Divergence in tracing the flavors of astrophysical neutrinos: can JUNO help IceCube?

Zhi-zhong Xing[1, 2, 3, *]

[1] *Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[2] *School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[3] *Center of High Energy Physics, Peking University, Beijing 100871, China*

We derive the flavor fractions $(\eta_e, \eta_\mu, \eta_\tau)$ for ultrahigh-energy neutrinos originating from a remote astrophysical source by using their flavor ratios (f_e, f_μ, f_τ) observed at a neutrino telescope, and find that a potential divergence may appear in inferring the values of η_μ and η_τ as an unavoidable consequence of the approximate μ-τ interchange symmetry of lepton flavor mixing. We illustrate such an effect by applying our formula to the IceCube all-sky neutrino flux data ranging from 5 TeV to 10 PeV in the naive assumption that the relevant sources have a common flavor composition. We show that only η_e and $\eta_\mu + \eta_\tau$ can be extracted from the measurements of f_e, f_μ and f_τ if the μ-τ flavor symmetry is exact, and the JUNO and Daya Bay precision antineutrino oscillation data will help a lot in this connection.

1. INTRODUCTION

It is known that the flavor oscillations of high-energy astrophysical neutrinos over a sufficiently long distance will lose their energy- and baseline-dependent information, but may allow a terrestrial neutrino telescope to trace their flavor composition at the source [1–4]. The key point is that there exists a linear relation between the original neutrino fluxes of different flavors $(\phi_e, \phi_\mu, \phi_\tau)$ and the observed ones $(\Phi_e, \Phi_\mu, \Phi_\tau)$ in the standard three-flavor oscillation scheme [1]:

$$\begin{bmatrix} \Phi_e \\ \Phi_\mu \\ \Phi_\tau \end{bmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix} \begin{bmatrix} \phi_e \\ \phi_\mu \\ \phi_\tau \end{bmatrix} , \tag{1}$$

where the oscillation probabilities are simple functions of the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix U [5–7] and satisfy

$$P_{\alpha\beta} = P_{\beta\alpha} = \sum_{i=1}^{3} \left(|U_{\alpha i}|^2 \, |U_{\beta i}|^2 \right) , \tag{2}$$

where the Greek subscripts α and β run over the flavor indices e, μ and τ for three active neutrinos ν_e, ν_μ and ν_τ. The unitarity of U assures $\Phi_0 \equiv \Phi_e + \Phi_\mu + \Phi_\tau = \phi_e + \phi_\mu + \phi_\tau$ to hold, implying that the total neutrino flux Φ_0 is conserved. In practice, the measurements of Φ_α (for $\alpha = e, \mu, \tau$) and Φ_0 are expected to involve quite large systematic uncertainties, but the latter can be largely cancelled out in the flux ratios

$$f_e \equiv \frac{\Phi_e}{\Phi_0} , \quad f_\mu \equiv \frac{\Phi_\mu}{\Phi_0} , \quad f_\tau \equiv \frac{\Phi_\tau}{\Phi_0} , \tag{3}$$

which characterize the three-flavor composition of high-energy astrophysical neutrinos observed at a neutrino telescope like the IceCube [8]. Namely, these three flavor ratios are the *working* observable quantities that can help determine the original neutrino flavor fractions

$$\eta_e \equiv \frac{\phi_e}{\Phi_0} , \quad \eta_\mu \equiv \frac{\phi_\mu}{\Phi_0} , \quad \eta_\tau \equiv \frac{\phi_\tau}{\Phi_0} , \tag{4}$$

for a remote astrophysical source. For example, $\eta_e = 1/3$, $\eta_\mu = 2/3$ and $\eta_\tau = 0$ have been conjectured for those astrophysical neutrinos produced from the decays of charged pions that originate from very energetic proton-proton and proton-photon collisions [1–4], but whether such a conjecture is true or not needs an experimental test in the precision era of neutrino astronomy.

This work is intended to derive the general expressions of η_α in terms of f_β, $|U_{\alpha i}|^2$ and $|U_{\beta i}|^2$ (for $i = 1, 2, 3$ and $\alpha, \beta = e, \mu, \tau$), so as to infer the possible flavor composition of an astrophysical source from a precision measurement of the corresponding flavor distribution at a terrestrial neutrino telescope. Such a calculation is important but has been lacking. We find that the result is rather nontrivial as a potential divergence may appear in inferring the values of η_μ and η_τ due to an approximate μ-τ interchange symmetry associated with the PMNS matrix U. To illustrate this effect, we apply our formula to the IceCube all-sky neutrino flux data ranging from 5 TeV to 10 PeV [8] by naively assuming that the relevant sources have the same flavor composition. We show that only η_e and $\eta_\mu + \eta_\tau$ can be extracted from the measurements of f_e, f_μ and f_τ if the μ-τ flavor symmetry is exact, and the latest JUNO and Daya Bay precision antineutrino oscillation data on the flavor mixing angles θ_{12} [9] and θ_{13} [10] will help a lot in this connection.

2. THE FORMULA IN THE GENERAL CASE

Let us begin with a normalization of the neutrino fluxes appearing in Eq. (1) with the total neutrino flux Φ_0, as done in Eqs. (3) and (4). Then we are left with a system

[1] Note that ϕ_e (or Φ_e) stands for a sum of both the electron neutrino flux and the electron antineutrino flux at the astrophysical source (or the telescope), so do ϕ_μ and ϕ_τ (or Φ_μ and Φ_τ).

of three linear equations in three variables η_e, η_μ and η_τ:

$$\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix} \begin{bmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{bmatrix} = \begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix} . \quad (5)$$

Following Gabriel Cramer's rule, one may solve the above equations by calculating the determinants

$$\begin{aligned} D_0 &= \begin{vmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{vmatrix} , \\ D_e &= \begin{vmatrix} f_e & P_{e\mu} & P_{e\tau} \\ f_\mu & P_{\mu\mu} & P_{\mu\tau} \\ f_\tau & P_{\tau\mu} & P_{\tau\tau} \end{vmatrix} , \\ D_\mu &= \begin{vmatrix} P_{ee} & f_e & P_{e\tau} \\ P_{\mu e} & f_\mu & P_{\mu\tau} \\ P_{\tau e} & f_\tau & P_{\tau\tau} \end{vmatrix} , \\ D_\tau &= \begin{vmatrix} P_{ee} & P_{e\mu} & f_e \\ P_{\mu e} & P_{\mu\mu} & f_\mu \\ P_{\tau e} & P_{\tau\mu} & f_\tau \end{vmatrix} . \end{aligned} \quad (6)$$

Under the condition of $D_0 \neq 0$, the solutions to Eq. (5) turn out to be

$$\eta_e = \frac{D_e}{D_0} , \quad \eta_\mu = \frac{D_\mu}{D_0} , \quad \eta_\tau = \frac{D_\tau}{D_0} . \quad (7)$$

It is quite obvious that $f_e = f_\mu = f_\tau = 1/3$ corresponds to a trivial solution $\eta_e = \eta_\mu = \eta_\tau = 1/3$ as guaranteed by the unitarity of U or the conservation of probability for neutrino oscillations. This solution is by no means interesting as it is fully independent of the PMNS flavor mixing parameters. In contrast, an exact μ-τ interchange symmetry associated with the second and third rows of the PMNS matrix U leads to a well-known nontrivial result $f_e = f_\mu = f_\tau = 1/3$ at a neutrino telescope provided the astrophysical source has $\eta_e = 1/3$, $\eta_\mu = 2/3$ and $\eta_\tau = 0$ for its flavor composition [11–14] (More comprehensive analyses can be found, e.g., in Refs. [15–19]). But here we focus on the general solutions to Eq. (5).

Taking account of Eq. (2), we get the expression of D_0:

$$\begin{aligned} D_0 &= \Big[|U_{e1}|^2 \left(|U_{\mu 2}|^2 |U_{\tau 3}|^2 - |U_{\mu 3}|^2 |U_{\tau 2}|^2 \right) \\ &+ |U_{e2}|^2 \left(|U_{\mu 3}|^2 |U_{\tau 1}|^2 - |U_{\mu 1}|^2 |U_{\tau 3}|^2 \right) \\ &+ |U_{e3}|^2 \left(|U_{\mu 1}|^2 |U_{\tau 2}|^2 - |U_{\mu 2}|^2 |U_{\tau 1}|^2 \right) \Big]^2 . \end{aligned} \quad (8)$$

We see that $D_0 \geq 0$ must hold. Whether $D_0 = 0$ could result from an exact cancellation among the three terms of D_0 remains an open question, as the moduli of the nine PMNS matrix elements have not been determined to a sufficiently good degree of accuracy [20–22]. A more interesting possibility of $D_0 = 0$ arises from the exact μ-τ interchange flavor symmetry [23, 24]

$$|U_{\mu 1}| = |U_{\tau 1}| , \quad |U_{\mu 2}| = |U_{\tau 2}| , \quad |U_{\mu 3}| = |U_{\tau 3}| , \quad (9)$$

which is a natural consequence of a lot of neutrino mass models based on simple flavor symmetry groups [25]. This possibility will be discussed in some detail later on.

After some straightforward calculations, we obtain the analytical results of η_e, η_μ and η_τ:

$$\begin{bmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{bmatrix} = \frac{1}{D_0} \begin{pmatrix} C_{ee} & C_{e\mu} & C_{e\tau} \\ C_{\mu e} & C_{\mu\mu} & C_{\mu\tau} \\ C_{\tau e} & C_{\tau\mu} & C_{\tau\tau} \end{pmatrix} \begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix} , \quad (10)$$

where

$$\begin{aligned} C_{ee} &= \left(|U_{\mu 1}|^2 |U_{\tau 2}|^2 - |U_{\mu 2}|^2 |U_{\tau 1}|^2 \right)^2 \\ &+ \left(|U_{\mu 2}|^2 |U_{\tau 3}|^2 - |U_{\mu 3}|^2 |U_{\tau 2}|^2 \right)^2 \\ &+ \left(|U_{\mu 3}|^2 |U_{\tau 1}|^2 - |U_{\mu 1}|^2 |U_{\tau 3}|^2 \right)^2 , \\ C_{\mu\mu} &= \left(|U_{\tau 1}|^2 |U_{e2}|^2 - |U_{\tau 2}|^2 |U_{e1}|^2 \right)^2 \\ &+ \left(|U_{\tau 2}|^2 |U_{e3}|^2 - |U_{\tau 3}|^2 |U_{e2}|^2 \right)^2 \\ &+ \left(|U_{\tau 3}|^2 |U_{e1}|^2 - |U_{\tau 1}|^2 |U_{e3}|^2 \right)^2 , \\ C_{\tau\tau} &= \left(|U_{e1}|^2 |U_{\mu 2}|^2 - |U_{e2}|^2 |U_{\mu 1}|^2 \right)^2 \\ &+ \left(|U_{e2}|^2 |U_{\mu 3}|^2 - |U_{e3}|^2 |U_{\mu 2}|^2 \right)^2 \\ &+ \left(|U_{e3}|^2 |U_{\mu 1}|^2 - |U_{e1}|^2 |U_{\mu 3}|^2 \right)^2 , \end{aligned} \quad (11)$$

together with

$$\begin{aligned} C_{e\mu} = C_{\mu e} &= \left(|U_{\mu 1}|^2 |U_{\tau 2}|^2 - |U_{\mu 2}|^2 |U_{\tau 1}|^2 \right) \\ &\times \left(|U_{\tau 1}|^2 |U_{e2}|^2 - |U_{\tau 2}|^2 |U_{e1}|^2 \right) \\ &+ \left(|U_{\mu 2}|^2 |U_{\tau 3}|^2 - |U_{\mu 3}|^2 |U_{\tau 2}|^2 \right) \\ &\times \left(|U_{\tau 2}|^2 |U_{e3}|^2 - |U_{\tau 3}|^2 |U_{e2}|^2 \right) \\ &+ \left(|U_{\mu 3}|^2 |U_{\tau 1}|^2 - |U_{\mu 1}|^2 |U_{\tau 3}|^2 \right) \\ &\times \left(|U_{\tau 3}|^2 |U_{e1}|^2 - |U_{\tau 1}|^2 |U_{e3}|^2 \right) , \\ C_{e\tau} = C_{\tau e} &= \left(|U_{e1}|^2 |U_{\mu 2}|^2 - |U_{e2}|^2 |U_{\mu 1}|^2 \right) \\ &\times \left(|U_{\mu 1}|^2 |U_{\tau 2}|^2 - |U_{\mu 2}|^2 |U_{\tau 1}|^2 \right) \\ &+ \left(|U_{e2}|^2 |U_{\mu 3}|^2 - |U_{e3}|^2 |U_{\mu 2}|^2 \right) \\ &\times \left(|U_{\mu 2}|^2 |U_{\tau 3}|^2 - |U_{\mu 3}|^2 |U_{\tau 2}|^2 \right) \\ &+ \left(|U_{e3}|^2 |U_{\mu 1}|^2 - |U_{e1}|^2 |U_{\mu 3}|^2 \right) \\ &\times \left(|U_{\mu 3}|^2 |U_{\tau 1}|^2 - |U_{\mu 1}|^2 |U_{\tau 3}|^2 \right) , \\ C_{\mu\tau} = C_{\tau\mu} &= \left(|U_{\tau 1}|^2 |U_{e2}|^2 - |U_{\tau 2}|^2 |U_{e1}|^2 \right) \\ &\times \left(|U_{e1}|^2 |U_{\mu 2}|^2 - |U_{e2}|^2 |U_{\mu 1}|^2 \right) \\ &+ \left(|U_{\tau 2}|^2 |U_{e3}|^2 - |U_{\tau 3}|^2 |U_{e2}|^2 \right) \\ &\times \left(|U_{e2}|^2 |U_{\mu 3}|^2 - |U_{e3}|^2 |U_{\mu 2}|^2 \right) \\ &+ \left(|U_{\tau 3}|^2 |U_{e1}|^2 - |U_{\tau 1}|^2 |U_{e3}|^2 \right) \\ &\times \left(|U_{e3}|^2 |U_{\mu 1}|^2 - |U_{e1}|^2 |U_{\mu 3}|^2 \right) . \end{aligned} \quad (12)$$

These coefficients depend on three flavor mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and one CP-violating phase (δ_ν) in the parametrization of U that has been advocated by the

Particle Data Group [20]. The direct precision measurement of θ_{12} has recently been implemented in the JUNO reactor antineutrino oscillation experiment [9], and the value of θ_{13} has been precisely determined by the Daya Bay Collaboration [10]. In comparison, the octant of θ_{23} and the quadrant of δ have not well been fixed by current neutrino oscillation data [20–22], and hence they constitute the main uncertainties of $C_{\alpha\beta}/D_0$ in bridging the gap between the original flavor fractions η_α and the observed flavor ratios f_β (for $\alpha, \beta = e, \mu, \tau$).

The nine moduli of the PMNS matrix elements appearing in Eqs. (11) and (12) can be expressed, in terms of θ_{12}, θ_{13}, θ_{23} and δ_ν [20], as follows:

$$
\begin{aligned}
|U_{e1}|^2 &= c_{12}^2 c_{13}^2 \; , \quad |U_{e2}|^2 = s_{12}^2 c_{13}^2 \; , \quad |U_{e3}|^2 = s_{13}^2 \; , \\
|U_{\mu 1}|^2 &= s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu \; , \\
|U_{\mu 2}|^2 &= c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu \; , \\
|U_{\tau 1}|^2 &= s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu \; , \\
|U_{\tau 2}|^2 &= c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu \; , \\
|U_{\mu 3}|^2 &= c_{13}^2 s_{23}^2 \; , \quad |U_{\tau 3}|^2 = c_{13}^2 c_{23}^2 \; ,
\end{aligned}
\tag{13}
$$

For the sake of illustration, let us take the best-fit values $s_{12}^2 = 3.03 \times 10^{-1}$, $s_{13}^2 = 2.23 \times 10^{-2}$, $s_{23}^2 = 4.73 \times 10^{-1}$ and $\delta_\nu = 1.20 \times \pi$ obtained by Capozzi *et al* [21] as the typical inputs to calculate the coefficient matrices in Eqs. (1) and (10). We obtain the numerical results

$$
\begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix} \simeq \begin{pmatrix} 0.5526 & 0.2125 & 0.2348 \\ 0.2125 & 0.4078 & 0.3797 \\ 0.2348 & 0.3797 & 0.3855 \end{pmatrix} \begin{bmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{bmatrix} \; , \tag{14}
$$

and

$$
\begin{bmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{bmatrix} \simeq \begin{pmatrix} 2.505 & 1.392 & -2.897 \\ 1.392 & 30.41 & -30.80 \\ -2.897 & -30.80 & 34.70 \end{pmatrix} \begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix} \; . \tag{15}
$$

Given $\eta_e = 1/3$, $\eta_\mu = 2/3$ and $\eta_\tau = 0$ at a remote astrophysical source of high-energy neutrinos, for example, we simply arrive at an approximate "flavor democracy" at a terrestrial neutrino telescope with the help of Eq. (14): $f_e \simeq 0.326$, $f_\mu \simeq 0.343$ and $f_\tau \simeq 0.331$. In contrast, making use of Eq. (15) to infer η_μ and η_τ from f_e, f_μ and f_τ may involve significant cancellations due to the very fact $C_{\mu\mu} \simeq C_{\tau\tau} \simeq -C_{\mu\tau}$, whose magnitudes are much larger than the other coefficients.

Note that $C_{\mu\mu} = C_{\tau\tau} = -C_{\mu\tau} \neq 0$ holds as a straightforward consequence of the exact μ-τ interchange symmetry, as can be easily seen from Eqs. (9), (11) and (12). But this special case gives rise to $D_0 = 0$ and thus an unavoidable divergence associated with the expressions of η_μ and η_τ. Since $C_{ee} = C_{e\mu} = C_{\mu e} = C_{e\tau} = C_{\tau e} = 0$ can also result from the μ-τ flavor symmetry, η_e turns out not to be that sensitive to this interesting symmetry limit. So it is not difficult to understand the numerical results obtained in Eq. (15), namely $C_{\mu\mu} \simeq 30.41$, $C_{\tau\tau} \simeq 34.70$ and $C_{\mu\tau} \simeq -30.80$, as the input value of θ_{23} has already implied an approximate μ-τ interchange symmetry hidden in the PMNS matrix U [2].

To illustrate, let us apply Eq. (15) to the IceCube all-sky neutrino flux data ranging from 5 TeV to 10 PeV by naively assuming that all the relevant neutrino sources have a common flavor composition. In this case the IceCube best-fit flavor ratios $f_e = 0.30$, $f_\mu = 0.37$ and $f_\tau = 0.33$ [8] lead us to the following flavor composition at those sources: $\eta_e \simeq 0.31$, $\eta_\mu \simeq -1.50$ and $\eta_\tau \simeq -0.81$. Such outputs of η_μ and η_τ are no doubt meaningless. The problem may simply arise from our assumption of the same flavor composition for the astrophysical neutrino sources under discussion, which appears too naive to be true. The problem is also likely to originate from the values of f_μ and f_τ reported by the IceCube Collaboration, because their uncertainties may easily fine-tune how the significant cancellations in η_μ and η_τ evolve. In any case, a sufficiently accurate measurement of f_α (for $\alpha = e, \mu, \tau$) at the neutrino telescopes is highly desired [28, 29].

3. THE FORMULA IN THE μ-τ SYMMETRY

Now we come back to the limit of an exact μ-τ flavor symmetry associated with the PMNS lepton flavor mixing matrix U, as this special case remains consistent with current neutrino oscillation data at least at the 2σ confidence level [21, 22]. Taking account of Eq. (9), which is equivalent to $\theta_{23} = \pi/4$ and $\delta_\nu = -\pi/2$ in the standard parametrization of U, we immediately arrive at

$$
P_{e\mu} = P_{e\tau} \; , \quad P_{\mu\mu} = P_{\mu\tau} = P_{\tau\tau} \; , \tag{16}
$$

from Eq. (2). As a result, the three linear equations in Eq. (5) are reduced to two independent linear equations in two variables η_e and $(\eta_\mu + \eta_\tau)$:

$$
\begin{aligned}
P_{ee}\eta_e + P_{e\mu}\left(\eta_\mu + \eta_\tau\right) &= f_e \; , \\
P_{e\mu}\eta_e + P_{\mu\mu}\left(\eta_\mu + \eta_\tau\right) &= f_\mu \; ,
\end{aligned}
\tag{17}
$$

[2] Combining Eq. (9) with Eq. (13), one obtains either $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ or $\theta_{23} = \pi/4$ and $\delta_\nu = \pm\pi/2$. The former has been ruled out by $\theta_{13} \neq 0$ [26], but the latter remains viable (especially for $\delta_\nu = -\pi/2$ [27]). Because the most poorly known CP-violating phase δ_ν is always associated with the sine function of the smallest flavor mixing angles θ_{13} in the standard parametrization of U, as shown in Eq. (13), it is actually $\theta_{23} \sim \pi/4$ and $s_{13} \ll 1$ that assure the approximate μ-τ flavor symmetry relations $|U_{\mu i}| \simeq |U_{\tau i}|$ (for $i = 1, 2, 3$) to hold.

as $f_\tau = f_\mu$ holds in this symmetry limit. Then we obtain

$$\begin{aligned}\eta_e &= \frac{P_{\mu\mu}}{P_{ee}P_{\mu\mu} - P_{e\mu}^2} f_e - \frac{P_{e\mu}}{P_{ee}P_{\mu\mu} - P_{e\mu}^2} f_\mu \\ &= \frac{1 - c_{12}^2 s_{12}^2 c_{13}^4 - c_{13}^2 s_{13}^2}{1 - 3c_{12}^2 s_{12}^2 c_{13}^4 - 3c_{13}^2 s_{13}^2} f_e \\ &- \frac{2\left(c_{12}^2 s_{12}^2 c_{13}^4 + c_{13}^2 s_{13}^2\right)}{1 - 3c_{12}^2 s_{12}^2 c_{13}^4 - 3c_{13}^2 s_{13}^2} f_\mu \ , \end{aligned} \tag{18}$$

and

$$\begin{aligned}\eta_\mu + \eta_\tau &= \frac{P_{ee}}{P_{ee}P_{\mu\mu} - P_{e\mu}^2} f_\mu - \frac{P_{e\mu}}{P_{ee}P_{\mu\mu} - P_{e\mu}^2} f_e \\ &= 2\left[\frac{1 - 2c_{12}^2 s_{12}^2 c_{13}^4 - 2c_{13}^2 s_{13}^2}{1 - 3c_{12}^2 s_{12}^2 c_{13}^4 - 3c_{13}^2 s_{13}^2} f_\mu \right. \\ &- \left. \frac{c_{12}^2 s_{12}^2 c_{13}^4 + c_{13}^2 s_{13}^2}{1 - 3c_{12}^2 s_{12}^2 c_{13}^4 - 3c_{13}^2 s_{13}^2} f_e \right] \ , \end{aligned} \tag{19}$$

where $\theta_{23} = \pi/4$ and $\delta_\nu = -\pi/2$ have been input. It is clear that the exact μ-τ flavor symmetry only allows us to infer a sum of the μ- and τ-flavor fractions of high-energy neutrinos at the astrophysical source from the observed flavor ratios at a neutrino telescope.

It is certainly improper to apply the global-analysis best-fit values of θ_{12} and θ_{13} to Eqs. (18) and (19), as they are inconsistent with the exact μ-τ symmetry conditions. But direct measurements of θ_{12} and θ_{13} in the JUNO and Daya Bay experiments have nothing to do with θ_{23} and δ_ν, and thus they are applicable to the numerical calculations of η_e and $\eta_\mu + \eta_\tau$ by means of Eqs. (18) and (19). Given the latest best-fit results $s_{12}^2 = 3.092 \times 10^{-1}$ and $s_{13}^2 = 2.175 \times 10^{-2}$ reported respectively by JUNO [9] and Daya Bay [10], we are left with

$$\begin{aligned} &\eta_e \simeq 2.398 f_e - 1.398 f_\mu \ , \\ &\eta_\mu + \eta_\tau \simeq 3.398 f_\mu - 1.398 f_e \ . \end{aligned} \tag{20}$$

So the "flavor democracy" $f_e = f_\mu = f_\tau = 1/3$ at a terrestrial neutrino telescope will automatically lead us to $\eta_e = 1/3$ and $\eta_\mu + \eta_\tau = 2/3$ at the astrophysical source.

4. SUMMARY

With no assumption of any specific flavor composition pattern for a remote astrophysical source of high-energy neutrinos, we have calculated their original flavor fractions $(\eta_e, \eta_\mu, \eta_\tau)$ in terms of the flavor ratios (f_e, f_μ, f_τ) observed at a neutrino telescope. We find that a potential divergence is expected to appear in inferring the values of η_μ and η_τ, as an unavoidable consequence of the observed approximate μ-τ interchange symmetry of lepton flavor mixing. We have illustrated this effect by applying our formula to the recent IceCube all-sky neutrino flux data ranging from 5 TeV to 10 PeV by naively assuming that the relevant sources have a common flavor composition. In addition, we have shown that only η_e and $\eta_\mu + \eta_\tau$ can be extracted from the precision measurements of f_e, f_μ and f_τ if the μ-τ flavor symmetry is exact, and emphasized that the JUNO and Daya Bay data will help a lot in this connection.

We conclude that a precise determination of the μ-τ flavor symmetry breaking effects in the upcoming long-baseline accelerator neutrino oscillation experiments is crucial, in order to more reliably trace the true flavor distribution of high-energy astrophysical neutrinos with the terrestrial neutrino telescopes.

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* xingzz@ihep.ac.cn

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