# On Coalition Graphs and Coalition Count of Graphs

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**Abstract**

Let $G$ be graph with vertex set $V(G)$ and order $n$. A set $S \subseteq V(G)$ is a dominating set of a graph $G$ if every vertex in $V(G)\backslash S$ is adjacent to at least one vertex in $S$. A coalition in a graph $G$ consists of two disjoint sets of vertices $V_1$ and $V_2$, neither of which is a dominating set but whose union $V_1 \cup V_2$ is a dominating set. A coalition partition, abbreviated $c$-partition, in a graph $G$ is a vertex partition $\pi = \{V_1, V_2, \ldots, V_k\}$ such that every set $V_i$ of $\pi$ is either a singleton dominating set, or is not a dominating set but forms a coalition with another set $V_j$ in $\pi$. The sets $V_i$ and $V_j$ are coalition partners in $G$. The coalition number $C(G)$ equals the maximum order $k$ of a $c$-partition of $G$. For any graph $G$ with a $c$-partition $\pi = \{V_1, V_2, \ldots, V_k\}$, the coalition graph $CG(G,\pi)$ of $G$ is a graph with vertex set $V_1, V_2, \ldots, V_k$, corresponding one-to-one with the set $\pi$, and two vertices $V_i$ and $V_j$ are adjacent in $CG(G,\pi)$ if and only if the sets $V_i$ and $V_j$ are coalition partners in $\pi$. In [4], authors proved that for every graph $G$ there exist a graph $H$ and $c$-partition $\pi$ such that $CG(H,\pi) \cong G$, and raised the question: Does there exist a graph $H^*$ of smaller order $n^*$ and size $m^*$ with a $c$-partition $\pi^*$ such that $CG(H^*,\pi^*) \cong G$?. In this paper, we constructed a graph $H^*$ of small order and size and a $c$- partition $\pi^*$ such that $CG(H^*,\pi^*) \cong G$. Recently, Haynes et al. [5] defined the coalition count $c(G)$ of a graph $G$ as the maximum number of different coalition in any $c$-partition of $G$. We characterize all graphs $G$ with $c(G) = 1$. Further, imposing some suitable conditions on coalition number, we study the properties of coalition count of graph.

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# 1 Introduction

Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$, edge set $E(G)$, order $n(G) = |V(G)|$, and size $m(G) = |E(G)|$. A set $S \subseteq V(G)$ is a *dominating set* of a graph $G$ if every vertex in $V(G) \backslash S$ is adjacent to at least one vertex in $S$. A detailed account of domination in graphs is presented in the recent books on domination theory [8, 9]. The open neighborhood $N(v)$ of a vertex $v$ in $G$ is the set of vertices adjacent to $v$, while the closed neighborhood of $v$ is the set $N[v] = \{v\} \cup N(v)$. A vertex of degree one is a *pendant vertex.*

The concept of coalitions in graphs was introduced by Haynes et.al. [5] in 2020 as follows. A *coalition* in a graph $G$ consists of two disjoint sets of vertices $V_1$ and $V_2$, neither of which is a dominating set but whose union $V_1 \cup V_2$ is a dominating set. A *coalition partition*, henceforth called a $c$-partition, in a graph $G$ is a vertex partition $\pi = \{V_1, V_2, \ldots, V_k\}$ such that every set $V_i$ of $\pi$ is either a singleton dominating set, or is not a dominating set but forms a coalition with another set $V_j$ in $\pi$. The *coalition number* $C(G)$ equals the maximum order $k$ of a $c$-partition of $G$, and a $c$-partition of $G$ having order $C(G)$ is called a $C(G)$*-partition.* In [6], Haynes et al. established upper bounds on the coalition number in terms of the minimum and maximum degree. In [4], they proved that every graph is the coalition graph of some graph, while in [7], they investigated coalition graphs of trees, paths, and cycles. Furthermore, in [5], the authors posed the open problem of characterizing all graphs $G$ of order $n$ with coalition number $C(G) = n$. This problem was partially addressed by Bakhshesh et al. [3], where the authors characterized all graphs of order $n$ with $\delta(G) \leq 1$ that satisfy $C(G) = n$, and identified all trees whose coalition number equals $n - 1$. In addition, several variants of coalition have been introduced and studied in [1, 2, 10, 11].

In the paper [5] while introducing coalitions, the authors suggest several related areas for future study, one of which is coalition count of $G$ defined as follows. The coalition count $c(G)$ of a graph $G$ is equal to the maximum number of different coalition in any $c$-partition of $G$.

**Example 1.1.** *Consider the cycle $C_4 = (v_1, v_2, v_3, v_4)$.*
*The partition $\pi_1 = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}$ is a c-partition of $C_4$. No individual set in $\pi_1$ is a dominating set. However the following different coalition exist:*

- *The set $\{v_1\}$ forms coalition with $\{v_2\}$, $\{v_3\}$ and $\{v_4\}$.*
- *The set $\{v_2\}$ forms coalition with $\{v_3\}$ and $\{v_4\}$.*
- *The set $\{v_3\}$ form coalition with $\{v_4\}$.*

*Thus, every set in $\pi_1$ forms a coalition with at least one other set. Hence, $C(C_4) = 4$, and $\pi_1$ is a $C(C_4)$-coalition partition. But, the total number of different possible coalitions is 6, thus $c(C_4) = 6$.*

## 2 Coalition graphs

In [4] Haynes et al. defined coalition graph of $G$ as follows. Let $G$ be a graph with a $c$-partition $\pi = \{V_1, V_2, \dots, V_k\}$. The coalition graph $CG(G, \pi)$ of $G$ is a graph with vertex set $V_1, V_2, \dots, V_k$, and two vertices $V_i$ and $V_j$ are adjacent in $CG(G, \pi)$ if and only if the sets $V_i$ and $V_j$ are coalition partners in $\pi$. In [4] Haynes et.al proved that, for every graph $G$, there is a graph $H$ and some $c$-partition $\pi$ of $H$, such that $CG(H, \pi) \cong G$, that is, for every graph $G = (V, E)$ having $n$ non-isolated vertices and $t$ isolated vertices with $|E| = m$, they constructed a graph $H$ of order $n + m + \binom{n}{2} + t$ and size $m = \binom{n}{2} + 2m(n-1) + \overline{m}(n-2) + t(n(H) - 1)$, where $n(H)$ is order of $H$, and the $c$-partition $\pi$ such that $CG(H, \pi) \cong G$.
Further, they raised the following question: Does there exist a graph $H^*$ of smaller order $n^*$ and size $m^*$ with a $c$-partition $\pi^*$ such that $CG(H^*, \pi^*) \cong G$?.

**Theorem 2.1.** *For every graph $G$, there exist a graph $H^*$ and c-partition $\pi^*$ such that $CG(H^*, \pi^*) \cong G$.*

*Proof.* Let $G$ be a graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\} \cup \{w_1, w_2, \dots, w_t\}$, where each $v_i$ has degree at least one and each $w_i$ is an isolate. Let $G' = G - \{w_1, \dots, w_t\}$. Then $|E(G)| = |E(G')| = m$ and $|E(\overline{G'})| = \overline{m}_{G'}$, where $m + \overline{m}_{G'} = \binom{n}{2}$. To construct $H^*$, we begin with a complete graph $K_n$ on vertices $\{v_1, \dots, v_n\}$, corresponding to the non-isolates of $G$. If $n$ is even, partition these vertices into $\frac{n}{2}$ disjoint pairs and delete the edge within each pair; if $n$ is odd, form $\frac{n-1}{2}$ such pairs and delete the edge within each pair and retain all edges incident with $v_n$. These $n$ vertices are referred to as the base vertices of $H^*$. The partition $\pi^*$ is initially taken as the singleton partitions $V_i = \{v_i\}$, $1 \le i \le n$. We will add to the sets in the partition as we build $H^*$.

Now we consider the following cases:
Case 1: Let $G \cong K_n \cup tK_1$. If $n$ is even and $t \neq 0$, we construct $H^*$ by adding each isolate vertex $w_i \in V(G)$ to $H^*$ and connecting $w_i$ to every other vertex of $H^*$. Then, the partition $\pi^*$ is extended by adding singleton sets $W_i = \{w_i\}$ for $n \le i \le t$. In this construction, no set $V_i$ in $\pi^*$ is a dominating set, but every pair of sets $V_i$ in $\pi^*$ forms a coalition. Therefore, the coalition graph satisfies $CG(H^*, \pi^*) \cong K_n \cup tK_1$, where $n$ is even.
If $n$ is odd and $t \neq 0$, we first extend $H^*$ by introducing a new vertex $u_n$ associated with vertex $v_n$, such that $u_n$ is adjacent to all base vertices except $v_n$. The vertex $u_n$ is assigned to a set $V_k$ in $\pi^*$, where $k \neq n$, and the edge $u_n v_k$ is removed from $H^*$. Next, for every isolate $w_i \in V(G)$, we add $w_i$ to $H^*$ and connect it to every other vertex of $H^*$. The partition $\pi^*$ is then extended by singleton sets $W_i = \{w_i\}$ for $n \le i \le t$. Further, none of the sets $V_i$ is a dominating set, but every pair of sets $V_i$ in $\pi^*$ forms a coalition. Thus, $CG(H^*, \pi^*) \cong K_n \cup tK_1$, where $n$ is odd.

Case 2: If $G \cong G' \cup tK_1$, where $G' \ncong K_n$, then without loss of generality, we assume that $v_n$ is not a full vertex in $G$. For each edge $v_j v_k \in E(\overline{G'})$, introduce a new vertex $v_{jk}$ adjacent to every base vertex except $v_j$ and $v_k$, and place $v_{jk}$ into some set of $\pi^*$ other than $V_j$ or $V_k$ and then remove edges between the vertices in the same set $V_i$ in $\pi^*$. Here it is important to note that none of the base vertices of $H^*$ is a full vertex. Finally, add each isolate $w_i \in V(G)$ to $H^*$ and insert an edge from $w_i$ to every other vertices of $H^*$. Now, extend $\pi^*$ by a singleton sets $W_i = \{w_i\}$, $1 \leq i \leq n$.

In the resulting graph, every edge of $H^*$ is incident with either a base vertex or a dominating vertex, and no set $V_j$ of $\pi^*$ is itself a dominating set. For any edge $v_jv_k \in E(G)$. Since $v_j$ and $v_k$ collectively dominate every vertex in $H^*$, $V_j \cup V_k$ is a coalition in $H^*$. Further, if $v_jv_k \notin E(G)$, then $V_j \cup V_k$ is not a dominating set as there is no vertex in $V_j \cup V_k$ that will dominate the vertex $v_{jk}$. Each $w_i$ is dominating vertex in $H^*$, so $w_i$ is an isolate in $G$. Hence the partition $\pi^* = \{V_1, \ldots, V_n, W_1, \ldots, W_t\}$ is a $c$-partition of $H^*$ satisfying $CG(H^*, \pi^*) \cong G$. □

Properties of graph $H^*$ constructed above

| Graph $G$ | $n(H^*)$ | $m(H^*)$ |
|---|---|---|
| $G \cong K_n \cup tK_1$, $n$ is even | $n + t$ | $\binom{n}{2} - \frac{n}{2} + t(n(H^*) - 1)$ |
| $G \cong K_n \cup tK_1$, $n$ is odd | $n + t + 1$ | $\binom{n}{2} - \frac{n-1}{2} + n - 2 + t(n(H^*) - 1)$ |
| $G \cong G' \cup tK_1$, where $G' \ncong K_n$ | $n + \overline{m}_{G'} + t$ | $\binom{n}{2} - \lfloor \frac{n}{2} \rfloor + \overline{m}_{G'}(n-3) + t(n(H^*) - 1)$ |

# 3 Properties of coalition count $c(G)$ of graph $G$

The following observations on coalition count $c(G)$ of graph $G$ motivated us to study the properties of $c(G)$.

(1) The coalition number $C(G)$ and coalition count $c(G)$ of graph $G$ are not comparable. For example, Consider complete graph $K_n$, $C(K_n) = n > 0 = c(K_n)$, but for $C_4$, $c(C_4) = 6 > C(C_4) = 4$ and for $P_4$, $c(P_4) = C(P_4) = 4$.

(2) The $c$-partition used to count the coalition number and coalition count need not be unique. For example, consider $P_6 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. The partition $\pi_2 = \{\{v_2\}, \{v_4\}, \{v_1, v_6\}, \{v_3\}, \{v_5\}\}$ is a $c$-partition of $P_6$. Thus, $C(P_6) = 5$ but the total number of different possible coalitions is 3. Now, consider $\pi_3 = \{\{v_2, v_4\}, \{v_6\}, \{v_1, v_3\}, \{v_5\}\}$ is a $c$-partition of $P_6$, where $c(G) = 5$.

(3) The maximum number of edges among all the graphs $CG(G, \pi)$ is the coalition count of $G$.

The *domatic partition* is a partition of the vertex set into dominating sets. The *domatic number* $d(G)$ is equal to the maximum order $k$ of the vertex partition, called domatic partition, $P = \{V_1, V_2, \ldots, V_k\}$, such that every set $V_i$ is a dominating set in $G$.
In a graph $G$ of order $n$, a vertex of degree $n-1$ is called a *full vertex*. A subset $V_i$ is called a *singleton set* if $|V_i| = 1$. Note that any full vertex forms a singleton dominating set.

**Theorem 3.1.** *For any graph $G$ with no isolated vertices and $f$ full vertices, $c(G) \geq d(G) - f$.*

*Proof.* Assume $P = \{V_1, V_2, \ldots, V_k\}$ is a domatic partition of $G$ with $k = d(G)$. Without loss of generality, suppose that $V_1, V_2, \ldots, V_{k-1}$ are minimal dominating sets. If any $V_i$ (for $1 \leq i \leq k-1$) is not minimal, we replace it with a minimal dominating set $V_i' \subseteq V_i$, and add the vertices in $V_i \setminus V_i'$ to $V_k$. Since $G$ has $f$ full vertices, there will be $f$ singleton dominating sets. Now, consider a domatic-partition $P_1 = \left\{ \underbrace{V_1', \ldots, V_f'}_{f \text{ full vertices}}, \underbrace{V_{f+1}', \ldots, V_{k-1}'}_{\text{minimal dominating sets}}, V_k' \right\}$. Then each of $k - f - 1$ minimal dominating sets can be partitioned into two non-empty, non-dominating subsets $V_{i,1}$ and $V_{i,2}$, where $i = f+1, \ldots, k-1$, whose union is dominating. Thus, these minimal dominating sets contribute at least $k - f - 1$ coalitions.

Consider the vertex partition $P_2 = \left\{V_1', \ldots, V_f', V_{f+1,1}, V_{f+2,2}, \ldots, V_{k-1,1}, V_{k-2,2}, V_k'\right\}$. Further, if $V_k'$ is also a minimal dominating set, then $V_k'$ can be partitioned into two non-empty, non-dominating subsets whose union is dominating. Therefore, by replacing $\{V_k'\}$ in $P_2$ using these two sets, we get a $c$-partition of $G$. Thus, $c(G) \geq k - f - 1 + 1 = k - f$.

Suppose $V_k'$ is not a minimal dominating set. Let $V_k'' \subseteq V_k'$ be a minimal dominating set, and let $V_k'' = V_{k,1} \cup V_{k,2}$, where $V_{k,1}$ and $V_{k,2}$ are non-empty, non-dominating sets that together form a coalition. Define $W = V_k' \setminus V_k''$. If $W$ is a dominating set, then there exist at least $k+1$ disjoint dominating sets in $G$, contradicting $d(G) = k$. Thus, $W$ is not a dominating set in $G$. Now, by replacing $V_k'$ in $P_2$ by $V_{k,1}, V_{k,2}$ and $W$, we obtain a $c$-partition of $G$. If $W$ can form a coalition with another non-dominating set then $c(G) \geq k - f + 1$. If $W$ does not form a coalition with any set, then consider the union $V_{k,2} \cup W$ as a set in $c$-partition. This set remains non-dominating but forms a coalition with $V_{k,1}$. Thus, $c(G) \geq k - f$. Therefore, $c(G) \geq k - f = d(G) - f$. □

The bound of Theorem 3.1 is sharp for stars $K_{1,n-1}$ , for $n \geq 3$, as $c(K_{1,n-1}) = 1$, $d(G) = 2$ and $f = 1$. Also, the bound is sharp for complete graph, where $c(K_n) = 0$ and $d(K_n) = f = n$.
An *independent set* is a set of vertices in $G$ such that no two vertices in the set are adjacent. The *independence number* $\alpha(G)$ of a graph $G$ is the cardinality of the largest independent set in $G$.
In next theorem, we characterize the graphs whose coalition count is exactly one.

**Theorem 3.2.** *Let $G$ be a graph with $f$ full vertices. Then $c(G) = 1$ if and only if $\alpha(G) = n - f$.*

*Proof.* Consider a graph $G$ with $f$ full vertices $U = \{v_1, v_2, \ldots, v_f\}$ and $c(G) = 1$. Then there exist exactly two sets $V_i$ and $V_j$ which are not dominating sets, but $V_i \cup V_j$ is a dominating set. Since $V_i$ is not a dominating set, there exists a vertex $x \in V_j$, such that $(N[x] \backslash U) \cap V_i = \phi$.
Claim: $N(x) \backslash U = \phi$.
Suppose $N(x) \backslash U \neq \phi$. Then by adding the vertices in $V_j \backslash (N[x])$ to $V_i$, it follows that

the set $V_i$ is not a dominating set and $V_j = N[x]$. Now consider the sets $V_{j_1} = \{x\}$ and $V_{j_2} = N(x)$. We note that the set $V_i$ form coalition with the sets $\{x\}$ and $N(x)$. Thus, $c(G) \geq 2$, a contradiction. In similar way one can prove that if there exist a vertex in $y \in V_i$ such that $N(y)\backslash U \neq \phi$, then $c(G) \geq 2$. Thus, $N(x)\backslash U = \phi$. Therefore, $\alpha(G) = n - f$.

Conversely, suppose $G$ has $f$ full vertices and $\alpha(G) = n-f$. Then, there are $n-f$ vertices which are not adjacent to each other, and all these $n-f$ vertices is of degree $f$. Thus, in order to form a coalition partition, these $n-f$ vertices must be divided into exactly two sets $V_i$ and $V_j$ which are not dominating sets but $V_i \cup V_j$ is a dominating set. If we partition these $n-f$ vertices into at least three sets $V_i$, $V_j$ and $V_k$. Then clearly, $V_i$ do not form coalition with $V_j$ as there is no vertices either in $V_i$ or in $V_j$ which can dominate the vertices of $V_k$. Similarly, $V_i$ cannot form coalition with $V_k$. Thus, $c(G) = 1$. □

The union of two graphs $G_1$ and $G_2$ is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. It is denoted by $G_1 \cup G_2$. The join of two graphs $G_1$ and $G_2$ is the graph obtained by taking the union of $G_1$ and $G_2$, and adding edges between every vertex of $G_1$ and every vertex of $G_2$. It is denoted by $G_1 + G_2$.
Using Theorem 3.2, we deduce the following result:

**Corollary 3.3.** *Let $G$ be a graph of order $n$ with $f$ full vertices. Then $c(G) = 1$ if and only if $G \cong (K_f + pK_1) \cup qK_1$, where $f + p + q = n$.*

**Theorem 3.4.** *Let $G$ be a graph with exactly one full vertex and $\delta(G) = 1$. If $C(G) = s(s \geq 2)$, then $c(G) = s - 2$. Further, $CG(G, \pi) \cong K_1 \cup K_{1,s-2}$.*

*Proof.* Consider a graph $G$ with $\delta(G) = 1$ and a full vertex $u_1$. For $s = 2, 3$, clearly result holds. For $s \geq 4$, let $u_2, u_3, \ldots, u_p$ are pendant vertices attached to $u_1$. Then $\pi = \{\{u_1\}, W\}$, where $W = \{V_1, V_2, \ldots, V_{s-1}\}$ is the partition of the $V(G)\backslash \{u_1\}$ such that every $V_i$ is not a dominating set but forms coalition with some other set $V_j$ in $\pi$. Here it is important to note that the pendant vertices must form a single set otherwise, it is impossible to form coalition with other sets as they are independent to each other. Without loss of generality let $\{u_2, u_3, \ldots, u_p\} \subseteq V_1$. Therefore, all other sets in $W$ must form coalition with the unique set $V_1$. If not, then there exist no vertex other than $u_1$ that will dominate the vertices $u_2, u_3, \ldots, u_p$. Since $\{u_1\}$ is a dominating set, it do not form coalition with any set in $W$. Thus, $c(G) = s - 2$.
Since $G$ has only one full vertex and $\delta(G) = 1$, the coalition graph $CG(G, \pi)$ will have exactly two components. Further, as $CG(G, \pi)$ has $s$ vertices out of which the vertex corresponding to the set $\{u_1\}$ do not form coalition with any other vertices, it will be an isolated vertex in $CG(G, \pi)$. Let $H$ be the another component of $CG(G, \pi)$. Then the remaining $s-1$ vertices out of which one vertex corresponding to the set $\{u_2, u_3, \ldots, u_p\}$ form coalition with all $s-2$ vertices and as $c(G) = s-2$, $H \cong K_{1,s-2}$. Thus, $CG(G, \pi) \cong K_1 \cup K_{1,s-2}$. □

A coalition partition $\pi$ of $G$ is a singleton coalition partition if every set in $\pi$ consists of a single vertex. If a graph $G$ has a singleton coalition partition, then $G$ is referred to

as a singleton-partition graph (SP-graph).
Next, we relate coalition number $C(G)$ of graph $G$ with coalition count of $G$. We omit the proof as it is straight forward.

**Remark 3.5.** *For any graph $G$ with $f$ full vertices having coalition number $C(G)$ and coalition count $c(G)$, $c(G) \geq \lceil \frac{C(G)-f}{2} \rceil$.*

**Theorem 3.6.** *For any SP-graph $G$ of order $n$ with no full vertices, $c(G) \geq \alpha(G)$.*

*Proof.* Let $G$ is a singleton-partition graph. Then $\pi = \{\{v_1\}, \{v_2\}, \ldots, \{v_n\}\}$ is a $c$-partition. Since $G$ has no full vertex, each set $V_i$ must form a coalition with some set $V_j$, $1 \leq i < j \leq n$. By remark 3.5, $c(G) \geq \lceil \frac{n}{2} \rceil$. If $\alpha(G) \leq \lceil \frac{n}{2} \rceil$, then clearly result holds. Suppose $\alpha(G) \geq \lceil \frac{n}{2} \rceil + 1$. Then there are at least $\lceil \frac{n}{2} \rceil + 1$ vertices in $G$ are independent. Let $I = \{v_1, v_2, \ldots, v_\alpha\}$ is the maximum independent set. Further, it is easy to observe that any set in $\pi$ corresponding to the vertices of $I$ will form coalition with the set corresponding to the vertex not in $I$. Thus, there will be at least $\alpha$ coalitions. Therefore, $\alpha(G) \leq c(G)$. $\square$

# References


[1] S. Alikhani, D. Bakhshesh, and H. Golmohammadi. Total coalitions in graphs. Quaestiones Mathematicae. **47**(11): 2283–2294, 2024.

[2] S. Alikhani, D. Bakhshesh, H. Golmohammadi, and S.Klavžar. On independent coalition in graphs and independent coalition graphs. Discuss. Math. Graph Theory. **45**(2): 533–544, 2025.

[3] D. Bakhshesh, M. A. Henning, and D. Pradhan. On the coalition number of trees. Bull. Malays. Math. Sci. Soc. **46**(3): 95, 2023.

[4] T. W. Haynes, J. T. Hedetniemi, S. T. Hedetniemi, A. McRae, and R. Mohan. Coalition graphs. Commun. Comb. Optim. **8**(2): 423–430, 2023.

[5] T. W. Haynes, J. T. Hedetniemi, S. T. Hedetniemi, A. A. McRae, and R. Mohan. Introduction to coalitions in graphs. AKCE Int. J. Graphs Comb. **17**(2): 653–659, 2020.

[6] T. W. Haynes, J. T. Hedetniemi, S. T. Hedetniemi, A. A. McRae, and R. Mohan. Upper bounds on the coalition number. Australas. J. Combin. **80**(3): 442–453, 2021.

[7] T. W. Haynes, J. T. Hedetniemi, S. T. Hedetniemi, A. A. McRae, and R. Mohan. Coalition graphs of paths, cycles, and trees. Discuss. Math. Graph Theory. **43**(4): 931–946, 2023.

[8] T. W. Haynes, S. Hedetniemi, and P. Slater. Fundamentals of domination in graphs. CRC press, 2013.

[9] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning. Topics in domination in graphs. Springer, 2020.

[10] M. A. Henning and D. A. Mojdeh. Double coalitions in graphs. Bull. Malays. Math. Sci. Soc. **48**(2): 51, 2025.

[11] M. R. Samadzadeh, D. A. Mojdeh, and R. Nadimi. Paired coalition in graphs. AKCE Int. J. Graphs Comb. **22**(1): 43–54, 2025.