

Multiterminal Ballistic Josephson Effect in Monocrystalline Gold

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We report on the realization of a planar, quasi-ballistic Josephson junction array using a Au micron-sized single-crystal. The system exhibits a nonlocal, multiterminal Josephson effect, where the supercurrent between any two superconducting leads is governed by the phase coherence across the entire crystal. Key evidence includes a non-monotonic dependence of the critical current on junction length and magnetic interference patterns with periods corresponding to the shared normal-metal area. Nonlocal transport measurements further confirm that the supercurrent between two electrodes depends on the phase configuration of all the others. Our results, supported by a developed theoretical model, establish a platform for exploring complex superconducting phenomena in multiterminal ballistic systems.

Introduction.—The Josephson effect continues to inspire fundamental research and technological applications since its discovery in 1962 [1]. Planar Josephson junction technology offers a promising route to miniaturize weak-link regions to 100–200 nm while maintaining high critical currents I_c and characteristic voltages $V_c = I_c R_N$. Typically, such Josephson junctions comprise a normal metal (N) nanowire or film with two superconductor electrodes (S) deposited on top [2–20], forming complex SN/N/SN Josephson junctions (JJs) in bridge geometry. Most experimental and theoretical studies [21–26] address the diffusive regime, where the mean free path l is less than the superconducting coherence length ξ_N .

Planar ballistic JJs, enabling Josephson coupling over micron-scale distances, provide a unique platform for studying proximity-induced superconductivity in the ballistic regime. Up to now, the ballistic JJs have been mainly implemented and studied experimentally in heterostructures of graphene [27–32]. Moreover, in more complex S/N/S structures, where a normal metal is proximitized by several superconducting electrodes, novel effects come into play. In particular, the energy spectrum of a multi-terminal Josephson device can emulate a band structure supporting Weyl points, where the superconducting phases play the role of quasi-momenta [33–35]. Thus, multi-terminal Josephson junctions are expected to be a platform for designing and studying the topological matter of arbitrary dimensions. Several groups have already implemented mesoscopic JJs with three or four terminals in diffusive metallic junctions [36–38], hybrid semiconductor-superconductor heterostructures [39–42], and topological materials [43]. Ballistic JJs are particularly attractive for multiterminal systems, as they facilitate efficient coupling between leads separated by several microns, as demonstrated in graphene [44–46].

In this Letter, we present a combined experimental and theoretical study of planar, quasi-ballistic multi-terminal JJs comprising Al superconducting leads on a gold single

crystal. We investigate supercurrent flow as a function of temperature and magnetic field and develop a microscopic quasiclassical theory for quasiballistic transport in multi-terminal planar S/N/S structures. The theory agrees quantitatively with experiments, confirming the realization of a planar multi-terminal Josephson system in the quasi-ballistic regime.

Sample fabrication and characterization.— We fabricate ballistic Josephson junctions by depositing aluminum ($d_{\text{Al}} = 175$ nm) through an electron-beam lithography mask onto a mechanically transferred monocrystalline Au flake ($d_{\text{Au}} = 80$ nm, lateral size $14 \mu\text{m}$) on a SiO_2/Si substrate. The monocrystalline Au flakes were synthesized using a modified Brust-Schiffrin method followed by thermolysis, as described in Ref. [47–49]. The Au flake was transferred onto the substrate using an all-dry viscoelastic stamping technique with a polyvinyl chloride/polydimethylsiloxane (PVC/PDMS) stamp [50]. Figure 1(a) shows a false-colored SEM image of the device, with eight Al electrodes (labeled A_1 – A_4 and B_1 – B_4) contacting the Au crystal for standard four-probe dc measurements. The inset schematically shows the additional four-probe dc measurement configuration. Transport measurements are performed in a dilution refrigerator using filtered dc lines. From resistivity measurements, $\rho(300 \text{ K}) = 20.70 \text{ n}\Omega \cdot \text{m}$ and $\rho(4 \text{ K}) = 1.35 \text{ n}\Omega \cdot \text{m}$, we extract a low-temperature electron mean free path, $l = 680$ nm. The junction lengths are $L_1^{JJ} = 94$ nm, $L_2^{JJ} = 258$ nm, $L_3^{JJ} = 614$ nm, and $L_4^{JJ} = 1008$ nm. Each lead has a length of $\sim 7 \mu\text{m}$ and a width $w = 570$ nm, with an inter-lead spacing of $W = 1 \mu\text{m}$.

Zero-field critical currents.— Figure 1(b) displays the temperature dependence of the critical current I_c for A_i – B_i junctions ($i = 1$ – 4). Strikingly, I_c depends non-monotonically on interlayer length L_i^{JJ} : the critical currents of the two middle JJs $I_c^{A_2-B_2}(T = 200 \text{ mK}) \approx I_c^{A_3-B_3}(T = 200 \text{ mK}) = I_c^m$ are very close to each other and exceed the currents of the two outer JJs $I_c^{A_1-B_1}(T =$

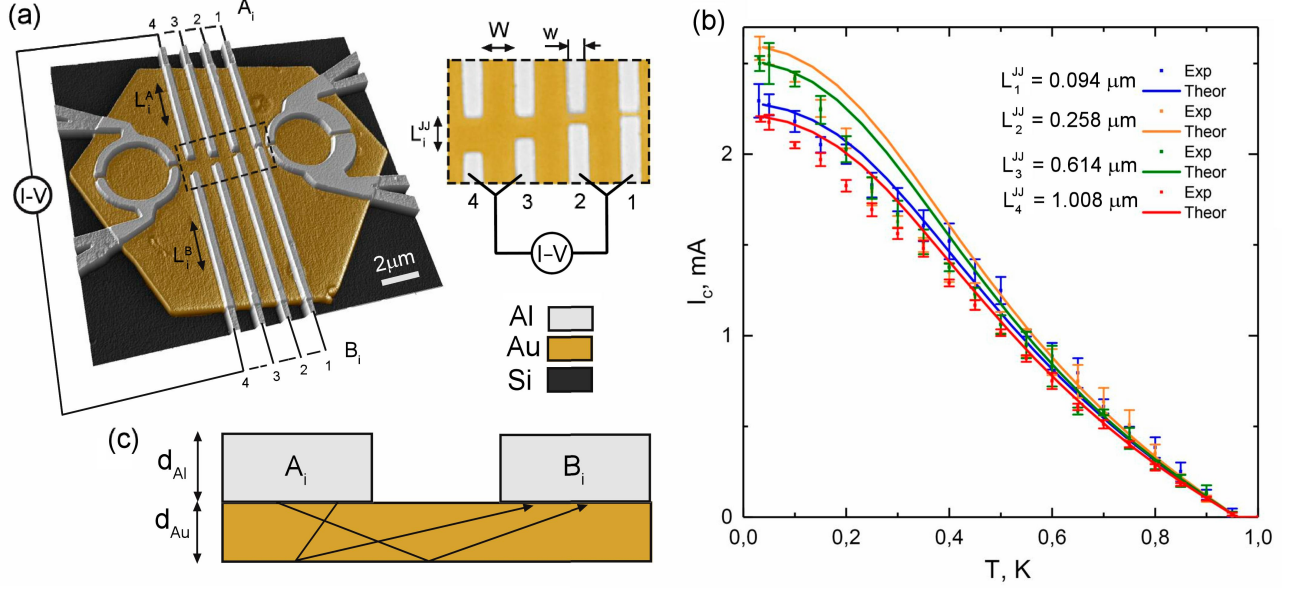


FIG. 1. (a) False-colored SEM image of the Josephson junction array with Al electrodes (labeled A_1 – A_4 , B_1 – B_4) on a Au crystal. Inset: measurement configuration schematic. (b) Critical current $I_c^{A_i-B_i}$ versus temperature: experimental data (dots) and theoretical fits (lines) with interface transmission $D = 0.03$. (c) Model for Josephson current calculation in a single junction.

$200\text{mK}) \approx I_c^{A_4-B_4}(T = 200\text{mK}) = I_c^o$, which are also close to each other and are smaller than I_c^m by a factor of $I_c^o/I_c^m \approx 7/8$. This occurs despite the monotonic increase of L_i^{JJ} from A_1 – B_1 to A_4 – B_4 , indicating that the four JJs are not independent. Using the Au Fermi velocity $v_F = 1.4 \times 10^6 \text{ m s}^{-1}$ and Al critical temperature $T_c = 0.95 \text{ K}$, the ballistic coherence length is $\xi = v_F/2\pi T_c = 1.7 \mu\text{m}$, exceeding all L_i^{JJ} and comparable to inter-electrode distances. Thus, Josephson coupling occurs between all superconducting lead pairs.

Existing theories for planar SN/N/NS structures assume either diffusive transport or purely ballistic trajectories, neither of which is applicable here. The Au crystal thickness d is less than the mean free path, and its width significantly exceeds the electrode dimensions. Moreover, no direct ballistic trajectories connect the leads; only trajectories involving reflections contribute [Fig. 1(b)]. Previous ballistic junction models [51, 52] assume translationally invariant infinite leads, which are inadequate for multiterminal structures. We develop a theoretical approach for strongly coupled planar Josephson structures on large crystals, detailed in a companion work [53]. Here, we apply it to our Al/Au/Al junctions.

If the thickness of the normal metal is large compared to the mean free path and the coherence length, that is $d \gtrsim (\xi, l)$, then the main contribution to the Josephson current is given by the impurity scattering in the normal layer. If the opposite case $d \ll (\xi, l)$ occurs, the main contribution to the current is provided via the mechanism of reflections from the surfaces of the normal crystal. Our experiment corresponds to the second case, and

for this reason, we focus on this limit, while a more complete theory is discussed in the companion paper [53]. The calculation of the Josephson current is based on the Eilenberger equation for the quasiclassical Green's function $\check{g}(\mathbf{n}, \mathbf{r}, \omega)$, which is a matrix 2×2 in the particle-hole space [54, 55]

$$[i\omega\tau_z + \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix}, \check{g}] + i\mathbf{v}_F \nabla \check{g} = 0, \quad (1)$$

where ω is the fermionic Matsubara frequency, Δ is the superconducting order parameter, which is nonzero only in the leads, and τ_z is the Pauli matrix in the particle-hole space. We neglect the impurity scattering in the normal layer; for this reason, the impurity self-energy does not appear in Eq. (1). Assuming that the transmission D of the interface between superconductor and normal metal (S/N) is small, $D \ll 1$, as confirmed by subsequent comparisons of the calculated results with experimental data for the critical current amplitudes, the boundary conditions at the S/N interface take the form [56]:

$$\check{g}_N^{\text{out}} - \check{g}_N^{\text{in}} = \frac{D}{2}[\check{g}_N^0, \check{g}_S^0], \quad (2)$$

where $\check{g}^{\text{in(out)}}$ is the Green's function incoming to (or outgoing from) the N side of the S/N interface, and $\check{g}_{N(S)}^0$ is the Green's function in the normal metal (or superconductor) at $D = 0$. The complete system of equations required to calculate the Green's function in the normal layer and the Josephson current also contains boundary conditions at the upper and bottom surfaces of the N

layer. Depending on their roughness, the reflection process at these surfaces may contain both a specular and a diffusive component. For surface roughness, we consider specular reflection and isotropic diffusive scattering from the bottom N surface [Fig. 1(c)]. The diffusive scattering model agrees better with the experiment, suggesting relatively diffusive surfaces. The bottom surface boundary condition is:

$$\tilde{g}_N^{out} = \langle \tilde{g}_N^{in} \rangle, \quad (3)$$

where $\langle \tilde{g}_N^{in} \rangle$ is the Green's function averaged over all the incoming directions of the electron velocity $\mathbf{v}_F$. The Josephson current between two arbitrarily small superconducting elements of the whole Josephson structure, presented in Fig. 1(a), takes the form [53]:

$$I_{S \rightarrow R} = T \sum_{\omega} \frac{D^2 e |\Delta|^2 d S_S d S_R N_F \lambda^2 v_F \sin(\phi_2 - \phi_1)}{8\pi(|\Delta|^2 + \omega^2)} \times \quad (4)$$

$$\int_0^\pi \int_{-\pi/2}^{\pi/2} \frac{e \tilde{d} - \sqrt{\tilde{d}^2 + \tilde{L}^2 - 2\tilde{L}\tilde{d}\cos\theta} \tilde{d}}{(\tilde{d}^2 + \tilde{L}^2 - 2\tilde{L}\tilde{d})^{3/2}} \sin\theta d\theta d\varphi$$

where $dS_{S,R}$ are areas of the corresponding superconducting elements, N_F is the electron density of states at the Fermi surface in the normal layer, $|\Delta|$ is the absolute value of the superconducting order parameter, which is assumed to be the same in all superconducting leads, $\phi_{1,2}$ are the phases of the superconducting order parameter of the considered elements, L is the distance between the elements, and $\lambda = 2\omega/v_F$. $\tilde{L} = \lambda L$, $\tilde{d} = \frac{\lambda d}{\cos\varphi \sin\theta}$, θ , and φ are spherical angles of the scattering point in a coordinate system with the origin at one of the superconducting elements and the polar axis along the line connecting these elements. A good approximation is:

$$I_{S \rightarrow R} = T \sum_{\omega} \frac{D^2 e |\Delta|^2 d S_S d S_R N_F \lambda^2 v_F \sin(\phi_2 - \phi_1)}{8\pi(|\Delta|^2 + \omega^2)} \times \quad (5)$$

$$\frac{2}{\lambda^2(L^2 + 4d^2)} (2e^{-\lambda\sqrt{L^2 + 4d^2}} + e^{-\lambda(L+2d)})$$

In this model, the multiple diffusive reflections from the N layer surfaces are disregarded. This approximation works well if $\xi < d < l$ or $L \lesssim 2d$. The first of the conditions is not applicable to the current experiment, but the second condition is fulfilled for part of the JJs.

To calculate the net critical current $I_c^{A_i-B_i}$ corresponding to the current flow from A_i to B_i electrodes, we first calculate the critical currents $i_c^{A(B)_i-A(B)_j}$ between all possible pairs of $A(B)_i - A(B)_j$ and $A_i - B_j$ by summing the currents over all pairs of small superconducting elements belonging to the corresponding leads (28 JJs in total). Then the superconducting phases are set at two

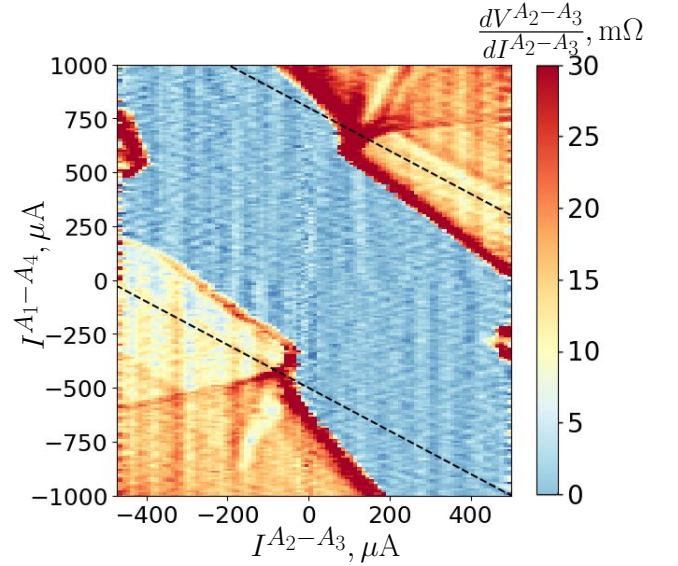


FIG. 2. Differential resistance dV/dI at the A_2-A_3 junction with simultaneous currents $I^{A_2-A_3}$ and $I^{A_1-A_4}$. $T = 0.7$ K.

leads through which the current is passed, and the other phases are determined numerically from the condition of current conservation [53].

In Fig. 1(b), the calculated $I_c^{A_i-B_i}$ JJs for $i = 1 - 4$ are superimposed on the experimental data. The model provides very good quantitative agreement for the low-temperature values of all four critical currents $I_c^{A_i-B_i}$. The overall nearly linear temperature dependence of the critical current is also reproduced, although the quantitative agreement between theory and experiment at intermediate temperatures is not ideal. In the experiment, the deviation of the $I_c(T)$ from linearity occurs at lower temperatures than the theory suggests. In our opinion, this is due to the fact that, in the experiment, the interlayer lengths of most JJs are of the same order as the mean free path l ; therefore, the ballistic limit condition is not fully satisfied. That is, in addition to diffusive reflection from the surfaces of the N layer, there is also a certain amount of reflection from impurities in the N layer itself, which changes the temperature dependence of the coherence length.

Multiterminal transport.—Fig. 2 shows the resistance dV/dI measured at the $A_2 - A_3$ JJ when the current $I^{A_2-A_3}$ is applied from A_2 to A_3 and the current $I^{A_1-A_4}$ is applied from A_1 to A_4 simultaneously. These data provide evidence of the multi-terminal behavior of our system, demonstrating that $I^{A_2-A_3}$ depends not only on the phase difference at the $A_2 - A_3$ JJ but also on the phases at other superconducting electrodes. Indeed, if $I^{A_i-A_j}$ is only determined by the phase difference at the $A_i - A_j$ JJ between the nearest neighbor electrodes, then the system of A_1, A_2, A_3 , and A_4 electrodes on top of the Au crystal can be viewed as a chain of three JJs in

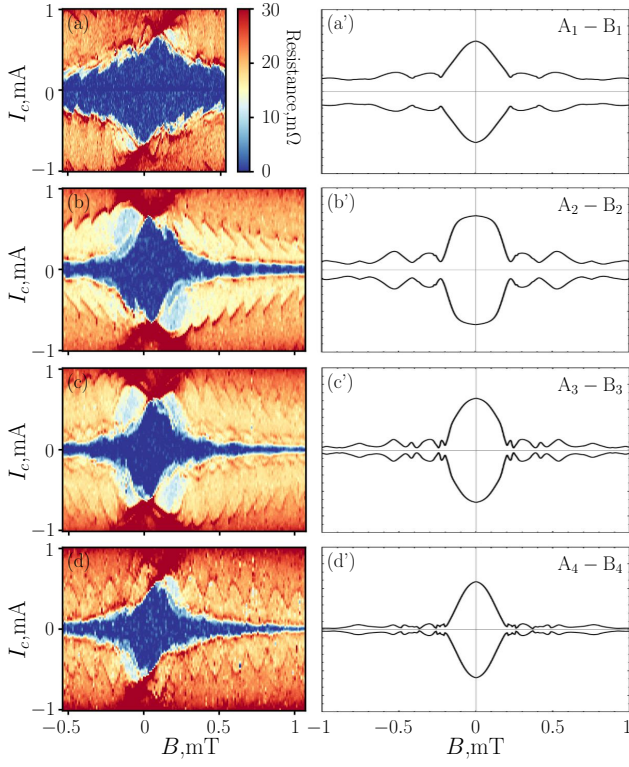


FIG. 3. (a)-(d) Experimental data for $I_c^{A_i-B_i}$ (dark blue region boundaries) versus perpendicular magnetic field. (a')-(d') Theoretical results for $I_c^{A_i-B_i}$ versus perpendicular magnetic field. $T = 0.7$ K.

series: $A_1 - A_2$, $A_2 - A_3$, and $A_3 - A_4$. In this case, the boundary of the blue region is determined by the condition $I^{A_2-A_3} + I^{A_1-A_4} = \pm i_c^{A_2-A_3}$. This means that the slopes of the blue region boundaries should be equal to -1 (shown as black dotted lines in Fig. 2), and the observed slopes are ≈ -1.7 . It indicates that, for example, $I_c^{A_1-A_3} \neq 0$ and, therefore, $I^{A_2-A_3}$ depend on the phases at all the electrodes.

Magnetic field response.—The experimental data for the dependence of $I_c^{A_i-B_i}$ on the applied magnetic field B are presented in Figs. 3(a)-(d). First of all, one can see that the period of the Fraunhofer-like oscillations of $I_c^{A_i-B_i}$ is approximately two orders of magnitude smaller than expected for an independent JJ $B_p = \Phi_0/(L_i^{JJ}w)$, where $\Phi_0 = \pi\hbar c/e$ is the flux quantum. $B_p = 3 \sim 30$ mT for $i \in [1, \dots, 4]$. The observed period approximately corresponds to the large area of the normal interlayers of the $A(B)_i - A(B)_j$ JJs between the nearest neighbors i and j . It once again proves our main statement that, due to the quasi-ballistic regime of transport corresponding to the large coherence length $\xi \approx 1.7\mu\text{m}$, the Josephson coupling occurs between all the superconducting elements in the system. The asymmetry of positive and negative maxima of I_c can be explained by the magnetic self-fields of the Josephson currents, which are not taken into account in our theoretical consideration.

In order to calculate the dependence $I_c(B)$, the expressions for the critical current should be generalized in a standard way [57] by adding the integral of the magnetic field vector potential $\mathbf{A}$ to the superconducting phase difference:

$$\phi_2 - \phi_1 \rightarrow \phi_2 - \phi_1 + \frac{2\pi}{\Phi_0} \int \mathbf{A}(s) \cdot \mathbf{n} ds \quad (6)$$

where $\mathbf{n}$ is the unit vector along the trajectory and s is the coordinate along it. The results of the calculation [53] are presented in Figs. 3(a')-(d'). The theory reproduces well both the maximum current amplitude at $B = 0$ and all the main features of the curves: a sharp drop in the critical current at a field of ~ 0.2 mT, which is the same for all JJ, followed by a weakly decaying behavior on which quasi-periodic oscillations are superimposed. The initial sharp drop at $B \sim 0.2$ mT is attributed to the first minimum of the Fraunhofer pattern of $i_c^{A(B)_i-A(B)_j}$ and is located at $B \approx \Phi_0/(L_i^{A(B)}w)$. At larger fields, the main contribution to the current is produced by $A_i - B_i$ JJs, which have a much smaller area of the normal inter-layer region and a much weaker, non-oscillating dependence on the field [53]. It is important that at $B > 0.2$ mT, when the contribution of the $A(B)_i - A(B)_j$ JJs is strongly suppressed, the $A_i - B_i$ JJs become practically independent. For this reason, the average critical current at $B > 0.2$ mT, which corresponds to $i_c^{A_i-B_i}$, monotonically decreases with increasing L_i^{JJ} from $i = 1$ to 4. The quasiperiodic character of the oscillations is the result of the superimposition of the Fraunhofer patterns corresponding to $A(B)_i - A(B)_j$ JJs, which correspond to slightly different $L_i^{A(B)}$.

Conclusions.—We demonstrate a multi-terminal superconducting system on a Au single crystal in the quasi-ballistic regime, where the superconducting coherence length is comparable to inter-electrode distances. The nonmonotonic critical current dependence on inter-layer length, distinctive magnetic field response showing a sharp regime crossover, and direct multiterminal transport measurements provide compelling evidence for strong coupling among all superconducting leads. Our results establish a platform for exploring multi-terminal Josephson effects, including artificial topological matter in metallic mesoscopic devices, where the entire Au crystal serves as the normal scattering region [53]. The developed theoretical approach for strongly coupled planar Josephson structures on large quasi-ballistic crystals agrees well with experiments, strongly supporting our interpretation.

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