Chiral Evolution and Femtoscopic Signatures of the $K_1(1270)$ Resonance

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We present a comprehensive study of the axial-vector resonance $K_1(1270)$ within the unitarized chiral perturbation theory, focusing on its two-pole structure and manifestation in femtoscopic observables. By considering the dominant ρK and $K^*\pi$ coupled channels, we reproduce the well-established double-pole structure and trace the chiral evolution of both poles as functions of the pion mass, using the vector-meson mass trajectories fitted to lattice-QCD data and experimental values. The lower pole, dominantly coupled to $K^*\pi$, evolves from an above-threshold resonance to a virtual or bound state with increasing pion mass. In comparison, the higher pole, dominantly coupled to ρK, moves downward in energy, reflecting the strengthening of the chiral attraction. The influence of the finite vector-meson widths is systematically examined, showing that their inclusion smooths the pole trajectories without altering their qualitative behavior. Furthermore, femtoscopic CFs are calculated for all relevant vector-pseudoscalar channels in both charged sectors. The results exhibit distinct resonance and bound-state features consistent with the two-pole dynamics. The weak impact of higher channels, such as $\omega\bar{K}$, $\bar{K}^*\eta$, and $\phi\bar{K}$, confirms that the simplified two-channel treatment captures the essential dynamics of the $K_1(1270)$ resonance. This study demonstrates that combining chiral extrapolation and femtoscopic correlation analyses provides a powerful and complementary framework for connecting lattice-QCD calculations, chiral effective theory, and experimental measurements, offering new insights into the molecular nature and chiral origin of the $K_1(1270)$ resonance.

I. INTRODUCTION

Understanding the internal structure of hadronic resonances remains one of the most challenging problems in nonperturbative QCD [1–13]. Among them, the low-lying axial-vector mesons, such as $a_1(1260)$, $f_1(1285)$, and $K_1(1270)$ [14, 15], provide an essential probe into chiral symmetry breaking and the dynamics of vector-pseudoscalar (VP) interactions. In the conventional quark model [16], these resonances are described as the $L = 1$ orbital excitations of $q\bar{q}$ states, forming the 1^3P_1 and 1^1P_1 nonets. The strangeness partners $K_1(1270)$ and $K_1(1400)$ are usually interpreted as mixtures of the SU(3) eigenstates K_{1A} $(C = +1)$ and K_{1B} $(C = -1)$ due to SU(3)-flavor symmetry breaking.

However, certain long-standing experimental inconsistencies—such as reaction-dependent line shapes, channel-dependent branching ratios ($K^*\pi$ vs. ρK), and interference with $K_1(1400)$—challenge this simple mixing picture [17]. The coexistence of multiple peaks in diffractive and non-diffractive Kp reactions suggests that the internal structure of $K_1(1270)$ is more complex than a single Breit-Wigner resonance [11].

The development of unitarized chiral perturbation theory (UChPT) provides a compelling alternative [18–24]. By unitarizing chiral VP interactions [15, 25], UChPT dynamically generates axial-vector states as hadronic molecules rather than conventional $q\bar{q}$ excitations. Notably, UChPT [15] predicted two nearby poles associated with $K_1(1270)$, originating from the coupled-channel dynamics among $K^*\pi$, ρK, ωK, $K^*\eta$, and ϕK. The analysis of the high-statistics WA3 data on $K^-p \to K^-\pi^+\pi^-p$ [17] further supports the existence of two poles: a lower one around $1195 - 123i$ MeV dominantly coupling to $K^*\pi$, and a higher one around $1284 - 73i$ MeV mainly coupling to ρK. This provided strong evidence that $K_1(1270)$ is a superposition of two dynamically generated states rather than a simple K_{1A}–K_{1B} mixture.

Subsequent studies reinforced this picture. Wang et al. [26] analyzed $D^0 \to \pi^+VP$ decays, showing that different final states manifest different poles—$K^*\pi$ the lower one, and ρK the higher. Later, their study of semileptonic $D^+ \to \nu e^+VP$ decays [27] confirmed this channel dependence. Dias et al. [28] extended the analysis to $\bar{B} \to J/\psi\rho\bar{K}$ and $\bar{B} \to J/\psi\bar{K}^*\pi$, finding both poles in the line shapes of the ρK and $K^*\pi$ spectra. Roca et al. [29] further showed that the PDG branching ratios for $K_1(1270) \to \pi K_0^*(1430)$ [30] are inconsistent with a single-state interpretation, reinforcing the two-pole picture.

Together, these theoretical and phenomenological studies establish a coherent framework in which the $K_1(1270)$ resonance emerges as two dynamically generated poles arising from vector-pseudoscalar interactions—an analogue of the well-known two-pole nature of the $\Lambda(1405)$ [31–36]. Despite these advances, direct experimental confirmation remains elusive, and the dependence of the pole structure on light-quark masses—crucial for connecting lattice QCD simulations to experiments—has yet to be systematically explored [37–40].

Meanwhile, the two-particle femtoscopic correlation technique has become a quantitative and model-sensitive probe of hadron-hadron interactions at low relative momenta [13, 41–58]. Femtoscopy provides direct access to scattering lengths, effective ranges, and near-threshold resonance structures that are otherwise unreachable in conventional scattering experiments. Kamiya et al. [41] achieved a major step forward by developing a coupled-channel femtoscopic framework for the K^-p system considering the $\bar{K}N - \pi\Sigma - \pi\Lambda$ dynamics based on chiral SU(3) theory. Their results, including Coulomb and coupled-channel effects, successfully reproduced the ALICE K^-p correlation data, providing stringent empirical constraints on the $\bar{K}N$ interaction responsible for the $\Lambda(1405)$. Following this, Liu et al. [46] extended the approach to the DK system, showing that theoretically predicted correlation functions (CFs) are consistent with interpreting the $D^*_{s0}(2317)$ as a hadronic molecule dynamically generated from the DK interaction, and highlighting that future femtoscopic measurements could directly test this scenario.

Given these developments, applying the femtoscopic framework to the VP interactions offers a natural and timely opportunity to probe the exotic resonance $K_1(1270)$. Since direct ρK and $K^*\pi$ scattering measurements are experimentally inaccessible due to the short lifetimes of vector mesons, information on $K_1(1270)$ must be inferred from the $K\pi\pi$ invariant-mass spectra [17, 26, 27], where the coupled VP subsystems act as intermediate states. In contrast, femtoscopy provides a direct means of accessing the underlying two-body interaction kernels via near-threshold correlation-function observables. A systematic study of VP femtoscopic CFs across different channels can thus provide crucial constraints on the two-pole dynamics of $K_1(1270)$ and yield new insights into its internal structure and chiral origin.

In this work, we present a quantitative study of the $K_1(1270)$ resonance within the two-pole scenario, combining chiral extrapolation and femtoscopic CF analyses. First, we employ UChPT to trace the mass evolution of the two poles as a function of the pion mass, linking theoretical predictions to future lattice-QCD simulations. Second, we analyze femtoscopic CFs of VP systems to probe near-threshold interactions and assess the impact of the two poles on observable correlation functions.

The paper is organized as follows. Sec. II A outlines the theoretical framework of VP interactions in UChPT and presents the extrapolation formulas for vector-meson masses. Sec. II B introduces the formalism of femtoscopic CFs and their application to the VP system. In Sec. III A, we fit the chiral trajectories of the vector mesons ρ and K^* using lattice-QCD and experimental data, followed by an analysis of the two-pole evolution with increasing pion mass in Sec. III B. Sec. III C presents the computed femtoscopic CFs for various coupled channels. Finally, Sec. IV summarizes our findings and outlines prospects for future theoretical and experimental studies.

II. FORMALISM

A. UChPT and mass evolution

For the vector-pseudoscalar (VP) interaction, the leading-order (LO) chiral Lagrangian [15, 25] takes the following form

$$\mathcal{L}_{\rm VP}^{\rm WT} = -\frac{1}{4f^2}\,{\rm Tr}([\mathcal{V}^\mu, \partial^\nu \mathcal{V}_\mu][\Phi, \partial_\nu \Phi])\,, \tag{1}$$

from which one obtains the Weinberg-Tomozawa (WT) interaction kernel, projected onto the S wave, which reads [15, 17]

$$\begin{aligned} V_{ij}(s) = -\,\epsilon^i\!\cdot\!\epsilon^j\,\frac{C_{ij}}{8f^2}\Bigg[&3s - (M_i^2 + m_i^2 + M_j^2 + m_j^2) \\ &- \frac{1}{s}\,(M_i^2 - m_i^2)(M_j^2 - m_j^2)\Bigg], \end{aligned} \tag{2}$$

where C_{ij} are the channel-dependent coupling coefficients determined by SU(3) symmetry [15, 17], f is the decay constant, M and m are the masses of the vector and pseudoscalar mesons, respectively, and ϵ^i (ϵ^j) denotes the polarization vector of the initial (final) vector meson.

Within the unitarized chiral perturbation theory (UChPT) [22], the on-shell unitarized scattering amplitude is given by

$$T = \big(1 - VG\big)^{-1} V, \tag{3}$$

where G is a diagonal matrix whose elements $G_i(\sqrt{s})$ correspond to the loop functions of the individual coupled channels. In this work, the logarithmic divergence of $G_i(\sqrt{s})$ is regularized using the dimensional-regularization (DR) scheme, which ensures a proper analytic continuation to higher energies. Further details of this formalism can be found in Ref. [22].

An important refinement arises when dealing with intermediate states containing unstable particles with sizable decay widths. In such cases, the standard loop functions need to be modified to account for the finite-width effects, which can be consistently incorporated through the Källén–Lehmann spectral representation of the vector-meson propagators as

$$\begin{aligned} \tilde{G}_i(\sqrt{s}) = \frac{1}{C}\int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V\, G_i(\sqrt{s}, \sqrt{s_V}, m_i) \\ \times \left(-\frac{1}{\pi}\right) {\rm Im}\left\{\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\right\}, \end{aligned} \tag{4}$$

where the normalization factor C is given by

$$C = \int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V \left(-\frac{1}{\pi}\right) {\rm Im}\left\{\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\right\}. \tag{5}$$

This treatment effectively replaces the narrow-width approximation and leads to a more realistic description of the VP dynamics, as discussed in Refs. [15, 17].

We now turn to the chiral extrapolation of the masses of the relevant particles involved in the ρK and $K^*\pi$ coupled-channel system. As the pion mass increases, the kaon mass approximately follows a linear relation in terms of m_π^2, which can be expressed as

$$m_K^2 = a + b\, m_\pi^2 + c\, a_{\rm lat}^2, \tag{6}$$

where the coefficients a, b, and c are determined by fitting to the lattice-QCD and experimental data. This form is consistent with the standard chiral extrapolation of pseudoscalar meson masses [59].

For the vector mesons, both the ρ and K^* masses, as well as their couplings relevant to the dominant decay channels $\rho\to\pi\pi$ and $K^*\to\pi K$, are parameterized in a similar quadratic form [60–62],

$$\begin{aligned} m_{\rm V} &= c_0 + c_1\, m_\pi^2 + c_2\, a_{\rm lat}^2, \\ g_{\rm VPP} &= \tilde{c}_0 + \tilde{c}_1\, m_\pi^2 + \tilde{c}_2\, a_{\rm lat}^2, \end{aligned} \tag{7}$$

where the coefficients $(c_i, \tilde{c}_i)$ are obtained from global fits combining lattice-QCD results and experimental data. Furthermore, the decay width for the $\rho\to\pi\pi$ decay channel can be written as

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi m_\rho^2}\left(\frac{m_\rho^2}{4} - m_\pi^2\right)^{3/2}, \tag{8}$$

and an analogous expression holds for the $K^*\to\pi K$ decay. In particular, our treatment in Eq. (7) reproduces the IR-regularized chiral EFT result under the same truncation-keeping terms up to $\mathcal{O}(m_\pi^2)$ and discarding higher-order/non-analytic pieces [63]. Finally, the inclusion of the $a_{\rm lat}^2$ term effectively accounts for finite-lattice-spacing artifacts up to $\mathcal{O}(a_{\rm lat}^2)$ and ensures a smooth extrapolation to the continuum limit [61].

Note that the ρ and K^* mesons can be dynamically generated within the chiral unitary approach or the inverse-amplitude method (IAM) by employing the chiral Lagrangians for meson-meson scattering, as demonstrated in Refs. [64, 65]. The pion-mass dependence of the ρ meson has been investigated in this framework in Refs. [66–68]. In the present work, we focus on tracing the movement of the pole dynamically generated by the WT term of Eq. (2) in the VP meson interaction. For this purpose, a simplified parametrization of the pion-mass dependence of the ρ and K^* mesons is sufficient.

B. Femtoscopic correlation functions

The femtoscopic CF provides a powerful means of extracting information on hadron-hadron interactions at low relative momenta from two-particle momentum correlations. Within the standard Koonin–Pratt formalism [69, 70], the CF between two particles with a relative momentum $\vec{p}$ can be expressed as

$$C(\vec{p}) = \int d^3r\, S(\vec{r})\, \left|\Psi^{(-)}(\vec{p},\vec{r})\right|^2, \tag{9}$$

where $S(\vec{r})$ denotes the source distribution of the relative separation between the two emitted particles at chemical freeze-out, and $\Psi^{(-)}(\vec{p},\vec{r})$ is the outgoing two-body wave function that encodes the final-state interaction (FSI). Assuming a spherically symmetric Gaussian source,

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{4R^2}\right), \tag{10}$$

where R characterizes the source size, the CF depends only on the modulus of the relative momentum $p = |\vec{p}|$.

For coupled-channel systems, such as the VP meson pairs considered here, Eq. (9) generalizes to [49]

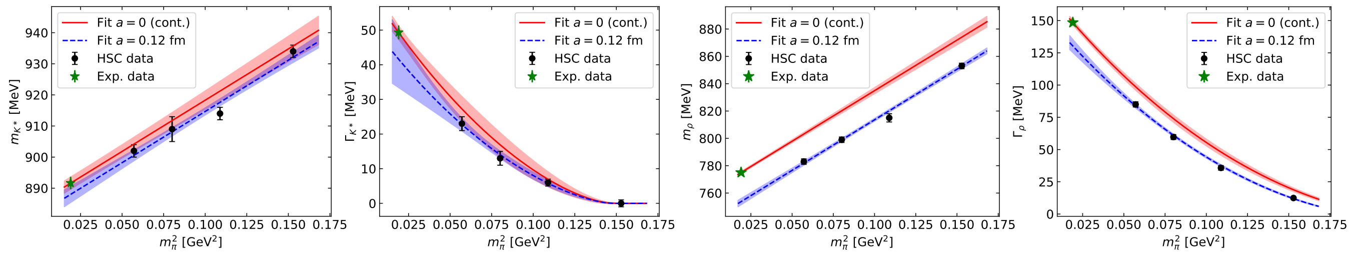
$$C_i(p) = \sum_j w_j \int d^3r\, S_j(r) \left|\Psi_{ji}^{(-)}(p,r)\right|^2, \tag{11}$$

where the indices i and j label the detection and coupled intermediate channels, respectively. The outgoing wave function $\Psi_{ji}^{(-)}$ satisfies the Lippmann–Schwinger (Bethe–Salpeter) equation with an interaction potential V_{ji} derived from the on-shell unitarized T-matrix in the chiral framework, ensuring consistency between the femtoscopic analysis and the scattering amplitudes that generate the $K_1(1270)$ poles. The explicit form of $\Psi_{ji}^{(-)}$ reads

$$\begin{aligned} \Psi_{ji}^{(-)}(p,r) &= \delta_{ji}\, j_0(pr) \\ &\quad + \int_0^{q_{\rm max}} \frac{d^3q}{(2\pi)^3}\, \frac{E(q)+\omega(q)}{E(q)\,\omega(q)} \\ &\quad \times \frac{T_{ji}(\sqrt{s},q,p)}{s(p) - [E(q)+\omega(q)]^2 + i\epsilon}\, j_0(qr). \end{aligned} \tag{12}$$

Here, j_0 denotes the spherical Bessel function of the first kind. The quantities $E(q) = \sqrt{q^2+M_j^2}$ and $\omega(q) = \sqrt{q^2+m_j^2}$ represent the energies of the heavy matter particle (the vector meson) and the pseudoscalar meson in the initial-state channel j, respectively. The function $s(p)$ corresponds to the squared center-of-mass energy of the final-state channel i with relative momentum p. $q_{\rm max}$ represents a sharp ultraviolet cutoff that suppresses the high-energy components of the propagator.

In the present work, the strong interaction between the PV pairs is fully incorporated through the coupled-channel T-matrix $T_{ji}^{\rm S}(\sqrt{s})$ of the unitarized chiral perturbation theory in Eq. (3), while the Coulomb interaction $T_{ii}^{\rm C}(\sqrt{s},q,p)$ in charged channels (e.g., ρ^+K^- and $K^{*-}\pi^+$) is included perturbatively $T_{ii}(\sqrt{s},q,p) = T_{ii}^{\rm S}(\sqrt{s}) + T_{ii}^{\rm C}(\sqrt{s},q,p)$ as described in Refs. [45, 53].



(a) (b)

FIG. 1. (a) Chiral extrapolation of the K^* mass and decay width; (b) chiral extrapolation of the ρ mass and decay width. The solid and dashed lines denote the continuum-limit and the discrete lattice QCD fits, respectively, and the shaded regions correspond to the 1σ uncertainty bands.

TABLE I. The tabulated quantities correspond to the pion mass m_π, kaon mass m_K, and the extracted ρ and K^* vector-meson pole parameters (all quantities in units of MeV) obtained from the Hadron Spectrum Collaboration (HSC) studies [71–75].

m_π	m_K	m_ρ	Γ_ρ	m_{K^*}	Γ_{K^*}
239	507.7 ± 2.4	783 ± 2	85.0 ± 2.0	902 ± 2	23 ± 2
283	519.4 ± 2.6	799 ± 2	59.7 ± 1.9	909 ± 4	13 ± 2
330	527.8 ± 2.8	815 ± 3	35.8 ± 1.8	914 ± 2	6 ± 1
391	549.1 ± 1.2	853 ± 2	12.4 ± 0.6	934 ± 2	0 ± 1

III. RESULTS AND DISCUSSION

A. Fits of vector meson masses

According to the $\pi\pi$ and πK scattering phase shifts obtained from lattice QCD simulations performed by the Hadron Spectrum Collaboration (HSC) over the past decade [71–75], the masses of the ρ and K^* vector mesons can be extracted from the pole positions of parametrized scattering amplitudes evaluated at different pion and kaon masses.

Based on the results summarized in Table I, we carry out a global fit that combines the HSC lattice QCD results with the corresponding experimental measurements, following the strategy outlined in Sec. II A [1]. In this fit, the kaon trajectory is described by the linear squared–mass relation in Eq. (6), whereas the ρ and K^* are modeled by the quadratic forms in Eq. (7) for their masses and couplings, which effectively capture the impact of their sizable decay widths. Since the HSC simulations were performed at a finite spatial lattice spacing $a_{\rm lat} \sim 0.12$ fm (temporal spacing a_t is used for the scale setting) [71–75], we include an $a_{\rm lat}^2$ term to account for discretization corrections and to provide continuum limit inputs to the UChPT framework. The fit parameters obtained from this global analysis are listed in Table II, and the corresponding curves with the 1σ uncertainty bands are shown in Fig. 1.

[1] Note that the HSC does not provide scattering data at different lattice spacings, and therefore this assumption in Eq. (7) should be verified in future lattice QCD simulations.

TABLE II. Fit coefficients for the m_π-dependence of pseudoscalar and vector meson masses and couplings in the continuum limit (all quantities in units of GeV).

Particle	Parameter	Value $\pm$ Uncertainty	Unit
K	a_K	0.23697 ± 0.00048	GeV^2
	b_K	0.454 ± 0.025	1
	c_K	-0.0137 ± 0.0075	GeV^4
ρ	$c_0^{(\rho)}$	0.7609 ± 0.0010	GeV
	$c_1^{(\rho)}$	0.740 ± 0.027	GeV^{-1}
	$c_2^{(\rho)}$	-0.0576 ± 0.0070	GeV^3
	$\tilde{c}_0^{(\rho)}$	5.939 ± 0.054	1
	$\tilde{c}_1^{(\rho)}$	0.60 ± 1.8	GeV^{-2}
	$\tilde{c}_2^{(\rho)}$	-0.71 ± 0.34	GeV^2
K^*	$c_0^{(K^*)}$	0.8854 ± 0.0021	GeV
	$c_1^{(K^*)}$	0.330 ± 0.029	GeV^{-1}
	$c_2^{(K^*)}$	-0.0101 ± 0.0091	GeV^3
	$\tilde{c}_0^{(K^*)}$	5.61 ± 0.21	1
	$\tilde{c}_1^{(K^*)}$	-1.6 ± 9.1	GeV^{-2}
	$\tilde{c}_2^{(K^*)}$	-1.4 ± 1.4	GeV^2

The resulting masses and couplings as functions of m_π are then used as input to the coupled-channel UChPT study in the next section.

B. Two-pole trajectories

In Ref. [17], the coupled-channel analysis of the vector-pseudoscalar (VP) interaction was performed within the leading-order unitarized chiral perturbation theory. Two poles were dynamically generated, located just below the ρK threshold and slightly above the $K^*\pi$ threshold, respectively. When the three higher channels are neglected, the essential features of the spectrum remain essentially unchanged. By slightly readjusting the subtraction constants to $a_{K^*\pi} = -2.21$ and $a_{\rho K} = -2.44$ while keeping $\mu = 900$ MeV and $f = 115$ MeV, one obtains nearly identical pole positions at $W_H = 1269.5 - 12.0i$ MeV and $W_L = 1198.5 - 123.2i$ MeV for the zero-width case [34]. For comparison, if we keep

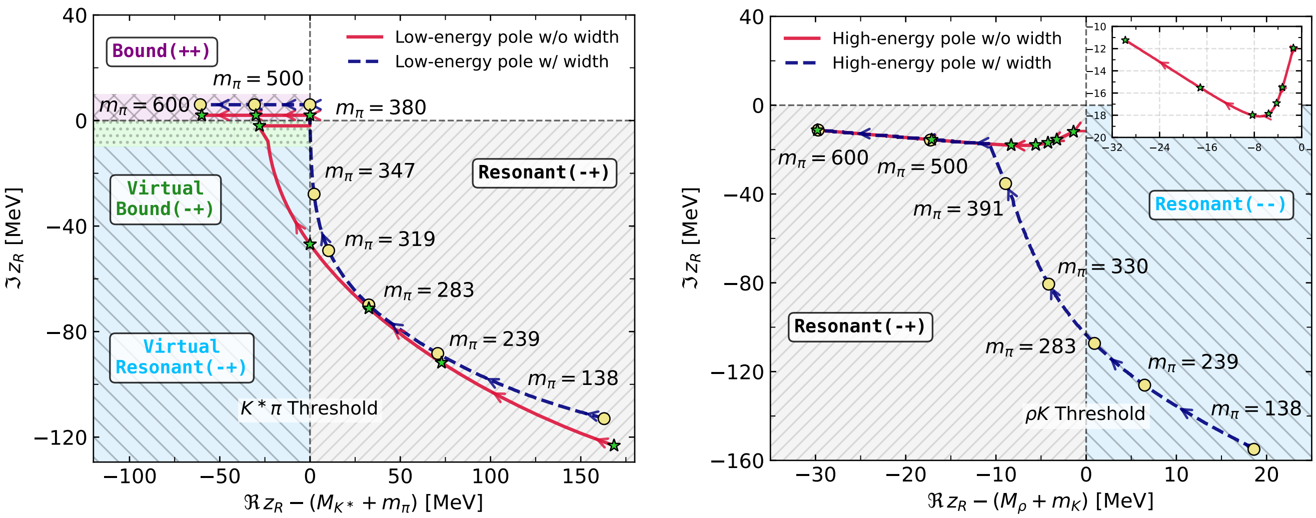


FIG. 2. Chiral evolution of the two-pole structure of the $K_1(1270)$ resonance in the ρK–$K^*\pi$ coupled-channel system. The left and right panels correspond to the lower and higher poles, mainly coupled to $K^*\pi$ and ρK, respectively. Solid and dashed lines denote the results without and with finite vector-meson widths, respectively. Markers indicate pole positions at different pion masses, with the critical masses explicitly labeled.

the same set of subtraction constants as above, the pole positions shift to $W_H = 1289.2 - 154.0i$ MeV and $W_L = 1192.6 - 113.3i$ MeV in the finite-width case. The sizable shift of the higher pole is of kinematical origin and does not affect the underlying two-channel dynamics [17], as will be illustrated by the pole trajectories. This demonstrates that the dominant dynamics of the $K_1(1270)$ resonance are already well captured by the reduced two-channel system composed of ρK and $K^*\pi$.

Motivated by this observation, we perform the present calculation in the same unitarized chiral framework, restricting ourselves to the ρK and $K^*\pi$ coupled channels. The rationale for this simplification is twofold. First, as will be demonstrated in Sec. II B through the study of femtoscopic CFs, the dominant contributions to the femtoscopic line shapes arise from these two channels, whose distinct line shapes reflect their different coupling strengths [34]. Second, from the lattice-QCD perspective, simulating multi-channel scattering involving unstable vector mesons such as ρ and K^* is extremely challenging, and current simulations are limited to systems composed of stable hadrons at relatively low pion masses [38, 39]. Therefore, the two-channel approximation provides both a physically motivated and practically tractable description of the $K_1(1270)$ dynamics.

Furthermore, the essential difference between the $K_1(1270)$ system and the other well-known two-pole structure, the $\Lambda(1405)$ [34], lies in the fact that the heavy scattering constituents in the former case, ρ and K^*, are themselves resonances with large decay widths and short lifetimes. To clarify the chiral trajectories of the lower and higher poles, we perform calculations in two scenarios: one neglecting and one incorporating the vector meson decay widths. We expect that the qualitative evolution behavior of the poles is largely independent of whether or not the vector-meson widths are explicitly included. Fortunately, the unitarized chiral framework provides a straightforward way to incorporate the finite widths and spin properties of the ρ and K^* mesons by modifying the analytic form of the loop functions as described in Sec. II A.

In Fig. 2, we show the resulting chiral trajectories of the two poles. The left panel corresponds to the lower pole, which couples predominantly to the $K^*\pi$ channel, while the right panel corresponds to the higher pole, mainly associated with the ρK channel. The evolution of both poles is consistent with that reported in Ref. [34] for the $\Lambda(1405)$, highlighting their common origin from chiral dynamics. In both subfigures, different pion-mass values along the trajectories are indicated by stars or circles.

In the left panel, the lower pole exhibits a characteristic behavior: as the pion mass increases, the strength of the Weinberg–Tomozawa (WT) interaction grows, causing the pole to move from an above-threshold resonance to a below-threshold virtual or bound state. In the case without vector-meson widths, the trajectory undergoes a “zigzag” evolution: the resonance first becomes a below-threshold state located on the second Riemann sheet $(-+)$, then evolves into a virtual state as the width decreases to zero, and eventually crosses to the first Riemann sheet $(++)$ as a zero-energy bound state before becoming a deeply bound state as the binding energy increases [34]. When the widths are included, the pole instead evolves directly from an above-threshold resonance to a bound state without exhibiting the zigzag pattern, in agreement with the general expectation discussed in Ref. [76].

The right panel shows a much simpler pattern for the higher pole, consistent with the findings of Ref. [34]. As the pion mass increases, the attraction in the ρK channel strengthens,

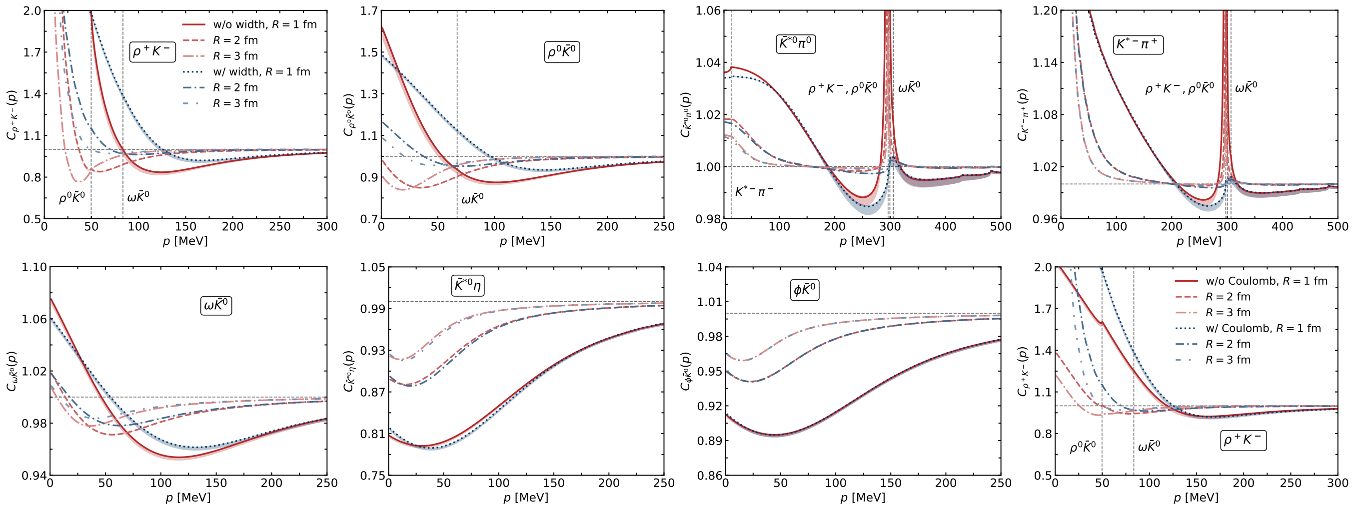


FIG. 3. Femtoscopic correlation functions $C_i(p)$ for the $Q = 0$ sector, corresponding to the coupled-channel system $\{\rho^+K^-,\ \rho^0\bar{K}^0,\ \bar{K}^{*0}\pi^0,\ K^{*-}\pi^+,\ \omega\bar{K}^0,\ \bar{K}^{*0}\eta,\ \phi\bar{K}^0\}$. The red and blue curves represent results without and with vector-meson widths, respectively, and different line transparencies correspond to source sizes of $R = 1$, 2, and 3 fm.

and the higher pole moves downward in energy, following a more intuitive trajectory. The main difference between the two scenarios lies in the physical starting point: in the case without width, the higher pole initially appears as a shallow below-threshold resonance, while in the width-included case, it starts as an above-threshold resonance with a larger imaginary part. Nevertheless, both treatments yield essentially the same qualitative two-pole trajectories, demonstrating that the inclusion of finite widths does not significantly alter the overall chiral evolution of the $K_1(1270)$ poles.

Since the vector-meson mass evolution formulae include uncertainties propagated from the fits to the lattice-QCD data [71–75], we also take into account the corresponding uncertainties in the pole trajectories. However, we do not explicitly display the error bands in the trajectory Fig. 2, because the variations of the pole positions are relatively small—less than 5 MeV in both mass and width—and thus visually indistinguishable on the scale of the figure.

C. Femtoscopic correlation functions

Since the femtoscopic CF is a direct experimental observable, it is necessary to convert the theoretical amplitudes from the isospin basis, which is more suitable for lattice-QCD studies with an $N_f = 2 + 1$ flavor setup [37], to the charge basis, which corresponds to experimentally measurable final states [13]. The $K_1(1270)$ forms an isospin doublet with two members, $K_1^0(1270)$ [27, 28] and $K_1^-(1270)$ [17, 26]. For $I_3 = +1/2$ $(Q = 0)$, the channels are $\{\rho^+K^-,\ \rho^0\bar{K}^0,\ \bar{K}^{*0}\pi^0,\ K^{*-}\pi^+,\ \omega\bar{K}^0,\ \bar{K}^{*0}\eta,\ \phi\bar{K}^0\}$, while for $I_3 = -1/2$ $(Q = -1)$ they are $\{\rho^0K^-,\ \rho^-\bar{K}^0,\ K^{*-}\pi^0,\ \bar{K}^{*0}\pi^-,\ \omega K^-,\ K^{*-}\eta,\ \phi K^-\}$.

To include the finite-width effects of the vector mesons, we adopt the formalism developed in Ref. [52], which consistently accounts for the widths both in the T-matrix and in the asymptotic final-state relative wave functions, ultimately yielding modified femtoscopic CFs. For the $I_3 = +1/2$ case, in addition to the strong interactions that are common to both isospin components, the electromagnetic Coulomb interaction must also be considered due to the presence of the charged channels ρ^+K^- and $K^{*-}\pi^+$. The implementation of the Coulomb contribution follows the prescriptions given in Sec. II B, where it is incorporated perturbatively by adding the Coulomb amplitudes to the diagonal strong scattering amplitudes in the same manner as a Born-term correction.

We summarize here the relevant parameters employed in the calculation of the femtoscopic CFs. First, the source size R is taken to be three representative values $R = 1, 2, 3$ fm. Second, the channel weights w_j, which quantify the relative contributions from various coupled channels, are set to unity for simplicity. Third, the subtraction constants, renormalization scale, and decay constant are chosen to be consistent with those adopted in Ref. [17], namely

$$\mu = 900\ \text{MeV}, \quad a(\mu) = -1.85, \quad f = 115\ \text{MeV}, \qquad (13)$$

where μ denotes the renormalization scale, $a(\mu)$ is the universal subtraction constant, and f represents the pion decay constant. These parameter choices follow the convention established in previous analyses of the $K_1(1270)$ system [26–28] and ensure consistency with the chiral interaction framework used throughout this study. Finally, to estimate the theoretical uncertainties, we vary the cutoff $q_{\rm max}$ in Eq. (12) within a reasonable range of 0.6–1.4 GeV. This variation is consistent with the typical choice of the subtraction constant $a(\mu)$, and corresponds to the equivalent cutoff Λ in the cutoff regularization scheme of the T-matrix [15].

In Figs. 3 and 4, we show the calculated CFs for the

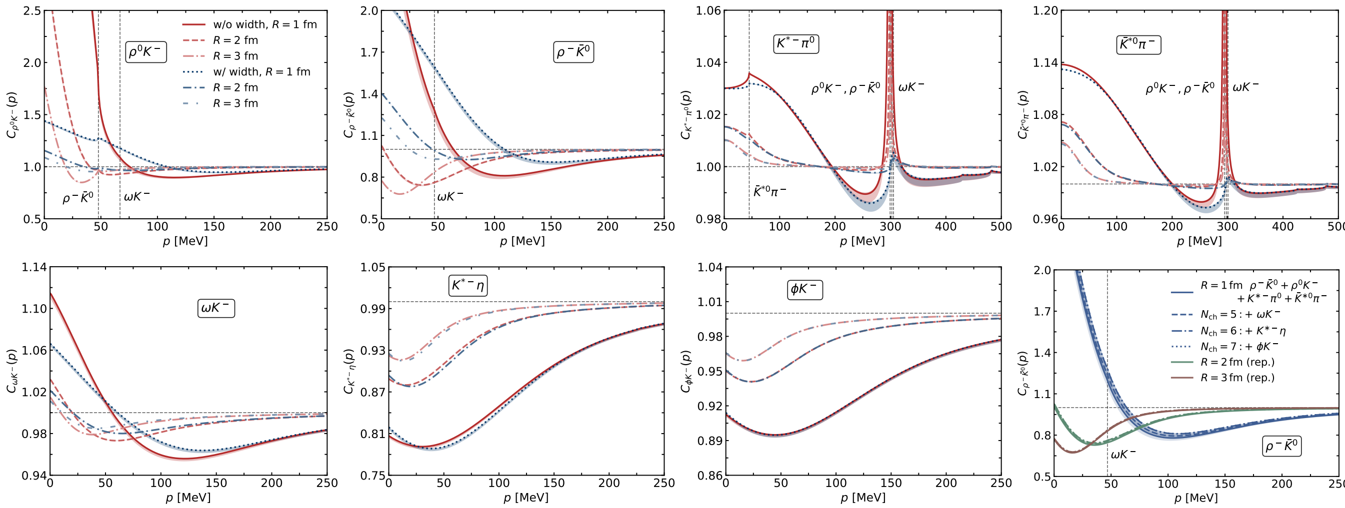


FIG. 4. Femtoscopic correlation functions $C_i(p)$ for the $Q=-1$ sector, corresponding to the coupled-channel system $\{\rho^0K^-,\ \rho^-\bar{K}^0,\ K^{*-}\pi^0,\ \bar{K}^{*0}\pi^-,\ \omega K^-,\ K^{*-}\eta,\ \phi K^-\}$. The red and blue curves denote calculations without and with the finite vector-meson widths, respectively, and different line transparencies correspond to source sizes of $R=1$, 2, and 3 fm.

seven final-state channels in the two charge bases $Q=0$ and $Q=-1$, respectively. For each channel, the CF $C_i(p)$ $(i=1,\ldots,7)$ is plotted over the relative momentum range $(0,250)$ MeV, except for the $\bar{K}^*\pi$ channels, for which the range $(0,500)$ MeV is adopted to illustrate better the threshold cusps induced by the coupled $\rho\bar{K}$ and $\omega\bar{K}$ channels. These cusp structures are significantly smoothened when the vector-meson decay widths are included. The threshold effects from the higher channels $\bar{K}^*\eta$ and $\phi\bar{K}$ are omitted in all plots, as their influence on the line shapes of the CFs is negligible.

In both Figs. 3 and 4, two sets of curves are shown using distinct color schemes: the red curves correspond to calculations without including the vector-meson widths, and the blue curves include the width effects. Within each color set, different source sizes $R=1,\ 2,\ 3$ fm are plotted with decreasing opacity. From the first five subfigures, one can observe that the red curves are much sharper than the blue ones. This behavior reflects that incorporating the finite widths of the intermediate vector mesons effectively smears the loop functions, resulting in smoother scattering amplitudes. The only exceptions are the highest-threshold channels, $\bar{K}^*\eta$ and $\phi\bar{K}$, where the couplings to the lower channels are weak, as evident from the potential-coefficient matrix C_{ij} in Eq. (2).

Within each color system, increasing the source size R systematically weakens the interaction strength, leading to a corresponding reduction in the magnitude of the CF, as expected from the standard source-size dependence in Femtoscopy [41, 47].

Comparing the subfigures for different final-state channels, one can see that for the five lower-threshold channels, the CFs exhibit resonant-like behavior, consistent with the findings of Refs. [46, 51]. In contrast, for the last two channels, $\bar{K}^*\eta$ and $\phi\bar{K}$, the line shapes resemble those of weakly bound states due to the small coupling between the two poles and these higher channels.

Turning to the $Q=0$ sector in Fig. 3, a pronounced enhancement appears in the low-momentum region of the CFs in channels ρ^+K^- and $K^{*-}\pi^+$, as compared with their isospin-partner channels ρ^0K^- and $\bar{K}^{*0}\pi^-$ in Fig. 4 [41]. This enhancement originates predominantly from the Coulomb interaction rather than from small isospin-breaking effects. To further illustrate this behavior, the last subfigure in Fig. 3 shows the ρ^+K^- CF with and without the Coulomb interaction, both calculated including the finite vector-meson widths. The comparison confirms that the Coulomb force produces a strong near-threshold enhancement, whereas the finite widths mainly smooth the overall correlation pattern.

In the last subfigure of the $Q=-1$ sector of Fig. 4, three color groups (blue, green, and yellow) represent source sizes of $R=1,\ 2,\ 3$ fm, respectively. Within each group, we sequentially add channels to the coupled system with finite widths: starting with only $\rho\bar{K}$ and $\bar{K}^*\pi$, and including $\omega\bar{K}$, $\bar{K}^*\eta$, and $\phi\bar{K}$ [41, 47], one by one. The resulting curves lie almost on top of each other, showing only minor increases in magnitude as additional channels are introduced. This confirms that restricting the analysis to the dominant two channels, $\rho\bar{K}$ and $\bar{K}^*\pi$, is a justified and physically sound approximation [34].

Finally, to clarify more precisely the connection between the two-pole structure and the femtoscopic CFs, we focus on two representative channels, $\rho^-\bar{K}^0$ and $\bar{K}^{*0}\pi^-$, from the $Q=-1$ system, and calculate the individual contributions from the $\rho\bar{K}$ and $\bar{K}^*\pi$ components, respectively. From Fig. 5, one can clearly observe the distinct behavior of the two poles in different final states. In the left panel, the $\rho\bar{K}$ contribution is primarily governed by the higher pole, so that the red line exhibits a typical resonant behavior. In contrast, the $\bar{K}^*\pi$

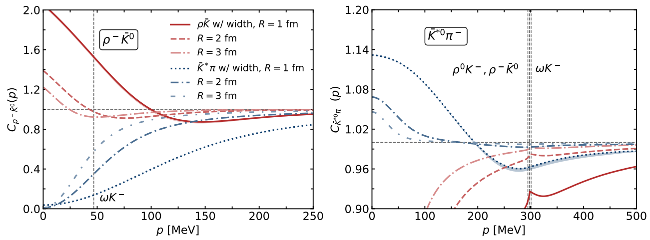


FIG. 5. Femtoscopic correlation functions $C_i(p)$ for the two channels $\rho^-\bar{K}^0$ and $\bar{K}^{*0}\pi^-$ in the $Q = -1$ system. The red and blue lines represent the individual contributions to $C(p)$ from the $\rho\bar{K}$ and $\bar{K}^*\pi$ channels, respectively. Different line transparencies correspond to source radii of $R = 1$, 2, and 3 fm.

contribution is mainly dominated by the lower pole, where the blue line gradually increases from zero and approaches unity, showing a clear signature of a deeply bound state relative to the $\rho^-\bar{K}^0$ threshold [51]. In the right panel, the situation is essentially reversed: the blue line almost entirely dominates the low-momentum region of the total CFs shown in Fig. 4, simply because the lower pole plays a crucial role in the $\bar{K}^*\pi$ component.

IV. CONCLUSION AND OUTLOOK

In this work, we have performed a comprehensive study of the axial-vector resonance $K_1(1270)$ within the two-pole scenario predicted by unitarized chiral perturbation theory (UChPT). Using a coupled-channel framework including the dominant ρK and $K^*\pi$ interactions, we reproduced the well-known two-pole structure originally identified in Refs. [17, 34] and investigated its dependence on the light-quark masses. The fitted pion-mass evolution of the ρ and K^* vector mesons, constrained by both lattice-QCD and experimental data, enabled us to trace the trajectories of the two poles in the complex-energy plane with increasing pion mass. We further examined the influence of vector-meson decay widths by comparing calculations with and without width effects, demonstrating that including finite widths smooths the pole evolution but does not alter the qualitative two-pole pattern.

The lower pole, dominantly coupled to the $K^*\pi$ channel, evolves from an above-threshold resonance to a bound or virtual state as the pion mass increases, while the higher pole, mainly associated with ρK, moves downward in energy, showing a strengthened attraction consistent with chiral expectations. These results establish a consistent and physically intuitive picture of the chiral evolution of the $K_1(1270)$ two-pole structure.

In addition, we have carried out a detailed analysis of the femtoscopic CFs for all relevant vector-pseudoscalar (VP) channels in both charge sectors. The calculated CFs clearly reflect the two-pole dynamics and exhibit distinct resonance and bound-state features in different channels. We have also shown that the vector meson finite widths play a smoothing role in the CF line shapes, and that the Coulomb interaction produces sizable low-momentum enhancements in the charged channels ρ^+K^- and $K^{*-}\pi^+$. Moreover, the negligible influence of the higher channels such as $\omega\bar{K}$, $\bar{K}^*\eta$, and $\phi\bar{K}$ further confirms that the simplified two-channel description (ρK, $K^*\pi$) captures the essential dynamics of the $K_1(1270)$ system.

Future directions include the extension of the present framework to incorporate additional coupled channels with improved lattice QCD constraints, as well as the direct comparison of the predicted CFs with experimental measurements from ALICE and future facilities. Such developments will allow a more precise determination of the $K_1(1270)$ pole parameters and a deeper understanding of its molecular components, offering valuable insight into the nonperturbative dynamics of chiral symmetry breaking and resonance formation in QCD.

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