

Probing Dark Sector Particles Coupling to Neutrinos with Double Beta Decay

Noor-Ines Boudjema,^{1,*} Frank F. Deppisch,^{1,†} Antonio
Herrero-Brocal,^{2,‡} Chayan Majumdar,^{1,3,§} and Supriya Senapati^{4,5,¶}

¹*Department of Physics and Astronomy,*

University College London, London WC1E 6BT, United Kingdom

²*Instituto de Física Corpuscular, CSIC-Universitat de València, 46980 Paterna, Spain*

³*Institute of Modern Physics, Chinese Academy of sciences, Lanzhou, 730000, China*

⁴*Amherst Center for Fundamental Interactions, Department of Physics,
University of Massachusetts, Amherst, MA 01003, USA*

⁵*Department of Applied Physics and MIIT Key Laboratory
of Semiconductor Microstructure and Quantum Sensing,*

Nanjing University of Science and Technology, Nanjing 210094, China

Abstract

Motivated by the observation of non-zero neutrino masses and the potential for discovering physics beyond the Standard Model, numerous experiments are actively searching for neutrinoless double beta ($0\nu\beta\beta$) decay. In all of these searches, a substantial amount of data on two-neutrino double beta ($2\nu\beta\beta$) decay has been collected. In this work, we explore the sensitivity of current and future double beta decay experiments to a massive Majoron-like scalar particle coupled to neutrinos and potentially dark sector fermions, and compare their reach to the relevant cosmological constraints. On- and off-shell production of such scalar particles leads to characteristic distortions in the double beta decay electron spectrum. We investigate how these distortions would manifest in current and future double beta decay experiments, deriving the sensitivity to such scenarios. We project the reach of future experiments which can probe scalar-neutrino couplings of $|a_\nu| \approx 2 \times 10^{-6}$ for sub-MeV scalar particles and remain sensitive to off-shell production above the Q -value of double beta decay isotopes.

*Electronic address: noor-ines.boudjema.19@ucl.ac.uk

†Electronic address: f.deppisch@ucl.ac.uk

‡Electronic address: antonio.herrero@ific.uv.es

[§]Electronic address: chayanmajumdar@impcas.ac.cn

[¶]Electronic address: ssenapati@umass.edu

I. INTRODUCTION

Double beta decay processes provide a sensitive window into physics beyond the Standard Model (SM). The two-neutrino double beta ($2\nu\beta\beta$) decay, the only mode permitted within the SM and experimentally observed, is among the rarest processes ever detected, with half-lives on the order of $T_{1/2}^{2\nu} \sim 10^{19}$ years or longer [1]. In contrast, the neutrinoless double beta ($0\nu\beta\beta$) decay, characterized by the absence of missing energy, is a clear signature of lepton number violation and provides a direct test of the Majorana nature and absolute mass scale of light neutrinos, m_ν . Current experimental sensitivities reach half-lives of $T_{1/2}^{0\nu} \sim (100 \text{ meV}/m_\nu)^2 \times 10^{26}$ years, making this process a critical probe of new physics scenarios that violate lepton number by two units [2–4].

Although the primary goal of double beta decay experiments is the search for $0\nu\beta\beta$ decay, they also yield precise measurements of the $2\nu\beta\beta$ decay rates and spectra across multiple isotopes. For instance, KamLAND-Zen has measured the $2\nu\beta\beta$ spectrum in ^{136}Xe with high statistics [5], while it is limited to the summed electron energy. In contrast, the NEMO-3 experiment, with its electron tracking capability, has enabled measurements of the individual electron energy distributions and their opening angles. This has resulted in detailed spectral data for isotopes such as ^{96}Zr [6], ^{150}Nd [7], ^{48}Ca [8], ^{82}Se [9] and especially ^{100}Mo [10], the latter with approximately 5×10^5 recorded $2\nu\beta\beta$ decay events. These high-statistics measurements are crucial for refining theoretical inputs, such as the effective axial coupling g_A , which influence the interpretation of $0\nu\beta\beta$ decay limits [11]. As the experimental exposure continues to increase, enhancing the $0\nu\beta\beta$ sensitivity, the precision of $2\nu\beta\beta$ data is also expected to improve, although the involved systematic errors and background need to improve in line with the decreased statistical uncertainty. This allows $2\nu\beta\beta$ data itself to be used to search for signals of new physics beyond the SM (BSM).

This may involve non-standard interactions of the SM particles involved, especially the neutrinos [12, 13], modifying the decay’s spectral shape or the production and emission of neutral exotic particles such as sterile neutrinos [14] and light scalars [15]. With $2\nu\beta\beta$ Q values of the order $Q \sim 2 - 4$ MeV, depending on the respective isotope, such particles can be produced if massless or lighter than a few MeV. The best-studied class of such scenarios involves the emission of a scalar particle known as the Majoron S . The original Majoron

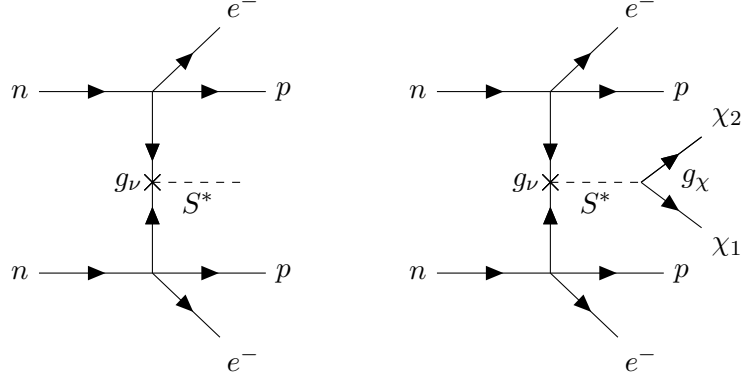


FIG. 1: Feynman diagrams for neutrinoless double beta decay with scalar Majoron(-like) emission (left) and neutrinoless double beta decay with exotic fermion-pair emission ($2\chi\beta\beta$) via s -channel scalar exchange (right).

model proposed a massless Goldstone boson arising from the spontaneous breaking of lepton number symmetry [16, 17], which couples to neutrinos via an interaction of the form $g_\nu \bar{\nu}^c \nu S^*$ from which it can be produced in double beta decay as shown in Fig. 1 (left). The term “Majoron” has since been generalized to refer to any electrically neutral scalar (Goldstone or otherwise), or even vector particle [18]. While the original models assumed a massless Majoron, more recent frameworks allow for it to be a light but massive particle [12, 15, 19–21], potentially serving as a viable dark matter candidate [22–24]. Current experimental searches for neutrinoless double beta decay with scalar emission ($0\nu\beta\beta S$) are sensitive to half-lives of the order $T_{1/2}^{0\nu S} \sim (10^{-5}/|g_\nu|)^2 \times 10^{24}$ years [10, 25–27], assuming a massless Majoron.

We here focus on the modification of the double beta decay spectrum from the subsequent production of the scalar S and its decay to two exotic fermions. The right diagram in Fig. 1 represents such a portal to probe new fermions. In addition to this diagram, S will also decay to two SM neutrinos. If S can be produced on-shell, $m_S < Q$, the spectrum will be well approximated by the usual Majoron-like spectrum for electron kinetic energies $T = E_{e_1}^{\text{kin}} + E_{e_2}^{\text{kin}} < Q - m_S$. On the other hand, the finite decay width of S (especially if the exotic χ particles couple strongly to S) and the possibility of off-shell S production means that the spectrum is also modified for $T > Q - m_S$, which includes the scenario where S is too heavy to be produced on-shell, $m_S > Q$. If the χ particles are massive but can still be produced on-shell, $m_{\chi_1} + m_{\chi_2} < Q$, we expect a distinctive kink in the spectrum. Searches

for such exotic signatures in double beta decay are essential for advancing our understanding of neutrino properties.

Furthermore, this exotic fermion species χ can be considered as a viable dark matter (DM) candidate owing to its stability in presence of some discrete symmetry in the framework. The interaction between active neutrinos and such light DM, their couplings with the mediator S as well as their masses can be constrained by astrophysical and cosmological bounds, i.e., coming from supernova, high energy neutrino sources; cosmological constraints arising from Cosmic Microwave Background (CMB), Big Bang Nucleosynthesis (BBN), collisional damping and present-day thermal relic density for DM particles as well as the laboratory bounds considering light meson decays to exotic particles. For a recent comprehensive overview on these constraints, interested readers can look into Ref. [28].

The paper is organized as follows. In Sec. II, we construct simplified models of a scalar particle couplings to neutrinos and exotic fermions. While not limited to such a case, we focus on scenarios where the light neutrinos are Dirac fermions and total lepton number is conserved, thus no standard $0\nu\beta\beta$ decay would be expected. In Sec. III, we outline the calculation of the double beta decay spectrum in the models considered. The constraints on the light S and χ particles from cosmology and astrophysics are discussed in Sec. IV, along with an overview of the most relevant laboratory-based experimental constraints. Following a discussion of our statistical limit-setting approach in Sec. V, we present the sensitivity of current and future experiments to select model scenarios in Sec. VI. We conclude our work in Sec. VII.

II. SIMPLIFIED MODELS

In the standard $0\nu\beta\beta$ mechanism, two neutrinos annihilate due to a Majorana mass term, leaving only electrons in the final leptonic states. The existence of new BSM neutral fermions, denoted generically by χ , could lead to a new variant of $\beta\beta$ decay with no neutrinos in the final state.

This hypothetical double beta decay, involving two χ particles in the final state, can be first motivated by a four-fermion interaction, arising from a heavy mediator in an effective field theory (EFT) formulation. Since neutrinos produced in double beta decay arise from the SM Fermi interaction, where the leptonic part of the current is given by

$J_\ell^\mu = -G_F/\sqrt{2}\bar{e}\gamma^\mu P_L\nu$, we need the combination $\overline{\nu\nu}$ or, equivalently, $\overline{\nu^c\nu^c}$. Therefore, the EFT operators that give rise to the exotic double beta decay are

$$\mathcal{L}_S = \frac{1}{\Lambda^2}\overline{\chi^c}_i\chi_j\overline{\nu^c}\nu, \quad (1)$$

$$\mathcal{L}_P = \frac{1}{\Lambda^2}\overline{\chi^c}_i\gamma^5\chi_j\overline{\nu^c}\gamma^5\nu, \quad (2)$$

$$\mathcal{L}_V = \frac{1}{\Lambda^2}\overline{\chi^c}_i\gamma^\mu\chi_j\overline{\nu^c}\gamma_\mu\nu, \quad (3)$$

$$\mathcal{L}_A = \frac{1}{\Lambda^2}\overline{\chi^c}_i\gamma^5\gamma^\mu\chi_j\overline{\nu^c}\gamma^5_\mu\nu, \quad (4)$$

$$\mathcal{L}_T = \frac{1}{\Lambda^2}\overline{\chi^c}_i\sigma^{\mu\nu}\chi_j\overline{\nu^c}\sigma_{\mu\nu}\nu, \quad (5)$$

where the subscripts S, P, V, A and T correspond to scalar, pseudoscalar, vector, axial-vector and tensor interactions, respectively. Depending on the charge assignment, we can have only one ($i = j$) or different ($i \neq j$) generations of BSM fermion χ involved in the process. Here, Λ corresponds to the new physics scale which is a function of the mediator mass and its coupling with ν and χ . Considering a generic symmetry group $U(1)_X$ under which ν and χ_i have non-trivial charges, Q_X^ν and $Q_X^{\chi_i}$, respectively, in order to construct $U(1)_X$ -invariant EFT vertices in Eqs. (1) - (5), we have, for example, $Q_X^\nu = -Q_X^{\chi_i} = -Q_X^{\chi_j}$.

These interactions, when probed in double beta decay experiments, could provide a promising window for discovering new physics. However, when computing the decay rate for $2\chi_S\beta\beta$ decay, some of the operators in Eqs. (1) - (5) lead to a suppression by the small neutrino masses. In general, integrating out a mediator generates the effective operators shown in Eqs. (1) - (5). These operators can be realized through two distinct tree-level diagrams, via an s -channel or t/u -channel exchange. For the s -channel scenario with scalar mediator S , the Lagrangian takes the general form

$$-\mathcal{L}^S \supset \overline{\nu^c}(a_\nu P_L + b_\nu P_R)\nu S^* + \overline{\chi^c}(a_\chi P_L + b_\chi P_R)\chi S^* + \text{h.c.}, \quad (6)$$

The resulting S -mediated $2\nu_S\beta\beta$ and $2\chi_S\beta\beta$ decay is not chirally suppressed by the light active neutrino masses. Moreover, the process can be dominated by the emission of S , i.e., scalar emission [24], provided it is kinematically accessible. We can recover the purely scalar and pseudoscalar interactions by imposing $a_i = \pm b_i$, respectively. In the latter case, we reproduce the ordinary Majoron term $a_\nu\overline{\nu^c}\gamma_5\nu S^* + \text{h.c.}$. The classical scenario with Majorana neutrinos is easily reproduced by taking $\nu = \nu^c$. In this case, S must be real

and the coupling includes a normalization factor of $1/2$ to avoid double-counting. Then $\frac{a_\nu}{2} \bar{\nu}^c \gamma_5 \nu S^* + \text{h.c.} = i \text{Im}(a_\nu) \bar{\nu} \gamma_5 \nu S$.

For an t/u -channel exchange, the relevant Lagrangian is given by

$$-\mathcal{L}_{UV}^{t/u} \supset \bar{\nu}^c (a_{\nu\chi} P_L + b_{\nu\chi} P_R) \chi S^* + \text{h.c.} \quad (7)$$

Although this setup also avoids chiral suppression of the $2\chi\beta\beta$ decay, the structure of the couplings connecting one neutrino and one χ precludes the emission of a scalar or pseudoscalar boson, making the $0\nu\beta\beta S$ channel inaccessible in this case. However, due to the t/u -channel nature of the interaction, the decay amplitude acquires a nontrivial dependence on the neutrino momentum transfer q . This leads to integrals over q with a different structure compared to standard $0\nu\beta\beta$ decay, necessitating dedicated nuclear matrix element computations. A detailed treatment of such nuclear physics effects lies beyond the scope of this work.

The additional operators, Eqs. (3) - (5), arise when the mediating boson is a vector or tensor field. As before, both s - and t/u -channel completions are possible. In the s -channel case, a gauge boson mediates interactions between two neutrinos and two χ fermions. To reproduce the effective operator structure in Eq. (3), the neutrino must be of Majorana type, since only then can a current-current interaction involving two neutrinos be realized. In this case, the vector interaction necessitates a chirality flip for neutrinos, leading to a suppression proportional to the neutrino mass. In contrast, the t/u -channel setup involves a gauge boson interacting with one neutrino and one χ fermion at the same vertex. This interaction does not require a chirality flip, and hence the resulting decay rate is not neutrino mass suppressed. Moreover, the axial-vector and tensor-mediated cases are also possible which resemble the phenomenology of vector and scalar mediated scenarios, respectively.

In the following, we focus on scenarios involving s -channel mediation by a scalar field S , as described in Eq. (6). In this work, the lepton number assignments of the newly introduced scalar and neutral fermion fields are chosen such that total lepton number is conserved at each interaction vertex. We work in the broken electroweak phase, thereby preserving the residual gauge symmetry group $SU(3)_C \times U(1)_{\text{EM}}$. Under these assumptions, neutrinos acquire Dirac masses via lepton number-conserving Yukawa interactions, and consequently, the conventional Majorana mass-induced mechanism for $0\nu\beta\beta$ is absent in this framework. Furthermore, we have considered the scalar S does not acquire a vacuum expectation value.

However, depending on the lepton number assignments, different combinations of couplings between S and the neutral fermions are possible. Therefore, in the following, we present three distinct UV-complete models, each characterized by specific lepton number assignments for the additional fields and their corresponding interaction structures. We do not attempt to explain the origin of the scalar mass, especially why it is comparatively light. Massive Majorons as pseudo-Goldstone boson were for example discussed in [23, 29]. While our frameworks are based Dirac neutrinos, the interaction $\nu\nu S$ is equivalent to that of ordinary Majoron production and our calculations will cover the usually considered classes IA, IB and IIB of ordinary Majoron production [20], i.e., with a spectral index of $n = 1$ and being dependent on the $0\nu\beta\beta$ decay nuclear matrix element $\mathcal{M}_{0\nu}$, as limiting cases.

A. Model I

We construct a model featuring a neutral fermion sector composed of left- and right-handed neutrinos (ν'_L, ν'_R) and additional singlet fermions (χ'_L, χ'_R), along with a scalar singlet S . The fields are assigned lepton numbers as follows: $L_{\nu'_L} = +1 = L_{\nu'_R}, L_{\chi'_L} = +1 = L_{\chi'_R}, L_S = 2$. With these charge assignments, the relevant Lagrangian takes the form as

$$\begin{aligned}
-\mathcal{L} = & m_\nu \overline{\nu'_R} \nu'_L + m_{\nu\chi} \overline{\chi'_R} \nu'_L + m_{\chi\nu} \overline{\nu'_R} \chi'_L + M \overline{\chi'_R} \chi'_L \\
& + (a_\nu \overline{\nu'_L} \nu'_L S^* + b_\nu \overline{\nu'_R} \nu'_R S^* + a_{\nu\chi} \overline{\nu'_L} \chi'_L S^* + b_{\nu\chi} \overline{\nu'_R} \chi'_R S^* + a_\chi \overline{\chi'_L} \chi'_L S^* + b_\chi \overline{\chi'_R} \chi'_R S^*) + \text{h.c.}
\end{aligned} \tag{8}$$

For simplicity, we consider a single generation of each fermion species. The neutral fermion mass matrix in the basis of $(\nu'_L, \nu'^c_R, \chi'_L, \chi'^c_R)^T$ can be written as,

$$\mathcal{M} = \begin{pmatrix} 0 & m_\nu & 0 & m_{\nu\chi} \\ m_\nu & 0 & m_{\chi\nu} & 0 \\ 0 & m_{\chi\nu} & 0 & M \\ m_{\nu\chi} & 0 & M & 0 \end{pmatrix}. \tag{9}$$

However, for simplicity, assuming a special scenario $m_{\chi\nu} = m_{\nu\chi}$ and considering all relevant Yukawa couplings to be real, the mass matrix $\mathcal{M}$ can be diagonalized yielding two Dirac mass eigenstates with masses (generalised discussion with $m_{\nu\chi} \neq m_{\chi\nu}$ can be found in [30]),

$$m_{\text{light,heavy}} = \frac{1}{2} \left| (M + m_\nu) \mp \sqrt{(M - m_\nu)^2 + 4m_{\nu\chi}^2} \right|. \tag{10}$$

In the hierarchical regime where $(M - m_\nu) \gg m_{\nu\chi}$, the masses are given by

$$m_{\text{light}} \approx \left| m_\nu - \frac{m_{\nu\chi}^2}{M - m_\nu} \right|, \quad m_{\text{heavy}} \approx M. \quad (11)$$

In the mass basis, the flavor eigenstates can be written as

$$\begin{pmatrix} \nu'_{L,R} \\ \chi'_{L,R} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \nu_{L,R} \\ \chi_{L,R} \end{pmatrix}, \quad (12)$$

where primed and un-primed bases correspond to interaction and mass eigenstates, respectively. These Dirac mass eigenstates are defined as, $\nu = \nu_L + \nu_R$ and $\chi = \chi_L + \chi_R$ with masses m_{light} and m_{heavy} , respectively. Now, in the limit of negligible light-heavy mixing $\theta \rightarrow 0$, we recover the active neutrinos and heavier exotic Dirac particles as light ($\nu_{L,R} \approx \nu'_{L,R}$) and heavy ($\chi_{L,R} \approx \chi'_{L,R}$) eigenstates, respectively. As an example, we can consider $M \sim 1$ MeV, $m_{\text{light}} \sim 10^{-2}$ eV and $m_{\nu\chi} \sim 10^2$ eV yields a lighthheavy mixing angle of

$$\theta \approx \frac{m_{\nu\chi}}{M} = 10^{-4}. \quad (13)$$

In the mass basis, the interactions, can be written as

$$\begin{aligned} -\mathcal{L} = & \bar{\nu}^c \{ (a_\nu - a_{\nu\chi}\theta + a_\chi\theta^2) P_L + (b_\nu - b_{\nu\chi}\theta + b_\chi\theta^2) P_R \} \nu S^* \\ & + \bar{\nu}^c \{ (a_{\nu\chi} + (a_\nu - a_\chi)\theta) P_L + (b_{\nu\chi} + (b_\nu - b_\chi)\theta) P_R \} \chi S^* \\ & + \bar{\chi}^c \{ (a_\chi + a_{\nu\chi}\theta + a_\nu\theta^2) P_L + (b_\chi + b_{\nu\chi}\theta + b_\nu\theta^2) P_R \} \chi S^* + \text{h.c.}, \end{aligned} \quad (14)$$

where we have used the relation $g_\psi \bar{\psi}_1^c \psi_2 = g_\psi^T \bar{\psi}_2^c \psi_1$, following from the properties of the charge conjugation operator. We also retained the second-order mixing term θ^2 , otherwise ignored, accompanied by $a_{\nu,\chi}$ and $b_{\nu,\chi}$ as these couplings could enhance it.

This above simplified model provides a minimal realization of Dirac neutrino mass through a tree-level Dirac seesaw mechanism, via mixing with an additional neutral Dirac fermion χ . Here, m_{light} is the light active neutrino masses and the heavier state can be considered as a heavy sterile neutrino which is a Dirac particle. Due to sizeable mixing between these light and heavy states, χ can be unstable and decays to three active neutrino final states as, $\chi \rightarrow \nu S^* \rightarrow 3\nu$.

B. Model I'

To ensure the stability of χ and allow it to be a viable dark matter candidate (or component of dark matter), we impose a discrete $\mathcal{Z}_2$ symmetry in Model I under which χ'_L and

χ'_R are odd, while all other fields are even. This forbids terms that couple ν to χ , leading to the Lagrangian

$$-\mathcal{L} = m_\nu \overline{\nu'_L} \nu'_R + M \overline{\chi'_L} \chi'_R + (a_\nu \overline{\nu'_L} \nu'_L + b_\nu \overline{\nu'^c_R} \nu'_R + a_\chi \overline{\chi'_L} \chi'_L + b_\chi \overline{\chi'^c_R} \chi'_R) S^* + \text{h.c.} \quad (15)$$

The masses are already diagonal in this case and there is no mass mixing between ν and χ . This then simply yields two Dirac pairs with masses $m_{\text{light}} = m_\nu$ and $m_{\text{heavy}} = M$. The interaction Lagrangian in this basis becomes

$$-\mathcal{L} = \overline{\nu^c} (a_\nu P_L + b_\nu P_R) \nu S^* + \overline{\chi^c} (a_\chi P_L + b_\chi P_R) \chi S^* + \text{h.c.} \quad (16)$$

As a consequence, the lightest Z_2 -odd particle, χ , is kinematically forbidden from decaying into lighter SM particles.

C. Model II

Finally, we consider a scenario involving two distinct, exotic neutral Dirac fermions denoted as χ'_1 and χ_2 , with the lepton number assignments $L_{\nu'_L} = +1 = L_{\nu'_R}, L_{\chi'_{1L}} = -2 = L_{\chi'_{1R}}, L_{\chi_2} = 0, L_S = 2$. The active SM neutrinos retain the canonical lepton number values. The Lagrangian respecting these assignments is given by

$$\begin{aligned} -\mathcal{L} = & m_\nu \overline{\nu'_L} \nu'_R + M_1 \overline{\chi'_{1L}} \chi'_{1R} + M_2 \overline{\chi_2^c} \chi_2 \\ & + (a_\nu \overline{\nu'^c_L} \nu'_L + b_\nu \overline{\nu'^c_R} \nu'_R + a_{\chi_1 \chi_2} \overline{\chi'_{1R}} \chi_2 + b_{\chi_1 \chi_2} \overline{\chi'_{1L}} \chi_2) S^* + \text{h.c.} \end{aligned} \quad (17)$$

The corresponding neutral fermion mass matrix in the basis of $(\nu'_L, \nu'^c_R, \chi'_{1L}, \chi'^c_{1R}, \chi_2)^T$ takes the form

$$\mathcal{M} \sim \begin{pmatrix} 0 & m_\nu & 0 & 0 & 0 \\ m_\nu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_1 & 0 \\ 0 & 0 & M_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_2 \end{pmatrix} \quad (18)$$

In this framework, we have three mass-decoupled neutral fermions: the active neutrinos form a Dirac state $\nu' = \nu = \nu'_L + \nu'^c_R$ with mass m_ν , the field $\chi'_1 = \chi_1 = \chi'_{1L} + \chi'^c_{1R}$ becomes a Dirac state with mass $m_{\chi_1} = M_1$ and χ_2 is a Majorana fermion with mass M_2 . In the mass basis, the interactions are

$$-\mathcal{L} = \overline{\nu^c} \{a_\nu P_L + b_\nu P_R\} \nu S^* + \overline{\chi_1^c} \{a_{\chi_1 \chi_2} P_L + b_{\chi_1 \chi_2} P_R\} \chi_2 S^* + \text{h.c.} \quad (19)$$

This framework is mainly intended to highlight that interactions involving both Dirac and Majorana neutral fermions through the scalar mediator are possible.

D. Scalar Decay Width

Before discussing double beta decay in detail, we determine the total decay width of the scalar S in all three models. Considering ϕ and $\psi_{1,2}$ as generic scalar and fermions, respectively, with the coupling $\overline{\psi}_1^c(aP_L + bP_R)\psi_2\phi^* + \text{h.c.}$, the corresponding partial decay width of $\phi \rightarrow \psi_1\psi_2$ is given by,

$$\Gamma(\phi \rightarrow \psi_1\psi_2) = \frac{m_\phi}{16\pi} \{ (|a|^2 + |b|^2) (1 - x_1^2 - x_2^2) - 4\text{Re}[a^*b] x_1x_2 \} \times \sqrt{1 - (x_1 - x_2)^2} \sqrt{1 - (x_1 + x_2)^2}, \quad (20)$$

with the mass ratios $x_i = m_{\psi_i}/m_\phi$ ($i = 1, 2$). The decay width to a pair of identical fermions $\psi_1 = \psi_2 = \psi$ includes a symmetry factor and can be written as

$$\Gamma(\phi \rightarrow \psi\psi) = \frac{m_\phi}{32\pi} \{ (|a|^2 + |b|^2) (1 - 2x^2) - 4\text{Re}[a^*b] x^2 \} \sqrt{1 - 4x^2}, \quad (21)$$

with $x = m_\psi/m_\phi$. Therefore, the total decay widths for the scalar S in our models are

- Model I:

$$\Gamma_S = \Gamma(S \rightarrow \nu\nu) + \Gamma(S \rightarrow \nu\chi) + \Gamma(S \rightarrow \chi\chi). \quad (22)$$

Furthermore, the heavier state χ is not stable as it can decay via an off-shell S , $\chi \rightarrow \nu S^* \rightarrow 3\nu$ as well as through the SM currents due to the mixing.

- Model I':

$$\Gamma_S = \Gamma(S \rightarrow \nu\nu) + \Gamma(S \rightarrow \chi\chi). \quad (23)$$

Due to the imposed Z_2 symmetry, there is no $\chi\nu S$ interaction and χ is stable. This affects the cosmological bounds on the model parameter space as will be discussed in Sec. IV.

- Model II:

$$\Gamma_S = \Gamma(S \rightarrow \nu\nu) + \Gamma(S \rightarrow \bar{\chi}_1\chi_2). \quad (24)$$

In this model, the heavier fermion state, say, χ_2 can decay via an off-shell S , $\chi_2 \rightarrow \chi_1 S^* \rightarrow \chi_1 + 2\nu$.

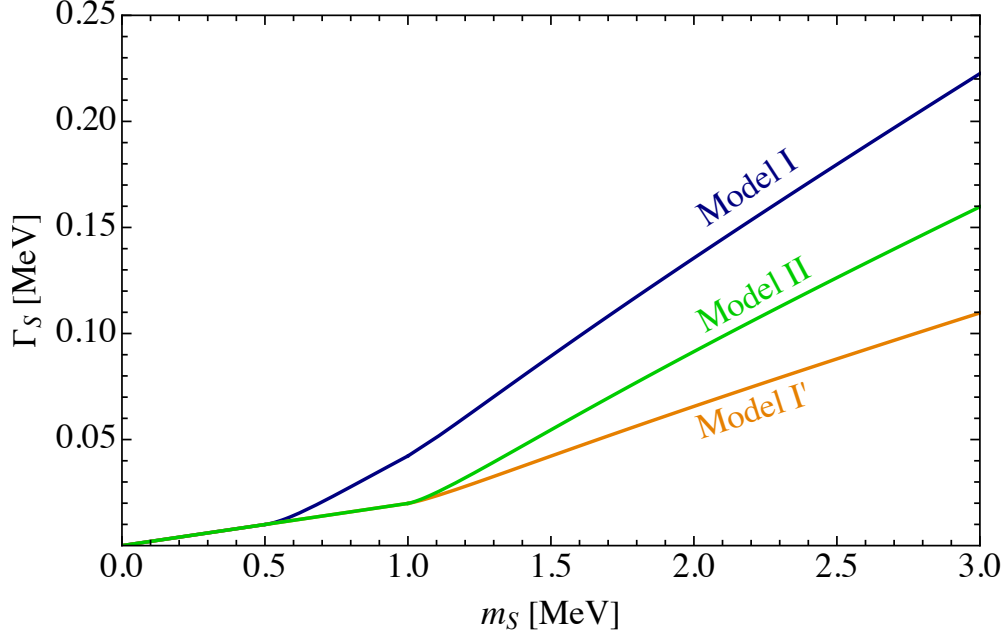


FIG. 2: Total decay width of the scalar particle S as a function of its mass m_S in the three different models considered. The active neutrino is considered massless whereas we take $m_{\chi_{1,2}} = 0.5$ MeV. In each model, all a_i and b_i couplings are set to unity. The active-sterile mixing in Model I is set to $\theta = 10^{-4}$.

In Fig. 2, we plot the total decay widths of S as a function of its mass m_S in the three different models considered. Here, the active neutrino is considered massless whereas we take $m_{\chi_{1,2}} = 0.5$ MeV for the heavier fermion(s). The active-sterile mixing in Model I is set at $\theta = 10^{-4}$ and all the couplings a_i, b_i ($i = \nu, \chi, \nu\chi, \chi_1\chi_2$ as discussed for each model) are considered to be unity. As expected for an MeV-scale scalar, the widths are small, reaching at most $\Gamma_S \sim 220$ keV in Model I due its higher number of decay channels with thresholds at $m_S = 0.5$ MeV ($\nu\chi$ accessible) and 1 MeV ($\chi\chi$ accessible). The total decay width of S for Model I' and Model II remains the same for $m_S < 1$ MeV (due to only accessible $\nu\nu$ channel), beyond that Model II decay width dominates over Model I', which is evident from the expressions (23) and (24).

III. DOUBLE BETA DECAY

In Sec. II, we saw that the presence of BSM neutral fermions could lead to new phenomenology in double beta decay experiments. In particular, Sec. II revealed that the

most promising scenario involves a scalar mediator in an s -channel. Let us now analyze this possibility. We will work with two different χ particles and with different couplings for the right and left-handed fermions for the sake of generality.

From Fig. 1, it can be observed that the nuclear structure of both processes ($\chi_1\chi_2\beta\beta$ decay and scalar emission) shares the same nuclear structure as the $0\nu\beta\beta$ decay, since the corresponding Feynman diagrams are identical except for the emission of a scalar particle. Consequently, all the constants involved in these processes are those appearing in $0\nu\beta\beta$ decay, with the addition of the new interaction terms. We therefore denote these common constants generically as $\mathcal{N}_{0\nu}$, which we match to the conventions used in the literature in order to ensure the correct normalization of the nuclear matrix elements (NMEs). See Appendix B for further details. We here present the derivation of the decay width for the generic $\chi_1\chi_2\beta\beta$ decay via an s -channel scalar. We also show the results for the interference with the standard $2\nu\beta\beta$ decay, in the case $S \rightarrow \bar{\nu}\bar{\nu}$, i.e., for the emission of two active anti-neutrinos. For comparison and later analysis we also show the rates for scalar particle emission $0\nu\beta\beta S$ (with ordinary Majoron emission $0\nu\beta\beta J$ as massless limit) and the SM $2\nu\beta\beta$ decay. Here note that we omit the calculation of $2\nu_{t\text{-channel}} s\beta\beta$ via the exchange of a t -channel scalar as we are here mainly interested in the modification of the spectrum near the scalar production threshold and this contribution is expected to be sub-dominant due to the higher momentum $|q^2| \approx (100 \text{ MeV})^2$ in the propagator [13]. It would also require the calculation of new types of NMEs.

A. $\chi_1\chi_2\beta\beta$ Decay

The effective Lagrangian can thus be written as

$$\mathcal{L} = -\bar{\nu}^c (a_\nu P_L + b_\nu P_R) \nu S^* - \bar{\chi}_i^c (a_\chi P_L + b_\chi P_R) \chi_j S^* + \text{h.c.} \quad (25)$$

As we noted in Sec. II, we can recover the pure scalar and pseudoscalar interactions by imposing $a_\nu = \pm b_\nu$. In this section, we will treat χ_i and χ_j as different particles. If they are identical particles, a symmetry factor of 1/2 must be included.

The interactions above lead to two different scenarios depending on whether the mediator is produced on- or off-shell. The corresponding Feynman diagrams are shown in Figure 1.

The diagram on the left represents $0\nu\beta\beta S$ decay, which occurs when the production of the scalar mediator S is kinematically allowed, i.e., $m_S < Q$. The second diagram represents a potential portal to probe new fermions. Here, we focus on the case where the χ particles are massive, which leads to a distinctive kink in the decay spectrum, provided that the condition $m_{\chi_1} + m_{\chi_2} < Q$ is satisfied. We begin by computing the amplitude for the process as depicted in Figure 1 (right). The leptonic part of the matrix element for the $\chi_1\chi_2\beta\beta$ decay is given by

$$L_{2\chi}^{\mu\nu} = \frac{\mathcal{N}_{0\nu}}{p^2 - m_S^2 + im_S\Gamma_S} \left[\bar{u}(p_1)\gamma^\mu P_L \frac{\not{q} + m_\nu}{q^2 - m_\nu^2} (a_\nu^* P_R + b_\nu^* P_L) \frac{\not{p} - \not{q} + m_\nu}{(p - q)^2 - m_\nu^2} \gamma^\nu P_R v(p_2) \right] \\ \times [\bar{u}(p_3) (a_\chi^* P_R + b_\chi^* P_L) v(p_4)] . \quad (26)$$

The outgoing electrons have momenta p_1 and p_2 whereas the exotic fermions χ_1 and χ_2 have momenta p_3 and p_4 , respectively. The neutrino mass is m_ν , and m_S and Γ_S are the mass and decay width for the scalar mediator S , respectively, which has momentum $p = p_3 + p_4$. Due to the small $Q \sim \text{MeV}$ value in double beta decay isotopes and the tiny neutrino masses $m_\nu \lesssim 0.01 \text{ eV}$, we can assume that the final state lepton momenta and the neutrino masses are negligible compared to the momentum exchanged through the neutrino propagators, $|q^2| \approx (100 \text{ MeV})^2 \gg |p^2|, m_\nu^2$, and the amplitude simplifies

$$L_{2\chi}^{\mu\nu} \simeq \frac{\mathcal{N}_{0\nu} a_\nu^*}{q^2} \frac{g^{\mu\nu}}{p^2 - m_S^2 + im_S\Gamma_S} [\bar{u}(p_1) P_R v(p_2)] [\bar{u}(p_3) (a_\chi^* P_R + b_\chi^* P_L) v(p_4)] . \quad (27)$$

Here, we have used the symmetry under the $\mu \leftrightarrow \nu$ exchange of the hadronic contribution.

To compute the leptonic phase space of the process, we adopt the $S_{1/2}$ approximation for the outgoing electrons. For each electron, we include the Fermi function $F_0(Z_f, E_i)$ ($i = 1, 2$), which depends on the energy of the electron. It describes the modification of the electron wave function by the electromagnetic potential of the final state nucleus with charge $Z_f = Z + 2$ [31],

$$F_0(Z_f, E) = 4 \frac{(2R\sqrt{E^2 - m_e^2})^{2(\gamma_0 - 1)}}{\Gamma^2(1 + 2\gamma_0)} e^{\pi y} |\Gamma(\gamma_0 + iy)|^2 . \quad (28)$$

Here, $R = 1.2 \text{ fm} \cdot A^{1/3}$ denotes the nuclear radius, $\Gamma(x)$ is the Gamma function, $\gamma_0 = \sqrt{1 - (\alpha Z_f)^2}$, and $y = \alpha Z_f E / \sqrt{E^2 - m_e^2}$, where $\alpha = 1/137$ is the fine-structure constant. As in $0\nu\beta\beta$ decay, we assume the closure approximation, which allows us to factorize the amplitude of the process into separate leptonic and hadronic contributions. The squared

amplitude can then be expressed as

$$|iM_{2\chi}|^2 = \left| \frac{\mathcal{N}_{0\nu} a_\nu}{p^2 - m_S^2 + im_S \Gamma_S} \right|^2 F_0(p_1) F_0(p_2) |\mathcal{M}_{0\nu}|^2 \\ \times 4p_1 \cdot p_2 \left[(|a_\chi|^2 + |b_\chi|^2) p_3 \cdot p_4 - 2\text{Re}[(a_\chi)^* b_\chi] m_{\chi_1} m_{\chi_2} \right], \quad (29)$$

where the NME, $\mathcal{M}_{0\nu}$, is identical to the one in standard $0\nu\beta\beta$ decay, due to the same hadronic structure. We will write the total $\chi_1\chi_2\beta\beta$ decay width as

$$\Gamma_{2\chi} = \mathcal{G}_{2\chi} |\mathcal{M}_{0\nu}|^2, \quad (30)$$

The phase space factor $\mathcal{G}_{2\chi}$ is calculated by integrating

$$\mathcal{G}_{2\chi} = |\mathcal{N}_{0\nu}|^2 |a_\nu|^2 \\ \times \int \prod_{i=1}^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} 4p_1 \cdot p_2 F_0(p_1) F_0(p_2) \frac{(|a_\chi|^2 + |b_\chi|^2) p_3 \cdot p_4 - 2\text{Re}[a_\chi^* b_\chi] m_{\chi_1} m_{\chi_2}}{|p^2 - m_S^2 + im_S \Gamma_S|^2} \\ \times \delta(Q - T - E_3 - E_4), \quad (31)$$

over the phase space of the final state fermions with $p_i = (E_i, \mathbf{p}_i)$. First, we integrate over the phase space of the invisible fermions χ_1 and χ_2 , yielding two integrals

$$I_1 = \int \prod_{i=3}^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \frac{p_3 \cdot p_4}{|(p_3 + p_4)^2 - m_S^2 + im_S \Gamma_S|^2} \delta(Q - T - E_3 - E_4), \\ I_2 = m_{\chi_1} m_{\chi_2} \int \prod_{i=3}^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \frac{1}{|(p_3 + p_4)^2 - m_S^2 + im_S \Gamma_S|^2} \delta(Q - T - E_3 - E_4), \quad (32)$$

with the total kinetic energy of the electrons $T = E_1 + E_2 - 2m_e$. Defining

$$x = \frac{2|\mathbf{p}_3||\mathbf{p}_4|}{m_S^2 - m_{\chi_1}^2 - m_{\chi_2}^2 - 2E_3 E_4}, \quad (33)$$

with $|\mathbf{p}_3| = (E_3^2 - m_{\chi_1}^2)^{1/2}$, $|\mathbf{p}_4| = (E_4^2 - m_{\chi_2}^2)^{1/2}$ and $E_4 = Q - T - E_3$, we can integrate analytically over E_4 and the angles of $\mathbf{p}_3$ and $\mathbf{p}_4$, to express the integrals as

$$I_1(T) = \frac{1}{4(2\pi)^4} \int_{m_{\chi_1}}^{Q-T-m_{\chi_2}} dE_3 \left\{ \frac{m_S^2 - m_{\chi_1}^2 - m_{\chi_2}^2}{2m_S \Gamma_S} [\arctan \zeta_+(E_3) + \arctan \zeta_-(E_3)] \right. \\ \left. + \frac{1}{4} \log \left[\frac{\zeta_-^2(E_3) + 1}{\zeta_+^2(E_3) + 1} \right] \right\}, \quad (34)$$

and

$$I_2(T) = \frac{1}{4(2\pi)^4} \frac{m_{\chi_1} m_{\chi_2}}{m_S \Gamma_S} \int_{m_{\chi_1}}^{Q-T-m_{\chi_2}} dE_3 [\arctan \zeta_+(E_3) + \arctan \zeta_-(E_3)]. \quad (35)$$

Here,

$$\zeta_{\pm}(E_3) = \frac{2|\mathbf{p}_3||\mathbf{p}_4|(x \pm 1)}{m_S \Gamma_S x}. \quad (36)$$

In Eqs. (34) and (35), we expressed the integrands using the relation

$$\arctan \zeta_+ + \arctan \zeta_- = \arctan \left(\frac{\zeta_+ + \zeta_-}{1 - \zeta_+ \zeta_-} \right) \quad (37)$$

as this turns out to be more numerically stable for physical, i.e., small values of Γ_S when evaluating the integrands.

Then, $\mathcal{G}_{2\chi}$ can be expressed as

$$\begin{aligned} \mathcal{G}_{2\chi} &= |\mathcal{N}_{0\nu}|^2 |a_\nu|^2 \\ &\times \int \prod_{i=1}^2 \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} 4p_1 \cdot p_2 F_0(p_1) F_0(p_2) \left[(|a_\chi|^2 + |b_\chi|^2) I_1(T) - 2\text{Re} [a_\chi^* b_\chi] I_2(T) \right]. \end{aligned} \quad (38)$$

It is convenient to transform this expression into one that depends on the electron kinetic energies, $T_1 = E_1 - m_e$ and $T_2 = E_2 - m_e$, and the angle θ_{12} between the emitted electrons, with $\mathbf{p}_1 \cdot \mathbf{p}_2 = |\mathbf{p}_1||\mathbf{p}_2| \cos \theta_{12}$. The fully differential decay rate can then be written as

$$\begin{aligned} \frac{d\Gamma_S^{2\chi}}{dT_1 dT_2 d\cos \theta_{12}} &= \kappa_{0\nu} \left(\frac{m_e}{2R} \right)^2 |\mathcal{M}_{0\nu}|^2 |\mathbf{p}_1||\mathbf{p}_2| E_1 E_2 F_0(p_1) F_0(p_2) \left(1 - \frac{|\mathbf{p}_1||\mathbf{p}_2|}{E_1 E_2} \cos \theta_{12} \right) \\ &\times |a_\nu|^2 \left\{ (|a_\chi|^2 + |b_\chi|^2) I_1(T_1 + T_2) - 2\text{Re} [a_\chi^* b_\chi] I_2(T_1 + T_2) \right\}, \end{aligned} \quad (39)$$

where we have explicitly extracted the factor $m_e/(2R)$ to emphasize that $\mathcal{M}_{0\nu}$ is dimensionless and that the phase space factor $\mathcal{G}_{2\chi}$ has units of yr^{-1} , to conform to the usual conventions employed in $0\nu\beta\beta$ decay calculations. Here $|\mathbf{p}_i|$ and E_i are understood to be expressed in terms of the T_i and the factor $\kappa_{0\nu}$ has been obtained using the correct normalization for $\mathcal{N}_{0\nu}$,

$$\kappa_{0\nu} = \frac{|\mathcal{N}_{0\nu}|^2 R^2}{2\pi^4 m_e^2} = \frac{G_F^4 \cos^4 \theta_C}{8\pi^5 m_e^2}, \quad (40)$$

with the Cabibbo angle θ_C .

Most double beta decay experiments measure only the differential decay rate $d\Gamma/dT$ with respect to the total electron kinetic energy $T = T_1 + T_2$. Integrating Eq. (39) over the angle and energy difference $\Delta T = T_1 - T_2$ yields the result

$$\begin{aligned} \frac{d\Gamma_S^{2\chi}}{dT} &= \kappa_{0\nu} \left(\frac{m_e}{2R} \right)^2 |\mathcal{M}_{0\nu}|^2 |a_\nu|^2 \left\{ (|a_\chi|^2 + |b_\chi|^2) I_1(T) - 2\text{Re} [a_\chi^* b_\chi] I_2(T) \right\} g(T) \\ &\times \Theta(Q - m_{\chi_1} - m_{\chi_2} - T), \end{aligned} \quad (41)$$

with the Heaviside function $\Theta(x)$ and

$$g(T) = \int_{-T}^T d\Delta T |\mathbf{p}_1| |\mathbf{p}_2| E_1 E_2 F_0(|\mathbf{p}_1|) F_0(|\mathbf{p}_2|), \quad (42)$$

where we express the integrand as $E_1 = \frac{1}{2}(T + \Delta T) + m_e$, $E_2 = \frac{1}{2}(T - \Delta T) + m_e$ and $|\mathbf{p}_i| = (E_i^2 - m_e^2)^{1/2}$ ($i = 1, 2$). Finally, to compute the total decay width, we integrate over T ,

$$\Gamma_S^{2\chi} = \int_0^{Q-m_{\chi_1}-m_{\chi_2}} dT \frac{d\Gamma_{2\chi}}{dT}. \quad (43)$$

The above generic calculation for distinct, massive χ_1, χ_2 is applicable to all contributions considered here, with an additional symmetry factor of $1/2$ for identical particles, i.e., $2\nu_S\beta\beta$ and $2\chi_S\beta\beta$. The above expressions can be simplified for numerical evaluation if one or both χ are massless, due to $|\mathbf{p}_i| = E_i$.

B. Interference between SM $2\nu\beta\beta$ and Scalar-induced $2\nu_S\beta\beta$

In the case of the scalar S producing two active anti-neutrinos, its contribution will interfere with the standard $2\nu\beta\beta$ decay. This interference is determined by the complex-valued integral

$$\begin{aligned} I_{\text{int}}(T) &= \int \prod_{i=3}^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \frac{p_3 \cdot p_4}{(p_3 + p_4)^2 - m_S^2 + i m_S \Gamma_S} \delta(Q - T - E_3 - E_4) \\ &= \frac{m_S^2 - i \Gamma_S m_S}{8(2\pi)^4} \int_0^{Q-T} dE_3 \left\{ \frac{4E_3 E_4}{m_S^2 - i \Gamma_S m_S} + \frac{1}{2} \log \left(\frac{(m_S^2 - 4E_3 E_4)^2 + \Gamma_S^2 m_S^2}{m_S^2 (\Gamma_S^2 + m_S^2)} \right) \right. \\ &\quad \left. + i \left[\text{arccot} \left(\frac{\Gamma_S m_S}{m_S^2 - 4E_3 E_4} \right) - \text{arccot} \left(\frac{\Gamma_S}{m_S} \right) \right] \right\}. \quad (44) \end{aligned}$$

Here, we note that the convention for $\text{arccot}(x)$ is such that it has the range $-\frac{\pi}{2} < \text{arccot}(x) < \frac{\pi}{2}$.

The contribution of the decay rate from this interference is given by

$$\frac{d\Gamma_{\text{int}}^{2\nu}}{dT} = 4\pi \kappa_{0\nu} \left(\frac{m_e}{2R} \right) \text{Re} \left[a_\nu^2 \mathcal{M}_{0\nu} \mathcal{M}_{2\nu}^* I_{\text{int}}(T) \right] g(T) \Theta(Q - T). \quad (45)$$

It thus depends on both the $0\nu\beta\beta$ decay NME $\mathcal{M}_{0\nu}$ and the $2\nu\beta\beta$ decay NME $\mathcal{M}_{2\nu}$. As an interference between a SM and exotic contribution, it is proportional to the small exotic coupling to second power only. It is also enhanced relative to SM $2\nu\beta\beta$ decay by the

appearance of the larger NME $M_{0\nu}$. We still expect that its contribution will be small as the interference requires a sizeable width Γ_S or off-shell S production, $T > Q - m_S$, which is strongly suppressed compared to the on-shell production.

C. Scalar Emission

Taking the limit of zero width, $\Gamma_S \rightarrow 0$ and considering the emission of S as a real particle, we recover the case of scalar emission $0\nu\beta\beta S$,

$$\frac{d\Gamma_S}{dT} = \frac{\kappa_{0\nu}}{8\pi^2} \left(\frac{m_e}{2R}\right)^2 |\mathcal{M}_{0\nu}|^2 |a_\nu|^2 g(T) \sqrt{(Q-T)^2 - m_S^2} \Theta(Q - m_S - T). \quad (46)$$

We note that this limit is, strictly speaking, not self-consistent as at least the decay to massless active neutrinos is always allowed if $m_S > 0$ and thus $\Gamma_S > 0$. If all couplings involved are small, this is still a well justified approximation. For $m_S = 0$, the above expression gives the standard Majoron contribution $0\nu\beta\beta J$ [20]. As we express our result as distributions over T and ΔT instead of the electron energies, an additional factor of $1/2$ appears in the pre-factor compared to the expression commonly found in the literature [20, 24].

D. Standard Model $2\nu\beta\beta$ Decay

Finally, we give the spectrum for standard $2\nu\beta\beta$ decay as the irreducible background to the exotic contributions presented above. Its calculation is non-trivial and it receives corrections to the spectral shape due the dependence of the corresponding NME on the final lepton momenta [11]. For consistency with the above calculation, we ignore such corrections and use the usual expression, decoupling the NME and the phase space integral,

$$\frac{d\Gamma_{\text{SM}}^{2\nu}}{dT} = \frac{\kappa_{0\nu}}{2\pi^2} |M_{2\nu}|^2 g(T) \frac{1}{30} (Q-T)^5 \Theta(Q-T), \quad (47)$$

matching with the existing literature in this approximation [11].

E. Spectral Shapes

In Fig. 3, we show the decay distribution of $2\chi\beta\beta$ for increasing values of the s -channel mediator mass $m_S = 0.5$ MeV (blue), 2.8 MeV (green) and 3.5 MeV (red) in Model I' for

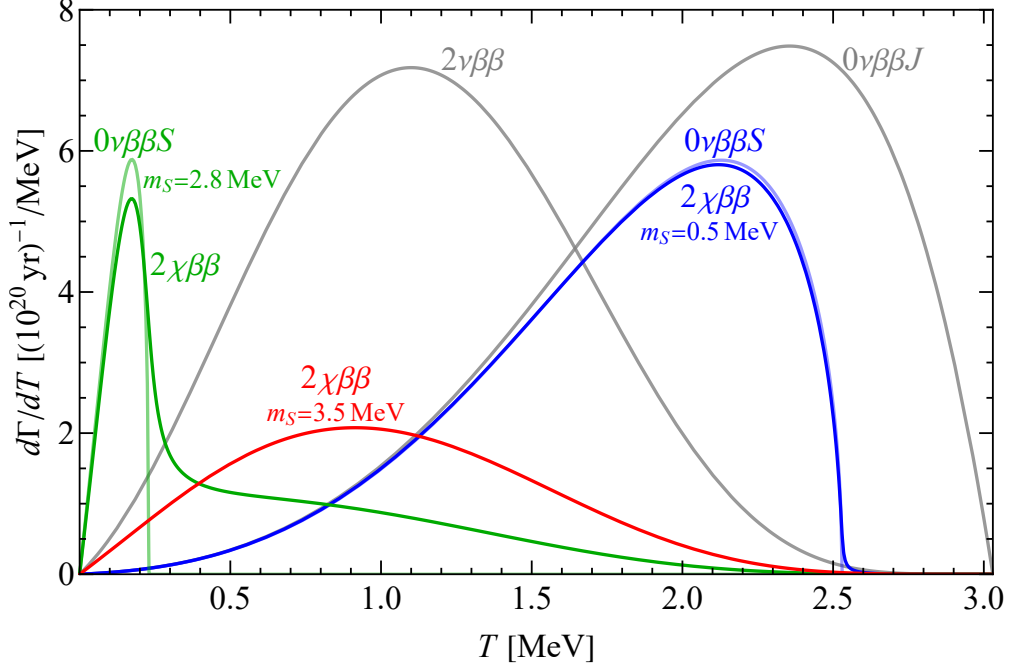


FIG. 3: Differential distribution $d\Gamma/dT$ of $2\chi\beta\beta$ decay of ^{100}Mo with respect to the total electron kinetic energy T in Model I' for three different parameter sets: $(m_S, a_\nu, a_\chi) = (0.5 \text{ MeV}, 3 \times 10^{-3}, \sqrt{2})$ (blue), $(2.8 \text{ MeV}, 7 \times 10^{-2}, \sqrt{2})$ (green) and $(3.5 \text{ MeV}, 2 \times 10^{-1}, \sqrt{2})$ (red). All other couplings are set to zero and the χ are massless. For comparison, the scalar emission contributions $0\nu\beta\beta S$ are also shown, for the corresponding values of m_S and a_ν . The SM $2\nu\beta\beta$ and ordinary Majoron emission $0\nu\beta\beta$ (with $a_\nu = 3 \times 10^{-3}$) are shown in grey. The relevant NMEs are given in Table I.

massless χ . The spectra, for ^{100}Mo having the largest Q value $Q = 3.03 \text{ MeV}$ among the isotopes considered here, are not normalized but the couplings of S to neutrinos and χ are chosen such that the features are visually enhanced: $(a_\nu, a_\chi) = (3 \times 10^{-3}, \sqrt{2})$ (blue), $(7 \times 10^{-2}, \sqrt{2})$ (green) and $(2 \times 10^{-1}, \sqrt{2})$ (red). Thus, the coupling a_χ to χ is large and always dominant compared to a_ν . Thus, S almost exclusively decays to χ with a branching ratio $\text{Br}(S \rightarrow \chi\chi) \approx 1$.

This can be most easily seen by comparing with the scalar-emission spectrum $0\nu\beta\beta S$ for $m_S = 0.5 \text{ MeV}$ that is also shown and which almost overlaps with $2\chi\beta\beta$ as we effectively have

$$\frac{d\Gamma_S^{2\chi}}{dT} \approx \frac{d\Gamma_S}{dT} \times \text{Br}(S \rightarrow \chi\chi), \quad (48)$$

in narrow-width approximation. Only a slight reduction and small tail above the scalar emission threshold $T = Q - m_S \approx 2.53$ MeV can be seen due to finite width effects from the total decay width $\Gamma_S \approx 10^{-2}$ MeV. By comparing with the ordinary (massless) Majoron emission spectrum $0\nu\beta\beta J$ (for the same $a_\nu = 3 \times 10^{-3}$), we can also see the common phase space reduction due to the massive S .

In the green scenario, $m_S = 2.8$ MeV, S is near the threshold to be produced on-shell, leading to a peak at small electron energies with a nominal threshold at $Q - m_S \approx 0.23$ MeV. In many double beta decay experiments, backgrounds become dominant in this regime and such a peak would be difficult to detect. The width of S becomes sizeable, though, with $\Gamma_S \approx 0.06$ MeV and, most importantly, $2\chi\beta\beta$ decay can be induced by an off-shell S , i.e., for electron energies $T > Q - m_S \approx 0.23$ MeV, leading to a contribution in the higher energy region where it is more easily detectable.

Finally, in the red scenario, the scalar S has a mass of 3.5 MeV and it cannot be produced on-shell in ^{100}Mo double beta decay. The resulting spectrum of $2\chi\beta\beta$ decay has a shape similar to SM $2\nu\beta\beta$, increasingly so with larger m_S where the scalar mediation can be described by an effective $\nu\nu\chi\chi$ operator, $m_S \gg Q$ [13].

In Fig. 3, we always had $m_\chi = 0$. Instead, in Fig. 4 we show the dependence of the $2\chi_S\beta\beta$ on m_χ for a fixed $m_S = 0.5$ MeV. The blue, green and red distributions are for $m_\chi = 0.1$ MeV, 0.5 MeV and 1.0 MeV, respectively, where the a_ν coupling is changed accordingly to show a visible spectrum. In the blue case, S can still be produced on-shell and there is no discernible effect compared to the massless χ case in Fig 3. On the other hand, for $m_\chi = 0.5$ MeV and 1.0 MeV, the mass threshold for χ production is more restrictive than that for S . The scalar can thus not be produced on-shell. Because of this, the depicted spectra are expected to be difficult to observe as they compete with the always accessible, and resonantly enhanced, $2\nu_S\beta\beta$ contribution.

In Fig. 5, we show an example of the full $2\nu\beta\beta$ decay spectrum, including contributions from SM $2\nu\beta\beta$, S -mediated $2\nu_S\beta\beta$ and their interference (green). The scalar mass is $m_S = 2.8$ MeV and the coupling is $a_\nu = 7 \times 10^{-2}$. All other couplings are zero, i.e., there is no

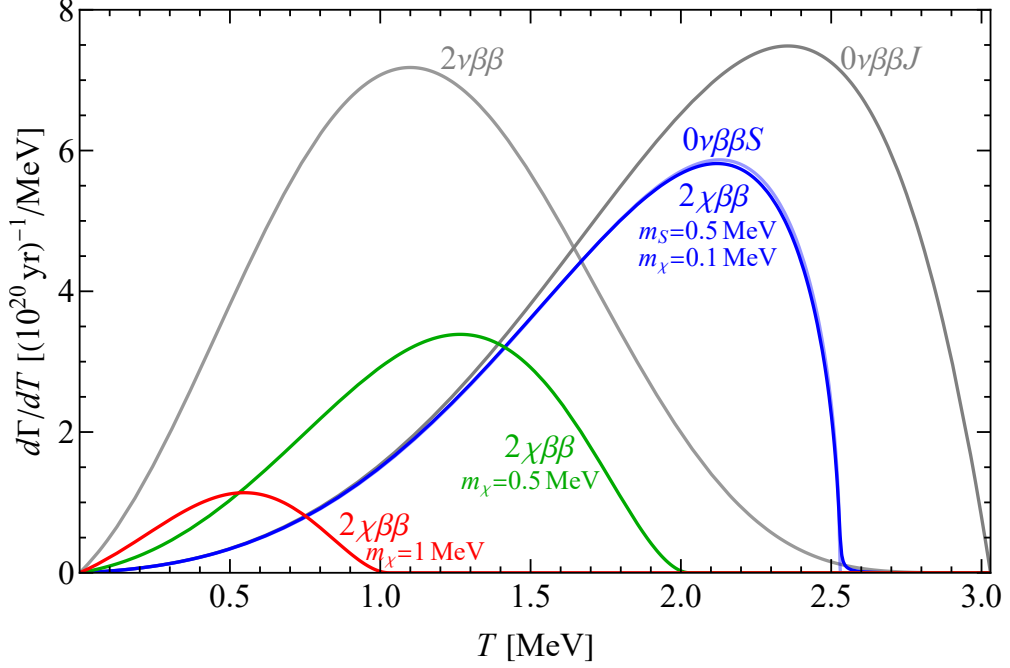


FIG. 4: As Fig. 3, but showing $d\Gamma_S^{2\chi}/dT$ for $m_S = 0.5$ MeV and $m_\chi = 0.1$ MeV (blue), 0.5 MeV (green) and 1.0 MeV (red). The non-zero couplings are $(a_\nu, a_\chi) = (3 \times 10^{-3}, \sqrt{2})$ (blue), $(7 \times 10^{-2}, \sqrt{2})$ (green) and $(3 \times 10^{-1}, \sqrt{2})$ (red).

coupling to χ and S exclusively decays to neutrinos in this case. The contributions add as

$$\begin{aligned}
\frac{d\Gamma^{2\nu}}{dT} &= |\mathcal{M}_{2\nu}|^2 \frac{d\Gamma_{\text{SM}}^{2\nu}}{dT} \\
&+ |a_\nu|^2 |\mathcal{M}_{0\nu}|^2 \frac{d\Gamma_S^{2\nu} \text{ on-shell}}{dT} + |a_\nu|^4 |\mathcal{M}_{0\nu}|^2 \frac{d\Gamma_S^{2\nu} \text{ off-shell}}{dT} \\
&+ 2 \text{Re} (a_\nu^2 \mathcal{M}_{0\nu} \mathcal{M}_{2\nu}^*) \frac{d\Gamma_{\text{int}}^{2\nu}}{dT},
\end{aligned} \tag{49}$$

where we have extracted the direct scaling of a_ν and the NMEs. Here, we have split up the on- and off-shell parts of the S -mediated contributions, where the latter scales with Γ_S^{-1} , leading to an overall proportionality with $|a_\nu|^2$ in this case. We see that the interference depends on both the $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs and their relative signs. We here take both NMEs to be positive which leads to a negative interference above the threshold and a suppression of the spectrum compared to pure SM $2\nu\beta\beta$ decay as shown in Fig. 5.

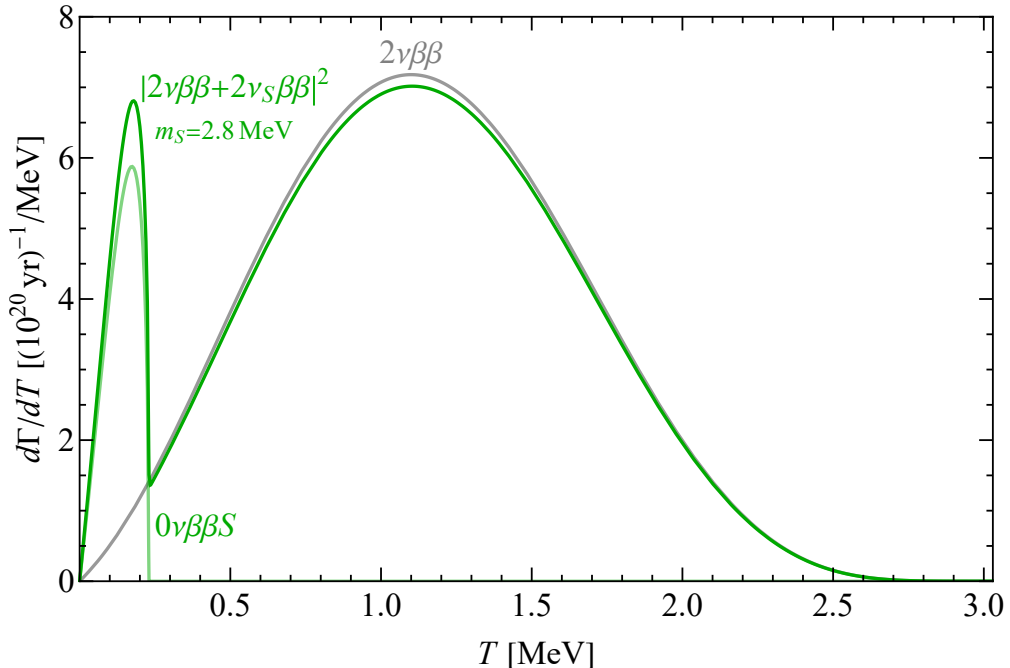


FIG. 5: Differential distribution $d\Gamma/dT$ of $2\nu\beta\beta$ decay of ^{100}Mo with respect to the total electron kinetic energy T including contributions from SM $2\nu\beta\beta$, S -mediated $2\nu_S\beta\beta$ and their interference (green). The scalar mass is $m_S = 2.8$ MeV and $a_\nu = 7 \times 10^{-2}$ with all other couplings set to zero. For comparison, the scalar emission contribution $0\nu\beta\beta S$ (using the same parameters) and SM $2\nu\beta\beta$ are also shown. The relevant NMEs are given in Table I.

IV. COSMOLOGY AND LABORATORY CONSTRAINTS

Interactions between neutrinos and other light particles produce a range of observable effects which can be constrained by imposing astrophysical, cosmological and laboratory bounds. We begin by reviewing the existing constraints on the frameworks presented in Sec. II, specifically Model I' as the exotic Dirac fermion χ is stable under the presence of a discrete Z_2 symmetry. These include limits on neutrino and exotic fermion (self-)interactions as well as their couplings to mediator particles. The constraints arise from a variety of sources, including cosmological observations such as those from Cosmic Microwave Background (CMB), Big Bang Nucleosynthesis (BBN), collisional damping, and thermal relic density considerations, as well as laboratory searches involving meson decays.

A. Cosmology Constraints

1. Thermal Relic

Considering equal number densities for DM particles and their antiparticles, the thermally averaged annihilation cross section (multiplied by the relative velocity) required to account for the observed DM abundance in present day universe, which can be expressed by a series expansion in relative velocity v_r (considering non-relativistic DM scenario with $v_r \ll c$) is [32, 33],

$$\langle \sigma v_r \rangle = a_r + b_r v_r^2 + d_r v_r^4. \quad (50)$$

To avoid DM overabundance, the upper limit on the DM thermal relic density [34], $\Omega_\chi h^2 \leq 0.120 \pm 0.001$ can be approximately translated to the lower limit [35, 36] on $\langle \sigma v_r \rangle \geq 3 \times 10^{-26} \text{cm}^3/\text{s}$ for a constant cross section at leading order in the expansion stated above, where σ corresponds to the annihilation cross section of DM species to active neutrino pair, $\chi\chi \rightarrow \nu\nu$. Additionally, if the cross section, at leading-order, depends on v_r^2 or v_r^4 , the required value at freeze-out will be $\langle \sigma v_r \rangle \simeq 6 \times 10^{-29} \text{cm}^3/\text{s}$ or $\langle \sigma v_r \rangle \simeq 9 \times 10^{-26} \text{cm}^3/\text{s}$, respectively [37].

2. Collisional Damping

Interactions between neutrinos and DM can significantly affect the distribution of matter and radiation in the Universe, particularly by affecting the CMB spectrum and the formation of large-scale structures (LSS) [38–40]. There are two primary effects to consider in this perspective: the considered DM is no longer collisionless due to its interaction with neutrinos and the mediator particle, and the neutrinos remain collisional for a longer period compared to the standard cosmological scenario. This reduces the length of DM free streaming and enhances their ability to cluster more efficiently which eventually contributes to the formation of large-scale structures. By comparing theoretical predictions with CMB and LSS observations, constraints can be placed on the strength of ν -DM interactions. In this regard, Planck data suggests that the ν -DM elastic scattering cross section for the process $\nu\chi \rightarrow \nu\chi$ must be [28, 41]

$$\sigma_{\text{el}} < 10^{-29} \left(\frac{m_\chi}{\text{MeV}} \right) \text{cm}^2. \quad (51)$$

3. Neutrino Reheating Bounds

DM annihilations into neutrinos after they have decoupled from electrons, at temperatures $T \lesssim T_D \sim 1$ MeV (where T_D is the decoupling temperature of DM from electrons) [42–50], can reheat the neutrino sector, producing observable effects. This change in neutrino energy density effectively increases the number of relativistic degrees of freedom, N_{eff} , in the early Universe which can be expressed as

$$\rho_\nu \equiv \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^{4/3} N_{\text{eff}} \right], \quad (52)$$

where ρ_γ represents the photon energy density and T_ν, T_γ correspond to the temperature of neutrino and photons, respectively. In our framework, because of interactions with neutrinos, the DM particle remains in thermal equilibrium for a longer time. Therefore, the effective number of light relativistic species becomes [47, 51]

$$N_{\text{eff}} = N_\nu \left[1 + \frac{1}{N_\nu} \frac{g_\chi}{2} F \left(\frac{m_\chi}{T_D} \right)^{4/3} \right], \quad (53)$$

where $N_\nu = 3.046$ [52, 53] and $g_\chi = 4$ as the considered DM is a Dirac particle, the function $F(x)$ is defined as

$$F(x) = \frac{30}{7\pi^4} \int_x^\infty dy \frac{(4y^2 - x^2) \sqrt{y^2 - x^2}}{e^y \pm 1}. \quad (54)$$

4. CMB-NSI

Neutrino non-standard interactions (NSI) influence the epoch of neutrino free streaming and their coupling to the photonbaryon fluid in the early Universe. Such effects manifest as modifications in the phase shift and amplitude of the matter power spectrum, leading to stringent cosmological bounds. These constraints apply not only to neutrino self-interactions but also to interactions with a light dark sector, with implications for alleviating long-standing tensions in cosmological data, such as the H_0 and $\sum m_\nu$ tensions [28]. The most recent CMB analysis excludes effective four-neutrino interactions with strength G_{eff} [54] as

$$G_{\text{eff}} \simeq \frac{y_\nu^2}{m_S^2} < 5.6 \times 10^{-5} \text{ MeV}^{-2}, \quad (55)$$

where y_ν, m_S correspond to generic neutrino coupling and mediator mass, respectively.

5. CMB- ν DM

Additional constraints on ν -DM interactions arise from cosmological observables including the Lyman- α forest [55], structure formation [56] and small-scale CMB data [57]. The upper limits on the DM-neutrino scattering cross section can be parameterized as [55]

$$\sigma_{\nu\chi} < \sigma_n \left(\frac{m_\chi}{\text{GeV}} \right) \left(\frac{T_\nu}{T_\nu^0} \right)^n, \quad (56)$$

with $T_\nu^0 = 6.1$ K, $\sigma_0 = 2 \times 10^{-28}$ cm², $\sigma_2 = 2 \times 10^{-39}$ cm². We have also considered $n = 2$, as discussed in Ref. [28].

6. BBN

If both the mediator and light particles are at the MeV scale, additional constraints arise from Big Bang Nucleosynthesis observation. For light particles, the annihilation process $\nu\nu \rightarrow \chi\chi$, via s -channel scalar mediation, keeps the dark fermion in equilibrium with the SM plasma, leading to a significant contribution to N_{eff} . Considering the neutrino energy $E_\nu \sim T$, the interaction rate is given by $\Gamma_{\nu\chi} = \sigma(\nu\nu \rightarrow S \rightarrow \chi\chi)n_\nu(T)$, where $n_\nu(T)$ is the neutrino number density at temperature T . The relevant Hubble expansion rate is $H \approx 1.66\sqrt{g_*}\frac{T^2}{m_{\text{Pl}}}$ with $g_* = 10.75$ at $T \approx 1$ MeV. We have considered the neutrinos remain in thermal equilibrium with the SM plasma around this temperature. The BBN constraint follows from requiring $\frac{\Gamma_{\nu\chi}}{H} < 1$ [28] which eventually excludes the parameter space where exotic particle remains in equilibrium with the SM plasma and effectively increasing the N_{eff} .

B. Meson Decays

For a light scalar mediator S coupled to neutrinos and an exotic fermion, rare meson decays such as kaon decays, $K^\pm \rightarrow \ell^\pm + \nu/\chi + S$ provide strong constraints on the relevant couplings involved [58]. At tree level, these three-body decays suffer from infrared divergences in the limit $m_S \rightarrow 0$, with the divergent terms proportional to $m_\ell^2 \log(m_S^2/m_K^2)$. The divergences cancel one-loop corrections to the standard 2-body decay $K^\pm \rightarrow \ell^\pm \nu$ are included, in accordance with the Kinoshita-Lee-Nauenberg (KLN) theorem. For a more detailed analysis, we refer to [28]. Using kaon partial width data from the PDG [59], stringent

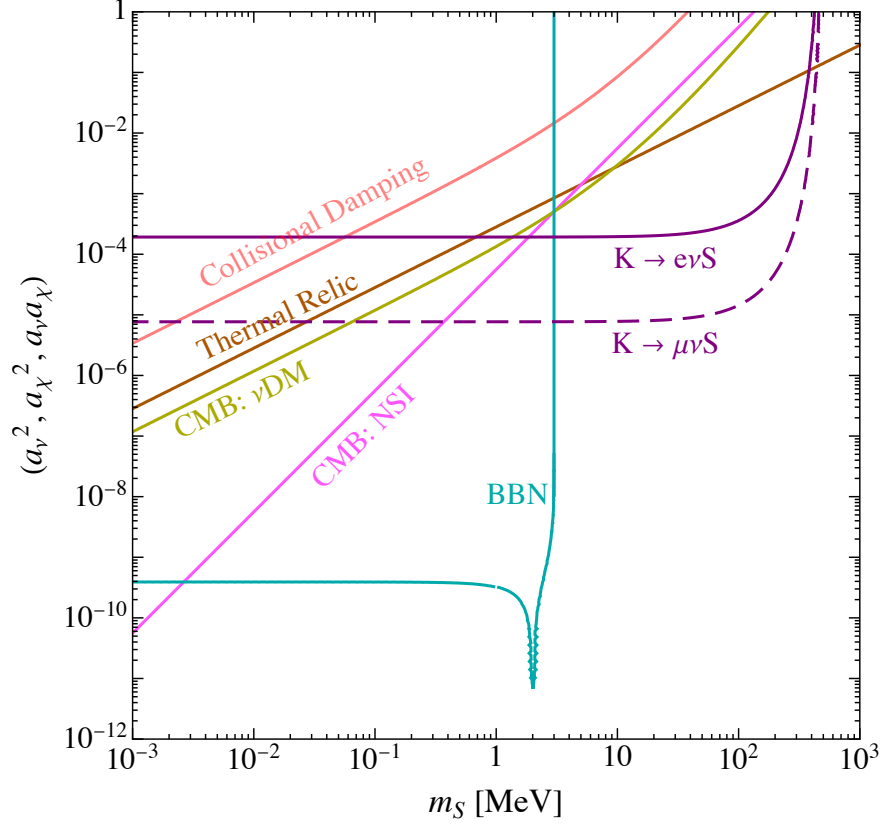


FIG. 6: Upper limit on (products of) couplings from cosmological (DM thermal relic, collisional damping, CMB and BBN) and kaon decay bounds as a function of the scalar mass m_S . The χ mass is set at $m_\chi = m_S/3$.

bounds can be placed on scalar-neutrino or scalar-exotic fermion interactions [28, 60]. We specifically consider the rare Kaon decay to constrain the parameter space as it gives the most stringent limit on the couplings for our masses of interest. There are other laboratory constraints arising from invisible Z decays, modification of the Fermi constant G_F and others. For a detailed overview, see [28].

C. Impact on Parameter Space

With the above considerations, we summarize the impact on the model parameters. For Model I' in our study, the relevant processes are : (i) DM self-annihilation to neutrino pair $\chi\chi \rightarrow \nu\nu$, (ii) DM-neutrino scattering $\nu\chi \rightarrow \nu\chi$ and (iii) DM pair-production from neutrinos $\nu\nu \rightarrow \chi\chi$. The corresponding matrix squared amplitudes and other relevant quantities for

such processes are given in Appendix A.

Fig. 6 shows the upper limit on couplings or products of couplings as a function of the scalar mass m_S . The combinations of couplings $a_\nu^2, a_\chi^2, a_\nu a_\chi$ are involved in different processes, in the simplified parameter space where we set $b_{\nu,\chi} = 0$. We also set $m_\chi = m_S/3$. The brown curve labeled "Thermal Relic" shows the coupling required to achieve the observed relic density for χ . Cosmological limits arise from the CMB, with yellow (CMB: ν DM) and purple (CMB: NSI) curves excluding parameter regions where scalar interactions distort the CMB anisotropy spectrum. The light red curve arises from collisional damping, where excessive interactions of S with neutrinos and DM χ interrupts structure formation. Laboratory constraints are dominated by rare kaon decays, $K \rightarrow e\nu S$ (solid magenta) and $K \rightarrow \mu\nu S$ (dashed magenta), which probe couplings down to $\sim 10^{-2}$ and $\sim 10^{-2.5}$, respectively, for $m_S \lesssim 100$ MeV. We here note, that $K \rightarrow \mu\nu S$ involves coupling to μ , i.e., not directly comparable to the electron coupling involved in double beta decay. BBN via the N_{eff} bound, excludes scalars lighter than ~ 3 MeV, indicated by the teal contour. This BBN bound imposes the most stringent limit on this parameter space for $m_S \leq \text{few MeV}$. For sufficiently larger values of couplings, the denominator of Eq. (A3) becomes proportional to $m_S^2 \Gamma_S^2$, then the BBN limit becomes independent of couplings, which has been shown by the vertical line. Similarly for smaller value of m_S , the cross section becomes independent of mediator mass, which has been presented as the horizontal line in the plot. Furthermore, the dip in this contour happens due to the resonance evaluating the scattering cross section $\nu\nu \rightarrow S \rightarrow \chi\chi$.

V. STATISTICAL PROCEDURE

Our goal is to estimate the sensitivity of current and future double beta decay experiments to the different models discussed in Sec. II, and we follow the standard frequentist approach [61]. The total double beta decay rate with respect to the electrons' kinetic energy T in our case is then generally written as

$$\frac{d\Gamma(\mathbf{x})}{dT} = \frac{d\Gamma_{\text{SM}}^{2\nu}}{dT} + \frac{d\Gamma_{\text{BSM}}(\mathbf{x})}{dT}. \quad (57)$$

Here, $d\Gamma_{\text{BSM}}^{2\nu}/dT$ represents the standard $2\nu\beta\beta$ decay contribution in the SM, which will act as background to the search for an exotic contribution $d\Gamma_{\text{BSM}}(\mathbf{x})/dT$. Here, $\mathbf{x}$ represents

the parameters of the model in question. For example, in the simplest scenario of scalar emission, $\mathbf{x} = (a_\nu, m_S)$. The BSM contribution will vanish in a certain parameter limit $\mathbf{x} \rightarrow \mathbf{0}$, e.g., for $a_\nu \rightarrow 0$ in the scalar emission case.

Experiments observe events of double electron emission distributed over energy bins. Theoretically, the expected number of events within an energy interval $T_i < T < T_i + \Delta T$ is calculated as

$$N_{\text{exp}}^{(i)}(\mathbf{x}) = N_A m_{\text{iso}}^{-1} \mathcal{E} \int_{T_i}^{T_i + \Delta T} dT \frac{d\Gamma(\mathbf{x})}{dT}. \quad (58)$$

The factor in front of the integral gives the effective number of decaying nuclei with Avogadro's number $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$, the molar mass m_{iso} of the isotope in question, and the fiducial exposure $\mathcal{E}$ of the experiment in units of kg·yr.

Having observed a set of events, the log-likelihood, or $\chi^2(\mathbf{x})$, of the binned data $\mathbf{D} = (N_{\text{obs}}^{(i)})$ given a BSM hypothesis is

$$\begin{aligned} -2 \log \mathcal{L}(\mathbf{D}|\mathbf{x}) &= 2 \sum_i^{N_{\text{bins}}} \left\{ N_{\text{exp}}^{(i)}(\mathbf{x}) - N_{\text{obs}}^{(i)} + N_{\text{obs}}^{(i)} \log \left(\frac{N_{\text{obs}}^{(i)}}{N_{\text{exp}}^{(i)}(\mathbf{x})} \right) \right\} \\ &\approx \chi^2(\mathbf{x}) = \sum_i^{N_{\text{bins}}} \frac{\left(N_{\text{obs}}^{(i)} - N_{\text{exp}}^{(i)}(\mathbf{x}) \right)^2}{N_{\text{exp}}^{(i)}(\mathbf{x})}, \end{aligned} \quad (59)$$

where the second line applies if there are a sufficiently many events per bin [62]. For the standard $2\nu\beta\beta$ decay as common contribution, this is the case for the current and future experimental exposures considered below.

In order to estimate the sensitivity of an experiment, we assume that it does not see a distribution that differs significantly from the SM prediction, setting the number of observed events to $N_{\text{obs}}^{(i)} = N_{\text{exp}}^{(i)}(\mathbf{0})$. This represents the so-called Asimov data set [63] which provides a good approximation to Monte Carlo simulations of the experiment that would otherwise be required [64].

In addition to the statistical uncertainty, we include two types of systematic errors:

1. Systematic experimental errors, assumed proportional to the number of events in a bin, $\sigma_{\text{sys}}^{(i)} = \sigma_f N_{\text{exp}}^{(i)}$, are added in quadrature to the statistical uncertainty. We take σ_f as constant for a given experiment.
2. The main theoretical uncertainty arises from the $2\nu\beta\beta$ and $0\nu\beta\beta$ NMEs that enter the decay rate calculations. The standard $2\nu\beta\beta$ decay depends on $\mathcal{M}_{2\nu}$ whereas the

BSM contributions, due to their ordinary Majoron structure, mainly scale with $\mathcal{M}_{0\nu}$ (the interference of the scalar-mediated $2\nu_S\beta\beta$ with SM $2\nu\beta\beta$ will also depend on $\mathcal{M}_{2\nu}$). Calculations in different nuclear structure models differ by factors of 2-3 in a given isotope. In order to include this effect, we treat both $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$ as Gaussian-distributed nuisance parameters with best fit values and standard deviations, $\langle\mathcal{M}_{n\nu}\rangle \pm \delta\mathcal{M}_{n\nu}$, derived from a survey of NME values [65]. We also include a potential correlation between $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$. While this correlation is not direct and perfect, it is observed statistically in several NME calculations and is encoded by a statistical correlation factor ρ [66]. Jointly, $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$ are assumed to follow a bi-variate, potentially tilted, Gaussian distribution.

We thus have the χ^2 function

$$\chi^2(\mathbf{x}) = \sum_i^{N_{\text{bins}}} \frac{\left(N_{\text{exp}}^{(i)}(\mathbf{0}) - N_{\text{exp}}^{(i)}(\mathbf{x})\right)^2}{N_{\text{exp}}^{(i)}(\mathbf{x}) + (\sigma_{\text{sys}}^{(i)})^2} + \frac{1}{1 - \rho^2} \left[\frac{(\eta - \langle\mathcal{M}_{2\nu}\rangle)^2}{(\delta\mathcal{M}_{2\nu})^2} + \frac{(\kappa - \langle\mathcal{M}_{0\nu}\rangle)^2}{(\delta\mathcal{M}_{0\nu})^2} - 2\rho \frac{(\eta - \langle\mathcal{M}_{2\nu}\rangle)(\kappa - \langle\mathcal{M}_{0\nu}\rangle)}{\delta\mathcal{M}_{2\nu} \cdot \delta\mathcal{M}_{0\nu}} \right], \quad (60)$$

where the set of parameters $\mathbf{x}$ now includes the two NME nuisance parameters η, κ , and the Asimov data set is calculated using the best fit value $\eta = \langle\mathcal{M}_{2\nu}\rangle$. Thus, by construction, $\chi^2(\mathbf{0}) = 0$ for the Asimov data set, i.e., the SM $2\nu\beta\beta$ decay case with $\langle\mathcal{M}_{2\nu}\rangle$ as NME.

We are mainly interested in the overall sensitivity to the BSM scenarios considered. We thus do not attempt fit all model parameters at the same time. Instead, we choose one coupling constant g as fitting parameter while keeping all other parameters fixed, specifically the particle masses involved. For example, in the simple scalar emission model, we calculate χ^2 using a_ν for a fixed scalar mass m_S . In order to find the 90% confidence level (CL) limit on such a coupling constant g , we first minimize over the NME nuisance parameters for a given g ,

$$\chi^2(g) = \min_{\eta, \kappa} [\chi^2(g, \eta, \kappa)]. \quad (61)$$

This marginalized profile function then follows a chi-squared distribution with one degree of freedom and to find the 90% CL upper limit on g , we set $\chi^2(g) = 2.71$.

Isotope	m_{iso} [g/mol]	Q [MeV]	$(T_{1/2}^{2\nu})_{\text{exp}}$ [yr]	$\langle \mathcal{M}_{2\nu} \rangle$	$\delta \mathcal{M}_{2\nu}$	$\langle \mathcal{M}_{0\nu} \rangle$	$\delta \mathcal{M}_{0\nu}$
$^{76}_{32}\text{Ge}$	72.6	2.04	1.9×10^{21}	0.10	0.22	4.3	1.2
$^{100}_{42}\text{Mo}$	96.0	3.03	7.1×10^{18}	0.19	0.27	4.9	1.5
$^{136}_{54}\text{Xe}$	131	2.46	2.2×10^{21}	0.016	0.19	2.6	1.1

TABLE I: Molar mass m_{iso} , Q value and experimental $2\nu\beta\beta$ decay half-life $(T_{1/2}^{2\nu})_{\text{exp}}$ for the isotopes considered in this work. Also given are the average NMEs $\langle \mathcal{M}_{2\nu} \rangle$, $\langle \mathcal{M}_{0\nu} \rangle$ and the standard deviations $\delta \mathcal{M}_{2\nu}$, $\delta \mathcal{M}_{0\nu}$.

VI. SENSITIVITY OF DOUBLE BETA DECAY EXPERIMENTS

In Tab. I, we provide the relevant data for the three isotopes considered in this work, ^{76}Ge , ^{100}Mo and ^{136}Xe . As described above, we treat the $2\nu\beta\beta$ and $0\nu\beta\beta$ NMEs as individual random variables, based on the spread in different nuclear structure calculations. We fix the best fit value $\langle \mathcal{M}_{2\nu} \rangle$ using the experimentally measured $2\nu\beta\beta$ decay half lives via Eq. (47). For the NME $\mathcal{M}_{0\nu}$, we compute the mean of the values provided in Table I of [65] which includes calculations from the Shell Model, QRPA, EDF and IBM frameworks. The corresponding uncertainty $\delta \mathcal{M}_{0\nu}$ is evaluated as the standard deviation of this set, providing a measure of the spread among theoretical predictions. As discussed in [66], the matrix elements $\mathcal{M}_{0\nu}$ and $\mathcal{M}_{2\nu}$ are correlated and can be approximated by a linear relation, $\mathcal{M}_{2\nu}/q^2 = a + b \cdot A^{-1/6} \mathcal{M}_{0\nu}$ where a and b are real constants and q is a factor capturing the potential axial-coupling quenching. We incorporate this correlation into our uncertainty estimates, by propagating the variation in $\mathcal{M}_{0\nu}$ through this expression. The resulting uncertainty in $\mathcal{M}_{2\nu}$ is therefore given by

$$\delta \mathcal{M}_{2\nu} = b A^{-1/6} \delta \mathcal{M}_{0\nu}, \quad (62)$$

where we take typical value $a = -0.25$ and $b = 0.18$ [66].

We will assume two different NME scenarios:

1. For the existing experimental searches at GERDA II, NEMO-3 and KamLAND-Zen, we assume the NME values in Tab. I and without correlation between $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$.
2. For the future searches at LEGEND-1000, CUPID and nEXO, take the same values above as a pessimistic scenario (no future improvement in NME calculations) but also

Isotope	Experiment	$\mathcal{E}$ [kg · yr]	$T_{\min}$ [keV]	ΔT [keV]	σ_f [%]
^{76}Ge	GERDA II [67]	104	500	15	1.9
	LEGEND-1000 [68]	7.0×10^3	100	2.5	0.5
^{100}Mo	NEMO-3 [10]	34	500	100	1.8
	CUPID-1T [69]	1.7×10^3	100	5	0.5
^{136}Xe	KamLAND-Zen [5]	126	100	50	0.3
	nEXO [70]	1.9×10^4	100	5	0.5

TABLE II: Current or expected future exposure $\mathcal{E}$, minimum electron energy $T_{\min}$, energy resolution (bin width) ΔT and relative systematic uncertainty σ_f for the double beta decay experiments considered in this work.

consider an optimistic scenario as follows: We keep the best fit values from Tab. I but we reduce the uncertainties by a factor of five, $\delta\mathcal{M}_{n\nu} \rightarrow 0.2 \times \delta\mathcal{M}_{n\nu}$. At the same time, we also include a statistical correlation between $\mathcal{M}_{0\nu}$ and $\mathcal{M}_{2\nu}$, with a correlation factor $\rho = 0.85$ [66]. This is intended as an optimistic choice for the state of NME calculations at the time of the experimental search.

We will estimate the sensitivity for different experimental setups as described in Tab. II. For each of the relevant isotopes ^{76}Ge , ^{100}Mo and ^{136}Xe , we consider an existing $0\nu\beta\beta$ decay search (at GERDA II, NEMO-3 and KamLAND-Zen, respectively) and at a future planned experiment (LEGEND-1000, CUPID and nEXO, respectively). We have extracted the key experimental metrics for our statistical approach from the references listed in Table II and [65], specifically, the assumed exposure $\mathcal{E}$, the minimum usable electron energy $T_{\min}$, the energy resolution (bin width) ΔT and the relative systematic uncertainty σ_f . In determining the binned spectrum, we also exclude the highest 100 keV below the $2\nu\beta\beta$ endpoint, mainly to ensure that there are enough events in each bin to justify the use of the χ^2 function in Eq. (59). Thus, we assume a usable spectrum over the interval $T_{\min} < T < Q - 0.1$ MeV. Our statistical approach described in Sec. V is not intended to exactly reproduce the experimental sensitivities, but to provide an estimate that mainly incorporates the statistical uncertainty in measuring the SM $2\nu\beta\beta$ spectrum, with some theoretical and experimental systematic errors captured.

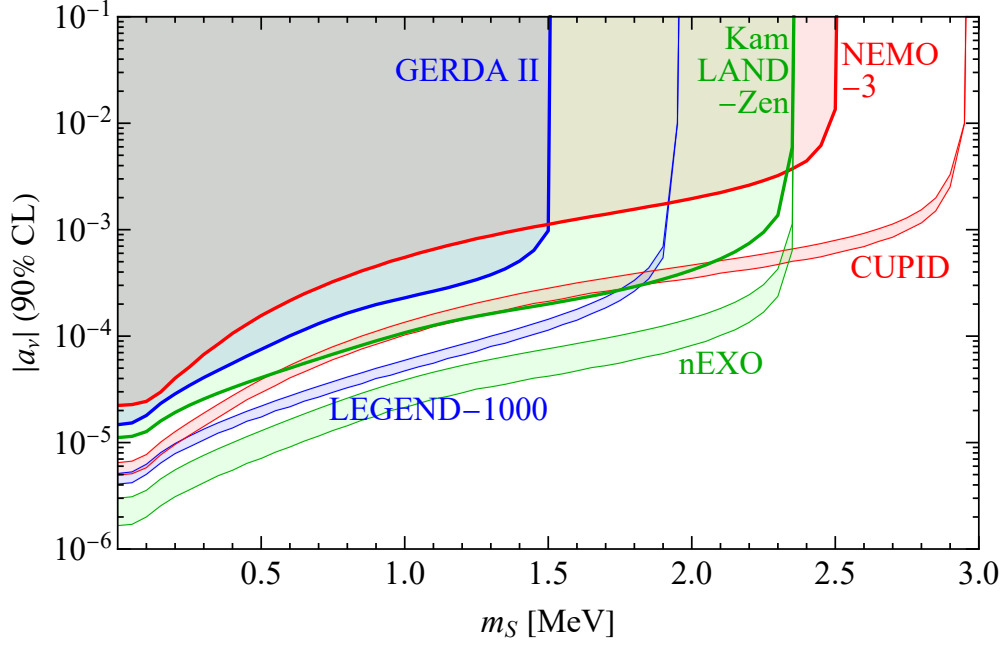


FIG. 7: Estimated upper limits and sensitivity on the coupling $|a_\nu|$ at 90% CL in scalar emission $0\nu\beta\beta S$ decay as a function of the scalar mass m_S . The experimental setups for the current and planned future experiments are given in Table II. For the sensitivity of future experiments, the coloured bands indicate the use of two sets of NMEs are used: A pessimistic scenario with current uncertainties as given in Table I and without correlation; and an optimistic scenario where uncertainties are reduced by a factor of 5 and a correlation $\rho = 0.85$.

A. Scalar Emission

We first consider simple scalar particle emission, $0\nu\beta\beta S$ as the BSM signal, i.e., S is assumed to be emitted as a real particle in the final state. The measured electron energy spectrum is then given by

$$\frac{d\Gamma(\mathbf{x})}{dT} = \frac{d\Gamma_{\text{SM}}^{2\nu}}{dT} + \frac{d\Gamma_S(\mathbf{x})}{dT}. \quad (63)$$

with SM $2\nu\beta\beta$ and $0\nu\beta\beta S$ spectra given in Eq. (47) and Eq. (46), respectively. The model parameters are the scalar mass m_S and the coupling a_ν to active neutrinos, $\mathbf{x} = (m_S, a_\nu)$.

Following our statistical approach, using the experimental setups and NME scenarios described above, we estimate the sensitivity to a_ν at 90% CL for a given scalar mass m_S . The result is shown in Fig. 7. The coloured blue, green and red regions at its top would be excluded by the existing searches at GERDA II, KamLAND-Zen and NEMO-3, respectively.

The maximally testable m_S is determined by the isotopes Q value and the minimum usable $T_{\min}$ of the given experiment, $m_S < Q - T_{\min}$. Thus, despite NEMO-3 having the smallest exposure, it still provides a relevant constraint due the large Q value of ^{100}Mo . As mentioned, for the future searches, we take a pessimistic and optimistic set of NMEs, corresponding to the blue, green and red bands for LEGEND-1000, nEXO and CUPID, respectively. In addition to the higher exposures, we also take a uniform value of $T_{\min} = 100$ keV, e.g., extending the sensitivity of CUPID to $m_S \lesssim 2.9$ MeV. We note that the sensitivity to scalar emission $0\nu\beta\beta S$ (massive Majoron) was estimated in [24] but only taking into account the phase space suppression of the total $0\nu\beta\beta S$ decay rate due to a massive scalar. We here instead include the spectrum shape as well. Specifically, as m_S increases, the phase space is suppressed and the $0\nu\beta\beta S$ spectrum shifts to lower electron energies, becoming more similar to SM $2\nu\beta\beta$ decay, see Fig. 3. We thus see that our sensitivity worsens more rapidly for increasing m_S until m_S becomes large enough for the spectrum to peak to the left of the SM $2\nu\beta\beta$ spectrum.

B. Model I'

We now extend the scope and explore the sensitivity towards Model I' (c.f., Sec. IIB), i.e., where the scalar S couples to active neutrinos and a dark fermion χ as $\nu\nu S$ and $\chi\chi S$. The measured electron energy spectrum can then be expressed as

$$\frac{d\Gamma(\mathbf{x})}{dT} = \frac{d\Gamma_{\text{SM}}^{2\nu}}{dT} + \frac{d\Gamma_S^{2\nu}(\mathbf{x})}{dT} + \frac{d\Gamma_{\text{int}}^{2\nu}(\mathbf{x})}{dT} + \frac{d\Gamma_S^{2\chi}(\mathbf{x})}{dT}, \quad (64)$$

where the $2\nu_S\beta\beta$ and $2\chi_S\beta\beta$ spectra are determined using Eq. (41) with $m_{\chi_1} = m_{\chi_2} = 0$ and m_χ , respectively, and including a symmetry factor of $1/2$. The interference term is given in Eq. (45). In full generality, the model parameters are described by the exotic particle masses m_S , m_χ and the full set of couplings $a_\nu, b_\nu, a_\chi, b_\chi$, $\mathbf{x} = (m_S, m_\chi, a_\nu, b_\nu, a_\chi, b_\chi)$. For simplicity and to explore the full range by which the above scalar emission case can be modified, we take $b_\nu = b_\chi = 0$ and a_χ , i.e., we only keep the left-handed couplings and assume that S couples dominantly to the dark fermion χ . Because of this, S almost exclusively decays to χ , $\text{Br}(S \rightarrow \chi\chi) \approx 1$ for $a_\nu \lesssim 10^{-1}$ and sufficiently light χ , $m_\chi \lesssim m_S/3$. Given these choices, we determine the 90% CL constraint on a_ν for given masses m_S and m_χ .

In Fig. 8, we show the modification of the a_ν sensitivity at the three existing experiments

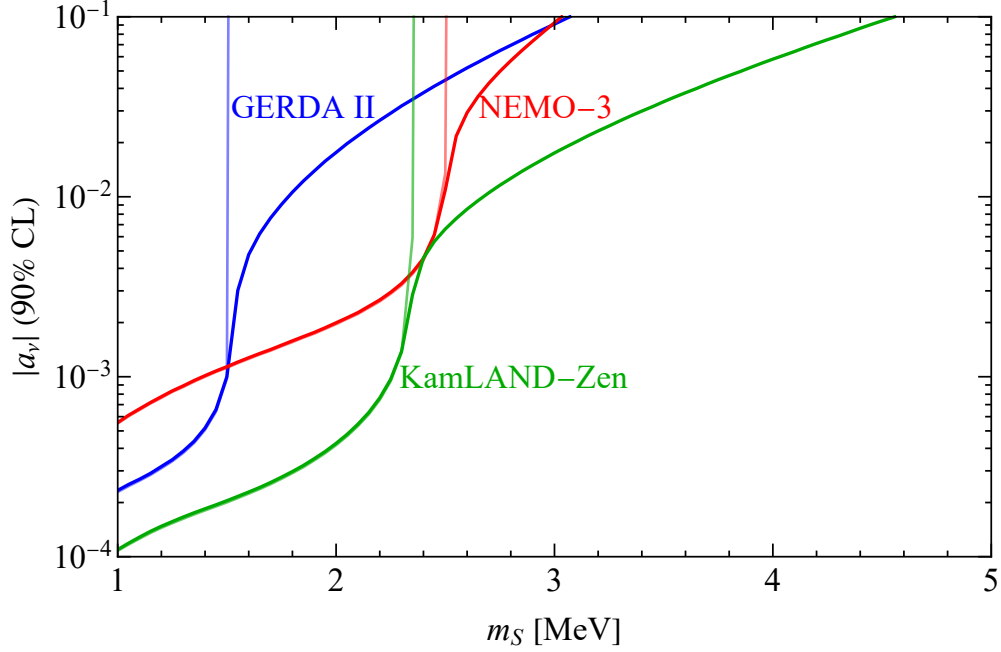


FIG. 8: Modification of the upper limit on a_ν in Model I' as a function of m_S , with a massless χ with coupling $a_\chi = 1$ for the three current experiments as indicated. The light-coloured (vertical) lines indicate the constraint as given in Fig. 7.

as a function of m_S , assuming a massless χ . The light-coloured blue, green and red vertical lines correspond to the experimental thresholds in Fig. 7 for GERDA II, KamLAND-Zen and NEMO-3, respectively, whereas dark coloured curves show the modification from allowing S to decay to $\nu\nu$ and mostly $\chi\chi$ in Model I'. For small $|a_\nu|$, with m_S below the experimental threshold the resulting constraints are equivalent, i.e., the curves asymptotically match. This is because $0\nu\beta\beta S$ and $2\chi_S\beta\beta$ are largely indistinguishable as long as S can be produced on-shell within the experimentally accessible electron spectrum and the total decay width Γ_S remains small. On the other hand, above the threshold, $0\nu\beta\beta S$ produces no events whereas $2\chi_S\beta\beta$ is allowed and sizeable, especially for the large coupling $a_\chi = 1$ considered. For larger a_ν , the interference between SM $2\nu\beta\beta$ and $2\nu_S\beta\beta$ also becomes sizeable, scaling as $\propto a_\nu^2$. Thus, the sensitivity to $|a_\nu|$ extends both beyond the experimental threshold $m_S < Q - T_{\min}$ and the ultimate m_S threshold $m_S < Q$. For example, KamLAND-Zen provides a meaningful sensitivity, $|a_\nu| \lesssim 10^{-1}$ for masses up to $m_S \lesssim 4.6$ MeV.

Of course, this increased sensitivity on a_ν is model-dependent, specifically on the choice of the other parameters of the model. This can be captured by an extended χ^2 analysis

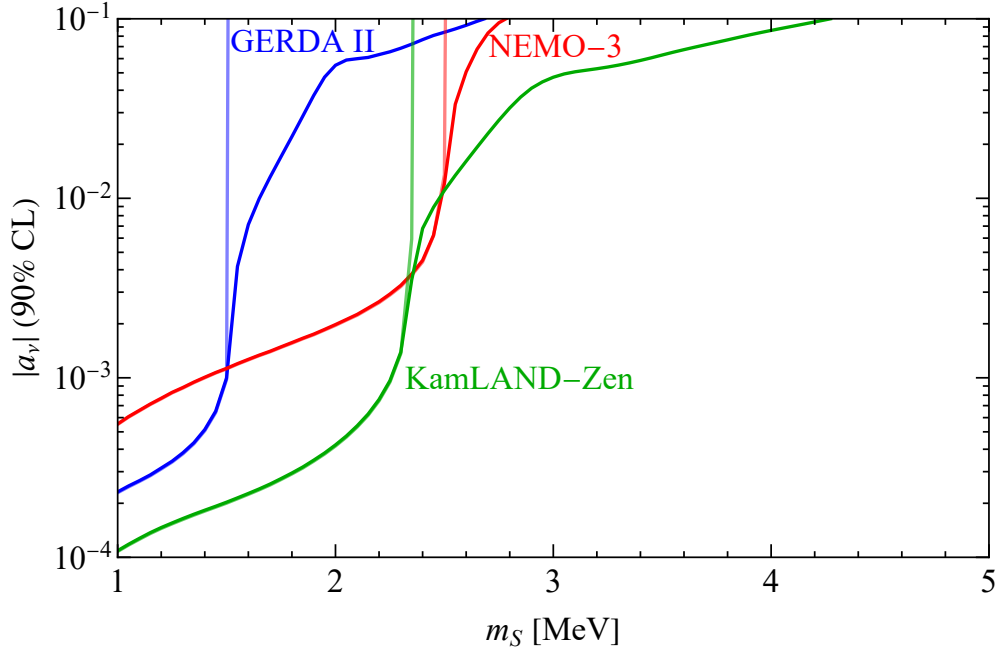


FIG. 9: As Fig. 8, but for $m_\chi = m_S/3$.

incorporating two or more model parameters at a time. This is beyond the scope of this work but in Fig. 9, we show the equivalent modification for $m_\chi = m_S/3$ instead of massless χ . The overall effect is reduced due to the phase-space suppression from m_χ , but the sensitivity is extended here as well.

VII. CONCLUSION

Double beta decay processes are sensitive avenues to probe physics beyond the SM. While the neutrinoless mode, $0\nu\beta\beta$ is the key process to probe the Majorana nature of neutrinos, in presence of new light particles, one can also expect distinct signatures over the SM $2\nu\beta\beta$ decay spectrum. In this work, we have explored a class of scenarios where a light scalar, S , couples both to active neutrinos and new fermionic states χ . Owing to these new interactions, on-shell and off-shell production of S followed by its decay into neutrinos or exotic fermions, a spectral distortion in the double beta decay spectrum can be observed. We have constructed simplified model scenarios that generate the effective four-fermion operators responsible for exotic double beta decay with the emission of χ . Focusing on scalar s -channel mediation, we constructed two Dirac neutrino scenarios (Model I and I')

differing by the presence or absence of $\nu\chi$ mixing, the latter enforced through a stabilizing $\mathcal{Z}_2$ symmetry that makes χ a potential dark matter candidate. We further introduced a third model (Model II) involving both Dirac and Majorana exotic fermions to illustrate the broader range of possible interactions.

We analyzed how the double beta decay spectrum is modified by the presence of mediator scalar S and the exotic fermion species χ , highlighting the dependence of the spectral shape on their masses and couplings. For massless χ , we showed that when S is sufficiently light to be produced on-shell, the resulting $2\chi_S\beta\beta$ spectrum closely follows the scalar emission shape, with only minor deviations arising from finite-width effects of the mediator scalar. As m_S approaches the kinematic threshold or becomes too heavy to be produced on-shell, the spectrum shifts toward lower electron energies or enters an off-shell regime in which the dominant contributions move to larger T , eventually matching the SM $2\nu\beta\beta$ distribution in the $m_S \gg Q$ limit. Introducing non-zero χ masses further restricts on-shell production of the mediator and suppresses visible distortions in the electron spectrum, making heavier χ scenarios increasingly challenging to probe. Finally, in the case where S couples exclusively to neutrinos, we demonstrated that the full double beta decay spectrum receives both on-shell and off-shell scalar-mediated contributions, including destructive interference with the SM amplitude, leading to characteristic spectral suppression near threshold. These results collectively illustrate the diverse range of spectral signatures that light scalars and exotic fermions can imprint on double beta decay observables.

In Model I', the χ can be considered as a potential dark matter candidate (or a component of the dark sector) due to the absence of mixing with the active neutrino sector, due to a $\mathcal{Z}_2$ symmetry. Here, we discussed cosmological and laboratory constraints. Collisional damping, CMB, and BBN limits, and relic-density requirements collectively require lower bounds on light DM and mediator masses, while rare kaon decays further restrict the viable couplings for larger mediator scalar mass. Together, these complementary constraints significantly narrow the parameter space in which the light scalarfermion dark sector remains consistent with current observations, as shown in Fig. 6.

Incorporating statistical, experimental and theoretical uncertainties on the double beta decay spectrum, we have estimated the sensitivity of a set of current and future double beta decay experiments towards exotic scalar emission $0\nu\beta\beta S$ as well as massless and massive exotic fermion production from the mediator scalar decay $2\chi_S\beta\beta$. This is summarized in

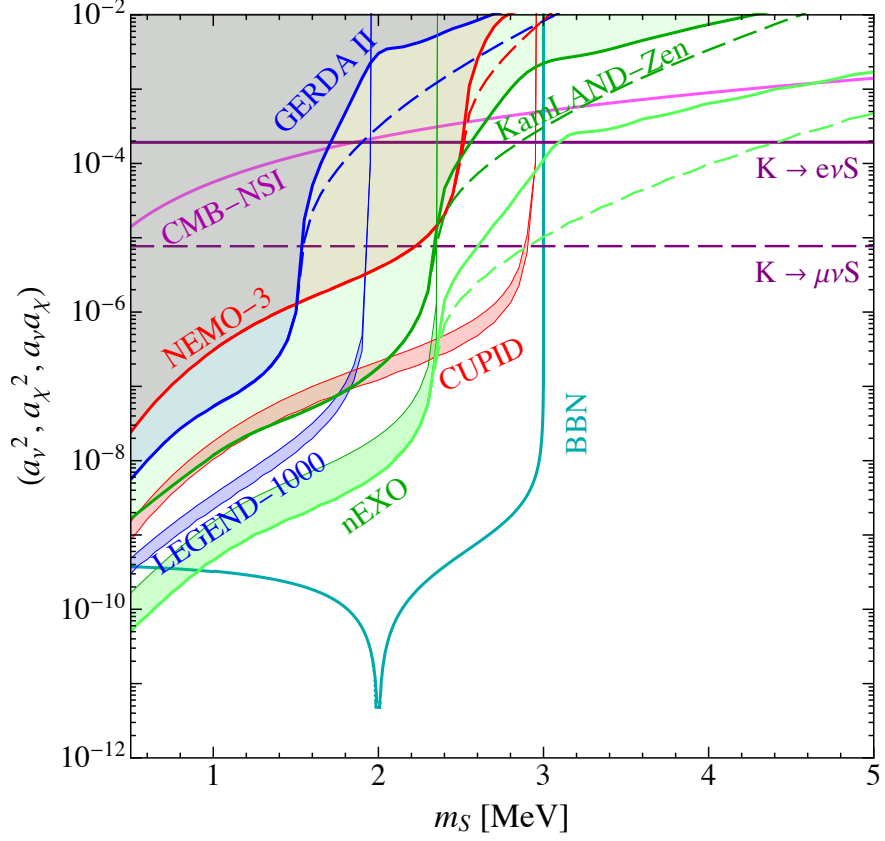


FIG. 10: Summary of double beta decay sensitivities as discussed in Sec. VI compared with the most relevant BBN and kaon constraints, c.f. Sec. IV.

Fig. 10, comparing the double beta decay constraints and future sensitivity with the most relevant cosmological and laboratory bounds. The darker blue, green and red solid curves illustrate the exclusion bounds on $|a_\nu|$ in Model I' for $a_\chi = 1$ and $m_\chi = m_S/3$, derived from the three current experimental searches, GERDA II, KamLAND-Zen and NEMO-3, respectively, at 90% CL. In contrast, the case of a massless χ with $a_\chi = 1$ is represented by the dashed curves. The lighter coloured bands indicate the projected sensitivities of the future experiments LEGEND-1000, nEXO and CUPID-1T, using different nuclear matrix element uncertainties. While BBN and kaon decay bounds impose stringent bounds, future double beta decay experiment, here represented by the nEXO sensitivity can probe the parameter space for $m_S < 1$ MeV for couplings $a_\nu \lesssim \mathcal{O}(10^{-5})$ and $3 \text{ MeV} \lesssim m_S \lesssim 4 \text{ MeV}$ with $a_\nu < \mathcal{O}(10^{-2})$. Double beta decay experiments thus provide the most stringent laboratory bounds and have the potential to probe light scalars coupling to neutrinos as well as to light fermions in a dark sector, extending beyond the double beta decay Q values. The

experimental sensitivities are in a parameter space relevant for cosmological considerations, providing independent, laboratory tests of light exotic physics.

Note

As we were finalizing this manuscript, we became aware of an independent experimental analysis by the PandaX collaboration on a similar topic [71].

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Appendix A: Calculation of scattering rates

The squared amplitude, summing over final spins and averaging over initial spins, for annihilation channel of DM χ to neutrinos via s-channel scalar mediation, $\chi(p_1) + \chi(p_2) \rightarrow S \rightarrow \nu(p_3) + \nu(p_4)$, can be written as,

$$\sum_{\text{spins}} |\mathcal{M}_s|^2 = \frac{1}{(s - m_S^2)^2 + m_S^2 \Gamma_S^2} \left[(|a_\chi|^2 + |b_\chi|^2)(p_1 \cdot p_2) - 2m_\chi^2 \text{Re}(a_\chi b_\chi^*) \right] \times \left[(|a_\nu|^2 + |b_\nu|^2)(p_3 \cdot p_4) - 2m_\nu^2 \text{Re}(a_\nu b_\nu^*) \right], \quad (\text{A1})$$

where $p_1 \cdot p_2 = (s - 2m_\chi^2)/2$, $p_3 \cdot p_4 = (s - 2m_\nu^2)/2$ and $\Gamma_S = \Gamma(S \rightarrow \nu\nu) + \Gamma(S \rightarrow \chi\chi)$ is the total decay width of scalar S . Following the technique developed in [32, 33], we have estimated the thermally averaged annihilation cross section $\langle \sigma v_{\text{rel}} \rangle$ which will be constrained

by present-day relic density criteria for DM candidates in the universe, where we have considered $v_{\text{rel}} = 10^{-3} \ll c$.

Similarly, the squared amplitude for elastic scattering between DM χ and neutrinos, $\chi(p_1) + \bar{\nu}(p_2) \rightarrow \bar{\chi}(p_3) + \nu(p_4)$ via t-channel scalar mediation can be expressed as,

$$\sum_{\text{spins}} |\mathcal{M}_t|^2 = \frac{1}{2(t - m_S^2)^2} [(p_3 \cdot p_1)(|a_\chi|^2 + |b_\chi|^2) + (p_4 \cdot p_2)(|a_\nu|^2 + |b_\nu|^2) - 2m_\chi^2 \text{Re}(a_\chi b_\chi^*) - 2m_\nu^2 \text{Re}(a_\nu b_\nu^*)] \quad (\text{A2})$$

where $p_3 \cdot p_1 = (t - 2m_\chi^2)/2$ and $p_4 \cdot p_2 = (t - 2m_\nu^2)/2$. Using this, the elastic scattering cross section can be estimated as $\sigma_{\text{el}} = 1/(16\pi s) \sum_{\text{spins}} |\mathcal{M}_t|^2$ and this is constrained by the bounds from collisional damping [38–40].

Furthermore, we have obtained the scattering cross section for $\nu(p_1) + \nu(p_2) \rightarrow \chi(p_3) + \chi(p_4)$ via s-channel scalar mediation can be expressed as,

$$\begin{aligned} \sigma(\nu\nu \rightarrow S \rightarrow \chi\chi) &= \frac{1}{32\pi s [(s - m_S^2)^2 + m_S^2 \Gamma_S^2]} \sqrt{\frac{s - 4m_\chi^2}{s - 4m_\nu^2}} \\ &\times [(|a_\nu|^2 + |b_\nu|^2)(p_1 \cdot p_2) - 2m_\nu^2 \text{Re}(a_\nu b_\nu^*)] \\ &\times [(|a_\chi|^2 + |b_\chi|^2)(p_3 \cdot p_4) - 2m_\chi^2 \text{Re}(a_\chi b_\chi^*)], \end{aligned} \quad (\text{A3})$$

where $p_1 \cdot p_2 = (s - 2m_\nu^2)/2$ and $p_3 \cdot p_4 = (s - 2m_\chi^2)/2$.

Appendix B: Normalization for $\beta\beta$ decay widths

The nuclear structure appearing in our processes, i.e., $\chi_1\chi_2\beta\beta$ and Majoron decay, is the same as that involved in neutrinoless double beta decay. Therefore, we can make use of the NMEs of that process. To do so, our normalization must be matched to that used in the literature in order to consistently employ the numerical results for $\mathcal{M}_{0\nu}$.

Within our approach, it is straightforward to recover the standard results for $0\nu\beta\beta$. The process amplitude can be factorized into leptonic and hadronic contributions:

$$\mathcal{A}_{0\nu} = \sum_n L_{\mu\nu} H^{\mu\nu}, \quad (\text{B1})$$

where L and H denote the leptonic and hadronic contributions, respectively, and the sum over n accounts for the nuclear intermediate states. Assuming the closure approximation

both factors decouple and, using, the symmetry under $\mu \leftrightarrow \nu$ in the hadronic sector, the leptonic part can be written as

$$L_{\mu\nu} = \mathcal{N}_{0\nu} m_{\alpha\alpha} g_{\mu\nu} \bar{u}_1 P_R v_2, \quad (\text{B2})$$

where $\mathcal{N}_{0\nu}$ is a normalization factor chosen to match the standard results in the literature. The corresponding decay width is then expressed as

$$\Gamma_{0\nu} = \left| \frac{m_{\alpha\alpha}}{m_e} \right|^2 |\mathcal{M}_{0\nu}|^2 \mathcal{G}_{0\nu}. \quad (\text{B3})$$

where $\mathcal{G}_{0\nu}$ and $\mathcal{M}_{0\nu}$ denote the phase space factor and the nuclear matrix element, respectively. The phase space factor can be computed as

$$\begin{aligned} \mathcal{G}_{0\nu} &= |\mathcal{N}_{0\nu}|^2 \int \prod_{i=1}^2 \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} 2 p_1 \cdot p_2 F_0(p_1) F_0(p_2) \delta(Q - T) \\ &= \frac{2 |\mathcal{N}_{0\nu}|^2}{(2\pi)^4} \int_{m_e}^{Q+m_e} |\mathbf{p}_1| |\mathbf{p}_2| E_1 E_2 F_0(p_1) F_0(p_2) dE_1, \end{aligned} \quad (\text{B4})$$

where $F_0(p)$ is the Fermi function, and we have used $E_2 = Q + 2m_e - E_1$ and $|\mathbf{p}_i| = \sqrt{E_i^2 - m_i^2}$. Comparing with the standard result in the literature, we obtain

$$|\mathcal{N}_{0\nu}|^2 = \frac{G_F^4 \cos^4(\theta_c)}{4\pi R^2}, \quad (\text{B5})$$

in such a way the decay width is given by

$$\Gamma_{0\nu} = \left| \frac{m_{\alpha\alpha}}{m_e} \right|^2 \frac{G_F^4 \cos^4(\theta_c) m_e^2}{32\pi^5 R^2} |\mathcal{M}_{0\nu}|^2 \int_{m_e}^{Q+m_e} |\mathbf{p}_1| |\mathbf{p}_2| E_1 E_2 F_0(p_1) F_0(p_2) dE_1, \quad (\text{B6})$$

and reproduce the known result.

For the case where χ_1 and χ_2 are antineutrinos, one must also account for the interference with standard $2\nu\beta\beta$. Following a similar approach, the leptonic contribution can be written as

$$L_{\mu\nu} = \mathcal{N}_{2\nu} m_{\alpha\alpha} g_{\mu\nu} \bar{u}_1 P_R v_2 \bar{u}_4 P_R v_3, \quad (\text{B7})$$

where p_1 and p_2 correspond to the electron momenta, while p_3 and p_4 correspond to the neutrinos. The decay width is

$$\Gamma_{2\nu} = |\mathcal{M}_{2\nu}|^2 \mathcal{G}_{2\nu}, \quad (\text{B8})$$

with the phase space factor

$$\begin{aligned}\mathcal{G}_{2\nu} &= |\mathcal{N}_{2\nu}|^2 \int \prod_{i=1}^4 \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} 4 p_1 \cdot p_2 F_0(p_1) F_0(p_2) p_3 \cdot p_4 \delta(Q - T - E_3 - E_4) \\ &= \frac{4 |\mathcal{N}_{2\nu}|^2}{(2\pi)^8} \int \frac{(Q - T)^5}{30} |\mathbf{p}_1| |\mathbf{p}_2| E_1 E_2 F_0(p_1) F_0(p_2) dE_1 dE_2.\end{aligned}\quad (\text{B9})$$

Comparison with the standard literature then gives

$$|\mathcal{N}_{2\nu}|^2 = 8\pi \frac{G_F^4 \cos^4(\theta_c)}{m_e^2}.\quad (\text{B10})$$

Thus, the decay width is given by

$$\Gamma_{2\nu} = \frac{G_F^4 \cos^4(\theta_c)}{8\pi^7 m_e^2} |\mathcal{M}_{2\nu}|^2 \int \frac{(Q - T)^5}{30} |\mathbf{p}_1| |\mathbf{p}_2| E_1 E_2 F_0(p_1) F_0(p_2) dE_1 dE_2,\quad (\text{B11})$$

recovering the known result.

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