$ without conflicting with stringent bounds from FCNC and CP-violating observables. A pragmatic requirement is only that the most dangerous observables, such as K–$\bar{K}$, D–$\bar{D}$ mixing, $\mu \to e\gamma$, and electric dipole moments (EDMs) receive strong suppression, as these processes probe energies far above the TeV scale.

We formulate this question by searching for the smallest continuous subgroup $\mathcal{H} \subset U(3)^5$ that must remain a

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good approximate symmetry at the TeV scale, while all other directions in flavor space are allowed to be maximally violated, i.e. an $\mathcal{O}(1)$ breaking of $U(3)^5 \to \mathcal{H}$. The residual symmetry $\mathcal{H}$ is then broken by a set of small spurions, whose sizes control and reproduce the observed SM flavor parameters. This symmetry–spurion structure then sets the power counting of flavor violation in baryon-number–conserving dimension-6 SMEFT operators, again our proxy for generic short-distance new physics, either through explicit spurion insertions or via rotations to the mass basis. We seek the smallest continuous flavor symmetry that admits spurions capable of reproducing the SM flavor parameters while simultaneously providing structural protection for generic TeV-scale new physics against dangerous FCNC.

To sum up, we adopt a pragmatic viewpoint: introduce only the necessary flavor suppression and nothing more. This framework, which we name *Minimal Flavor Protection* (MFP)[1] as a structural alternative to MFV and $U(2)^5$, has immediate implications for ongoing flavor experiments at bottom, charm, tau, muon and kaon facilities [26–36], EDMs [37–39] as well as high-p_T flavor studies at the LHC [40, 41] and future colliders [42–45]. It naturally predicts the possibility of $\mathcal{O}(1)$ violations of flavor universality. In addition, it allows a class of sizable flavor-changing decays, which we term *flavor-charged currents*, that are forbidden or extremely suppressed in MFV due to correlations with neutral-meson mixing. This motivates a systematic experimental program to search for rich TeV-scale flavor dynamics.

Another motivation for reducing the starting flavor symmetry group is a tendency to transition from merely accommodating SM flavor hierarchies to explaining them within a dynamical framework. In this regard, even $U(2)^5$ is only a partial step: it leaves unexplained the hierarchical eigenvalues of the bifundamental spurions $\Delta_{u,d,e}$ that generate masses for the first two generations, while MFV leaves the SM flavor hierarchies entirely unexplained. As we will show, in our framework all four spurions naturally appear at the same order, $\mathcal{O}(0.01)$, with no hierarchies between them nor within their components. This uniformity greatly simplifies the task of identifying a dynamical origin of these spurions, *e.g.*, through the minimization of a scalar potential. This is known to be non-trivial task for larger symmetry groups [46–52].

Regarding EDMs, it is well known that flavor symmetries alone are insufficient; for example, they do not forbid the purely bosonic operator $GG\widetilde{G}$ [53], nor do they provide sufficient protection for the electron EDM [54, 55], which calls for additional CP symmetry. In this letter, we assume that CP is broken solely by the flavor spurion responsible for the CKM phase. This assumption, however, is not required for FCNC, which may maximally violate CP in our setup.

[1] The nomenclature MFP was previously used in the narrow context of warped extra dimensions [25].

2. The necessity of $SU(2)_q$

The smallest continuous subgroup of $U(3)^5$, a single $U(1)$, can address the SM flavor puzzle via the classic Froggatt–Nielsen (FN) mechanism [56–59]. This $U(1)$ may be viewed as a particular linear combination of the Cartan generators, giving fermions family-dependent charges.[2] The resulting Yukawa hierarchies are then generated by appropriate powers of a single small spurion that breaks the symmetry. However, it is well known that the resulting selection rules on SMEFT operators typically induce large FCNC, leading to bounds far above the TeV scale, see e.g. [60–62].

Here is a simple argument illustrating that a single $U(1)$ (or multiple $U(1)$ factors) is insufficient. Consider a four-fermion operator with left-handed quarks,

$$\mathcal{L} \supset \frac{\mathcal{C}_{ijk\ell}}{\Lambda_{\text{eff}}^2}\,(\bar{q}_i\gamma_\mu q_j)(\bar{q}_k\gamma^\mu q_\ell)\,. \tag{1}$$

To avoid unsuppressed contributions to $K-\bar{K}$ and $D-\bar{D}$ mixing, one must assign different charges to the first two families, $X_{q_1} \neq X_{q_2}$. This predicts $\mathcal{C}_{1111}$, $\mathcal{C}_{1122}$, $\mathcal{C}_{1221}$, and $\mathcal{C}_{2222}$ to be uncorrelated $\mathcal{O}(1)$ numbers. The rotation from the interaction to the mass basis must reproduce the Cabibbo angle, whether it is generated in the up sector, the down sector, or shared between them. This choice determines whether K- or D-mixing sets the dominant bound, but it cannot evade both. Indeed, as shown in Table 6 of Ref. [63], all these $\mathcal{O}(1)$ structures are constrained at the level $\Lambda_{\text{eff}} \gtrsim 300$ TeV, with very little sensitivity to the CKM alignment. In the *shared* case, the bound is stronger by up to a factor of ~ 10, owing to the presence of a possible 1–2 rotation phase [64].

Achieving a *structured* cancellation among operators with $\Lambda_{\text{eff}} = \mathcal{O}(\text{TeV})$ requires an $SU(2)_q$ flavor symmetry under which $q \equiv (q_1, q_2)^T$ transforms as a doublet. For $SU(2)_q$-invariant operators, the Cabibbo rotation becomes unphysical and flavor violation proceeds only through the third family, as in MFV. Importantly, $SU(2)_q$ is the minimal continuous symmetry: reducing it to $O(2)_q$ is not viable, since $O(2)_q$ allows operators of the form $(\bar{q}_a\varepsilon_{ab}q_b)^2$, which induce unsuppressed mixing.

Thus, an $SU(2)_q$ symmetry is unavoidable. For the remaining SM fermions, we will show that, perhaps surprisingly, a *single* $U(1)_X$ taken as an appropriate linear combination of the remaining Cartan generators, $U(1)_X \subset U(1)^{14} \subset U(3)^5$, is sufficient.

Interestingly, a single $U(2) = SU(2)\times U(1)$ flavor symmetry [65–67] and its recent revivals [55, 68–73] offer an elegant minimal solution to the SM flavor puzzle. However, all $SU(2)\times U(1)$ constructions proposed so far predict sizable FCNC, see Fig. 1 of [55] for the bounds on selected SMEFT operators. These findings in the single-$U(2)$ literature, together with the established paradigms

[2] The Cartan of $U(3)^5$ has rank 15, corresponding to $U(1)^{15}$.

SM fields	d_1, u_1	d_2	u_2	d_3	q_3, u_3	ℓ_i	e_i
$U(1)_X$ charge	$-X_V - 2X_z$	$X_V + X_z$	$-X_W$	$X_z - X_U$	$2X_z - X_U$	X_i	$X_i - (4-i)X_z$

TABLE I. Assignment of *non-zero* $U(1)_X$ charges to the SM fields. See Section 3 for details.

of $U(2)^5$ and MFV, make our result somewhat unexpected.

3. Minimal setup for SM and BSM flavor puzzles

As a starting point, consider the five gauge representations of the SM matter fields, q_i, u_i, d_i, ℓ_i, and e_i ($i = 1, 2, 3$), whose kinetic terms are invariant under the global $U(3)^5$ flavor symmetry. In this work, we impose a smaller flavor subgroup,

$$\mathcal{H}_{\rm MFP} = SU(2)_q \times U(1)_X \,. \tag{2}$$

Among the SM fermions, only the two left-handed quark fields transform non-trivially under the $SU(2)_q$ part, forming a doublet $q \equiv (q_1, q_2)^T \sim \mathbf{2}$, while the fields with non-zero $U(1)_X$ charges are summarized in Table I.[3]

The symmetry-breaking spurions necessary to reproduce the observed fermion masses and mixings are given in Table II. We assume that spurions enter the effective Lagrangian only through positive integer powers. Since doublet spurions will play different roles, we require $|X_V| \neq |X_W| \neq |X_U|$. All $U(1)_X$ charges are taken to be integers, without loss of generality.

Quarks. For the charge assignment in Table I, the leading terms in SM quark Yukawa Lagrangian are[4]

$$\begin{aligned} -\mathcal{L} \supset\, & y_3^u\, \bar{q}_3\, u_3\, \widetilde{H} + y_2^u\, \bar{q}\, \boldsymbol{W}\, u_2\, \widetilde{H} + y_1^u\, \bar{q}\, \boldsymbol{V} \boldsymbol{z}^2\, u_1\, \widetilde{H} \\ & + y_3^d\, \bar{q}_3\, \boldsymbol{z}\, d_3\, H - y_2^d\, \bar{q}\, \widetilde{\boldsymbol{V}} \boldsymbol{z}^*\, d_2\, H + y_1^d\, \bar{q}\, \boldsymbol{V} \boldsymbol{z}^2\, d_1\, H \\ & + y_q^d\, \bar{q}\, \boldsymbol{U} \boldsymbol{z}^*\, d_3\, H + \text{h.c.}\,, \end{aligned} \tag{3}$$

where $\widetilde{H} = i\sigma_2 H^*$ and $\widetilde{\boldsymbol{V}} = i\sigma_2 \boldsymbol{V}^*$. All $y_x^{u,d}$ are assumed to be $\mathcal{O}(1)$ parameters, while hierarchies are entirely encoded in the spurions $\boldsymbol{V}$, $\boldsymbol{W}$, $\boldsymbol{U}$, and $\boldsymbol{z}$. Schematically, up to $\mathcal{O}(1)$ parameters, we have

$$Y_u = \begin{pmatrix} \boldsymbol{V}\boldsymbol{z}^2 & \boldsymbol{W} & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \quad Y_d = \begin{pmatrix} \boldsymbol{V}\boldsymbol{z}^2 & \widetilde{\boldsymbol{V}}\boldsymbol{z}^* & \boldsymbol{U}\boldsymbol{z}^* \\ 0 & 0 & \boldsymbol{z} \end{pmatrix}\,. \tag{4}$$

The explicit values for the $U(1)_X$ charges of the spurions can be chosen to preserve this structure to a sufficiently high order, as will be detailed later.

[3] In general, the doublet q may also carry non-zero $U(1)_X$ charge.
[4] The term in the up sector $\bar{q}\, \boldsymbol{U}(\boldsymbol{z}^*)^2\, u_3$ is omitted; the CKM V_{3i} elements are mainly generated in the down sector. Alternatively, one could have written two terms $\bar{q}\, \boldsymbol{U}\boldsymbol{z}\, d_3$ and $\bar{q}\, \boldsymbol{U}\, u_3$, in which case the charges of q_3, u_3 become -X_U while that of d_3 is -$(X_U + X_z)$. In this case, both terms contribute similarly to the CKM elements V_{td} and V_{ts}.

Spurions	$SU(2)_q \times U(1)_X$
$\boldsymbol{V}^a$	$(\mathbf{2},\, X_V)$
$\boldsymbol{W}^a$	$(\mathbf{2},\, X_W)$
$\boldsymbol{U}^a$	$(\mathbf{2},\, X_U)$
$\boldsymbol{z}$	$(\mathbf{1},\, X_z)$

TABLE II. Spurion representations under $\mathcal{H}_{\rm MFP}$.

Exploiting $\mathcal{H}_{\rm MFP}$ transformations, as well as other global phases broken by Eq. (3), we may, *without* loss of generality, represent the breaking as

$$\boldsymbol{z} = z,\ \boldsymbol{V} = v \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \boldsymbol{W} = w \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix},\ \boldsymbol{U} = u \begin{pmatrix} \sin\beta\, e^{i\phi} \\ \cos\beta \end{pmatrix}, \tag{5}$$

where z, v, w, u are real parameters all of $\mathcal{O}(10^{-2})$. To give a brief overview, z, v, w yield mass hierarchies, u sets the size of V_{ts}, and $\tan\beta\, e^{i\phi} \approx V_{td}^*/V_{ts}^*$. Analogous to MFV, we assume the phase of $\boldsymbol{U}$ is the only source of CP violation. Crucially, our choice of spurions yields *down alignment* in the 1-2 sector. This means the 2×2 block of Y_d is exactly diagonal, so the Cabibbo angle is generated entirely in the up sector from the angle α. As up-sector right-handed rotations are m_u/m_c suppressed, this arrangement avoids large contributions to kaon and charm mixing from right-handed flavor rotations.

Charged Leptons. The absence of charged-lepton flavor violation (cLFV), together with the smallness of the charged-lepton Yukawas, can be simultaneously accounted for by the set of $U(1)_X$ charges listed in Table I. When

$$(X_i - X_j) \bmod X_z \neq 0 \quad \forall\, i \neq j\,, \tag{6}$$

the charged-lepton Yukawa interactions take the form

$$-\mathcal{L} \supset y_3^e\, \bar{\ell}_3\, \boldsymbol{z}\, e_3\, H + y_2^e\, \bar{\ell}_2\, \boldsymbol{z}^2\, e_2\, H + y_1^e\, \bar{\ell}_1\, \boldsymbol{z}^3\, e_1\, H + \text{h.c.}\,, \tag{7}$$

where $y_x^e \sim \mathcal{O}(1)$ can be taken real due to the global chiral $U(1)^3$. Up to $\mathcal{O}(1)$ parameters, Y_e reads

$$Y_e = \begin{pmatrix} \boldsymbol{z}^3 & 0 & 0 \\ 0 & \boldsymbol{z}^2 & 0 \\ 0 & 0 & \boldsymbol{z} \end{pmatrix}\,. \tag{8}$$

The off-diagonal terms vanish to all orders in $\boldsymbol{z}$ due to Eq. (6), perfectly aligning the mass and interaction bases. The condition in Eq. (6) may be relaxed, but we choose to adopt it such that 2-fermion SMEFT operators preserve an accidental $U(1)_e \times U(1)_\mu \times U(1)_\tau$ symmetry to all orders in $\boldsymbol{z}$. Furthermore, 4-fermion operators can violate this symmetry only for non-identical and flavor-violating currents. Examples are $[Q_{\ell e}]_{1212}$, or when all three flavors participate simultaneously, e.g. $[Q_{\ell\ell}]_{2123}$. As a result, the most dangerous processes such as $\mu \to 3e, e\gamma$ from $\boldsymbol{z}$ insertions are absent.

When including additional spurions from Table II, the $SU(2)$ structure implies that singlet combinations arise at second order as $\boldsymbol{a}^\dagger \boldsymbol{b}$ or $\boldsymbol{a}^\dagger \widetilde{\boldsymbol{b}}$ ($\boldsymbol{a}, \boldsymbol{b} = \boldsymbol{V}, \boldsymbol{W}, \boldsymbol{U}$ and $\boldsymbol{a} \neq \boldsymbol{b}$). Their charge assignments, $\pm(X_a \pm X_b)$, should be chosen such that the $Y_e^{12,21}$ elements are suppressed at least to quartic order to avoid bounds from $\mu \to e$ transitions reported in Table V of [74]. No further constraints are needed for Y_e^{a3} and Y_e^{3a} ($a = 1, 2$) given Eq. (6).

Producing SM parameters. Next, we perform a perturbative singular value decomposition, $Y_f = L_f \hat{Y}_f R_f^\dagger$, to reproduce the observed fermion masses and CKM mixing $V_{\rm CKM} = L_u^\dagger L_d$, thereby fixing the spurion values and confirming that all $y_x^{u,d,e}$ are indeed $\mathcal{O}(1)$.

All quark and charged lepton masses are controlled by the z, v and w spurions:

$$\begin{aligned} y_u &\sim v z^2\,, & y_c &\sim w\,, & y_t &\sim 1\,,\\ y_d &\sim v z^2\,, & y_s &\sim v z\,, & y_b &\sim z\,,\\ y_e &\sim z^3\,, & y_\mu &\sim z^2\,, & y_\tau &\sim z\,. \end{aligned} \tag{9}$$

To reproduce the CKM matrix, we match the Wolfenstein parameterization at leading order [75]

$$\alpha \approx -\lambda\,,\ \tan\phi \approx \frac{\eta}{1-\rho}\,,\ \beta \approx -\lambda\,\frac{1-\rho}{\cos\phi}\,,\ \frac{y_q^d\, u}{y_3^d} \approx A\lambda^2\,, \tag{10}$$

where we set $\lambda = 0.225$, $A = 0.826$, $\rho = 0.163$ and $\eta = 0.357$ following PDG [76]. A good fit to these and observed y_f, taken from Eq. (A8) in [69], is obtained for

$$z = 0.02\,,\ \ v = 0.015\,,\ \ w = 0.006\,,\ \ u = 0.04\,. \tag{11}$$

For this choice of spurions, remarkably, all nine $y_x^{u,d,e}$ are $\mathcal{O}(1)$ with the ratio of the largest to the smallest $\lesssim 5$. As advertised, the spurions lie within at most one order of magnitude of each other, providing a solid starting point for dynamically addressing the SM flavor puzzle.

Alignment conditions. This construction is a promising framework to address the new physics flavor puzzle for two main reasons. First, the non-universality of $U(1)_X$ charges largely suppresses flavor-changing dimension-6 SMEFT operators in the interaction basis, with notable exceptions of flavor-charged currents that depend on the charges. Secondly, the mass and interaction bases are nearly perfectly aligned, while still reproducing the CKM matrix. This feature is crucial to prevent FCNC induced by rotations of flavor non-universal operators from the interaction to the mass basis.

Since the Cabibbo angle comes entirely from the up sector in our framework, the corresponding left-handed up-quark mixing between the interaction and mass bases is $\theta_{u_L}^{12} \approx \lambda$. This in turn predicts a right-handed up-quark 1-2 mixing, $\theta_{u_R}^{12} = \lambda\, m_u/m_c \sim 5\times 10^{-4}$, which is an unavoidable feature of this construction. Nevertheless, it suffices to pass limits on right-handed $D-\bar{D}$ mixing. The left-handed mixing angles in the down sector are $(\theta_{d_L}^{13}, \theta_{d_L}^{23}) \approx (V_{td}, V_{ts})$ while the right-handed counterparts $(\theta_{d_R}^{13}, \theta_{d_R}^{23}) \approx (V_{td}\frac{m_d}{m_b}, V_{ts}\frac{m_s}{m_b})$. Down-sector 1-2 mixing arises only as $\theta_{d_L}^{12} \sim \theta_{d_L}^{13}\theta_{d_L}^{23} \sim V_{td}V_{ts}$, with $\theta_{d_R}^{12}$ further suppressed by m_d/m_s. This provides sufficient alignment to avoid excessive contributions to right-handed kaon mixing.

These rotation angles correspond to the exact structure in Eq. (4). However, depending on the $U(1)_X$ charges, higher spurion insertions may spoil the alignment and lead to excessively large rotation angles. Following Ref. [63], starting from a flavor non-universal four-fermion operator, neutral-meson mixing imposes stringent bounds on the rotation angles listed above. A careful inspection shows that these bounds require forbidding higher-order spurion insertions: specifically, $Y_{d,u}^{21}$ must be absent up to fourth order, while Y_d^{12}, Y_d^{31}, and Y_d^{32} must be absent up to third order. This translates into a set of inequalities on the spurion charges that we provide in the Supplemental Material. Interestingly, a single condition $|X_a| + |X_b| < X_z$ is sufficient to satisfy all these inequalities and preserve the rotation angles predicted by the exact form of Eq. (4).

Conditions on EFT spurion insertions. In addition to suppressed FCNC from small rotations applied to $\Delta F = 0$ operators, one must also verify that direct spurion insertions leading to $\Delta F \neq 0$ EFT operators in the interaction basis appear only at sufficiently high order. By systematically examining the most stringent flavor observables, one can derive a set of inequalities that constrain the allowed charge assignments. For instance, operators like $[Q_{qd}]_{1212}$ must be forbidden up to quartic order in spurion insertions to adequately suppress K- and D-meson mixing. The complete set of resulting inequalities is given in the Supplemental Material.

After outlining the key conditions, we scan over $U(1)_X$ charge assignments to identify consistent solutions, defined by an effective scale $\Lambda_{\rm eff} \lesssim 10\,\text{TeV}$ for all dimension-6 SMEFT $\Delta B = 0$ operators in the Warsaw basis [19]. The effective scale is defined as[5]

$$\mathcal{L} \supset \frac{\mathcal{C}}{\Lambda_{\rm eff}^2} Q\,, \tag{12}$$

where $\mathcal{C}$ encodes *only* the flavor spurion power counting. For dipole operators, we do not assume a loop factor but do include one gauge coupling for each field-strength tensor. It is worth emphasizing that the $U(1)_X$ factor is not

[5] The relation between $\Lambda_{\rm eff}$ and new mass thresholds is, of course, UV-dependent. Regarding dimension-8 operators [77], under flavor anarchy with $\mathcal{C}_8 \sim 1$, only a small subset can probe scales as high as $\Lambda_{\rm eff,8} \lesssim 100\,\text{TeV}$ [78]. However, in our setup, the flavor-power-counting suppressing dimension-6 also suppresses all such contributions.

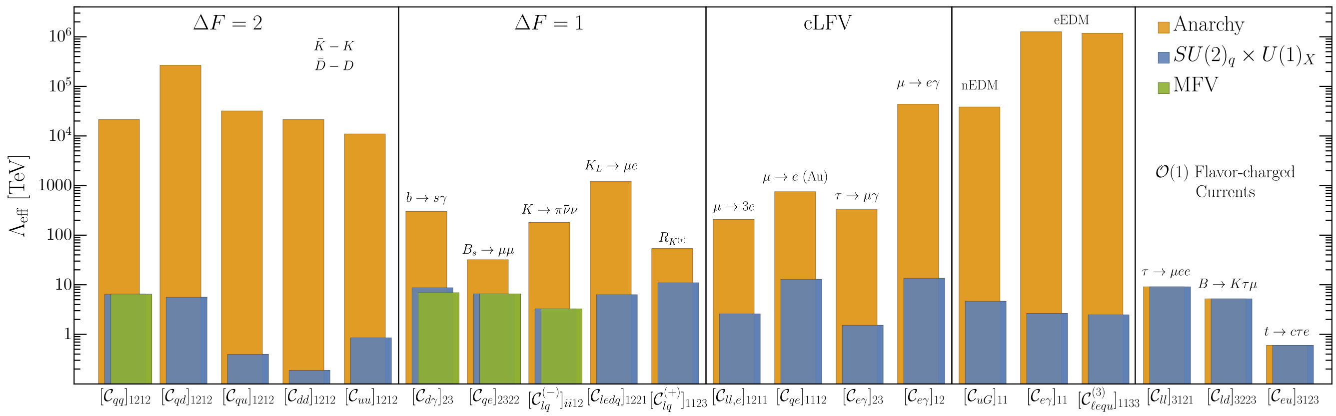

FIG. 1. Bounds on selected SMEFT operators: Anarchy (yellow), $SU(2)_q \times U(1)_X$ (blue), MFV (green). See Sec. 4 for details.

unique: there is a wide range of possibilities, even when imposing a moderate upper bound on the largest charge. A comprehensive exploration of this space is left for future work. In the next section, as a proof of principle, we focus on a representative benchmark.

4. SMEFT analysis for a selected benchmark

In this section, we examine a benchmark charge assignment in detail that satisfies all previously imposed conditions:

$$\{X_V,\, X_W,\, X_U,\, X_z\} = \{1,\, \text{-}2,\, \text{-}3,\, 7\}\,,$$
$$\{X_1,\, X_2,\, X_3\} = \{\text{-}2,\, \text{-}3,\, \text{-}1\}\,. \tag{13}$$

Following the methodology of Ref. [22], we systematically construct the basis of dimension-6, baryon-number-conserving SMEFT operators order-by-order in the spurion expansion for this benchmark. This basis is reported up to quadratic order in the Supplemental Material.

In Table III, we compare the number of flavor-symmetric operator structures that require no spurion insertions (i.e. $\mathcal{O}(1)$) in three scenarios: $U(3)^5$, $U(2)^5$, and our framework, $SU(2)_q \times U(1)_X$. The comparison highlights how our symmetry structure generically admits more flavor-universality violation than $U(2)^5$ unless the operator features only q^a. Nevertheless, the number of allowed operators remains well under control, representing only 8% of the full 2499 operators in the dimension-6, baryon-number-conserving SMEFT. For completeness, we also provide the corresponding counting for $SU(2)_q$ alone, which does not solve the BSM flavor puzzle but serves to illustrate the role of the additional $U(1)_X$ factor. We note that there are 176 self-conjugate operators in both the $SU(2)_q \times U(1)_X$ and $SU(2)_q$ scenarios, so the extra operators that appear when $U(1)_X$ is removed necessarily correspond to 546 new non-hermitian structures generally yielding $\mathcal{O}(1)$ violations of both flavor and CP.

Next, we allow spurion insertions and examine the resulting flavor-changing operators individually to confirm

Operator class	$\boldsymbol{U(3)^5}$		$\boldsymbol{U(2)^5}$		$\boldsymbol{SU(2)_q \times U(1)_X}$		$\boldsymbol{SU(2)_q}$	
CP	even	odd	even	odd	even	odd	even	odd
Bosonic	9	6	9	6	9	6	9	6
$\boldsymbol{\psi^2 H^3}$	-	-	3	3	1	1	15	15
$\boldsymbol{\psi^2 X H}$	-	-	8	8	3	3	36	36
$\boldsymbol{\psi^2 H^2 D}$	7	-	15	1	20	1	43	24
$\boldsymbol{(\bar{L}L)(\bar{L}L)}$	8	-	23	-	32	1	61	30
$\boldsymbol{(\bar{R}R)(\bar{R}R)}$	9	-	29	-	61	1	255	195
$\boldsymbol{(\bar{L}L)(\bar{R}R)}$	8	-	32	-	58	1	195	138
$\boldsymbol{(\bar{L}R)(\bar{R}L)}$	-	-	1	1	1	1	27	27
$\boldsymbol{(\bar{L}R)(\bar{L}R)}$	-	-	4	4	-	-	90	90
Total	41	6	124	23	185	15	731	561

TABLE III. Number of independent, $\Delta B = 0$ dimension-6 SMEFT operators that are invariant under the specified flavor symmetry at $\mathcal{O}(1)$, i.e. without spurion insertions. For $SU(2)_q \times U(1)_X$, the counting assumes Eq. (13).

that FCNC are sufficiently suppressed. More precisely, for every independent operator at a fixed order in spurion expansion, one applies the appropriate rotation matrices from the interaction to the mass basis and confronts the experimental bounds. We review the compendium of bounds on effective operators compiled in [53, 63, 74, 79–89]. As previously mentioned, our analysis assumes that the only source of CP violation is the spurion $\boldsymbol{U}$, which is necessary only to pass bounds on EDMs. The leading EDM invariants are given in the Supplemental Material.

Fig. 1 summarizes the FCNC and EDM bounds on $\Lambda_{\rm eff}$ defined in Eq. (12) for a selected set of operators and compares them with the two limiting cases of MFV and flavor anarchy. We see that $SU(2)_q \times U(1)_X$ preserves the rich pattern of flavor-changing phenomena exhibited in the anarchy case, while remaining compatible with new dynamics as light as $\Lambda_{\rm eff} \lesssim 10\,\text{TeV}$. A similar limit holds for MFV, but it severely restricts the allowed flavor dynamics. Interestingly, the benchmark in Eq. (13) permits the following flavor-charged currents at $\mathcal{O}(1)$:

$$[Q_{eu}]_{3123}\,,\quad [Q_{\ell\ell}]_{3121}\,,\quad [Q_{\ell d}]_{3223}\,, \tag{14}$$

which induce unsuppressed $t \to c\tau^- e^+$ [90], $\tau^- \to \mu^+ e^- e^-$ [82], and $B^+ \to K^+\tau^+\mu^-$ [86] transitions. Note that these decays predict particular charge assignments, *e.g.* tau decays to same-sign electrons. Mediators generating these interactions are necessarily charged under $\mathcal{H}_{\rm MFP}$, which motivates the term flavor-charged currents.

5. Outlook and Conclusions

In this letter, we introduce a novel mechanism that ensures the structural protection of generic TeV-scale new physics by identifying the minimal flavor symmetry that must be approximately conserved. In contrast to the MFV paradigm, our *Minimal Flavor Protection* framework allows for genuinely rich TeV-scale flavor dynamics while satisfying the most stringent experimental constraints.

Our framework predicts a broad spectrum of flavor-changing and universality-violating dynamics, as well as unconventional flavor-charged currents, strongly motivating a diverse flavor physics program. In particular, our work provides a symmetry-based argument that the set of flavor-violating dynamics accessible to current and future colliders is much larger than previously understood. This provides strong support for a future high-energy physics strategy that will explore complementarity between the intensity and energy frontiers, such as the FCC program [42, 91].

This work opens several avenues for further exploration. On the phenomenological side, our framework expands the space of well-motivated higher-dimensional operators relevant for collider studies and calls for a broader program to target flavor-non-universal interactions. It also enables a systematic classification of heavy mediators that couple linearly to the SM, organized as irreducible representations of the underlying flavor symmetry, analogous to the MFV analysis of Ref. [92]. Such a classification would define a comprehensive set of TeV-scale resonances to be searched for at the LHC, substantially extending the scope of simplified-model studies. The same logic can be applied to structured UV scenarios, including MSSM soft terms [93, 94] and composite Higgs models with partial compositeness [95–97], thereby opening new phenomenological possibilities beyond the standard benchmarks.

The MFP framework also offers a fresh perspective on the flavor puzzle, motivating new model-building efforts to explain the dynamical origin of the approximate symmetries identified. In addition, while we presented one explicit set of charges as a proof of principle, it is not unique. Indeed, we have verified that departing from the charge assignments given in Table I allows for gauge-anomaly-free solutions, though alignment may not be guaranteed. A systematic exploration of all viable charge configurations is warranted, particularly to map out the space of distinctive flavor-changing processes.

Acknowledgments

We thank Wolfgang Altmannshofer, Stefan Antusch, Claudia Cornella, Gian F. Giudice, Stefania Gori, Javier Fuentes-Martín, Claudio A. Manzari, David Marzocca, Anders E. Thomsen, Alessandro Valenti, and Luca Vecchi for helpful comments and discussions. This work has received funding from the Swiss National Science Foundation (SNF) through the Eccellenza Professorial Fellowship "Flavor Physics at the High Energy Frontier," project number 186866. The work of BAS is supported by a CDEIGENT grant from the Generalitat Valenciana with no. CIDEIG/2023/35.

End Matter: Towards an ultraviolet completion of the Yukawa sector

The EFT construction points naturally toward a UV framework capable of explaining the origin of the observed flavor hierarchies. A simple renormalizable realization can be obtained by introducing a vector-like fermion (VLF) "chain" structure that communicates small symmetry breaking through sequential mixings. In this setup, the Yukawa textures in Eqs. (4) and (8) arise from a *softly* broken global $SU(2)_q \times U(1)_X$ symmetry. The symmetry breaking enters only through mass-mixing terms, which may, in turn, originate from the vacuum expectation values of scalar fields in a Froggatt–Nielsen-like mechanism. For clarity, we first explain Yukawa generation through mass mixings, and then comment on how flavons dynamically generate these mixings via marginal interactions and a consistent scalar potential.

The model is sketched in Fig. 2. To generate the structure of Y_e in Eq. (8), we introduce vector-like leptons E_i^p transforming as e_R under the SM gauge group. The flavor index $i = 1, 2, 3$ labels the e, μ, τ sectors, while p denotes the position in a chain. The chains have lengths $k_i = (3, 2, 1)$ for (e, μ, τ), respectively. Each E_i^p carries a $U(1)_X$ charge

$$X_{E_i^p} = X_i - (p-1)\,X_z\,, \qquad p = 1, \ldots, k_i\,. \qquad (15)$$

such that only nearest-neighbor mass mixings are allowed by symmetry. Taking all VLF masses to be of order M, and all allowed mass mixings to be of order $M \cdot z$ with z given in Eq. (5), reproduces the charged-lepton Yukawa texture of Eq. (8) after integrating out the VLFs.[6]

The VLF content in the quark sector consists of right-handed quark partners U and D, which form $SU(2)_q$ doublets each and are neutral under $U(1)_X$, together with the singlets U_1^r and D_1^r $(r = 1, 2)$ with charges -$X_V - (r-1)X_z$, as well as, D_2^1, D_3^1 and D_3 with charges X_V, -X_U, and $2X_z - X_U$, respectively. As before, all

[6] Imposing exchange parity $E_{iL}^p \leftrightarrow E_{iR}^p$ removes all physical phases and prevents CP violation.

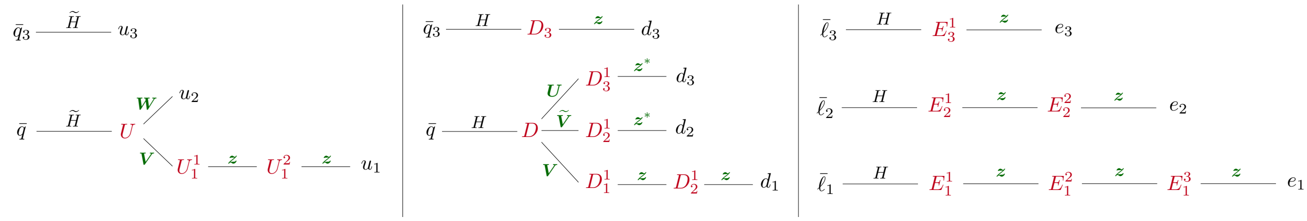

FIG. 2. The chains of vector-like fermions yielding $Y_{u,d,e}$ in the TeV-scale flavor model introduced in Section 5. Flavor-spurion mixing is shown in green, and the vector-like fermions in red.

VLF masses are of order M, while the allowed mass mixings are of order $M \cdot x$, with $x = z, v, w$, or u as defined in Eq. (5). Assuming that the only CP-breaking spurion appears in the $\bar{D}D_3^1$ mixing term, the quark Yukawa texture in Eq. (4) follows directly, after integrating out VLFs.

The origin of VLF mass mixings could be flavor-symmetry-invariant marginal interactions with spurions promoted to dynamical scalar fields, flavons. Including soft-breaking terms in the flavon scalar potential, which are needed in any case to avoid massless Goldstone modes, we find that the minimization of the potential can naturally yield the vacuum structure of Eq. (5). A detailed study of this and related constructions is left for future work.

Finally, our minimal flavor protection mechanism permits this model to be realized at the TeV scale, $M = \mathcal{O}(\text{TeV})$, making this theory of flavor accessible to collider searches.[7]

Neutrinos. In this work, we focused on quarks and charged leptons, as the impact of tiny neutrino masses on the observables of interest is negligible. Here, we briefly comment on how neutrino flavor fits into this framework.

To incorporate neutrino masses, we study the flavor spurion insertions in the Weinberg operator, $\frac{c_{ij}}{\Lambda}\ell_i\ell_j HH$. When the condition

$$(X_i + X_j) \bmod X_z \neq 0 \quad \forall\, i, j\,, \tag{16}$$

holds, the Weinberg operator is never populated by z insertions. This is indeed the case for the benchmark charge assignment in Eq. (13). In such cases, the leading contributions to c_{ij} arise from doublet–spurion bilinears $\boldsymbol{a}^\dagger \boldsymbol{b}$ or $\boldsymbol{a}^\dagger \widetilde{\boldsymbol{b}}$, with $\boldsymbol{a}, \boldsymbol{b} \in \{\boldsymbol{V}, \boldsymbol{W}, \boldsymbol{U}\}$ and $\boldsymbol{a} \neq \boldsymbol{b}$. When

$$|X_i + X_j| = \left|[\boldsymbol{a}^\dagger \boldsymbol{b}] \text{ or } [\boldsymbol{a}^\dagger \widetilde{\boldsymbol{b}}]\right| \quad \forall\, i, j\,, \tag{17}$$

all entries of the Weinberg operator are populated at $\mathcal{O}(s^2)$ in the spurion expansion. As a result, we expect c_{ij} to be anarchic, a welcome feature that arises because $\boldsymbol{V}, \boldsymbol{W}, \boldsymbol{U}$ are of comparable size, as indicated in Eq. (11).

For the benchmark charge assignment in Eq. (13), all entries of c_{ij} arise at $\mathcal{O}(s^2)$ except for c_{22}, which is additionally suppressed and first appears at $\mathcal{O}(s^3)$. Nevertheless, this texture naturally accommodates the large (apparently anarchic) PMNS mixing and the observed neutrino mass splittings. The overall scale, $\Lambda \sim \mathcal{O}(10^{11}\,\text{GeV})$, follows from fitting the mass splittings together with the cosmological upper bound on the absolute mass scale and is compatible with standard thermal leptogenesis [99], see also [100]. To sum up, our framework straightforwardly accommodates the observed neutrino masses and mixings, in contrast with the minimally-broken $U(2)^5$ symmetry, which inevitably imprints hierarchical structures onto the PMNS matrix that must then be undone to match data.

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Supplemental material

1. Conditions on $U(1)_X$ charges

Here we list the conditions employed in the scan of $U(1)_X$ charges:

1. **Alignment conditions for quark and charged lepton Yukawas**
   (a) Demanding a zero in Y_d^{31} up to third order leads to the following conditions: $X_U - 4X_z - X_V \neq \{0, \pm X_z, \pm 2X_z, \pm(X_a \pm X_b)\}$ where $a \neq b$, and $a, b = \{U, V, W\}$. Note that $\boldsymbol{a}^\dagger \boldsymbol{b}$ and $\boldsymbol{a}^\dagger \widetilde{\boldsymbol{b}}$ form $SU(2)_q$ singlets.
   (b) Demanding a zero in Y_d^{32} up to third order leads to the following conditions: $X_U - X_z + X_V \neq \{0, \pm X_z, \pm 2X_z, \pm(X_a \pm X_b)\}$ where $a \neq b$, and $a, b = \{U, V, W\}$.
   (c) Demanding a zero in Y_d^{12} up to third order leads to the following conditions: $X_V + X_z \neq \{0, \pm X_a, \pm(X_a \pm X_z)\}$ where $a = \{U, V, W\}$ and excluding $\widetilde{\boldsymbol{V}} \boldsymbol{z}^*$.
   (d) Demanding a zero in $Y_{d,u}^{21}$ up to fourth order leads to the following conditions: $X_V + 2X_z \neq \{0, \pm X_a, \pm(X_a \pm X_z), \pm(X_a \pm 2X_z), \pm(X_a \pm X_b \pm X_c)\}$ where $a, b, c = \{U, V, W\}$ and $a \neq b$.
   (e) Demanding a zero in $Y_e^{12,21}$ up to fourth order leads to the following conditions: $X_2 - X_1 - 2X_z$ and $X_1 - X_2 - 3X_z \neq \{0, z, z^2, z^3, \pm(X_a \pm X_b), \pm(X_a \pm X_b \pm X_z)\}$ where $a \neq b$ and $a, b = \{U, V, W\}$.

2. **Conditions for $\boldsymbol{\Delta F = 2}$ spurion insertions**
   (a) For $[Q_{qd}]_{1212}$ and $[Q_{qu}]_{1212}$ operators to be forbidden at quadratic level in spurions
   $$|X_a \pm X_b| \neq |2X_V + 3X_z|\,, \qquad |X_a \pm X_b| \neq |X_V + 2X_z - X_W|\,, \tag{S1}$$
   should hold for $\forall a, b \in \{U, V, W\}$.
   (b) For $[Q_{dd}]_{1212}$ and $[Q_{uu}]_{1212}$ operators to be allowed, we need the following number of $\boldsymbol{z}$ insertions
   $$N = \left|6 + \frac{4X_V}{X_z}\right|, \qquad N = \left|4 + \frac{2(X_V - X_W)}{X_z}\right|, \tag{S2}$$
   such that $N > 3$ or N is not an integer.
   (c) For $[Q_{qd}]_{1212}$ and $[Q_{qu}]_{1212}$ operators of the form $\boldsymbol{a}\boldsymbol{b}\boldsymbol{z}^N$ to be allowed, we need the following number of $\boldsymbol{z}$ insertions
   $$N = \pm\left|\frac{X_a \pm X_b}{X_z}\right| \pm \left|3 + \frac{2X_V}{X_z}\right|, \qquad N = \pm\left|\frac{X_a \pm X_b}{X_z}\right| \pm \left|2 + \frac{(X_V - X_W)}{X_z}\right|, \tag{S3}$$
   where for both conditions $\forall a, b \in \{U, V, W\}$. Passing the bounds requires either $|N| > 2$ (1) for qd (qu) or that N does not take an integer value. Configurations producing $N = 0$ can be ignored as they are covered by condition (a).

3. **Conditions for Semi-leptonic $\boldsymbol{\mu - e}$ LFV**
   (a) To forbid the semileptonic $\mu - e$ LFV operators such as $[Q_{\ell d, ed}]_{12st}$ from appearing at $\mathcal{O}(1)$, one should have
   $$|X_2 - X_1 + X_z| \text{ or } |X_2 - X_1| \neq \begin{cases} |2X_V + 3X_z|\,, & (s,t) = (2,1) \\ |X_U + X_V|\,, & (s,t) = (2,3) \\ |3X_z - X_U + X_V|\,, & (s,t) = (1,3) \end{cases}. \tag{S4}$$
   Similarly, the operator $[Q_{\ell edq}]_{12s3,21s3}$ is subject to analogous $\mathcal{O}(1)$ conditions. We find
   $$0 \neq \begin{cases} X_r + rX_z - X_p - X_U + X_V\,, & s = 1 \\ X_r - (3-r)X_z - X_p - X_U - X_V\,, & s = 2 \\ X_r - (3-r)X_z - X_p\,, & s = 3 \end{cases}, \tag{S5}$$

where one should consider $(p,r)=(1,2)$ or $(p,r)=(2,1)$. In the case of $K_L\to\mu e$, we also require that $[Q_{\ell d,ed}]_{1212}$ operators be forbidden at the level of linear spurion insertions, which requires

$$N=\pm\left|3+\frac{2X_V}{X_Z}\right|\pm\left|1+\frac{X_2-X_1}{X_Z}\right|,\qquad N=\pm\left|3+\frac{2X_V}{X_Z}\right|\pm\left|\frac{X_2-X_1}{X_Z}\right|,\tag{S6}$$

such that $|N|>1$ or N is not an integer.

(b) Linear realizations of the $\boldsymbol{V}$, $\boldsymbol{U}$ and $\boldsymbol{W}$ spurions in operators of the form $[Q_{\ell edq}]_{prst}$ must be eliminated, since choices such as $(prst)=(1212)$, along with their permutations, generate contributions to $K_L\to\mu e$. The conditions read

$$0\neq\begin{cases}\pm X_a+X_V+X_r-(2-r)X_z-X_p\,, & s=1\\ \pm X_a-X_V+X_r-(5-r)X_z-X_p\,, & s=2\end{cases},\tag{S7}$$

where $a=\{V,U,W\}$ and $(p,r)=(1,2)$ or $(p,r)=(2,1)$. Additionally, quadratic terms of the form $\boldsymbol{V}\boldsymbol{z}$, $\boldsymbol{U}\boldsymbol{z}$ could also be borderline for this operator. These can be forbidden with

$$N=\begin{cases}\pm\frac{1}{X_z}\left[\pm X_a+X_V+X_r-(2-r)X_z-X_p\right]\,, & s=1\\ \pm\frac{1}{X_z}\left[\pm X_a-X_V+X_r-(5-r)X_z-X_p\right]\,, & s=2\end{cases},\tag{S8}$$

where $a=\{V,U\}$ and $(p,r)=(1,2)$ or $(p,r)=(2,1)$ such that $|N|>1$ or that N does not take an integer value. Finally, $\boldsymbol{U}^2$ terms must be forbidden for operators with left-handed quarks, such as $[Q_{\ell q}]_{1212}$ and $[Q_{qe}]_{1212}$. This requirement is satisfied by imposing the following conditions

$$|2X_U|\neq|X_2-X_1|\,,\qquad |2X_U|\neq|X_2-X_1-X_Z|\,.\tag{S9}$$

2. EDM flavor invariants

In Table IV we collect the leading flavor invariants that contribute to the electron and neutron EDMs, assuming charges in Eq. (13), and corresponding to the bounds shown in Fig. 1.

Operator	**Order**	**Invariants**
$[Q_{e(B,W)}]_{11}$	$\mathcal{O}(\boldsymbol{U}\boldsymbol{V}^2\boldsymbol{W}\boldsymbol{z}^2)$	$\boldsymbol{z}^2(\boldsymbol{U}^\dagger\boldsymbol{V})(\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{e(B,W)}]_{11}$
	$\mathcal{O}(\boldsymbol{U}^2\boldsymbol{V}\boldsymbol{W}\boldsymbol{z}^2)$	$\boldsymbol{z}^2(\varepsilon^{ac}\boldsymbol{U}_a^\dagger\boldsymbol{V}_c^\dagger)(\varepsilon^{bd}\boldsymbol{U}_b^\dagger\boldsymbol{W}_d^\dagger)[Q_{e(B,W)}]_{11}$
$[Q_{u(B,W,G)}]_{11}$	$\mathcal{O}(\boldsymbol{U}^2\boldsymbol{W}\boldsymbol{z})$	$\boldsymbol{z}(\varepsilon^{ab}\boldsymbol{U}_b^\dagger)(\varepsilon^{cd}\boldsymbol{U}_c^\dagger\boldsymbol{W}_d^\dagger)[Q_{u(B,W,G)}]_{a1}$
$[Q_{d(B,W,G)}]_{11}$	$\mathcal{O}(\boldsymbol{U}^2\boldsymbol{W}\boldsymbol{z})$	$\boldsymbol{z}(\varepsilon^{ab}\boldsymbol{U}_b^\dagger)(\varepsilon^{cd}\boldsymbol{U}_c^\dagger\boldsymbol{W}_d^\dagger)[Q_{d(B,W,G)}]_{a1}$
$[Q_{\ell equ}^{(1,3)}]_{1133}$	$\mathcal{O}(\boldsymbol{U}\boldsymbol{V}^2\boldsymbol{W}\boldsymbol{z}^2)$	$\boldsymbol{z}^2(\boldsymbol{U}^\dagger\boldsymbol{V})(\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{\ell equ}^{(1,3)}]_{1133}$
	$\mathcal{O}(\boldsymbol{U}^2\boldsymbol{V}\boldsymbol{W}\boldsymbol{z}^2)$	$\boldsymbol{z}^2(\varepsilon^{ac}\boldsymbol{U}_a^\dagger\boldsymbol{V}_c^\dagger)(\varepsilon^{bd}\boldsymbol{U}_b^\dagger\boldsymbol{W}_d^\dagger)[Q_{\ell equ}^{(1,3)}]_{1133}$

TABLE IV. Leading flavor invariants relevant for the generation of electron and neutron EDMs with the associated constraints shown in Fig. 1. Notice that all invariants necessarily include $\boldsymbol{U}$ spurion, which uniquely encodes the CP violation, see Eq. (5).

3. SMEFT operator basis up to second order in spurion expansion

Class	Operator	Order	Invariants	# CP even	# CP odd
$(\bar{\boldsymbol{L}}\boldsymbol{R})(\bar{\boldsymbol{L}}\boldsymbol{R})$	$Q_{quqd}^{(1,8)}$	$\mathcal{O}(\boldsymbol{z})$	$(\boldsymbol{z}\varepsilon_{ab})[Q_{quqd}^{(1,8)}]_{a1b2} \oplus \boldsymbol{z}[Q_{quqd}^{(1,8)}]_{3333}$	2	2
		$\mathcal{O}(\boldsymbol{V}^2)$	$\widetilde{\boldsymbol{V}}^a\widetilde{\boldsymbol{V}}^b[Q_{quqd}^{(1,8)}]_{a3b1}$	1	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$\boldsymbol{U}^a\boldsymbol{V}^b[Q_{quqd}^{(1,8)}]_{a3b1} \oplus \boldsymbol{U}^b\boldsymbol{V}^a[Q_{quqd}^{(1,8)}]_{a3b1}$	2	2
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{z})$	$\boldsymbol{z}^*\widetilde{\boldsymbol{V}}^a[Q_{quqd}^{(1,8)}]_{a332} \oplus \boldsymbol{z}^*\widetilde{\boldsymbol{V}}^a[Q_{quqd}^{(1,8)}]_{33a2}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$\varepsilon_{ab}(\boldsymbol{U}^\dagger\widetilde{\boldsymbol{W}})[Q_{quqd}^{(1,8)}]_{a1b3} \oplus \widetilde{\boldsymbol{U}}^a\widetilde{\boldsymbol{W}}^b[Q_{quqd}^{(1,8)}]_{a1b3} \oplus \widetilde{\boldsymbol{U}}^b\widetilde{\boldsymbol{W}}^a[Q_{quqd}^{(1,8)}]_{a1b3}$	3	3
		$\mathcal{O}(\boldsymbol{W}\boldsymbol{z})$	$\boldsymbol{z}\boldsymbol{W}^a[Q_{quqd}^{(1,8)}]_{a233} \oplus \boldsymbol{z}\boldsymbol{W}^a[Q_{quqd}^{(1,8)}]_{32a3}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{z})$	$\boldsymbol{z}^*\boldsymbol{U}^a[Q_{quqd}^{(1,8)}]_{a333} \oplus \boldsymbol{z}^*\boldsymbol{U}^a[Q_{quqd}^{(1,8)}]_{33a3}$	2	2
	$Q_{\ell equ}^{(1,3)}$	$\mathcal{O}(\boldsymbol{V})$	$\widetilde{\boldsymbol{V}}^a[Q_{\ell equ}^{(1,3)}]_{32a3}$	1	1
		$\mathcal{O}(\boldsymbol{W})$	$\boldsymbol{W}^a[Q_{\ell equ}^{(1,3)}]_{12a3}$	1	1
		$\mathcal{O}(\boldsymbol{U})$	$\boldsymbol{U}^a[Q_{\ell equ}^{(1,3)}]_{22a3} \oplus \widetilde{\boldsymbol{U}}^a[Q_{\ell equ}^{(1,3)}]_{21a3} \oplus \widetilde{\boldsymbol{U}}^a[Q_{\ell equ}^{(1,3)}]_{23a2}$	3	3
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{\ell equ}^{(1,3)}]_{3333}$	1	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{U}^\dagger\widetilde{\boldsymbol{W}})[Q_{\ell equ}^{(1,3)}]_{2333}$	1	1
		$\mathcal{O}(\boldsymbol{W}\boldsymbol{z})$	$\boldsymbol{z}\boldsymbol{W}^a[Q_{\ell equ}^{(1,3)}]_{31a3} \oplus \boldsymbol{z}\boldsymbol{W}^a[Q_{\ell equ}^{(1,3)}]_{33a2}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{z})$	$\boldsymbol{z}\boldsymbol{U}^a[Q_{\ell equ}^{(1,3)}]_{11a3} \oplus \boldsymbol{z}\boldsymbol{U}^a[Q_{\ell equ}^{(1,3)}]_{13a2} \oplus \boldsymbol{z}^*\boldsymbol{U}^a[Q_{\ell equ}^{(1,3)}]_{33a3}$	3	3
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{\ell equ}^{(1,3)}]_{2233}$	1	1
$(\bar{\boldsymbol{L}}\boldsymbol{R})(\bar{\boldsymbol{R}}\boldsymbol{L})$	$Q_{\ell edq}$	$\mathcal{O}(1)$	$[Q_{\ell edq}]_{3333}$	1	1
		$\mathcal{O}(\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{V}^b)[Q_{\ell edq}]_{321a} \oplus (\boldsymbol{V}^\dagger)^a[Q_{\ell edq}]_{221a}$	2	2
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{\ell edq}]_{2233} \oplus \boldsymbol{z}[Q_{\ell edq}]_{3223}$	2	2
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell edq}]_{1333} \oplus (\boldsymbol{V}^\dagger\boldsymbol{W})[Q_{\ell edq}]_{1323}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell edq}]_{2333} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell edq}]_{3323} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{\ell edq}]_{2323}$	3	3
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{z})$	$\boldsymbol{z}(\boldsymbol{V}^\dagger)^a[Q_{\ell edq}]_{111a} \oplus \boldsymbol{z}^*(\boldsymbol{V}^\dagger)^a[Q_{\ell edq}]_{331a}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell edq}]_{1333} \oplus (\boldsymbol{U}^\dagger\widetilde{\boldsymbol{W}})[Q_{\ell edq}]_{2223}$	2	2
		$\mathcal{O}(\boldsymbol{W}\boldsymbol{z})$	$(\boldsymbol{z}\varepsilon_{ab}\boldsymbol{W}^b)[Q_{\ell edq}]_{211a} \oplus (\boldsymbol{z}^*\varepsilon_{ab}\boldsymbol{W}^b)[Q_{\ell edq}]_{131a}$	2	2
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{z})$	$(\boldsymbol{z}^*\varepsilon_{ab}\boldsymbol{U}^b)[Q_{\ell edq}]_{231a}$	1	1
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{\ell edq}]_{1133}$	1	1

TABLE VIII. Non-redundant flavor invariants associated with the $(\bar{L}R)(\bar{L}R)$ and $(\bar{L}R)(\bar{R}L)$ operator classes, constructed up to second order in spurion expansion for the $SU(2)_q \times U(1)_X$ flavor symmetry with charges given in Eq. (13). The first column specifies the SMEFT class, the second lists the operators, the third indicates the spurion order, the fourth collects the corresponding flavor invariants, and the fifth (sixth) reports the number of CP-even (CP-odd) independent parameters.

Class	Operator	Order	Invariants	# CP even	# CP odd
$(\bar{\boldsymbol{L}}\boldsymbol{L})(\bar{\boldsymbol{L}}\boldsymbol{L})$	$Q_{qq}^{(1,3)}$	$\mathcal{O}(1)$	$[Q_{qq}^{(1,3)}]_{aabb} \oplus [Q_{qq}^{(1,3)}]_{abba} \oplus [Q_{qq}^{(1,3)}]_{3aa3} \oplus [Q_{qq}^{(1,3)}]_{33aa} \oplus [Q_{qq}^{(1,3)}]_{3333}$	5	0
		$\mathcal{O}(\boldsymbol{V}^2)$	$\boldsymbol{V}^a(\boldsymbol{V}^\dagger)^c[Q_{qq}^{(1,3)}]_{abbc} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qq}^{(1,3)}]_{abcc} \oplus (\boldsymbol{V}^\dagger)^a\boldsymbol{V}^b[Q_{qq}^{(1,3)}]_{3ab3} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qq}^{(1,3)}]_{33ab}$	4	0
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^c[Q_{qq}^{(1,3)}]_{abbc} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qq}^{(1,3)}]_{abcc} \oplus (\boldsymbol{W}^\dagger)^a\boldsymbol{W}^b[Q_{qq}^{(1,3)}]_{3ab3} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qq}^{(1,3)}]_{33ab}$	4	0
		$\mathcal{O}(\boldsymbol{U}^2)$	$\boldsymbol{U}^a(\boldsymbol{U}^\dagger)^c[Q_{qq}^{(1,3)}]_{abbc} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qq}^{(1,3)}]_{abcc} \oplus (\boldsymbol{U}^\dagger)^a\boldsymbol{U}^b[Q_{qq}^{(1,3)}]_{3ab3} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qq}^{(1,3)}]_{33ab}$	4	0
	$Q_{\ell\ell}$	$\mathcal{O}(1)$	$[Q_{\ell\ell}]_{1111} \oplus [Q_{\ell\ell}]_{2222} \oplus [Q_{\ell\ell}]_{3333} \oplus [Q_{\ell\ell}]_{1122} \oplus [Q_{\ell\ell}]_{1133}$ $\oplus [Q_{\ell\ell}]_{2233} \oplus [Q_{\ell\ell}]_{1221} \oplus [Q_{\ell\ell}]_{1331} \oplus [Q_{\ell\ell}]_{2332} \oplus \boxed{[Q_{\ell\ell}]_{3121}}$	10	1
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{1311} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{2111} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{2213} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{2221}$ $\oplus(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{2312} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{3123} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{3313} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell\ell}]_{3321}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{\ell\ell}]_{3212} \oplus (\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{\ell\ell}]_{3231}$	10	10
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{1313} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{2113} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{2121} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{2311}$ $\oplus(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{2322} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell\ell}]_{3323} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{2323}$	7	7
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{1311} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{2111} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{2213} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{2221}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{2312} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{3123} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{3313} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell\ell}]_{3321}$	8	8
	$Q_{\ell q}^{(1,3)}$	$\mathcal{O}(1)$	$[Q_{\ell q}^{(1,3)}]_{11aa} \oplus [Q_{\ell q}^{(1,3)}]_{22aa} \oplus [Q_{\ell q}^{(1,3)}]_{33aa} \oplus [Q_{\ell q}^{(1,3)}]_{1133} \oplus [Q_{\ell q}^{(1,3)}]_{2233} \oplus [Q_{\ell q}^{(1,3)}]_{3333}$	6	0
		$\mathcal{O}(\boldsymbol{V}^2)$	$(\varepsilon_{bc}\boldsymbol{V}^a\boldsymbol{V}^c)[Q_{\ell q}^{(1,3)}]_{32ab} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{11ab} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{22ab} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{33ab}$	4	1
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{bc}\boldsymbol{V}^b\boldsymbol{W}^c)[Q_{\ell q}^{(1,3)}]_{13aa} \oplus (\varepsilon_{bc}\boldsymbol{V}^a\boldsymbol{W}^c)[Q_{\ell q}^{(1,3)}]_{13ab} \oplus (\varepsilon_{bc}\boldsymbol{V}^c\boldsymbol{W}^a)[Q_{\ell q}^{(1,3)}]_{13ab} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell q}^{(1,3)}]_{1333}$ $\oplus(\varepsilon_{bc}\boldsymbol{V}^b\boldsymbol{W}^c)[Q_{\ell q}^{(1,3)}]_{21aa} \oplus (\varepsilon_{bc}\boldsymbol{V}^a\boldsymbol{W}^c)[Q_{\ell q}^{(1,3)}]_{21ab} \oplus (\varepsilon_{bc}\boldsymbol{V}^c\boldsymbol{W}^a)[Q_{\ell q}^{(1,3)}]_{21ab} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell q}^{(1,3)}]_{2133}$	8	8
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{bc}\boldsymbol{U}^b\boldsymbol{V}^c)[Q_{\ell q}^{(1,3)}]_{23aa} \oplus (\varepsilon_{bc}\boldsymbol{U}^a\boldsymbol{V}^c)[Q_{\ell q}^{(1,3)}]_{23ab} \oplus (\varepsilon_{bc}\boldsymbol{U}^c\boldsymbol{V}^a)[Q_{\ell q}^{(1,3)}]_{23ab} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell q}^{(1,3)}]_{2333}$	4	4
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{11ab} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{22ab} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{33ab}$	3	0
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell q}^{(1,3)}]_{13aa} \oplus \boldsymbol{U}^a(\boldsymbol{W}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{13ab} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell q}^{(1,3)}]_{1333}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell q}^{(1,3)}]_{21aa} \oplus \boldsymbol{U}^a(\boldsymbol{W}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{21ab} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell q}^{(1,3)}]_{2133}$	6	6
		$\mathcal{O}(\boldsymbol{U}^2)$	$\boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{11ab} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{22ab} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{\ell q}^{(1,3)}]_{33ab}$	3	0
$(\bar{\boldsymbol{R}}\boldsymbol{R})(\bar{\boldsymbol{R}}\boldsymbol{R})$	Q_{uu}	$\mathcal{O}(1)$	$[Q_{uu}]_{1111} \oplus [Q_{uu}]_{2222} \oplus [Q_{uu}]_{3333} \oplus [Q_{uu}]_{1122} \oplus [Q_{uu}]_{1133}$ $\oplus[Q_{uu}]_{2233} \oplus [Q_{uu}]_{1221} \oplus [Q_{uu}]_{1331} \oplus [Q_{uu}]_{2332}$	9	0
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{uu}]_{3212}$	1	1
	Q_{dd}	$\mathcal{O}(1)$	$[Q_{dd}]_{1111} \oplus [Q_{dd}]_{2222} \oplus [Q_{dd}]_{3333} \oplus [Q_{dd}]_{1122} \oplus [Q_{dd}]_{1133}$ $\oplus[Q_{dd}]_{2233} \oplus [Q_{dd}]_{1221} \oplus [Q_{dd}]_{1331} \oplus [Q_{dd}]_{2332}$	9	0
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{dd}]_{2113} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{dd}]_{2311} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{dd}]_{2322}$ $\oplus(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{dd}]_{3323} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{dd}]_{2323}$	5	5
	$Q_{ud}^{(1,8)}$	$\mathcal{O}(1)$	$[Q_{ud}^{(1,8)}]_{1111} \oplus [Q_{ud}^{(1,8)}]_{1122} \oplus [Q_{ud}^{(1,8)}]_{1133} \oplus [Q_{ud}^{(1,8)}]_{2211} \oplus [Q_{ud}^{(1,8)}]_{2222}$ $\oplus[Q_{ud}^{(1,8)}]_{2233} \oplus [Q_{ud}^{(1,8)}]_{3311} \oplus [Q_{ud}^{(1,8)}]_{3322} \oplus [Q_{ud}^{(1,8)}]_{3333}$	9	0
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{ud}^{(1,8)}]_{3113}$	1	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ud}^{(1,8)}]_{1123} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ud}^{(1,8)}]_{2223} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ud}^{(1,8)}]_{3323}$	3	3
	Q_{ee}	$\mathcal{O}(1)$	$[Q_{ee}]_{1111} \oplus [Q_{ee}]_{1122} \oplus [Q_{ee}]_{1133} \oplus [Q_{ee}]_{2222} \oplus [Q_{ee}]_{2233} \oplus [Q_{ee}]_{3333}$	6	0
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{ee}]_{3212}$	1	1
	Q_{eu}	$\mathcal{O}(1)$	$[Q_{eu}]_{1111} \oplus [Q_{eu}]_{1122} \oplus [Q_{eu}]_{1133} \oplus [Q_{eu}]_{2211} \oplus [Q_{eu}]_{2222}$ $\oplus[Q_{eu}]_{2233} \oplus [Q_{eu}]_{3311} \oplus [Q_{eu}]_{3322} \oplus [Q_{eu}]_{3333} \oplus \boxed{[Q_{ee}]_{3123}}$	10	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{eu}]_{3112}$	1	1
	Q_{ed}	$\mathcal{O}(1)$	$[Q_{ed}]_{1111} \oplus [Q_{ed}]_{1122} \oplus [Q_{ed}]_{1133} \oplus [Q_{ed}]_{2211} \oplus [Q_{ed}]_{2222}$ $\oplus[Q_{ed}]_{2233} \oplus [Q_{ed}]_{3311} \oplus [Q_{ed}]_{3322} \oplus [Q_{ed}]_{3333}$	9	0
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{ed}]_{3223}$	1	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ed}]_{1123} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ed}]_{2223} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{ed}]_{3323} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{ed}]_{1232}$	4	4
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{ed}]_{2321}$	1	1

TABLE V. Non-redundant flavor invariants associated with the $(\bar{L}L)(\bar{L}L)$ and $(\bar{R}R)(\bar{R}R)$ operator classes, constructed up to second order in spurion expansion for the $SU(2)_q \times U(1)_X$ flavor symmetry with charges given in Eq. (13). The first column specifies the SMEFT class, the second lists the operators, the third indicates the spurion order, the fourth collects the corresponding flavor invariants, and the fifth (sixth) reports the number of CP-even (CP-odd) independent parameters. Highlighted entries denote $\mathcal{O}(1)$ flavor-charged currents.

Class	Operator	Order	Invariants	# CP even	# CP odd
$(\bar{\boldsymbol{L}}\boldsymbol{L})(\bar{\boldsymbol{R}}\boldsymbol{R})$	$Q_{qu}^{(1,8)}$	$\mathcal{O}(1)$	$[Q_{qu}^{(1,8)}]_{aa11} \oplus [Q_{qu}^{(1,8)}]_{aa22} \oplus [Q_{qu}^{(1,8)}]_{aa33} \oplus [Q_{qu}^{(1,8)}]_{3311} \oplus [Q_{qu}^{(1,8)}]_{3322} \oplus [Q_{qu}^{(1,8)}]_{3333}$	6	0
		$\mathcal{O}(\boldsymbol{W})$	$\boldsymbol{W}^a[Q_{qu}^{(1,8)}]_{a332}$	1	1
		$\mathcal{O}(\boldsymbol{V}^2)$	$\boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab11} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab22} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab33}$	3	0
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab11} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab22} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab33}$	3	0
		$\mathcal{O}(\boldsymbol{U}^2)$	$\boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab11} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab22} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qu}^{(1,8)}]_{ab33}$	3	0
	$Q_{qd}^{(1,8)}$	$\mathcal{O}(1)$	$[Q_{qd}^{(1,8)}]_{aa11} \oplus [Q_{qd}^{(1,8)}]_{aa22} \oplus [Q_{qd}^{(1,8)}]_{aa33} \oplus [Q_{qd}^{(1,8)}]_{3311} \oplus [Q_{qd}^{(1,8)}]_{3322} \oplus [Q_{qd}^{(1,8)}]_{3333}$	6	0
		$\mathcal{O}(\boldsymbol{V}^2)$	$(\varepsilon_{bc}\boldsymbol{V}^a\boldsymbol{V}^c)[Q_{qd}^{(1,8)}]_{ab32} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab11} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab22} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab33}$	4	1
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{bc}\boldsymbol{U}^b\boldsymbol{V}^c)[Q_{qd}^{(1,8)}]_{aa23} \oplus (\varepsilon_{bc}\boldsymbol{U}^a\boldsymbol{V}^c)[Q_{qd}^{(1,8)}]_{ab23} \oplus (\varepsilon_{bc}\boldsymbol{U}^c\boldsymbol{V}^a)[Q_{qd}^{(1,8)}]_{ab23} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{qd}^{(1,8)}]_{3323}$	4	4
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{z})$	$(\boldsymbol{z}\boldsymbol{V}^a)[Q_{qd}^{(1,8)}]_{a331} \oplus (\boldsymbol{z}^*\varepsilon_{ab}\boldsymbol{V}^b)[Q_{qd}^{(1,8)}]_{3a12}$	2	2
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab11} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab22} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab33}$	3	0
		$\mathcal{O}(\boldsymbol{U}^2)$	$\boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab11} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab22} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qd}^{(1,8)}]_{ab33}$	3	0
	$Q_{\ell e}$	$\mathcal{O}(1)$	$[Q_{\ell e}]_{1111} \oplus [Q_{\ell e}]_{1122} \oplus [Q_{\ell e}]_{1133} \oplus [Q_{\ell e}]_{2211} \oplus [Q_{\ell e}]_{2222}$ $\oplus[Q_{\ell e}]_{2233} \oplus [Q_{\ell e}]_{3311} \oplus [Q_{\ell e}]_{3322} \oplus [Q_{\ell e}]_{3333}$	9	0
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{\ell e}]_{1221} \oplus \boldsymbol{z}[Q_{\ell e}]_{2332} \oplus \boldsymbol{z}[Q_{\ell e}]_{3121}$	3	3
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{1311} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{1322} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{1333}$ $\oplus(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{2111} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{2122} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell e}]_{2133}$	6	6
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell e}]_{2311} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell e}]_{2322} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell e}]_{2333} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{\ell e}]_{3212}$	4	4
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{W}^b)[Q_{\ell e}]_{1212} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{W}^b)[Q_{\ell e}]_{3112} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{1311} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{1322}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{1333} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{2111} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{2122} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell e}]_{2133}$	8	8
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{\ell e}]_{1331} \oplus \boldsymbol{z}^2[Q_{\ell e}]_{2131}$	2	2
	$Q_{\ell u}$	$\mathcal{O}(1)$	$[Q_{\ell u}]_{1111} \oplus [Q_{\ell u}]_{1122} \oplus [Q_{\ell u}]_{1133} \oplus [Q_{\ell u}]_{2211} \oplus [Q_{\ell u}]_{2222}$ $\oplus[Q_{\ell u}]_{2233} \oplus [Q_{\ell u}]_{3311} \oplus [Q_{\ell u}]_{3322} \oplus [Q_{\ell u}]_{3333}$	9	0
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{1311} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{1322} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{1333}$ $\oplus(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{2111} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{2122} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell u}]_{2133}$	6	6
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell u}]_{2311} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell u}]_{2322} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell u}]_{2333}$	3	3
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{1311} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{1322} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{1333}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{2111} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{2122} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell u}]_{2133}$	6	6
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{\ell u}]_{1332} \oplus \boldsymbol{z}^2[Q_{\ell u}]_{2132}$	2	2
	$Q_{\ell d}$	$\mathcal{O}(1)$	$[Q_{\ell d}]_{1111} \oplus [Q_{\ell d}]_{1122} \oplus [Q_{\ell d}]_{1133} \oplus [Q_{\ell d}]_{2211} \oplus [Q_{\ell d}]_{2222}$ $\oplus[Q_{\ell d}]_{2233} \oplus [Q_{\ell d}]_{3311} \oplus [Q_{\ell d}]_{3322} \oplus [Q_{\ell d}]_{3333} \oplus \boxed{[Q_{\ell d}]_{3223}}$	10	1
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{W})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{1223} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{1311} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{1322} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{1333}$ $\oplus(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{2111} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{2122} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{2133} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{\ell d}]_{3123}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{\ell d}]_{1323} \oplus (\boldsymbol{W}^\dagger\boldsymbol{V})[Q_{\ell d}]_{3132}$	10	10
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{V})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{1123} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{2223} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{2311} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{2322}$ $\oplus(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{2333} \oplus (\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{\ell d}]_{3323} \oplus (\boldsymbol{V}^\dagger\boldsymbol{U})[Q_{\ell d}]_{2323}$	7	7
		$\mathcal{O}(\boldsymbol{U}\boldsymbol{W})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{1223} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{1311} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{1322} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{1333}$ $\oplus(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{2111} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{2122} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{2133} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{\ell d}]_{3123}$	8	8
	Q_{qe}	$\mathcal{O}(1)$	$[Q_{qe}]_{aa11} \oplus [Q_{qe}]_{aa22} \oplus [Q_{qe}]_{aa33} \oplus [Q_{qe}]_{3311} \oplus [Q_{qe}]_{3322} \oplus [Q_{qe}]_{3333}$	6	0
		$\mathcal{O}(\boldsymbol{W})$	$\boldsymbol{W}^a[Q_{qe}]_{a331}$	1	1
		$\mathcal{O}(\boldsymbol{V}^2)$	$\boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qe}]_{ab11} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qe}]_{ab22} \oplus \boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{qe}]_{ab33}$	3	0
		$\mathcal{O}(\boldsymbol{V}\boldsymbol{z})$	$(\boldsymbol{z}\varepsilon_{ab}\boldsymbol{V}^b)[Q_{qe}]_{3a23}$	1	1
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qe}]_{ab11} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qe}]_{ab22} \oplus \boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{qe}]_{ab33}$	3	0
		$\mathcal{O}(\boldsymbol{U}^2)$	$(\varepsilon_{bc}\boldsymbol{U}^a\boldsymbol{U}^c)[Q_{qe}]_{ab12} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qe}]_{ab11} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qe}]_{ab22} \oplus \boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{qe}]_{ab33}$	4	1

TABLE VI. Non-redundant flavor invariants associated with the $(\bar{L}L)(\bar{R}R)$ operator class, constructed up to second order in spurion expansion for the $SU(2)_q \times U(1)_X$ flavor symmetry with charges given in Eq. (13). The first column specifies the SMEFT class, the second lists the operators, the third indicates the spurion order, the fourth collects the corresponding flavor invariants, and the fifth (sixth) reports the number of CP-even (CP-odd) independent parameters. Highlighted entry denotes $\mathcal{O}(1)$ flavor-charged current.

Class	Operator	Order	Invariants	# CP even	# CP odd
$\boldsymbol{\psi^2 H^3}$	Q_{uH}	$\mathcal{O}(1)$	$[Q_{uH}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{W})$	$\boldsymbol{W}^a[Q_{uH}]_{a2}$	1	1
	Q_{dH}	$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{dH}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{Vz})$	$(\boldsymbol{z}^*\widetilde{\boldsymbol{V}}^a)[Q_{dH}]_{a2}$	1	1
		$\mathcal{O}(\boldsymbol{Uz})$	$(\boldsymbol{z}^*\boldsymbol{U}^a)[Q_{dH}]_{a3}$	1	1
	Q_{eH}	$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{eH}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{UW})$	$(\boldsymbol{U}^\dagger\widetilde{\boldsymbol{W}})[Q_{eH}]_{23}$	1	1
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{eH}]_{22}$	1	1
$\boldsymbol{\psi^2 XH}$	$Q_{u(B,W,G)}$	$\mathcal{O}(1)$	$[Q_{u(B,W,G)}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{W})$	$\boldsymbol{W}^a[Q_{u(B,W,G)}]_{a2}$	1	1
	$Q_{d(B,W,G)}$	$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{d(B,W,G)}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{Vz})$	$(\boldsymbol{z}^*\widetilde{\boldsymbol{V}}^a)[Q_{d(B,W,G)}]_{a2}$	1	1
		$\mathcal{O}(\boldsymbol{Uz})$	$(\boldsymbol{z}^*\boldsymbol{U}^a)[Q_{d(B,W,G)}]_{a3}$	1	1
	$Q_{e(B,W)}$	$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{e(B,W)}]_{33}$	1	1
		$\mathcal{O}(\boldsymbol{UW})$	$(\boldsymbol{U}^\dagger\widetilde{\boldsymbol{W}})[Q_{e(B,W)}]_{23}$	1	1
		$\mathcal{O}(\boldsymbol{z}^2)$	$\boldsymbol{z}^2[Q_{e(B,W)}]_{22}$	1	1
$\boldsymbol{\psi^2 H^2 D}$	$Q_{Hq}^{(1,3)}$	$\mathcal{O}(1)$	$[Q_{Hq}^{(1,3)}]_{aa} \oplus [Q_{Hq}^{(1,3)}]_{33}$	2	0
		$\mathcal{O}(\boldsymbol{V}^2)$	$\boldsymbol{V}^a(\boldsymbol{V}^\dagger)^b[Q_{Hq}^{(1,3)}]_{ab}$	1	0
		$\mathcal{O}(\boldsymbol{W}^2)$	$\boldsymbol{W}^a(\boldsymbol{W}^\dagger)^b[Q_{Hq}^{(1,3)}]_{ab}$	1	0
		$\mathcal{O}(\boldsymbol{U}^2)$	$\boldsymbol{U}^a(\boldsymbol{U}^\dagger)^b[Q_{Hq}^{(1,3)}]_{ab}$	1	0
	Q_{Hu}	$\mathcal{O}(1)$	$[Q_{Hu}]_{11} \oplus [Q_{Hu}]_{22} \oplus [Q_{Hu}]_{33}$	3	0
	Q_{Hd}	$\mathcal{O}(1)$	$[Q_{Hd}]_{11} \oplus [Q_{Hd}]_{22} \oplus [Q_{Hd}]_{33}$	3	0
		$\mathcal{O}(\boldsymbol{UV})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{Hd}]_{23}$	1	1
	Q_{Hud}	$\mathcal{O}(1)$	$[Q_{Hud}]_{11}$	1	1
		$\mathcal{O}(\boldsymbol{z})$	$\boldsymbol{z}[Q_{Hud}]_{33}$	1	1
	$Q_{H\ell}^{(1,3)}$	$\mathcal{O}(1)$	$[Q_{H\ell}^{(1,3)}]_{11} \oplus [Q_{H\ell}^{(1,3)}]_{22} \oplus [Q_{H\ell}^{(1,3)}]_{33}$	3	0
		$\mathcal{O}(\boldsymbol{VW})$	$(\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{H\ell}^{(1,3)}]_{13} \oplus (\varepsilon_{ab}\boldsymbol{V}^a\boldsymbol{W}^b)[Q_{H\ell}^{(1,3)}]_{21}$	2	2
		$\mathcal{O}(\boldsymbol{UV})$	$(\varepsilon_{ab}\boldsymbol{U}^a\boldsymbol{V}^b)[Q_{H\ell}^{(1,3)}]_{23}$	1	1
		$\mathcal{O}(\boldsymbol{UW})$	$(\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{H\ell}^{(1,3)}]_{13} \oplus (\boldsymbol{W}^\dagger\boldsymbol{U})[Q_{H\ell}^{(1,3)}]_{21}$	2	2
	Q_{He}	$\mathcal{O}(1)$	$[Q_{He}]_{11} \oplus [Q_{He}]_{22} \oplus [Q_{He}]_{33}$	3	0

TABLE VII. Non-redundant flavor invariants associated with the $\psi^2 H^3$, $\psi^2 XH$ and $\psi^2 H^2 D$ operator classes, constructed up to second order in spurion expansion for the $SU(2)_q \times U(1)_X$ flavor symmetry with charges given in Eq. (13). The first column specifies the SMEFT class, the second lists the operators, the third indicates the spurion order, the fourth collects the corresponding flavor invariants, and the fifth (sixth) reports the number of CP-even (CP-odd) independent parameters.