

Leptophilic Gauge Interactions in the SMEFT Era

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We develop a *precision-ready*, model-independent EFT framework that connects general leptophilic gauge interactions to their low-energy manifestations in SMEFT and LEFT. Starting from a broad $U(1)'_\ell$ extension in which leptons carry family-dependent charges (q_e, q_μ, q_τ) while quarks remain neutral, quantum consistency is ensured through a minimal set of vectorlike leptons—chiral under $U(1)'_\ell$ but vectorlike under the SM—together with singlet scalars responsible for symmetry breaking and the masses of both Z_ℓ and the heavy leptons.

In the heavy-mediator limit, we integrate out Z_ℓ at tree level and derive compact, analytic SMEFT Wilson coefficients for four-lepton operators and Higgs-current structures, including hypercharge- $U(1)'_\ell$ kinetic mixing proportional to $\text{Tr}(YQ')$. Renormalization-group evolution down to the electroweak scale and matching onto LEFT produce closed-form expressions directly applicable to $e^+e^- \rightarrow \ell^+\ell^-$, neutrino trident production, νe scattering, parity-violating Møller observables, and muon-decay parameters.

A single parameter combination,

$$\Lambda_{\text{eff}} = \frac{M_{Z_\ell}}{g_\ell \sqrt{|q q'|}},$$

governs all leading EFT signatures, offering a unified way to map UV charge assignments onto precision observables. We delineate the validity domain of this EFT and outline its use as a “plug-and-play” interface for global fits, collider recasts, and future-collider projections.

I. INTRODUCTION

The observation of neutrino oscillations [1] established that the individual lepton flavour numbers L_e , L_μ and L_τ are not conserved, providing one of the clearest indications of physics beyond the Standard Model (SM). Among the theoretically economical extensions motivated by this fact, an additional Abelian gauge factor acting on leptons stands out. In this work we consider the most general leptophilic $U(1)'_\ell$, under which the three lepton families carry arbitrary charges (q_e, q_μ, q_τ) while quarks remain neutral. Special cases include the flavour-universal assignment $q_e = q_\mu = q_\tau$ and the anomaly-free $L_\mu - L_\tau$ choice $(0, +1, -1)$ [2]. The corresponding gauge boson, which we denote by Z_ℓ , couples to leptons at tree level and provides a clean portal to precision electroweak, neutrino, and charged-lepton observables.

Such gauge extensions are theoretically well-motivated [3] and phenomenologically testable across a wide energy range. However, a fully consistent construction requires anomaly cancellation, a viable symmetry-breaking sector, and control over possible gauge-kinetic mixing. Once these ingredients are specified, the low-energy effects of the heavy mediator may be described in a model-independent way using the Standard Model Effective Field Theory (SMEFT) and its low-energy limit, the LEFT. Leptophilic interactions are particularly attractive in this context: they produce a restricted set of purely leptonic operators, allowing analytic matching and transparent parametric control.

In this paper we develop a complete and UV-agnostic EFT bridge between leptophilic gauge interactions and their manifestations in precision observables. Our analysis proceeds in four steps: (i) we formulate an anomaly-free leptophilic $U(1)'_\ell$ with a minimal set of vectorlike leptons; (ii) we integrate out the heavy gauge boson Z_ℓ and derive compact analytic SMEFT Wilson coefficients for four-lepton and Higgs-current operators, including kinetic mixing proportional to $\text{Tr}(YQ')$; (iii) we evolve these coefficients to the electroweak scale and match them onto LEFT; and (iv) we identify the resulting patterns in key high- and low-energy probes such as $e^+e^- \rightarrow \ell^+\ell^-$, neutrino trident production, νe scattering, parity-violating Møller observables, and muon decay.

A central outcome of our analysis is the emergence of a single effective interaction scale,

$$\Lambda_{\text{eff}} = \frac{M_{Z_\ell}}{g_\ell \sqrt{|q q'|}}, \tag{1}$$

which controls the size of all induced SMEFT and LEFT operators. This scale provides a unified interpretation framework for current and future precision data—including those from LEP, MOLLER, neutrino facilities, and upcoming lepton colliders—and enables direct comparison of broad classes of UV completions.

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Organization. Section II introduces the gauge structure, vectorlike spectrum, and anomaly constraints. Section III presents the tree-level SMEFT matching, Section IV the RG evolution and matching onto LEFT, and Section V the resulting phenomenology. Section VI summarizes benchmark scenarios, while Section VII discusses the validity of the EFT and theoretical consistency. We conclude in Section VIII. Technical derivations are collected in the Appendices.

II. GENERAL LEPTOPHILIC MODEL AND ANOMALY CANCELLATION

A. Field content and charges

We extend the SM gauge group as

$$SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'_\ell.$$

Leptons carry family-dependent charges (q_e, q_μ, q_τ) under $U(1)'_\ell$, while quarks remain neutral, realizing a genuinely “leptophilic” gauge interaction [4, 5]. The associated gauge boson is denoted Z_ℓ with coupling g_ℓ . Spontaneous symmetry breaking is achieved through one or more SM-singlet scalar fields S_i with charges Q_{S_i} , whose vacuum expectation values generate

$$M_{Z_\ell} = g_\ell \sqrt{\sum_i Q_{S_i}^2 v_{S_i}^2},$$

and induce masses for the heavy vectorlike leptons via Yukawa terms $y_S S_i \bar{L}'_L L'_R$. This setup represents the minimal UV structure required to produce a well-defined low-energy EFT.

B. Anomaly conditions

We work in a left-handed Weyl basis so that all chiralities transform consistently. The vanishing of gauge and mixed anomalies requires

$$\mathcal{A}_{[U(1)'_\ell]^3} = 0, \quad (2)$$

$$\mathcal{A}_{[SU(2)_L]^2 U(1)'_\ell} = 0, \quad (3)$$

$$\mathcal{A}_{[U(1)_Y]^2 U(1)'_\ell} = 0, \quad (4)$$

$$\mathcal{A}_{U(1)_Y [U(1)'_\ell]^2} = 0, \quad (5)$$

$$\mathcal{A}_{\text{grav}-U(1)'_\ell} = 0. \quad (6)$$

These relations yield linear constraints among the $U(1)'_\ell$ charges of SM leptons and the additional vectorlike fields. A minimal anomaly-free completion can be achieved by adding one vectorlike generation (L', E', N') that is vectorlike under the SM but chiral under $U(1)'_\ell$, leading to the compact condition

$$q_e + q_\mu + q_\tau = 0.$$

as emphasized in [2, 6]. Kinetic mixing between the two Abelian factors, $-\frac{\epsilon}{2} B_{\mu\nu} X^{\mu\nu}$, is unavoidably induced at one loop [7]. Radiative stability of a small ϵ follows if the trace condition $\text{Tr}(Y\bar{Q}') = 0$ holds at the field-theory level, a feature naturally realized in several of our benchmark charge assignments.

C. Masses and interactions

After symmetry breaking, the charged- and neutral-lepton mass matrices take the schematic form

$$\mathcal{L}_{\text{mass}} \supset \bar{E}_L M_E E_R + \bar{N}_L M_N N_R + \text{h.c.},$$

where $M_{E,N}$ receive both SM-like and singlet-induced contributions. Biunitary diagonalization, $E_{L,R} = U_{L,R}^E E'_{L,R}$, $N_{L,R} = U_{L,R}^N N'_{L,R}$, yields the couplings of Z_ℓ to mass eigenstates:

$$\mathcal{L} \supset g_\ell Z_{\ell\mu} \left[\bar{E}' \gamma^\mu (Q_L^E P_L + Q_R^E P_R) E' + \bar{N}' \gamma^\mu (Q_L^N P_L + Q_R^N P_R) N' \right]. \quad (7)$$

Mixing between SM leptons and the heavy vectorlike states is constrained by electroweak precision tests and is taken to satisfy $\sin \theta_{\text{mix}} \lesssim 10^{-2}$. Consequently their impact on effective operators enters only at higher order (see e.g. [8, 9]). Explicit charge solutions and mixing formulas are collected in [Sections A and B](#).

D. Connection to the EFT description

In the low-energy regime $E \ll M_{Z_\ell}, M_{L', E', N'}$, the heavy fields can be integrated out, leading to contact interactions among SM leptons of the schematic form

$$\frac{g_\ell^2}{M_{Z_\ell}^2} (\bar{\ell} \gamma_\mu \ell) (\bar{\ell}' \gamma^\mu \ell').$$

These represent the leading $d = 6$ contributions in the SMEFT expansion and form the starting point for the matching analysis in [Section III](#). This section therefore establishes the bridge between the anomaly-free UV theory and its precision-EFT representation.

III. TREE-LEVEL MATCHING ONTO SMEFT

In the regime $p^2 \ll M_{Z_\ell}^2$, the heavy mediator Z_ℓ can be integrated out at tree level, leading to a universal and compact set of dimension-six operators. The starting point is the leptophilic current

$$J'_\mu = \sum_{\alpha=e,\mu,\tau} [q_\alpha^L \bar{L}_\alpha \gamma_\mu L_\alpha + q_\alpha^e \bar{e}_\alpha \gamma_\mu e_\alpha], \quad (8)$$

whose quadratic contraction arises from the equation of motion for Z_ℓ ,

$$\mathcal{L}_{\text{eff}}^{(6)} \simeq -\frac{g_\ell^2}{2M_{Z_\ell}^2} J'_\mu J'^\mu. \quad (9)$$

This interaction generates the Warsaw-basis four-lepton structures $\mathcal{O}_{ll}$, $\mathcal{O}_{le}$, and $\mathcal{O}_{ee}$ [10], with family-diagonal coefficients

$$\frac{[C_{ll}]_{\alpha\alpha\beta\beta}}{\Lambda^2} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^L q_\beta^L, \quad \frac{[C_{le}]_{\alpha\alpha\beta\beta}}{\Lambda^2} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^L q_\beta^e, \quad \frac{[C_{ee}]_{\alpha\alpha\beta\beta}}{\Lambda^2} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^e q_\beta^e. \quad (10)$$

Beyond these current-current operators, hypercharge- $U(1)'_\ell$ kinetic mixing $-\frac{\epsilon}{2} B_{\mu\nu} X^{\mu\nu}$ induces Higgs-current structures whenever $\text{Tr}(YQ') \neq 0$. Integrating out Z_ℓ in this case produces [7, 11]

$$\mathcal{O}_{H\ell, \alpha}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{L}_\alpha \gamma^\mu L_\alpha), \quad \mathcal{O}_{He, \alpha} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{e}_\alpha \gamma^\mu e_\alpha), \quad (11)$$

with leading coefficients

$$\frac{[C_{H\ell}^{(1)}]_\alpha}{\Lambda^2} \simeq -\epsilon \frac{g_Y g_\ell}{M_{Z_\ell}^2} Y_H q_\alpha^L, \quad \frac{[C_{He}]_\alpha}{\Lambda^2} \simeq -\epsilon \frac{g_Y g_\ell}{M_{Z_\ell}^2} Y_H q_\alpha^e, \quad (Y_H = +1). \quad (12)$$

Taken together, these operators encapsulate the complete low-energy imprint of a heavy leptophilic gauge boson. The four-lepton terms dominate high-energy processes such as $e^+e^- \rightarrow \ell^+\ell^-$ and neutrino trident production, while the Higgs-current operators govern electroweak precision observables, parity-violating Møller scattering, and the small induced couplings of Z_ℓ to quarks through Z - Z_ℓ mixing [12, 13]. These expressions therefore form the backbone for the RGE evolution and LEFT matching discussed in [Section IV](#), and constitute the universal EFT dictionary for precision probes of leptophilic interactions.

IV. RUNNING AND MATCHING TO LEFT

The tree-level SMEFT coefficients obtained at the matching scale $\mu = M_{Z_\ell}$ do not directly correspond to the low-energy observables measured in neutrino, Møller, or charged-lepton scattering experiments. Between M_{Z_ℓ} and the

electroweak scale m_Z , the coefficients evolve according to the full one-loop anomalous-dimension matrices of SMEFT [14–16]. This renormalization-group evolution induces electroweak corrections, including mild flavor mixing between the four-lepton operators and the Higgs-current structures generated by kinetic mixing. The running preserves the simple charge-factor dependence of the EFT, so that each Wilson coefficient remains proportional to combinations such as $q_\alpha^L q_\beta^L$ or $q_\alpha^L q_\beta^e$ throughout the evolution.

At the electroweak scale $\mu = m_Z$, the heavy SM fields W , Z , and H are integrated out. Using the standard LEFT basis of Refs. [17, 18], the SMEFT operators at m_Z match onto a set of purely leptonic and semileptonic LEFT operators. For leptophilic gauge extensions, the dominant structures are the vector-vector interactions,

$$\mathcal{O}_{\nu_\alpha \ell_\beta}^{V,LL} = (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha)(\bar{\ell}_\beta \gamma^\mu P_L \ell_\beta), \quad \mathcal{O}_{\nu_\alpha \ell_\beta}^{V,LR} = (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha)(\bar{\ell}_\beta \gamma^\mu P_R \ell_\beta), \quad (13)$$

whose Wilson coefficients inherit the same parametric suppression

$$C \sim \frac{g_\ell^2}{M_{Z_\ell}^2} (q_\alpha^L q_\beta^L, q_\alpha^L q_\beta^e),$$

up to small loop corrections from the SMEFT running. These operators provide a unified EFT description of several key probes: neutrino-electron scattering, neutrino trident production, parity-violating Møller asymmetries, and the extraction of G_F from muon decay. Because they interfere directly with SM weak interactions, these observables offer some of the most sensitive tests of leptophilic gauge sectors in the region well below the Z_ℓ pole.

The complete analytic expressions for the LEFT coefficients, along with the G_F -scheme subtleties and subleading kinetic-mixing effects, are collected in Section C. This scale-separated treatment ensures that the final low-energy EFT retains the model-independent structure emphasized throughout this work, enabling direct application to precision data and future global analyses.

V. PHENOMENOLOGY IN THE EFT LIMIT

Because Λ_{eff} controls all Wilson coefficients in the EFT limit, the phenomenology across all channels can be discussed in a unified manner. The EFT obtained after integrating out Z_ℓ predicts a set of leptonic contact interactions whose leading effects scale as $g_\ell^2/M_{Z_\ell}^2$ times charge products. Across all observables, the phenomenology is governed by the effective interaction scale $\Lambda_{\text{eff}} = M_{Z_\ell}/(g_\ell \sqrt{|q q'|})$, which directly maps onto LEP-II bounds and future lepton-collider sensitivities. Below we summarize the key probes in the EFT regime $E \ll M_{Z_\ell}$.

A. $e^+e^- \rightarrow \ell^+\ell^-$ and contact interactions

In terms of the standard LEP chiral contact structures $(\bar{e}_i \gamma_\mu e_i)(\bar{\ell}_j \gamma^\mu \ell_j)$ with $i, j = L, R$, the effective interactions induced by Z_ℓ exchange correspond to

$$\frac{4\pi}{\Lambda_{LL,e\beta}^2} \eta_{LL}^{e\beta} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_e^L q_\beta^L, \quad \frac{4\pi}{\Lambda_{LR,e\beta}^2} \eta_{LR}^{e\beta} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_e^L q_\beta^e, \quad \text{etc.} \quad (14)$$

Interference with γ/Z exchange modifies both the differential cross section and the forward-backward asymmetry A_{FB} linearly in $s/M_{Z_\ell}^2$. This mapping allows for a direct reinterpretation of existing LEP-II limits [12, 13, 19]. Future e^+e^- machines (ILC, FCC-ee, CEPC) can extend the reach in Λ_{eff} by nearly an order of magnitude.

B. Neutrino trident production and νe scattering

For $\nu_\mu N \rightarrow \nu_\mu \mu^+ \mu^- N$, integrating out Z_ℓ yields

$$\mathcal{L}_{\text{eff}} \supset \frac{g_\ell^2}{M_{Z_\ell}^2} (\bar{\nu}_\mu \gamma_\mu P_L \nu_\mu) [\bar{\mu} \gamma^\mu (\alpha_L P_L + \alpha_R P_R) \mu], \quad (15)$$

with coefficients $\alpha_L \propto q_\mu^L q_\mu^L$ and $\alpha_R \propto q_\mu^L q_\mu^e$. The resulting interference modifies the SM prediction at order $\mathcal{O}(g_\ell^2/M_{Z_\ell}^2)$, providing a powerful probe of muon-philic scenarios [20, 21].

Neutrino-electron scattering constrains the combinations (q_e^L, q_e^e) through both the four-lepton operators of Section III and the Higgs-current operators $\mathcal{O}_{H\ell}^{(1)}$ and $\mathcal{O}_{He}$. Current bounds are dominated by CHARM-II and TEXONO data, complementing the e^+e^- constraints.

C. Parity-violating Møller scattering and muon decay

The operators $C_{H\ell}^{(1)}$ and C_{He} shift the weak charges and hence modify parity-violating observables in polarized electron–electron scattering, such as the E158 result and the upcoming MOLLER experiment [22, 23]. The same structures, together with C_{ll} and C_{le} , induce universal corrections to muon decay and to the extraction of G_F , scaling as $g_\ell^2 v^2 / M_{Z_\ell}^2$. These effects are small but become relevant in high-precision fits.

D. Muon ($g - 2$): qualitative expectations

At tree level no dipole operator is generated. Loop-induced SMEFT renormalization mixes the four-lepton and Higgs–current operators into $\mathcal{O}_{e\gamma}$, leading to the parametric estimate

$$\Delta a_\mu \sim \frac{m_\mu^2}{16\pi^2} \frac{g_\ell^2}{M_{Z_\ell}^2} (\text{charge factors}) \ln \frac{M_{Z_\ell}^2}{m_\mu^2}, \quad (16)$$

as discussed in Section E. A full computation requires specifying the vectorlike-lepton sector and associated thresholds [24, 25]. Within the pure EFT, Δa_μ remains loop- and $m_\mu^2 / M_{Z_\ell}^2$ -suppressed, making it subleading relative to the vector–vector probes above.

VI. BENCHMARK STRUCTURE AND EFT IMPACT

To quantify the parametric reach of leptophilic gauge interactions in a model-independent manner, it is useful to express all leading effects in terms of a single effective scale,

$$\Lambda_{\text{eff}}^{(\alpha\beta)} \equiv \frac{M_{Z_\ell}}{g_\ell \sqrt{|q_\alpha q_\beta|}}, \quad (17)$$

which controls the normalization of the four-lepton and Higgs–current operators appearing in SMEFT. This scale captures the entire phenomenological impact of the heavy mediator in the EFT limit, allowing different UV charge assignments to be compared on equal footing.

Benchmark strategies. We concentrate on two representative and theoretically well-motivated scenarios:

- (B1) Universal leptonic charges** $q_e = q_\mu = q_\tau \equiv q_\ell$. This case is maximally symmetric and illustrates how a flavor-blind leptophilic current feeds into purely leptonic SMEFT operators. It also highlights the necessity of additional chiral fermions required for anomaly cancellation.
- (B2) L_μ – L_τ gauge symmetry** $(q_e, q_\mu, q_\tau) = (0, +1, -1)$. This anomaly-free realization provides a clean and minimal example in which only muon and tau currents couple to Z_ℓ , sharply delineating the structure of LEFT coefficients at low energies.

In both scenarios, quarks remain neutral and integrating out Z_ℓ at $\mu = M_{Z_\ell}$ generates only leptonic dimension-six operators. The resulting SMEFT Wilson coefficients are summarized in Table I, showing the dominant family-diagonal entries that control LEP-II dilepton constraints, neutrino scattering, and Møller observables.

Phenomenological implications. The structure in Table I cleanly determines the pattern of deviations in all key observables: LEP-II dilepton spectra, neutrino trident rates, νe scattering, Møller asymmetries, and precision muon-decay observables. For each channel, the EFT prediction is simply proportional to $g_\ell^2 / M_{Z_\ell}^2$ times a product of the relevant charges. This transparency is precisely what makes the leptophilic EFT approach well suited for global fits and reinterpretation across different UV completions.

Finally, the effective scale Λ_{eff} provides a universal parameter for comparing present and future constraints. Projected improvements in e^+e^- data at FCC-ee, next-generation trident searches, and ultra-precise Møller scattering measurements can collectively probe $\Lambda_{\text{eff}} \sim \mathcal{O}(10\text{--}30 \text{ TeV})$, pushing well into the region where new gauge bosons would remain inaccessible to direct production.

TABLE I. Leading SMEFT Wilson coefficients induced at tree level after integrating out Z_ℓ at $\mu = M_{Z_\ell}$. Only the family-diagonal structures relevant for LEP-II and low-energy probes are displayed. For compactness we write $\Lambda^{-2} = 1/\Lambda^2$.

Case	Coefficient	Expression	Note
(B1)	$[C_{ll}]_{ee\mu\mu}/\Lambda^2$	$-g_\ell^2 q_\ell^2/M_{Z_\ell}^2$	universal
	$[C_{le}]_{ee\mu\mu}/\Lambda^2$	$-g_\ell^2 q_\ell^2/M_{Z_\ell}^2$	universal
	$[C_{ee}]_{ee\mu\mu}/\Lambda^2$	$-g_\ell^2 q_\ell^2/M_{Z_\ell}^2$	universal
(B2)	$[C_{ll}]_{ee\mu\mu}/\Lambda^2$	0	$q_e = 0$
	$[C_{le}]_{ee\mu\mu}/\Lambda^2$	0	$q_e = 0$
	$[C_{ee}]_{ee\mu\mu}/\Lambda^2$	0	$q_e = 0$
	$[C_{ll}]_{\mu\mu\mu\mu}/\Lambda^2$	$-g_\ell^2/M_{Z_\ell}^2$	muonic
	$[C_{le}]_{\mu\mu\mu\mu}/\Lambda^2$	$-g_\ell^2/M_{Z_\ell}^2$	muonic
	$[C_{ee}]_{\mu\mu\mu\mu}/\Lambda^2$	$-g_\ell^2/M_{Z_\ell}^2$	muonic

VII. VALIDITY DOMAIN AND THEORETICAL CONSISTENCY

The EFT description applies for partonic energies $s, |t|, |u| \ll M_{Z_\ell}^2$ and below the masses of any heavy vectorlike leptons that complete the ultraviolet theory. In this regime, the exchange of Z_ℓ reduces to local four-fermion and Higgs-current structures, while higher-dimensional operators are suppressed by additional powers of $E^2/M_{Z_\ell}^2$. Near the Z_ℓ pole, or whenever the momentum transfer approaches the heavy-lepton thresholds, one must revert to the full UV completion.

Perturbativity further requires $g_\ell \lesssim \mathcal{O}(1)$ so that the running gauge coupling remains under control up to the symmetry-breaking scale and does not encounter a Landau pole below the UV completion. The scalar sector that breaks $U(1)_\ell'$ is assumed to remain weakly coupled in the same domain, ensuring a stable hierarchy between M_{Z_ℓ} and the electroweak scale.

Flavor consistency follows from anomaly cancellation and the smallness of mass mixing between SM leptons and the vectorlike states. As long as $\sin \theta_{\text{mix}} \ll 1$, flavor off-diagonal SMEFT coefficients are parametrically suppressed, and the effective theory preserves the charged-lepton flavor structure inherent in the UV model. Explicit charge solutions and mixing formulas are collected in [Sections A and B](#).

a. Remark. Within this validity window, the EFT description offers a model-independent interface between ultraviolet leptophilic gauge dynamics and precision observables at both high and low energies. Its predictivity rests entirely on the two quantities $(g_\ell, q_\alpha/M_{Z_\ell})$ and is therefore ideally suited for global analyses, collider reinterpretations, and future precision programs at lepton machines.

VIII. CONCLUSIONS

We have presented a universal and UV-agnostic framework that systematically connects a broad class of leptophilic $U(1)_\ell'$ gauge extensions to their low-energy manifestations in SMEFT and LEFT. By formulating an anomaly-free completion, integrating out the heavy mediator Z_ℓ at tree level, and tracing the evolution of the resulting operators down to the electroweak and sub-electroweak scales, we obtain a compact EFT description that isolates the essential charge and coupling combinations governing all phenomenological signatures.

A central outcome of this analysis is the identification of the effective interaction scale

$$\Lambda_{\text{eff}} \equiv \frac{M_{Z_\ell}}{g_\ell \sqrt{|q_\alpha q_\beta|}},$$

which provides a model-independent handle on a wide range of observables. In particular, the EFT representation derived here maps directly onto LEP-II dilepton spectra, neutrino trident production, νe scattering, parity-violating Møller observables, and precision muon-decay measurements, while maintaining full compatibility with any anomaly-free ultraviolet completion.

The framework developed in this work therefore enables a unified reinterpretation of current data and a seamless interface with forthcoming precision programs at ILC, CEPC, and FCC-ee. Because the EFT depends only on a minimal set of leptonic charge products, entire classes of UV scenarios—such as universal leptonic charges or the $L_\mu - L_\tau$ symmetry—can be confronted on equal footing, making this approach well suited for global analyses and collider recasts.

Looking ahead, the EFT bridge constructed here forms the foundation for a wider research program that links precision electroweak measurements, neutrino scattering data, and future lepton-collider sensitivities. Dedicated numerical fits, including combined LEP-II–LEFT constraints and detailed projections for FCC-ee, will be presented in forthcoming work. The resulting framework provides a robust and flexible tool for exploring leptophilic gauge interactions across energy scales, and for identifying the parameter regions where new physics may first emerge.

Appendix A: Anomaly equations and example solutions

This appendix summarizes the gauge- and gravity-anomaly conditions for $U(1)'_\ell$ in a left-handed Weyl basis, following the conventions used in Secs. II–VII. We also provide minimal sets of vectorlike leptons that render the theory anomaly-free for the benchmark scenarios (B1) and (B2).

1. Anomaly equations

For leptons with $U(1)'_\ell$ charges q_α^L for the doublets L_α and q_α^e for the singlets e_α (with $\alpha = e, \mu, \tau$), the relevant anomalies are:

$[SU(2)_L]^2 U(1)'_\ell$ Only lepton doublets contribute:

$$\mathcal{A}_{[SU(2)_L]^2 U(1)'_\ell} = \sum_{\alpha=e,\mu,\tau} q_\alpha^L = 0. \quad (\text{A1})$$

$[U(1)_Y]^2 U(1)'_\ell$ Each fermion contributes $Y_f^2 q_f$:

$$\mathcal{A}_{[U(1)_Y]^2 U(1)'_\ell} = \sum_{\alpha} \left[2 \left(-\frac{1}{2} \right)^2 q_\alpha^L + (-1)^2 q_\alpha^e \right] = \frac{1}{2} \sum_{\alpha} q_\alpha^L + \sum_{\alpha} q_\alpha^e = 0. \quad (\text{A2})$$

$U(1)_Y [U(1)'_\ell]^2$ Each fermion contributes $Y_f (q_f)^2$:

$$\mathcal{A}_{U(1)_Y [U(1)'_\ell]^2} = \sum_{\alpha} \left[2 \left(-\frac{1}{2} \right) (q_\alpha^L)^2 + (-1) (q_\alpha^e)^2 \right] = - \sum_{\alpha} (q_\alpha^L)^2 - \sum_{\alpha} (q_\alpha^e)^2 = 0. \quad (\text{A3})$$

$[U(1)'_\ell]^3$

$$\mathcal{A}_{[U(1)'_\ell]^3} = \sum_{\alpha} [2(q_\alpha^L)^3 + (q_\alpha^e)^3] = 0. \quad (\text{A4})$$

Gravitational– $U(1)'_\ell$

$$\mathcal{A}_{\text{grav} - U(1)'_\ell} = \sum_{\alpha} [2q_\alpha^L + q_\alpha^e] = 0. \quad (\text{A5})$$

If all SM leptons carry nontrivial $U(1)'_\ell$ charge, Eqs. (A1)–(A5) cannot be satisfied without additional fermions. A minimal solution is obtained by introducing a **vectorlike generation** (L', E', N') which is vectorlike under the SM gauge group but **chiral** under $U(1)'_\ell$:

$$L'_L(2, -\tfrac{1}{2}, q'_L), \quad L'_R(2, -\tfrac{1}{2}, q'_L), \quad E'_L(1, -1, q'_e), \quad E'_R(1, -1, q''_e), \quad N'_L(1, 0, q'_N), \quad N'_R(1, 0, q''_N).$$

Their charges enter in the same way as SM leptons in the anomaly equations.

2. Illustrative solutions for benchmarks (B1) and (B2)

(B1) Universal charges For $q_\alpha^L = q_\alpha^e = q_\ell$ for all $\alpha = e, \mu, \tau$, the SM contribution to the anomaly is proportional to

$$\{3q_\ell, \tfrac{3}{2}q_\ell, -3q_\ell^2, 3q_\ell^3, 3q_\ell\},$$

which cannot vanish simultaneously. A minimal solution introduces one vectorlike generation with charges chosen such that

$$q'_L - q''_L = -3q_\ell, \quad q'_e - q''_e = -3q_\ell, \quad q'_N - q''_N = -3q_\ell,$$

ensuring that all linear and cubic anomaly sums cancel. The resulting theory is anomaly-free and compatible with the EFT construction in Sec. II.

(B2) $L_\mu - L_\tau$ model For the charge assignment

$$(q_e, q_\mu, q_\tau) = (0, +1, -1),$$

the SM itself satisfies Eqs. (A1)–(A5). In particular:

$$q_e + q_\mu + q_\tau = 0, \quad (q_\mu)^3 + (q_\tau)^3 = 1 - 1 = 0,$$

and all mixed hypercharge- $U(1)'_\ell$ anomalies cancel as well. Therefore no additional fermions are needed in this benchmark, and the EFT matching reduces to the muon-tau subsector.

3. Remarks

The above solutions illustrate that: (i) universal charge models require additional chiral fermions, (ii) $L_\mu - L_\tau$ is the unique anomaly-free choice among family-dependent lepton symmetries without introducing new fermion states, and (iii) vectorlike leptons in Sec. II provide the minimal UV completion consistent with all anomaly conditions.

Appendix B: Vectorlike leptons and mixing effects

In this appendix we summarize the mass mixing between the SM leptons and the vectorlike states introduced to cancel anomalies in Sec. II. We work in a left-handed Weyl basis and assume that all mixings are small, $\sin \theta_{\text{mix}} \ll 1$, as required by electroweak precision data.

1. Charged-lepton mass matrix

Let (e_α, E') denote the SM and vectorlike charged leptons, where $E'_L(1, -1, q'_e)$ and $E'_R(1, -1, q''_e)$ are vectorlike under the SM but chiral under $U(1)'_\ell$. After electroweak symmetry breaking and $U(1)'_\ell$ breaking, the charged-lepton mass terms take the schematic form

$$\mathcal{L}_{\text{mass}}^{(e)} \supset (\bar{e}_{\alpha L} \quad \bar{E}'_L) \begin{pmatrix} m_\alpha^{\text{SM}} & \delta_\alpha \\ 0 & M_E \end{pmatrix} \begin{pmatrix} e_{\alpha R} \\ E'_R \end{pmatrix} + \text{h.c.}, \quad (\text{B1})$$

where $-m_\alpha^{\text{SM}} = y_\alpha v / \sqrt{2}$, $-M_E$ is the vectorlike mass, $-\delta_\alpha = y_{S,\alpha} v_S / \sqrt{2}$ arises from the $U(1)'_\ell$ -breaking scalar S .

Diagonalization proceeds via biunitary rotations,

$$e_{L,R} = U_{L,R}^e e'_{L,R}, \quad E_{L,R} = U_{L,R}^E E'_{L,R},$$

with small mixing angles

$$\theta_{\alpha L} \simeq \frac{\delta_\alpha}{M_E}, \quad \theta_{\alpha R} \simeq \frac{m_\alpha^{\text{SM}} \delta_\alpha}{M_E^2}. \quad (\text{B2})$$

Typically $\theta_{\alpha R} \ll \theta_{\alpha L}$.

2. Neutral-lepton sector

If the vectorlike multiplet includes a neutral state $N'_{L,R}(1, 0, q'_N)$, the neutral-lepton mass matrix is

$$\mathcal{L}_{\text{mass}}^{(\nu)} \supset (\bar{\nu}_{\alpha L} \quad \bar{N}'_L) \begin{pmatrix} 0 & \delta_\alpha^N \\ 0 & M_N \end{pmatrix} \begin{pmatrix} \nu_{\alpha R} \\ N'_R \end{pmatrix} + \text{h.c.}, \quad (\text{B3})$$

where δ_α^N is again controlled by the $U(1)'_\ell$ -breaking vev. Mixing angles are

$$\theta_\alpha^N \simeq \frac{\delta_\alpha^N}{M_N}, \quad (\text{B4})$$

with no right-handed mixing if ν_R is absent.

3. Impact on gauge interactions

In the gauge basis, the Z_ℓ current contains

$$J'_\mu \supset q_\alpha^L \bar{e}_{\alpha L} \gamma_\mu e_{\alpha L} + q_\alpha^e \bar{e}_{\alpha R} \gamma_\mu e_{\alpha R} + q_e' \bar{E}'_L \gamma_\mu E'_L + q_e'' \bar{E}'_R \gamma_\mu E'_R.$$

After moving to the mass eigenbasis, the SM-like couplings shift as

$$q_\alpha^{L,\text{phys}} \simeq q_\alpha^L + (q_e' - q_\alpha^L) \theta_{\alpha L}^2, \quad (\text{B5})$$

$$q_\alpha^{R,\text{phys}} \simeq q_\alpha^e + (q_e'' - q_\alpha^e) \theta_{\alpha R}^2. \quad (\text{B6})$$

Thus **mixing shifts the physical $U(1)'_\ell$ charges only at order** $\mathcal{O}(\theta^2)$, ensuring flavor safety as long as $\theta_{\text{mix}} \ll 1$.

4. Subleading impact on SMEFT coefficients

At tree level, integrating out Z_ℓ yields (Sec. III)

$$C \sim \frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha q_\beta.$$

Mixing corrections appear only through the shifted charges of Eq. (B6). Therefore

$$\frac{\Delta C}{C} \simeq \mathcal{O}(\theta_{\alpha L}^2, \theta_{\alpha R}^2), \quad (\text{B7})$$

which is negligible for $\theta \lesssim 10^{-2}$, as required by electroweak precision data.

At one loop, integrating out the vectorlike leptons themselves induces additional dimension-six operators (e.g. $\mathcal{O}_{H\ell}^{(1)}$, $\mathcal{O}_{He}$), but the coefficients scale as

$$C_{\text{loop}} \sim \frac{1}{16\pi^2} \frac{\delta^2}{M^2} \sim \frac{\theta^2}{16\pi^2},$$

which is safely subleading relative to the tree-level Z_ℓ contribution.

Vectorlike leptons provide the minimal and anomaly-free UV completion of the leptophilic $U(1)'_\ell$ framework. Mixing with SM leptons is strongly constrained by electroweak precision observables, implying $\theta_{\text{mix}} \ll 1$, which in turn guarantees that their impact on the SMEFT coefficients is only of second order in the mixing angles and fully negligible throughout the parameter space considered in this work.

Appendix C: LEFT matching formulas

In this appendix we collect the explicit matching relations between the SMEFT operators generated at $\mu = M_{Z_\ell}$ and the low-energy effective theory (LEFT) obtained after integrating out W , Z , and H at $\mu = m_Z$. All expressions are given in the G_F scheme, where the muon decay rate is used to define v and the leading SMEFT corrections to G_F are absorbed into the normalization of four-fermion operators.

1. LEFT operator basis

We focus on purely leptonic operators relevant for neutrino scattering, neutrino trident production, parity-violating Møller observables, and muon decay. Following the LEFT conventions of Ref. [18], the relevant dimension-six operators are:

Vector–vector structures

$$\mathcal{O}_{\nu_\alpha \ell_\beta}^{V,LL} = (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) (\bar{\ell}_\beta \gamma^\mu P_L \ell_\beta), \quad (C1)$$

$$\mathcal{O}_{\nu_\alpha \ell_\beta}^{V,LR} = (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) (\bar{\ell}_\beta \gamma^\mu P_R \ell_\beta). \quad (C2)$$

Four-charged-lepton operators

$$\mathcal{O}_{\ell_\alpha \ell_\beta}^{V,LL} = (\bar{\ell}_\alpha \gamma_\mu P_L \ell_\alpha) (\bar{\ell}_\beta \gamma^\mu P_L \ell_\beta), \quad (C3)$$

$$\mathcal{O}_{\ell_\alpha \ell_\beta}^{V,RR} = (\bar{\ell}_\alpha \gamma_\mu P_R \ell_\alpha) (\bar{\ell}_\beta \gamma^\mu P_R \ell_\beta), \quad (C4)$$

$$\mathcal{O}_{\ell_\alpha \ell_\beta}^{V,LR} = (\bar{\ell}_\alpha \gamma_\mu P_L \ell_\alpha) (\bar{\ell}_\beta \gamma^\mu P_R \ell_\beta). \quad (C5)$$

2. Matching of SMEFT operators to LEFT

At tree level, the SMEFT operators generated by integrating out Z_ℓ in Sec. III,

$$\mathcal{O}_{ll}, \quad \mathcal{O}_{le}, \quad \mathcal{O}_{ee}, \quad \mathcal{O}_{H\ell}^{(1)}, \quad \mathcal{O}_{He},$$

match onto LEFT after electroweak symmetry breaking via exchange of W , Z , and through the redefinition of G_F .

The matching splits naturally into two pieces:

$$C^{\text{LEFT}} = C_{\text{direct}}^{\text{SMEFT}}(m_Z) + C_{G_F}^{\text{SMEFT}}(m_Z),$$

where $-C_{\text{direct}}^{\text{SMEFT}}$ come directly from integrating out W/Z , $-C_{G_F}^{\text{SMEFT}}$ arise from the shift in G_F .

3. Direct matching at the Z pole

The four-lepton SMEFT operators contribute directly as

$$C_{\nu_\alpha \ell_\beta}^{V,LL} = -\frac{1}{2} C_{H\ell\beta}^{(1)} + \frac{1}{2} C_{ll\alpha\alpha\beta\beta}, \quad (C6)$$

$$C_{\nu_\alpha \ell_\beta}^{V,LR} = -\frac{1}{2} C_{He\beta} + \frac{1}{2} C_{le\alpha\alpha\beta\beta}. \quad (C7)$$

Similarly, for purely charged-lepton operators,

$$C_{\ell_\alpha \ell_\beta}^{V,LL} = C_{ll\alpha\alpha\beta\beta}, \quad (C8)$$

$$C_{\ell_\alpha \ell_\beta}^{V,RR} = C_{ee\alpha\alpha\beta\beta}, \quad (C9)$$

$$C_{\ell_\alpha \ell_\beta}^{V,LR} = C_{le\alpha\alpha\beta\beta}. \quad (C10)$$

4. G_F -scheme corrections

Muon decay receives SMEFT corrections from $\mathcal{O}_{ll}$ and $\mathcal{O}_{H\ell}^{(3)}$ (the latter is absent here). The net effect is a redefinition of v :

$$v^2 \rightarrow v^2 (1 + \delta G_F),$$

with

$$\delta G_F = \frac{v^2}{\sqrt{2}} C_{ll}^{\mu ee\mu}, \quad (\text{in our model only } C_{ll} \text{ contributes}). \quad (C11)$$

This shifts all LEFT Wilson coefficients involving a factor of G_F :

$$C_X^{\text{LEFT}} \rightarrow C_X^{\text{LEFT}} - \delta G_F C_X^{\text{SM}}. \quad (C12)$$

For neutrino scattering and trident production this correction is numerically small, but it must be included in high-precision Møller and muon-decay observables.

5. Result for the leptophilic model

Using the tree-level SMEFT coefficients from Sec. III,

$$C \sim -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha q_\beta,$$

Eqs. (C6)–(C7) yield the LEFT coefficients

$$C_{\nu_\alpha \ell_\beta}^{V,LL} = -\frac{g_\ell^2}{2M_{Z_\ell}^2} \left(q_\beta^L q_\alpha^L + q_\beta^L Y_H \epsilon \frac{g_Y}{g_\ell} \right), \quad (\text{C13})$$

$$C_{\nu_\alpha \ell_\beta}^{V,LR} = -\frac{g_\ell^2}{2M_{Z_\ell}^2} \left(q_\beta^e q_\alpha^L + q_\beta^e Y_H \epsilon \frac{g_Y}{g_\ell} \right). \quad (\text{C14})$$

For purely charged lepton interactions,

$$C_{\ell_\alpha \ell_\beta}^{V,LL} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^L q_\beta^L, \quad (\text{C15})$$

$$C_{\ell_\alpha \ell_\beta}^{V,RR} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^e q_\beta^e, \quad (\text{C16})$$

$$C_{\ell_\alpha \ell_\beta}^{V,LR} = -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha^L q_\beta^e. \quad (\text{C17})$$

The LEFT coefficients generated by a leptophilic $U(1)_\ell'$ gauge boson are dominated by the tree-level structures proportional to $g_\ell^2/M_{Z_\ell}^2$. Kinetic mixing induces subleading contributions through $C_{H\ell}^{(1)}$ and C_{He} , and G_F renormalization generates a universal, numerically suppressed shift. These formulas form the basis for the phenomenological analyses in Sec. V.

Appendix D: Contact-interaction map and $e^+e^- \rightarrow \ell^+\ell^-$

This appendix summarizes the dictionary between the SMEFT operators generated at $\mu = M_{Z_\ell}$ and the LEP-II contact-interaction (CI) parameters usually denoted by $(4\pi/\Lambda_{ij}^2)\eta_{ij}$ with $i, j \in \{L, R\}$. We also provide the analytic expressions for the interference terms in the differential cross section and the forward-backward asymmetry A_{FB} .

1. LEP-II contact interaction conventions

LEP parameterized new physics affecting $e^+e^- \rightarrow \ell^+\ell^-$ through the effective Lagrangian

$$\mathcal{L}_{\text{CI}} = \sum_{i,j=L,R} \frac{4\pi}{\Lambda_{ij}^2} \eta_{ij} (\bar{e}_i \gamma_\mu e_i) (\bar{\ell}_j \gamma^\mu \ell_j), \quad (\text{D1})$$

where $\eta_{ij} = \pm 1$ encodes constructive/destructive interference. Only s -channel contributions appear for $\ell \neq e$; for $\ell = e$ both s - and t -channel contributions exist, but the mapping below focuses on the s -channel terms consistent with the SMEFT operators relevant for Z_ℓ exchange.

The chiral currents are

$$\bar{e}_L \gamma_\mu e_L, \quad \bar{e}_R \gamma_\mu e_R, \quad \bar{\ell}_L \gamma_\mu \ell_L, \quad \bar{\ell}_R \gamma_\mu \ell_R.$$

2. SMEFT $\rightarrow$ Contact-interaction dictionary

After integrating out Z_ℓ (Sec. III), the SMEFT Wilson coefficients are:

$$C_{ll}, \quad C_{le}, \quad C_{ee}.$$

For family-diagonal external states ($e\beta$), the mapping to the LEP parameters is

$$\frac{4\pi}{\Lambda_{LL,e\beta}^2} \eta_{LL}^{e\beta} = C_{ll}^{ee\beta\beta}, \quad (D2)$$

$$\frac{4\pi}{\Lambda_{LR,e\beta}^2} \eta_{LR}^{e\beta} = C_{le}^{ee\beta\beta}, \quad (D3)$$

$$\frac{4\pi}{\Lambda_{RR,e\beta}^2} \eta_{RR}^{e\beta} = C_{ee}^{ee\beta\beta}. \quad (D4)$$

Using the model expressions

$$C \sim -\frac{g_\ell^2}{M_{Z_\ell}^2} q_e^i q_\beta^j,$$

we obtain

$$\eta_{ij}^{e\beta} = -\text{sign}(q_e^i q_\beta^j), \quad (D5)$$

$$\Lambda_{ij,e\beta} = \sqrt{\frac{4\pi}{|C_{ij}^{ee\beta\beta}|}} = \frac{\sqrt{4\pi} M_{Z_\ell}}{g_\ell \sqrt{|q_e^i q_\beta^j|}} = \sqrt{4\pi} \Lambda_{\text{eff}}^{(e\beta)}. \quad (D6)$$

Thus **LEP limits are directly limits on $\Lambda_{\text{eff}}^{**}$ defined in Sec. VI.

3. Differential cross section with CI interference

The unpolarized differential cross section for

$$e^+ e^- \rightarrow \ell^+ \ell^-,$$

including SM plus contact interactions, is

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma_{\text{SM}}}{d\cos\theta} + \frac{s}{8\pi} \Delta_{\text{int}}(\cos\theta) + \mathcal{O}\left(\frac{s^2}{\Lambda^4}\right). \quad (D7)$$

Only the **interference** term is required at leading order:

$$\Delta_{\text{int}}(\cos\theta) = \sum_{i,j=L,R} \frac{4\pi}{\Lambda_{ij}^2} \eta_{ij} [g_i^e g_j^\ell (1 + \cos\theta) + h_i^e h_j^\ell (1 - \cos\theta)], \quad (D8)$$

where

$$g_L^f = T_3^f - Q_f \sin^2 \theta_W, \quad g_R^f = -Q_f \sin^2 \theta_W,$$

and

$$h_{L,R}^f = g_{L,R}^f,$$

following the LEP s-channel chiral decomposition.

For family off-diagonal final states ($\ell = \mu, \tau$), only s-channel terms contribute.

4. Forward-backward asymmetry

The forward-backward asymmetry is defined as

$$A_{FB} = \frac{\int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\sigma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta \frac{d\sigma}{d\cos\theta}}.$$

The CI contribution enters linearly:

$$A_{FB} = A_{FB}^{\text{SM}} + \frac{s}{8\pi\sigma_{\text{tot}}^{\text{SM}}} \left[\frac{8\pi}{3} \sum_{i,j=L,R} \frac{\eta_{ij}}{\Lambda_{ij}^2} (g_i^e g_j^\ell - h_i^e h_j^\ell) \right] + \mathcal{O}\left(\frac{s^2}{\Lambda^4}\right). \quad (\text{D9})$$

For $\ell \neq e$, the t -channel singularities are absent and the expression above is directly applicable to LEP-II data.

The mapping in Eqs. (D4)–(D9) provides a direct dictionary between the leptophilic $U(1)'_\ell$ EFT and the LEP-II contact-interaction language. Experimental bounds on $(\Lambda_{ij}, \eta_{ij})$ thus translate immediately into constraints on the model parameters $(g_\ell, q_\alpha, M_{Z_\ell})$ via the effective scale Λ_{eff} introduced in Sec. VI.

Appendix E: Sketch of dipole mixing and $(g-2)_\mu$

For completeness, we provide a qualitative estimate of the loop-induced contribution of the leptophilic Z_ℓ to the muon anomalous magnetic moment, $(g-2)_\mu$, within the SMEFT framework. Our discussion is schematic and intended to indicate the parametric size of the effect rather than to provide a full numerical calculation.

1. Dipole operator and $(g-2)_\mu$

In SMEFT, charged-lepton dipole moments are encoded in the operator

$$\mathcal{O}_{e\gamma}^{\alpha\beta} = (\bar{L}_\alpha \sigma^{\mu\nu} e_\beta) H F_{\mu\nu}, \quad (\text{E1})$$

with Wilson coefficient $C_{e\gamma}^{\alpha\beta}$. After electroweak symmetry breaking, this operator generates a dipole term

$$\mathcal{L}_{\text{dipole}} \supset -\frac{v}{\sqrt{2}} C_{e\gamma}^{\alpha\beta} \bar{\ell}_\alpha \sigma^{\mu\nu} P_R \ell_\beta F_{\mu\nu} + \text{h.c.}$$

For the muon, the anomalous magnetic moment is proportional to the real part of $C_{e\gamma}^{\mu\mu}$:

$$\Delta a_\mu \simeq -\frac{4m_\mu^2}{e} \text{Re}[C_{e\gamma}^{\mu\mu}(m_\mu)], \quad (\text{E2})$$

up to convention-dependent signs (see, e.g., Refs. [24, 25]).

At the matching scale $\mu = M_{Z_\ell}$, our tree-level analysis yields

$$C_{e\gamma}^{\alpha\beta}(M_{Z_\ell}) = 0,$$

so any contribution to $(g-2)_\mu$ must arise from loop-induced mixing and/or threshold corrections in the UV completion.

2. Schematic RGE mixing

The dominant source of $C_{e\gamma}$ in our setup comes from RGE mixing between the four-lepton and Higgs-current operators generated by Z_ℓ exchange and the dipole operator. Schematically, the one-loop RGE for $C_{e\gamma}$ can be written as

$$\mu \frac{d}{d\mu} C_{e\gamma}^{\alpha\beta} \sim \frac{e}{16\pi^2} y_\alpha \left[a_1 C_{H\ell\alpha}^{(1)} + a_2 C_{He\alpha} + a_3 C_{ll}^{\alpha\alpha\alpha\alpha} + a_4 C_{le}^{\alpha\alpha\alpha\alpha} \right], \quad (\text{E3})$$

where y_α is the charged-lepton Yukawa coupling and $a_{1,\dots,4} = \mathcal{O}(1)$ are numerical coefficients that depend on the operator basis and renormalization scheme.

Inserting the tree-level matching from Sec. III,

$$C \sim -\frac{g_\ell^2}{M_{Z_\ell}^2} q_\alpha q_\beta,$$

and integrating Eq. (E3) from $\mu = M_{Z_\ell}$ down to $\mu = m_Z$, we obtain the parametric estimate

$$C_{e\gamma}^{\mu\mu}(m_Z) \sim \frac{e}{16\pi^2} y_\mu \frac{g_\ell^2}{M_{Z_\ell}^2} (\text{charge factors}) \ln \frac{M_{Z_\ell}^2}{m_Z^2}. \quad (\text{E4})$$

Evolving further from m_Z to m_μ introduces only logarithmic corrections, which do not change the overall parametric scaling.

3. Parametric size of Δa_μ

Combining the above estimate with the relation between $C_{e\gamma}$ and Δa_μ , we arrive at

$$\Delta a_\mu \sim \frac{m_\mu^2}{16\pi^2} \frac{g_\ell^2}{M_{Z_\ell}^2} (\text{charge factors}) \ln \frac{M_{Z_\ell}^2}{m_Z^2}. \quad (\text{E5})$$

This expression makes explicit that the contribution is loop-suppressed and proportional to $m_\mu^2/M_{Z_\ell}^2$, as expected on general grounds.

Numerically, for $g_\ell = \mathcal{O}(1)$, $M_{Z_\ell} \gtrsim \mathcal{O}(\text{TeV})$, and $\mathcal{O}(1)$ charge factors, the induced Δa_μ is typically well below the current experimental sensitivity unless M_{Z_ℓ} lies close to the electroweak scale or additional light states in the UV completion enhance the mixing.

A dedicated two-scale analysis, including the full set of SMEFT anomalous dimensions and model-dependent threshold effects from the vectorlike leptons, is beyond the scope of this work and will be left for future study.

In summary, the leptophilic $U(1)_\ell'$ considered here does not generate a dipole operator at tree level. Loop-induced contributions to $(g-2)_\mu$ arise only through SMEFT RGE mixing and are parametrically suppressed by both a loop factor and the ratio $m_\mu^2/M_{Z_\ell}^2$. Consequently, the most sensitive probes of the model are the vector–vector observables discussed in Sec. V, rather than the muon anomalous magnetic moment.

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