Noname manuscript No.
(will be inserted by the editor)

Constraining Yukawa-type interaction and coupling constant of axionlike particles to nucleons from recent measurement of the Casimir-Polder interaction

G. L. Klimchitskaya[1,2], **V. M. Mostepanenko**[1,2,a]

[1] Central Astronomical Observatory at Pulkovo of the Russian Academy of Sciences, St.Petersburg, 196140, Russia
[2]Peter the Great Saint Petersburg Polytechnic University, Saint Petersburg, 195251, Russia

Received: 28 February 2025 / Accepted: 22 April 2025

Abstract We derive constraints on the parameters of the Yukawa-type interaction and on the coupling constant of axionlike particles to nucleons from the results of recent diffraction experiment on measuring the Casimir-Polder interaction between Ar atoms and a silicon nitride nanograting. It is shown that within the interaction range from 1 to 2 nm the obtained constraints are by up to a factor of 33.4 stronger than all the other ones found previously from measurements of the Casimir force. The derived constraints are weaker in strength than those deduced from the experiments on neutron scattering. The constraints on the coupling constants of axionlike particles to nucleons following from the diffraction experiment are up to a factor of 24.2 stronger withing the range of axion masses from 32.4 to 100 eV than the previously derived ones from experiments on measuring the Casimir force. They are weaker only in comparison to the constraints found from the experiment using the beams of molecular hydrogen. The potential of the Casimir effect for obtaining stronger constraints on the parameters of hypothetical interactions is discussed.

1 Introduction

The interaction of Yukawa type between two point masses arises due to an exchange of light scalar particles having the mass m [1]. The existence of such particles (dilatons, moduli, chameleons, etc.) is predicted by the Kaluza-Klein theory, string theory, and supergravity [2–8]. The Yukawa-type potential is also predicted in the multi-dimensional models [9–12]. The interaction range λ of the Yukawa-type potential is caused by either the Compton wavelength of a scalar particle $\lambda = \hbar/(mc)$ or by the characteristic size of the compact manifold $\lambda \sim R_\star$.

[a]e-mail: vmostepa@gmail.com (corresponding author)

The strongest constraints on the parameters α and λ of the Yukawa-type potential were obtained from different experiments. Thus, for λ varying from 2×10^{-2} nm to 17.4 nm the strongest constraints are placed by the experiment on neutron scattering [13], from 17.4 nm to 40 nm – by measurements of the effective Casimir pressure using the micromechanical torsional oscillator [14], from 40 nm to 8 μm – by the differential force measurements [15], from 8 μm to 1 cm – by the Cavendish-type experiments [16–20], and for $\lambda > 1$ cm – by the Eötvös-type experiments [21,22].

Axions are the pseudoscalar particles. Originally, they were predicted in [23,24] as a consequence of violation of the Peccei-Quinn symmetry, which was postulated [25] for a solution of strong CP problem in QCD by making the CP-violating θ-term sufficiently small. In this approach, axions are the Nambu-Goldstone bosons, which become massive because the Peccei-Quinn symmetry is explicitly broken. As a result, axions can be considered as an extension of the Standard Model of elementary particles which awaits its experimental confirmation.

The axion field interacts with the fermion fields by means of the pseudovector coupling [26,27]. Along with axions, the Grand Unified Theories introduce the axionlike particles, which are not the Goldstone bosons and interact with fermions via the preudoscalar coupling [26–28]. An exchange of one axion or axionlike particle between two nucleons results in the spin-dependent interaction potential [29,30], whereas an exchange of two axionlike particles leads to the potential independent of the spins of nucleons [31].

For m_a varying from 10^{-4} μeV to 1 μeV the strongest constraints on g are placed by measuring the spin-dependent forces between neutrons by means of magnetometer [32], from 1 μeV to 0.676 meV – by the Cavendish-type experiments [18,19] measuring the spin-independent forces, from 0.676 meV to 4.9 meV – by measuring the spin-independent forces of the gravitational strength using the torsional oscillator [33–35], from 4.9 meV to 0.5 eV – by the differential measurements of the spin-independent forces [15,36], from 0.5 eV to 200 eV – by measuring the spin-dependent forces between protons in the beam of molecular hydrogen [37,38], and from 200 eV to 1000 eV – by measuring these forces between nucleons in deuterated molecular hydrogen [38]. In addition to the laboratory experiments, strong constraints on the coupling constant of axionlike particles to nucleons were obtained from the astrophysical data [27,39,40]. Numerous experiments are also devoted to constraining the coupling constants of axions and axionlike particles to electrons and photons (see [40–42] for a review).

In this paper, we obtain constraints on the Yukawa-type potential and on the coupling constant of axionlike particles to nucleons, which are placed by the recent diffraction experiment on measuring the Casimir-Polder interaction between Ar atoms and a silicon nitride nanograting [43]. It is shown that the metrological accuracy reached in the experiment [43] makes it possible to strengthen the constraints on the Yukawa-type interaction within the interaction range from $\lambda = 1$ nm to $\lambda = 2$ nm by up to a factor of 33.4 as compared to other constraints following from measurements of the Casimir force. It is also shown that in the region of masses of axionlike particles 32.2 eV $< m_a <$ 100 eV the experiment [43] on measuring the Casimir-Polder force leads to by up to a factor 24.2 stronger constraints on the coupling constant of axionlike particles to nucleons than those obtained previously from measurements of the Casimir force.

The paper is organized as follows. In Sect. 2, we briefly discuss the main features of the experiment [43] used for obtaining the constraints on hypothetical interactions which may coexist with the Casimir-Polder interaction. In Sect. 3, the constraints on the Yukawa-type interaction are obtained. Section 4 is devoted to the derivation of constraints on the coupling constant of axionlike particles to nucleons. In Sect. 5, our conclusions and a discussion are presented.

Below we use the system of units where $\hbar = c = 1$.

2 Accurate measurement of the Casimir-Polder interaction in diffraction experiment

In [43], the Casimir-Polder interaction potential was investigated by means of diffraction of slow Ar atoms (with velocities below 16 m/s) on a Si_3N_4 nanograting. An analysis of the measurement data of metrological quality using the advanced statistical methods and a careful account of various systematic effects were based on the previously developed description of atomic diffraction on a nanograting [44–46].

It has been known that in the nonrelativistic limit (actually, at separations $z \lesssim 3$ nm) the atom plate interaction potential is given by [47]

$$V_{\text{C-P}}(z) = -\frac{C_3}{z^3}, \quad C_3 = \frac{1}{4\pi}\int_0^\infty d\xi\alpha(i\xi)\frac{\varepsilon(i\xi)-1}{\varepsilon(i\xi)+1}, \tag{1}$$

where $C_3 =$ const, $\alpha(\omega)$ and $\varepsilon(\omega)$ are the frequency-dependent dynamic polarizability of an atom and dielectric permittivity of wall material calculated along the imaginary frequency axis. At separations $z \lesssim 1$ nm there are corrections to the potential (1) due to the discrete atomic structure of the plate surface and the quantum effect of exchange repulsion described by the Lennard-Jones potential $V_{\text{L-J}} = C_{\text{rep}}/z^9$. Their role was included in the balance of systematic errors.

At separations $z > 3$ nm the relativistic effects come into play and the Casimir-Polder interaction between an atom and a plate is given by the Lifshitz formula [47,48], which can be represented in the form

$$V_{\text{C-P}}(z) = -\frac{C_3 f(z)}{z^3}, \tag{2}$$

where

$$f(z) = \frac{1}{8\pi C_3}\int_0^\infty d\xi\alpha(i\xi)\int_{2z\xi}^\infty dy e^{-y} \times\left[(y^2-2z^2\xi^2)r_{\text{TM}}(i\xi,y) - 2z^2\xi^2 r_{\text{TE}}(i\xi,y)\right], \tag{3}$$

and the reflection coefficients for the transverse magnetic (TM) and transverse electric (TE) polarizations of the electromagnetic field are given by

$$r_{\text{TM}}(i\xi,y) = \frac{\varepsilon(i\xi)y - \sqrt{y^2+4z^2\xi^2[\varepsilon(i\xi)-1]}}{\varepsilon(i\xi)y + \sqrt{y^2+4z^2\xi^2[\varepsilon(i\xi)-1]}},$$
$$r_{\text{TE}}(i\xi,y) = \frac{y - \sqrt{y^2+4z^2\xi^2[\varepsilon(i\xi)-1]}}{y + \sqrt{y^2+4z^2\xi^2[\varepsilon(i\xi)-1]}}. \tag{4}$$

Note that for the gratings used in the experiment [43] the diffraction picture is formed at separations $z \lesssim 100$ nm from the grating faces. Because of this, the thermal effects, which are not taken into account in (2) and (3), are very small and can be neglected. Computations show (see, for instance, [49]) that the function $f(z)$ defined in (3) satisfies the condition $f(z) \leqslant 1$.

The measurement results in [43] were presented as the value of C_3 obtained from two measurement sets and its total error $\varDelta C_3$ determined at the 95% confidence level. Besides the systematic errors mentioned above, the error analysis took into account the systematic effects arising from the finite area of the nanograting faces, deviations of the nanograting profile from the rectangular geometry, surface roughness, losses due to collisions of Ar atoms with the nanograting faces etc. The resulting total systematic error was combined in quadrature with the random error leading to

$$C_3 = 6.9\ \text{meV}\,\text{nm}^3 \pm \varDelta C_3,$$
$$\varDelta C_3 = 1.2\ \text{meV}\,\text{nm}^3, \tag{5}$$

i.e., to 17.4% relative error. The measurement result (5) is consistent with computations by means of the Lifshitz formulas (1)–(3) using the optical data of Si_3N_4 which result in the values 7.37 [50] and 7.42 meV nm^3 [51].

3 Constraints on the Yukawa-type potential from measuring the Casimir-Polder interaction in diffraction experiment

The Yukawa-type interaction potential between the two point-like masses m_1 and m_2 is usually presented in relation to the Newtonian gravitational potential [1]

$$V_{\mathrm{Yu}}(r) = \alpha e^{-r/\lambda}\, V_{\mathrm{N}}(r), \qquad V_{\mathrm{N}}(r) = -\frac{Gm_1m_2}{r}. \tag{6}$$

Here, α is the dimensionless strength of the Yukawa interaction, r is the distance between masses m_1 and m_2, and G is the gravitational constant.

Now we calculate the Yukawa-type interaction energy $V_{\mathrm{Yu}}^{\mathrm{AP}}$ between an Ar atom at the height z and a nanograting face at $z = 0$ approximated by the plate of infinite large area and of $D \approx 100$ nm thickness. Integrating V_{YU} (6) over the plate volume, one obtains [47]

$$V_{\mathrm{Yu}}^{\mathrm{AP}}(z) = -2\pi G m_{\mathrm{A}}\rho_{\mathrm{P}}\alpha\lambda^2 e^{-z/\lambda}\left(1 - e^{-D/\lambda}\right). \tag{7}$$

Here, the mass of an Ar atom $m_{\mathrm{A}} = 6.63 \times 10^{-23}$ g and the density of a silicon nitride nanograting is $\rho_{\mathrm{P}} = 3.17$ g/cm^3 [52].

The constraints on the parameters α and λ of the Yukawa-type interaction (7) can be obtained from the fact that in the experiment [43] within the limits of the measurement error this interaction was not observed, i.e.,

$$\left| V_{\mathrm{Yu}}^{\mathrm{AP}}(z) \right| < \frac{\Delta C_3\, f(z)}{z^3}. \tag{8}$$

To be conservative, we replace $f(z)$ with its maximum value and obtain the sought for constraints from the inequality

$$\left| V_{\mathrm{Yu}}^{\mathrm{AP}}(z) \right| < \frac{\Delta C_3}{z^3}. \tag{9}$$

The resulting constraints are shown by the line labeled C-P in Fig. 1 within the interaction range from $\lambda = 1$ nm to $\lambda = 15$ nm. For each λ, the value of z in (7) and (9) leading to the strongest constraint on α was chosen. Here and below the region of the (λ, α) plane above the line is excluded by the measurement data and below the line is allowed.

For comparison purposes, in Fig. 1 we also present the constraints obtained from other measurements of the Casimir force and from experiments on neutron scattering. Thus, the constraints obtained [53] from the experiment on measuring the Casimir force between a SiC plate and a borosilicate microsphere [54] are shown by the line 1. The constraints of the line 2 were derived [55] from measurements of the lateral Casimir force between the sinusoidally corrugated surfaces [56]. In a similar way, the constraints of the line 3 were derived [57] from measurements of the normal to corrugated surfaces Casimir force [58,59]. Finally, by the line 4 we show the constraints following from measurements of the effective Casimir pressure [14]. As to the lines labeled n_1, n_2, and n_3, they show the constraints obtained from the experiments on measuring the scattered intensity of neutrons on the xenon and helium gases [60], the gas of xenon atoms [61], and on the crystal of silicon [13], respectively.

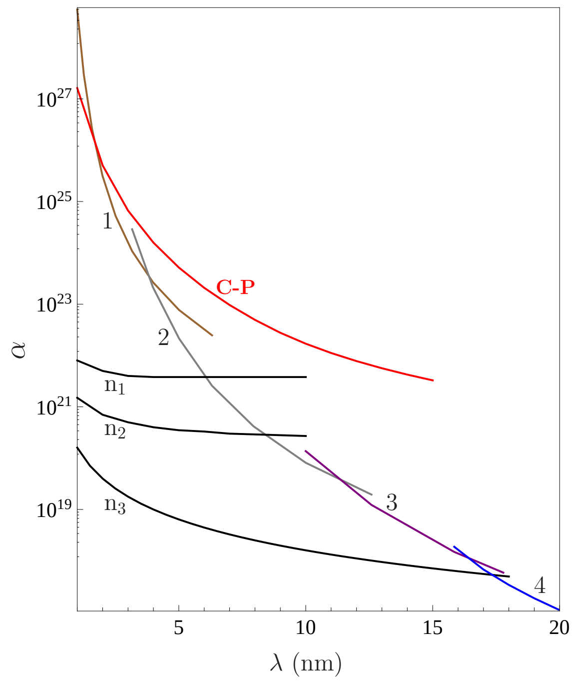

Fig. 1 Constraints on the interaction constant of the Yukawa-type potential obtained from measurements of the Casimir-Polder interaction in the diffraction experiment are shown by the line labeled C-P versus the interaction range. The previously obtained constraints from measuring the Casimir force between a SiC plate and a borosilicate microsphere, the lateral and normal Casimir forces between corrugated surfaces, the effective Casimir pressure, and from the experiments on neutron scattering are shown by the lines 1, 2, 3, 4, and n_1, n_2, n_3, respectively. The measurement data of each mentioned experiment exclude the region of the (λ, α) plane above the corresponding line and allow the region below it.

As indicated in Fig. 1, the novel constraints of the line C-P are stronger than the constraints following from other measurements of the Casimir force (lines 1, 2, and 3) over the interaction range from $\lambda = 1$ nm to 2 nm. Thus, the largest strengthening, as compared to the line 1, attained at 1 nm is by the factor of 33.4.

A comparison with the constraints following from the experiments on neutron scattering shows that the constraints of the lines labeled n_1, n_2, and n_3 remain stronger.

4 Constraints on the coupling constant of axionlike particles to nucleons from measuring the Casimir-Polder interaction in diffraction experiment

We consider the spin-independent interaction potential between two nucleons spaced r apart, which is caused by the exchange of two axionlike particles [31]

$$V_a(r) = -\frac{g^4}{32\pi^3 M^2}\,\frac{m_a}{r^2}\, K_1(2m_a r), \tag{10}$$

where $M=(m_p+m_n)/2$ is the mean nucleon mass and $K_1(x)$ is the modified Bessel function of the second kind. Here and below, following [30], we assume that the coupling constants of the axionlike particles to neutrons and protons are equal.

The measurement of the Casimir-Polder interaction between ^{87}Rb atoms and a SiO_2 plate [62] has already been used for constraining the axion-to-nucleon interaction [63]. Later it was shown, however, that the Casimir experiment [14] result in stronger constraints [64].

Here, we calculate an addition to the Casimir-Polder interaction potential which arises due to the exchange of two axionlike particles between the protons and neutrons belonging to an Ar atom at the height z and a nanograting face approximated by a plate in the same way as in Sect. 3. For this purpose, we integrate expression (10) in the polar coordinates over the plate volume and obtain

$$V_a^{\rm AP}(z)=-\frac{m_a}{M^2M_{\rm H}}\left(\frac{g^2}{4\pi}\right)^2C_{\rm A}C_{\rm P}\times\int_z^D dz_1\int_0^\infty\rho d\rho\frac{K_1\left(2m_a\sqrt{\rho^2+z_1^2}\right)}{\rho^2+z_1^2}. \qquad (11)$$

Here the coefficients $C_{\rm A,P}$ for an Ar atom and for a Si_3N_4 plate are defined as

$$C_{\rm A}=Z_{\rm A}+N_{\rm A}, \qquad C_{\rm P}=\rho_{\rm P}\left(\frac{Z_{\rm P}}{\mu_{\rm P}}+\frac{N_{\rm P}}{\mu_{\rm P}}\right), \qquad (12)$$

$Z_{\rm A,P}$ and $N_{\rm A,P}$ are the numbers of protons and the mean numbers of neutrons, respectively, in an Ar atom and a silicon nitride molecule. In doing so

$$Z_{\rm P}=3Z_{\rm Si}+4Z_{\rm N}, \qquad N_{\rm P}=3N_{\rm Si}+4N_{\rm N}, \qquad (13)$$

where the numerical data for the quantities $Z_{\rm Ar}$, $Z_{\rm Si}$, $Z_{\rm N}$, $N_{\rm Ar}$, $N_{\rm Si}$, $N_{\rm N}$ can be found in table 2.1 in [1],

$$\mu_{\rm P}=3\mu_{\rm Si}+4\mu_{\rm N}, \qquad \mu_{\rm Si}\equiv\frac{M_{\rm Si}}{M_{\rm H}}, \qquad \mu_{\rm N}\equiv\frac{M_{\rm N}}{M_{\rm H}}, \qquad (14)$$

$M_{\rm H}$ is the mass of atomic hydrogen, $M_{\rm Si}$ and $M_{\rm N}$ are the mean masses of Si and N atoms, and the density of silicon nitride $\rho_{\rm P}$ was given in Sect. 3.

Substituting the values of all these parameters in (12), one obtains

$$C_{\rm A}=39.985, \qquad C_{\rm P}=3.196\times10^3\,{\rm kg\,m^{-3}}. \qquad (15)$$

Introducing the new integration variable $t=\sqrt{\rho^2+z_1^2}$ instead of ρ in (11) and using the integral representation for the modified Bessel function [65]

$$K_1(x)=x\int_1^\infty du\,\sqrt{u^2-1}\,e^{-ux}, \qquad (16)$$

we arrive at

$$V_a^{\rm AP}(z)=-\frac{2m_a^2}{M^2M_{\rm H}}\left(\frac{g^2}{4\pi}\right)^2C_{\rm A}C_{\rm P}\times\int_z^D dz_1\int_{z_1}^\infty dt\int_1^\infty du\,\sqrt{u^2-1}e^{-2m_atu}. \qquad (17)$$

Integrating here with respect to t and then with respect to z_1, the final result takes the form

$$V_a^{\rm AP}(z)=-\frac{1}{2M^2M_{\rm H}}\left(\frac{g^2}{4\pi}\right)^2C_{\rm A}C_{\rm P}\int_1^\infty du\frac{\sqrt{u^2-1}}{u}\times\left(e^{-2m_azu}-e^{-2m_aDu}\right). \qquad (18)$$

Now, taking into account that the theoretical value for the Casimir-Polder constant C_3 was confirmed by the measurement data within the limits of total error $\varDelta C_3$, the constraints on the parameters of axionlike particles g and m_a can be obtained from the inequality

$$\left|V_a^{\rm AP}(z)\right|<\frac{\varDelta C_3\,f(z)}{z^3}<\frac{\varDelta C_3}{z^3}. \qquad (19)$$

Similar to the case of Yukawa-type interaction, for each value of m_a, the value of z in (18) and (19) leading to the strongest constraint on g was chosen. The obtained constraints on the quantity $g^2/(4\pi)$ are shown in Fig. 2 by the line labeled C-P. Note that according to recent results [66], under certain assumptions, the constraints of this line are approximately valid not for only axionlike particles, but also for massive QCD axions whose interaction with nucleons is described by the pseudovector coupling. In the same figure, the constraints derived [36] from the differential force measurements, where the contribution of the Casimir force

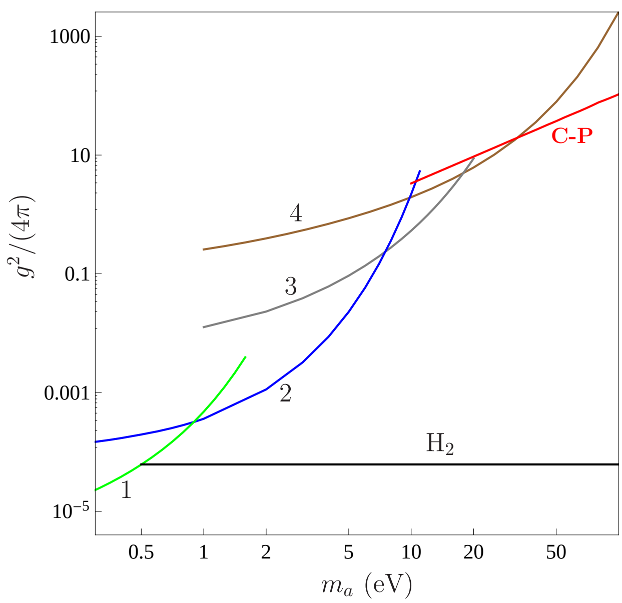

Fig. 2 Constraints on the coupling constant of axionlike particles to nucleons obtained from measurements of the Casimir-Polder interaction in the diffraction experiment are shown by the line labeled C-P versus the axion mass. The previously obtained constraints from the differential force measurements, measuring the effective Casimir pressure, the lateral Casimir force between corrugated surfaces, the Casimir force between a SiC plate and a borosilicate microsphere, and the spin-dependent forces between protons in the beam of molecular hydrogen are shown by the lines 1, 2, 3, 4, and H_2, respectively. The measurement data of each mentioned experiment exclude the region of the $(m_a,\ g^2/(4\pi))$ plane above the corresponding line and allow the region below it.

was nullified [15], are shown by the line 1. By the line 2 in Fig. 2 the constraints are shown found [64] from measurements of the effective Casimir pressure using the micromechanical oscillator [14]. The constraints of the line 3 follow [67] from measurements of the lateral Casimir force between the sinusoidally corrugated surfaces [56]. Finally, the line 4 shows the constraints derived [53] from the experiment on measuring the Casimir force between a SiC plate and a borosilicate microsphere. All these constraints obtained from measuring the Casimir force between the unpolarized bodies were obtained using the process of the exchange of two axionlike particles between nucleons. As to the line in Fig. 2 labeled H_2, it was derived [38] from measurements of the spin-dependent forces between protons in the beam of molecular hydrogen [37]. These constraints exploit the exchange of one axion or axionlike particle and, as a result, are much stronger. Note also that the possibility of constraining the coupling of axions to both electrons and nuclei in the 10^{-4} eV mass range using atomic transitions was proposed in [68].

Although the constraints shown by the line C-P in Fig. 2 cannot compete with those shown by the line labeled H_2, within the range of masses of axionlike particles from 32.2 eV to 100 eV they are stronger than the constraints of line 4 obtained previously from measurements of the Casimir force. The major strengthening by the factor of 24.2 is reached for the axionlike particles of 100 eV mass.

5 Conclusions and discussion

In the foregoing, we have considered the constraints on the Yukawa-type interaction predicted in many extensions of the Standard Model and on the coupling constant of axionlike particles to nucleons. Both these subjects are of considerable interest because, by constraining the Yukawa-type interaction, we obtain a valuable information about the proposed multi-dimensional models, whereas the stronger constraints on axionlike particles may help to clarify their status as the most probable constituents of dark matter.

After a brief listing of the strongest constraints obtained so far on the strength of Yukawa-type potential and the coupling constant of axionlike particles to nucleons from the laboratory experiments, we discussed the recent diffraction experiment [43] on measuring the Casimir-Polder interaction between Ar atoms and a silicon nitride nanograting. It was emphasized that this experiment has many points in its favor by using the slow atomic beam of noble argon gas and the metrological study of all possible sources of both systematic and random errors.

According to our results, the measure of agreement between the prediction of the Lifshitz theory for the Casimir-Polder coefficient C_3 and its experimental value makes it possible to strengthen the constraints on the interaction constant of the Yukawa-type interaction within the interaction range from 1 to 2 nm, as compared to all previous measurements of the Casimir force. The major strengthening by the factor of 33.4 holds at $\lambda = 1$ nm.

We have also used the same measure of agreement between experiment and theory for constraining the coupling constant of axionlike particles to nucleons. It was shown that in the range of masses of axionlike particles from 32.2 to 100 eV the obtained constraints are stronger by up to a factor of 24.2 than those found previously from measurements of the Casimir force. In this case, however, the constraints following from the diffraction experiment on measuring the Casimir-Polder interaction cannot compete with the constraints found using the exchange of one axion between protons in the beam of molecular hydrogen [38].

It should be underlined that by decreasing the contribution of systematic errors to the total error in the diffraction experiment (currently they contribute 16.2% to the total relative error of 17.2% leaving only 1% for the random errors [43]) it will be possible to strengthen the constraints on both the Yukawa-type interaction and the coupling constant of axionlike particles to nucleons following from this experiment. That is why measurements of the Casimir force preserve their potential for obtaining stronger constraints on the parameters of hypothetical interactions.

Acknowledgements This work was supported by the State Assignment for Basic Research (project FSEG-2023-0016).

References

1. E. Fischbach, C. L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer, New York, 1999)
2. K.-i. Maeda, D. Wands, Dilaton gravity on the brane. Phys. Rev. D **62**, 124009 (2000)
3. Y. Fujii, Mass of the Dilaton and the Cosmological Constant. Progr. Theor. Phys. **110**, 433 (2003)
4. P. Brax, C. Burrage, J. A. R. Cembranos, P. Valageas, Invisible dilaton. Phys. Rev. D **107**, 095015 (2023)
5. S. Dimopoulos, G. F. Giudice, Macroscopic forces from supersymmetry. Phys. Lett. B **379**, 105 (1996)
6. M. Dine, R. Kitano, A. Morisse, Yu. Shirman, Moduli decays and gravitinos. Phys. Rev. D **73**, 123518 (2006)
7. J. Khoury, A. Weltman, Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space. Phys. Rev. Lett. **93**, 171104 (2004)
8. J. Khoury, A. Weltman, Chameleon cosmology. Phys. Rev. D **69**, 044026 (2004)
9. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, New dimensions at a millimeter to a fermi and superstrings at a TeV. Phys. Lett. B **436**, 257 (1998)
10. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phenomenology, astrophysics, and cosmology of theories with millimeter dimensions and TeV scale quantum gravity. Phys. Rev. D **59**, 086004 (1999)
11. E. G. Floratos, G. K. Leontaris, Low scale unification, Newton's law and extra dimensions. Phys. Lett. B **465**, 95 (1999)
12. A. Kehagias, K. Sfetsos, Deviations from $1/r^2$ Newton law due to extra dimensions. Phys. Lett. B **472**, 39 (2000)

13. B. Heacock, T. Fujiie, R. W. Haun, A. Henins, K. Hirota, T. Hosobata, M. G. Huber, M. Kitaguchi, D. A. Pushin, H. Shimizu, et al., Pendellösung interferometry probes the neutron charge radius, lattice dynamics, and fifth forces. Science **373**, 1239 (2021)
14. R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, V. M. Mostepanenko, Novel constraints on light elementary particles and extra-dimensional physics from the Casimir effect. Eur. Phys. J. C **51**, 963 (2007)
15. Y.-J. Chen, W. K. Tham, D. E. Krause, D. López, E. Fischbach, R. S. Decca, Stronger Limits on Hypothetical Yukawa Interactions in the 30–8000 nm Range. Phys. Rev. Lett. **116**, 221102 (2016)
16. J. K. Hoskins, R. D. Newman, R. Spero, J. Schultz, Experimental tests of the gravitational inverse-square law for mass separations from 2 to 105 cm. Phys. Rev. D **32**, 3084 (1985)
17. S. J. Smullin, A. A. Geraci, D. M. Weld, J. Chiaverini, S. Holmes, A. Kapitulnik, Constraints on Yukawa-type deviations from Newtonian gravity at 20 microns. Phys. Rev. D **72**, 122001 (2005)
18. D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, H. E. Swanson, Tests of the Gravitational Inverse Square Law below the Dark-Energy Length Scale. Phys. Rev. Lett. **98**, 021101 (2007)
19. E. G. Adelberger, B. R. Heckel, S. Hoedl, C. D. Hoyle, D. J. Kapner, A. Upadhye, Particle-Physics Implications of a Recent Test of the Gravitational Inverse-Square Law. Phys. Rev. Lett. **98**, 131104 (2007)
20. W.-H. Tan, A.-B. Du, W.-C. Dong, S.-Q. Yang, C.-G. Shao, S.-G. Guan, Q.-L. Wang, B.-F. Zhan, P.-S. Luo, L.-C. Tu, J. Luo, Improvement for Testing the Gravitational Inverse-Square Law at the Submillimeter Range. Phys. Rev. Lett. **124**, 051301 (2020)
21. G. L. Smith, C. D. Hoyle, J. H. Gundlach, E. G. Adelberger, B. R. Heckel, H. E. Swanson, Short-range tests of the equivalence principle. Phys. Rev. D **61**, 022001 (2000)
22. S. Schlamminger, K.-J. Choi, T. A. Wagner, J. H. Gundlach, E. G. Adelberger, Test of the Equivalence Principle Using a Rotating Torsion Balance. Phys. Rev. Lett. **100**, 041101 (2008)
23. S. Weinberg, A New Light Boson? Phys. Rev. Lett. **40**, 223 (1978)
24. F. Wilczek, Problem of Strong P and T Invariance in the Presence of Instantons. Phys. Rev. Lett. **40**, 279 (1978)
25. R. D. Peccei, H. R. Quinn, CP Conservation in the Presence of Pseudoparticles. Phys. Rev. Lett. **38**, 1440 (1977)
26. J. E. Kim, Light pseudoscalars, particle physics and cosmology. Phys. Rep. **150**, 1 (1987)
27. G. G. Raffelt, Axions—motivation, limits and searches. J. Phys. A: Math. Theor. **40**, 6607 (2007)
28. J. E. Moody, F. Wilczek, New macroscopic forces? Phys. Rev. D **30**, 130 (1984)
29. A. Bohr, B. R. Mottelson, *Nuclear Structure*, Vol. 1 (Benjamin, New York, 1969)
30. E. G. Adelberger, E. Fischbach, D. E. Krause, R. D. Newman, Constraining the couplings of massive pseudoscalars using gravity and optical experiments. Phys. Rev. D **68**, 062002 (2003)
31. F. Ferrer, M. Nowakowski, Higgs- and Goldstone-boson-mediated long range forces. Phys. Rev. D **59**, 075009 (1999)
32. G. Vasilakis, J. M. Brown, T. R. Kornak, M. V. Romalis, Limits on New Long Range Nuclear Spin-Dependent Forces Set with a K-^{3}He Comagnetometer. Phys. Rev. Lett. **103**, 261801 (2009)
33. J. C. Long, H. W. Chan, A. B. Churnside, E. A. Gulbis, M. C. M. Varney, J. C. Price, Upper limits to submillimetre-range forces from extra space-time dimensions. Nature **421**, 922 (2003)
34. J. C. Long, V. A. Kostelecký, Search for Lorentz violation in short-range gravity. Phys. Rev. D **91**, 092003 (2015)
35. S. Aldaihan, D. E. Krause, J. C. Long, W. M. Snow, Calculations of the dominant long-range, spin-independent contributions to the interaction energy between two nonrelativistic Dirac fermions from double-boson exchange of spin-0 and spin-1 bosons with spin-dependent couplings. Phys. Rev. D **95**, 096005 (2017).
36. G. L. Klimchitskaya, V. M. Mostepanenko, Improved constraints on the coupling constants of axion-like particles to nucleons from recent Casimir-less experiment, Eur. Phys. J. C **75**, 164 (2015)
37. N. F. Ramsey, The tensor force between two protons at long range. Physica A **96**, 285 (1979)
38. M. P. Ledbetter, M. V. Romalis, D. F. Jackson Kimball, Constraints on Short-Range Spin-Dependent Interactions from Scalar Spin-Spin Coupling in Deuterated Molecular Hydrogen. Phys. Rev. Lett. **110**, 040402 (2013)
39. M. Kawasaki, K. Nakayama, Axions: Theory and Cosmological Role. Annu. Rev. Nucl. Part. Sci. **63**, 69 (2013)
40. P. Sikivie, Invisible axion search methods. Rev. Mod. Phys. **93**, 015004 (2021)
41. L. J. Rosenberg, K. A. van Bibber, Searches for invisible axions. Phys. Rep. **325**, 1 (2000)
42. I. G. Ivastorza, J. Redondo, New experimental approaches in the search for axion-like particles. Progr. Part. Nucl. Phys. **102**, 89 (2018)
43. J. Lecoffre, A. Hadi, M. Bruneau, C. Garcion, N. Fabre, É. Charron, N. Gaaloul, G. Dutier, Q. Bouton, Measurement of Casimir-Polder interaction for slow atoms through a material grating. Phys. Rev. Research **7**, 013232 (2025)
44. C. Garcion, N. Fabre, H. Bricha, F. Perales, S. Scheel, M. Ducloy, G. Dutier, Intermediate-Range Casimir-Polder Interaction Probed by High-Order Slow Atom Diffraction. Phys. Rev. Lett. **127**, 170402 (2021)
45. J. Morley, R. Flack, B. J. Hiley, P. F. Barker, Characterising a tunable, pulsed atomic beam using matter-wave interferometry. J. Phys. B: At. Mol. Opt. Phys. **54**, 155301 (2021)
46. C. Garcion, Q. Bouton, J. Lecoffre, N. Fabre, E. Charron, G. Dutier, N. Gaaloul, Quantum description of atomic diffraction by material nanostructures. Phys. Rev. Res. **6**, 023165 (2024)
47. M. Bordag, G. L. Klimchitskaya, U. Mohideen, V. M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, Oxford, 2015)
48. E. M. Lifshitz, L. P. Pitaevskii, *Statistical Physics*, Pt. II (Pergamon, Oxford, 1980)
49. A. O. Caride, G. L. Klimchitskaya, V. M. Mostepanenko, S. I. Zanette, Dependences of the van der Waals atom-wall interaction on atomic and material properties. Phys. Rev. A **71**, 042901 (2005)
50. C. Karam, N. Wipf, J. Grucker, F. Perales, M. Boustimi, G. Vassilev, V. Bocvarski, C. Mainos, J. Baudon, J. Robert, Atom diffraction with a natural metastable atom nozzle beam. J. Phys. B: At. Mol. Opt. Phys. **38**, 2691 (2005)
51. J. Morley, R. Flack, B. J. Hiley, P. F. Barker, Characterising a tunable, pulsed atomic beam using matter-wave interferometry. J. Phys. B: At. Mol. Opt. Phys. **54**, 155301 (2021)
52. *CRC Handbook of Chemistry and Physics*, ed. W. M. Haynes (CRC Press, Boca Raton, 2011)
53. G. L. Klimchitskaya, P. Kuusk, V. M. Mostepanenko, Constraints on non-Newtonian gravity and axionlike particles from measuring the Casimir force in nanometer separation range. Phys. Rev. D **81**, 055003 (2020)
54. M. Sedighi, V. B. Svetovoy, G. Palasantzas, Casimir force measurements from silicon carbide surfaces. Phys. Rev. B **101**, 056013 (2020)
55. V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, C. Romero, Advances and prospects in constraining the Yukawa-type corrections to Newtonian gravity from the Casimir effect. Phys. Rev. D **81**, 055003 (2010)
56. H.-C. Chiu, G. L. Klimchitskaya, V. N. Marachevsky, V. M. Mostepanenko, U. Mohideen, Lateral Casimir force between sinusoidally corrugated surfaces: Asymmetric profiles, deviations from the proximity force approximation, and comparison with exact theory. Phys. Rev. B **81**, 115417 (2010)

57. G. L. Klimchitskaya, U. Mohideen, V. M. Mostepanenko, Constraints on corrections to Newtonian gravity from two recent measurements of the Casimir interaction between metallic surfaces. Phys. Rev. D **87**, 125031 (2013)
58. A. A. Banishev, J. Wagner, T. Emig, R. Zandi, U. Mohideen, Demonstration of Angle-Dependent Casimir Force between Corrugations. Phys. Rev. Lett. **110**, 250403 (2013)
59. A. A. Banishev, J. Wagner, T. Emig, R. Zandi, U. Mohideen, Experimental and theoretical investigation of the angular dependence of the Casimir force between sinusoidally corrugated surfaces. Phys. Rev. B **89**, 235436 (2014)
60. C. C. Haddock, N. Oi, K. Hirota, T. Ino, M. Kitaguchi, S. Matsumoto, K. Mishima, T. Shima, H. M. Shimizu, W. M. Snow, T. Yoshioka, Search for deviations from the inverse square law of gravity at nm range using a pulsed neutron beam. Phys. Rev. D **97**, 062002 (2018)
61. Y. Kamiya, K. Itagami, M. Tani, G. N. Kim, S. Komamiya, Constraints on New Gravitylike Forces in the Nanometer Range. Phys. Rev. Lett. **114**, 161101 (2015)
62. J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, E. A. Cornell, Measurement of the Temperature Dependence of the Casimir-Polder Force. Phys. Rev. Lett. **98**, 063201 (2007)
63. V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, C. Romero, Constraints on the parameters of an axion from measurements of the thermal Casimir-Polder force. Phys. Rev. D **89**, 035010 (2014)
64. V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, C. Romero, Constraining axion-nucleon coupling constants from measurements of effective Casimir pressure by means of micromachined oscillator. Eur. Phys. J. C **74**, 2859 (2014)
65. I. S. Gradshtein, I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1980)
66. M. Bauer, G. Rostagni, Forces from QCD Axions Scale Differently. Phys. Rev. Lett. **132**, 101802 (2024)
67. V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, C. Romero, Constraints on axion-nucleon coupling constants from measuring the Casimir force between corrugated surfaces. Phys. Rev. D **90**, 055013 (2014)
68. P. Sikivie, Axion Dark Matter Detection Using Atomic Transitions. Phys. Rev. Lett. **113**, 201301 (2014)