Enhancement of T_c in Oxide Superconductors: Double-Bridge Mechanism of High-T_c Superconductivity and Bose-Einstein Condensation of Cooper Pairs

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The cuprate $Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$ exhibits the highest superconducting transition temperature T_c of 138 K under ambient pressure. Achieving superconductivity at even higher temperatures, up to room temperature, represents the ultimate dream of humanity. As temperature increases, Cooper pairs formed through weak electron-phonon coupling will be disintegrated by the thermal motion of electrons, severely limiting the enhancement of T_c. It is thus imperative to explore new strong-coupling pairing pictures and establish novel condensation mechanism of Cooper pairs at higher temperatures. Based on our recently proposed groundbreaking idea of electron e^- (hole h^+) pairing bridged by oxygen O (metal M) atoms, namely, the eV-scale ionic-bond-driven atom-bridge (bridge-I) **e^--O-e^-** (**h^+-M-h^+**) strong-coupling itinerant Cooper pairing formed at pseudogap temperature $T^* > T_c$ in ionic oxide superconductors, we further discover that there is an attractive interaction between two Cooper pairs induced by the bridge atom (bridge-II) located between them. It is precisely this attraction mediated by the bridge-II atoms that promotes all the Cooper pairs within the CuO_2 plane to hold together and enter the superconducting state at T_c finally. Moreover, according to the Bose-Einstein condensation theory, we find that T_c is inversely proportional to the effective mass m^* of Cooper pairs, directly proportional to $n_s^{2/3}$ (n_s: the density of Cooper pairs), and linearly increases with the scattering length a <0 due to attraction between two Cooper pairs. Therefore, according to our double-bridge mechanism of high-T_c superconductivity, increasing the attraction between the Cooper pairs and the bridge-II atoms, ensuring that n_s takes the optimal value, and minimizing the effective mass of the Cooper pairs are the main approaches to enhancing T_c of ionic-bonded superconductors, which opens up a new avenue with clear direction for designing higher T_c superconductors.

Since the discovery of high-critical temperature T_c superconductivity in cuprates in 1986 [1], it has always been one of the most fascinating research fields in physics. Many challenging issues have attracted the extensive research interest among physicists across generations. Realizing superconductivity at higher temperatures, even at room temperature, has become the ultimate dream of humanity. Due to the lack of guidance from a widely accepted microscopic theory of high-T_c superconductivity, current superconducting research exhibits multiple divergent directions, such as cuprates [2], nickelates [3, 4], iron-based superconductors [5], multi-hydrogen superconductors under ultrahigh-pressure (∼200 GPa) [6], organic superconductors [7], heavy-fermion superconductors [8], kagome superconductors [9, 10], 'magic angle' twisted bilayer graphene (MATBG) [11], and so on. Among so many superconductors, which one is the most promising high-T_c superconducting material is still unclear at present. No one knows exactly how to effectively increase T_c.

In practical applications of superconducting materials, not only is a high critical temperature T_c important, but also a high critical magnetic field, a high critical current, and good ductility are crucial. Moreover, the ambient pressure is also highly desirable. That is, it requires '*3 highs + ductility + ambient pressure.*' Regrettably, none of the existing superconducting materials can simultaneously meet all these requirements. The search of high-T_c superconducting materials with practical value must take all these factors into account, which urgently demands a clear theoretical framework to guide the right direction of exploration. With applications as the ultimate goal, the pursuit of high-T_c superconductivity must remain on the correct path, making relentless efforts to move forward. Any research that does not take into account the practical application of this ultimate goal into account may have no future, just as Norman pointed out [12].

Quite recently, considering the dominance of eV-scale ionic bonding, affinity of O^- (1.46 eV) and O^{2-} (-8.08 eV) [13] and large two-electron ionization energy (∼15-28 eV) of metal atoms [14], we propose a transformative idea of electron e^- (hole h^+) pairing bridged by oxygen O (metal M) atoms named as bridge-I, i.e., the ionic-bond-driven **e^--O-e^-** (**h^+-M-h^+**) itinerant Cooper pairing formed at pseudogap temperature $T^* > T_c$, by following the principle of "tracing electron footprints to explore pairing mechanisms" and by standing on the solid foundation of the chemical-bond→structure→property relationship [15]. This is a universal carrier-pairing picture for all ionic-bonded superconductors including cuprates, nickelates, iron-based and other potential ionic superconductors. Here, we use this universal carrier-pairing image

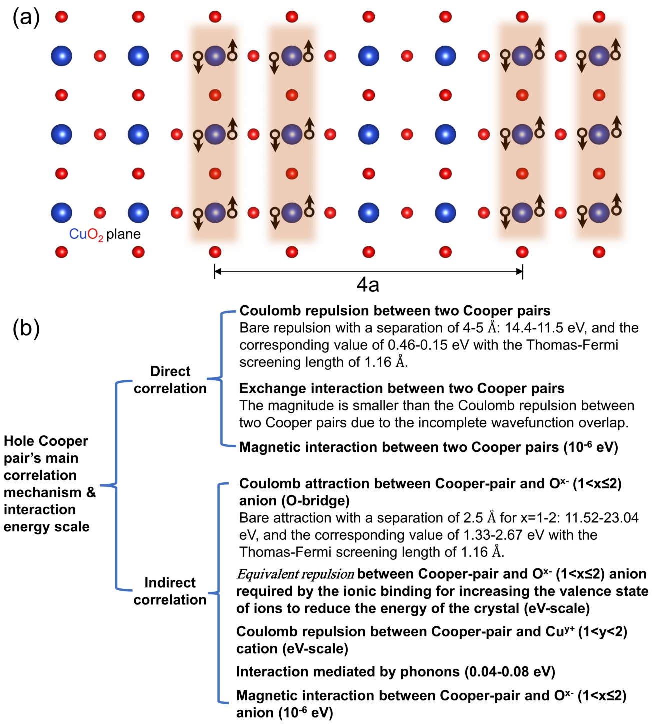

FIG. 1. (a) The $4a$-period Cu-bridged (**bridge-I**) hole Cooper-pair $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ stripe phase in the CuO_2 plane [15, 17, 18], in which two Cooper pairs are attracted to each other through the O^{x-} ($1< x\leq2$) anion as a bridge (**bridge-II**). (b) The major direct and O-bridged correlation mechanisms between two $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ hole Cooper pairs and the corresponding interaction energy scale. Similar to the $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ pairs, the correlation mechanisms can also be found for the Cu-bridged correlation of the O-bridged $\mathbf{e^-}$**-O-**$\mathbf{e^-}$ electron Cooper pairs.

to explore new methods for enhancing T_c by combining the layered crystal structure characteristics of high-T_c superconducting materials [16] and the $4a$-periodic Cooper pair charge stripe phase feature [17, 18]. Through detailed analysis of the dominant direct and indirect interactions between Cooper pairs, we pave a new way with a clear direction, i.e., increasing the attraction between Cooper pairs and reducing the effective mass of Cooper pairs under the optimal carrier concentration, for maximizing T_c within the theoretical framework of Bose-Einstein condensation (BEC) [19, 20], which provides a clear and directional pathway for the exploration of higher T_c superconductivity.

Correlation mechanisms of Cooper pairs and their interaction energy scale—Before delving deeply into the BEC of Cooper pairs in high-T_c oxides [19, 20], let us first analyze the possible interacting mechanisms between two Cooper pairs. Figure 1(a) clearly illustrates that not only direct interactions but also indirect interactions mediated by oxygen anions (bridge-II) exist between two Cu-bridged (bridge-I) $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ hole Cooper pairs, which can be named as the double-bridge mechanism of high-T_c superconductivity (see Fig. 2). We know that a $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ hole Cooper pair carries a charge of $+2e$ and possesses a non-zero orbital magnetic moment with the angular momentum quantum number l=2 (d-wave superconductivity) and the spin quantum number s=0 (Boson) [15]. It is evident that direct Coulomb repulsion exists between two $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ Cooper pairs, with the shortest distance between them being the lattice constant $a\approx$4-5 Å of the CuO_2 plane [2], and the corresponding bare repulsion energy is 14.4-11.5 eV. Considering the strongest screening scenario, if we take the electron concentration of 1×10^{21} cm^{-3}, the Thomas-Fermi screening length is approximately 1.16 Å, and the corresponding Coulomb repulsion energy decays exponentially to about 0.46-0.15 eV (Fig. 1(b)). Due to the overlap of wave functions, exchange interactions also exist between two Cooper pairs, with the magnitude depending on the degree of wave function overlap. According to quantum mechanics [21], the exchange interaction is generally smaller than the direct Coulomb repulsion between two Cooper pairs due to the incomplete overlap of wave functions. If we regard a $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ Cooper pair with d-wave symmetry as a magnetic dipole with an orbital magnetic moment of M_z=$2\mu_B$, the magnetic interaction between them can be estimated to be on the order of 10^{-6} eV, which is significantly smaller than the Coulomb repulsion energy between two Cooper pairs.

In addition to the direct Coulomb repulsion, the oxygen anion acts as a correlation-bridge directly connecting two $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ Cooper pairs (Fig. 1(a)), and there also exists an eV-scale Coulomb attraction between oxygen anion and its nearest-neighboring Cooper pairs. Based on our prior analysis, the oxygen anion O^{x-} has a valence state of $1<x\leq2$ [15] and is located approximately half a lattice constant away from the Cooper pairs (Fig. 1(a)), that is, $a/2\approx2.5$ Å. We can thus estimate that the bare Coulomb attractive energy between a Cooper pair with a charge of $+2e$ and oxygen anions with charges of $-e$ or $-2e$ ranges from 11.52 to 23.04 eV. As a conservative estimate, if we take the strongest screening with the Thomas-Fermi screening length of 1.16 Å [15], the corresponding Coulomb attractive energy decreases exponentially to 1.33-2.67 eV (Fig. 1(b)), which is still approximately one order of magnitude larger than the direct Coulomb repulsion between two Cooper pairs, as analyzed above. Moreover, there is a *equivalent eV-scale repulsion* between $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ Cooper-pair and O^{x-} anion required by the ionic binding for increasing the valence state of ions to reduce the total energy of the crystal and to maintain structural stability (see Fig. 2(a) [15]). The eV-scale Coulomb repulsion between $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ and Cu^{y+} ($1<y<2$) cation also exists. As for the electron-phonon coupling, we know from experiments that the electron-phonon coupling strength in high-T_c cuprates is 0.04-0.08 eV approximately [22, 23]. If we further assume the mag-

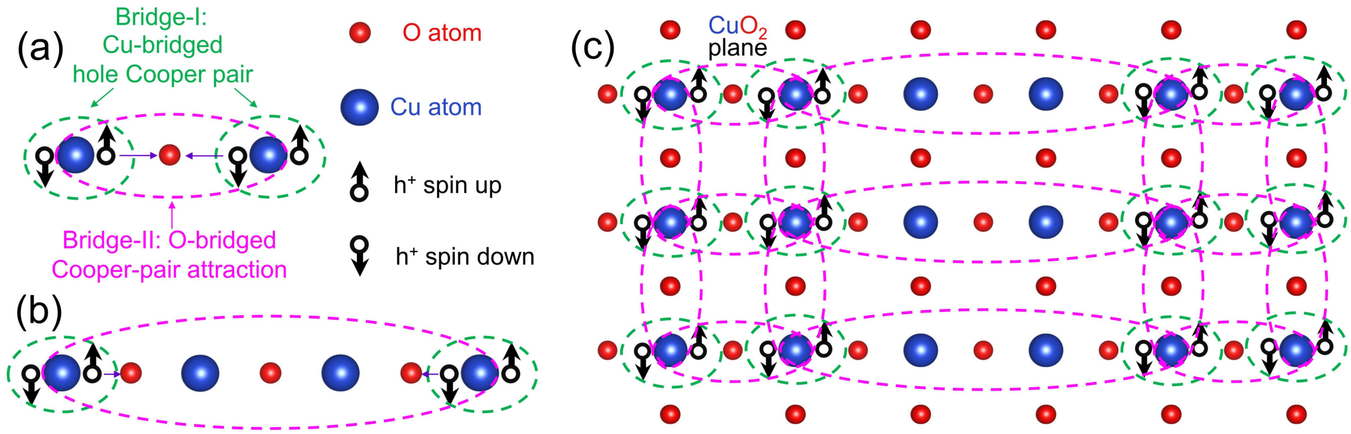

FIG. 2. The double-bridge mechanism of high-T_c superconductivity in cuprates. (a) The strong Coulomb attraction exerted by the O^{x-} ($1< x\leq 2$) anion, acting as a bridge (**bridge-II**), on its two nearest-neighboring Cu-bridged (**bridge-I**) hole Cooper pairs **h$^+$-Cu-h$^+$**, together with the strong Coulomb repulsion between Cu^{y+} ($1< y <2$) cation and the Cooper pair around it to push the **h$^+$-Cu-h$^+$** hole pair towards the nearest-neighboring O^{x-} anion, is equivalent to an indirect strong mutual attraction between the two **h$^+$-Cu-h$^+$** pairs. When $T_c < T < T^*$ (T^*: the pseudogap temperature), this indirect strong mutual attraction induced by the bridge-II atom reaches a balance with the direct Coulomb repulsion between the two **h$^+$-Cu-h$^+$** pairs and the *equivalent repulsion* between the O^{x-} anions and the **h$^+$-Cu-h$^+$** pairs due to the requirement of ionic binding for increasing the valence state of ions to reduce the total energy of the crystal and to keep the structural stability (see Fig. 2(a) [15]), as described in Fig. 1(b). When $T\leq T_c$, the decrease in the kinetic energy of electrons in the electron cloud surrounding the O^{x-} anions weakens the electrons' ability to escape the binding of the oxygen nuclei. This leads to a slight increase in the charge amount Q (negative valence state) carried by the O^{x-} anions, i.e., $Q\rightarrow Q+\delta Q$ with δQ a small quantity, which will also be influenced by the transition of Cooper pairs from their excited states to the ground state (see text). The original balance is thus disrupted. Similar to O^{x-} anion, the positive valence state of the Cu^{y+} cation will also be enhanced slightly at $T\leq T_c$. The Coulomb attraction between the O^{x-} anion and its two nearest-neighboring **h$^+$-Cu-h$^+$** Cooper pairs thus increases, causing a net attraction between two **h$^+$-Cu-h$^+$** Cooper pairs. According to the BCS-BEC crossover theory (see Fig. 3 in End Matter) [19, 20], it is precisely under the driving of this net attraction that BEC occurs in the two **h$^+$-Cu-h$^+$** Cooper pairs. (b) According to Coulomb's law, for two **h$^+$-Cu-h$^+$** Cooper pairs located on the same -Cu-O-Cu- chain and separated by several lattice constants, their strong mutual attractions with their respective nearest-neighboring O^{x-} anions are much greater than the interactions with other more distant ions. The net effect is equivalent to an indirect mutual attraction between the two **h$^+$-Cu-h$^+$** Cooper pairs. Similar to (a), at $T\leq T_c$, these two **h$^+$-Cu-h$^+$** Cooper pairs can also undergo BEC through the O-bridge. (c) Combining (a) and (b), and taking into account the hole stripe phase, all Cu-bridged (bridge-I) **h$^+$-Cu-h$^+$** Cooper pairs within the CuO_2 planes "hold hands" with each other through the oxygen bridges (bridge-II) and undergo BEC to enter the superconducting state—the double-bridge mechanism of high-T_c superconductivity. This picture fully demonstrates the true physical beauty of the high-T_c superconducting mechanism. For O-bridged (bridge-I) **e$^-$-O-e$^-$** Cooper pairs, a similar double-bridge picture can also be established by way of Cu^{y+}($1<y<2$) cations as bridges (bridge-II) for BEC of **e$^-$-O-e$^-$** pairs.

netic moment of the oxygen anion O^{x-} to be $0.2\mu_B$ [24], we can estimate that the magnetic interaction energy between O^{x-} and **h$^+$-Cu-h$^+$** is on the order of 10^{-6} eV, which is negligible. A similar physical analysis can also be conducted for the O-bridged (bridge-I) **e$^-$-O-e$^-$** electron Cooper pairs.

Double-bridge mechanism of high-T_c superconductivity—Comprehensively considering all the major eV-scale correlation mechanisms summarized in Fig. 1(b), we can derive an elegant double-bridge mechanism of high-T_c superconductivity detailed in Fig. 2 in cuprates, in which the Cu-bridge (**bridge-I**) for hole pairing **h$^+$-Cu-h$^+$** and O-bridge (**bridge-II**) for BEC of **h$^+$-Cu-h$^+$** pairs [19, 20]. It is precisely the net attraction between two **h$^+$-Cu-h$^+$** Cooper pairs induced by the O-bridge (bridge-II), as analyzed in Fig. 2, that causes the Cooper pairs' condensation through "holding hands together" within the whole CuO_2 plane, which directly contributes to the enhancement of the BEC temperature T_{BEC}, namely T_c, in high-T_c cuprates (see Eq. (3)). Based on the physical essence of ionic bonding in high-T_c superconductors, we believe this conclusion is universal and can be directly extended to strongly ionic-bonded high-T_c superconducting materials including cuprates, nickelates, iron-based and new potential ionic superconductors, providing a new guideline for further enhancing T_c of high-T_c superconductors. The BEC picture of **e$^-$-O-e$^-$** electron Cooper pairs can also be set up similarly.

Enhancement of the Coulomb attraction between Cooper pairs and bridge-II atoms due to the transition of Cooper pairs from their excited states to the ground state: superconducting fluctuations—To elucidate the mechanism of enhanced Coulomb attraction between Cooper pairs and bridge-II atoms, let us now analyze the case of hole carriers (the dominant charge carriers in high-T_c cuprates) as an example. According to our previously proposed ionic-bond-driven atom-bridged (bridge-I) hole pairing mechanism [15], the strong ionic bonds in crystal overcome the Coulomb repulsion between two holes, forming excited-state hole Cooper pairs $\mathbf{h^+}$**-M-**$\mathbf{h^+}$ at the pseudogap temperature $T^* > T_c$. As the temperature further decreases to T_c, these Cooper pairs release energy E_{sc}, i.e., the energy of the superconducting gap, undergoing BEC and transitioning from their excited states to the ground state, thereby entering the superconducting state. For the high-T_c cuprates, E_{sc} is about 40 meV [25]. Where does this energy go? What impact will it have on the system's properties?

Based on the process of atomic bonding in high-T_c oxides (Fig. 2 [15]), we believe that the total energy released by the transitions of Cooper pairs from their excited states to the ground state, with similar function to the released energy due to the combination of crystals, will cause ionization of few metal cations. These newly ionized electrons will further accumulate toward few oxygen anions under the drive of the eV-scale ionic bonds, thereby increasing their valence state slightly (Fig. 2), which may cause the superconducting fluctuations. The exact mechanism still requires further investigation. The increasing of the O^{x-} valence state directly enhances the Coulomb attraction between O^{x-} (bridge-II) and its nearest-neighbor Cu-bridged (bridge-I) hole Cooper pair $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$, ultimately resulting in a net mutual attraction between two hole Cooper pairs. Consequently, T_c is further increased (see Eq. (3)). A similar explanation applies to O-bridged electron Cooper pairs $\mathbf{e^-}$**-O-**$\mathbf{e^-}$. As one of the causes of superconducting fluctuations, the energy released during the transition of Cooper pairs from their excited states to the ground state will help to improve T_c^{onset} of oxide superconductors [4].

The BEC theoretical framework for the interacting Cooper pairs—It is well-known that high-T_c cuprates exhibit typical BEC-like characteristics (see End Matter) [20, 26–28], such as strong-coupling Cooper pairs, small pair sizes, large pairing or superconducting fluctuations, two gaps (pseudogap and superconducting gap), preformed pairs at pairing-onset temperature $T^* > T_c$, large pseudogap, the dome-like superconducting phase diagram, the strong localization of holes and electrons, and the strong ionic bonding, among others [2, 15, 25]. The Cooper pairs, Bosons with zero spin, can undergo the BEC phase transition, transforming from a Bose liquid to a superconducting state (Fig. 3) [2, 20, 29]. The basic idea of BEC was first proposed by Einstein in 1925 and the corresponding basic theory of BEC had been established before 1995 [19]. According to the well-known two-fluid model [30], the delocalized charge carriers (N) in the superconducting state can be divided into condensed superconducting carriers (N_s) and uncondensed normal carriers (N_n) at finite temperatures ($T < T_c$). We thus have the following density of carriers,

$$n = \frac{N}{V} = \frac{N_s + N_n}{V} = \frac{N_s}{V} + \frac{N_n}{V} = n_s + n_n, \quad (1)$$

where $n_s = N_s/V$ is the number density of Cooper pairs, i.e., superconducting carriers, and $n_n = N_n/V$ is the density of the uncondensed normal carriers. The BEC critical temperature $T_{\rm BEC}$, i.e., the superconducting transition temperature T_c, for the ideal Cooper-pair Bose liquid without interactions can be calculated as follows [19],

$$T_c^0 = \frac{h^2}{2\pi m^* k_B}\left(\frac{n_s}{2.612}\right)^{2/3}, \quad (2)$$

where k_B is the Boltzmann's constant, h is the Planck's constant, m^* is the effective mass of one Cooper pair. The superscript "0" indicates the ideal Cooper-pair Bose liquid without interactions. Equation (2) clearly indicates that T_c^0 is inversely proportional to m^* and directly proportional to $n_s^{2/3}$. This is precisely the physical reason why heavy-fermion superconductors, with the electron effective mass $m^* \sim 200 m_e$, exhibit low critical temperatures (T_c <1 K) [8, 31]. Compared with high-T_c cuprates with carrier density of 10^{21} cm^{-3} [2, 32], it is the low carrier density ($\sim 10^{17}$-10^{18} cm^{-3}) that leads to the small T_c ($\sim$1.7 K) in MATBG [11, 20]. In order to maximize T_c, m^* must be reduced as much as possible under the condition of maintaining the optimal Cooper-pair concentration n_s.

The BEC critical temperature T_c for interacting Cooper pairs can be estimated by treating the interaction in the mean-filed approximation as follows [19],

$$T_c = T_c^0\left(1 - 3.426\frac{a}{\lambda_0}\right), \quad (3)$$

where a is the scattering length, an indicator for the interacting strength, between two Cooper pairs with a <0 for attractive interaction and a >0 for repulsive interaction, respectively. Generally, the $|a|$ can be larger than the interparticle distance. Equation (3) clearly shows that T_c increases linearly with a for attractive interactions between two Cooper pairs, which paves a new way to improve T_c with great significance for the exploration of higher T_c superconductivity. As an open-ended question, the maximization of a is definitely a highly challenging and complex material design engineering. The thermal de Broglie wavelength λ_0 at the critical temperature T_c^0, depending solely on the number density n_s of Cooper pairs, can be calculated as follows,

$$\lambda_0 = \frac{h}{\sqrt{2\pi m^* k_B T_c^0}} = \left(\frac{2.612}{n_s}\right)^{1/3}. \quad (4)$$

Equations (2-4) indicate that T_c just depends on the effective mass m^* and the number density n_s of Cooper pairs, as well as the scattering length a. Moreover, the condensate fraction of Cooper pairs at temperature T is given by [19],

$$\frac{N_s}{N} = \begin{cases} 1 - \left(\frac{T}{T_c}\right)^{3/2}, & T \leq T_c \\ 0, & T > T_c. \end{cases} \tag{5}$$

Conclusion—Based on our recently proposed eV-scale ionic-bond-driven atom-bridged (bridge-I) room-temperature itinerant Cooper pairing picture $\mathbf{h^+}$**-M-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) formed at pseudogap temperature $T^* > T_c$ [15], and through a meticulous analysis of several primary interaction mechanisms between Cooper pairs, we find that the indirect Coulomb attraction between Cooper pairs $\mathbf{h^+}$**-M-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) mediated by the O-bridge (M-bridge), named as bridge-II, can overcome the direct Coulomb repulsion between two Cooper pairs, constituting the most significant interaction among them. According to our double-bridge superconducting mechanism (Fig. 2) and the BCS-BEC crossover theory of interacting Cooper pairs (Fig. 3), we find that the net attraction between Cooper pairs can effectively enhance the critical temperature T_c of superconductors. Generally, T_c of ionic-bonded superconductors is directly proportional to $n_s^{2/3}$ and inversely proportional to the effective mass m^* of Cooper pairs, and increases linearly with the scattering length $|a|$ (a <0) of the net attraction between Cooper pairs via bridge-II atoms, which paves a new way with a clear direction for maximizing T_c and will be of great importance to exploring higher T_c superconductivity.

Based on the experimentally observed dome-like superconducting phase diagram and our previous theoretical analysis [15], we know that there exists an optimal superconducting carrier concentration n_s in high-T_c oxide superconductors. If this optimal carrier concentration is exceeded, the surplus unpaired carriers will dissociate Cooper pairs, which is harmful to improve the T_c. Therefore, maintaining n_s at the optimal value is very important for acquiring high-T_c superconductivity. According to the BEC theory, the smaller the effective mass m^* of Cooper pairs, the higher the superconducting transition temperature T_c ($T_c \propto 1/m^*$). If n_s, m^* and a can be ingeniously designed (Table I in End Matter), the superconducting critical temperature T_c can be appropriately enhanced (Fig. 4 in End Matter), opening a new avenue for further raising T_c. For high-T_c oxide superconductors bound by strong ionic bonds, our eV-scale ionic-bond-driven atom-bridged (bridge-I) pairing mechanism confirms that Cooper pairs can exist at room temperature, laying a solid foundation for achieving room-temperature superconductivity. If ingenious regulation of n_s, m^* and a can be realized to further enhance T_c, the ultimate realization of higher T_c even room-temperature superconductivity may even be possible in ionic oxide superconductors. We hope our double-bridge superconducting mechanism, shown in Fig. 2, can arouse the strong interest of superconducting scientists, thereby further promoting the development of superconducting science and ultimately realizing the dream of higher T_c even room-temperature superconductivity for the benefit of all mankind.

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Data availability—The data supporting this study's findings are available within the Letter.

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End Matter

Attraction between Cooper pairs in cuprates: the driving force from BCS to BEC—Quite recently, the electronic properties of CuO_2 planes in an electron-doped infinite-layer cuprate superconductor $Sr_{1-x}Nd_xCuO_2$ has been investigated by using the spectroscopic-imaging scanning tunneling microscopy (SI-STM), a powerful tool [26]. The experimental results strongly suggest the emergence of BEC-like behaviors prior to the macroscopic superconducting condensation in $Sr_{1-x}Nd_xCuO_2$ and provide a direct real-space experimental evidence for the BCS-BEC crossover on CuO_2 planes due to the negative chemical potential $\mu<0$ [26, 27]. Considering that the T_c dome with overlain coherence length provides a rather ideal prototype for BCS-BEC crossover physics, Ref. [28] confirmed the existence of BCS-BEC crossover in high-T_c cuprates. The BEC of Cooper pairs in high-T_c cuprates has also been verified in Refs. [2, 29]. As is well known, only when there exists mutual attraction between two Cooper pairs can it facilitate the transition of Cooper pairs from the BCS regime to the BEC regime (Fig. 3), and drive the system to undergo a BEC phase transition and enter the superconducting state finally [19, 20]. Obviously, *as a successful and reasonable high-T_c superconducting mechanism, it must simultaneously address two key issues: (1) the strong attractive pairing mechanism between two holes (electrons); (2) the attraction mechanism between two Cooper pairs as a driving force for their condensation.* In Ref. [15], we proposed the eV-scale ionic-bond-driven atom-bridged (bridge-I) $\mathbf{h^+}$**-M-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) strong itinerant Cooper pairing picture with d-wave holes (electrons) and small pair-size. This Letter addresses the key issue of the BEC phase transition of Cooper pairs $\mathbf{h^+}$**-M-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) through the bridging effect of O (Cu) atoms (bridge-II). It is the attraction between Coopers induced by the bridge-II atoms

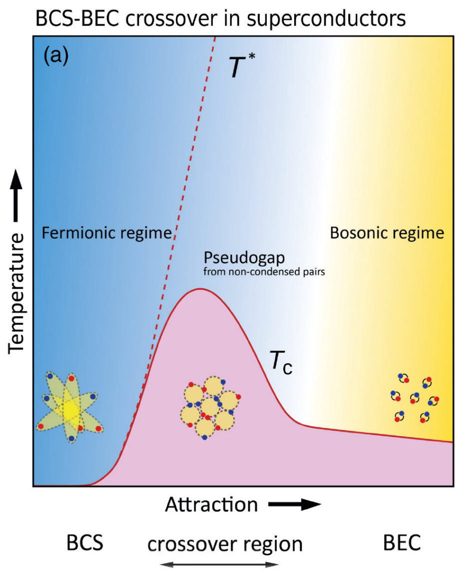

FIG. 3. The BCS-BEC crossover phase diagram in high-T_c cuprates [20], in which the attraction between the Cooper pairs with zero spin is of crucial importance for the condensation of Cooper pairs.

TABLE I. The density n_s and effective mass m^* of Cooper pairs, the thermal wavelength λ_0, and the calculated BEC critical temperature T_c^0 and the corresponding experimental value T_c^{Exp} in cuprates: $YBa_2Cu_3O_7$ (I), $YBa_2Cu_4O_8$ (II), $Bi_2Sr_2CaCu_2O_8$ (III), $Tl_2Ba_2CaCu_2O_8$ (IV), $Tl_2Ca_2Ba_2Cu_3O_{10}$ (V), and $Tl_{0.5}Pb_{0.5}Sr_2CaCu_2O_7$ (VI).

Cuprate	n_s (10^{21} cm^{-3})	m^*/m_e [a]	λ_0 (nm)	T_c^0 (K)	T_c^{Exp} (K) [a]
I	0.22	12.0	2.28	89	92
II	0.033	3.8	4.29	79	80
III	0.11	7.8	2.87	86	89
IV	0.15	8.4	2.59	98	99
V	0.117	5.7	2.82	123	125
VI	0.025	3.2	4.71	78	80

[a] From Ref. [32].

that leads the system to enter the superconducting state at $T_c < T^*$ finally (T^*: the pairing-onset temperature, i.e., the pseudogap temperature).

Estimation of T_c in cuprates—All experiments indicate the carrier density $n \sim 10^{21}$ cm^{-3} in high-T_c cuprates [2, 32, 33]. Considering the density n_s of Cooper pairs is smaller than n (see Eqs. (1) and (5)), we can thus choose $n_s \sim 10^{19}$-10^{20} cm^{-3} at finite temperature ($T < T_c$) as a reasonable estimation, larger than 10^{18} cm^{-3} suggested in Ref. [29]. All the relevant material parameters are summarized in Table I for six high-T_c cuprates. The measured T_c^{Exp} and the effective masses m^* of Cooper pairs are taken from Ref. [32]. The calculated T_c values as a function of the scattering length $|a|$ are shown in Fig. 4.

We can find from Fig. 4 that the BEC temperature

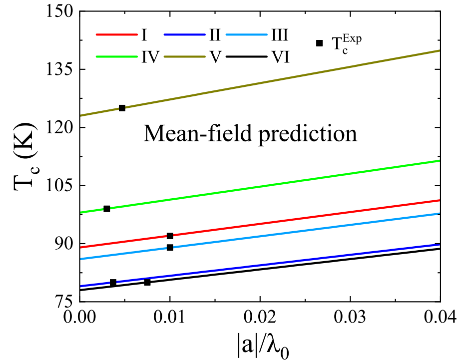

FIG. 4. The superconducting critical temperature T_c as a function of the scattering length $|a|$ (a <0) due to the indirect net attraction between two Cu-bridged (bridge-I) $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ hole Cooper pairs mediated by the O-bridge (bridge-II), derived from Eq. (3), by using the parameters summarized in Table I for cuprates I-VI. It indicates a new and clear way to enhance T_c, i.e., by improving the indirect attraction between two Cooper pairs with small effective mass m^* under the optimal concentration n_s. This would be extremely encouraging for maximizing T_c.

T_c of interacting Cooper pairs sensitively depends on the density n_s and effective mass m^* of Cooper pairs as well as the scattering length a. To obtain reliable T_c values, the precise values of these important parameters are of crucial importance, which is a significant challenge. Generally, the critical BEC temperature T_c^0 of ideal Cooper pairs is proportional to $n_s^{2/3}/m^*$ ($T_c^0 \propto n_s^{2/3}/m^*$). Therefore, increasing n_s appropriately and decreasing m^* are beneficial for improving T_c. Moreover, T_c linearly depends on the scattering length a of interacting Cooper pairs. The attractive interaction of $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) Cooper pairs mediated by the O-bridge (Cu-bridge), named as bridge-II, is crucial to enhance T_c. Conversely, the repulsive interaction of $\mathbf{h^+}$**-Cu-**$\mathbf{h^+}$ ($\mathbf{e^-}$**-O-**$\mathbf{e^-}$) Cooper pairs is harmful to the improvement of T_c, which will reduce T_c linearly.

As we previously analyzed, Cooper pairs can exist at room temperature in ionic oxide superconductors [15], which is highly desirable for room-temperature superconductivity. Figure 4 shows that if the net attraction between Cooper pairs can be appropriately enhanced and the material parameters can be cleverly designed (see Table I), the enhancement of T_c in oxide superconductors remains promising, though it is an extremely challenging task. The significance of this Letter lies in pointing out a new direction that requires further effort to increase T_c appropriately, i.e., the optimal Cooper-pair concentration n_s, the smallest possible effective mass m^* of Cooper pairs and an appropriately increased scattering length $|a|$ (a <0).