Analytical Inverse Kinematic Solution for "Moz1" NonSRS 7-DOF Robot arm with novel arm angle

Ke CHEN

Abstract

This paper presents an analytical solution to the inverse kinematic problem(IKP) for the seven degree-of-freedom (7-DOF) Moz1 Robot Arm with offsets on wrist. We provide closed-form solutions with the novel arm angle . it allow fully self-motion and solve the problem of algorithmic singularities within the workspace. It also provides information on how the redundancy is resolved in a new arm angle representation where traditional SEW angle faied to be defined and how singularities are handled. The solution is simple, fast and exact, providing full solution space (i.e. all 16 solutions) per pose.

Index Terms

7-DOF redundant robot arm, Closed-form Inverse kinematics, NonSRS structure, novel redundancy representation

I. Introduction

Research on light-weight redundant manipulators, has grown in various directions like human robot interaction [1] or machine learning [2]. While some of these applications use the inverse kinematics solver which is integrated in the robot's control system, others use a numerical solver. The former solution is only available in position control mode and the code of the robot control unit is closed source and information on the used algorithms and therefore the redundancy resolution can hardly be found. Numerical solvers, on the other hand, always come with a risk of violating real-time constraints or not finding a solution at all, although solutions exist in the workspace [3].

Finding an analytical solution for the IKP of serial robotic manipulators is one of the fundamental basic problems in robotics and there is still no general solution available. There exist many analytical solutions for specific classes of robots like 7R anthropomorphic robot arms with spherical shoulder, spherical wrist(SRS structure), and three intersecting joint axes in the elbow [4], [5]. Although NonSRS structure manipulation(like Franka Emika Panda) is also a 7R anthropomorphic robot arm, the previously mentioned solutions do not apply for the robot since there are linear offsets in its elbow and wrist joints. These offsets make the analytical solution more complex compared to other anthropomorphic arms like the KUKA iiwa, where all joints are aligned in the zero configuration. More details on the robot's structure are given in section II.

For SRS strucure, The IKP is solved based on the SEW angle as an redundancy represent [6], The IKP of some non-zero link offsets robot(NonSRS sturcture) could also be analytically solved based on the SEW angle method [7], it required that the shoulder and wrist must be static during self-motion(we call it static-SW 7dof robot) However, for those nonstatic-SW robot, To the best of the author's knowledge, there is no analytical IKP implementation available. This paper provides both the analytical solution as well as a real-time safe implementation for the nonstatic-SW 7dof robot. More details about inverse kinematic are given in section III, first in a specific configuration then promote it to general case.

To validate the solutions presented in this paper, a C++ version of the approach has been implemented . Details on the implementation and used libraries with visualization of results are presented in section IV. A conclusion of this work is given in section V, where also some other robots with open IKP are mentioned and possible future work based on this solution is illustrated.

II. KINEMATIC STRUCTURE

The Moz1 robot consists of eight links connected by seven joints, which make the serial robot a redundant manipulator. figure 1 show the MDH parameters and figure 2 illustrates its kinematic structure. The shoulder is constructed similar to other anthropomorphic arms like the KUKA iiwa, i.e., it is also a spherical shoulder. the first six joint is Similar to KUKA iiwa while there is an offset bettwen sixth joint and seventh joint, so there axis are not intersecting. which can be seen in figure 2. the offset awr = 0.0905 m .

The fig. 2 shows the seven joint frames F_i for $0 \leq i \leq 6$ and j = i + 1 as well as the base frame F_0. A joint frame is represented by a homogeneous transformation matrix as

$$^{i}T_j = \begin{pmatrix} ^{i}R_j & ^{i}P_j \\ 0 & 1 \end{pmatrix} \tag{1}$$

it describes the transformation between the joint connecting links i and j. Additionally, some special frames are defined which are important for later calculations. For analogies with anthropomorphic arms, a shoulder frame F_S = $^{0}T_2$, an elbow frame F_E = $^{0}T_4$, and a wrist frame F_W = $^{0}T_6$ are defined.

MDH parameter	a	alpha	d	theta
1	0	0	dbs	θ_1
2	0	-pi/2	0	θ_2 -pi/2
3	0	pi/2	-dse	θ_3 +pi/2
4	0	pi/2	0	θ_4
5	0	-pi/2	-dew	θ_5 -pi/2
6	0	-pi/2	0	θ_6 +pi/2
7	awr	pi/2	0	θ_7

Fig. 1: Robot MDH parameters

III. INVERSE KINEMATICS

A. *Definition of the novel arm angle*

From Figure 2, there are four important points shoulder center S, elbow center E, wrist center W and center point of 7th axis C. Obvously, in the self motion of the robot arm, the wrist point W would move in a circle around point C, which is not suitable using SEW angle where the S and W is static during the self motion. here we provide a new arm anlge using point S, E and C as SC are staic during the self motion. the reference plane is spaned by the vector SC and $\overrightarrow{z_7}$ while the arm plane is spaned by the vector SC and vector SE, hence the arm angle would be the angle bettwen reference plane and arm plane around SC

$$\psi = \angle plane < SC, \overrightarrow{z_7} > \;\; plane < SC, SE >$$

B. *Special configuration*

First, let's consider a special configuration where the wrist center point C is located on the base z-axis straight above the shoulder S, and the base x-axis is in the reference plane. As the figure 3 shows, the angle from SC to $\overrightarrow{z_7}$ around base y-axis is denoted as q, dsc is the length of vector SC, xc2 $= y_0 \times z_7$ and al is the angle from xc2 to xc1 $= x_7$ around z_7. In such configuration, we could present endframe in only 3 dof, the parameter dsc, q, and al as follow:

$$\begin{aligned} {}^0T_7 &= Trans(0, 0, dbs + dsc) * Roty(q) * Rotz(al) \\ &= \begin{bmatrix} cos(al) * cos(q) & -cos(q) * sin(al) & sin(q) & 0 \\ sin(al) & cos(al) & 0 & 0 \\ -cos(al) * sin(q) & sin(al) * sin(q) & cos(q) & dbs + dsc \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{2}$$

given the endframe ${}^0T_7(dsc, q, al)$ and arm angle ψ, we present the analytical IK by algebraic method.

1) solve q6 and q7

III-B1a arm equation

denote the point E in base frame ${}^0E = (E_x, E_y, E_z, 1)$ According to the figure 3, we get the arm angle constrains

$$\psi = atan2(E_y, E_x) + \pi \tag{3}$$

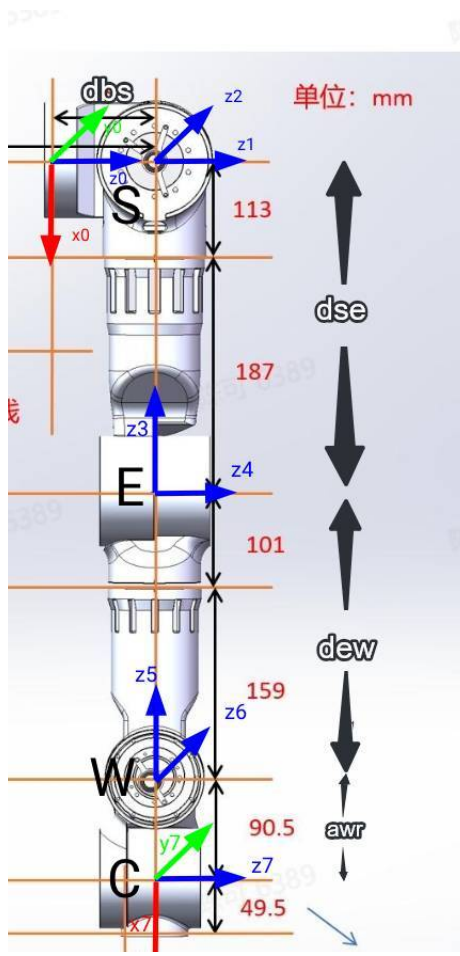

Fig. 2: Robot kinematic structure

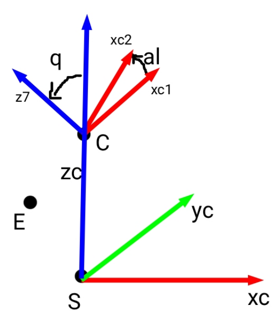

Fig. 3: specific configuration

E in Frame F_7 is

$$^7E = {}^7T_6 * {}^6T_5 * {}^5T_4 * \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -t_6 cos(q7) \\ t_6 sin(q7) \\ -r_6 \\ 1 \end{bmatrix}$$

where $t_6 = a_{wr} + d_{ew}cos(q6)$ and $r_6 = d_{ew}sin(q6)$
hence

$$^0E = \ ^0T_7 \ ^7E = \begin{bmatrix} -r_6 * sin(q) - t_6cos(q)cos(q7-al) \\ t_6sin(q7-al) \\ dbs + dsc - r_6cos(q) + t_6sin(q)cos(q7-al) \\ 1 \end{bmatrix} \tag{4}$$

subs (4) in to (3), we get

$$\begin{aligned} \psi &= atan2(t_6sin(q7-al), -r_6sin(q) - t_6cos(q)cos(q7-al)) + pi \\ &= atan2(t_6sin(q8), -r_6sin(q) - t_6cos(q)cos(q8)) + pi \end{aligned} \tag{5}$$

where $q8 = q7 - al$

III-B1b pose equation

denote the point S in frame F_6, $^6S = (S_x, S_y, S_z, 1)$ according to DH,

$$^3S = col_4(^3T_2) = \begin{bmatrix} 0 \\ 0 \\ d_{se} \\ 1 \end{bmatrix} \tag{6}$$

Additionally,

$$^3S =^3 T_4 *^4 T_5 *^5 T_6 * \ ^6S \tag{7}$$

combine (6) and (7) and square both side we get

$$S_x^2 + S_y^2 + S_z^2 + 2d_{ew}S_xcos(q6) - 2d_{ew}S_ysin(q6) + d_{ew}^2 - d_{se}^2 = 0 \tag{8}$$

and we have

$$^0S = (0, 0, d_{bs}, 1)$$

hence

$$^6S = \ ^6T7 * \ ^0T_7^{-1} * \ ^0S = \begin{bmatrix} a_{wr} + d_{sc}sin(q)cos(q8) \\ d_{sc}cos(q) \\ d_{sc}sin(q)sin(q8) \\ 1 \end{bmatrix} \tag{9}$$

subs (9) in to (8) we get:

$$a_{wr}t_6 - d_{sc}(r_6cos(q) - t_6cos(q8)sin(q)) - k = 0 \tag{10}$$

where

$$k = (a_{wr}^2 + d_{se}^2 - d_{sc}^2 - d_{ew}^2)/2$$

III-B1c quartic equation

according the two constrains (5),(10) we get two equations about q6 and q8 we figure that these equations would reduced into a quartic equation about q6. subs (10) into (5) to reduce $cos(q8)$ get

$$sin(q8) = \frac{(xcot(q) - d_{sc}r_6sin(q)) * tan(psi)}{d_{sc}t_6} \tag{11}$$

where $x = a_{wr} * t_6 - k - d_{sc} * r_6 * cos(q)$
combine (10),(11)with $sin(q8)^2 + cos(q8)^2 = 1$ we get

$$x^2cos(psi)^2 + (xcos(q)sin(psi) - d_{sc}r_6sin(psi)sin(q)^2)^2 - (d_{sc}t_6cos(psi)sin(q))^2 = 0$$

expand the above equation and notice that $(t6 - a_{wr})^2 + r_6^2 = d_{ew}^2$ we get

$$tm1 + t_6tm2 + t_6^2tm3 = -r_6 * y * (k - a_{wr}t_6) \tag{12}$$

where

$$y = 2 * d_{sc}cos(q)$$

$$tm1 = cos(psi)^2(k^2 - (a_{wr}^2 - d_{ew}^2)d_{sc}^2cos(q)^2) + 1/2(-2a_{wr}^2d_{sc}^2 + 2d_{ew}^2d_{sc}^2 + k^2 + k^2cos(2q))sin(psi)^2$$

$$tm2 = -2a_{wr}cos(psi)^2(k - d_{sc}^2 cos(q)^2) + a_{wr}(2d_{sc}^2 - k - kcos(2q))sin(psi)^2$$

$$tm3 = 1/8(6a_{wr}^2 - 8d_{sc}^2 + 2a_{wr}^2 cos(2psi) - a_{wr}^2 cos(2(psi - q)) + 2a_{wr}^2 cos(2q) - a_{wr}^2 cos(2(psi + q)))$$

square (12), we get

$$g_4 t_6^4 + g_3 t_6^3 + g_2 t_6^2 + g_1 t_6 + g_0 = 0 \tag{13}$$

where

$$g_4 = tm3^2 + a_{wr}^2 y^2$$

$$g_3 = 2tm2tm3 - 2a_{wr}(a_{wr}^2 + k)y^2$$

$$g_2 = tm2^2 + 2tm1tm3 + (a_{wr}^4 - a_{wr}^2(d_{ew}^2 - 4k) + k^2)y^2$$

$$g_1 = 2tm1tm2 - 2a_{wr}k(a_{wr}^2 - d_{ew}^2 + k)y^2$$

$$g_0 = tm1^2 + (a_{wr}^2 - d_{ew}^2)k^2 y^2$$

(13) is a quartic equation about t6, so using quartic solver we could analytically solve the equation and get four solution of q6, the (12) could be used to get the sign of q6.

subs q6 into (10),(11), we get sin(q8), cos(q8);

$$q8 = atan2(sin(q8), cos(q8))$$

$$q7 = q8 + al$$

2) solve q4 q5

we have

$${}^6S = col_4({}^6T_5\ {}^5T_4\ {}^4T_3\ {}^3T_2) = \begin{bmatrix} -d_{se} * (cos(q4) * cos(q6) + sin(q4) * sin(q5) * sin(q6)) - d_{ew} * cos(q6) \\ d_{se} * (cos(q4) * sin(q6) - cos(q6) * sin(q4) * sin(q5)) + d_{ew} * sin(q6) \\ d_{se} * cos(q5) * sin(q4) \\ 1 \end{bmatrix} \tag{14}$$

square the two side we get

$$cos(q4) = (S_x^2 + S_y^2 + S_z^2 - d_{ew}^2 - d_{se}^2)/(2 * d_{se} * d_{ew})$$

combined (9) then we get two solution of q4

$$q4 = \pm acos((S_x^2 + S_y^2 + S_z^2 - d_{ew}^2 - d_{se}^2)/(2 * d_{se} * d_{ew})) \tag{15}$$

get the third rows of (14) we get

we have

$${}^5S = col_4({}^5T_4\ {}^4T_3\ {}^3T_2) = \begin{bmatrix} d_{se} * sin(q4) * sin(q5) \\ d_{se} * cos(q5) * sin(q4) \\ d_{ew} + d_{se} * cos(q4) \\ 1 \end{bmatrix} \tag{16}$$

$$= {}^5T_6 * {}^6S = \begin{bmatrix} -S_y * cos(q6) - S_x * sin(q6) \\ S_z \\ S_y * sin(q6) - S_x * cos(q6) \\ 1 \end{bmatrix} \tag{17}$$

from the first two rows of (17) we get

$$q5 = atan2((-S_x * sin(q6) - S_y * cos(q6)), S_z) \tag{18}$$

3) solve q1 q2 q3

first we have

$$\begin{aligned} {}^0R_3 &= {}^0R_1(q_1) * {}^1R_2(q_2) * {}^2R_3(q_3) \\ &= \begin{bmatrix} -c_1s_2s_3 - c_3s_1 & s_1s_3 - c_1c_3s_2 & -c_1c_2 \\ c_1c_3 - s_1s_2s_3 & -c_3s_1s_2 - c_1s_3 & -c_2s_1 \\ -c_2s_3 & -c_2c_3 & s_2 \end{bmatrix} \end{aligned} \tag{19}$$

with $s_i = sin(q_i), c_i = cos(q_i)$ Additionally,

$$\begin{aligned} {}^0R_3 &= {}^0R_7 * {}^7R_3 \\ &= {}^0R_7 * {}^7R_6(q_7) * {}^6R_5(q_6) * {}^5R_4(q_5) * {}^4R_3(q_4) \\ &\overset{\text{denoted}}{=} \{r_{ij}\}_{3*3} \end{aligned} \tag{20}$$

combine (19),(20) we get two solution of q2

$$q_2 = \pm acos(r_{33}) + \frac{\pi}{2} \tag{21}$$

denote $sgn2 = sign(cos(q_2))$, from r_{13}, r_{23}

$$q_1 = atan2(-r_{23} * sgn2, -r_{13} * sgn2) \tag{22}$$

from r_{31}, r_{32}

$$q_3 = atan2(-r_{31} * sgn2, -r_{32} * sgn2) \tag{23}$$

totally, 16(4*2*2) solutions would be get

C. general configuration

The ik method above is only valid for a very small subset of all possible end effector poses in the work space where the wrist center C is located on the base z-axis and the 7th joint axis z7 is located in the base xz-plane. However, a closer look on the robot's kinematics shows that all possible end effector poses can be reduced to one of those special poses. [8] shows an example of how general poses can be reduced to a special pose which is similar to our case. By freeing the first 3 joint (sphere joint). it would rotate the robot around the sphere until it become a special configuration, it is easy to see that q4 q5 q6 q7 is state in such motion. so we could get the q4 to q7 with the same method, then using q4 to q7 to solve a sphere rotation to get q1 q2 q3. the key step is get the 3 dof d_{sc}, q and al from given endpose;

1) calculate dsc, q, al

given general end frame ${}^0T_7 = \begin{pmatrix} {}^0R_7 & {}^0P_7 \\ 0 & 1 \end{pmatrix}$

$$SC = {}^0P_7 - (0, 0, d_{bs})^T$$

$$d_{sc} = norm(SC), zv = SC/d_{sc}, z7 = col_3({}^0R_7)$$

$$q = -acos(zv \cdot z7)$$

$$yv = z7 \times zv, yv = yv/norm(yv)$$

$$x72 = yv \times z7$$

$$x7 = col_1({}^0R_7)$$

$$al = atan2(dot(x72 * x7, z7), dot(x72, x7));$$

with the same method as in special configuration we get all the joints value, notice that in (20), the ${}^0R_7$ comes from the original given ${}^0T_7$

IV. Singularity analysis

the singularity analysis is also a basic part in robot, especilly in redundency robot, there are no simple way to get all singularities.Here we use the reciprocity-based method [9] and give the result of the singularity:

- Kinematic Singularities
  - $q4 = \{0, \pi\}$
  - $q2 = \pm\pi/2$ && $q3 = \{0, \pi\}$
  - $q2 = \pm\pi/2$ && $q6 = \pm acos(-awr/dew)$
  - $q5 = \{0, \pi\}$ && $q6 = \pm acos(-awr/dew)$
- Algorithmic Singularities
  - $q2 = \pm\pi/2$
  - $q6 = \pm acos(-awr/dew)$
  - (dew + awr*cos(q6))*sin(q4) - awr*cos(q4)*sin(q5)*sin(q6)=0

V. IMPLEMENTATION AND RESULTS

One main reason for the development of an analytical solution for the IKP of the redundent robot is the ability to have a real-time safe deterministic implementation that can be used for moving the robot in any of its provided control modes. A disadvantage of numerical solvers for the IKP of redundant robots is, for example, their use of loops, which makes the number of calculations unpredictable if no maximum of iterations is specified. If such a maximum is specified, there is guarantee that the algorithm finds a solution within a given error threshold. With the proposed analytical approach, the user will definitely get a correct solution if one exists and the maximum number of calculations is always predictable regardless of the given target. Even if the target lies outside the workspace, the algorithm gives a solution for the IKP that is based on geometric calculations being reasonable and predictable. Another drawback of numerical solvers is the need for a sophisticated initial guess which should be already close to the solution. If the user just gives an arbitrary configuration like the zero posture as initial joint configuration, the joint values often diverge quite fast and end up in some fanciful regions. The C++ implementation only uses Eigen as external library, which is known and popular for its real-time safety. The methods and functions used for the implementation of the proposed approach do also have a fixed number of calculations and do not violate any real-time requirements.

References

[1] R. J. Kirschner, L. Burr, M. Porzenheim, H. Mayer, S. Abdolshah, and S. Haddadin, "Involuntary motion in human-robot interaction: Effect of interactive user training on the occurrence of human startle-surprise motion," in *2021 IEEE International Conference on Intelligence and Safety for Robotics (ISR)*, pp. 28–32, IEEE, 2021.

[2] V. Petrík, M. Tapaswi, I. Laptev, and J. Sivic, "Learning object manipulation skills via approximate state estimation from real videos," in *Conference on Robot Learning*, pp. 296–312, PMLR, 2021.

[3] P. Beeson and B. Ames, "Trac-ik: An open-source library for improved solving of generic inverse kinematics," in *2015 IEEE-RAS 15th International Conference on Humanoid Robots (Humanoids)*, pp. 928–935, IEEE, 2015.

[4] W. Liu, D. Chen, and J. Steil, "Analytical inverse kinematics solver for anthropomorphic 7-dof redundant manipulators with human-like configuration constraints," *Journal of Intelligent & Robotic Systems*, vol. 86, no. 1, pp. 63–79, 2017.

[5] B. Bongardt, "Inverse kinematics of anthropomorphic arms yielding eight coinciding circles," in *Computational Kinematics: Proceedings of the 7th International Workshop on Computational Kinematics that was held at Futuroscope-Poitiers, France, in May 2017*, pp. 525–534, Springer, 2017.

[6] M. Shimizu, H. Kakuya, W.-K. Yoon, K. Kitagaki, and K. Kosuge, "Analytical inverse kinematic computation for 7-dof redundant manipulators with joint limits and its application to redundancy resolution," *IEEE Transactions on robotics*, vol. 24, no. 5, pp. 1131–1142, 2008.

[7] H. H. An, W. I. Clement, and B. Reed, "Analytical inverse kinematic solution with self-motion constraint for the 7-dof restore robot arm," in *2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pp. 1325–1330, IEEE, 2014.

[8] S. Tittel, "Analysis of panda's elbow motion in null space and a combined analytical-numerical ik with intuitive redundancy resolution," in *2023 21st International Conference on Advanced Robotics (ICAR)*, pp. 1–6, IEEE, 2023.

[9] S. B. Nokleby and R. P. Podhorodeski, "Reciprocity-based resolution of velocity degeneracies (singularities) for redundant manipulators," *Mechanism and machine theory*, vol. 36, no. 3, pp. 397–409, 2001.