

Models for differential cross section in neutron-proton scattering and their implications

Muhammad Saad Ashraf¹ and Nosheen Akbar¹

¹*Department of Physics, COMSATS University Islamabad, Lahore Campus, Lahore, Pakistan.**

Abstract A few analytic exponential models of elastic differential cross section, constructed as purely phenomenological models, are proposed and tested. The models incorporate energy-dependent exponential slopes, power-law prefactors, and localized Gaussian modifications which are built to reproduce the observed dip region, supplemented in some cases by logarithmic t -dependent slopes. Simple additive sub-leading exponential contributions that represent charge conjugation and isospin roles are introduced in the models to increase applicability and quality of fit across elastic differential cross section data of np , $n\bar{p}$, pp , and $p\bar{p}$ elastic scattering. The models remain analytic, differentiable, and integrable in both s and t and reproduce the characteristic features of the elastic scattering data such as the dip-bump structure, shrinkage of the forward peak, and controlled curvature that is localized around the dip. Parameters of the models are found by fitting the experimental data of elastic np differential cross section from Argonne National Laboratory Zero Gradient Synchrotron (ZGS), CERN Proton Synchrotron (PS), BNL Alternating Gradient Synchrotron (AGS), and from Fermilab (FNAL) in an energy range of $\sqrt{s} = 3.36$ GeV to 26.02 GeV, across a momentum range of $0.065 \leq |t| \leq 5.341 \text{ GeV}^2$. The parameter values and their ranges, obtained by χ^2 minimization are found within their assumed expected bounds with the np data fitting. The total cross section σ_{tot} , the slope parameter B , the interaction radius R , the total elastic cross section σ_{el} , the inelastic cross section σ_{inel} , the ratios $\sigma_{\text{el}}/\sigma_{\text{tot}}$, and $\sigma_{\text{inel}}/\sigma_{\text{tot}}$ are predicted by the models for the np scattering at all the energies, which show accurate quantitative agreement with their reference values. The ranges of the predicted values are compared with their reference value ranges, and the least difference is identified. The results show that the proposed models not only provide accurate quantitative description of np elastic differential cross section but also yield estimates of the observables that are consistent with theoretical expectations from Regge phenomenology and high-energy scattering constraints. The findings of this study provide a basis for future investigations of the differential cross section in other hadronic scattering processes at various collider energies.

Keywords: Phenomenological models, Nucleon-nucleon (NN) interaction, Proton, Antiproton, Neutron, Antineutron, Elastic scattering, Differential cross section, Total elastic cross section, Total cross section, Inelastic cross section

I. INTRODUCTION

Elastic scattering of hadrons ($h_1 h_2 \rightarrow h_1 h_2$) is one of the most fundamental processes in particle physics. In an elastic collision process, the interacting particles emerge in the same internal states as they entered with their quantum numbers conserved, but no new particles are produced. Its simplicity surrounds rich physics involving the analysis of elastic differential cross sections $\frac{d\sigma}{dt}$ which is very important to investigate the spatial and dynamical structure of the interaction, the role of exchange trajectories (Pomerons, Reggeons, mesons), and the interference between hadronic and Coulomb contributions. Hadron-hadron elastic scattering is governed by strong interactions and should be explained by QCD. However, the understanding of the elastic processes is less developed due to absence of QCD description from first principles [1–3], especially in non-perturbative regime of QCD where the quark-gluon dynamics is involved [4]. In this regime, phenomenological models based on Regge theory and effective approaches are useful to study the experimental data of hadron-hadron elastic scattering [4, 5]. The physical quantity, elastic differential cross section, $\frac{d\sigma}{dt}$, is very critical to investigate the elastic scattering data, because it allows to probe the elastic scattering and estimate several important observables. Precise and consistent parameterizations for the differential cross section for elastic scattering processes enhance our understanding of hadron structure and the emergence of collective phenomena like gluon saturation [6]. These models of the differential cross section can be used to describe and reproduce the experimental data of elastic scattering from Large Hadron Collider (LHC), CERN-Intersecting Storage Rings (ISR), Tevatron at Fermilab, and other colliders [7, 8]. Such parameterizations also find applications in cosmic ray physics, hadronic Monte Carlo generators, and the development of theoretical frameworks such as Regge theory, and QCD-driven saturation approaches.

Neutron-proton (np) (or equivalently proton-neutron pn) elastic scattering is a less-explored but equally important frontier while much of high-energy elastic scattering effort has focused on proton-proton (pp) and proton-antiproton ($p\bar{p}$) collisions, because of collider availability. The existence of isospin symmetry [9] between proton and neutron allows to probe isospin dependence of the strong force by the elastic np scattering [10]. Important complementary constraints are offered by this dependence on nucleon-nucleon (NN) potentials [11] and allows to test the universality of Pomeron or Regge descriptions beyond purely proton-based systems [12]. Another important symmetry is of charge conjugation that exists between a particle and its antiparticle due to their opposite electric charges, and allows one to probe neutron-proton (np) and neutron-antiproton ($n\bar{p}$) systems, and also proton-proton (pp) and proton-antiproton ($p\bar{p}$) in the scattering phenomena [5, 10]. In

* msaadashraf99@gmail.com; nosheenakbar@cuilahore.edu.pk (corresponding author)

this study, important experimental measurements of the np elastic scattering that provide significant experimental data and the related theoretical contexts are discussed with an emphasis on the considerations of phenomenological modeling of the differential cross section.

A. Experimental and Theoretical Contexts

In the several decades in past, the fundamental features of hadronic interactions in both the diffraction and transition regimes of QCD have been investigated by conducting a series of high-precision measurements of np elastic scattering. Numerous earlier experiments [14–19] have measured neutron-proton (np) differential cross sections across a wide range of incident momenta. Small-angle diffraction peak and the backward (charge-exchange) region are focused on most of these investigations. Distinct aspects of the hadronic scattering amplitude are probed by these features. The measurements at Brookhaven National Laboratory [14] covered the momentum interval $5 \leq p_{\text{lab}} \leq 30 \text{ GeV}/c$ for $0.2 \lesssim -t \lesssim 1.2 (\text{GeV}/c)^2$. In these measurements the np elastic cross sections at small $|t|$ have been found which shows close resemblance with those of pp scattering. It indicates that at forward angles, the elastic cross sections are dominated by diffractive exchange due to Pomeron contributions.

Experiments at the CERN Proton Synchrotron [15] and IHEP Serpukhov [16] extend the np measurements up to $p_{\text{lab}} \approx 70 \text{ GeV}/c$ in a four-momentum transfer range of $0.2 < -t < 2.8 (\text{GeV}/c)^2$, as a subsequent effort to prior measurements. These measurements consist of important results in a region beyond the diffraction cone region. The only data available prior to these efforts can be found in [17]. This np elastic scattering data gives measurements at large angles above $1 \text{ GeV}/c$, where the range $2 < p_{\text{lab}} < 7 \text{ GeV}/c$ is explored. In these results a smooth behavior of $d\sigma/dt$ below $7 \text{ GeV}/c$ is indicated. And monotonic decrease in cross sections is found with increasing $|t|$.

In [18], high-statistics measurements of the np charge-exchange process are performed [18] using the Argonne Zero Gradient Synchrotron (ZGS). These measurements provide complementary information in the momentum interval $3 \leq p_{\text{lab}} \leq 12 \text{ GeV}/c$ in $1 \text{ GeV}/c$ bins. Nearly complete angular distributions for the elastic np scattering is provided in these results together with the large angle np elastic data [27]. Valuable constraints on spin-dependent components of the scattering amplitude are offered in this energy range.

Elastic pp scattering studies [20–22] are also important to be considered, in parallel to the np measurements. These pp studies cover momenta from 3 to $24 \text{ GeV}/c$ in which below $7 \text{ GeV}/c$, no distinct structure in $d\sigma/dt$ is revealed. This exhibits a smooth exponential decay with $|t|$. However, above $7 \text{ GeV}/c$, a shoulder is found to emerge near $|t| \approx 1.4 (\text{GeV}/c)^2$. This shoulder feature evolves into a pronounced diffraction minimum or “dip” observed in ISR and Fermilab measurements which are conducted at higher energies [23, 24]. Moreover, in the c.m. frame following $\sigma \propto s^{-n}$ for $s \gtrsim 8 \text{ GeV}^2$, a steepening of the logarithmic slope for $|t| < 0.2 (\text{GeV}/c)^2$ and a power-law dependence of the cross section at 90° is shown in analyses of [20, 21, 25, 26]. At $|t| \gtrsim 2.6 (\text{GeV}/c)^2$, an onset of “large angle scattering regime” is marked by this behavior. This regime suggests the gradual transition from diffraction dominated to perturbative QCD dynamics.

Early systematic studies performed at the Argonne National Laboratory Zero Gradient Synchrotron provide a comprehensive dataset of np differential cross sections in the intermediate energy range that covers incident neutron momenta from 4.5 to $12.5 \text{ GeV}/c$ ref. [27]. These measurements were based on approximately 470,000 recorded events and extend over a wide angular range up to 145° in the center-of-mass frame and correspond to $0.14 < -t \lesssim 19 (\text{GeV}/c)^2$. This allows one to make very detailed comparisons with corresponding pp data due to complete angular distributions. These results also compare well with theoretical predictions related to Regge phenomenology. It becomes crucial to conduct such comparisons for identifying effects related to charge symmetry breaking and to study the role of isovector exchanges in hadronic amplitudes. Enhanced statistical precision and in a broad range of the $|t|$ -range of $0.06 < |t| < 3 (\text{GeV}/c)^2$ provide measurements at high energies between 10 and $24 \text{ GeV}/c$ in a broad range of the $|t|$ -range of $0.06 < |t| < 3 (\text{GeV}/c)^2$ ref. [28]. These measurements allow further advancements in np elastic scattering. In this domain, the forward region is dominated by Pomeron exchange. This occurs due to the remarkable similarity of np elastic differential cross section to that of the pp system. A high degree of universality in hadronic diffraction phenomena is also an indication of such similarities. However, at intermediate $|t|$ subtle differences occur between pp and np elastic scattering. Residual contributions from C-odd (Odderon-like) and isospin-dependent exchanges result in such subtleties in the elastic nucleon-nucleon scattering [10, 29]. Experiments performed at the Brookhaven National Laboratory Alternating Gradient Synchrotron (AGS) cover broad energy range with incident neutron momenta from 5 to $30 \text{ GeV}/c$. These experiments achieved unprecedented angular and momentum resolution ref. [30], and demonstrate that a qualitative similarity exists between np and pp differential cross sections but are not identical. These results reinforced that a characteristic “shrinkage” with increasing incident momentum occurs in the diffraction peak in the np system. This result remains consistent with expectations related to Regge trajectory [5]. In these results no auxiliary oscillatory structure is observed which indicates that a smoother difference exists in transition for np as compared to pp elastic differential cross sections, when measured at comparable energies. An extension of these measurements occurs in incident neutron momenta from 70 to $400 \text{ GeV}/c$ and gives insights in elastic scattering behavior at high energies ref.[31]. The logarithmic slope parameter is found to be consistent with existing pp parameterizations, when evaluated near $-t = 0.2 (\text{GeV}/c)^2$, over the range $0.15 < -t < 3.6 (\text{GeV}/c)^2$. This demonstrates the persistence of shrinkage of the diffraction cone even at larger GeV domains. A distinct diffractive minimum (dip) is also found around $-t \approx 1.4 (\text{GeV}/c)^2$ for incident neutron momenta above $200 \text{ GeV}/c$. This dip structure gives an indication of the onset of interference between Pomeron and Odderon. However, it is reported in ref. [31] the data favors a Pomeron-Reggeon interfer-

ence term and the data are inadequate to distinguish clearly between Pomeron-Reggeon and Reggeon-Reggeon interference terms. Elastic np cross sections exceed the corresponding pp data in the intermediate $|t|$ region ($0.7 \lesssim |t| \lesssim 1.3$ (GeV/c)²) for momenta below approximately 280 GeV/c. This becomes challenging to most theoretical predictions that are based on Regge theory. In these studies, factors such as subtle interplay of charge conjugation, isospin dependence, and nonperturbative QCD effects play their role in defining the elastic scattering amplitude across a wide range of energies. These results are important to develop phenomenological models for investigations of the differential cross section.

A variety of theoretical and phenomenological frameworks that span the low-energy, diffraction, and high-energy regimes of hadronic physics exist that model elastic np scattering in close correlation with elastic pp scattering. The differential cross section for np elastic scattering, $d\sigma/dt$, is useful probe of the spatial and dynamical structure of strong interactions allows to understand non-perturbative and transitional QCD regimes [4]. The evolution of $d\sigma/dt$ across various energies and momentum ranges provides insights into mechanisms related to confinement and the onset of gluonic degrees of freedom. At low energies ($\sqrt{s} \lesssim 2.5$ GeV), scattering is dominated by meson-exchange dynamics and partial-wave interference (specifically S- and P-waves). This is demonstrated by partial-wave analyses, these exchanges lead to production of strong angular oscillations and near-threshold enhancements related to the deuteron bound state [32]. In an intermediate-energy region ($2.5 \lesssim \sqrt{s} \lesssim 6$ GeV), an exponentially decreasing behavior at small $|t|$ is observed along with a diffractive cone. This is parameterized as $d\sigma/dt(s,t) \simeq A(s)e^{B(s)t}$. Here, $B(s)$ is the forward slope parameter which is also related to the interaction radius. At intermediate $|t|$ ($0.5 \lesssim |t| \lesssim 1.5$ GeV²), a characteristic dip-bump structure arises from interference between even- and odd-under-crossing exchanges such as the Pomeron and Odderon [33, 34]. The dip-bump structure smooths out as energy increases. At high energies ($\sqrt{s} \gtrsim 6$ GeV), the elastic scattering is characterized by forward peak and exhibits a nearly exponential fall-off at small $|t|$. At large $|t|$, differential cross section exhibits a power-law tail that gives signals of partonic scattering which is governed by quark-gluon dynamics [35]. The systematic evolution of the diffraction pattern is characterized by an increasing slope $B(s)$ and diminishing oscillations. This provides crucial insight into the underlying phenomena in elastic scattering. Amplitude models based on Regge poles require one to construct the hadronic scattering amplitude in such a way that it can be written as a sum of components involving Regge exchanges i.e. Pomeron (P), secondary Reggeon (R), and Odderon (O). In such models a structured t -dependence is brought in the differential cross sections through form factors and signature factors, and explicit energy scaling $\sim s^{\alpha(t)}$ [4, 6, 37]. In holographic QCD and string-inspired models with Reggeized glueball and vector meson exchanges, a similar decomposition is implemented and recently applied to describe np or $n\bar{p}$ differential cross sections [12]. The amplitude is expressed in terms of Pomeron and Reggeon trajectories by early Regge-pole treatments and have underpinned many total and differential cross-section fits [5, 13, 38–40]. Asymptotic forms for total cross sections (σ_{tot}) and related constraints are obtained by several Regge fits that offer physical interests by linking s and t behaviors and allow to extrapolate to higher energies [6, 33, 34, 41]. Insight into the geometric origin of diffraction cones and quantitative descriptions of differential cross sections can be obtained in moderate $|t|$ [42] by impact-parameter (eikonal) models, such as the Bourrely-Soffer-Wu (BSW) model that has Fermi-Dirac-type opaqueness in b -space [42]. Regge-eikonal approach in a wide kinematic region describes elastic diffractive scattering of nucleons where the eikonal of pp scattering can be approximated by the sum of two exchange terms of Reggeon (soft Pomeron and f-Reggeon contributions) [43]. Nucleon-structure based models that aim to describe the nucleon structure in elastic scattering by providing insights on outer cloud or inner core of nucleons, often link form factors and spatial distributions of matter to large- $|t|$ tails [4, 44]. Models that describe the C-odd (odderon) exchange, explain subtle differences between particle-particle and particle-antiparticle elastic scattering and are relevant when comparing np , $n\bar{p}$, pp , and $p\bar{p}$ processes. Various recent parametrizations are used to describe total, elastic and diffractive cross sections across various energies that provide good description but involve many parameters that make the description less compact for several phenomenological applications [36]. This fact is also highlighted and discussed for elastic pp scattering in ref. [8] and it is important to correlate for the development of phenomenological models in elastic nucleon-nucleon scattering. To develop compact models with lesser parameters that can be used to effectively reproduce experimental data while providing an effective description for the underlying mechanisms of the elastic phenomena, a necessity of simple models emerges that can reflect the generalized behavior of scattering amplitudes. In this view, the simplest and generalized empirical description can be a single exponential form for the forward diffraction cone, and it is mostly applicable to extract the slope $B(s)$ and the forward value. This form is reliable for a local description at very small $|t|$ but the curvature or dip-bump structures cannot be reproduced and described by it. More complex forms, such as double or multi-exponential and interference models can be used to model the dip-bump structure ref. [7, 8]. These models have been used in pp phenomenology and can be adapted for np fits where dip behavior is suspected.

As a continuation of previous study [8], this study introduces a family of parameterizations as purely phenomenological models for the differential cross section that are constructed to fit elastic differential cross section data of four processes of elastic scattering (np , $n\bar{p}$, pp , $p\bar{p}$), and specifically utilizes them to fit the experimental data of the elastic np scattering. The methodical approach towards modeling the elastic differential cross section $\frac{d\sigma}{d|t|}$ is discussed in Section II. The results of the fitting of our models to all the data are written in Section III. The predicted results of the total, inelastic, and elastic cross sections, slope parameter, interaction radius, and other quantities by our models are also mentioned in this section. The χ^2 for the curve fitting are also written in Section III. Section IV provides the conclusion of this study. Additionally, Section V offers recommendations for future investigations in the highlighted directions of this study.

II. METHODOLOGY

In this study, five different models based on their ability to reproduce key features of the differential cross-section, including the forward peak, diffractive dip, and large- $|t|$ behavior, are constructed to fit four datasets of the experimental data of elastic neutron-proton (np) differential cross-section. The dataset 1 in this work are experimental measurements of the differential cross section made at the Argonne National Laboratory Zero Gradient Synchrotron for 8 different momentum ranges between 4.5 and 12.5 GeV/c, reported in ref. [27]. The dataset 2 comprises of measurements of the differential cross section for incident momenta between 10 and 24 GeV/c using wire spark chambers performed at the CERN Proton Synchrotron (PS) reported in ref. [28]. The dataset 3 consists of the differential cross section measurements in the diffraction region with an incident neutron momenta between 5 and 30 GeV/c from an optical-spark-chamber-counter experiment conducted at the Brookhaven National Laboratory alternating gradient synchrotron, reported in ref. [30]. The dataset 4 consists of the experimental measurements of the differential cross section from an experiment over a range in t from 0.15 to $\sim 3.6(\text{GeV}/c)^2$ for incident neutron momenta from 70 to 400 GeV/c at Fermilab energies, reported in ref. [31]. The available experimental data of the differential cross sections are taken in the $|t|$ ranges given in Table I at their corresponding center-of-mass (c.m.) energies, with a total of 937 datapoints. In these ranges, the data contains the forward peak, dip structure and most part of the large- $|t|$ tail.

TABLE I: Summary of the experimental data for the neutron-proton (np) elastic scattering with $|t|$ -ranges at each $\sqrt{s}$ value

Dataset 1 [27]			Dataset 2 [28]			Dataset 3 [30]			Dataset 4 [31]		
$\sqrt{s}$ GeV	$ t $ GeV ²	No. of points	$\sqrt{s}$ GeV	$ t $ GeV ²	No. of points	$\sqrt{s}$ GeV	$ t $ GeV ²	No. of points	$\sqrt{s}$ GeV	$ t $ GeV ²	No. of points
3.363	$0.134 \leq t \leq 5.281$	36	4.74	$0.065 \leq t \leq 1.68$	42	3.466-4.409	$0.174 \leq t \leq 0.934$	12	13.748	$0.152 \leq t \leq 2.093$	29
3.628	$0.134 \leq t \leq 5.341$	38	5.120	$0.065 \leq t \leq 2.48$	47	4.409-5.187	$0.175 \leq t \leq 1.476$	14	16.823	$0.152 \leq t \leq 3.404$	33
3.876	$0.134 \leq t \leq 4.355$	42	5.473	$0.065 \leq t \leq 2.80$	49	5.187-5.863	$0.175 \leq t \leq 1.515$	14	19.416	$0.152 \leq t \leq 3.401$	33
4.109	$0.134 \leq t \leq 5.207$	43	5.805	$0.065 \leq t \leq 2.80$	49	5.863-6.47	$0.223 \leq t \leq 1.638$	13	21.471	$0.151 \leq t \leq 3.606$	33
4.329	$0.13 \leq t \leq 5.113$	44	6.119	$0.065 \leq t \leq 2.80$	49	6.47-7.024	$0.227 \leq t \leq 1.185$	12	22.956	$0.151 \leq t \leq 3.328$	33
4.54	$0.147 \leq t \leq 5.214$	44	6.418	$0.065 \leq t \leq 2.8$	49	7.024-7.538	$0.325 \leq t \leq 0.881$	8	24.536	$0.151 \leq t \leq 3.479$	33
4.741	$0.175 \leq t \leq 5.283$	41	6.704	$0.37 \leq t \leq 2.80$	26				26.019	$0.175 \leq t \leq 3.662$	32
4.935	$0.175 \leq t \leq 5.208$	39									

A. Models for the Differential Cross Section

The models of elastic differential cross section, constructed from various generalized functional forms, reflect the approximated behavior of elastic scattering amplitude to reproduce the elastic differential cross section data. These models have significant pedigree in elastic hadron-hadron scattering phenomenology. The models at their core contain a diffractive cone $\exp[B(s)t]$ with a $B(s)$, that increases with $\ln s$. This structure reflects a generalized Regge-inspired picture of the Pomeron dominance and the shrinkage of the diffraction cone. This form is an essential constituent used in Regge fits and in several subsequent amplitude analyses [4, 5]. The double-exponential and two-scale mixtures used in the models 2 and 3, follow from the two two-component parametrizations that are introduced to fit curvature in the forward cone. And the phenomenological observation that a single slope is insufficient to describe both the forward region and intermediate $|t|$ curvature [7, 45]. The localized modifications of the diffractive envelope by multiplicative Gaussians of the forms, $[1 + \varepsilon[1 - \exp(-(t - t_d)^2/\Delta^2)]]$, and $[1 - \varepsilon \exp(-(t - t_d)^2/2\Delta^2)]$ are used in the models 1, 3, 4, and 5. These Gaussian forms descend from the models that write the differential cross section as a sum of exponential terms with relative phase in the amplitude [7]. A logarithmic t -dependence, $b_1 \ln(1 + t/t_0)t$, which is conceptually related to effective trajectory is used in the model 4 [46, 47]. Finally, the explicit C-odd term with a threshold suppression factor $(1 - e^{-\gamma|t|})$ is employed in the model 5. This factor is used to suppress C-odd signature contributions at $t = 0$, and allows them to contribute at finite $|t|$. In our model this factor follows from general symmetry arguments, and by the experimental finding that C-odd effects are small at the optical point and become relevant around the dip region [10, 48].

The models are constructed to fit hadron-hadron and hadron-antihadron elastic scattering data of four processes (np , $n\bar{p}$, pp , $p\bar{p}$) which can have dip-bump structure, steepening and moving of dip position with increasing center of mass energy and other features at a broad $|t|$. Most of these models can be considered in two major parts. The first part is a phenomenological construct to fit the forward peak, dip-bump structure, and dip (diffractive minimum) or a shoulder. And the second part consists of two additive exponential terms which enhances the fitting with the first part. In this study these models are fitted to np data of Table I. However, these proposed models can also fit well with other data of the $n\bar{p}$, pp , $p\bar{p}$ elastic scattering especially not only in the Regge or high energy limit ($s \gg t$) but also in the intermediate- $|t|$ region, where a dip structure is observed in both pp , and $p\bar{p}$ data at both ISR, and higher GeV energy domains of refs. [12, 48, 54]. These models can also give very good fit in the dip-bump structure that becomes very prominent in the recent pp data of [55–59] at LHC energies. For each model, non-linear fitting is performed with the experimental data of Table I by first fitting the forward peak (low- $|t|$) and dip region by the first part of the model and the fitting is further refined in the intermediate

to high- $|t|$ regions by the additive exponential terms. The fitting procedure relies on the most optimal set of parameter values in an appropriate range for each model across a c.m. energy for which the curve fitting can be performed with least χ^2 and the calculations can have most agreement with their reference values. The models are parameterized using simple parameterizations, to keep smaller number of parameters. This makes these models different from traditional Regge models [6, 60, 61] that often have many parameters. The models are described in the following:

1. Model 1

This model introduces a *Regge-inspired exponential structure* to fit diffraction cone and the dip-bump structure with additional correction terms. The model is developed within the following generalized form:

$$\frac{d\sigma(s,t)}{dt} = T_D(s,t) + T_O(s,t) + T_I(s,t). \quad (1)$$

It is built from three analytic functions, each built from products of energy-dependent power laws and convex exponentials in t . The function $T_D(s,t)$ is defined as,

$$T_D(s,t) = A_0 \left(\frac{s}{s_0} \right)^\alpha e^{B_{\text{eff}}(s)t} F(s,t), \quad B_{\text{eff}}(s) = B_0 + 2\beta \ln(s/s_0), \quad F(s,t) = 1 + \varepsilon(s) \left(1 - e^{-(t-t_d)^2/\Delta^2} \right),$$

where s is the squared center-of-mass energy, t is the four-momentum transfer, and the Regge parameter s_0 is conventionally taken as a fixed scale (typically 1 GeV² used to make the dimensionless ratio (s/s_0)) refs. [4, 5, 30]. The factor $\varepsilon(s)$ can have mild energy dependence and for simplicity it is taken as dimensionless ε_0 which allows one to consider $F(t)$ instead of $F(s,t)$. The factor $A_0(s/s_0)^\alpha$ represents the energy evolution of the differential cross-section. It is positive, continuous, and monotonically increasing (decreasing) in s depending on the sign of α . As a power law it is analytic on $(0, \infty)$ and exhibits slow asymptotic growth typical of Regge amplitudes refs. [4, 6, 37]. In other models, the exponent α corresponds to an effective Regge intercept difference between the leading even- C (Pomeron) and subleading trajectories. In the Regge formalism, the scattering amplitude behaves as $A(s,t) \propto s^{\alpha_P(t)}$, with $\alpha_P(t) \simeq \alpha_P(0) + \alpha'_P t$. Hence, the power-law scaling $(s/s_0)^\alpha$ ensures that the total and differential cross sections evolve consistently with observed high-energy behavior, where the Pomeron exchange dominates [4, 6, 37]. In this study, α is treated as a free parameter, determined through fitting the model with the data at all the energies of Table I. However, it can be parameterized according to specific physical conditions from Regge theory.

The parameterization for the effective slope parameter $B_{\text{eff}}(s)$ is inspired from refs. [4, 46, 47]. We treat β as a free parameter and its value is determined by data fitting. It quantifies the *shrinkage* of the diffraction cone with c.m. energy. This logarithmic dependence is motivated by the Regge slope relation, $B(s) = B_0 + 2\alpha'_P \ln(s/s_0)$, where $\alpha'_P \approx \beta$ corresponds to the slope of the Pomeron trajectory ref. [4]. Physically, this accounts for the experimentally observed broadening of the forward peak with rising energy which is an important feature of diffraction-dominated elastic scattering. The term $e^{(B_0 + 2\beta \ln(s/s_0))t}$ in the model describes the exponential fall of the differential cross section in the forward (small- $|t|$) region and represents the diffractive cone. Its slope has the property that for fixed s it is a convex function of t , decreasing for $t < 0$ (the physical region) and bounded between $(0, 1]$ as $t \rightarrow -\infty$. It is differentiable and increasing in $\ln s$ when $\beta > 0$, which mathematically guarantees a progressive narrowing of the t -distribution at higher energies. It is consistent with the observed energy-dependent steepening of np elastic scattering in the forward region. Since $t < 0$ in the physical region, the exponential is strictly decreasing in $|t|$ and satisfies: i) $0 < \exp[B_{\text{eff}}(s)t] \leq 1$, and ii) $\lim_{t \rightarrow -\infty} \exp[B_{\text{eff}}(s)t] = 0$, and ensures integrability at large $|t|$. The deformation factor $F(t)$ is an even function around $t = t_d$ with the properties (i) $\lim_{|t| \rightarrow \infty} F(t) = 1 + \varepsilon_0$, (ii) $F(t_d) = 1$, (iii) $0 < e^{-(t-t_d)^2/\Delta^2} \leq 1$. These properties imply boundedness as (i) $1 \leq F(t) \leq 1 + \varepsilon_0$ ($\varepsilon_0 \geq 0$), (ii) $1 + \varepsilon_0 \leq F(t) \leq 1$ ($\varepsilon_0 < 0$). Since $e^{-(t-t_d)^2/\Delta^2}$ is a Gaussian kernel, $F(t)$ is analytic and rapidly decreasing in its second derivative which localizes curvature around the dip region.

At $t = 0$ yields $T_D(s,0) = A_0(s/s_0)^\alpha F(0)$, since the exponential equals unity at $t = 0$. Thus the forward-scattering normalization depends solely on the power-law factor and on $F(0) = 1 + \varepsilon_0(1 - e^{-t_d^2/\Delta^2})$, which lies strictly between 1 and $1 + \varepsilon_0$. The partial derivative with respect to t at $t = 0$ gives the local slope in which $F'(t)$ is exponentially suppressed by the Gaussian. Since $e^{-t_d^2/\Delta^2} \ll 1$ for a physically realistic dip, the leading contribution to the forward slope is $B_{\text{eff}}(s)$ which ensures a logarithmic growth in s . This is consistent with the constraint of optical theorem that relates the forward amplitude to growth of total cross-section. Because $T_D(s,t) \sim \exp[B_{\text{eff}}(s)t]$ for $|t| \rightarrow \infty$, the integral $\int_{-\infty}^0 T_D(s,t) dt$ converges absolutely, with value proportional to $1/B_{\text{eff}}(s)$. Thus, the model predicts that the integrated diffractive contribution decreases in agreement with narrowing of the diffraction cone at higher energies.

The factor $[1 + \varepsilon_0(1 - e^{-(t-t_d)^2/\Delta^2})]$ modifies the simple exponential form to reproduce the dip-bump structure observed in intermediate- $|t|$ regions. Here, ε_0 determines the amplitude of the deviation from a purely exponential shape, t_d specifies the position of the dip, and Δ controls its width. This Gaussian modulation ensures a smooth and analytic deformation of the forward cone without introducing discontinuities. These are pragmatic devices to reproduce local deviations (a dip) and allow them to preserve a global exponential trend. They are practical when one wants a small number of parameters but to allow dip-like structure in the differential cross section. The factor is infinitely differentiable, bounded, and symmetric about

$t = t_d$. It interpolates monotonically between 1 (as $t \rightarrow \pm\infty$) and $1 + \varepsilon_0$ at $t = t_d$. The Gaussian core has a negative second derivative at $t = t_d$, which creates a localized concavity in the overall cross section, enabling a controlled dip or bump. The width parameter Δ regulates the curvature of the function through a characteristic scale for the second and higher derivatives. It ensures smooth structural modification rather than a discontinuous breakpoint. Since the full leading term is the product of three analytic functions in both s and t , it is itself analytic and free from nonphysical singularities. This smoothness is essential for fitting np scattering data, where the dip region is broad and requires a differentiable model to capture the gradual rather than abrupt departure from exponential behavior. Physically, it represents the transition between the coherent diffractive regime (forward peak) and the interference region (dip-bump) caused by the interference of real and imaginary parts of the scattering amplitude. The parameters of the last two additive terms in the model are defined with a minimal choice of s dependent parameterizations as follows.

$$T_O(s, t) = O_0 \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_{\text{eff}}(s)t}, \quad D_{\text{eff}}(s) = D_0(s/s_0)^\mu,$$

$$T_I(s, t) = I_0 \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_{\text{eff}}(s)t}, \quad E_{\text{eff}}(s) = E_0(s/s_0)^\mu.$$

These simple additive terms $O_0(s/s_0)^\alpha \xi_C e^{D_0(s/s_0)^\mu t}$ and $I_0(s/s_0)^\alpha \xi_I e^{E_0(s/s_0)^\mu t}$ represent the contributions from charge conjugation and isospin-dependent exchanges, respectively. The model is expressed as follows,

$$\frac{d\sigma(s, t)}{dt} = A_0 \left(\frac{s}{s_0} \right)^\alpha e^{(B_0 + 2\beta \ln(s/s_0))t} \left[1 + \varepsilon_0 (1 - e^{-(t-t_d)^2/\Delta^2}) \right] + O_0 \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_0(s/s_0)^\mu t} + I_0 \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_0(s/s_0)^\mu t}. \quad (2)$$

The $T_O(s, t)$ term is significant in distinguishing np and $n\bar{p}$, and the pp and $p\bar{p}$ scattering, as it contributes with opposite signs for each of the two cases. Similarly, the isospin term $T_I(s, t)$ accounts for the differences between isospin channels that remains quite relevant for np and $n\bar{p}$ scattering. Together, these two terms ensure that the model remains flexible enough to fit most features of the data of the four elastic processes (pp , $p\bar{p}$, np , and $n\bar{p}$). In this study, $\xi_C = +1$ and $\xi_I = -1$ values are fixed for this model and for the other models for fitting the np data of Table I. The parameters ξ_C and ξ_I represent complex phase signature factors that are significant key features of scattering amplitudes in Regge theory [4, 5, 10], and encode fundamental properties like crossing symmetry and C-parity. These factors would disappear when squared but in the cross section enter through interference and allow the model to simulate oscillatory or non-oscillatory interference patterns. Although the cross section itself is real and positive, these phases influence the curvature by modulating the strength and sign of interference with the leading term. In these terms, slopes are scaled as power laws in s , unlike the logarithmic behavior of the leading term. The slopes $D_{\text{eff}}(s)$ and $E_{\text{eff}}(s)$ have markedly different mathematical behavior from $B_{\text{eff}}(s)$ and follow power laws in s . These functions are convex, monotonic in t for $t < 0$, and analytic positive. And remain monotonic in s , and accept a controlled asymptotic behavior, when $\mu > 0$ they cause a strong energy-dependent steepening of the t -slope, while for $\mu < 0$ they flatten with increase in energy. Because these terms enter additively, they contribute linearly independent exponential curvatures in t and produce a multi-slope profile. This ensures that the combined model belongs to the class of sums of convex exponentials, which are known to approximate a wide variety of monotonic concave-upward shapes on $t < 0$.

The parameters, δ_O and δ_I are free parameters which are determined through fitting. The model inherits uniform boundedness for $t < 0$, monotonic decrease with respect to $|t|$ in the forward region, exponentially decaying tail, and well-controlled curvature that is tunable through $(B_0, D_0, E_0, \beta, \mu, \Delta)$. These properties allow the model to reproduce the characteristic exponential forward peak, the mid- t dip-bump structure, and the multi-slope behavior observed in np elastic scattering. Its analytic nature also ensures numerical stability and differentiability. These features are crucial both for fitting procedures, and for interpretation of the underlying amplitude structure. The fitted parameter values of this model with the data of Table I are written in Table III.

TABLE II: ξ_C and ξ_I parameters for all the models of this work

Process	ξ_C	ξ_I
$np \rightarrow np$	+1	-1
$n\bar{p} \rightarrow n\bar{p}$	-1	-1
$pp \rightarrow pp$	+1	+1
$p\bar{p} \rightarrow p\bar{p}$	-1	+1

2. Model 2

This model introduces a *Regge-inspired interference double-exponential structure* instead of Gaussian modification to reproduce the dip-bump structure and the other features. The model is developed within Eq. (1), and it is composed of three analytic functions. The leading component in this model is expressed as follows,

$$T_D(s, t) = A_0 \left(\frac{s}{s_0} \right)^\alpha \left[a_1 e^{B_1(s)t} + a_2 e^{B_2(s)t} \right], \quad B_{1,2}(s) = B_{1,2}^{(0)} + 2\beta \ln(s/s_0),$$

It is a linear combination of two exponentials, each analytic and convex in t . Because the slopes $B_{1,2}(s)$ increase monotonically with $\ln s$ for $\beta > 0$, the exponentials become progressively narrower in t as s increases that leads the full diffractive term to shrink in width while rising in forward height. For $t < 0$ both exponentials satisfy $0 < e^{B_{1,2}(s)t} \leq 1$ and vanish as $t \rightarrow -\infty$ and ensure absolute integrability. The coefficients a_1 and a_2 are real, satisfy $a_1 > a_2 > 0$, and control the relative strength of the two slopes. Since the sum of entire functions is entire, $T_D(s, t)$ is infinitely differentiable and holomorphic in both variables. The difference between the slopes $B_1(s)$ and $B_2(s)$ introduces curvature into $\ln(d\sigma/dt)$ as a function of t . Because the logarithmic derivative is the smooth interpolating expression which shifts from the smaller slope at small $|t|$ to the larger slope at larger $|t|$, convexity and prevention of artificial oscillatory behaviour is ensured.

At $t = 0$, the diffractive term reduces smoothly to $T_D(s, 0) = A_0 (s/s_0)^\alpha (a_1 + a_2)$, and the forward slope becomes the convex combination $(a_1 B_1(s) + a_2 B_2(s))/(a_1 + a_2)$ that ensures its logarithmic growth with $\ln s$. Since the second derivative $\partial_t^2 \ln T_D$ is positive unless $B_1 = B_2$. Therefore, the model guarantees full convexity of the diffractive sector across all t . The integrated diffractive strength follows directly from exponential integrability, $\int_{-\infty}^0 T_D(s, t) dt = A_0 \left(\frac{s}{s_0} \right)^\alpha \left(\frac{a_1}{B_1(s)} + \frac{a_2}{B_2(s)} \right)$, which decreases with energy due to the growth of the slopes $B_{1,2}(s)$ that represents the narrowing of the diffraction cone.

The subleading terms of $T_O(s, t)$ and $T_I(s, t)$ are same as in model 1, and involve slopes $D_{\text{eff}}(s)$ and $E_{\text{eff}}(s)$, which grow as power laws in s . The model is expressed as follows

$$\frac{d\sigma(s, t)}{dt} = A_0 \left(\frac{s}{s_0} \right)^\alpha \left[a_1 e^{(B_1^{(0)} + 2\beta \ln(s/s_0))t} + a_2 e^{(B_2^{(0)} + 2\beta \ln(s/s_0))t} \right] + O_O \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_O(s/s_0)\mu t} + I_O \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_O(s/s_0)\mu t}. \quad (3)$$

For $\mu > 0$, the exponentials $e^{D_{\text{eff}}(s)t}$ and $e^{E_{\text{eff}}(s)t}$ decay much more rapidly as $t \rightarrow -\infty$ than the diffractive components. It makes these terms sharply localized near the forward region. Their convexity is correspondingly stronger, since their curvature scales as $D_{\text{eff}}^2(s) e^{D_{\text{eff}}(s)t}$ (and analogously for E_{eff}). At the forward point, they reduce to the simple forms, $T_O(s, 0) = O_O (s/s_0)^{\delta_O} \xi_C$, and $T_I(s, 0) = I_O (s/s_0)^{\delta_I} \xi_I$, and their slopes at $t = 0$ equal exactly $D_{\text{eff}}(s)$ and $E_{\text{eff}}(s)$, respectively. This rapid energy dependence ensures that non-diffractive contributions are significant only in a restricted forward interval which is an essential for modelling np scattering behavior where isovector exchanges are enhanced at small $|t|$.

Global integrability of the full model follows from the fact that each contribution decays at least as $e^{c(s)t}$ with $c(s) > 0$, so $\int_{-\infty}^0 \frac{d\sigma}{dt}(s, t) dt < \infty$. Since the asymptotic decay is governed by the smallest slope, the diffractive term dominates the behaviour of the integrated cross section, which scales essentially as $(a_1/B_1(s) + a_2/B_2(s))$, with subleading corrections suppressed by powers of $s^{-\mu}$. Every term in the model is an entire function of t and s and ensures analyticity and differentiability in the physical domain. Convexity in t is ensured for each exponential, and all curvature variations originate from the interplay of different slopes in the diffractive sector. It also gives the model the flexibility to reproduce the characteristic curvature of np elastic scattering without introducing singularities or unphysical oscillatory structure.

Physically, the first exponential ($a_1 e^{B_1 t}$) dominates the small- $|t|$ region. It describes the steep forward peak, while the second exponential ($a_2 e^{B_2 t}$) contributes significantly at larger $|t|$ and reproduces curvature and interference effects. The additive terms ($C_O(s) \xi_C e^{D(s)t} + C_I(s) \xi_I e^{E(s)t}$) are parameterized as in the model 1 for the same roles. The double-exponential interference naturally reproduces the observed dip-bump structure. Both exponentials share the same shrinkage parameter β which ensures a physically consistent energy evolution. The model remains analytic and well-behaved in both t and s , while physically it reflects the coherent superposition of two diffractive components with differing transverse scales. The simplicity and flexibility of this parameterization make it highly suitable for fitting broad datasets at broad energy ranges. The fitted parameter values of this model are written in Table IV.

3. Model 3

This model introduces a *Regge-inspired triple-component cone structure* with smooth transition with additive corrections. The model is developed within the Eq. (1) with the leading component $T_D(s, t)$, defined by,

$$T_D(s, t) = A_0 \left(\frac{s}{s_0} \right)^\alpha \left[f e^{B_1(s)t} + (1-f) e^{B_2(s)t} \right] F(t), \quad B_{1,2}(s) = B_{1,2}^{(0)} + 2\beta \ln(s/s_0), \quad F(t) = 1 + \varepsilon_0 \left(1 - e^{-(t-t_d)^2/\Delta^2} \right),$$

with the subleading contributions $T_O(s, t)$, and $T_I(s, t)$ as in the previous models. The model is expressed as follows

$$\frac{d\sigma(s, t)}{dt} = A_0 \left(\frac{s}{s_0} \right)^\alpha \left[f e^{(B_1^{(0)} + 2\beta \ln(s/s_0))t} + (1-f) e^{(B_2^{(0)} + 2\beta \ln(s/s_0))t} \right] \left[1 - \varepsilon_0 e^{-\frac{(t-t_d)^2}{2\Delta^2}} \right] + O_O \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_O(s/s_0)\mu t} + I_0 \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_0(s/s_0)\mu t}. \quad (4)$$

The model is built from products and sums of power laws in s , exponential functions and a Gaussian in t , so every term is an entire function of t (and smooth in $\ln s$). Consequently, the full $d\sigma/dt$ is in the physical domain ($s > 0, t \in \mathbb{R}$). The diffractive part is a convex mixture of two energy-dependent exponentials. For $t < 0$ and $B(s) > 0$ each exponential is positive, monotonically decreasing in $|t|$ and vanishes as $t \rightarrow -\infty$ due to which the diffractive part inherits positivity, monotonic decay and exponential integrability in t . The parameter $f \in [0, 1.5]$ controls the relative weight of the two scales. It also makes the local logarithmic slope $\partial_t \ln T_D(s, t)$ a smooth and continuous in t and in $\ln s$ and interpolates between $B_1(s)$ and $B_2(s)$ as $|t|$ grows. This interpolation produces a controlled two-scale curvature and ensures $\partial_t^2 \ln T_D(s, t) > 0$ unless $B_1 = B_2$, i.e. the diffractive contribution is globally log-convex and free of factitious oscillations.

In this model, the multiplicative Gaussian factor $F(t) = 1 - \varepsilon_0 e^{-(t-t_d)^2/(2\Delta^2)}$ is entire, bounded and tends to unity for $|t| \rightarrow \infty$. So, it implements a localized modulation (a smooth suppression when $\varepsilon_0 > 0$) of the diffractive envelope near $t = t_d$ without altering the exponential asymptotics at large $|t|$. Because the Gaussian and its derivatives decay super-polynomially away from t_d , the Gaussian modifies higher derivatives of the cross section only inside a region of width. And contributes finite, explicitly computable corrections to forward quantities. Particularly the forward value, and the forward slope receives the logarithmic growth from $2\beta \ln(s/s_0)$ via the convex combination of B_1 and B_2 . The forward slope is corrected by a bounded term coming from the Gaussian derivative which is suppressed when $e^{-t_d^2/(2\Delta^2)}$ is small. Thus, the Gaussian can produce a dip-like reduction or shoulder without destroying the forward peak or the principal energy dependence of the slope, provided $|\varepsilon_0| < 1$ so the multiplicative factor remains positive.

The subleading $T_O(s, t)$ and $T_I(s, t)$ components are likewise entire and convex in t for positive slopes. Because $D_{\text{eff}}(s)$ and $E_{\text{eff}}(s)$ scale as power laws in s , their slopes can increase faster with energy than the logarithmic $B_{1,2}(s)$. It makes these exponentials decay more strongly as $t \rightarrow -\infty$ at high s . This renders T_O and T_I increasingly localized near the forward region as energy grows, and their contributions to the integrated cross section are correspondingly suppressed at large s when δ_O, δ_I are not too large. At $t = 0$ they yield finite contributions proportional to $(s/s_0)^{\delta_O}$ and $(s/s_0)^{\delta_I}$ respectively, and their local slopes equal $D_{\text{eff}}(s)$ and $E_{\text{eff}}(s)$.

Global integrability is ensured because each term decays at least as $e^{c(s)t}$ with $c(s) > 0$, explicitly the diffractive t -integral equals $A_0(s/s_0)^\alpha (f/B_1(s) + (1-f)/B_2(s))$ plus finite Gaussian-corrected integrals of the form $\int_{-\infty}^0 e^{B_{1,2}t} e^{-(t-t_d)^2/(2\Delta^2)} dt$, and the T_O and T_I integrals converge even faster when $\mu > 0$. From the viewpoint of convexity and curvature, every pure exponential contributes a positive second derivative in t , and any inflection or local concavity arises exclusively from the Gaussian multiplicative factor. This provides a mathematically controlled mechanism for dip formation and meanwhile preserves overall log-convexity and avoids factitious oscillatory structure. Finally, for the model to remain physically and mathematically admissible one requires nonnegative slope functions $B_{1,2}(s), D_{\text{eff}}(s), E_{\text{eff}}(s)$ and $|\varepsilon| < 1$. If any slope becomes negative the monotone decay is lost, integrals diverge exponentially and the functional interpretation as a physically meaningful elastic cross section breaks down, although analyticity as an entire function of t is maintained. The fitted parameter values of this model are written in Table V.

4. Model 4

In this model, a *modified Regge-inspired core structure* with *log-enhanced slope* and saturating corrections are introduced. It is a modified form of the model 3. It is developed within Eq. (1) by taking $T_D(s, t)$ as follows:

$$T_D(s, t) = A_0 \left(\frac{s}{s_0} \right)^\alpha e^{B_{\text{eff}}(s, t)t} F(t), \quad B_{\text{eff}}(s, t) = B_0 + 2\beta \ln(s/s_0) + b_1 \ln \left(1 + \frac{t}{t_0} \right), \quad F(t) = 1 - \varepsilon_0 e^{-(t-t_d)^2/2\Delta^2},$$

with the subleading contributions $T_O(s, t)$ and $T_I(s, t)$ defined in the previous models. The model is expressed as follows

$$\frac{d\sigma(s, t)}{dt} = A_0 \left(\frac{s}{s_0} \right)^\alpha \exp \left[(B_0 + 2\beta \ln(s/s_0) + b_1 \ln(1 + t/t_0))t \right] \left[1 - \varepsilon_0 e^{-(t-t_d)^2/(2\Delta^2)} \right] + O_O \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_O(s/s_0)\mu t} + I_0 \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_0(s/s_0)\mu t}. \quad (5)$$

The model possesses several notable mathematical properties arising from the interplay between the exponential baseline and the logarithmic slope modification. The additional term in the B_{eff} , $b_1 \ln(1 + |t|/t_0)$ introduces a slow, saturating variation of the slope with t . It reflects the empirical observation that the diffraction cone curvature cannot be represented by a pure exponential function at larger $|t|$. This modification can be interpreted as a *logarithmic saturation effect* to account for

the curvature of the Regge trajectory or the onset of partonic saturation at moderate momentum transfers. Mathematically, this allows the exponential slope to vary smoothly with $|t|$ while preserving analyticity and stability of the fit. The term $b_1 t \ln(1+t/t_0)$, which introduces a branch point at $t = -t_0$, is analytic and infinitely differentiable only on domains excluding this point. However, for physical scattering where $t \leq 0$, it remains analytic and real-valued as long as the data range lies above $-t_0$. For $|t| < t_0$, the expansion $\ln(1+t/t_0) = t/t_0 - t^2/(2t_0^2) + \dots$ implies that the logarithmic correction contributes no linear term in t , so the forward slope remains $B_0 + 2\beta \ln(s/s_0)$, while the first nonvanishing contribution arises at order t^2 . This contribution modifies curvature through a positive or negative term proportional to $2b_1/t_0$. Thus, the parameter b_1 controls the second derivative of $\ln(d\sigma/dt)$ at the origin without altering its forward normalization or leading exponential falloff. The multiplicative dip-modifying factor $1 - \epsilon_0 e^{-(t-t_d)^2/(2\Delta^2)}$ is smooth, bounded between $1 - \epsilon_0$ and 1. For large negative t , the behavior of the logarithmic term becomes essential. Since $\ln(1+t/t_0) \sim \ln|t| + i\pi$ as $t \rightarrow -\infty$, the real part of the exponent behaves like $b_1 t \ln|t|$ that ensures super-exponential decay if $b_1 > 0$ and therefore even stronger integrability than the pure exponential case [46, 47]. If $b_1 < 0$, the term grows positively and may overpower the linear decay unless the coefficient $B_0 + 2\beta \ln(s/s_0)$ is sufficiently large, so physical consistency typically requires $b_1 \geq 0$. Both $T_O(s, t)$ and $T_I(s, t)$ interference terms preserve pure exponential structure in t and inherit monotonicity, convexity, and full integrability provided their slopes remain positive. Altogether the model remains infinitely differentiable, positive, and integrable on physical domains under mild parameter constraints. Its detailed curvature structure near and beyond the diffraction cone stems entirely from the logarithmic slope correction. While forward normalization and leading slope retain the stable and physically interpretable exponential behavior which is characteristic of diffractive scattering. The fitted parameter values of this model in Table VI.

5. Model 5

In this model, a *modified Regge-type exponential core structure* with a *threshold suppression* are introduced. This model is developed within Eq. (1) with the following parameterizations.

$$T_D(s, t) = A_0 \left(\frac{s}{s_0} \right)^\alpha e^{B_{\text{eff}}(s, t)t} F(t), \quad B_{\text{eff}}(s, t) = B_0 + 2\beta \ln(s/s_0), \quad F(t) = 1 - \epsilon_0 e^{-(t-t_d)^2/2\Delta^2},$$

$$T_O(s, t) = O_0 \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_{\text{eff}}(s)t} (1 - e^{-\gamma|t|}), \quad D_{\text{eff}}(s) = D_0(s/s_0)^\mu.$$

The term representing the isospin-dependent contributions, $T_I(s, t)$, is same as for the previous models. The model is expressed as follows:

$$\frac{d\sigma(s, t)}{dt} = A_0 \left(\frac{s}{s_0} \right)^\alpha e^{(B_0 + 2\beta \ln(s/s_0))t} \left[1 - \epsilon_0 e^{-\frac{(t-t_d)^2}{2\Delta^2}} \right] + O_0 \left(\frac{s}{s_0} \right)^{\delta_O} \xi_C e^{D_0(s/s_0)^\mu t} (1 - e^{-\gamma|t|}) + I_0 \left(\frac{s}{s_0} \right)^{\delta_I} \xi_I e^{E_0(s/s_0)^\mu t}. \quad (6)$$

The model is constructed from combinations of entire exponentials in t , a localized Gaussian deformation, and a short-range suppression factor $(1 - e^{-\gamma|t|})$. Each building block has precise mathematical properties that determine global behaviour, local structure, integrals, and numerical stability in fits. The model is analytic and smooth. For fixed $s > 0$ every term is real analytic for real t (infinitely differentiable) because exponentials and Gaussians are entire. The absolute value in $|t|$ is nonanalytic at $t = 0$, but the factor smooth away from $t = 0$ and has a well-defined one-sided derivative at 0. Expansion about $t = 0$ gives $1 - e^{-\gamma|t|} = \gamma|t| - \frac{\gamma^2}{2}t^2 + O(|t|^3)$, so while the derivative is continuous from each side, higher derivatives are not analytic at the origin because of the cusp in $|t|$. In practice this mild nonanalyticity is physically desirable as it enforces vanishing of the odd-signature term at $t = 0$. And also preserves differentiability which is sufficient for numerical fits. Everywhere else the model is differentiable in t and smooth in $\ln s$ because slopes are smooth functions of $\ln s$.

Behavior at the optical point $t = 0$ is controlled by the forward limits of the components. The term, $\xi_C O_0(s/s_0)^{\delta_O} e^{D_0(s/s_0)^\mu t} (1 - e^{-\gamma|t|})$ is modulated by a *threshold suppression factor* $(1 - e^{-\gamma|t|})$. The suppression factor provides two major benefits: 1) It can enforce a physically motivated threshold behavior where Odderon contributions vanish smoothly at $t = 0$ and produces a *forward limit* for applications of optical theorem. 2) It can separate central (Pomeron-dominated) and peripheral (C-odd and isospin-dependent) effects, allowing the model to represent differences among pp , $p\bar{p}$, pn , and $\bar{p}n$ scattering within one form. When $t \rightarrow 0$, the suppression factor $(1 - e^{-\gamma|t|}) \rightarrow 0$ which eliminates the contribution to the forward cross section.

Each pure exponential e^{Ct} has $\partial_t^2 \ln e^{Ct} = 0$ while the diffractive prefactor yields nontrivial curvature through the Gaussian multiplicative term, explicitly $\partial_t^2 \ln(e^{B(s)t} F(t)) = \partial_t^2 \ln F(t)$ because the exponential contributes only a constant first derivative, so the second derivative of Gaussian controls sign changes in curvature and is responsible for creating a local concavity (dip) near $t \approx t_d$ whenever ϵ_0 is sufficiently large. The exponentials in $T_O(s, t)$ and $T_I(s, t)$ contribute convex positive curvature in $|t|$ except that the $|t|$ -dependence in $(1 - e^{-\gamma|t|})$ can introduce a subleading inflection close to the origin. Overall multiple curvature scales are present in the model, and their interplay can reproduce the observed dip-bump structure of the data. For fixed s and $t \rightarrow -\infty$ the dominant suppression is exponential because the leading exponent behaves

as $\exp((B_0 + 2\beta \ln(s/s_0))t)$ with positive slope $B_0 + 2\beta \ln(s/s_0) > 0$ in physically allowed fits. The Gaussian deformation tends to one exponentially fast and thus does not change the leading tail. The $T_O(s, t)$ and $T_I(s, t)$ exponentials have slopes $D_O(s/s_0)^\mu$ and $E_O(s/s_0)^\mu$. If $\mu > 0$ these slopes typically increase with energy, making those components decay even faster at large $|t|$ as energy grows. Thus, the model guarantees absolute integrability $\int_{-\infty}^0 (d\sigma/dt) dt < \infty$ provided all effective slopes remain positive. If any slope were negative the corresponding term would diverge as $t \rightarrow -\infty$ and disfigure integrability. The multiplicative factors $(s/s_0)^\alpha$ and $(s/s_0)^{\delta_{O,I}}$ set power-law scaling of each term. The logarithmic slope growth $2\beta \ln(s/s_0)$ produces Regge-type shrinkage of the diffractive cone. The scaling in the $T_O(s, t)$ and $T_I(s, t)$ slopes as $(s/s_0)^\mu$ alter the relative importance of subleading terms with energy. For $\mu > 0$ these terms become more forward peaked and less relevant at larger $|t|$ as s increases. The fitted parameter values of this model are written in Table VII.

We employ these proposed models to calculate the total, elastic and inelastic cross sections, discussed in Section III. Other observables such as the logarithmic slope parameter and the interaction radius in np elastic scattering are predicted using these models. Chi-square is calculated to determine the accuracy of the models. The predicted values of these quantities are compared with their corresponding reference values and their estimated ranges are compared with their reference values.

III. RESULTS AND DISCUSSION

In this study, we performed a systematic fitting of elastic differential cross-section data for np scattering of Table I, using five distinct composite exponential models. The results of the obtained fits for model 1, model 2, model 3, model 4, and model 5 are shown in Figures 1, 2, 3, 4, and 5, respectively. Here the results of the data fitting and other predicted quantities from the models are discussed.

The differential cross section data for np elastic scattering measured in the four energy domains, fitted by the models, probes distinct dynamical regimes of the nucleon interaction (mentioned in Sections I and II) and thereby can be used to study complementary aspects of structure of proton and neutron [27, 28, 30, 31]. For the dataset 1, quite good fitting is obtained by all the five models. The dip structure is not sharply visible in the graphical representation of this data. However, the forward peak which is less pronounced exhibits shrinkage as the slope also increases from 3.363 GeV to 4.935 GeV. The low to high $|t|$ -regions are well fitted by the models. At the lowest set of c.m. energies the data retain substantial partial-wave character that exhibits strong angular structure at moderate $|t|$ and sensitivity to spin-isospin components which indicate that meson-exchange (ω , ρ and ϵ), and the resonance contributions still shape the profile of the differential cross section [64]. And the forward cone is relatively broad (smaller B) that reflects a compact interaction region of order ~ 1 fm. This is also expected from nucleon form factors and potential models [4, 5, 10]. In the dataset 2, the diffractive cone becomes more pronounced at small $|t|$ while a nascent dip structure appears at moderate $|t|$. These features are well reproduced by the models in their data fitting. The empirical growth of $B(s)$ with $\ln(s)$ that is also known as Regge “shrinkage” indicates an increasing transverse interaction radius and gradual dominance of soft Pomeron exchange over secondary Reggeons [33, 35, 69]. The dataset 3 is fitted well by the models. The plots of the fitting by the models predict the elastic differential cross section in the regions beyond the highest $|t|$ -values of the data where the systematic evolution of the diffraction cone parameters and the dip morphology can be observed. This also shows the shrinkage of the diffraction peak with increasing slope B as the c.m. energy increases from 3.466-4.409 GeV to 7.024-7.538 GeV. In this data no auxiliary oscillatory structure is observed and reported. However, the fits by our models show that diffraction peak narrows, and show the Regge shrinkage. And its height changes according to the total-cross-section normalization [30]. The dip structure also becomes more pronounced with increasing c.m. energy that is also predicted by our models. The models show the shift of the diffractive minimum towards smaller $|t|$ -values with increasing $\sqrt{s}$. This behavior remains consistent with Regge-trajectory expectations and geometric interpretations of the impact-parameter profile in eikonal frameworks in their graphical representation of the differential cross section results [4, 5, 42]. The dataset 4 is also well fitted by the models. Diffractive minimum (dip) is observed at $1.4 (\text{GeV}/c)^2$. The dip structure resembles the dip of elastic pp scattering at ISR energies [67], and it is not as pronounced to become a dip-bump structure observed at LHC energies as in elastic pp scattering [59]. The dip structure is less prominent at 13.748 GeV and 16.823 GeV and appears at 19.416 GeV. From 19.416 GeV the dip becomes sharper and narrower up to 26.019 GeV. From 13.748 GeV to 26.019 GeV the slope increases, and the shrinkage of the forward peak is also produced. The position of the dip slightly shifts towards lower $|t|$ -values with increase in c.m. energies. In this data the forward cone is sharply defined and Pomeron dynamics that largely controls the small- $|t|$ behavior. The dip becomes clearer (when present) and its depth and exact $|t|$ location reflect the interference of C-even and possible C-odd contributions (Odderon or 3-gluon exchange) as well as changing absorptive corrections in the eikonal unitarization. These contributions are significantly investigated in the pp and $p\bar{p}$ data [42, 49, 77], and can be important for spin-related nucleon-nucleon investigations of the np data [50]. The large- $|t|$ in the data, across all energies, encodes information on short-range components and nucleon substructure which is related to hard scattering or falloff of nucleon form-factors [4, 78, 79].

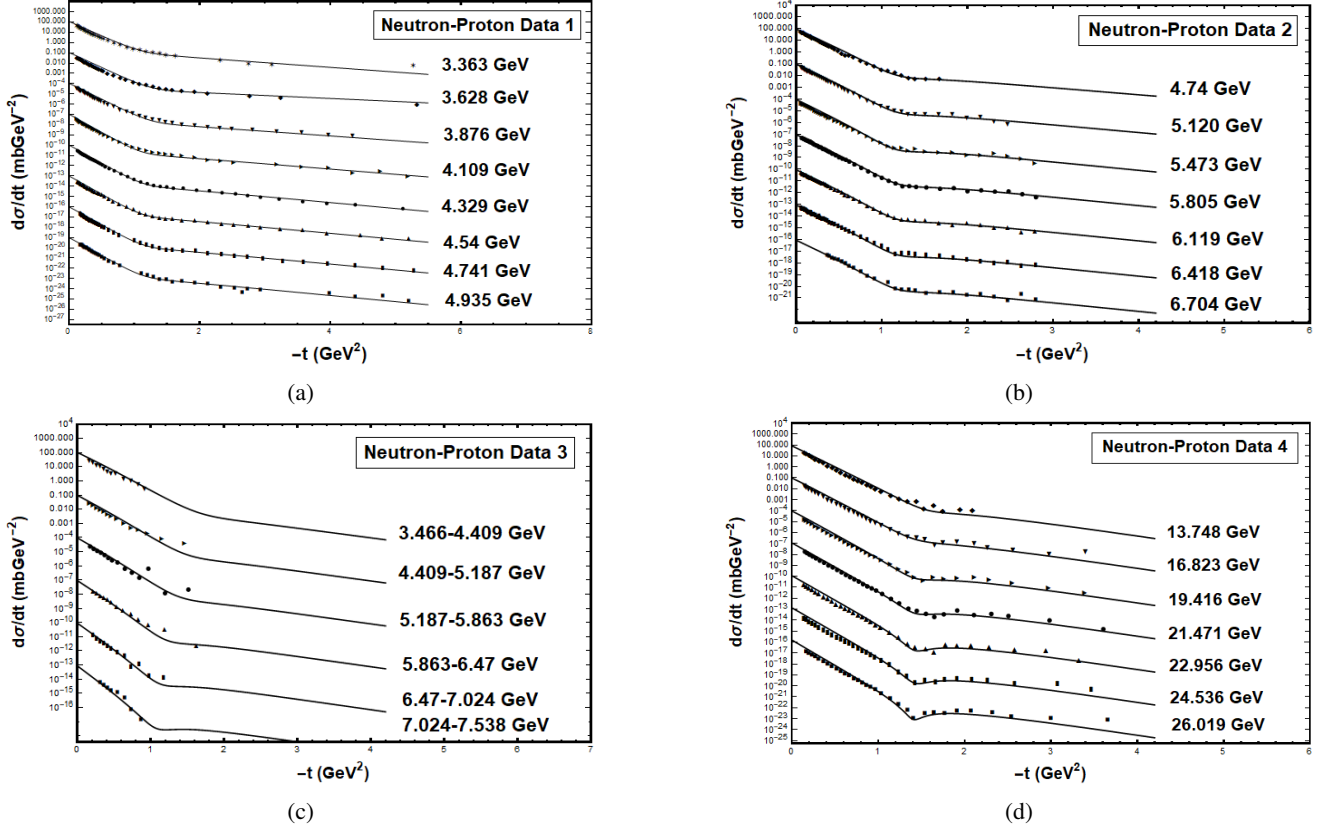


FIG. 1: (a) Model 1 fits on the np elastic differential cross section data of Dataset 1 ($3.363 \leq \sqrt{s} \leq 4.935 \text{ GeV}$) of Table I. Filled squares, filled rectangles, filled up triangles, filled circles, filled right triangles, filled down triangles, filled diamonds, and filled six-sided stars represent the experimental data of Dataset of I at 4.935, 4.741, 4.54, 4.329, 4.109, 3.876, 3.628, and 3.363 GeV respectively. The data and model values are multiplied by $10^{-3(n-1)}$, where n is the number of curve and corresponding data set starting from the top. The solid line represents the fit of our model to the data. (b) Model 1 fits on the np elastic differential cross section data of Dataset 2 ($4.74 \leq \sqrt{s} \leq 6.704 \text{ GeV}$) of Table I. Filled squares, filled rectangles, filled up triangles, filled circles, filled right triangles, filled down triangles, and filled diamonds represent the experimental data of Dataset of Table I at 6.704, 6.418, 6.119, 5.805, 5.473, 5.12, and 4.74 GeV, respectively. The data and model values are multiplied by $10^{-3(n-1)}$, where n is the number of curve and corresponding data set starting from the top. The solid line represents the fit of our model to the data. (c) Model 1 fits on the np elastic differential cross section data of Dataset 3 ($3.466 - 4.409 \leq \sqrt{s} \leq 7.024 - 7.538 \text{ GeV}$) of Table I. Filled squares, filled rectangles, filled up triangles, filled circles, filled right triangles, and filled down triangles, represent the experimental data of Dataset of Table I at 7.024-7.538, 6.47-7.024, 5.863-6.47, 5.187-5.863, 4.409-5.187, and 3.466-4.409 GeV, respectively. Data and model values are multiplied by $10^{-3(n-1)}$, where n is the number of curve and corresponding data set starting from the top. The solid line represents the fit of our model to the data. (d) Model 1 fits on the np elastic differential cross section data of Dataset 4 ($13.748 \leq \sqrt{s} \leq 26.019 \text{ GeV}$) of Table I. Filled squares, filled rectangles, filled up triangles, filled circles, filled right triangles, filled down triangles, and filled diamonds represent the experimental data of Dataset of Table I at 26.019, 24.536, 22.956, 21.471, 19.416, 16.823, and 13.748 GeV, respectively. Data and model values are multiplied by $10^{-3(n-1)}$, where n is the number of curve and corresponding data set starting from the top. The solid line represents the fit of our model to the data.

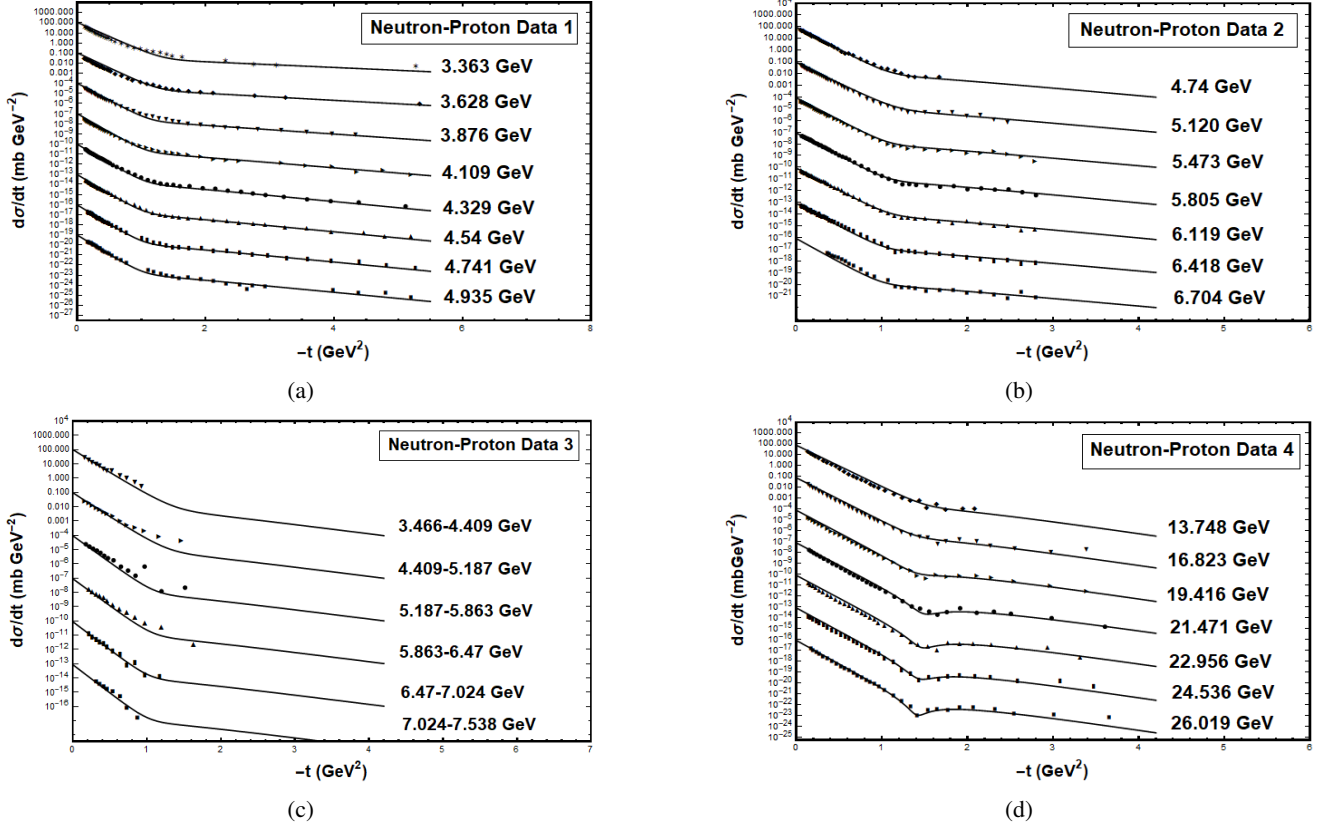


FIG. 2: Model 2 fits on the np elastic differential cross section data of Dataset 1 of Table I. Same legend for parts (a), (b), (c), and (d) as in the Figure 1.

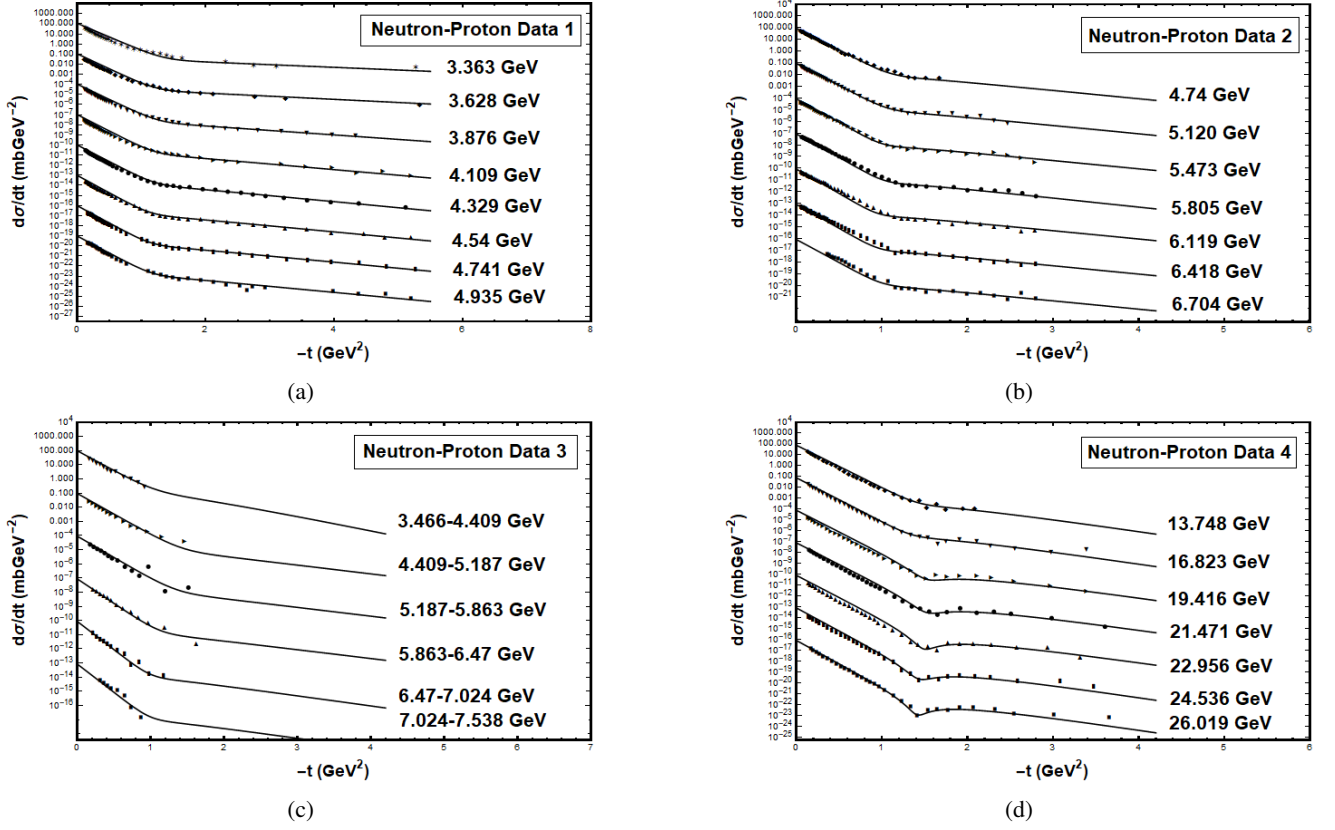


FIG. 3: Model 3 fits on the np elastic differential cross section data of Dataset 1 of Table I. Same legend for parts (a), (b), (c), and (d) as in the Figure 1.

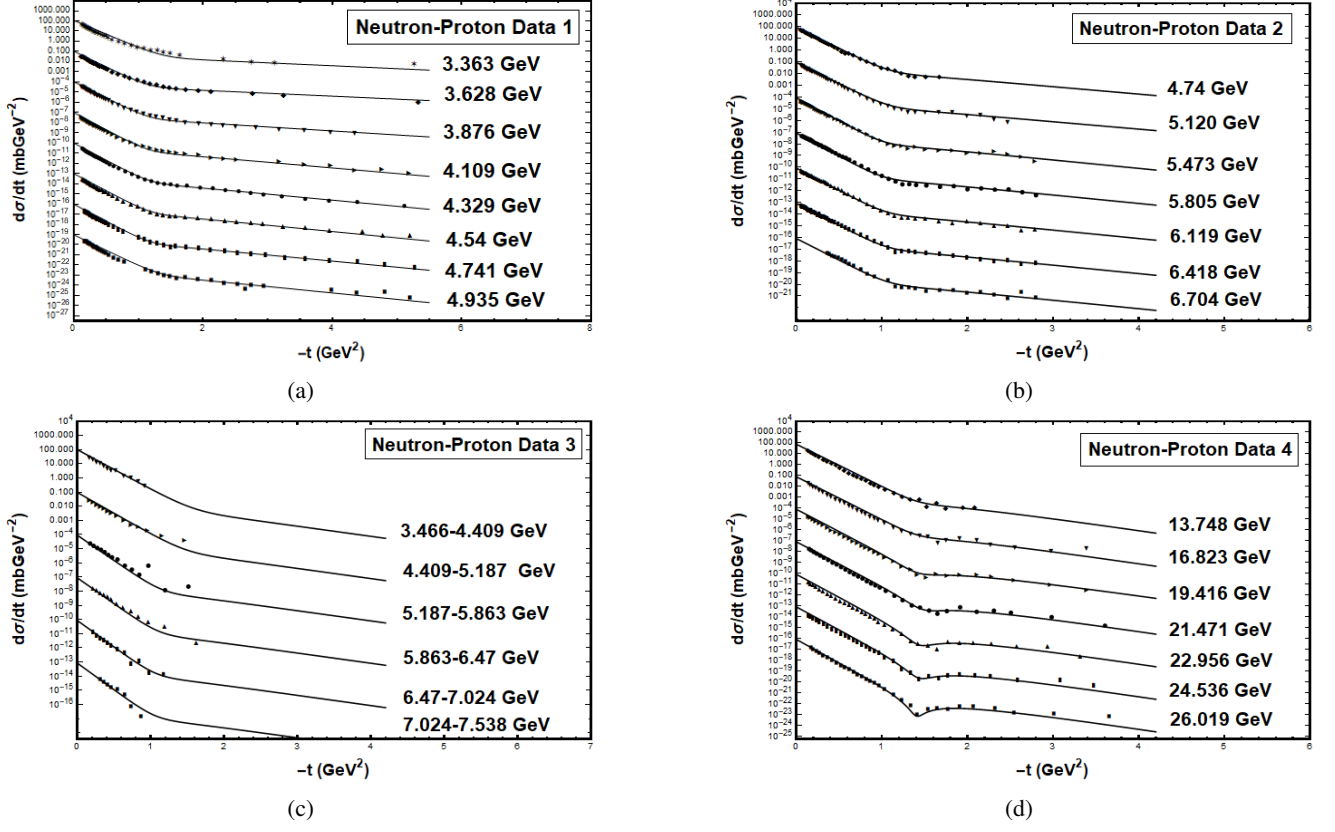


FIG. 4: Model 4 fits on the np elastic differential cross section data of Dataset 1 of Table I. Same legend for parts (a), (b), (c), and (d) as in the Figure 1.

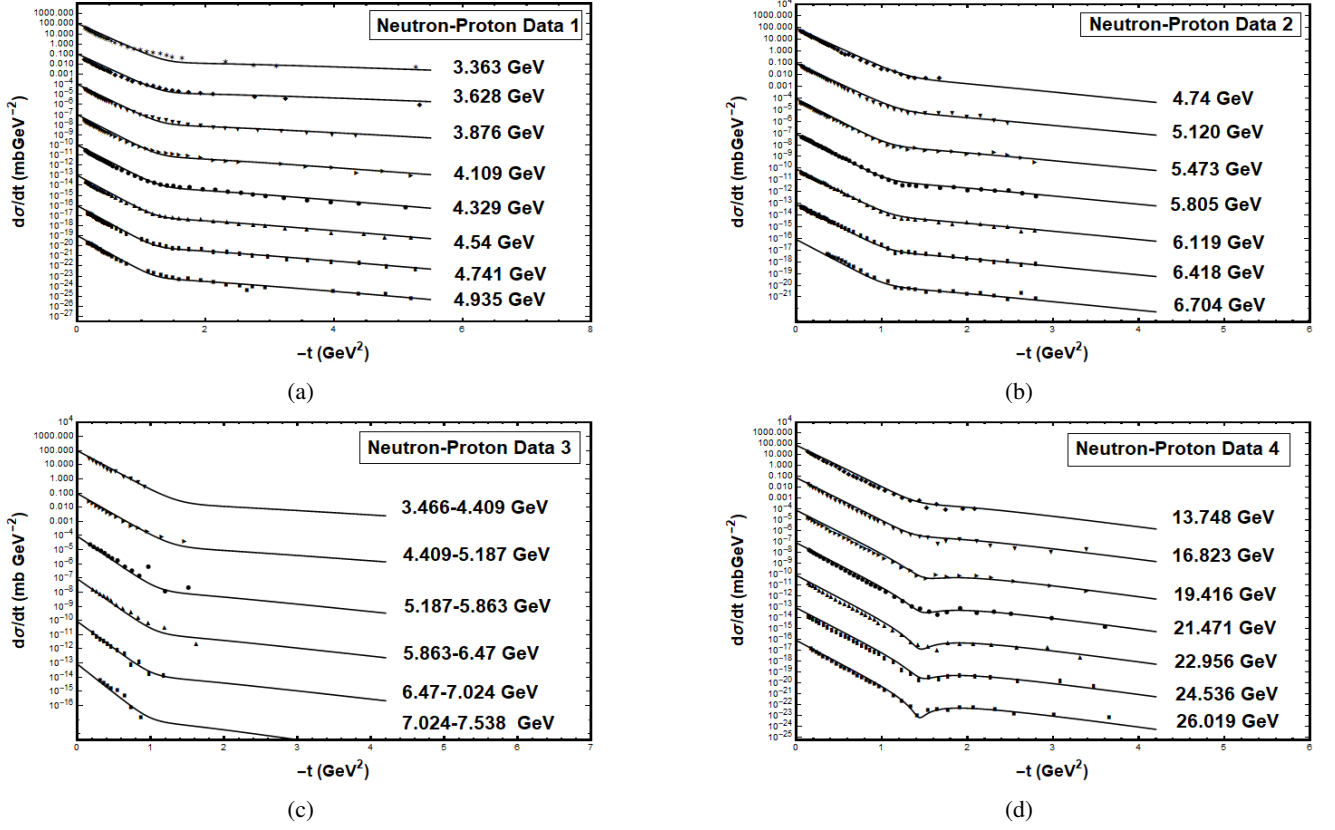


FIG. 5: Model 5 fits on the np elastic differential cross section data of Dataset 1 of Table I. Same legend for the parts (a), (b), (c), and (d) as in the Figure 1.

A. Fitted Parameters of the Models

We discuss the parameter values of the models, found by fitting all the data at the c.m. energies of Table I and define their reasonable limits or bounds to fit the np data. These ranges of the parameters, obtained by the np data fitting are important to employ these models with fitting of other elastic scattering data ($n\bar{p}$, pp , and $p\bar{p}$ data, mentioned in Section II), and for other correlation purposes. Each parameter value range found by the fitting in this study is correlated with our assumed expected bound. The trends in the parameter values of all the models with c.m. energy are evident from the parameter Tables III, IV, V, VI, and VII. The predicted ranges of the parameters, and the fixed values of all the parameters are mentioned in Table VIII. In this study, the comparison between the fitted parameter bounds or fixed values (shown in the Table VIII) with the assumed typical bounds is important as it allows one to investigate a collective behavior of the parameters across all nucleon–nucleon scattering processes (np , $n\bar{p}$, pp , $p\bar{p}$), especially to reproduce the experimental data by using the proposed models. It allows one to analyze global trends of the parameters, obtained from fitting the data over a wide range of energies, and to identify natural intervals for the forward slopes, energy exponents, deformation strengths, and the corrections related to short-range dynamics. In this way, stable and mutually consistent trends can be identified in elastic scattering data of various processes. These bounds are empirically inferred without relying on arbitrary or model-dependent priors and can act as universal phenomenological constraints that encode underlying common features of the elastic scattering data such as Pomeron-dominated diffraction, Reggeon contributions at intermediate $|t|$, and the mild C-odd and isospin effects. Agreement with such bounds can be essential to test a model across the data of a particular elastic scattering process and any deviation may indicate a distinct dynamical behavior or necessity of refining the parameterization.

TABLE III: Parameter values of model 1 for the neutron-proton (np) elastic scattering data of Table I

$\sqrt{s}$	A_0	α	B_0	β	ε_0	t_d	Δ	O_O	δ_O	D_O	I_0	δ_I	E_0	μ	χ^2
	(mb/GeV ²)		(GeV ⁻²)	(GeV ⁻²)		(GeV ²)	(GeV ²)	(mb/GeV ²)		(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)		
3.363 GeV	89.25	0.085	4.6	0.4	0.08	-0.8	0.2	0.2	0.12	0.85	0.06	0.12	2.9	0.09	0.189
3.628 GeV	86.0	0.075	4.7	0.4	0.08	-0.2	0.2	0.04	0.1	0.65	0.06	0.1	2.9	0.009	0.3151
3.876 GeV	82.5	0.085	5.0	0.4	0.08	-0.2	0.2	0.04	0.1	1.02	0.06	0.1	2.9	0.009	0.1623
4.109 GeV	71.5	0.12	5.9	0.4	0.08	-0.2	0.2	0.04	0.1	1.15	0.06	0.1	2.9	0.009	0.0404
4.329 GeV	69.25	0.12	5.8	0.4	0.08	-0.2	0.2	0.04	0.1	1.3	0.06	0.1	2.9	0.009	0.0736
4.54 GeV	66.8	0.12	5.8	0.4	0.08	-0.2	0.2	0.04	0.1	1.3	0.06	0.1	2.9	0.009	0.0752
4.741 GeV	65.0	0.12	5.6	0.4	0.08	-0.2	0.2	0.04	0.1	1.3	0.06	0.1	2.9	0.009	0.1084
4.935 GeV	64.0	0.12	5.5	0.4	0.08	-0.2	0.2	0.04	0.12	1.35	0.06	0.12	2.9	0.009	0.2733
4.74 GeV	61.5	0.13	5.5	0.4	0.32	-0.2	0.5	0.04	0.125	1.15	0.42	0.125	3.0	0.06	0.0709
5.12 GeV	58.5	0.135	5.8	0.4	0.32	-0.2	0.5	0.04	0.125	1.25	0.42	0.125	3.0	0.06	0.0348
5.473 GeV	56.473	0.135	6.02	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.4	0.125	3.0	0.06	0.0735
5.805 GeV	55.9	0.13	5.65	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0285
6.119 GeV	54.2	0.13	5.65	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.019
6.418 GeV	52.8	0.13	5.65	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0268
6.704 GeV	51.20	0.13	5.45	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0054
3.466-4.409 GeV	71.45	0.13	4.2	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.1987
4.409-5.187 GeV	64.0	0.13	4.45	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0861
5.187-5.863 GeV	60.15	0.13	4.65	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0629
5.863-6.4 GeV	56.7	0.13	5.45	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0003
6.47-7.024 GeV	50.650	0.13	5.85	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0085
7.024-7.538 GeV	49.0	0.13	6.25	0.4	0.32	-0.2	0.5	0.04	0.125	1.35	0.42	0.125	3.0	0.06	0.0141
13.748 GeV	41.8	0.123	4.7	0.4	0.08	-0.2	0.2	0.04	0.15	2.73	0.06	0.15	3.0	0.009	0.0447
16.823 GeV	40.2	0.123	4.9	0.4	0.3	-0.2	0.2	0.04	0.1	2.68	0.06	0.1	3.0	0.009	0.1919
19.416 GeV	38.2	0.123	5.25	0.4	0.3	-0.2	0.2	0.04	0.1	2.73	0.06	0.1	3.0	0.009	0.1963
21.471 GeV	38.5	0.123	5.25	0.4	0.4	-0.5	0.2	0.04	0.1	2.74	0.06	0.1	3.0	0.009	0.1651
22.956 GeV	38.3	0.123	5.3	0.4	0.4	-0.5	0.2	0.04	0.1	2.74	0.06	0.1	2.99	0.01	0.3596
24.536 GeV	38.0	0.125	5.3	0.4	0.6	-0.5	0.2	0.04	0.1	2.74	0.06	0.1	2.99	0.01	0.9788
26.019 GeV	32.0	0.15	5.45	0.4	0.8	-0.5	0.2	0.04	0.1	2.74	0.06	0.1	2.99	0.01	1.1303

TABLE IV: Parameter values of model 2 for the neutron-proton (np) elastic scattering data of Table I

$\sqrt{s}$	A_0	α	a_1	$B_1^{(0)}$	a_2	$B_2^{(0)}$	β	O_O	δ_O	D_O	I_0	δ_I	E_0	μ	χ^2
	(mb/GeV ²)			(GeV ⁻²)		(GeV ⁻²)	(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)		
3.363 GeV	33.6	0.12	1.5	4.6	1.0	4.4	0.4	0.04	0.1	0.65	0.04	0.12	2.75	0.001	0.138
3.628 GeV	31.25	0.12	1.5	4.75	1.0	4.8	0.4	0.04	0.1	0.8	0.04	0.12	2.75	0.001	0.2182
3.876 GeV	29.7	0.12	1.5	5.7	1.0	5.4	0.4	0.04	0.1	1.0	0.04	0.12	2.75	0.001	0.0368
4.109 GeV	28.65	0.12	1.5	6.75	1.0	5.43	0.4	0.04	0.1	1.2	0.04	0.12	2.75	0.001	0.0065
4.329 GeV	26.35	0.12	1.5	6.85	1.0	5.48	0.4	0.04	0.1	1.4	0.04	0.12	2.75	0.001	0.003
4.54 GeV	25.6	0.12	1.5	6.9	1.0	5.485	0.4	0.04	0.1	1.4	0.04	0.12	2.75	0.001	0.0026
4.741 GeV	24.9	0.12	1.5	6.95	1.0	5.495	0.4	0.04	0.1	1.4	0.04	0.12	2.75	0.001	0.0079
4.935 GeV	25.6	0.12	1.5	7.0	1.0	5.5	0.4	0.04	0.1	1.4	0.04	0.12	2.75	0.001	0.0447
4.74 GeV	24.5	0.13	1.5	5.9	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.016
5.12 GeV	23.6	0.13	1.5	5.0	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0156
5.473 GeV	22.7	0.13	1.5	5.7	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0295
5.805 GeV	22.0	0.13	1.5	5.8	1.0	6.5	0.4	0.04	0.1	1.6	0.06	0.12	2.75	0.001	0.0023
6.119 GeV	21.5	0.13	1.5	5.4	1.0	6.75	0.4	0.04	0.1	1.6	0.06	0.12	2.75	0.001	0.0018
6.418 GeV	20.8	0.13	1.5	5.0	1.0	6.95	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0035
6.704 GeV	20.35	0.13	1.5	5.5	1.0	7.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.1155
3.466-4.409 GeV	28.43	0.13	1.5	4.6	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0006
4.409-5.187 GeV	25.54	0.13	1.5	4.6	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0022
5.187-5.863 GeV	23.64	0.13	1.5	5.5	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0216
5.863-6.47 GeV	22.27	0.13	1.5	5.4	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0551
6.47-7.024 GeV	20.85	0.13	1.5	5.5	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0102
7.024-7.538 GeV	20.35	0.13	1.5	5.6	1.0	6.55	0.4	0.04	0.1	1.5	0.06	0.12	2.75	0.001	0.0185
13.748 GeV	21.22	0.12	1.0	4.2	1.0	5.08	0.4	0.04	0.06	2.65	0.06	0.06	3.0	0.008	0.0202
16.823 GeV	19.47	0.12	1.0	4.3	1.1	5.08	0.4	0.04	0.08	2.64	0.06	0.08	3.0	0.008	0.027
19.416 GeV	20.1	0.12	1.0	4.6	1.0	5.08	0.4	0.04	0.14	2.72	0.06	0.14	3.0	0.008	0.0572
21.471 GeV	19.8	0.12	1.0	4.2	1.0	5.08	0.4	0.04	0.14	2.75	0.06	0.14	3.0	0.002	0.0153
22.956 GeV	19.7	0.12	1.0	4.3	1.0	5.08	0.4	0.04	0.14	2.75	0.06	0.14	3.0	0.004	0.0758
24.536 GeV	19.6	0.12	1.0	4.4	1.0	5.08	0.4	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.163
26.019 GeV	19.5685	0.12	1.0	4.5	1.0	5.08	0.4	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.0924

TABLE V: Parameter values of model 3 for the neutron-proton (np) elastic scattering data of Table I

$\sqrt{s}$	A_0	α	f	$B_1^{(0)}$	$B_2^{(0)}$	β	ϵ_0	t_d	Δ	O_O	δ_O	D_O	I_0	δ_I	E_0	μ	χ^2
	(mb/GeV ²)			(GeV ⁻²)	(GeV ⁻²)	(GeV ⁻²)		(GeV ²)	(GeV ²)	(mb/GeV ²)		(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)		
3.363 GeV	88.2	0.12	1.1	4.3	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	0.6	0.06	0.14	3.0	0.008	0.1745
3.628 GeV	81.9	0.12	1.1	4.8	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	0.7	0.06	0.14	3.0	0.008	0.2191
3.876 GeV	78.6	0.12	1.1	4.8	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.0	0.06	0.14	3.0	0.008	0.1205
4.109 GeV	75.0	0.12	1.1	5.2	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.25	0.06	0.14	3.0	0.008	0.0535
4.329 GeV	72.25	0.12	1.1	5.2	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.0766
4.54 GeV	69.7	0.12	1.1	5.5	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.0514
4.741 GeV	68.8	0.12	1.1	5.4	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.1206
4.935 GeV	67.15	0.12	1.1	5.2	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.2420
4.74 GeV	66.0	0.12	1.1	5.8	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0193
5.12 GeV	63.4	0.12	1.1	5.8	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0099
5.473 GeV	61.0	0.12	1.1	6.2	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0183
5.805 GeV	59.5	0.12	1.1	6.1	4.2	0.4	0.045	-0.1	0.2	0.04	0.14	1.8	0.06	0.14	3.0	0.0001	0.0012
6.119 GeV	57.8	0.12	1.1	6.3	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0013
6.418 GeV	56.4	0.12	1.1	6.15	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0025
6.704 GeV	55.4	0.12	1.1	5.8	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0804
3.466-4.409 GeV	76.45	0.12	1.1	4.2	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	2.4	0.06	0.14	3.0	0.008	0.0510
4.409-5.187 GeV	69.0	0.12	1.1	4.2	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.4	0.06	0.14	3.0	0.008	0.0377
5.187-5.863 GeV	64.0	0.12	1.1	4.2	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.4	0.06	0.14	3.0	0.008	0.0334
5.863-6.47 GeV	60.4	0.12	1.1	5.0	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.4	0.06	0.14	3.0	0.008	0.0073
6.47-7.024 GeV	57.0	0.12	1.1	5.8	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0125
7.024-7.538 GeV	55.4	0.12	1.1	5.8	4.3	0.4	0.045	-0.1	0.2	0.04	0.14	1.6	0.06	0.14	3.0	0.008	0.0230
13.748 GeV	44.45	0.12	1.1	4.8	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.65	0.06	0.14	3.0	0.008	0.0136
16.823 GeV	42.77	0.12	1.1	4.8	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.65	0.06	0.14	3.0	0.008	0.0225
19.416 GeV	42.1	0.12	1.1	4.5	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.001	0.1021
21.471 GeV	41.5	0.12	1.1	4.35	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.0001	0.0352
22.956 GeV	41.25	0.12	1.1	4.45	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.0001	0.1124
24.536 GeV	41.0	0.12	1.1	4.7	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.1743
26.019 GeV	40.85	0.12	1.1	4.75	5.08	0.4	0.045	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.0996

TABLE VI: Parameter values of model 4 for the neutron-proton (np) elastic scattering data of Table I

$\sqrt{s}$	A_0	α	B_0	β	B_1	t_0	ε_0	t_d	Δ	O_O	δ_O	D_O	I_0	δ_I	E_0	μ	χ^2
	(mb/GeV ²)		(GeV ⁻²)	(GeV ⁻²)	(GeV ⁻²)	(GeV ²)		(GeV ²)	(GeV ²)	(mb/GeV ²)		(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)		
3.363 GeV	145.0	0.18	4.2	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	0.65	0.06	0.14	3.0	0.008	0.1887
3.628 GeV	133.9	0.18	5.0	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	0.65	0.06	0.14	3.0	0.008	0.1800
3.876 GeV	127.8	0.18	5.1	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	0.89	0.06	0.14	3.0	0.008	0.0853
4.109 GeV	120.5	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.25	0.06	0.14	3.0	0.008	0.0333
4.329 GeV	116.5	0.18	5.4	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.0651
4.54 GeV	112.95	0.18	5.3	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.4	0.06	0.14	3.0	0.008	0.0892
4.741 GeV	108.95	0.18	5.2	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.35	0.06	0.14	3.0	0.008	0.1669
4.935 GeV	105.95	0.18	4.45	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.425	0.06	0.14	3.0	0.008	0.5176
4.74 GeV	104.7	0.18	5.8	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.425	0.06	0.14	3.0	0.008	0.0235
5.12 GeV	99.8	0.18	5.7	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.425	0.06	0.14	3.0	0.008	0.0136
5.473 GeV	96.2	0.18	6.4	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0226
5.805 GeV	92.7	0.18	6.3	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0017
6.119 GeV	89.5	0.18	5.9	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0019
6.418 GeV	86.2	0.18	5.7	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0059
6.704 GeV	84.0	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0171
3.466-4.409 GeV	124.0	0.18	4.25	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0655
4.409-5.187 GeV	109.0	0.18	4.25	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0332
5.187-5.863 GeV	99.5	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0061
5.863-6.47 GeV	93.0	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0283
6.47-7.024 GeV	86.0	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0072
7.024-7.538 GeV	83.25	0.18	5.5	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	1.625	0.06	0.14	3.0	0.008	0.0003
13.748 GeV	61.95	0.18	4.75	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.65	0.06	0.14	3.0	0.008	0.0149
16.823 GeV	58.2	0.18	4.55	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.67	0.06	0.14	3.0	0.008	0.0367
19.416 GeV	56.0	0.18	4.7	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.72	0.06	0.14	3.0	0.001	0.0690
21.471 GeV	55.0	0.18	4.55	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.0207
22.956 GeV	54.25	0.18	4.75	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.0716
24.536 GeV	53.4	0.18	4.68	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.1756
26.019 GeV	52.8	0.18	4.78	0.4	0.1	10	0.5	-0.1	0.2	0.04	0.14	2.75	0.06	0.14	3.0	0.008	0.0931

TABLE VII: Parameter values of model 5 for the neutron-proton (np) elastic scattering data of Table I

$\sqrt{s}$	A_0	α	B_0	β	ε_0	t_d	Δ	O_O	δ_O	D_O	γ	I_0	δ_I	E_0	μ	χ^2
	(mb/GeV ²)		(GeV ⁻²)	(GeV ⁻²)		(GeV ²)	(GeV ²)	(mb/GeV ²)		(GeV ⁻²)	(GeV ⁻²)	(mb/GeV ²)		(GeV ⁻²)		
3.363 GeV	561.7	0.12	4.65	0.4	0.85	-0.1	0.1	0.04	0.14	0.452	0.5	0.06	0.14	3.0	0.08	0.1254
3.628 GeV	522.7	0.12	4.7	0.4	0.85	-0.1	0.1	0.04	0.14	0.49	0.5	0.06	0.14	3.0	0.08	0.2403
3.876 GeV	496.0	0.12	4.9	0.4	0.85	-0.1	0.1	0.04	0.14	0.69	0.5	0.06	0.14	3.0	0.08	0.0992
4.109 GeV	476.7	0.12	5.25	0.4	0.85	-0.1	0.1	0.04	0.14	0.9	0.5	0.06	0.14	3.0	0.08	0.0510
4.329 GeV	463.0	0.12	5.15	0.4	0.85	-0.1	0.1	0.04	0.14	1.0	0.5	0.06	0.14	3.0	0.08	0.0894
4.54 GeV	448.0	0.12	5.425	0.4	0.85	-0.1	0.1	0.04	0.14	1.0	0.5	0.06	0.14	3.0	0.08	0.0699
4.741 GeV	437.0	0.12	5.45	0.4	0.85	-0.1	0.1	0.04	0.14	1.0	0.5	0.06	0.14	3.0	0.08	0.1183
4.935 GeV	428.0	0.15	5.5	0.4	0.85	-0.1	0.1	0.04	0.14	1.0	0.5	0.06	0.14	3.0	0.08	0.3190
4.74 GeV	252.0	0.12	5.0	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0461
5.12 GeV	242.0	0.12	5.2	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0207
5.473 GeV	234.0	0.12	5.9	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0317
5.805 GeV	227.0	0.12	5.95	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0027
6.119 GeV	221.0	0.12	5.6	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0030
6.418 GeV	215.5	0.12	5.7	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0058
6.704 GeV	211.0	0.12	5.8	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0516
3.466-4.409 GeV	292.3	0.12	4.2	0.4	0.75	-0.1	0.1	0.04	0.14	0.6	1.0	0.06	0.14	3.0	0.08	0.0708
4.409-5.187 GeV	263.5	0.12	4.3	0.4	0.75	-0.1	0.1	0.04	0.14	0.7	1.0	0.06	0.14	3.0	0.08	0.0292
5.187-5.863 GeV	244.5	0.12	5.4	0.4	0.75	-0.1	0.1	0.04	0.14	0.95	1.0	0.06	0.14	3.0	0.08	0.0037
5.863-6.47 GeV	231.5	0.12	5.5	0.4	0.75	-0.1	0.1	0.04	0.14	1.0	1.0	0.06	0.14	3.0	0.08	0.0289
6.47-7.024 GeV	217.35	0.12	5.6	0.4	0.75	-0.1	0.1	0.04	0.14	1.0	1.0	0.06	0.14	3.0	0.08	0.0060
7.024-7.538 GeV	211.5	0.12	6.2	0.4	0.75	-0.1	0.1	0.04	0.14	1.25	1.0	0.06	0.14	3.0	0.08	0.0430
13.748 GeV	430.5	0.16	5.65	0.3	0.92	-0.1	0.1	0.04	0.02	2.4	1.0	0.06	0.02	3.0	0.001	0.0241
16.823 GeV	408.5	0.16	5.65	0.3	0.92	-0.1	0.1	0.04	0.02	2.4	1.0	0.06	0.02	3.0	0.001	0.0425
19.416 GeV	397.0	0.16	5.65	0.3	0.92	-0.1	0.1	0.04	0.02	2.6	1.0	0.06	0.02	3.0	0.001	0.1076
21.471 GeV	387.0	0.16	5.7	0.3	0.92	-0.1	0.1	0.04	0.02	2.6	1.0	0.06	0.02	3.0	0.001	0.0254
22.956 GeV	383.0	0.16	5.8	0.3	0.92	-0.1	0.1	0.04	0.02	2.6	1.0	0.06	0.02	3.0	0.001	0.0973
24.536 GeV	379.0	0.16	5.6	0.3	0.92	-0.1	0.1	0.04	0.02	2.6	1.0	0.06	0.02	3.0	0.001	0.2601
26.019 GeV	376.0	0.16	5.72	0.3	0.92	-0.1	0.1	0.04	0.02	2.6	1.0	0.06	0.02	3.0	0.001	0.1646

TABLE VIII: Typical bounds of the parameters of the models in this study, predicted via fitting the elastic np scattering data of Table I

Model 1				Model 2				Model 3			
Parameter	Dimension	Predicted Bound or Value	Expected Bound	Parameter	Dimension	Predicted Bound or Value	Expected Bound	Parameter	Dimension	Predicted Bound or Value	Expected Bound
A_0	mb/GeV ²	[32.0,89.25]	[10 ⁻⁶ ,10 ³]	A_0	mb/GeV ²	[19.347,33.6]	[10 ⁻⁶ ,10 ³]	A_0	mb/GeV ²	[40.85,88.20]	[10 ⁻⁶ ,10 ³]
α		[0.075,0.15]	[0,0.2]	α		[0.12,0.13]	[0,0.2]	α		0.12	[0,0.2]
B_0	GeV ⁻²	[4.20,6.25]	[2,30]	a_1		[1.0,1.5]	[0,10]	f		1.1	[0,1.5]
β	GeV ⁻²	0.4	[0,1.0]	$B_1^{(0)}$	GeV ⁻²	[4.2,7.0]	[2,30]	$B_1^{(0)}$	GeV ⁻²	[4.2,6.3]	[2,30]
ϵ_0		[0.08,0.8]	[0,1.0]	a_2		[1,1.1]	[0,10]	$B_2^{(0)}$	GeV ⁻²	[4.3,5.08]	[2,30]
t_d	GeV ²	[-0.8,-0.2]	[-1.2,-0.2]	$B_2^{(0)}$	GeV ⁻²	[4.4,7.55]	[2,30]	β	GeV ⁻²	0.4	[0,1.0]
Δ	GeV ²	[0.2,0.5]	[0.01,2]	β	GeV ⁻²	0.4	[0,1.0]	ϵ_0		0.045	[0,1.0]
O_O	mb/GeV ²	[0.04,0.2]	[10 ⁻⁸ ,10]	O_O	mb/GeV ²	0.04	[10 ⁻⁸ ,10]	t_d	GeV ²	-0.1	[-0.1,-0.2]
δ_O		[0.1,0.125]	[-0.1,1]	δ_O		[0.06,0.14]	[-0.1,1]	Δ	GeV ²	0.2	[0.01,2]
D_O	GeV ⁻²	[0.65,2.74]	[0.1,25]	D_O	GeV ⁻²	[0.65,2.75]	[0.1,25]	O_O	mb/GeV ²	0.04	[10 ⁻⁸ ,10]
I_0	mb/GeV ²	[0.06,0.42]	[10 ⁻⁸ ,10]	I_0	mb/GeV ²	[0.04,0.06]	[10 ⁻⁸ ,10]	δ_O		0.14	[-0.1,1]
δ_f		[0.1,0.125]	[-0.1,1]	δ_f		[0.06,0.14]	[-0.1,1]	D_O	GeV ⁻²	[0.6,2.75]	[0.1,25]
E_0	GeV ⁻²	[2.9,3.0]	[0.1,25]	E_0	GeV ⁻²	[2.75,3.0]	[0.1,25]	I_0	mb/GeV ²	0.06	[10 ⁻⁸ ,10]
μ		[0.009,0.09]	[0,1]	μ		[0.001,0.008]	[0,1]	δ_f		0.14	[-0.1,1]
								E_0	GeV ⁻²	3.0	[0.1,25]
								μ		[0.0001,0.008]	[0,1]

Model 4				Model 5			
Parameter	Dimension	Predicted Bound or Value	Expected Bound	Parameter	Dimension	Predicted Bound or Value	Expected Bound
A_0	mb/GeV ²	[52.8,145]	[10 ⁻⁶ ,10 ³]	A_0	mb/GeV ²	[211,561.7]	[10 ⁻⁶ ,10 ³]
α		0.18	[0,0.2]	α		[0.12,0.16]	[0,0.2]
B_0	GeV ⁻²	[4.2,6.4]	[2,30]	B_0	GeV ⁻²	[4.2,6.4]	[2,30]
β	GeV ⁻²	0.4	[0,1.0]	β	GeV ⁻²	[0.3,0.4]	[0,1.0]
B_1	GeV ⁻²	0.1	[0,5]	ϵ_0		[0.75,0.92]	[0,1.0]
t_0	GeV ²	10	[0.1,10]	t_d	GeV ²	-0.1	[-1.2,-0.2]
ϵ_0		0.5	[0,1.0]	Δ	GeV ²	0.1	[0.01,2]
t_d	GeV ²	-0.1	[-1.2,-0.2]	O_O	mb/GeV ²	0.04	[10 ⁻⁸ ,10]
Δ	GeV ²	0.2	[0.01,2]	δ_O		[0.02,0.14]	[-0.1,1]
O_O	mb/GeV ²	0.04	[10 ⁻⁸ ,10]	D_O	GeV ⁻²	[0.425,2.60]	[0.1,25]
δ_O		0.14	[-0.1,1]	γ	GeV ⁻²	[0.5,1.0]	[0.01,5.0]
D_O	GeV ⁻²	[0.65,2.75]	[0.1,25]	I_0	mb/GeV ²	0.06	[10 ⁻⁸ ,10]
I_0	mb/GeV ²	0.06	[10 ⁻⁸ ,10]	δ_f		[0.02,0.14]	[-0.1,1]
δ_f		0.14	[-0.1,1]	E_0	GeV ⁻²	3.0	[0.1,25]
E_0	GeV ⁻²	3.0	[0.1,25]	μ		[0.001,0.008]	[0,1]
μ		[0.001,0.008]	[0,1]				

The chi-square per degree of freedom (χ^2) values are reasonably small and indicate good fitting agreement of our models with the data. We have used the following expression for dimensionless χ^2 .

$$\chi^2 = \frac{\sum_{i=1}^N [(d\sigma/dt)_i^{\text{model}} - (d\sigma/dt)_i^{\text{data}}]^2}{\sum_{i=1}^N [(d\sigma/dt)_i^{\text{data}}]^2}, \quad (7)$$

with $i = 1, 2, 3, \dots, N$, number of data points. It is a normalized error measure which is particularly useful to determine goodness-of-fit when experimental uncertainties are not known. The χ^2 values for each model across all the energies along with the fitted parameter values are given in Tables III, IV, V, VI, and VII. The χ^2 with the dataset 1 for model 1 is found in the range ($0.0404 \leq \chi^2 \leq 0.3151$), for the model 2 in the range ($0.0026 \leq \chi^2 \leq 0.2182$), for the model 3 in the range ($0.0514 \leq \chi^2 \leq 0.2420$), for the model 4 in the range ($0.0333 \leq \chi^2 \leq 0.5176$), and for the model 5 in the range ($0.0510 \leq \chi^2 \leq 0.3190$). The χ^2 with the dataset 2 for model 1 is found in the range ($0.0054 \leq \chi^2 \leq 0.0735$), for the model 2 in the range ($0.0018 \leq \chi^2 \leq 0.1155$), for the model 3 in the range ($0.0012 \leq \chi^2 \leq 0.0804$), for the model 4 in the range ($0.0017 \leq \chi^2 \leq 0.0235$), and for the model 5 in the range ($0.0027 \leq \chi^2 \leq 0.0516$). The χ^2 with the dataset 3 for model 1 is found in the range ($0.0003 \leq \chi^2 \leq 0.1987$), for the model 2 in the range ($0.0006 \leq \chi^2 \leq 0.0551$), for the model 3 in the range ($0.0073 \leq \chi^2 \leq 0.0510$), for the model 4 in the range ($0.0003 \leq \chi^2 \leq 0.0655$), and for the model 5 in the range ($0.0037 \leq \chi^2 \leq 0.0708$). The χ^2 with the dataset 4 for model 1 is found in the range ($0.0447 \leq \chi^2 \leq 1.1303$), for the model 2 in the range ($0.0153 \leq \chi^2 \leq 0.163$), for the model 3 in the range ($0.0136 \leq \chi^2 \leq 0.1743$), for the model 4 in the range ($0.0149 \leq \chi^2 \leq 0.1756$), and for the model 5 in the range ($0.0241 \leq \chi^2 \leq 0.2601$).

The difference ($\chi_{\text{max}}^2 - \chi_{\text{min}}^2$) of the estimated range of χ^2 for each model is estimated across each dataset. Its values are mentioned in Table IX. This difference of the χ^2 is significant to determine the accuracy of data fitting of the models. As the models with small χ^2 ranges are more stable therefore, smaller range suggests higher consistency, showing that the fit quality for these models does not fluctuate significantly within that dataset. A larger range suggests variability, and the models with large χ^2 ranges show a significant difference in their minimum and maximum fit quality. The smallest range points to the most consistent fit. In this study, Model 4 with Dataset 2 is found with the smallest difference (0.0218) in range $0.0017 \leq \chi^2 \leq 0.0235$.

TABLE IX: Calculated differences across χ^2 ranges for all models across each dataset

Dataset	Model 1 Range	Model 2 Range	Model 3 Range	Model 4 Range	Model 5 Range
1	0.2747	0.2156	0.1906	0.4843	0.2680
2	0.0681	0.1137	0.0792	0.0218	0.0489
3	0.1984	0.0545	0.0437	0.0652	0.0671
4	1.0856	0.1477	0.1607	0.1607	0.2360

B. Total Cross Section

The observable total cross section (σ_{tot}) in np elastic scattering provides a quantitative characterization of the strength and spatial extension of the hadronic interaction ($np \rightarrow X$). The total cross section is related to the forward scattering amplitude through the optical theorem and serves as a global normalization constraint for all amplitude models [6, 33, 34, 41]. In the energy range $2 \text{ GeV} \lesssim \sqrt{s} \lesssim 6 \text{ GeV}$, the np total cross section lies approximately between 35 and 45 mb and decreases mildly with energy before showing the slow logarithmic rise which is predicted by Regge-Pomeron models [33, 69].

In this study, the total cross section σ_{tot} is predicted using the optical theorem [70] from the fitted models of the differential cross section by their extrapolation at $t = 0$:

$$\sigma_{\text{tot}}^2 = \frac{16\pi(\hbar c)^2}{1 + \rho^2} \left. \frac{d\sigma}{dt} \right|_{t=0}. \quad (8)$$

The conversion factor $(\hbar c)^2 \simeq 0.38 \text{ GeV}^2 \text{ mbarn}$ [80] is used in these σ_{tot} calculations. The ρ parameter values used for each c.m. energy along with the predicted total cross sections are given in Table XI. For Dataset 1 and 3, we used the ρ values from refs. [27] and [30], respectively. For Dataset 2, the values of the ρ parameter values are obtained by linear interpolation of the values reported in ref. [28] at the corresponding c.m. energies. Similarly, the values of the ρ parameter are obtained for Dataset 4 by linear interpolation from the values given in ref. [63] for pp scattering by assuming same values for np elastic scattering. The calculations of the σ_{tot} relied mostly on the outcome of the models after their extrapolation at $t = 0$ with the determined fitted parameters values and lesser on ρ values. The ρ for np elastic scattering has the same contribution as for pp scattering as mentioned in ref. [31]. In the absence of exact reference values for the σ_{tot} for all the c.m. energies mentioned in Table I, especially for the datasets 2 and 3 for which only fixed normalization σ_{tot} values of 39 mb and 38 mb are available in refs. [28, 30], we perform simple fits on the np σ_{tot} data in the c.m. energy range, $3.0 \text{ GeV} \leq \sqrt{s} \leq 26.4 \text{ GeV}$ of [65] by some parametrizations and report their extracted parameter values. In this way we extrapolate σ_{tot} results for all the energies and the obtained results by interpolation from these fits at the c.m. energies of Table I, are considered as the reference values to compare with σ_{tot} results obtained by our models of the differential cross section. In this study, we perform the following,

- Fit 1: the extended form of the parametrization of ref. [33] with $\varepsilon = 0.0808$ and $\eta = 0.4525$ is considered with one constant A added as follows:

$$\sigma_{\text{tot}}(s) = A + Bs^{-\eta} + Cs^\varepsilon. \quad (9)$$

In ref. [33] the terms containing s^ε and $s^{-\eta}$ give Pomeron rising contribution and the decreasing Reggeon contribution to the total cross section, respectively. The other two fits are obtained as extended or modified forms of the parametrization which give quite good quantitative agreement with it and with each other (as shown in the Figure 6).

- Fit 2: This form has a Froissart-like $\ln^2(s)$ growth term and Reggeon term with $\eta = 0.4525$ ensures correct low- s behavior. It contains one constant A, and a quadratic logarithmic energy dependent term as follows:

$$\sigma_{\text{tot}}(s) = A + B \text{Log}^2\left(\frac{s}{s_0}\right) + Cs^{-\eta}. \quad (10)$$

- Fit 3: It contains a Froissart bound type term, showing $\ln^2(s)$ growth [41], widely used at high energies:

$$\sigma_{\text{tot}}(s) = A + B \ln(s) + C \ln^2(s). \quad (11)$$

Results with these fits as the estimated σ_{tot} values are found to be very close to each other as shown in the Figure 6. The estimated best fit parameter values of these fits along with their χ^2 are mentioned in Table X. We have used the following

expression for dimensionless χ^2 for σ_{tot} fits,

$$\chi^2 = \frac{\sum_{i=1}^N [(\sigma_{\text{tot}})_i^{\text{model}} - (\sigma_{\text{tot}})_i^{\text{data}}]^2}{\sum_{i=1}^N [(\sigma_{\text{tot}})_i^{\text{data}}]^2}, \quad (12)$$

with $i = 1, 2, 3, \dots, N$, number of data points. These fits are used to separately estimate the σ_{tot} values and can be further improved after doing suitable modifications, according to the requirements related to specific physical constraints. Furthermore, it should be kept in view that in this study, these fits are used only to estimate σ_{tot} values for comparison with the predicted results obtained from the models of the differential cross section for np elastic scattering.

The σ_{tot} results, shown in Table XI, obtained by the proposed models are in quite good quantitative agreement with the reference values and with the values obtained by separate fits. The comparison plots for graphical visualization of the σ_{tot} results are shown in Figure 7. In this study, the least differences across the predicted ranges for σ_{tot} and the other observables, are estimated by,

$$D_i = |Ref_{\min} - (Model_{\min})_i| + |Ref_{\max} - (Model_{\max})_i|, \quad (13)$$

where $i = 1, 2, \dots, 5$ is the model number.

It is evident from the Table XII for the dataset 1 (3.363 GeV - 4.935 GeV), the reference values are in range 41.69 mb - 39.30 mb [27]. For the dataset 2 (4.74 GeV - 6.704 GeV), the reference value is a fixed normalization value 39.0 mb [28]. But if the resulting values by the fits 1, 2, and 3 are considered their values show a decrease in σ_{tot} values from 4.74 GeV to 6.704 GeV in the ranges 39.8246 mb - 39.0134 mb, 39.8423 mb - 39.0199 mb, and 39.9492 mb - 39.0726 mb, respectively. An average reference range of 39.0353 mb - 39.8720 mb is used across this dataset. For the dataset 3 (3.466-4.409 GeV to 7.024-7.538 GeV), the reference value is a fixed normalization value 38.0 mb [30]. The resulting values by fits 1, 2, and 3 show a decrease in σ_{tot} values from 3.466-4.409 GeV to 7.024-7.538 GeV in the ranges 40.5275 mb - 38.9018 mb, 40.5396 mb - 38.9037 mb, and 40.6055 mb - 38.931 mb, respectively. We consider the average value of the ranges of the three fits to be 40.5575-38.9122 mb. For the dataset 4 (13.748 GeV - 26.019 GeV), the reference values are in range 38.8 mb to 40.3 mb showing a gradual increase with increasing c.m. energy [31].

It is evident from the comparison plots of Figure 7 that a decreasing trend is found by all the models by their predicted values of σ_{tot} values from 3.363 GeV - 4.935 GeV, 4.74 GeV - 6.704 GeV, and 3.466-4.409 GeV to 7.024-7.538 GeV c.m. energies. And an increasing trend by all the models is found by their predicted values of σ_{tot} from 13.748 GeV - 26.019 GeV.

TABLE X: Estimated parameter values of the fits of the total cross section $\sigma_{\text{tot}}(s)$ on np data of [65] in range $3.0\text{GeV} \leq \sqrt{s} \leq 26.4\text{GeV}$

No.	Fit	A	B	C	ϵ	η	χ^2
		(mb)	(mb)	(mb)			Approx.
1	$\sigma_{\text{tot}}(s) = A + Bs^\epsilon + Cs^{-\eta}$	17.5118	12.3638	26.2255	0.0808	0.4525	0.00025
2	$\sigma_{\text{tot}}(s) = A + B\ln^2(s) + Cs^{-\eta}$	33.2611	0.1283	21.8285	-	0.4525	0.00026
3	$\sigma_{\text{tot}}(s) = A + B\ln(s) + C\ln^2(s)$	49.5012	-4.5454	0.4743	-	-	0.00036

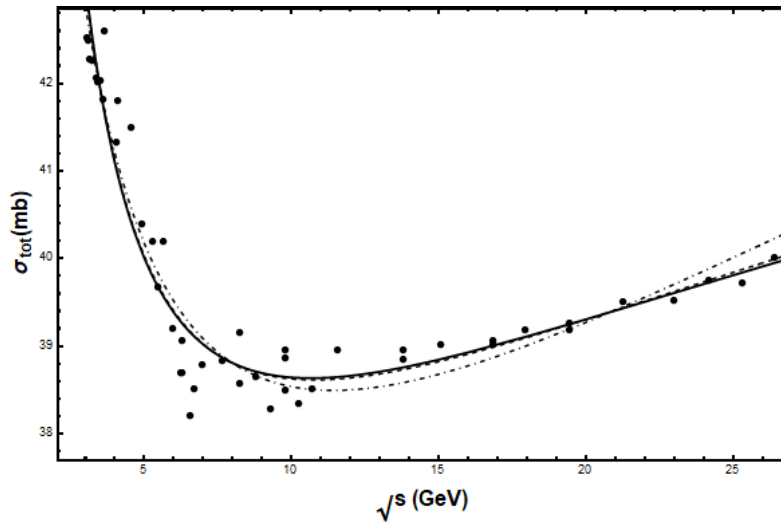


FIG. 6: Fits on the np total cross section data of ref. [65]. Fit 1, Fit 2, and Fit 3 are shown by solid line, dashed line, and dotted dashed line, respectively. The data points are filled black circles

TABLE XI: Results of the np total cross section σ_{tot} by extrapolation of our models at $t = 0$ with interpolations of separate fits performed on σ_{tot} reference data of [65]

$\sqrt{s}$	ρ	Model 1	Model 2	Model 3	Model 4	Model 5	Ref. values	Fit 1	Fit 2	Fit 3
		σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)
3.363 GeV	-0.4875 [27]	41.1787	41.6455	41.698	41.6056	41.6936	41.69 [27]	41.3032	41.2993	41.2663
3.628 GeV	-0.475 [27]	40.3198	40.7284	40.7468	40.7291	40.7854	40.77 [27]	40.9082	40.9135	40.9367
3.876 GeV	-0.4625 [27]	40.1488	40.2139	40.4284	40.4598	40.2522	40.25 [27]	40.5972	40.6082	40.6672
4.109 GeV	-0.45 [27]	39.9234	39.963	39.9577	39.8906	39.9272	39.92 [27]	40.347	40.3615	40.4431
4.329 GeV	-0.4375 [27]	39.7204	39.745	39.6476	39.7766	39.7801	39.70 [27]	40.142	40.1584	40.2538
4.54 GeV	-0.425 [27]	39.4135	39.5833	39.3429	39.6827	39.5181	39.53 [27]	39.9694	39.9868	40.0901
4.741 GeV	-0.4125 [27]	39.2557	39.4211	39.467	39.4539	39.4079	39.40 [27]	39.8239	39.8416	39.9485
4.935 GeV	-0.40 [27]	39.311	39.3164	39.35	39.3598	39.3589	39.30 [27]	39.6987	39.7162	39.8238
4.74 GeV	-0.31	39.9545	39.9879	39.9388	39.9601	39.9158	39.0 [28]	39.8246	39.8423	39.9492
5.12 GeV	-0.293	39.8659	39.8284	39.6939	39.7452	39.6645	39.0 [28]	39.5916	39.6086	39.7147
5.473 GeV	-0.277	39.7024	39.5678	39.4137	39.6595	39.4821	39.0 [28]	39.4157	39.4311	39.5302
5.805 GeV	-0.26	39.6157	39.4195	39.3692	39.5139	39.3292	39.0 [28]	39.279	39.2923	39.3813
6.119 GeV	-0.243	39.4309	39.3947	39.2059	39.3535	39.2087	39.0 [28]	39.1709	39.1819	39.2592
6.418 GeV	-0.227	39.3336	39.1648	39.1233	39.144	39.1123	39.0 [28]	39.084	39.0927	39.1577
6.704 GeV	-0.21	39.0505	39.0604	39.0826	39.033	39.0086	39.0 [28]	39.0134	39.0199	39.0726
3.466-4.409 GeV	-0.42 [30]	40.5963	40.5901	40.5792	40.5996	40.5856	38.0 [30]	40.5275	40.5396	40.6055
4.409-5.187 GeV	-0.39 [30]	39.8232	39.8865	39.8901	39.8566	39.8711	38.0 [30]	39.7857	39.8034	39.9107
5.187-5.863 GeV	-0.36 [30]	39.7038	39.4713	39.4602	39.4461	39.448	38.0 [30]	39.3926	39.4077	39.5054
5.863-6.47 GeV	-0.33 [30]	39.4587	39.222	39.2035	39.2584	39.2544	38.0 [30]	39.1561	39.1668	39.2422
6.47-7.024 GeV	-0.30 [30]	39.0703	39.0154	39.0949	39.0969	39.0443	38.0 [30]	39.0038	39.0099	39.0607
7.024-7.538 GeV	-0.27 [30]	38.9195	38.9484	38.9384	38.9072	38.9106	38.0 [30]	38.9018	38.9037	38.931
13.748 GeV	-0.094	38.8228	38.8165	38.8175	38.8052	38.8225	38.8 [31]	38.8426	38.8225	38.7074
16.823 GeV	-0.060	39.1336	39.1332	39.111	39.1053	39.1607	39.1 [31]	39.0602	39.0462	38.9572
19.416 GeV	-0.027	39.3658	39.5311	39.5338	39.4179	39.5581	39.5 [31]	39.2694	39.2656	39.2284
21.471 GeV	-0.012	39.5317	39.7229	39.7392	39.7898	39.7021	39.7 [31]	39.4409	39.4474	39.4653
22.956 GeV	0.0221	39.7477	39.9347	39.9316	39.9891	39.9145	39.9 [31]	39.5654	39.5803	39.6441
24.536 GeV	0.024	40.1718	40.1508	40.1278	40.1509	40.1291	40.1 [31]	39.6974	39.7217	39.8388
26.019 GeV	0.028	40.2835	40.3978	40.3331	40.3444	40.3429	40.3 [31]	39.82	39.8538	40.0245

TABLE XII: Predicted ranges of the total cross section σ_{tot}

Dataset 1				Dataset 2			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)		σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)
1	39.311-41.1787	39.3-41.69	0.5223	1	39.0505-39.9545	39.0353-39.8720	0.0977
2	39.3164-41.6455	39.3-41.69	0.0609	2	39.0604-39.9879	39.0353-39.8720	0.1410
3	39.35-41.698	39.3-41.69	0.0580	3	39.0826-39.9388	39.0353-39.8720	0.1141
4	39.3598-41.6056	39.3-41.69	0.1442	4	39.033-39.9601	39.0353-39.8720	0.0904
5	39.3589-41.6936	39.3-41.69	0.0625	5	39.0086-39.9158	39.0353-39.8720	0.0705
Min. $\approx$ 0.0580				Min. $\approx$ 0.0705			
Dataset 3				Dataset 4			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)		σ_{tot} (mb)	σ_{tot} (mb)	σ_{tot} (mb)
1	38.9195-40.5963	38.9122-40.5575	0.0461	1	38.8228-40.2835	38.8-40.3	0.0393
2	38.9484-40.5901	38.9122-40.5575	0.0688	2	38.8165-40.3978	38.8-40.3	0.1143
3	38.9384-40.5792	38.9122-40.5575	0.0479	3	38.8175-40.3331	38.8-40.3	0.0506
4	38.9072-40.5996	38.9122-40.5575	0.0471	4	38.8052-40.3444	38.8-40.3	0.0496
5	38.9106-40.5856	38.9122-40.5575	0.0297	5	38.8225-40.3429	38.8-40.3	0.0654
Min. $\approx$ 0.0297				Min. $\approx$ 0.0393			

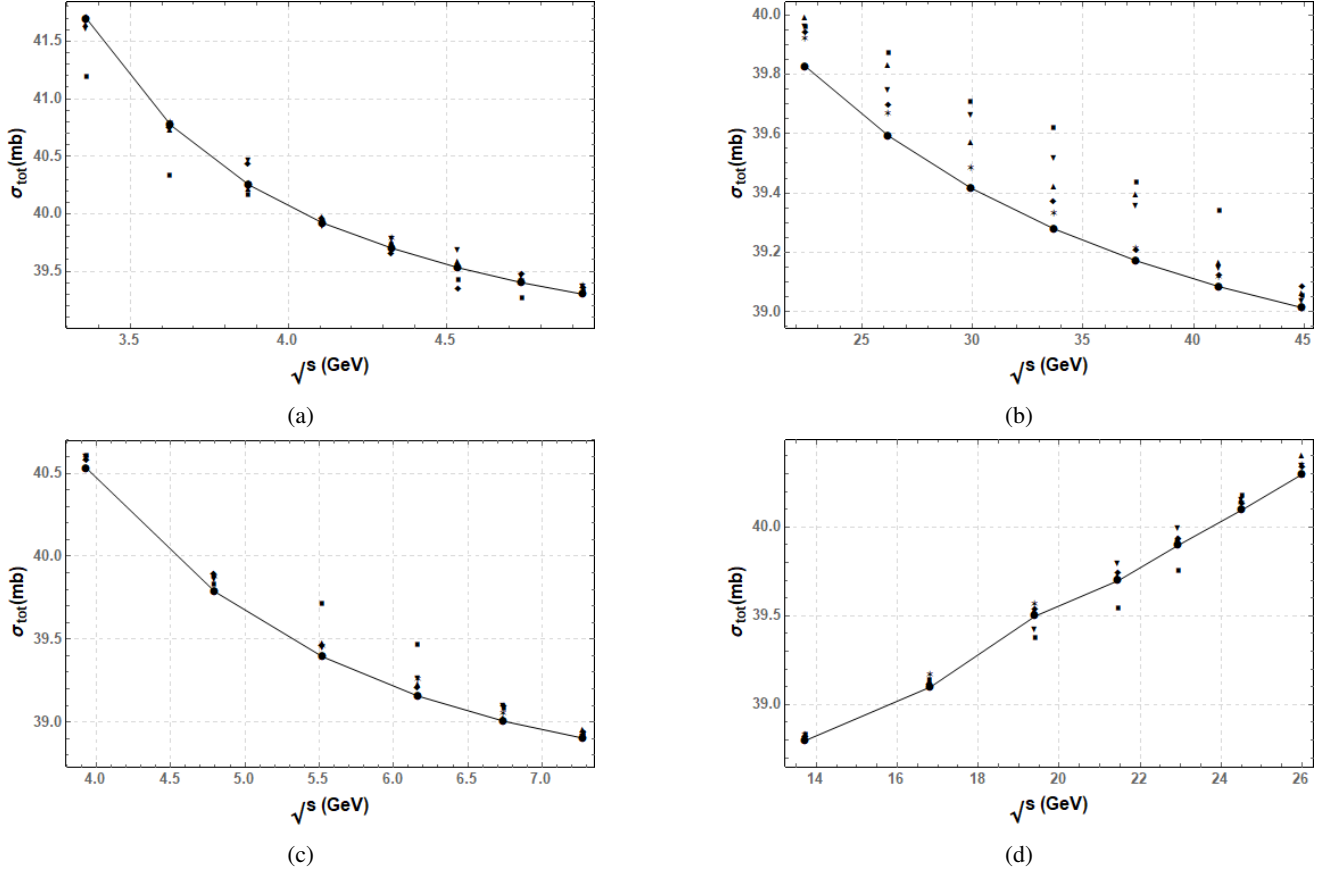


FIG. 7: (a) Graphical comparison plots of the results of the calculations of the total cross section (shown in Table XI) with the Dataset 1 of Table I. Total cross sections by model 1, model 2, model 3, model 4, and model 5 are represented by filled squares, filled up triangles, filled diamonds, filled down triangles, and six pointed stars, respectively. The reference values for the corresponding dataset are represented by filled circles which are joined by a solid curve. (b) Same legend as for the part (a) of this figure with the results across the Dataset 2. (c) Same legend as for the part (a) with the results across the Dataset 3. (d) Same legend as for the part (a) with the results across the Dataset 4

C. Slope Parameter and Interaction Radius

The logarithmic slope parameter $B(s)$ is a key dynamical quantity in elastic scattering of hadrons [46, 47, 62]. It characterizes the fall-off of the differential cross section in the forward region. The slope parameter B [6, 55] from the models, and its predicted values are shown in the Table XIII for a fixed s and extrapolation at $t = 0$. For the slope parameter we use the following:

$$B(s, t) = \frac{d}{dt} \left[\ln \left(\frac{d\sigma(s, t)}{dt} \right) \Big|_{t=0} \right]. \quad (14)$$

The comparison plots of $B(s)$ results are shown in Figure 8. The ranges of the predicted B values are compared with reference value ranges in the Table XIV. Physically, $B(s)$ characterizes the width of the diffractive cone and encodes information about the spatial distribution of the strong-interaction matter inside the nucleon. Across all five models, the extracted slope parameters exhibit an increasing trend with energy. It agrees with the well-established "shrinkage" of the diffractive peak, predicted by Regge theory [4, 5]. The logarithmic growth of $B(s)$ with s is especially well reproduced by the models that include explicit $\ln(s/s_0)$ dependence in their parametrizations. The broadening of the effective interaction region is indicated by this growth as the collision energy increases. It reflects the increasing relevance of gluon-mediated processes and the broadening of the impact-parameter distribution [42].

The interaction radius R [27] is estimated from the extracted values of the slope parameter and mentioned in Table XIII. For the calculations of the interaction radius, we use:

$$R = 2\sqrt{B}. \quad (15)$$

The comparison plots of R are shown in Figure 9. The ranges of the predicted R values are compared with reference value ranges in the Table XV. For np scattering, the interaction radius increases from approximately 0.9 fm at $\sqrt{s} \approx 2$ GeV to

about 1.2 fm at $\sqrt{s} \approx 6$ GeV, consistent with the expansion of the diffractive cone and the geometric growth of the hadronic overlap region. The increase of $R(s)$ with energy is a manifestation of the "shrinkage" effect that reflects the spreading of the impact-parameter profile as the diffractive interaction becomes more forward-focused [27]. The breadth of the forward cone is inversely related to the size of the interaction region.

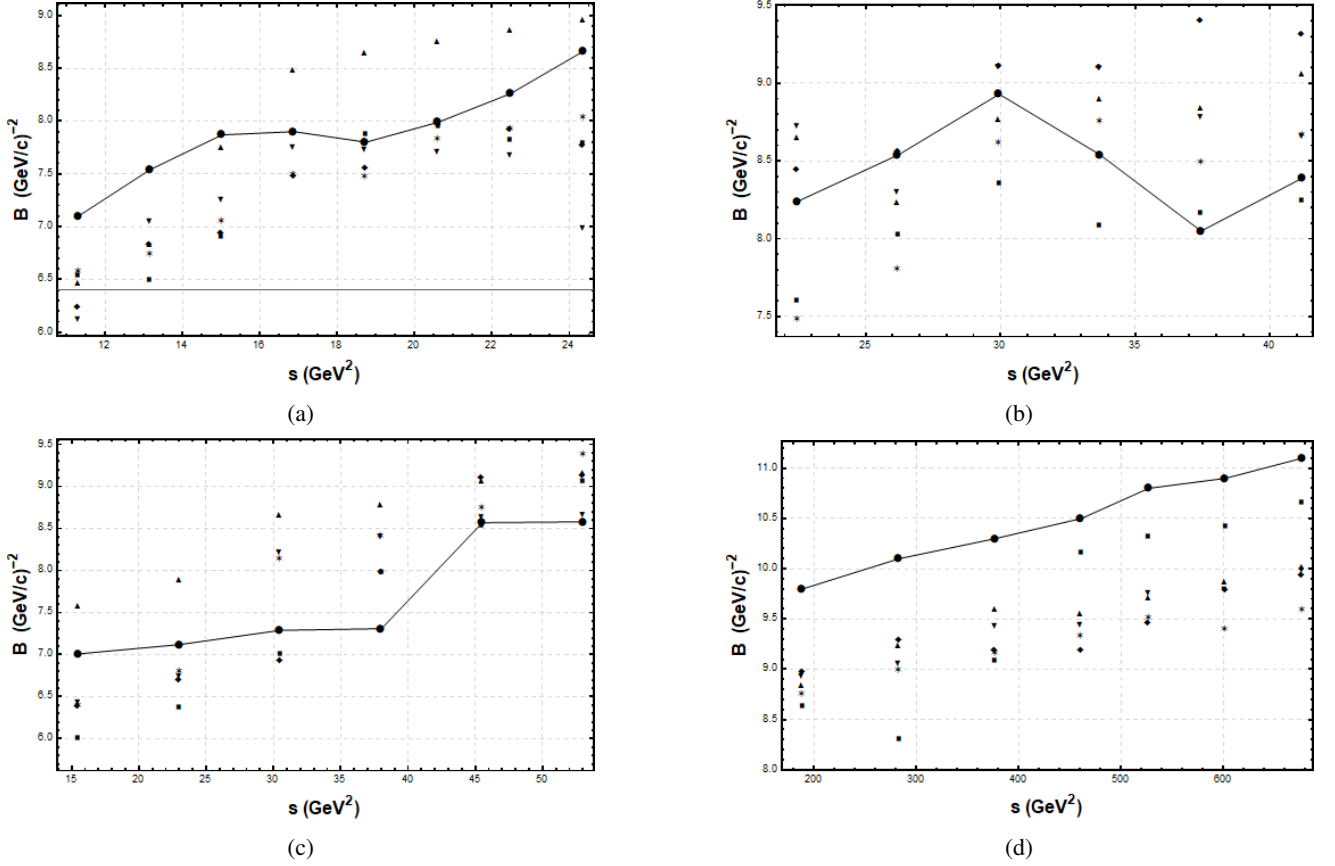


FIG. 8: (a) Graphical comparison plots of the results of the slope parameter calculations (shown in Table XIII) with Dataset 1 of Table I. Slope parameter values by model 1, model 2, model 3, model 4, and model 5 are represented by filled squares, filled up triangles, filled diamonds, filled down triangles, and six pointed stars, respectively. The reference values for the corresponding dataset are represented by filled circles which are joined by a solid curve. (b) Same legend as for the part (a) of this figure with the results across the Dataset 2. (c) Same legend as for part (a) of this figure with the results across the Dataset 3. (d) Same legend as for the part (a) with the results across the Dataset 4

TABLE XIV: Predicted ranges of the slope parameter $B(s)$

Dataset 1				Dataset 2			
Model	Predicted Range B (GeV/c) ⁻²	Ref. Range B (GeV/c) ⁻²	Least Difference B (GeV/c) ⁻²	Model	Predicted Range B (GeV/c) ⁻²	Ref. Range B (GeV/c) ⁻²	Least Difference B (GeV/c) ⁻²
1	6.5306-7.7745	6.98-8.76	1.4349	1	7.5948-8.3493	7.90-9.13	1.0859
2	6.4596-8.9536	6.98-8.76	0.714	2	8.2343-9.4263	7.90-9.13	0.6306
3	6.2392-6.9844	6.98-8.76	2.5164	3	8.4396-9.3986	7.90-9.13	0.8082
4	6.1201-7.7414	6.98-8.76	1.8785	4	8.2703-9.1009	7.90-9.13	0.3994
5	6.5641-8.0293	6.98-8.76	1.1466	5	7.4777-8.8344	7.90-9.13	0.7179
Min. ≈ 0.714				Min. ≈ 0.3994			
Dataset 3				Dataset 4			
Model	Predicted Range B (GeV/c) ⁻²	Ref. Range B (GeV/c) ⁻²	Least Difference B (GeV/c) ⁻²	Model	Predicted Range B (GeV/c) ⁻²	Ref. Range B (GeV/c) ⁻²	Least Difference B (GeV/c) ⁻²
1	5.9875-9.0428	6.58-9.2	0.7497	1	8.2903-10.6444	9.8-11.1	1.9653
2	7.5736-9.1583	6.58-9.2	1.0353	2	8.8351-10.0078	9.8-11.1	2.0571
3	6.3829-9.1268	6.58-9.2	0.2703	3	8.9671-9.9338	9.8-11.1	1.9991
4	6.4230-8.6575	6.58-9.2	0.6995	4	8.9260-9.9778	9.8-11.1	1.9962
5	6.3796-9.3671	6.58-9.2	0.3675	5	8.7418-9.5778	9.8-11.1	2.5804
Min. ≈ 0.2703				Min. ≈ 1.9653			

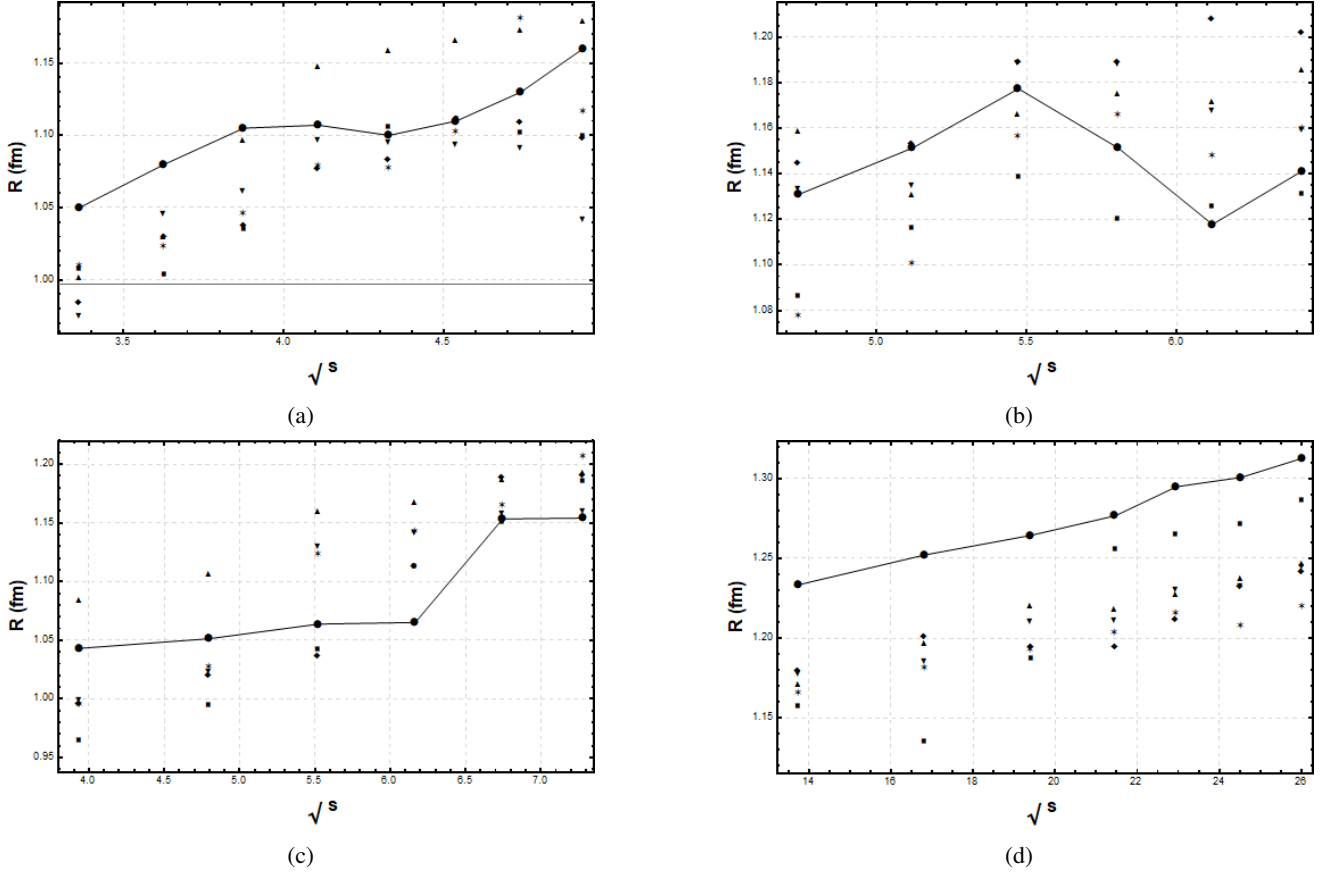


FIG. 9: (a) Graphical comparison plots of the results of the calculations of the interaction radius (shown in Table XIII) with the Dataset 1 of Table I. Interaction radius values by model 1, model 2, model 3, model 4, and model 5 are represented by filled squares, filled up triangles, filled diamonds, filled down triangles, and six pointed stars, respectively. The reference values for the corresponding dataset are represented by filled circles which are joined by a solid curve. (b) Same legend as for the part (a) of this figure with the results across the Dataset 2. (c) Same legend as for part (a) with the results across the Dataset 3. (d) Same legend as for the part (a) with the results across the Dataset 4

TABLE XV: Predicted ranges of the interaction radius R

Dataset 1				Dataset 2			
Model	Predicted Range R (fm)	Ref. Range R (fm)	Least Difference R (fm)	Model	Predicted Range R (fm)	Ref. Range R (fm)	Least Difference R (fm)
1	1.003-1.1102	1.05-1.16	0.0968	1	1.0857-1.1383	1.1178-1.1773	0.0711
2	1.0013-1.1788	1.05-1.16	0.0675	2	1.1304-1.2095	1.1178-1.1773	0.0448
3	0.9840-1.1116	1.05-1.16	0.1144	3	1.1445-1.2078	1.1178-1.1773	0.0572
4	0.9746-1.0961	1.05-1.16	0.1393	4	1.1329-1.1885	1.1178-1.1773	0.0263
5	1.0093-1.1163	1.05-1.16	0.0844	5	1.0773-1.1709	1.1178-1.1773	0.0469
Min. ≈ 0.0675				Min. ≈ 0.0263			
Dataset 3				Dataset 4			
Model	Predicted Range R (fm)	Ref. Range R (fm)	Least Difference R (fm)	Model	Predicted Range R (fm)	Ref. Range R (fm)	Least Difference R (fm)
1	0.9640-1.1847	1.0431-1.1847	0.1097	1	1.1343-1.2853	1.2334-1.3126	0.1264
2	1.0841-1.1922	1.0431-1.1847	0.0791	2	1.1710-1.2463	1.2334-1.3126	0.1287
3	0.9953-1.1902	1.0431-1.1847	0.0839	3	1.1797-1.2417	1.2334-1.3126	0.1246
4	0.9984-1.1592	1.0431-1.1847	0.0498	4	1.1770-1.2444	1.2334-1.3126	0.1246
5	0.9950-1.2057	1.0431-1.1847	0.0997	5	1.1648-1.2192	1.2334-1.3126	0.1620
Min. ≈ 0.0498				Min. ≈ 0.1246			

TABLE XIII: Results of the slope parameter and the interaction radius predicted by our models with the np data of Table I

$\sqrt{s}$	Model 1	Model 2	Model 3	Model 4	Model 5	Ref. values	Model 1	Model 2	Model 3	Model 4	Model 5	Ref. values
	B	B	B	B	B	B	R	R	R	R	R	R
	(GeV/c) $^{-2}$	(GeV/c) $^{-2}$	(GeV/c) $^{-2}$	(GeV/c) $^{-2}$	(GeV/c) $^{-2}$	(GeV/c) $^{-2}$	(fm)	(fm)	(fm)	(fm)	(fm)	(fm)
3.363 GeV	6.5306	6.45966	6.2392	6.1201	6.5641	7.10 ± 0.12 [27]	1.0068	1.0013	0.9840	0.9746	1.0093	1.05 [27]
3.628 GeV	6.4815	6.8311	6.8327	7.0417	6.7357	7.54 ± 0.10 [27]	1.0030	1.0297	1.0297	1.0454	1.0224	1.08 [27]
3.876 GeV	6.8876	7.7469	6.9387	7.2476	7.0419	7.87 ± 0.08 [27]	1.0339	1.0965	1.0377	1.0606	1.0454	1.105 [27]
4.109 GeV	7.8813	8.4824	7.4724	7.7414	7.4858	7.90 ± 0.08 [27]	1.1060	1.1474	1.0769	1.0961	1.0779	1.107 [27]
4.329 GeV	7.8648	8.6460	7.5559	7.7249	7.4693	7.80 ± 0.07 [27]	1.1048	1.1584	1.0829	1.0950	1.0767	1.10 [27]
4.54 GeV	7.9410	8.7541	7.9622	7.7011	7.8208	7.99 ± 0.07 [27]	1.1102	1.1656	1.1116	1.0933	1.1017	1.11 [27]
4.741 GeV	7.8103	8.8574	7.9215	7.6704	7.9144	8.26 ± 0.07 [27]	1.1010	1.1725	1.1088	1.0911	1.1083	1.13 [27]
4.935 GeV	7.7745	8.9536	7.7657	6.9844	8.0293	8.66 ± 0.10 [27]	1.0985	1.1788	1.0978	1.0411	1.1163	1.16 [27]
4.74 GeV	7.5948	8.651	8.4396	8.2703	7.4777	8.24 ± 0.26 [28]	1.0857	1.1587	1.1445	1.1329	1.0773	1.1309 [28]
5.12 GeV	8.0214	8.2343	8.5630	8.2937	7.8015	8.54 ± 0.23 [28]	1.1158	1.13047	1.1528	1.1345	1.1004	1.1513 [28]
5.473 GeV	8.3493	8.7612	9.1099	9.1009	8.6091	8.93 ± 0.20 [28]	1.1383	1.16608	1.1891	1.1885	1.1559	1.1773 [28]
5.805 GeV	8.0739	8.8956	9.1044	9.0951	8.7536	8.54 ± 0.17 [28]	1.1194	1.1750	1.1887	1.1881	1.1656	1.1513 [28]
6.119 GeV	8.1594	8.8400	9.3986	8.7793	8.4877	8.05 ± 0.15 [28]	1.1253	1.1713	1.2078	1.1673	1.1477	1.1178 [28]
6.418 GeV	8.2368	9.0564	9.3100	8.6557	8.6643	8.39 ± 0.14 [28]	1.1306	1.18556	1.2020	1.1590	1.1596	1.1412 [28]
6.704 GeV	8.1064	9.4263	8.9946	8.5254	8.8344	-	1.1217	1.2095	1.1815	1.1503	1.1709	-
3.466-4.409 GeV	5.9875	7.5736	6.3829	6.4230	6.3796	7.01 ± 0.43 [30]	0.9640	1.0841	0.9953	0.99843	0.9950	1.0431 [30]
4.409-5.187 GeV	6.3566	7.8901	6.6983	6.7394	6.7963	7.12 ± 0.33 [30]	0.9933	1.1065	1.0196	1.0227	1.0270	1.0513 [30]
5.187-5.863 GeV	6.9866	8.6562	6.9241	8.2157	8.1235	7.29 ± 0.35 [30]	1.0413	1.1590	1.0366	1.1292	1.1228	1.0638 [30]
5.863-6.47 GeV	7.9693	8.7721	7.9803	8.3915	8.3998	7.31 ± 0.35 [30]	1.1121	1.1668	1.1129	1.1412	1.1417	1.0652 [30]
6.47-7.024 GeV	8.5182	9.0673	9.0958	8.6265	8.7352	8.57 ± 0.53 [30]	1.1498	1.1863	1.1881	1.1571	1.1644	1.1534 [30]
7.024-7.538 GeV	9.0428	9.1583	9.1268	8.6575	9.3671	8.58 ± 0.62 [30]	1.1847	1.1922	1.1902	1.1592	1.2057	1.1541 [30]
13.748 GeV	8.6158	8.8351	8.9671	8.9260	8.7418	9.8 [31]	1.1564	1.1710	1.1797	1.177	1.1648	1.2334 [31]
16.823 GeV	8.2903	9.2270	9.2904	9.0492	8.9842	10.1 [31]	1.1343	1.1967	1.2008	1.1851	1.1808	1.2521 [31]
19.416 GeV	9.0701	9.5890	9.1900	9.4290	9.1563	10.3 [31]	1.1865	1.2199	1.1943	1.2097	1.1921	1.2644 [31]
21.471 GeV	10.145	9.5502	9.1860	9.4400	9.3271	10.5 [31]	1.2548	1.2175	1.1940	1.2104	1.2032	1.2767 [31]
22.956 GeV	10.302	9.7072	9.4581	9.7472	9.5075	10.8 [31]	1.26447	1.2274	1.2116	1.2299	1.2147	1.2948 [31]
24.536 GeV	10.404	9.8638	9.7848	9.7838	9.3872	10.9 [31]	1.27071	1.2373	1.2323	1.2323	1.2070	1.3007 [31]
26.019 GeV	10.6444	10.0078	9.9338	9.9778	9.5778	11.1 [31]	1.2853	1.2463	1.2417	1.2444	1.2192	1.3126 [31]

D. Elastic and Inelastic Cross Sections

The expression for elastic cross section σ_{el} is discussed in ref. [71, 72] in detail. It is mentioned as follows:

$$\sigma_{el} = \frac{\sigma_{tot}^2(1 + \rho^2)}{16\pi B}. \quad (16)$$

This expression is used for all the datasets to estimate the contribution of the forward region of the np elastic differential cross section to the elastic cross section σ_{el} . The values of ρ , the predicted values of the total cross section σ_{tot} , and the slope parameter B that are mentioned in Tables XI and XIII are used to estimate the σ_{el} . The references values for comparison of the calculations of the σ_{el} are estimated by using the Eq. 16 with the reference values of ρ , σ_{tot} , and B , written in Tables XI and XIII for each dataset. The predicted σ_{el} values are shown in Table XVI, and comparison of the predicted ranges of σ_{el} is given in Table XVII.

The total elastic cross section [71], evaluated through the diffractive-cone relation (Eq. 16) is one of the most sensitive observables of the forward scattering amplitude in np elastic scattering. This expression is derived under the assumption that the hadronic amplitude at small momentum transfer is dominated by a single exponential profile that emerges naturally from Regge pole dominance and diffraction models. Near to quadratic dependence on σ_{tot} stems from the optical theorem, making σ_{el} extremely responsive to even modest variations in the strength of the total interaction. The inverse dependence on the slope parameter B reveals a deep geometric interpretation [71, 72]. In the impact-parameter representation the quantity B is proportional to the mean squared transverse interaction radius which implies that a larger spatial extension of the interaction spreads the amplitude over broader t -space and suppresses the integrated elastic strength. Consequently, σ_{el} encodes an interplay between 1) the opacity of the hadronic core (or blackness of the core), which determines the amount of absorption and, the maximum possible elastic scattering, 2) the diffractive coherence of peripheral mesonic exchanges, and 2) the spatial localization of the scattering region that refers to the effective size and density profile of the interacting particles [71–74].

The integrated elastic cross section is predicted in the specified $|t|$ -ranges mentioned in Table I by integration of our fitted models by using:

$$\sigma_{el} = \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dt} dt. \quad (17)$$

The results of the integrated elastic cross section are given in Table XVI.

The inelastic cross section σ_{inel} is predicted by using predicted σ_{tot} and σ_{el} (by Eq. 16), mentioned in Tables XI and XVI

The σ_{inel} results are written in Table XVIII. Following relation is used for σ_{inel} [70, 75]:

$$\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}. \quad (18)$$

The ranges of the predicted values of the σ_{inel} are compared with their reference value ranges in Table XIX. The inelastic cross section provides critical insight into the absorptive, non-coherent elements of the hadronic interaction. Although its algebraic form is simple, it contains a quantitative contribution of all processes beyond pure elastic scattering, including excitation of internal nucleon degrees of freedom, charge-exchange processes, meson-production, and diffractive dissociation components that do not leave both nucleons intact [76]. The relative magnitudes of σ_{el} and σ_{inel} indicate a dominance of σ_{inel} at intermediate energies and reflect a predominantly absorptive core a typical feature of nucleon-nucleon interactions where short-range dynamics becomes important.

TABLE XVI: Results of the elastic cross section σ_{el} for the np data of Table I predicted by our models of the differential cross section

$\sqrt{s}$	σ_{el} from Eq. 16					σ_{el} from integration of the models					Reference values of σ_{el}
	Model 1 (mb)	Model 2 (mb)	Model 3 (mb)	Model 4 (mb)	Model 5 (mb)	Model 1 (mb)	Model 2 (mb)	Model 3 (mb)	Model 4 (mb)	Model 5 (mb)	Ref. (mb)
3.363 GeV	6.3933	6.6108	6.8617	6.9643	6.5207	7.7073	7.3785	7.8899	8.1177	7.1562	6.0275 ± 0.10 [27]
3.628 GeV	6.1157	5.921	5.9250	5.7441	6.0216	6.7682	6.2807	6.2923	5.9358	6.4580	5.3752 ± 0.07 [27]
3.876 GeV	5.6519	5.0413	5.6886	5.4546	5.5566	6.0072	4.7329	5.9373	5.4672	5.7035	4.9173 ± 0.05 [27]
4.109 GeV	4.8381	4.5041	5.1116	4.9174	5.0947	4.4639	3.8904	4.9633	4.6027	4.9215	4.8258 ± 0.05 [27]
4.329 GeV	4.7508	4.3305	4.9310	4.8546	5.0216	4.5363	3.5975	4.8773	4.6934	5.006	4.7894 ± 0.04 [27]
4.54 GeV	4.5947	4.2039	4.5661	4.8028	4.6901	3.7800	2.9788	3.7428	4.0869	3.9150	4.5936 ± 0.04 [27]
4.741 GeV	4.59317	4.0844	4.5776	4.7243	4.5679	3.0742	2.2392	3.0285	3.2624	3.0146	4.3751 ± 0.03 [27]
4.935 GeV	4.5872	3.9842	4.6015	5.1188	4.4524	3.0886	2.2640	3.1298	3.9790	3.1696	4.1158 ± 0.04 [27]
4.74 GeV	4.3784	4.0306	4.1215	4.2103	4.6463	7.7329	6.0661	6.2294	6.4811	7.5248	4.052 ± 0.12 [28]
5.12 GeV	4.1666	4.1616	3.9748	4.1145	4.3564	7.0204	6.5110	5.9632	6.3270	6.9101	3.8474 ± 0.10 [28]
5.473 GeV	3.8956	3.7383	3.5672	3.6155	3.7879	6.4905	5.7313	5.2692	5.4002	5.8381	3.6485 ± 0.08 [28]
5.805 GeV	4.1285	3.7101	3.6158	3.6461	3.7530	6.7459	5.4979	5.2141	5.3209	5.5964	3.7828 ± 0.07 [28]
6.119 GeV	4.0703	3.6989	3.4458	3.7167	3.8161	6.5228	5.5393	4.8757	5.5360	5.7899	3.9809 ± 0.07 [28]
6.418 GeV	4.0364	3.5431	3.4393	3.7032	3.6936	6.3392	5.2302	4.8954	5.5462	5.5305	3.7924 ± 0.06 [28]
6.704 GeV	4.0905	3.3620	3.5274	3.7121	3.5778	0.5154	0.3083	0.3304	0.4270	0.3657	-
3.466-4.409 GeV	6.4418	5.0912	6.0377	6.0061	6.0428	6.2019	3.7487	5.6534	5.1281	5.2059	4.8210 ± 0.29 [30]
4.409-5.187 GeV	5.7183	4.62154	5.4448	5.4026	5.3612	5.1774	3.2080	4.4489	4.3732	4.3077	4.6484 ± 0.21 [30]
5.187-5.863 GeV	4.8183	4.0447	5.0537	4.2562	4.3049	4.1138	2.3765	3.0748	2.6656	2.7423	4.4514 ± 0.21 [30]
5.863-6.47 GeV	4.3101	3.8688	4.2487	4.0518	4.0470	1.9946	1.4753	1.8924	1.6486	1.6453	4.3579 ± 0.20 [30]
6.47-7.024 GeV	3.8860	3.6404	3.6438	3.8424	3.7844	1.5301	1.2481	1.1994	1.4322	1.3763	3.6538 ± 0.22 [30]
7.024-7.538 GeV	3.5754	3.5355	3.5459	3.7321	3.4500	0.5259	0.4842	0.4668	0.5880	0.4316	3.5923 ± 0.25 [30]
13.748 GeV	3.51097	3.4227	3.3725	3.3859	3.4604	2.4942	2.3691	2.2679	2.2896	2.4053	3.0831 [31]
16.823 GeV	3.6882	3.3138	3.2874	3.3740	3.4081	2.7836	2.1565	2.1044	2.2389	2.2829	3.0222 [31]
19.416 GeV	3.4015	3.2445	3.3859	3.2807	3.4025	2.2424	1.9886	2.1970	2.0535	2.2193	3.0158 [31]
21.471 GeV	3.0650	3.2875	3.4206	3.3371	3.3626	2.4293	2.0564	2.2397	2.1052	2.1568	2.9867 [31]
22.956 GeV	3.0524	3.27	3.3556	3.26547	3.3353	2.3624	1.9940	2.1535	1.9662	2.0815	2.9340 [31]
24.536 GeV	3.0876	3.2533	3.2758	3.2799	3.4185	2.6870	1.9347	1.9620	1.9640	2.1704	2.9366 [31]
26.019 GeV	3.0353	3.2467	3.2605	3.2479	3.4003	2.2244	1.4839	1.5035	1.4853	1.6591	2.9131 [31]

E. Ratios of Elastic and Inelastic Cross Sections to Total Cross Section

The ratios $R_{\text{el}} \equiv \sigma_{\text{el}}/\sigma_{\text{tot}}$ and $R_{\text{inel}} \equiv \sigma_{\text{inel}}/\sigma_{\text{tot}} = 1 - R_{\text{el}}$ are compact, model-independent measures of how the total interaction probability is partitioned between elastic scattering and inelastic production. R_{el} quantifies the degree of coherence and opacity of the scattering system [73]. In the black-disk limit $R_{\text{el}} \rightarrow 1/2$, whereas a small R_{el} signals strong absorption into inelastic processes. Through the forward-cone relation Eq. 16, R_{el} also links the overall cross-section scale σ_{tot} to the diffraction slope B and the ratio ρ . Empirically, the energy dependence of R_{el} reveals the changing balance between diffraction and multiparticle production and constrains phenomenological models [10, 73].

In this study, the results of R_{el} and R_{inel} , shown in Table XX, are found to be in quite good quantitative agreement with their corresponding reference values for all the four datasets. It is evident from the Table XX that at the c.m. energy domains, $\sqrt{s} = 3.363 \text{ GeV} - 4.935 \text{ GeV}$, $\sqrt{s} = 4.74 \text{ GeV} - 6.704 \text{ GeV}$, $\sqrt{s} = 3.466-4.409 \text{ GeV}$ to $7.024-7.538 \text{ GeV}$, and $\sqrt{s} = 13.748 \text{ GeV} - 26.019 \text{ GeV}$, R_{el} values show a decreasing trend. And the R_{inel} values, predicted by the models for all these c.m. energy domains show an increasing trend. The ranges of the predicted values of R_{el} and R_{el} are compared with the reference value ranges in Tables XXI and XXII.

The ratio R_{el} in pp scattering tends to grow steadily with energy this reflects increasing diffractive coherence [66, 74]. While in np interactions this ratio is found to evolve more slowly which is indicative of a stronger absorptive (inelastic) component and of the persistence of meson-exchange mechanisms even at intermediate energies [37]. Correspondingly, the slope parameter B in np scattering increases with $\sqrt{s}$ but at a noticeably reduced rate compared to pp [27, 28, 30, 31].

TABLE XVII: Predicted ranges of the elastic cross section σ_{el} (via Eq. 16)

Dataset 1				Dataset 2			
Model	Predicted Range σ_{el} (mb)	Ref. Range σ_{el} (mb)	Least Difference σ_{el} (mb)	Model	Predicted Range σ_{el} (mb)	Ref. Range σ_{el} (mb)	Least Difference σ_{el} (mb)
1	4.5872-6.3933	4.0758-6.1275	0.7772	1	4.0364-4.3784	3.5685-4.172	0.6743
2	3.9842-6.6108	4.0758-6.1275	0.5749	2	3.3620-4.1616	3.5685-4.172	0.2169
3	4.5661-6.8617	4.0758-6.1275	1.2245	3	3.4393-4.1215	3.5685-4.172	0.1797
4	4.7243-6.9643	4.0758-6.1275	1.4853	4	3.6155-4.2103	3.5685-4.172	0.0853
5	4.4524-6.5207	4.0758-6.1275	0.7698	5	3.5778-4.6463	3.5685-4.172	0.4836
Min. ≈ 0.5749				Min. ≈ 0.0853			
Dataset 3				Dataset 4			
Model	Predicted Range σ_{el} (mb)	Ref. Range σ_{el} (mb)	Least Difference σ_{el} (mb)	Model	Predicted Range σ_{el} (mb)	Ref. Range σ_{el} (mb)	Least Difference σ_{el} (mb)
1	3.5754-6.4418	3.3423-5.111	1.5639	1	3.0353-3.6882	2.9131-3.0831	0.7273
2	3.5355-5.0912	3.3423-5.111	0.213	2	3.2445-3.4227	2.9131-3.0831	0.671
3	3.5459-6.0377	3.3423-5.111	1.1303	3	3.2605-3.4206	2.9131-3.0831	0.6849
4	3.7321-6.0061	3.3423-5.111	1.2849	4	3.2479-3.3859	2.9131-3.0831	0.6376
5	3.4500-6.0428	3.3423-5.111	1.0395	5	3.3353-3.4604	2.9131-3.0831	0.7
Min. ≈ 0.213				Min. ≈ 0.6376			

TABLE XVIII: Results of np inelastic cross section σ_{inel}

$symbol \sqrt{s}$	Model 1	Model 2	Model 3	Model 4	Model 5	Ref.
	σ_{inel} (mb)	σ_{inel} (mb)	σ_{inel} (mb)	σ_{inel} (mb)	σ_{inel} (mb)	σ_{inel} (mb)
3.363 GeV	34.7854	35.0347	34.8363	34.6413	35.1729	35.6625 ± 0.10 [27]
3.628 GeV	34.2041	34.8074	34.8218	34.985	34.7638	35.3948 ± 0.07 [27]
3.876 GeV	34.4969	35.1726	34.7398	35.0052	34.6956	35.3327 ± 0.05 [27]
4.109 GeV	35.0853	35.4589	34.8461	34.9732	34.8325	35.0942 ± 0.05 [27]
4.329 GeV	34.9696	34.4145	34.7166	34.922	34.7585	34.9106 ± 0.04 [27]
4.54 GeV	34.8188	35.3794	34.7768	34.8799	34.828	34.9364 ± 0.04 [27]
4.741 GeV	34.6625	34.3367	34.8894	34.7296	34.84	35.0249 ± 0.03 [27]
4.935 GeV	34.7238	35.3322	34.7485	34.241	34.9065	35.1842 ± 0.04 [27]
4.74 GeV	35.5761	35.9573	35.8173	35.7498	35.2695	34.948 ± 0.12 [28]
5.12 GeV	35.6993	35.6668	35.7191	35.6307	35.3081	35.1526 ± 0.10 [28]
5.473 GeV	35.8068	35.8295	35.8465	36.044	35.6942	35.3515 ± 0.08 [28]
5.805 GeV	35.4872	35.7094	35.7534	35.8678	35.5762	35.2172 ± 0.07 [28]
6.119 GeV	35.3606	35.6958	35.7601	35.6368	35.3926	35.0191 ± 0.07 [28]
6.418 GeV	35.2972	35.6217	35.684	35.4408	35.4187	35.2076 ± 0.06 [28]
6.704 GeV	34.96	35.6984	35.5552	35.3209	35.4308	-
3.466-4.409 GeV	34.1545	35.4989	34.5415	34.5935	34.5428	33.179 ± 0.29 [30]
4.409-5.187 GeV	34.1049	35.265	34.4453	34.454	34.5099	33.3516 ± 0.21 [30]
5.187-5.863 GeV	34.8855	35.4266	34.4065	35.1899	35.1431	33.5486 ± 0.21 [30]
5.863-6.47 GeV	35.1486	35.3532	34.9548	35.2067	35.2074	33.6421 ± 0.20 [30]
6.47-7.024 GeV	35.1843	35.375	35.4511	35.2545	35.2599	34.3462 ± 0.22 [30]
7.024-7.538 GeV	35.3441	35.4129	35.3925	35.1751	35.4606	34.4077 ± 0.25 [30]
13.748 GeV	35.7875	35.3939	35.445	35.4193	35.3621	35.7169 [31]
16.823 GeV	36.046	35.8194	35.8236	35.7313	35.7526	36.0778 [31]
19.416 GeV	36.3134	36.2866	36.1479	36.1372	36.1556	36.4842 [31]
21.471 GeV	36.4667	36.4354	36.3186	36.4527	36.3395	36.7133 [31]
22.956 GeV	36.6953	36.6648	36.576	36.7236	36.5792	36.966 [31]
24.536 GeV	37.0842	36.8976	36.852	36.871	36.7106	37.1634 [31]
26.019 GeV	37.3214	37.1512	37.0726	37.0965	36.9426	37.3869 [31]

This reduced rate signals a more gradual transverse expansion of the interaction radius R_{int} and a weaker approach to the black-disc limit [73, 74]. The predicted results of $B(s)$, R , and R_{el} in this study for np elastic scattering can be analyzed together with the results at much higher energies of ISR [67] and TOTEM at LHC [68] for pp and $p\bar{p}$ elastic scattering data. The energy trends predicted by our models respect these well-known qualitative features. As $\sqrt{s}$ increases, the total cross section exhibits the expected slow logarithmic growth, while the elastic component rises more modestly due to the interplay between the forward amplitude normalization and the t -slope. The ratio R_{el} , remains significantly below the corresponding pp values. It confirms that np scattering remains far from saturating the unitarity bound or achieving diffractive dominance over the studied range. Meanwhile, the inelastic fraction R_{inel} stays comparatively large which reveals that inelastic processes remain the principal contributors to the attenuation of the forward amplitude. This indicates that elastic scattering is a less dominant process relative to the total interaction cross-section in np collisions compared to pp

TABLE XIX: Predicted ranges of the inelastic cross section σ_{inel}

Dataset 1				Dataset 2			
Model	Predicted Range σ_{inel} (mb)	Ref. Range σ_{inel} (mb)	Least Difference σ_{inel} (mb)	Model	Predicted Range σ_{inel} (mb)	Ref. Range σ_{inel} (mb)	Least Difference σ_{inel} (mb)
1	34.2041-35.0853	34.8706-35.7625	1.3437	1	34.96-35.8068	34.828-35.315	0.5073
2	34.3367-35.4589	34.8706-35.7625	0.8375	2	35.6217-35.9573	34.828-35.315	1.3195
3	34.7166-34.8894	34.8706-35.7625	1.0271	3	35.5552-35.8465	34.828-35.315	1.1422
4	34.241-35.0052	34.8706-35.7625	1.3869	4	35.3209-36.044	34.828-35.315	1.1054
5	34.6956-35.1729	34.8706-35.7625	0.7646	5	35.2695-35.6942	34.828-35.315	0.7042
Min. ≈ 0.7646				Min. ≈ 0.5073			
Dataset 3				Dataset 4			
Model	Predicted Range σ_{inel} (mb)	Ref. Range σ_{inel} (mb)	Least Difference σ_{inel} (mb)	Model	Predicted Range σ_{inel} (mb)	Ref. Range σ_{inel} (mb)	Least Difference σ_{inel} (mb)
1	34.1049-35.3441	32.889-34.6577	1.0923	1	35.7875-37.3214	35.7169-37.3869	0.1361
2	35.265-35.4989	32.889-34.6577	3.2172	2	35.3939-37.1512	35.7169-37.3869	0.5587
3	34.4065-35.4511	32.889-34.6577	2.3109	3	35.445-37.0726	35.7169-37.3869	0.5862
4	35.2545-35.454	32.889-34.6577	3.1618	4	35.4193-37.0965	35.7169-37.3869	0.588
5	34.5099-35.4606	32.889-34.6577	2.4238	5	35.3621-36.9426	35.7169-37.3869	0.7991
Min. ≈ 1.0923				Min. ≈ 0.1361			

collisions within the studied energy range. The extracted slope parameter $B(s)$ grows slowly with energy, in accord with Regge expectations [4]. While the corresponding interaction radius displays a gradual expansion. It suggests the increasing role of peripheral contributions from long-range mesonic fields. Altogether, the description provided by the analysis in this study, using the proposed models apprehends the characteristic energy dependence of np scattering and aligns quantitatively with the established nucleon–nucleon systematics.

The correlation among σ_{tot} , σ_{el} , σ_{inel} , $B(s)$, and the ratio $\rho(s)$ is significant as a precise consistency test for phenomenological models of elastic scattering. Modern analyses, measurements [35, 41, 71, 72, 75], and recent holographic QCD approaches [12] emphasize that precise simultaneous predictions of these observables is essential to ensure the internal coherence of the theoretical description.

TABLE XX: Ratios of the elastic cross section to the total cross section (R_{el}) and the inelastic cross section to the total cross section (R_{inel})

$\sqrt{s}$	Model 1	Model 2	Model 3	Model 4	Model 5	Ref. values	Model 1	Model 2	Model 3	Model 4	Model 5	Ref. values
	R_{el}	R_{el}	R_{el}	R_{el}	R_{el}	R_{el}	R_{inel}	R_{inel}	R_{inel}	R_{inel}	R_{inel}	R_{inel}
3.363 GeV	0.1552	0.1587	0.1645	0.1673	0.1563	0.1445 ± 0.10 [27]	0.8448	0.8413	0.8355	0.8327	0.8437	0.8555 ± 0.10 [27]
3.628 GeV	0.1516	0.1453	0.1454	0.1410	0.1476	0.1318 ± 0.07 [27]	0.8484	0.8547	0.8546	0.859	0.8524	0.8682 ± 0.07 [27]
3.876 GeV	0.1407	0.1246	0.1465	0.1419	0.1380	0.1222 ± 0.05 [27]	0.8593	0.8754	0.8535	0.8581	0.862	0.8778 ± 0.05 [27]
4.109 GeV	0.1211	0.1127	0.1279	0.1232	0.1275	0.12 ± 0.05 [27]	0.8789	0.8873	0.8721	0.8768	0.8725	0.88 ± 0.05 [27]
4.329 GeV	0.1196	0.1089	0.1243	0.1220	0.1262	0.1206 ± 0.04 [27]	0.8804	0.8911	0.8757	0.878	0.8738	0.8794 ± 0.04 [27]
4.54 GeV	0.1165	0.1062	0.1160	0.1210	0.1186	0.1162 ± 0.04 [27]	0.8835	0.8938	0.884	0.879	0.8814	0.8838 ± 0.04 [27]
4.741 GeV	0.1170	0.1036	0.1159	0.1197	0.1159	0.1110 ± 0.03 [27]	0.883	0.8964	0.8841	0.8803	0.8841	0.889 ± 0.03 [27]
4.935 GeV	0.1166	0.1013	0.1169	0.1300	0.1131	0.1047 ± 0.10 [27]	0.8834	0.8987	0.8831	0.87	0.8869	0.8953 ± 0.10 [27]
4.74 GeV	0.1095	0.1007	0.12	0.1053	0.1164	0.1038 ± 0.26 [28]	0.8905	0.8993	0.88	0.8947	0.8836	0.8962 ± 0.26 [28]
5.12 GeV	0.1045	0.1044	0.1001	0.1035	0.1098	0.099 ± 0.10 [28]	0.8955	0.8956	0.8999	0.8965	0.8902	0.901 ± 0.10 [28]
5.473 GeV	0.098	0.0944	0.0905	0.0911	0.0959	0.0935 ± 0.08 [28]	0.902	0.9056	0.9095	0.9089	0.9041	0.9065 ± 0.08 [28]
5.805 GeV	0.1042	0.0941	0.0918	0.0922	0.0954	0.0969 ± 0.07 [28]	0.8958	0.9059	0.9082	0.9078	0.9046	0.9031 ± 0.07 [28]
6.119 GeV	0.1032	0.0938	0.0878	0.0944	0.0973	0.10 ± 0.07 [28]	0.8968	0.9062	0.9122	0.9056	0.9027	0.9 ± 0.07 [28]
6.418 GeV	0.1026	0.0905	0.0879	0.0946	0.0944	0.0972 ± 0.06 [28]	0.8974	0.9095	0.9121	0.9054	0.9056	0.9028 ± 0.06 [28]
6.704 GeV	0.1047	0.0860	0.0903	0.0951	0.0917	-	0.8953	0.914	0.9097	0.9049	0.9083	-
3.466-4.409 GeV	0.1586	0.1254	0.1487	0.1479	0.1488	0.1268 ± 0.29 [30]	0.8414	0.8746	0.8513	0.8521	0.8512	0.8732 ± 0.29 [30]
4.409-5.187 GeV	0.1435	0.1158	0.1364	0.1355	0.1344	0.1223 ± 0.21 [30]	0.8565	0.8842	0.8636	0.8645	0.8656	0.8777 ± 0.21 [30]
5.187-5.863 GeV	0.1213	0.1024	0.1280	0.1078	0.1091	0.1171 ± 0.21 [30]	0.8787	0.8976	0.872	0.8922	0.8909	0.8829 ± 0.21 [30]
5.863-6.47 GeV	0.1092	0.0986	0.1083	0.1032	0.1030	0.1146 ± 0.20 [30]	0.8908	0.9014	0.8917	0.8968	0.897	0.8854 ± 0.20 [30]
6.47-7.024 GeV	0.0994	0.0933	0.0932	0.0982	0.0969	0.0961 ± 0.22 [30]	0.9006	0.9067	0.9068	0.9018	0.9031	0.9039 ± 0.22 [30]
7.024-7.538 GeV	0.0918	0.0907	0.0910	0.0959	0.0887	0.0945 ± 0.25 [30]	0.9082	0.9093	0.909	0.9041	0.9113	0.9055 ± 0.25 [30]
13.748 GeV	0.0904	0.0881	0.0868	0.0872	0.0891	0.0794 [31]	0.9096	0.9119	0.9132	0.9128	0.9109	0.9206 [31]
16.823 GeV	0.0942	0.0846	0.0840	0.0862	0.0870	0.0772 [31]	0.9058	0.9154	0.916	0.9138	0.913	0.9228 [31]
19.416 GeV	0.0864	0.0820	0.0856	0.0832	0.0860	0.0763 [31]	0.9136	0.918	0.9144	0.9168	0.914	0.9237 [31]
21.471 GeV	0.0775	0.0827	0.0860	0.0838	0.0846	0.0752 [31]	0.9225	0.9173	0.914	0.9162	0.9154	0.9248 [31]
22.956 GeV	0.0767	0.0818	0.0840	0.0816	0.0835	0.0735 [31]	0.9233	0.9182	0.916	0.9184	0.9165	0.9265 [31]
24.536 GeV	0.0768	0.0810	0.0816	0.0816	0.0852	0.0732 [31]	0.9232	0.919	0.9184	0.9184	0.9148	0.9268 [31]
26.019 GeV	0.0753	0.0803	0.0808	0.0805	0.0842	0.07 [31]	0.9247	0.9197	0.9192	0.9195	0.9158	0.93 [31]

TABLE XXI: Predicted ranges of ratio of the elastic cross section to the total cross section R_{el}

Dataset 1				Dataset 2			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	R_{el}	R_{el}	R_{el}		R_{el}	R_{el}	R_{el}
1	0.1165-0.1552	0.1047-0.1445	0.0225	1	0.098-0.1095	0.0935-0.1038	0.0102
2	0.1013-0.1587	0.1047-0.1445	0.0176	2	0.086-0.1044	0.0935-0.1038	0.0081
3	0.1159-0.1645	0.1047-0.1445	0.0312	3	0.0878-0.12	0.0935-0.1038	0.0219
4	0.1197-0.1673	0.1047-0.1445	0.0378	4	0.0911-0.1053	0.0935-0.1038	0.0039
5	0.1131-0.1563	0.1047-0.1445	0.0202	5	0.0917-0.1164	0.0935-0.1038	0.0144
Min. ≈ 0.0176				Min. ≈ 0.0039			
Dataset 3				Dataset 4			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	R_{el}	R_{el}	R_{el}		R_{el}	R_{el}	R_{el}
1	0.0918-0.1586	0.0945-0.1268	0.0345	1	0.0753-0.0942	0.07-0.0794	0.0201
2	0.0907-0.1254	0.0945-0.1268	0.0052	2	0.0803-0.0881	0.07-0.0794	0.019
3	0.091-0.1487	0.0945-0.1268	0.0254	3	0.0808-0.0868	0.07-0.0794	0.0182
4	0.0959-0.1479	0.0945-0.1268	0.0225	4	0.0805-0.0872	0.07-0.0794	0.0183
5	0.0887-0.1488	0.0945-0.1268	0.0278	5	0.0835-0.0891	0.07-0.0794	0.0232
Min. ≈ 0.0052				Min. ≈ 0.0182			

TABLE XXII: Predicted ranges of ratio of the inelastic cross section to the total cross section R_{inel}

Dataset 1				Dataset 2			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	R_{inel}	R_{inel}	R_{inel}		R_{inel}	R_{inel}	R_{inel}
1	0.8448-0.8835	0.8555-0.8953	0.0225	1	0.8905-0.902	0.8962-0.9065	0.0102
2	0.8413-0.8987	0.8555-0.8953	0.0176	2	0.8956-0.914	0.8962-0.9065	0.0081
3	0.8355-0.8841	0.8555-0.8953	0.0312	3	0.88-0.9122	0.8962-0.9065	0.0219
4	0.8327-0.8803	0.8555-0.8953	0.0378	4	0.8947-0.9089	0.8962-0.9065	0.0039
5	0.8437-0.8869	0.8555-0.8953	0.0202	5	0.8836-0.9083	0.8962-0.9065	0.0144
Min. ≈ 0.0176				Min. ≈ 0.0039			
Dataset 3				Dataset 4			
Model	Predicted Range	Ref. Range	Least Difference	Model	Predicted Range	Ref. Range	Least Difference
	R_{inel}	R_{inel}	R_{inel}		R_{inel}	R_{inel}	R_{inel}
1	0.8414-0.9082	0.8732-0.9055	0.0348	1	0.9058-0.9247	0.9206-0.93	0.0201
2	0.8746-0.9093	0.8732-0.9055	0.0052	2	0.9119-0.9197	0.9206-0.93	0.019
3	0.8513-0.909	0.8732-0.9055	0.0254	3	0.9132-0.9192	0.9206-0.93	0.0182
4	0.8521-0.9041	0.8732-0.9055	0.0225	4	0.9128-0.9195	0.9206-0.93	0.0183
5	0.8512-0.9113	0.8732-0.9055	0.0278	5	0.9109-0.9165	0.9206-0.93	0.0232
Min. ≈ 0.0052				Min. ≈ 0.0182			

IV. CONCLUSION

The experimental data on elastic and soft diffractive scattering lacks global description from the first principles of QCD, which necessitates to perform model-independent analyses that provide useful phenomenological insights for the development of the theory in the soft sector. In this view, we have analyzed neutron-proton (np) elastic scattering by employing five distinct purely phenomenological models that also aim to fit the data of $n\bar{p}$, pp , and $p\bar{p}$ elastic differential cross section (mentioned in Section II). The diverse nature of these exponential parametrizations including logarithmic energy dependence, nonlinear t -modifications, deformation functions, and additional Regge-inspired components allows to reproduce the behavior of the elastic differential cross section data across various GeV energy domains ($\sqrt{s} = 3.363$ GeV - 26.019 GeV). Several key features of the models markedly include analyticity, differentiability, and integrability in both s and t which ensure their consistency and wide applicability to reproduce experimental and extrapolated data of elastic differential cross section at GeV and TeV energy domains. The modeling approach of this study allows not only to reproduce the main empirical features of the data but also to extract physically meaningful observables that give insights on the underlying mechanisms of np scattering. All five proposed models demonstrate an efficient capability to reproduce the forward-peaked

diffractive structure that is a characteristic of np elastic scattering. Specifically, important features of the experimental data such as the steep exponential decline at low $|t|$, the moderate deviations from a purely exponential shape at intermediate $|t|$ leading to the dip-bump structure, shrinkage of the forward peak, and shift of the diffractive minimum (a dip) towards lower $|t|$ -values with increasing c.m energies are well reproduced by the models. The differences among the models highlight the sensitivity of the np elastic scattering process to the detailed functional dependence of the scattering amplitude, reflected in our models, especially to energy-dependent modifications which are included in the effective slopes and simple correction terms that reflect the effects of multiple contributing exchange mechanisms in nucleon-nucleon (NN) interactions (related to charge conjugation and isospin) [4, 81]. The models contain energy dependence in both the prefactor and slope parameters that provide particularly good agreement with the observed energy evolution of the forward cone. It reflects the expected Regge-driven shrinkage of the diffractive peak. Limits or ranges of the model parameters are defined by fitting the elastic np data with χ^2 minimization. These estimated ranges occur within our assumed expected bounds of the parameter values. The models show very good fitting agreement with all the experimental data as the least χ^2 value is found to be very low. It is found that Model 4 with dataset 2 has the smallest range, with a value of 0.0218 in the range ($0.0017 \leq \chi^2 \leq 0.0235$).

In this study, a central outcome of the analysis is the evaluation of the total cross section, σ_{tot} that is obtained via the optical theorem from the extrapolation of the models of the differential cross section. The predicted σ_{tot} values exhibit a smooth, physically consistent energy dependence. The values are found to be decreasing moderately with $\sqrt{s}$ for datasets 1 to 3 and increasing for the dataset 4. The predicted values by our models are also in good quantitative and qualitative agreement with the reference values and with the outcomes of separate fits performed on the σ_{tot} reference data. For the dataset 1, the range for model 3 results has the least difference of 0.0580 mb with the reference value range, the range for model 5 results for the dataset 2 has the least difference of 0.0705 mb, the range for model 5 results for the dataset 3 has the least difference of 0.0297 mb, and the range for model 1 results for the dataset 4 has the least difference of 0.0393 mb.

The extracted slope parameters $B(s)$ and their corresponding interaction radii R , which quantify the geometric characteristics of the np elastic scattering are predicted using the fitted expressions of the models. The predicted results of slope parameter exhibit the show logarithmic increase with energy which is a signature of shrinkage of the diffractive cone. This shrinkage is also a manifestation of the increasing spatial extent of the interaction region. Results of $B(s)$ with model 2 have the closest estimates for the reference value range of the dataset 1 with a least difference of 0.714 GeV^{-2} , model 4 results show a least difference of 0.3994 GeV^{-2} with the dataset 2, model 3 results show a least difference of 0.2703 GeV^{-2} with the dataset 3, and model 1 results show a least difference of 1.9653 GeV^{-2} with the dataset 4. The associated interaction radii R results remain closer to the hadronic scale ($\sim 1 \text{ fm}$) and slowly expand with energy. This expansion reflects the effective broadening of the partonic distribution of nucleon as higher-energy exchanges become relevant. The predicted results of interaction radius for model 2 show the closest estimates for the dataset 1, with the least difference of 0.0675 fm with the reference value range, model 4 results show the closest estimates for the dataset 2 with the least difference of 0.0263 fm, model 4 results show the closest estimates for the dataset 3 with the least difference of 0.0498 fm, and models 3 and 4 results show the closest estimates for the dataset 4 with a least difference of 0.1246 fm.

The total elastic cross sections σ_{el} values are obtained by forward-cone relation. And a decreasing trend is found in σ_{el} results with energy which is found to be a smaller fraction of σ_{tot} . These predicted results are close to those obtained as reference values across each dataset. Model 2 has the least difference of 0.5749 mb with the reference value range of the dataset 1, model 4 has the least difference of 0.0853 mb with the dataset 2, model 2 has the least difference of 0.213 mb with the dataset 3, and the model 4 has the least difference of 0.6376 mb with the dataset 4. The inelastic cross section σ_{inel} contains a quantitative contribution of all processes beyond pure elastic scattering values. The predicted results of σ_{inel} are in good quantitative agreement with the reference values. Model 5 results for σ_{inel} across dataset 1 show a least difference of 0.7646 mb with respect to the reference value range, model 1 results across dataset 2 have a least difference of 0.5073 mb, model 1 results across dataset 3 have a least difference of 1.9023 mb, and model 1 results across dataset 4 have least difference of 0.1361 mb. Integrated elastic cross section values are also predicted by numerical integration of the fitted models in the momentum ranges of the Table I, across each dataset.

The ratios R_{el} and R_{inel} are evaluated to quantify the relative strengths of elastic and inelastic processes. Across the studied energies, the models consistently predict that R_{el} which remains well below unity. This illustrates the predominance of inelastic processes in np scattering. Predicted results of model 2 across the dataset 1 range has the least difference of 0.0176 with reference value range, model 4 across the dataset 2 range has the least difference of 0.0039, model 2 across the dataset 3 range has the least difference of 0.0052, and model 3 across the dataset 4 range has the least difference of 0.0182. For the dataset 1, the predicted range of R_{el} by model 2 has a least difference of 0.0176 with reference value range, model 4 across the dataset 2 has the least difference of 0.0039, model 2 across the dataset 3 has the least difference of 0.0052, and for the dataset 4, model 3 range has the least difference of 0.0182 with reference value range. The inelastic fraction R_{inel} grows steadily with energy conversely to the decreasing R_{el} . This behavior is a consequence of the Regge shrinkage observed in the elastic np data with $\ln(s)$. Consequently, it is consistently observed that $\sigma_{\text{el}}/\sigma_{\text{inel}} \sim (\log)^{-1}$ as $\log(s) \rightarrow \infty$ for all two-body elastic scattering processes ref. [5]. This leads to a finding that consistent results can be obtained by the models of this study, especially due to the Regge-inspired parameterizations utilized in the first effective slope B_{eff} in the $T_D(s, t)$ and also due to the other components of the models. In this way the $T_D(s, t)$ components of the models become accurately efficient to reproduce the experimental data not only in the diffraction peak but also in the other $-t$ -regions of the data. The predicted values for both the ratios respect the general phenomenological trends observed in pp and np scattering at comparable energies [5, 10, 82]. It is found that the exponential structures introduced in the five models can effectively test and accommodate various Regge parameterizations in place of the generalized parameters of the models. The analysis of

this study based on the proposed phenomenological models has provided a coherent and detailed phenomenological picture of np scattering in the studied energy domain, and a foundation for future investigation of the differential cross section in the elastic nucleon-nucleon scattering at various collider energies. The effectiveness of phenomenological modeling highlights the collective success in fitting the differential cross-section data and predicting physically plausible observables of the hadronic scattering.

V. RECOMMENDATIONS AND FUTURE WORK

The proposed models of this study can be extended or modified by using different parameterizations according to specific physical considerations, and can also be used to reproduce elastic scattering data of pp , $p\bar{p}$, and $n\bar{p}$ at various energies. For instance, these models can be modified by employing other approximate energy-dependent parameterizations in their effective slope parameters such as those used in the analyses of the slope data of elastic nucleon-nucleon and nucleon-antinucleon scattering, some choices can be found in [46, 47] that can work well with the models to study energy dependence in the slopes. We used simple choices to parameterize the additive correction terms that have the C-odd and isospin roles, due to considerations related to phenomenological parsimony in the models. However, these terms can be further parameterized depending on required physical considerations by taking insights from amplitude models which have been developed for elastic np , $n\bar{p}$, pp , and $p\bar{p}$ scattering [12, 48]. Furthermore, these models can be considered for other extensions with amplitude models based on Regge theory [6, 34, 41], polarization observables, comparisons with modern effective field theory and QCD-inspired approaches, which can be beneficial to gain insights in hadron-hadron scattering.

Data availability statement Associated data is available in the manuscript.

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