

Charged Higgs bosons associated with neutral gauge bosons at future multi–TeV muon colliders

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Abstract

The first results for charged Higgs pair production associated with neutral gauge bosons at future multi–TeV muon colliders are presented within the framework of the Two-Higgs-Doublet Model. In the phenomenological studies for the production processes, we first update the parameter space of the Type-X Two-Higgs-Doublet Model. From the viable regions of the parameter space, we compute the production cross sections, including all two-body decay modes of the charged Higgs boson. We mention that all two-body decay modes of the charged Higgs boson are evaluated with one-loop electroweak corrections, with two-loop QCD corrections also included for the decays of the charged Higgs into two fermions. The signal significances are also examined for several benchmark points within the updated parameter space. It is worth emphasizing that the production cross sections used for computing the significances in this work include initial-state radiation corrections up to two-loop order. With the help of the high integrated luminosity planned for future multi–TeV muon colliders, the production signals can be observed with a statistical significance exceeding 5σ for several selected points within the allowed parameter space.

Keywords: Scalar Higgs bosons phenomenology, Physics beyond the Standard Model, Physics at present and future multi–TeV muon colliders.

1. Introduction

Searches for additional scalar particles predicted in physics beyond the Standard Model (SM) are among the main goals of future colliders, including the High-Luminosity Large Hadron Collider (HL-LHC), the High-Energy LHC (HE-LHC), future lepton colliders such as the International Linear Collider, and multi–TeV muon colliders. The discovery of additional scalar particles could provide a direct signal of new physics and offer important insights for verifying the scalar potential in various particle models. Such data therefore provide crucial information to deepen our understanding of electroweak symmetry breaking in high-energy physics. Among all scalar particle productions, searches for charged Higgs boson production and its decay channels have recently been widely studied at current and future colliders. It is worth mentioning typical searches for charged Higgs bosons in the following paragraphs. At the LHC, charged Higgs bosons can be probed through top quark decays, as reported in [1, 2, 3]. In pp collisions at $\sqrt{s} = 8$ TeV, charged Higgs boson production has been measured, as reported in [4]. Measurements of charged Higgs bosons from di-top quark events using pp collision data at $\sqrt{s} = 7$ TeV have been performed with considering the sequential decay $H^+ \rightarrow \tau\nu$ in [5, 6, 7, 8], as

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well as the decay mode $H^+ \rightarrow c\bar{s}$ in [9, 10]. Investigations of the heavy charged Higgs mass regime through the process $pp \rightarrow tH^\pm \rightarrow t\bar{t}b$ have been conducted in [11, 12, 13]. Furthermore, charged Higgs productions via vector boson fusions, considering the decay mode $H^\pm \rightarrow W^\pm Z$, have been studied in [14, 15]. Measurements of charm and bottom quarks produced by charged Higgs decays have also been performed at the LHC [16]. In addition, charged Higgs decays to a heavy CP-even Higgs as well as to the SM-like Higgs boson at the LHC have been measured, as reported in [17, 18].

There exist many theoretical computations for charged Higgs production at current and future colliders. Single charged Higgs production via the process $pp \rightarrow tH^- \rightarrow tW^-b\bar{b}$ at the LHC has been studied in [19]. Investigations of charged Higgs pair production at the HL-LHC and the HE-LHC, including their subsequent bosonic decay channels, can be found in [21]. Within the MSSM scenarios, the dominant channel of single charged Higgs production, such as $gg \rightarrow tbH$, has been examined at the LHC, as reported in [22]. Light charged Higgs bosons decaying into bosonic modes have been investigated, including implications from LHC Run III data within the framework of the THDM, as published in [23]. Furthermore, searches for light charged Higgs at the LHC through the channels $pp \rightarrow H^\pm h/A$ and $pp \rightarrow H^+H^-$, with $H^\pm \rightarrow W^\pm h/A$, were performed in [24]. Other searches for charged Higgs at the LHC have been carried out in [25, 26, 27, 28, 29]. At future lepton colliders, including e^-e^+ and $\mu^-\mu^+$ machines, charged Higgs bosons have been studied in [30, 31, 32, 33, 34, 35]. In our previous works, we discussed the possibility of probing charged Higgs bosons, including $H^\pm \rightarrow W^\pm\gamma$ in [36] and $H^\pm \rightarrow W^\pm Z$ in [37] at future multi-TeV muon colliders.

Future lepton colliders are well known for providing a cleaner environment than hadron colliders, which have large QCD backgrounds, allowing for higher-precision measurements. Moreover, multi-TeV muon colliders could provide access to a higher-energy regime, planned for probing new physics. In comparison with e^-e^+ collisions, contributions from scalar particles exchanged in s -channels may be enhanced due to resonance effects, which highlight the advantage of multi-TeV muon colliders as important tools for exploring and discovering the scalar Higgs sectors. In the four types of THDM, type-I and type-X allow charged Higgs masses in a wide range, from $\mathcal{O}(100)$ GeV to $\mathcal{O}(1000)$ GeV, in comparison with the other types, for which charged Higgs masses are constrained above 500 GeV. Therefore, both type-I and type-X are more interesting for searches in the low-Higgs mass regions than the other types.

In this work, we present the first results for charged Higgs pair production associated with neutral gauge bosons at future multi-TeV muon colliders within the framework of the THDM. In the phenomenological studies for the production processes, we first update the parameter space of the Type-X THDM. From the viable regions of the parameter space, we compute the production cross sections, including all two-body decay modes of the charged Higgs boson. We emphasize that all two-body decay modes of the charged Higgs boson are evaluated at the one-loop level, with up to two-loop QCD corrections included for decays of the charged Higgs into two fermions. The signal significances are also examined for several benchmark points within the updated parameter space. It is worth mentioning that the production cross sections used for computing the significances in this work include initial-state radiation corrections up to two-loop order. With the help of the high luminosity planned at future multi-TeV muon colliders, the signals can be observed with a statistical significance exceeding 5σ for several benchmark points within the viable parameter space.

The layout of our paper is as follows. We briefly present the model under study, known as the THDM, and focus on its constraints for Type-X in the next section. Section 3 is devoted to the phenomenological study of charged Higgs pair production associated with neutral gauge bosons at future multi-TeV muon colliders. In Section 4, the same production is investigated

via $\gamma\gamma$ -fusion at future multi-TeV muon colliders. The signal significances are also examined for several benchmark points within the updated parameter space are computed in Section 5. Finally, the conclusions and outlooks of our work are presented in Section 6.

2. The Two-Higgs-Doublet Model and Its Constraints

In the model, the fermion and gauge sectors remain the same as in the SM. The scalar sector is extended by introducing an additional scalar Higgs field. For the THDM, we refer readers to Ref. [38] for detailed reviews and phenomenological studies. Moreover, we consider the scalar potential preserving the discrete Z_2 symmetry, given by $\Phi_1 \leftrightarrow \Phi_1$ and $\Phi_2 \leftrightarrow -\Phi_2$, with soft-breaking terms allowed, as seen in [39]. In the present study, we focus on the THDM with CP-conserving case. As a result, all parameters in the scalar potential are real variables. In the Yukawa sector, due to the Z_2 symmetry and depending on the Z_2 charge assignments, we classify the THDM into four types, known as Type-I, Type-II, Type-X, and Type-Y [39]. Consequently, the Lagrangian for Yukawa interactions can be parameterized as follows (taken from Refs. [36, 37]):

$$\mathcal{L}_Y = - \sum_{f=u,d,\ell} \left(\sum_{\phi_j=h,H} g_{\phi_j f f} \cdot \phi_j \bar{f} f + g_{A f f} \cdot A \bar{f} \gamma_5 f \right) \quad (1)$$

$$- \left[\bar{u}_i \left(g_{H^+ u_i d_j}^L m_{u_i} P_L + g_{H^+ u_i d_j}^R m_{d_j} P_R \right) d_j H^+ \right]$$

+ ...

$$= - \sum_{f=u,d,\ell} \left(\sum_{\phi_j=h,H} \frac{m_f}{v} \xi_{\phi_j}^f \phi_j \bar{f} f - i \frac{m_f}{v} \xi_A^f \bar{f} \gamma_5 f A \right) \quad (2)$$

$$- \frac{\sqrt{2}}{v} [\bar{u}_i V_{ij} (m_{u_i} \xi_A^u P_L + \xi_A^d m_{d_j} P_R) d_j H^+]$$

$$- \frac{\sqrt{2}}{v} \bar{\nu}_L \xi_A^\ell m_\ell \ell_R H^+ + \text{H.c.}$$

Here, $\ell_{L/R}$ and $\nu_{L/R}$ denote the left- and right-handed leptons and neutrinos, respectively. The factor V_{ij} represents an element of the CKM matrix. The projection operators are defined as $P_{L/R} = (1 \mp \gamma_5)/2$, which are used in the Lagrangian. We note that the coupling coefficients appearing in the Lagrangian are listed in Table 2 of Ref. [39].

We now present the constraints on the THDM. The models under investigation are restricted by both theoretical conditions and experimental data from collider measurements. These aspects are discussed in more detail in our previous works [36, 37]. In this study, we focus on the phenomenological analysis of the Type-X scenario. Accordingly, we only show the allowed parameter space for this type of THDM. We first choose the parameters for the Type-X THDM within the ranges $s_{\beta-\alpha} \in [0.97, 1]$, $t_\beta \in [0.5, 45]$, $m_H \in [130, 1000]$ GeV, $m_{A,H^\pm} \in [130, 1000]$ GeV, and $m_{12}^2 \in [0, 10^6]$ GeV², with the SM-like Higgs mass fixed at $m_h = 125.09$ GeV. The scanning of the parameters above is performed as follows. We first constrain the parameter space using theoretical requirements, including unitarity, perturbativity, and vacuum stability of the scalar potential. The surviving points are then tested against the electroweak precision observables (EWPOs). All remaining points are subsequently compared with data on the SM-like Higgs boson properties and the exclusion limits for BSM Higgs bosons measured at the LHC. For all the aforementioned constraints, we use 2HDMC-1.8.0 [40] to scan the parameter space. Finally, the

surviving parameter points are further constrained by flavor data, which are taken into account using the **SuperISO** package [43].

We now discuss the parameter space constrained by the following scatter plots. In Fig. 1, the left plot indicates for the correlations among the scalar mass splitting $m_{H^\pm} - m_H$, m_A , and $m_{H^\pm}$, while the right panel presents those among another scalar mass splitting and $m_H - m_A$, m_A , and $m_{H^\pm}$. For the left panel, when $m_A = m_{H^\pm}$, we observe that $m_{H^\pm} - m_H$ can vary from -600 GeV to 600 GeV. In the region where $m_A > m_{H^\pm}$, the data favor the range -100 GeV $\leq m_{H^\pm} - m_H \leq 100$ GeV. In the right panel, the points with $m_A > m_H$ are more favored than the other cases.

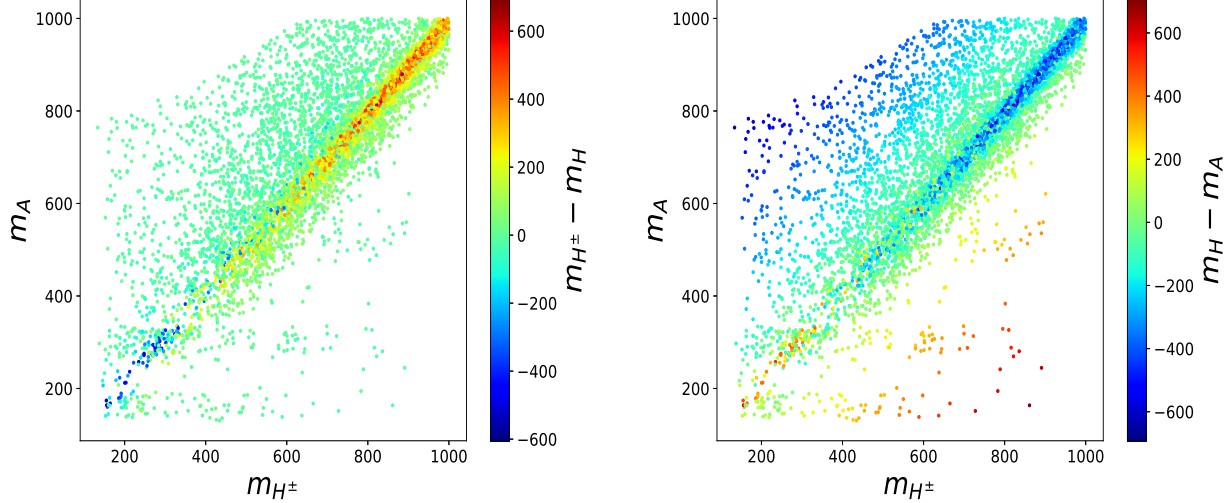


Figure 1: The scatter plots show correlations in the parameter space: $(m_A, m_{H^\pm}, m_H)$ in the left panel and $(m_A, m_{H^\pm}, m_H - m_A)$ in the right panel.

In Fig. 2, we show the correlations among the splitting mass $m_A - m_H$ with $m_{H^\pm}$ and $\tan \beta$ for the left panel, and among m_{12}^2 with $m_{H^\pm}$ and $\tan \beta$ for the right panel. The allowed regions indicate that the preferred valid points mainly lie in the pattern -100 GeV $\leq m_A - m_H \leq \sim 500$ GeV and $2 \leq \tan \beta \leq \sim 20$ in the left plot. The data shows that the viable regions are concentrated in $10^3 \leq m_{12}^2 \leq 5 \cdot 10^5$ and $2 \leq \tan \beta \leq \sim 20$ over the entire charged Higgs mass range in the right plot.

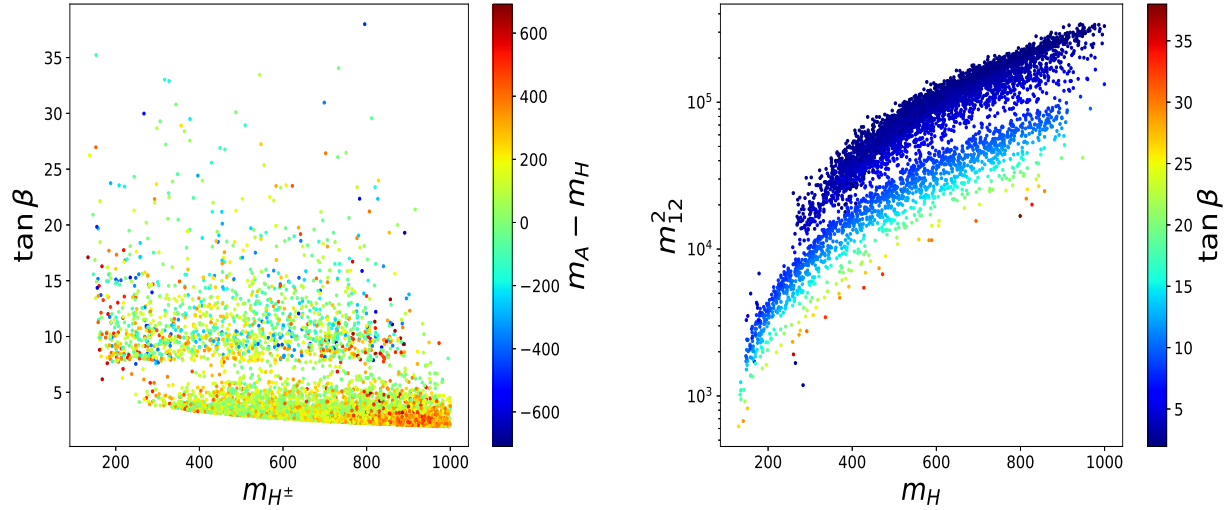


Figure 2: The scatter plots show the correlations among the splitting mass $m_A - m_H$ with $m_{H^\pm}$ and $\tan\beta$ for the left panel, and among m_{12}^2 with $m_{H^\pm}$, and $\tan\beta$ for the right panel.

After obtaining the viable parameter space for the Type-X THDM, we transfer the allowed points to the program **H-COUP** version 3 [44] to evaluate the branching ratios of the charged Higgs boson. All branching ratios presented in the following paragraphs are computed using **H-COUP** version 3 [44], which takes into account one-loop electroweak corrections as well as one- and two-loop QCD corrections for two-body decays of scalar particles; see [45] for more details.

We present the dominant decay modes of the charged Higgs in the Type-X THDM in the following paragraphs. In the Fig. 3, we examine the branching fractions of the charged Higgs boson into top and bottom quarks (left above), $H^\pm \rightarrow \tau\nu_\tau$ (right above), $H^\pm \rightarrow Wh$ (left below) and $H^\pm \rightarrow WH$ (right below). All scatter plots are generated in the plane of $m_{H^\pm}$ and t_β . Below the regions of $m_{H^\pm} \lesssim m_W + m_h \sim 220$ GeV, the decay rates $H^\pm \rightarrow \tau\nu_\tau$ are dominant when $\tan\beta \gtrsim 10$. Beyond the regions $m_{H^\pm} \gtrsim 220$ GeV (opening the decay of $H^\pm \rightarrow Wh$), the branching fractions of $H^\pm \rightarrow tb$ are dominant contributions in the regions of $\tan\beta \lesssim 5$. While the branching fractions of $H^\pm \rightarrow Wh$ become large contributions in comparison with those of $H^\pm \rightarrow tb$ when $\tan\beta \gtrsim 10$. Furthermore, we observe that when $\tan\beta \lesssim 2$, the branching fractions of $H^\pm \rightarrow WH$ also become dominant contributions. This explains the reason that the branching fractions of $H^\pm \rightarrow tb$ are small in the regions of $\tan\beta \lesssim 2$.

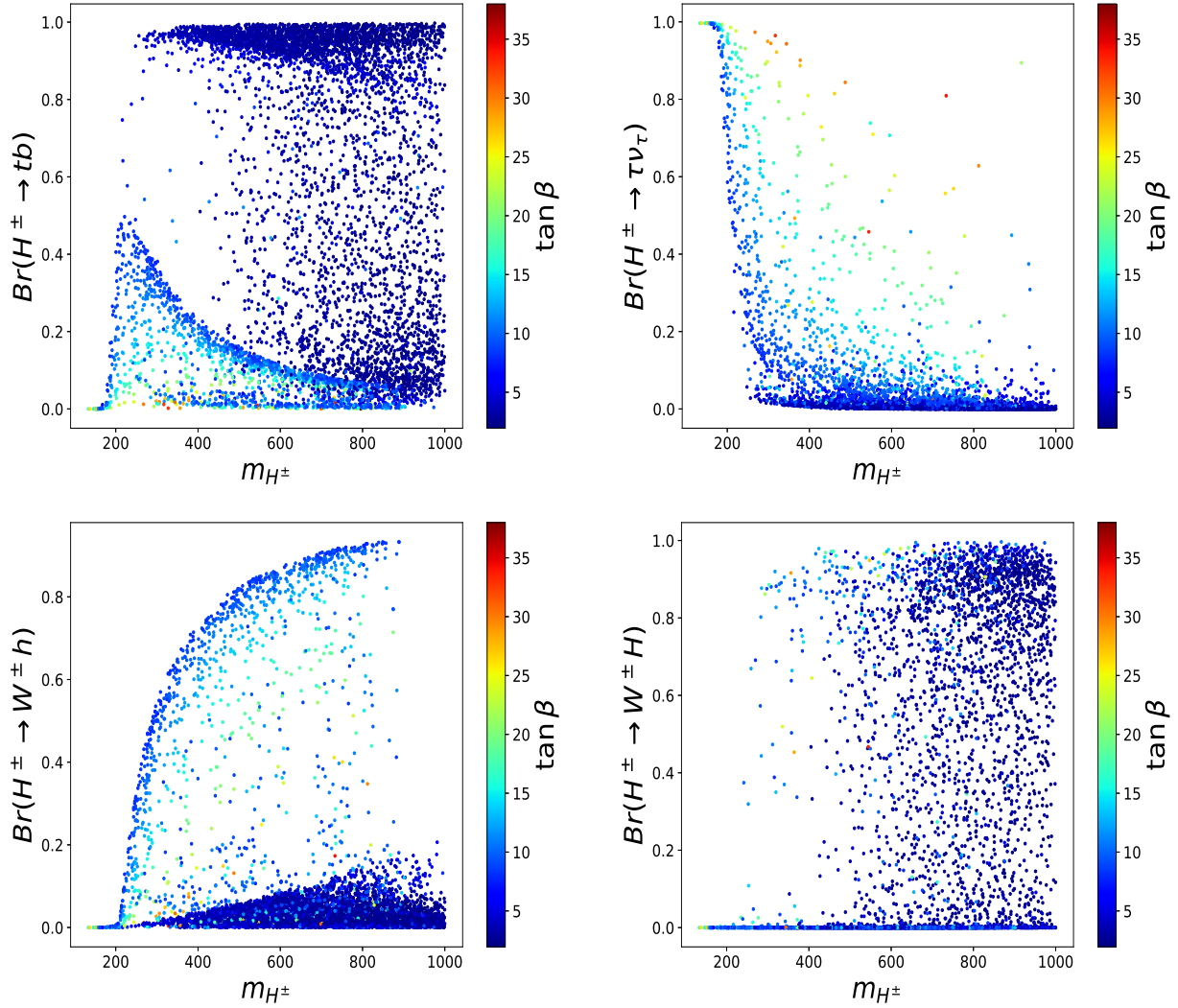


Figure 3: Branching ratios for $H^\pm \rightarrow tb$ (left above), $H^\pm \rightarrow \tau\nu_\tau$ (right above), $H^\pm \rightarrow Wh$ (left below), and $H^\pm \rightarrow WH$ (right below) are generated within the allowed parameter regions of the Type-X THDM.

Two decay modes, $H^\pm \rightarrow tb$ and $H^\pm \rightarrow Wh$, are of interest for computing the production cross sections in the next sections.

3. Productions $\mu^+\mu^- \rightarrow H^\pm H^\mp V$ at future multi-TeV muon colliders

In this section, we perform calculations for the production processes $\mu^+\mu^- \rightarrow H^\pm H^\mp V$, where $V \equiv \gamma, Z$, at future muon-TeV colliders. The computer packages **FeynArts/FormCalc** are employed to evaluate these processes. All tree-level Feynman diagrams and their corresponding amplitudes are presented in appendix A. The valid parameter space obtained in the previous section is then used to generate the cross sections. As mentioned earlier, there are four dominant decay channels such as $H^\pm \rightarrow tb, \tau\nu_\tau, Wh$, and WH . In this simulation, we focus on $H^\pm \rightarrow tb$ and $H^\pm \rightarrow Wh$ as two typical examples.

We now turn our attention to the cross section of $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma$ at $\sqrt{s} = 3$ TeV (left panel) and $\sqrt{s} = 5$ TeV (right panel), generated within the viable parameter space obtained above, including the subsequent decay $H^\pm \rightarrow tb$, as shown in Fig. 4. For the results, we apply angular and energy cuts on the external photon as follows: $-0.98 \leq \cos\theta_\gamma \leq 0.98$ and $E_\gamma \geq 5$ GeV. The

results indicate that the cross sections peak around $M_{H^\pm} \sim 300$ GeV. In the Type-X scenario, the coupling of the charged Higgs to top and bottom quarks is enhanced at small values of $\tan\beta$, which explains why the cross sections are larger in the low- $\tan\beta$ region compared to other regions. It is interesting to observe that the cross section can reach the order of $\mathcal{O}(1)$ fb. With the help of the high integrated luminosity planned at multi-TeV muon colliders, ranging from 500 fb^{-1} to 3000 fb^{-1} , we can expect approximately 500 to 3000 events to be observed at the colliders.

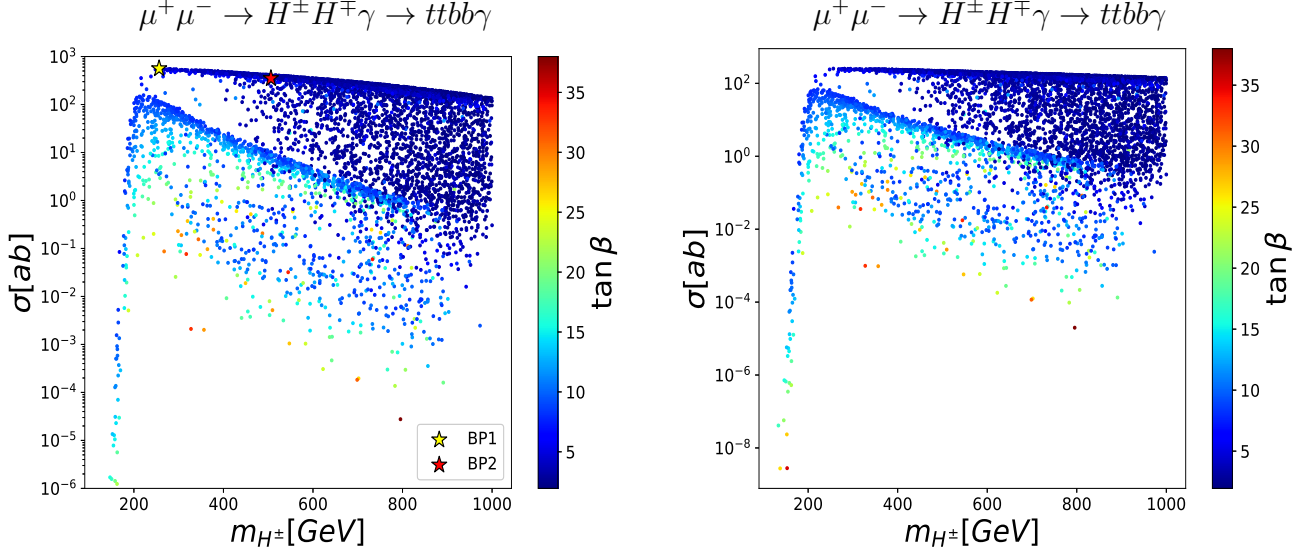


Figure 4: Cross-sections for the process $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma \rightarrow ttbb\gamma$ are generated in the allowed parameter spaces of the Type-X THDM at $\sqrt{s} = 3$ TeV in the left panel and at $\sqrt{s} = 5$ TeV in the right panel.

The cross sections for the process $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma \rightarrow WhWh\gamma$ at $\sqrt{s} = 3$ TeV (left panel) and at $\sqrt{s} = 5$ TeV (right panel), taking into account the sequential decay $H^\pm \rightarrow Wh$, are examined within the viable parameter space of the Type-X THDM. Over the entire range of charged Higgs masses, the cross sections exhibit similar behavior to the cases discussed above, as shown in Fig. 5. We find that the cross sections peak in the region $M_{H^\pm} \in [250, 400]$ GeV and are enhanced for $\tan\beta \gtrsim 10$. The cross sections can reach the order of $\mathcal{O}(0.5)$ fb. Consequently, one can expect approximately 250 to 1500 events for $\mathcal{L} = 500 \text{ fb}^{-1}$ to $\mathcal{L} = 3000 \text{ fb}^{-1}$ operating at future multi-TeV muon colliders.

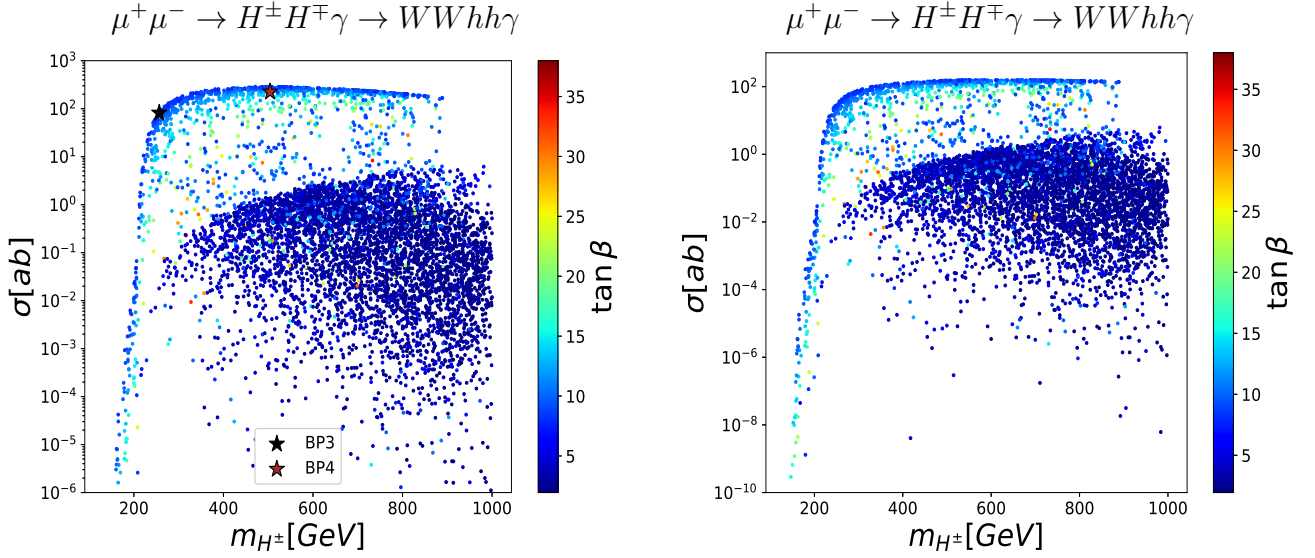


Figure 5: Cross-sections for the process $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma \rightarrow WWhh\gamma$ are generated in the allowed parameter spaces of the Type-X THDM at $\sqrt{s} = 3$ TeV in the left panel and at $\sqrt{s} = 5$ TeV in the right panel.

Another production channel considered in this work is $\mu^+\mu^- \rightarrow H^\pm H^\mp Z$. The corresponding cross sections at $\sqrt{s} = 3$ TeV (left panel) and at $\sqrt{s} = 5$ TeV (right panel) are generated within the viable parameter space obtained above, including the charged Higgs decay mode $H^\pm \rightarrow tb$, as shown in Fig. 6. The cross sections for this process are about one order of magnitude smaller than those of $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma \rightarrow tbtb\gamma$. Consequently, we can expect approximately 50 to 300 events at future multi-TeV muon colliders with the integrated luminosities from $\mathcal{L} = 500 \text{ fb}^{-1}$ to $\mathcal{L} = 3000 \text{ fb}^{-1}$.

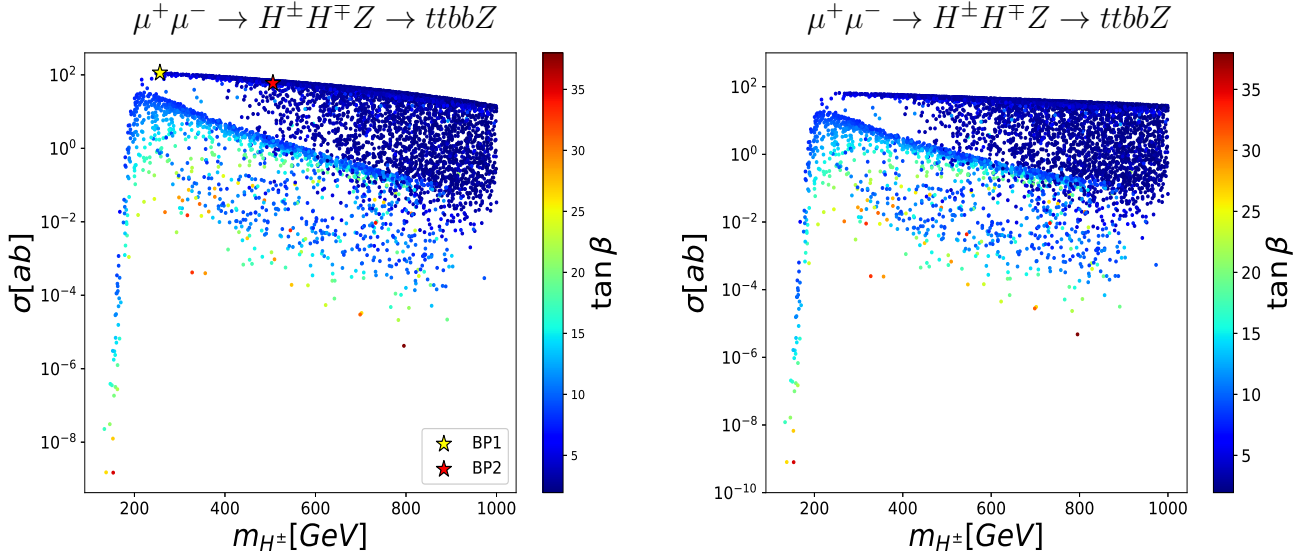


Figure 6: Cross-sections for $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow ttbbZ$ are generated in the allowed parameter spaces of the Type-X THDM at $\sqrt{s} = 3$ TeV in the left panel and at $\sqrt{s} = 5$ TeV in the right panel.

We then examine the production process considering the sequential decay $H^\pm \rightarrow Wh$, as shown in Fig. 7. Over the entire range of charged Higgs masses, we again find that the cross

sections are largest in the region $250 \text{ GeV} \leq m_{H^\pm} \leq 400 \text{ GeV}$ and are enhanced for $\tan\beta \gtrsim 10$. The cross sections can be obtained the order of $\mathcal{O}(0.07) \text{ fb}$. As a result, one can expect approximately 35 to 210 events for $\mathcal{L} = 500 \text{ fb}^{-1}$ to $\mathcal{L} = 3000 \text{ fb}^{-1}$ integrated at future multi-TeV muon colliders.

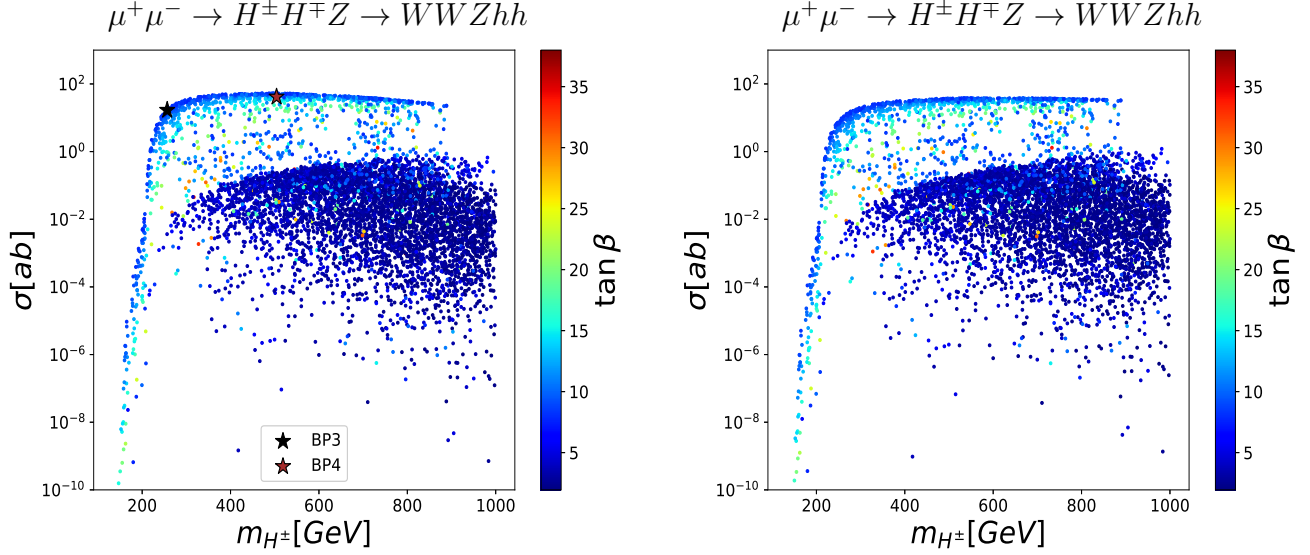


Figure 7: Cross-sections for $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow WW Zh h$ are generated in the allowed parameter spaces of Type-X THDM at $\sqrt{s} = 3 \text{ TeV}$ in the left panel and at $\sqrt{s} = 5 \text{ TeV}$ in the right panel.

4. Productions of $H^\pm H^\mp V$ via $\gamma\gamma$ fusion at future multi-TeV muon colliders

At future multi-TeV muon colliders, $\gamma\gamma$ -fusion is also promising option for probing scalar Higgs in many extension of the SM. For this reason, we also compute the production of $H^\pm H^\mp V$ via $\gamma\gamma$ -fusion at future multi-TeV muon colliders in this section. The total cross section can be calculated by integrating the photon splitting functions as follows:

$$\sigma(s) = \int_{\frac{2m_{H^\pm}}{\sqrt{s}}}^{x_{\max}} dz \left(2z \int_{z^2/x_{\max}}^{x_{\max}} \frac{dx}{x} f_{\gamma/\mu}(x) f_{\gamma/\mu}(z^2/x) \right) \hat{\sigma}(\hat{s} = z^2 s). \quad (3)$$

Here, the photon structure function $f_{\gamma/\mu}(x)$ is used, with x denoting the energy fraction of the photon emitted by the incoming lepton. The explicit formulas for $f_{\gamma/\mu}(x)$ are given in [48]. In the master formulas, we adopt $x_{\max} = 0.83$ as in [49].

We first study the cross section for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma$ at $\sqrt{s} = 3 \text{ TeV}$ (left panel) and at $\sqrt{s} = 5 \text{ TeV}$ (right panel) within the allowed parameter space obtained above for the Type-X THDM. In Fig. 8, the cross sections are included the sequential decay $H^\pm \rightarrow tb$. We have used the same cuts on external photon as $-0.98 \leq \cos\theta_\gamma \leq 0.98$ and $E_\gamma \geq 5 \text{ GeV}$. The cross sections peak around $m_{H^\pm} \sim 300 \text{ GeV}$. Since there are no diagrams with direct couplings of the charged Higgs boson to muons or their neutrinos, the cross sections in this process are inversely proportional to $\tan\beta$. This explains the different behavior of the cross sections in this process compared to those of $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma \rightarrow tb t b \gamma$. Around the peak regions, the cross sections can reach the order of $\mathcal{O}(1) \text{ ab}$. With considering the projected integrated luminosity of 10 ab^{-1} , only a few events are expected to be produced via $\gamma\gamma$ -fusion at future multi-TeV muon colliders.

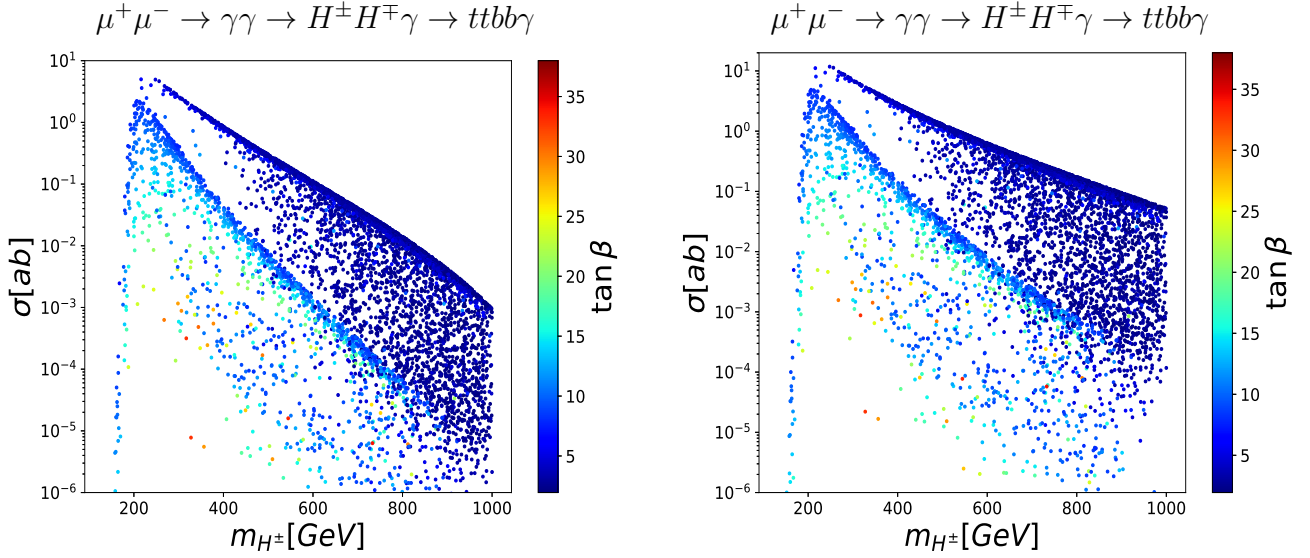


Figure 8: Cross-sections for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow ttbb\gamma$ are generated in the allowed parameter spaces of Type-X THDM at $\sqrt{s} = 3$ TeV in the left panel and at $\sqrt{s} = 5$ TeV in the right panel.

The production process $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow WWhh\gamma$ is investigated within the parameter space of the Type-X THDM, as shown in Fig. 9. In this channel, the cross sections are rather small. As a result, only a very small number of events are expected to be detected via $\gamma\gamma$ -fusion at future multi-TeV muon colliders.

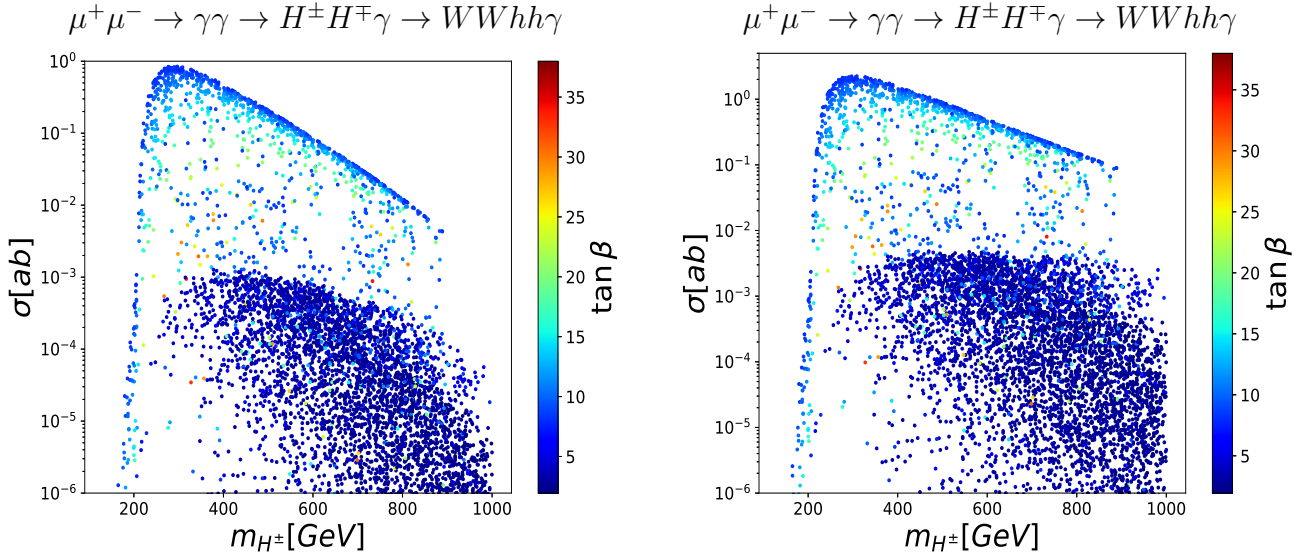


Figure 9: Cross-sections for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow WWhh\gamma$ are generated in the allowed parameter spaces of THDM type-X for $\sqrt{s} = 3$ TeV in the left panel and for $\sqrt{s} = 5$ TeV in the right panel.

Other production cross sections for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp Z$ at $\sqrt{s} = 3$ TeV (on the left panel) and at (on the right panel) are also computed in the parameter space of the Type-X THDM. The results are presented together with the branching ratios of decay modes such as $H^\pm \rightarrow tb$, Wh . In particular, the results are shown for the decay mode $H^\pm \rightarrow tb$ in Fig. 10 and

$H^\pm \rightarrow Wh$ in Fig. 11. The cross section are also very small. We expect to get few events of this proceses at future multi-TeV muon colliders.

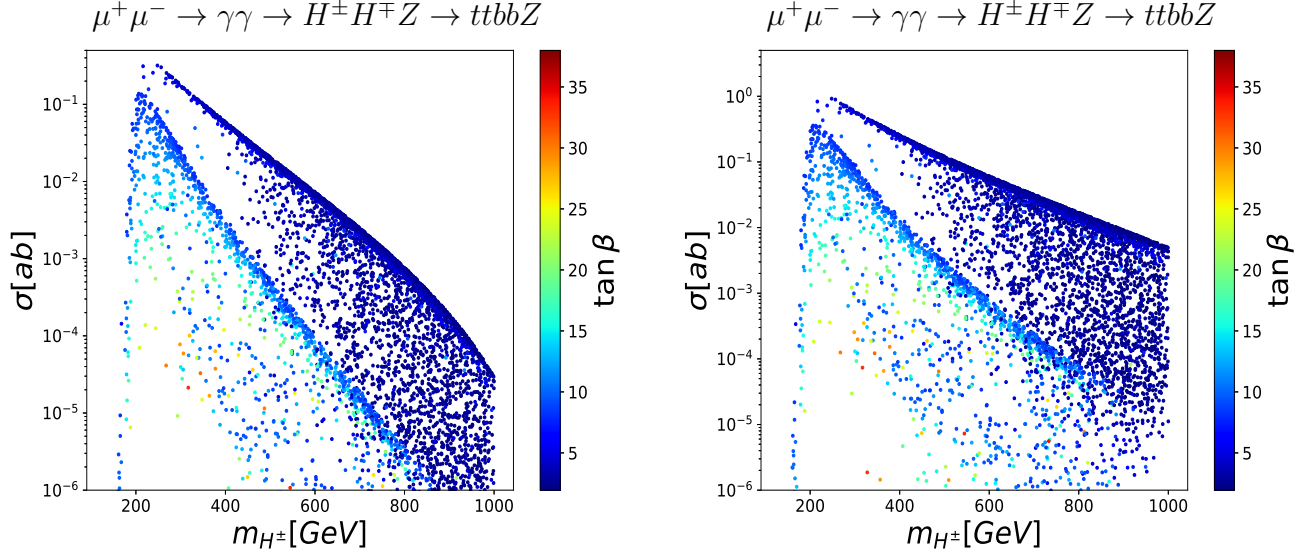


Figure 10: Cross-sections for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp Z \rightarrow t\bar{t}b\bar{b}Z$ are genrated in the allowed parameter spaces of THDM type-X for $\sqrt{s} = 3$ TeV in the left panel and for $\sqrt{s} = 5$ TeV in the right panel.

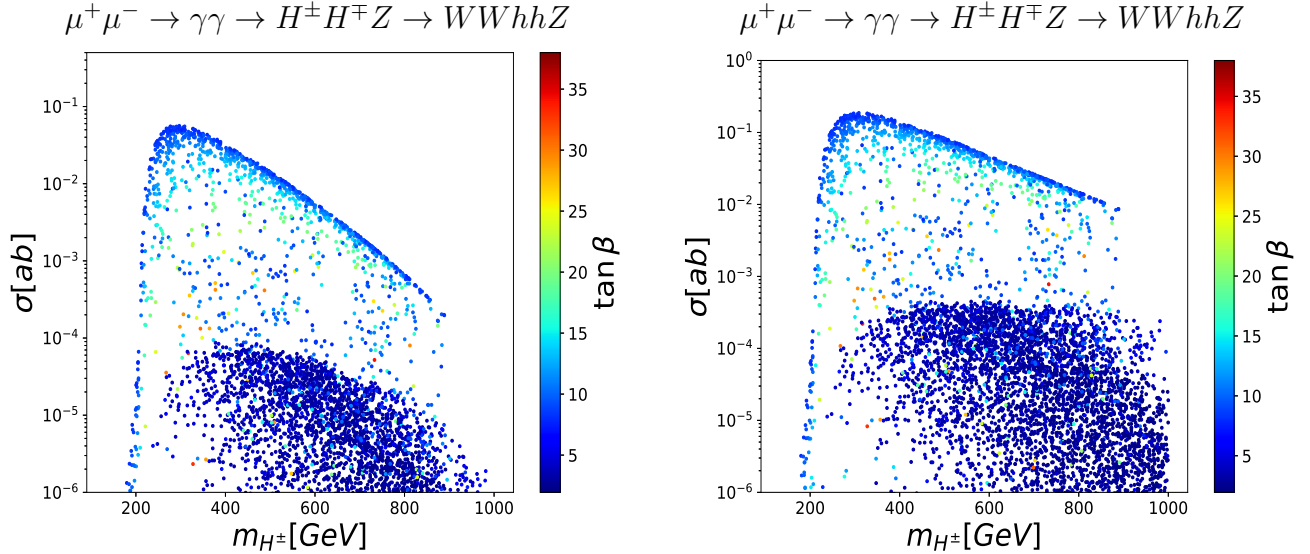


Figure 11: Cross-sections for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp Z \rightarrow WWZhZ$ are genrated in the allowed parameter spaces of THDM type-X for $\sqrt{s} = 3$ TeV in the left panel and for $\sqrt{s} = 5$ TeV in the right panel.

5. Signal significance for $H^\pm H^\mp V$ at future multi-TeV muon colliders

In this section, we compute the signal significances for $H^\pm H^\mp V$ production at future multi-TeV muon colliders. Since the cross sections for $\mu^+\mu^- \rightarrow H^\pm H^\mp V$ are greater than the corresponding ones through $\gamma\gamma$ -fusion. Additionally, both decay modes $H^\pm \rightarrow tb$ and $H^\pm \rightarrow Wh$ are

dominant in comparison with the other decay channels of the charged Higgs. For these reasons, we are interested in evaluating the significances of $\mu^+\mu^- \rightarrow H^\pm H^\mp V \rightarrow t\bar{t}bV$, $WhWhV$ in this section. The significances are evaluated for several benchmark points, marked as star symbols in the figures presented in the previous sections. These benchmark points are chosen from the regions where the cross sections are relatively large. The four selected benchmark points are summarized in Table below 1:

BP	M_H	Γ_H^{Total}	M_A	$M_{H^\pm}$	$s_{\beta-\alpha}$	$\tan\beta$	m_{12}^2
BP1	342.348	0.1538	348.134	256.356	0.9992	4.12749	23726.8
BP2	551.633	0.3839	678.245	506.019	0.999909	6.45338	44594.2
BP3	261.685	0.2755	731.413	256.640	0.970965	8.01807	7925.15
BP4	531.033	6.6749	619.362	504.104	0.988142	12.9283	21501.0

Table 1: Benchmark points used in the analysis. Masses and decay widths are in GeV.

Before presenting the signal significances for $H^\pm H^\mp V$ production at future multi-TeV muon colliders, we know that the initial-state radiation (ISR) plays an important role in the high-energy regime of such machines. To account for the ISR effects, we also compute the corresponding ISR corrections to the considered processes. The corrected cross sections, including ISR effects for $\mu^+\mu^- \rightarrow H^\pm H^\mp V$, are given as follows:

$$\sigma_{\text{ISR}}^{\mu^+\mu^- \rightarrow H^\pm H^\mp V}(s) = \int_0^1 dx H(x, s) \sigma_{\text{Tree}}^{\mu^+\mu^- \rightarrow H^\pm H^\mp V}(s(1-x)), \quad (4)$$

where s stands for the center-of-mass energy and x is for the energy fraction of an emitted photon from muon. The radiator function is taken in [46]

$$H(x, s) = \Delta_2 \beta x^{\beta-1} - \Delta_1 \beta \left(1 - \frac{x}{2}\right) + \frac{\beta^2}{8} \left[-4(2-x) \log x - \frac{1+3(1-x)^2}{x} \log(1-x) - 2x \right]. \quad (5)$$

In the formulas, we have

$$\begin{aligned} \beta &= \frac{2\alpha}{\pi} \left(\ln \frac{s}{m_\mu^2} - 1 \right), \quad \Delta_1 = 1 + \delta_1, \quad \Delta_2 = 1 + \delta_1 + \delta_2, \\ \delta_1 &= \frac{\alpha}{\pi} \left(\frac{3}{2} \ln \frac{s}{m_\mu^2} + \frac{\pi^2}{3} - 2 \right), \quad \delta_2 = \left(\frac{\alpha}{\pi} \ln \frac{s}{m_\mu^2} \right)^2 \left(-\frac{1}{18} \ln \frac{s}{m_\mu^2} + \frac{119}{72} - \frac{\pi^2}{3} \right). \end{aligned}$$

Following the perturbative calculations of initial-state photon emission diagrams up to two-loop order [46], in which the collinear approximation and the collinear logarithmic enhancement expansions are applied, we note that the terms proportional to α^2 in Eq. (6) correspond to the two-loop contributions.

We first study the effects of ISR corrections on the production cross sections. For this examination, we use the following benchmark point BP1 given in Table 1, as a typical example. In left panel of Fig. 12, cross sections are shown with respect to the center-of-mass energy from $\sqrt{s} = 650$ GeV to $\sqrt{s} = 6$ TeV. While cross section at $\sqrt{s} = 3000$ GeV with varying

charged Higgs masses from 180 GeV to 1000 GeV are studied in the right plot. In these Figures, the red line represents the tree-level cross sections without ISR corrections, while the blue line corresponds to the tree-level cross sections including ISR corrections. The corrections range up to approximately -10% contributions.

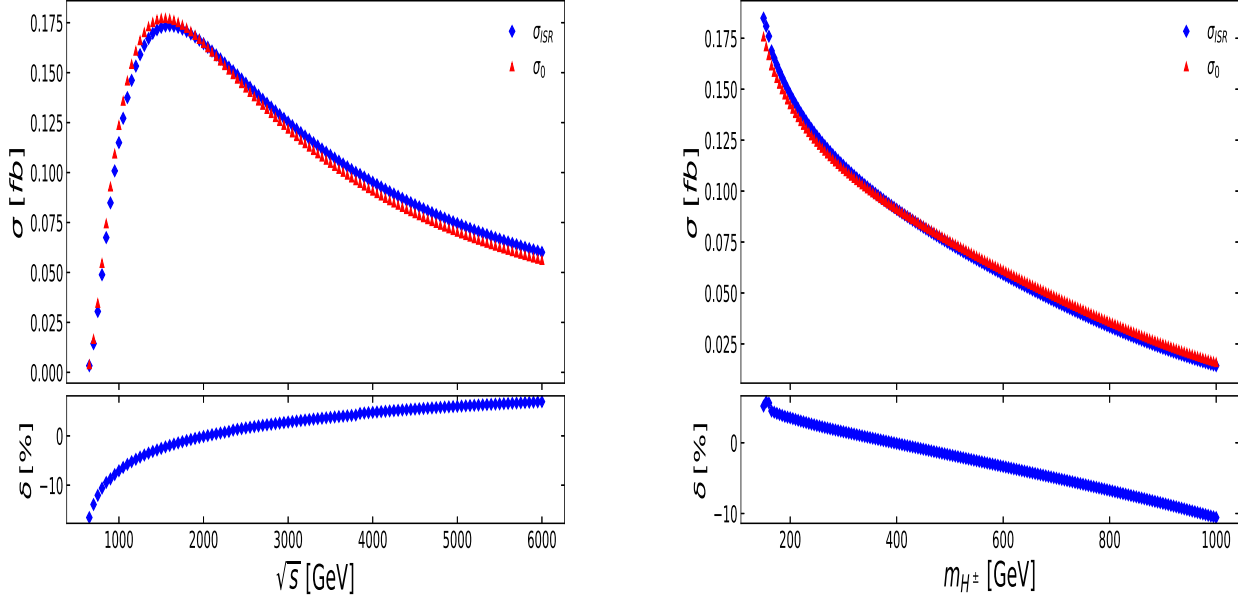


Figure 12: Effects of ISR on the production cross sections for $\mu^+\mu^- \rightarrow H^\pm H^\mp Z$ as functions of $\sqrt{s}$ (left) and fix $\sqrt{s} = 3$ TeV with varying $m_{H^\pm}$ (right).

We now discuss the electroweak corrections to both the signal processes and the corresponding Standard Model backgrounds. Based on the approximate Sudakov logarithmic behavior, the electroweak corrections can be estimated in the high-energy regime as follows [50]:

$$\delta_{\text{EW}} \approx -\frac{\alpha}{4\pi \sin^2 \theta_W} \kappa \ln\left(\frac{s}{M_W^2}\right). \quad (6)$$

The effective coefficient is taken as $\kappa = 2, 3, \dots$ for scattering processes of the type $2 \rightarrow 2$, $2 \rightarrow 3$, and so on. By taking the following input parameters, one can estimate electroweak corrections about 10% contributions at $\sqrt{s} = 3$ TeV.

For the SM background, we use MadGraph5MC@NLO [51, 52] for generating the process $\mu^-\mu^+ \rightarrow t\bar{t}b\bar{b}V$, $WWhhV$ with $V = \gamma, Z$. In order to reduce the SM background, we apply the following cuts:

$$p_T(b) \geq 20\text{GeV}, \quad |\eta_b| \leq 2.4 \quad p_T(\bar{b}) \geq 20\text{GeV}, \quad |\eta_{\bar{b}}| \leq 2.4. \quad (7)$$

Furthermore, one applies $E_\gamma \geq 5$ GeV and $|\cos \theta_\gamma| < 0.98$ for external photon.

In this work, the significances are evaluated as follows:

$$\mathcal{S} = \frac{N_S}{\sqrt{N_S + \varepsilon_B N_B}}. \quad (8)$$

Where $N_{S/B} = \mathcal{L}\sigma_{S/B}$ is corresponding for number of event and background. Where ε_B explains for systematic uncertainty fraction on the background yield. We take $\varepsilon_B = 1, 1.5$ in this work.

In Fig. 13, we present the signal significance for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow tbtb\gamma$ at the benchmark points BP1 (left panel) and BP2 (right panel). The significances are computed by varying the center-of-mass energy from 2 TeV to 6 TeV and for different integrated luminosities of $\mathcal{L} = 500$ (yellow line), $\mathcal{L} = 1000$ (green line) and $\mathcal{L} = 3000 \text{ fb}^{-1}$ (red line). In the Figures, the solid line corresponds to $\varepsilon_B = 1$, while the dashed line represents $\varepsilon_B = 1.5$, respectively. Our finding is that significances are above 5σ for all ranges of center-of-mass energies. Moreover, the signal significance reaches its maximum at $\sqrt{s} = 2 \text{ TeV}$ and decreases as the center-of-mass energy increases.

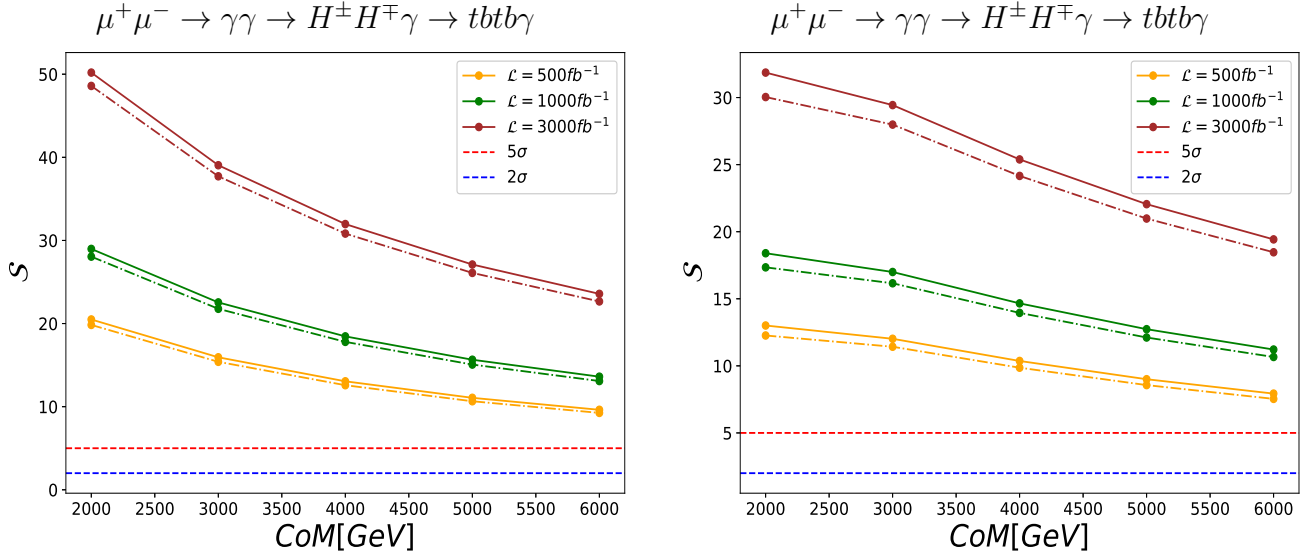


Figure 13: Signal significance for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow tbtb\gamma$ at the benchmark points BP1 (left panel) and BP2 (right panel).

In Fig. 14, we examine the signal significance for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow WWhh\gamma$ at the benchmark points BP3 (left panel) and BP4 (right panel). We use the same notations for lines presented in these Figures as previous cases. In general, we observe that the signal significances decrease with increasing center-of-mass energy. For BP3, the significance exceeds 5σ when $\sqrt{s} \leq 4 \text{ TeV}$. Across all energies considered, BP4 remains observable with a significance above 5σ .

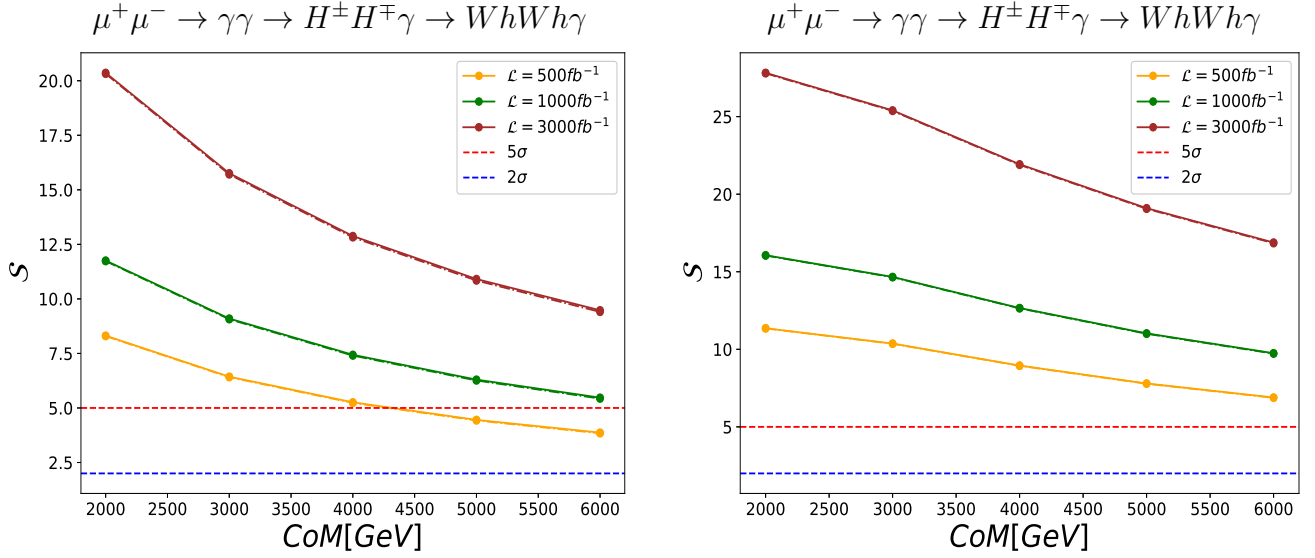


Figure 14: Signal significance for $\mu^+\mu^- \rightarrow \gamma\gamma \rightarrow H^\pm H^\mp \gamma \rightarrow WhWh\gamma$ at the benchmark points BP3 (left panel) and BP4 (right panel).

The significances for the process $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow t\bar{t}b\bar{b}Z$ are investigated at several benchmark points indicated in the previous sections. Figure 15 shows the signal significances for this process at the benchmark point BP1 (left) and BP2 (right). In the left panel, the results show that the production can be observed with a significance exceeding 5σ for integrated luminosities of $\mathcal{L} \geq 1000 \text{ fb}^{-1}$ for all energy ranges. For $\mathcal{L} = 500 \text{ fb}^{-1}$, the signal can be tested with a significance above 5σ when $\sqrt{s} \leq 4 \text{ TeV}$. At other ranges of center-of-mass energies, the production is difficult to test at future muon–TeV colliders because its cross section is smaller. In the right panel plot, the result indicates that the production can be probed with a significance exceeding 5σ for integrated luminosities of $\mathcal{L} = 3000 \text{ fb}^{-1}$ for all energy ranges. It is stressed that all notations for the lines shown in these Figures are the same as in the previous cases.

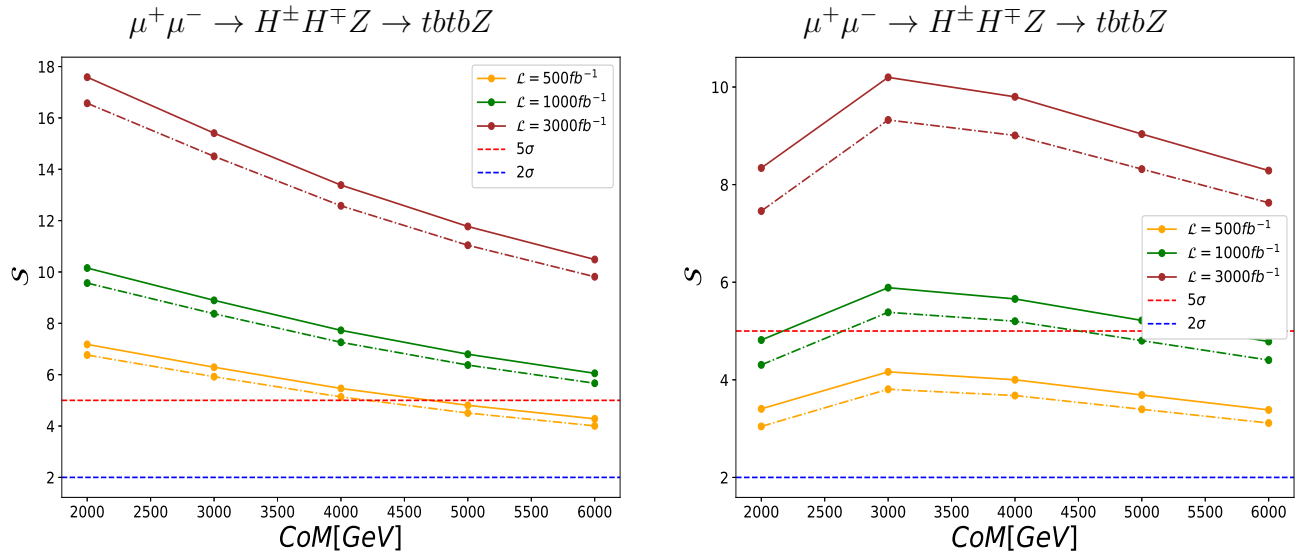


Figure 15: The signal significances of the process $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow t\bar{t}b\bar{b}Z$ are studied for the benchmark points BP1 (left) and BP2 (right).

Finally, we consider the significance for the production process $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow WWhhZ$ at future muon–TeV colliders. The signal significances are examined for two benchmark points, as shown in the scatter plots presented in the previous Figures. In the left plot, the significance for this process at BP3 is shown. It is found that the signal can exceed 5σ for $\mathcal{L} = 3000 \text{ fb}^{-1}$ in the energy range from 2 to 4 TeV. In other cases, it is difficult to test the production. In the right plot, the significance for this process at BP4 is presented. The significance is enhanced in the low-energy region, exceeding 5σ over the entire energy range for $\mathcal{L} \geq 1000 \text{ fb}^{-1}$. Again, we use the same labels for the lines as in previous plots.

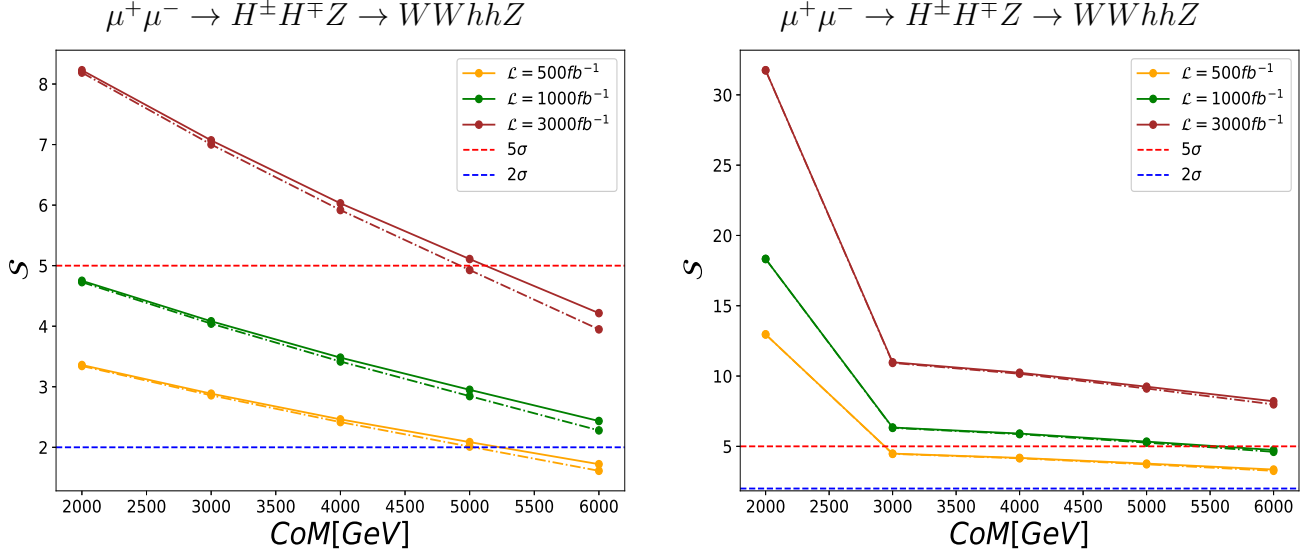


Figure 16: The signal significances of the process $\mu^+\mu^- \rightarrow H^\pm H^\mp Z \rightarrow WWhhZ$ are studied for the benchmark points BP3 (left) and BP4 (right).

6. Conclusions

Charged Higgs pair production associated with neutral gauge bosons at future multi–TeV muon colliders is first presented within the framework of THDM in this paper. In the phenomenological results for the production processes, the parameter space of the Type-X THDM is updated in this work. From the viable parameter regions for the model under consideration, we calculate the production cross sections, including all two-body decay modes of the charged Higgs boson. It is worth emphasizing that all two-body decays of the charged Higgs include one-loop contributions, with two-loop QCD corrections also included for decays into two fermions. The signal significances are also examined for several benchmark points within the updated parameter space. It is worth stressing that the production cross sections used to compute the significances in this work include initial-state radiation corrections up to two-loop order. Thanks to the high integrated luminosity planned at future multi–TeV muon colliders, the signals are expected to be observable with a statistical significance of up to 5σ .

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Process $\mu^+\mu^- \rightarrow H^\pm H^\mp V$ for $V = \gamma, Z$

Tree-level Feynman diagrams are generated using **FeynArts/FormCalc**. For the process $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma$, the external Z boson is replaced by an external photon. Subsequently,

diagrams 7 and 8 are omitted. For generality, we show the dominant Feynman diagrams for $\mu^+\mu^- \rightarrow H^\pm H^\mp Z$. It is emphasized that all Feynman diagrams are retained in our calculations. However, to present the amplitude expressions in a compact form, we only display the amplitudes in the limit $m_\mu \rightarrow 0$ in the formulas below.

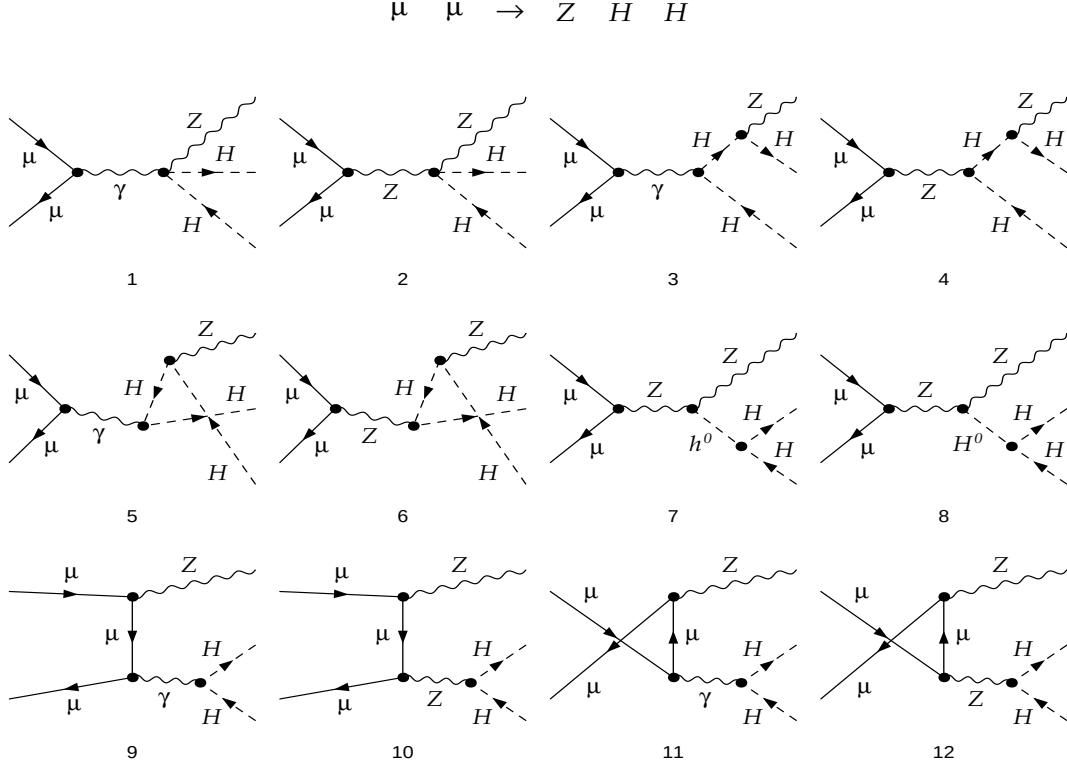


Figure 17: Tree-level Feynman diagrams are generated by `FeynArts/FormCalc`. For the process $\mu^+\mu^- \rightarrow H^\pm H^\mp \gamma$, we replace the external Z boson to external photon. Subsequently, we eliminate diagrams 7 and 8.

Amplitude for the process is presented as follows:

$$i\mathcal{M} = \sum_{k=1}^8 F_k \cdot \langle v|P_k|u \rangle. \quad (9)$$

The tree-level form factor F_k are functions of kinematic variables as: $s = s_{12} = (k_1 + k_2)^2$, $s_{34} = (k_3 + k_4)^2$, $t = t_{13} = (k_1 - k_3)^2$, $t_{14} = (k_1 - k_4)^2$, $t_{24} = (k_2 - k_4)^2$, $u = u_{23} = (k_2 - k_3)^2$. The detail expressions for form factor F_k is given by:

$$\begin{aligned} F_1 = & \frac{eg_{H^\pm H^\mp \gamma V}}{s_{12}} - \frac{g_{Vff}^L g_{H^\pm H^\mp V}}{s_{12} - m_Z^2} \\ & - \frac{(g_{Vff}^L)^2 g_{H^\pm H^\mp V}}{(t_{13} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - t_{13})}{(3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24})} \\ & + \frac{e^2 g_{Vff}^L}{(t_{13} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - t_{13})}{(3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24})} \\ & + \frac{g_{Vff}^L}{(s_{12} - m_Z^2)} \sum_{\phi_k=h,H} \frac{g_{\phi_k H^\pm H^\mp} g_{\phi_k VV}}{(m_{\phi_k}^2 - 3m_{H^\pm}^2 - 2m_\mu^2 - m_Z^2 + s_{34} + t_{14} + t_{24})} \end{aligned}$$

$$\begin{aligned}
& + \frac{(g_{Vff}^L)^2 g_{H^\pm H^\mp V}}{(u_{23} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - u_{23})}{(3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24})} \\
& - \frac{e^2 g_{Vff}^L}{(u_{23} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - u_{23})}{(3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24})}, \tag{10}
\end{aligned}$$

$$\begin{aligned}
F_2 &= \frac{e g_{H^\pm H^\mp \gamma V}}{s_{12}} - \frac{(g_{Vff}^R)^2}{(t_{13} - m_\mu^2)} \frac{g_{H^\pm H^\mp V} (m_{H^\pm}^2 + m_Z^2 - s_{34} - t_{13})}{(3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24})} \\
& - \frac{g_{Vff}^R g_{H^\pm H^\mp V}}{s_{12} - m_Z^2} + \frac{e^2 g_{Vff}^R}{(t_{13} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - t_{13})}{(3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24})} \\
& + \frac{g_{Vff}^R}{(s_{12} - m_Z^2)} \sum_{\phi_k=h,H} \frac{g_{\phi_k H^\pm H^\mp} g_{\phi_k VV}}{(m_{\phi_k}^2 - 3m_{H^\pm}^2 - 2m_\mu^2 - m_Z^2 + s_{34} + t_{14} + t_{24})} \\
& + \frac{(g_{Vff}^R)^2 g_{H^\pm H^\mp V}}{(u_{23} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - u_{23})}{(3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24})} \\
& - \frac{e^2 g_{Vff}^R}{(u_{23} - m_\mu^2)} \frac{(m_{H^\pm}^2 + m_Z^2 - s_{34} - u_{23})}{(3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24})}, \\
F_3 &= \frac{4 g_{H^\pm H^\mp V}^R g_{Vff}^R (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2) (s_{12} - m_Z^2)} - \frac{4e^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2) s_{12}} \\
& - \frac{4(g_{Vff}^R)^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_1)}{(t - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4e^2 g_{Vff}^R (\varepsilon^*(k_3) \cdot k_1)}{(t - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{24} - t_{14})} \\
& - \frac{4(g_{Vff}^R)^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_2)}{(u - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4e^2 g_{Vff}^R (\varepsilon^*(k_3) \cdot k_2)}{(u - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_5) [e^2 (s - m_Z^2) - g_{Vff}^R g_{H^\pm H^\mp V} s]}{s (s - m_Z^2) (m_{H^\pm}^2 + 2m_\mu^2 + 2m_Z^2 - s_{34} - t - u)}, \\
F_4 &= \frac{4g_{H^\pm H^\mp V}^R g_{Vff}^L (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2) (s - m_Z^2)} - \frac{4e^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2) s} \\
& - \frac{4(g_{Vff}^L)^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_1)}{(t - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4e^2 g_{Vff}^L (\varepsilon^*(k_3) \cdot k_1)}{(t - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{24} - t_{14})} \\
& - \frac{4(g_{Vff}^L)^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_2)}{(u - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4e^2 g_{Vff}^L (\varepsilon^*(k_3) \cdot k_2)}{(u - m_\mu^2) (3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{24} - t_{14})} \\
& + \frac{4g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_5) [e^2 (s - m_Z^2) - g_{Vff}^L g_{H^\pm H^\mp V} s]}{s (s - m_Z^2) (m_{H^\pm}^2 + 2m_\mu^2 + 2m_Z^2 - s_{34} - t - u)}, \tag{11}
\end{aligned}$$

$$\begin{aligned}
F_5 &= \frac{4g_{H^\pm H^\mp V}^2 g_{Vff}^L (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2)(s - m_Z^2)} - \frac{4e^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2)s} \\
&\quad + \left(\frac{\varepsilon^*(k_3) \cdot k_4}{u - m_\mu^2} - \frac{\varepsilon^*(k_3) \cdot k_4}{t - m_\mu^2} \right) \times \\
&\quad \times \left[\frac{2(g_{Vff}^L)^2 g_{H^\pm H^\mp V}}{3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24}} - \frac{2e^2 g_{Vff}^L}{3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24}} \right], \\
F_6 &= \frac{4g_{H^\pm H^\mp V}^2 g_{Vff}^R (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2)(s - m_Z^2)} - \frac{4e^2 g_{H^\pm H^\mp V} (\varepsilon^*(k_3) \cdot k_4)}{(s_{34} - m_{H^\pm}^2)s} \\
&\quad + \left(\frac{\varepsilon^*(k_3) \cdot k_4}{u - m_\mu^2} - \frac{\varepsilon^*(k_3) \cdot k_4}{t - m_\mu^2} \right) \times \\
&\quad \times \left[\frac{2(g_{Vff}^R)^2 g_{H^\pm H^\mp V}}{3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24}} - \frac{2e^2 g_{Vff}^R}{3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24}} \right], \\
F_7 &= \left(\frac{1}{u - m_\mu^2} + \frac{1}{t - m_\mu^2} \right) \times \\
&\quad \times \left[\frac{2(g_{Vff}^R)^2 g_{H^\pm H^\mp V}}{3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24}} - \frac{2e^2 g_{Vff}^R}{3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24}} \right], \\
F_8 &= \left(\frac{1}{u - m_\mu^2} + \frac{1}{t - m_\mu^2} \right) \times \\
&\quad \times \left[\frac{2(g_{Vff}^L)^2 g_{H^\pm H^\mp V}}{3m_{H^\pm}^2 + 2m_\mu^2 - s_{34} - t_{14} - t_{24}} - \frac{2e^2 g_{Vff}^L}{3m_{H^\pm}^2 + 2m_\mu^2 + m_Z^2 - s_{34} - t_{14} - t_{24}} \right], \quad (12)
\end{aligned}$$

The operators P_k are also taken the form of $P_1 = P_L \not{\varepsilon}^*(k_3)$, $P_2 = P_R \not{\varepsilon}^*(k_3)$, $P_3 = P_R \not{k}_4$, $P_4 = P_L \not{k}_4$, $P_5 = P_L \not{k}_3$, $P_6 = P_R \not{k}_3$, $P_7 = -P_R \not{\varepsilon}^*(k_3) \not{k}_3 \not{k}_4$, $P_8 = -P_L \not{\varepsilon}^*(k_3) \not{k}_3 \not{k}_4$. Here, $\varepsilon^*(k_3)$ is polarization vector of external vector V . When we consider $V = \gamma$, we take $g_{H^\pm H^\mp \gamma V} = g_{H^\pm H^\mp \gamma \gamma}$, $g_{H^\pm H^\mp V} = g_{H^\pm H^\mp \gamma}$ and $g_{hZV} = g_{HZV} = 0$. One then arrives at the corresponding amplitude for the process $\mu^+ \mu^- \rightarrow H^\pm H^\mp \gamma$. Cross section is calculated:

$$\sigma(s) = \frac{1}{\text{flux}} \int d\Phi_3 \frac{1}{4} \sum_{\text{spin}} |\mathcal{M}_{\mu^- \mu^+ \rightarrow H^\pm H^\mp V}|^2. \quad (13)$$

Partonic process $\gamma\gamma \rightarrow H^\pm H^\mp V$ for $V = \gamma, Z$

Tree-level Feynman diagrams for the process $\gamma\gamma \rightarrow H^\pm H^\mp V$ with $V = \gamma, Z$ are presented in Fig. 18. It is noted that we take $V = Z$ for generality.

$$\gamma \quad \gamma \rightarrow Z \quad H \quad H$$

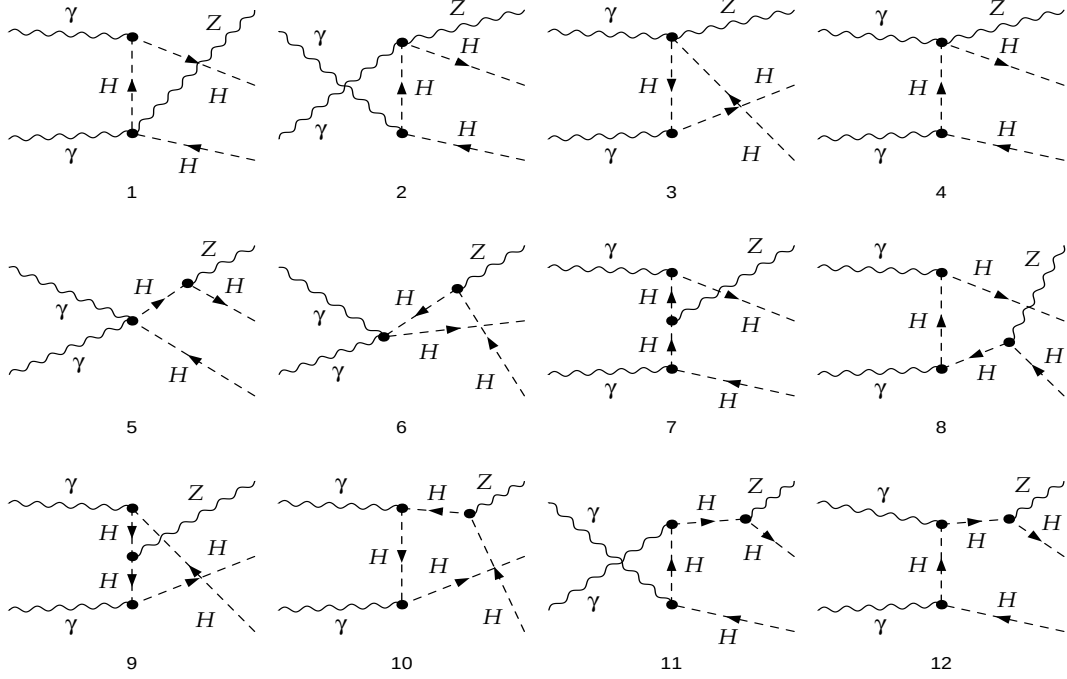


Figure 18: Tree-level Feynman diagrams are generated by **FeynArts/FormCalc**. The partonic process $\gamma\gamma \rightarrow H^\pm H^\mp \gamma$ will be obtained by approximately replacing the external Z by a photon.

Tree-level amplitude for partonic process $\gamma\gamma \rightarrow H^\pm H^\mp V$ is expressed as follows:

$$\begin{aligned}
 i\mathcal{M} = & \frac{2(\varepsilon(k_1) \cdot k_5)}{m_{H^\pm}^2 + m_Z^2 - s_{12} - t_{13} - t_{14}} \times \\
 & \times \left[e g_{H^\pm H^\mp V \gamma} \cdot (\varepsilon(k_2) \cdot k_5) - c_1 \frac{(4e^2) g_{H^\pm H^\mp V}}{s_{34} - m_{H^\pm}^2} \cdot (\varepsilon^*(k_3) \cdot k_4) \right] \\
 & - (2e) g_{H^\pm H^\mp V \gamma} \left[\frac{(\varepsilon(k_1) \cdot \varepsilon^*(k_3)) (\varepsilon(k_2) \cdot k_5)}{t_{14} - m_{H^\pm}^2} + \frac{(\varepsilon(k_1) \cdot k_4) (\varepsilon(k_2) \cdot \varepsilon^*(k_3))}{t_{24} - m_{H^\pm}^2} \right] \\
 & + \frac{2(\varepsilon(k_2) \cdot k_4)}{m_{H^\pm}^2 + m_Z^2 - s_{12} - t_{24} - u_{23}} \times \\
 & \times \left[e g_{H^\pm H^\mp V \gamma} (\varepsilon(k_1) \cdot k_4) - (4e^2) g_{H^\pm H^\mp V} \left(c_2 \frac{\varepsilon^*(k_3) \cdot k_4}{s_{34} - m_{H^\pm}^2} + c_6 \frac{\varepsilon(k_1) \cdot \varepsilon^*(k_3)}{t_{14} - m_{H^\pm}^2} \right) \right] \\
 & + (4e^2) g_{H^\pm H^\mp V} \times \\
 & \times \left[\frac{(\varepsilon(k_1) \cdot \varepsilon(k_2)) (\varepsilon^*(k_3) \cdot k_4)}{m_{H^\pm}^2 - s_{34}} + c_4 \frac{2(\varepsilon(k_2) \cdot \varepsilon^*(k_3)) (\varepsilon(k_1) \cdot k_5)}{(m_{H^\pm}^2 + m_Z^2 - s_{12} - t_{13} - t_{14}) (m_{H^\pm}^2 - t_{24})} \right. \\
 & \left. - \frac{2(\varepsilon^*(k_3) \cdot k_5)}{s_{34} + t_{13} + u_{23} - m_{H^\pm}^2 - 2m_Z^2} \times \right. \\
 & \left. \times \left(\frac{\varepsilon(k_1) \cdot \varepsilon(k_2)}{2} - c_3 \frac{\varepsilon(k_1) \cdot \varepsilon^*(k_3)}{m_{H^\pm}^2 - t_{14}} - c_5 \frac{\varepsilon(k_2) \cdot \varepsilon^*(k_3)}{m_{H^\pm}^2 - t_{24}} \right) \right].
 \end{aligned} \tag{14}$$

The polarization vectors of incoming photons are $\varepsilon(k_1)$, $\varepsilon(k_2)$ and of external vector V is $\varepsilon^*(k_3)$. In the amplitude, corresponding coefficients are given by

$$c_1 = \varepsilon(k_2) \cdot k_1 - \varepsilon(k_2) \cdot k_4, \tag{15}$$

$$c_2 = \varepsilon(k_1) \cdot k_2 - \varepsilon(k_1) \cdot k_5, \quad (16)$$

$$c_3 = \varepsilon^*(k_3) \cdot k_1 - \varepsilon^*(k_3) \cdot k_5, \quad (17)$$

$$c_4 = \varepsilon(k_2) \cdot k_1 - \varepsilon(k_2) \cdot k_5, \quad (18)$$

$$c_5 = \varepsilon^*(k_3) \cdot k_1 - \varepsilon^*(k_3) \cdot k_4, \quad (19)$$

$$c_6 = \varepsilon(k_1) \cdot k_2 - \varepsilon(k_1) \cdot k_4. \quad (20)$$

In the case of $V = \gamma$, we set the approximately couplings $g_{H^\pm H^\mp V} = g_{H^\pm H^\mp \gamma} = e$ and $g_{H^\pm H^\mp V \gamma} = g_{H^\pm H^\mp \gamma \gamma} = 2e^2$. Cross section is calculated by

$$\sigma(s) = \frac{1}{\text{flux}} \int d\Phi_3 \frac{1}{4} \sum_{\text{pol.}} |\mathcal{M}_{\gamma\gamma \rightarrow H^\pm H^\mp V}|^2. \quad (21)$$

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