

SL(2N,C) Hyperunification: Dynamical Tetrads, Induced Gravity, and Composite Families

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Abstract

A four-dimensional gauge-gravity unification based on the local $SL(2N, \mathbb{C})$ symmetry is developed in a universal Yang-Mills type setting. The accompanying tetrads are promoted to dynamical fields whose invertibility condition is interpreted as a nonlinear sigma-model type length constraint. This triggers tetrad condensation and spontaneously breaks $SL(2N, \mathbb{C})$ down to $SL(2, \mathbb{C}) \times SU(N)$, effectively filtering out unobserved non-compact directions. A special ghost-free curvature-squared Lagrangian provides a consistent quadratic sector for all gauge fields involved, while an Einstein-Cartan linear curvature term is shown to arise radiatively from fermion loops. The matter sector clearly points to a deeper elementarity of $SL(2N, \mathbb{C})$ spinors, which can be identified with preon constituents whose bound states form the observed quarks and leptons. Anomaly matching between preons and composites singles out $SL(16, \mathbb{C})$, accommodating precisely three composite quark-lepton families. The theory links noncompact unification, induced gravity, and fermion family structure in a single framework.

INTRODUCTION

Gauge formulations of gravity, starting from Utiyama and Kibble, place local Lorentz symmetry on the same footing as internal gauge symmetries by promoting the spin connection to a gauge field [1, 2]. This invites the idea that spacetime and internal interactions may be unified in a larger local group whose representations act on spinors, rather than on vectors in an extended spacetime. A natural candidate is a local $\text{SL}(2N, \mathbb{C})$ symmetry, where the spinorial $\text{SL}(2, \mathbb{C})$ of gravity is enlarged by an internal $\text{SU}(N)$ sector in a common noncompact setting, as in early work by Isham, Salam and Strathdee [3, 4] and more recent hyperunified proposals [5, 6].

Such programs face several persistent obstacles. The full $\text{SL}(2N, \mathbb{C})$ gauge multiplet contains vector, axial-vector and tensor submultiplets. Experiment only requires a compact internal vector sector; axial partners and extra hyperflavored tensors must be decoupled. Quadratic curvature terms needed for a Yang–Mills-like description typically introduce ghosts unless a special combination of curvature invariants is chosen [7, 8]. Matter multiplets constructed directly in $\text{SL}(2N, \mathbb{C})$ tend to be huge, with many multi-spin states that do not resemble observed quarks and leptons, unless some deeper compositeness is invoked. Finally, the Coleman–Mandula theorem severely constrains nontrivial Lie-algebraic unification of spacetime and internal symmetries [9].

In this Letter we show that these obstacles can be addressed in a unified way if the tetrad is treated as a genuine dynamical field whose invertibility plays the role of a nonlinear sigma-model constraint. The key points are: (i) a dynamical $\text{SL}(2N, \mathbb{C})$ tetrad with fixed length condenses and spontaneously breaks the symmetry down to $\text{SL}(2, \mathbb{C}) \times \text{SU}(N)$, lifting all noncompact directions; (ii) in the quadratic sector, a Neville-type ghost-free curvature-squared Lagrangian can be embedded into $\text{SL}(2N, \mathbb{C})$, giving a healthy spectrum for the tensor submultiplet; (iii) the usual Einstein–Cartan linear curvature term need not be fundamental: it can be induced at one loop from fermion bubbles that couple simultaneously to the tetrad and the tensor connection, in the spirit of Sakharov’s induced gravity [10–13]; (iv) when $\text{SL}(2N, \mathbb{C})$ acts on preons in fundamental spinor representations, ’t Hooft anomaly matching [14] for a chiral preon symmetry selects $N = 8$, and $\text{SL}(16, \mathbb{C})$ provides a natural hyperunified origin for three composite quark–lepton families [6, 15].

The resulting low-energy theory is a four-dimensional $\text{SL}(2, \mathbb{C}) \times \text{SU}(N)$ gauge theory with an emergent Einstein–Cartan sector and a filtered gauge spectrum, without recourse to extra dimensions, strings, or non-Lie group structures.

SL(2N,C) GAUGE FIELDS AND TETRADS

We start from a local $\text{SL}(2N, \mathbb{C})$ acting on Dirac spinors $\Psi(x)$ in the fundamental $2N$ -dimensional representation,

$$\Psi \rightarrow \Omega(x) \Psi, \quad \Omega = \exp \left\{ \frac{i}{2} [(\theta^k + \theta_5^k \gamma_5) \lambda^k + \frac{1}{2} \theta_{ab}^K \gamma^{ab} \lambda^K] \right\}, \quad (1)$$

where $\gamma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$ generate $\text{SL}(2, \mathbb{C})$ and λ^K form a basis of $\text{U}(1) \times \text{SU}(N)$ (with $K = 0, k$). The corresponding gauge connection I_μ decomposes into vector, axial-vector and tensor pieces,

$$I_\mu = \frac{1}{2} (V_\mu^k + A_\mu^k \gamma_5) \lambda^k + \frac{1}{4} T_{\mu[ab]}^K \gamma^{ab} \lambda^K, \quad (2)$$

and its field strength $I_{\mu\nu} = \partial_{[\mu} I_{\nu]} + \text{ig}[I_\mu, I_\nu]$ contains the usual Yang–Mills terms plus nontrivial tensor contributions.

The fermion kinetic term requires matrix-valued tetrads $e^\mu(x)$ that replace constant γ^μ and transform as $e^\mu \rightarrow \Omega e^\mu \Omega^{-1}$. In the hyperunified theory they may be written as

$$e_\mu = e_\mu^{aK} \gamma_a \lambda^K, \quad (3)$$

with inverse $e_a^{\mu K}$ satisfying generalized invertibility relations. However, consistency with a single four-dimensional metric strongly constrains the allowed structure. If one demands that the metric be $\text{SL}(2, \mathbb{C})$ -invariant and unique, the only invertible solution is

$$e_\mu^{aK} = e_\mu^a \delta^{K0}, \quad (4)$$

so that tetrads are neutral under $\text{SU}(N)$ and belong effectively to the $\text{SL}(2, \mathbb{C})$ subgroup. This is the only choice compatible with a standard tangent bundle and a single spacetime metric.

In the quadratic sector we adopt a ghost-free curvature-squared Lagrangian for the Lorentz tensor field, of the Neville type [7, 8]. In the pure $\text{SL}(2, \mathbb{C})$ case, writing T_μ^{ab} for the tensor connection and $T_{\mu\nu}^{ab}$ for its curvature,

$$e\mathcal{L}_G^{(2)} = \lambda \left(T_{ab}^{\mu\nu} T_{\mu\nu}^{ab} - 4 T_{ab}^{\mu\nu} T_{\rho\nu}^{ac} e_{\mu c} e^{\rho b} + T_{ab}^{\mu\nu} T_{\rho\sigma}^{cd} e_{\mu c} e_{\nu d} e^{\rho a} e^{\sigma b} \right). \quad (5)$$

This combination propagates a massless graviton plus a heavy massive mode without ghosts [7, 8]. We show that it can be embedded into $\text{SL}(2N, \mathbb{C})$ by taking suitable traces of $I_{\mu\nu}$ multiplied by tetrads. After fixing coefficients to reproduce the Neville form in the $\text{SL}(2, \mathbb{C})$ limit, a single gauge coupling g controls all quadratic terms, and vector and axial-vector submultiplets inherit canonical Yang–Mills kinetic terms.

At this stage V_μ^k , A_μ^k and T_μ^{abK} all appear on equal footing. The crucial step is to make the tetrad dynamical and impose a nonlinear length constraint that forces a symmetry-breaking vacuum.

LIFTING NONCOMPACT DIRECTIONS

We promote the tetrad to a dynamical field and rescale it to a dimensionless variable E_μ^{aK} ,

$$e_\mu^{aK} = M E_\mu^{aK}, \quad (6)$$

where M is a symmetry-breaking scale. The contracted invertibility condition is then taken as a nonlinear sigma-model constraint

$$\frac{1}{4} \text{Tr}(e_\mu e^\mu) = M^2, \quad (7)$$

leading to the tetrad condensation. This fixes the norm of the tetrad but not its orientation in the internal space.

A convenient parametrization of a vacuum configuration is

$$\bar{e}_\mu = v_\mu \otimes H, \quad v_\mu = M \delta_\mu^a \gamma_a, \quad H^\dagger = H. \quad (8)$$

All dependence on the internal (hyperflavor) orientation resides in the Hermitian $N \times N$ matrix H . The “hyperflavor-blind” direction corresponds to $H \propto \mathbf{1}_N$, which clearly satisfies (7). The key question is whether quantum effects select this orientation dynamically.

The tetrad acquires a canonical kinetic term

$$\mathcal{L}_e = -\frac{1}{4} (D_{[\mu} e_{\nu]}^{aK}) (D^{[\mu} e^{\nu]}_{aK}), \quad (9)$$

with the full $\text{SL}(2N, \mathbb{C})$ spin connection in the covariant derivative. Once $\mathcal{L}_e$ is present, the theory is automatically promoted from $\text{SL}(2N, \mathbb{C})$ to the corresponding Poincare-type group $\text{ISL}(2N, \mathbb{C})$: the hyperflavored tetrad is the gauge field of local hypertranslations.

Quantum fluctuations of gauge fields around a background $\bar{e}_\mu$ generate a Coleman–Weinberg potential $U(\bar{e})$ for the tetrad orientation. The dominant orientation-sensitive piece arises from the vector submultiplet. In Landau gauge [16] one finds

$$U_V(\bar{e}) = \frac{3}{64\pi^2} \sum_n m_n^4(\bar{e}) \left(\ln \frac{m_n^2(\bar{e})}{\mu^2} - \frac{5}{6} \right), \quad (10)$$

where m_n^2 are eigenvalues of the vector mass matrix $M_V^2(\bar{e})$. For generators $T_V^k = \lambda^k$ of $\text{SU}(N)$,

$$\begin{aligned} (M_V^2)_{kl}(\bar{e}) &= g^2 \text{Tr}([\lambda_k, \bar{e}_\mu][\lambda_l, \bar{e}^\mu]) \\ &= 4g^2 M^2 \text{Tr}([\lambda_k, H][\lambda_l, H]). \end{aligned} \quad (11)$$

It follows that M_V^2 vanishes precisely when $H \propto \mathbf{1}_N$; any deviation from a hyperflavor-blind matrix gives some nonzero eigenvalues and raises U_V . Thus the Coleman–Weinberg potential has a strict minimum at

$$\bar{e}_\mu = M \delta_\mu^a \gamma_a \lambda^0, \quad \lambda^0 = \sqrt{\frac{2}{N}} \mathbf{1}_N. \quad (12)$$

In this vacuum the tetrad is neutral under $\text{SU}(N)$ and aligned entirely along the $\text{SL}(2, \mathbb{C})$ factor. The spontaneous breaking pattern is

$$\text{SL}(2N, \mathbb{C}) \longrightarrow \text{SL}(2, \mathbb{C}) \times \text{SU}(N), \quad (13)$$

with all noncompact directions lifted.

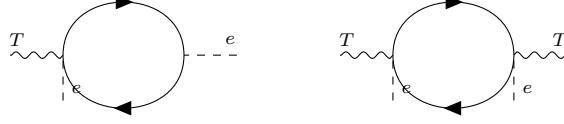


FIG. 1. Fermion-bubble diagrams generating the Einstein–Cartan term. Left: a mixed V_T – V_e insertion produces $e \wedge e \wedge dT$. Right: two V_T insertions produce $e \wedge e \wedge (gT \wedge T)$. Their sum reconstructs $e \wedge e \wedge R[T]$.

The spectrum reorganizes accordingly. The six Lorentz generators survive nonlinearly and still close on the $sl(2, \mathbb{C})$ algebra, so local Lorentz invariance is intact. The $SU(N)$ vector fields V_μ^k remain massless and mediate ordinary internal forces. By contrast, the axial-vector fields A_μ^k and all tensor fields T_μ^{abK} ($K = 0, k$) acquire masses of order gM from the tetrad condensate and decouple from low-energy physics.

Fluctuations of the tetrad can be decomposed into antisymmetric $\tilde{e}^{[ab]K}$ and symmetric $\tilde{e}^{\{ab\}K}$ components in the local frame. Antisymmetric (torsion-linked) modes are eaten by the tensor spin connections T_μ^{abK} ; among the symmetric modes, the $SU(N)$ -adjoint traces are eaten by axial vectors A_μ^k , while the remaining adjoint-traceless modes acquire radiative masses $\sim g^2 M / (4\pi)$ from axial and tensor loops. The only massless tetrad fluctuation is the $SU(N)$ -singlet symmetric traceless part, which provides the usual graviton. The effective low-energy spectrum thus consists of a neutral tetrad and the compact $SU(N)$ vector multiplet.

The tetrad kinetic term $\mathcal{L}_e$ and the curvature-squared sector imply that the spin connection and tetrad are *a priori* independent variables, so metricity $D_\mu e_\nu^a = 0$ is not imposed by hand. However, after tetrad condensation the heavy spin-connection modes acquire masses of order gM , and the equation of motion for T_μ^{ab} is algebraic to leading order in derivatives. Integrating out these heavy modes enforces metricity up to corrections of order p^2/M^2 , where p is a typical external momentum. In the low-energy regime $p \ll M$ the effective connection is thus Levi–Civita (plus the usual Einstein–Cartan torsion), while deviations are encoded in higher-derivative operators such as curvature-squared and torsion-squared terms suppressed by M^{-2} . Classical tests of GR are therefore unaffected, provided M lies well above the scales probed by present gravity experiments.

INDUCED EINSTEIN–CARTAN GRAVITY

After tetrad condensation, the quadratic curvature sector is governed by the ghost-free Lagrangian $\mathcal{L}_G^{(2)}$ embedded into $SL(2N, \mathbb{C})$. A separate linear curvature term of the Einstein–Cartan form,

$$\mathcal{L}_{\text{EC}} = \frac{1}{2\kappa} e e_{[a}{}^\mu e_{b]}{}^\nu R_{\mu\nu}^{ab}[T], \quad (14)$$

is allowed by symmetry but need not appear at tree level. Here T_μ^{ab} is the neutral tensor connection, $R_{\mu\nu}^{ab}[T]$ its curvature, and $e = \det(e_\mu^a)$. We show that (14) is generated radiatively by fermion loops in which both the tetrad and the tensor connection couple to matter.

The relevant interaction vertices follow from the fermion kinetic term and the $SL(2N, \mathbb{C})$ covariant derivative. In a schematic notation,

$$\begin{aligned} V_e : \quad & \frac{i}{M} e^\mu{}_a \bar{\Psi} \gamma^a \overleftrightarrow{\partial}_\mu \Psi, \\ V_T : \quad & \frac{ig}{4M} e^\mu{}_c \bar{\Psi} \gamma^c \gamma_{ab} \Psi T_\mu^{ab}, \end{aligned} \quad (15)$$

where each V_T vertex already contains one external tetrad leg. The minimal one-loop origin of (14) can be traced to two fermion bubbles (Fig. 1): the mixed diagram with one V_T and one V_e insertion produces an operator of the form $e \wedge e \wedge dT$, while the diagram with two V_T insertions produces $e \wedge e \wedge (gT \wedge T)$. Together they reconstruct $e \wedge e \wedge R[T]$ with $R[T] = dT + gT \wedge T$.

The coefficient of (14) depends on the fermion content and on the UV completion. Two complementary scenarios are illustrative. In a “threshold” scenario, vectorlike fermions with masses $m_\psi \sim M_{\text{Pl}}$ generate a regulator-independent contribution

$$\Delta \mathcal{L}_{\text{EC}} \sim \frac{N_f}{16\pi^2} m_\psi^2 e \wedge e \wedge R[T], \quad (16)$$

so that $M_{\text{Pl}}^2 \sim (N_f/16\pi^2)m_\psi^2$. In a “universal cutoff” scenario, quantum gravity itself provides a physical cutoff $\Lambda \sim M_{\text{Pl}}$, and even light (or massless) fermions induce

$$\Delta\mathcal{L}_{\text{EC}} \sim \frac{N_f}{16\pi^2} \Lambda^2 e \wedge e \wedge R[T], \quad (17)$$

with the Planck scale determined by the UV boundary condition. In both cases the emergent EC term is expressed in terms of the parameters of the underlying $\text{SL}(2N, \mathbb{C})$ gauge theory and its fermion sector. The quadratic curvature terms are already present via the ghost-free Neville-type Lagrangian, so the full low-energy gravitational sector is an Einstein–Cartan-plus-curvature-squared theory with the correct sign and no ghosts.

PREONS, $\text{SL}(16, \mathbb{C})$ AND THREE QUARK-LEPTON FAMILIES

The $\text{SL}(2N, \mathbb{C})$ gauge symmetry can also act on more fundamental constituents of quarks and leptons. A particularly economical scenario is a preon model in which quarks and leptons are composite states built from chiral preons transforming in the fundamental of $\text{SL}(2N, \mathbb{C})$ [6, 15]. At very short distances we assume $2N$ massless preons: N left-handed and N right-handed Weyl spinors,

$$P_{iaL}, \quad Q_{iaR} \quad (i = 1, \dots, N; a = 1, 2), \quad (18)$$

which share a common local metaflavor symmetry

$$G_{\text{MF}} = \text{SL}(2N, \mathbb{C}) \quad (19)$$

and transform in its fundamental representation. In addition, the preons carry metacolor charges under a chiral gauge group

$$G_{\text{MC}} = \text{SO}(n)^L \times \text{SO}(n)^R, \quad (20)$$

with left-handed and right-handed preons living in the vector representation of $\text{SO}(n)^L$ and $\text{SO}(n)^R$, respectively. Metacolor confinement at a scale Λ_{MC} binds preons into composite fermions, including quarks and leptons.

In the limit where metaflavor gauge interactions are formally switched off, the preon system enjoys an additional global chiral symmetry

$$K(N) = \text{SU}(N)_L \times \text{SU}(N)_R, \quad (21)$$

acting separately on left- and right-handed preons. Massless composite fermions can exist only if an appropriate subgroup of $K(N)$ survives in the infrared. The ’t Hooft anomaly matching (AM) condition then requires that the cubic anomalies $\text{SU}(N)_L^3$ and $\text{SU}(N)_R^3$ computed in terms of preons match those of the massless composite spectrum [14, 15].

The simplest and most restrictive possibility is that (i) only spin- $\frac{1}{2}$ composites remain massless; (ii) they are three-preon bound states of the metacolor interaction; and (iii) all such composites fit into a single irreducible representation of the residual metaflavor subgroup $\text{SL}(2, \mathbb{C}) \times \text{SU}(N)$. Under these assumptions, the candidate composite multiplets are third-rank tensors of $\text{SU}(N)$, schematically $\Psi^{\{ijk\}}$, $\Psi_{[ijk]}$, $\Psi_{\{[ij]k\}}$, $\Psi_{\{jk\}}^i$, $\Psi_{[jk]}^i$. Evaluating the anomaly coefficients $a(r)$ for these representations and imposing the AM condition for $\text{SU}(N)_L^3$ (and similarly for $\text{SU}(N)_R^3$) gives a nontrivial constraint

$$n a(N) = a(r_0), \quad (22)$$

where n is the number of metacolors, $a(N) = 1$ for preons in the fundamental, and r_0 is the representation of massless composites.

For three-preon states with $\text{SO}(3)^{L,R}$ metacolor, the unique integer solution is obtained when the massless composites transform as $\Psi_{[jk]}^i$ with

$$a(\Psi_{[jk]}^i) = 3 \implies N = 8, \quad (23)$$

selecting an eightfold chiral symmetry $\text{SU}(8)_L \times \text{SU}(8)_R$ in the infrared [15]. This points to $\text{SL}(16, \mathbb{C})$ as the preferred metaflavor group: at very short distances preons transform under local $\text{SL}(16, \mathbb{C})$, while at large distances the effective metaflavor symmetry acting on massless composites is $\text{SL}(2, \mathbb{C}) \times \text{SU}(8)$.

The surviving massless multiplet in the residual metaflavor symmetry can be organized into a $(\mathrm{SL}(2, \mathbb{C}) \times \mathrm{SU}(8))$ representation of dimension $216_{L,R}$, which decomposes under $\mathrm{SU}(5) \times \mathrm{SU}(3)_F \subset \mathrm{SU}(8)$ as

$$216_{L,R} = [(\bar{5} + 10, \bar{3}) + (45, 1) + (5, 8 + 1) + (24, 3) + (1, 3) + (1, \bar{6})]_{L,R}. \quad (24)$$

The first term $(\bar{5} + 10, \bar{3})_L$ contains three left-handed quark–lepton families as a triplet of a chiral family symmetry $\mathrm{SU}(3)_F$ [18], while the remaining components are vectorlike with respect to the Standard Model gauge group. A modest left–right asymmetry in the composite sector, generated by condensates of scalar composites, allows the right-handed multiplet to lose the $(5 + 10, \bar{3})_R$ piece while keeping the rest anomaly-free. All vectorlike states can then pair up and acquire large masses, leaving precisely three light chiral families of composite quarks and leptons at low energies [15].

Thus the same $\mathrm{SL}(2N, \mathbb{C})$ hyperunified symmetry that organizes the spin-connection and tetrad sectors also provides a natural home for preons. Tetrad condensation filters the gauge multiplet down to $\mathrm{SL}(2, \mathbb{C}) \times \mathrm{SU}(8)$, and anomaly matching in the preon sector selects $N = 8$ and three chiral families.

SUMMARY AND OUTLOOK

We have explored a four-dimensional hyperunified gauge framework based on a local $\mathrm{SL}(2N, \mathbb{C})$ symmetry, in which the spinor symmetry $\mathrm{SL}(2, \mathbb{C})$ underlying the spacetime spin structure is extended by an internal $\mathrm{SU}(N)$ sector, and the tetrad is treated as a dynamical field. The contracted tetrad invertibility condition is interpreted as a nonlinear sigma-model length constraint. When enforced on an $\mathrm{SL}(2N, \mathbb{C})$ tetrad with a kinetic term, it drives tetrad condensation and spontaneously breaks $\mathrm{SL}(2N, \mathbb{C}) \rightarrow \mathrm{SL}(2, \mathbb{C}) \times \mathrm{SU}(N)$, lifting all noncompact directions while preserving local Lorentz invariance. A Neville-type curvature-squared Lagrangian embedded into $\mathrm{SL}(2N, \mathbb{C})$ provides a ghost-free quadratic sector for the tensor submultiplet, and the usual Einstein–Cartan term is induced radiatively from fermion bubbles that couple both to the tetrad and to the tensor connection. At low energies only the neutral tetrad (graviton) and an $\mathrm{SU}(N)$ vector multiplet remain in the spectrum; axial-vector and tensor fields acquire masses of order gM .

Applying the same hyperunified symmetry to preons in fundamental spinor representations, together with 't Hooft anomaly matching for a chiral preon symmetry, singles out $N = 8$ and a residual $\mathrm{SL}(2, \mathbb{C}) \times \mathrm{SU}(8)$ metaflavor symmetry in the composite sector. The resulting 216-dimensional multiplet accommodates three composite quark–lepton families in a natural way, while extra states can be made heavy after a modest left–right asymmetry in the composite sector.

A single dynamical ingredient—the tetrad—thus simultaneously selects the vacuum, filters noncompact directions, and anchors an emergent Einstein–Cartan gravity scale within a four-dimensional hyperunified framework. Further work should derive the detailed mass spectrum of axial and tensor spin connections, compute the induced EC coefficient with explicit regulators in representative fermion sectors, and complete the preon model for $N = 8$, including family-symmetry breaking and fermion mass textures. If the tetrad-condensation scale M and the preon confinement scale Λ_{MC} lie well below M_{Pl} , remnants of the filtered hyperunified structure could leave characteristic signatures in the spectrum of heavy bosonic and fermionic composites.

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