Building far-from-equilibrium effective field theories using shift symmetries

Xin An,[1,*] Robbe Brants,[1,†] Michal P. Heller,[1,2,‡] and Yi Yin[3,§]

[1] *Department of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium*
[2] *Institute of Theoretical Physics and Mark Kac Center for Complex Systems Research, Jagiellonian University, 30-348 Cracow, Poland*
[3] *School of Science and Engineering, The Chinese University of Hong Kong (Shenzhen), Longgang, Shenzhen, Guangdong, 518172, China*

Contemporary understanding of thermalization in quantum field theory stems largely from understanding properties of transient excitations of equilibria. These nonhydrodynamic excitations are known to structurally differ between weakly- and strongly-coupled quantum field theories with no known results at intermediate values of the interaction strength. We demonstrate that all the known behaviors of transient excitations can be understood as a consequence of different realizations of a symmetry principle, the shift symmetry, applied at the level of the far from equilibrium generalization of the hydrodynamic effective action that we explicitly construct. Our approach naturally includes the effects of stochastic fluctuations outside the hydrodynamic regime and allows to explicitly construct hybrid models interpolating between weak- and strong-coupling behavior. We study properties of one such model motivated by thermalization in nuclear collisions in light of the QCD running coupling.

Introduction.– Hydrodynamics is often referred to as the universal low-energy effective description of many-body systems near equilibrium [1, 2]. Its applications are plentiful and in the context of high energy physics primarily concern modeling the quark-gluon plasma (QGP) in ultrarelativistic nuclear collisions at RHIC and LHC [3, 4].

Motivated by this experimental research program, studies of the emergence of hydrodynamic behavior in quantum field theories led to a simple yet deep realization: hydrodynamics becomes applicable when excitations not captured by it have decayed [5]. This triggered significant interest in understanding nonhydrodynamic degrees of freedom of holographic quantum field theories [6–13], followed by weak-coupling studies in kinetic theory [14–19].

Hydrodynamic excitations between the two classes of systems differ only in the associated values of transport coefficients. For example, for the diffusive mode

$$\omega = -iDq^2 + \mathcal{O}(q^4)\,, \tag{1}$$

the leading difference in hydrodynamic behavior between holography and kinetic theory is encoded in the diffusive constant D. The mode frequency ω receives corrections from higher powers of spatial momentum q that will be also theory-dependent, but the structure itself is not.

In contrast to Eq. (1), nonhydrodynamic sectors are structurally different and diverse. In particular, in strongly coupled quantum field theories, the excitations can be thought of as single pole singularities of Fourier-transformed retarded correlators of conserved currents and interpreted as transient quasinormal modes (QNMs) of holographic black holes [20]. In practical terms, they give rise to an exponential decay in time. In contrast, nonhydrodynamic degrees of freedom in kinetic theory appear as branch-cuts or their generalization: nonanalytic regions [14, 15]. The associated decay in time of such nonhydrodynamic sectors can be subexponential.

There is now a systematic effective field theory (EFT) for hydrodynamics based on the Schwinger-Keldysh (SK) formalism [21–23]. This triggers an important open question: can the purview of EFTs be extended to systematically capture the rich landscape of nonhydrodynamic excitations, such as the QNMs in holographic systems or the short-lived excitations in QGP and ultracold gases?

In the present letter we answer this question in a constructive manner utilizing structures inspired on one hand by holography and on the other by kinetic theory. Our study brings four main new results:

- We demonstrate that the shift symmetry known from the earlier studies of hydrodynamic EFTs can be used as a new principle in organizing the description of nonhydrodynamic excitations for a class of theories, including holographic-like and kinetic-theory like theories.
- We show that different ways of realizing this symmetry underlies the qualitative difference in excitations in holography vs. kinetic theory.
- Our approach allows for a construction of a new class of models of nonequilibrium dynamics phrased directly in the language of EFT that are hybrids of holography and kinetic theory. They are motivated by the QCD running coupling and earlier attempts to incorporate it in modeling QGP dynamics [24–26].
- The shift symmetry also allows us to systematically describe statistical fluctuations beyond the low-frequency hydrodynamic regime, which so far have eluded attention and understanding.

We refer our framework as the shift-symmetry based construction. These results open a new pathway for building effective theories for non-equilibrium quantum systems beyond hydrodynamics.

Setup.– Our goal is to develop the SK framework for describing both hydrodynamic and relevant nonhydrodynamic excitations and fluctuations. For demonstration, we focus on the sector with a conserved $\mathrm{U}(1)_V$ charge current J^μ in a symmetry-unbroken phase.

The construction of SK action involves three key steps [21, 23, 27]. The first step is to choose suitable dynamical fields, collectively denoted by $\chi^{r,a}$, in the r-a basis [28] where r-fields represent physical observables and a-fields encode fluctuations. Second, one must identify the symmetry of the action, both the microscopic symmetry, such as $\mathrm{U}(1)_V$, and importantly, the emergent symmetries that arise from the state under consideration. Finally, the action must be written down in a way that is consistent with these symmetries and other physical constraints (cf. Eq. (3) below), organized via a gradient expansion scheme.

In the EFT for diffusion [21, 22, 29–31], the $\mathrm{U}(1)_V$ phase fields $\chi = \psi, \chi^a = \psi^a$ (omitting the r subscript from now on) serve as dynamical variables. This choice is particularly convenient for implementing $\mathrm{U}(1)_V$ symmetry. Importantly, the action must be invariant under spatial shift transformation of the (r-)phase-field [21]

$$\delta_s \psi = \lambda(\boldsymbol{x}) \,. \tag{2}$$

This emergent *shift symmetry* is proposed to ensure that the fluid under study is in the symmetry-unbroken state, as it allows independent phase choices at different locations $\boldsymbol{x}$ [23, 27]. It prohibits unwanted propagating Goldstone modes (eliminating terms linear in q in Eq. (1)) in the EFT thus constructed, which correctly generates diffusive dynamics and fluctuations [31].

The central ingredient of our construction is the generalization of the shift symmetry underlying the EFT for diffusion. We propose that multiple nonhydrodynamic excitations can be described by the action of a system with multiple (global) U(1) symmetries. Some of those symmetries can be spontaneously broken and subsequently gauged to local ones. The dynamical fields include the phase fields of U(1)s and the dynamical gauge fields when the broken symmetries are gauged. There are three generic scenarios:

a) all U(1) symmetries are unbroken;

b) all U(1) symmetries are spontaneously broken into a diagonal U(1);

c) the mixtures of a) and b).

For each case, we formulate the appropriate and nontrivial shift symmetries (cf. Eqs. (4) and (6)) and demonstrate that the difference in those shift symmetries results in qualitatively distinctive features in excitations.

The action additionally satisfies several physical conditions [22, 23]

$$I[\chi, \chi_a = 0] = 0 \,, \quad I[\chi, -\chi_a] = -I^*[\chi, \chi_a] \,, \quad \mathrm{Im}\, I \geqslant 0 \,. \tag{3}$$

The first condition in Eq. (3) implies that the action can be organized as a series expanded in a-field: $I = I_a + I_{aa} + \dots$ where the subscript counts the power in χ_a. We shall first construct I_a, which suffices to determine excitations. Then we return to I_{aa}, which encodes Gaussian fluctuations, and demonstrate how shift symmetry is crucial for describing these fluctuations.

Unbroken U(1) and kinetic theory.– We begin by formulating the shift symmetry for scenario a) where the phase fields associated with multiple global $\mathrm{U}(1)_\alpha$ are denoted by ϕ_α with $\alpha = 1, \dots, N$. To ensure that those U(1)s are unbroken, we propose that the action in this scenario, I_{U}, is invariant under α-dependent spatial shift transformation

$$\delta_s \phi_\alpha = \lambda_\alpha(\boldsymbol{x}) \,, \tag{4}$$

which naturally generalizes the transformation (2). We further require the action to be invariant under $\mathrm{U}(1)_V$ transformations $\phi_\alpha \to \phi_\alpha + \Lambda_V(x)$, allowing for a vector current J^μ obtained via variation w.r.t. the background gauge field V_μ.

With this, we can construct the most general I_{U}, see our forthcoming paper [32]. For now, we highlight a specific but physically important case where $N \to \infty$ and the discrete index α is replaced by the single-particle velocity $\boldsymbol{v}$ (or momentum $\boldsymbol{p}$). In this limit, I_{U} recovers the action that describes collisionless linearized kinetic theory [33] (see Ref. [34] for the nonlinear cases) for the fluctuation of single particle distribution $n(x, \boldsymbol{p})$ around a homogeneous and static background $n_0(\epsilon_0)$, where the single particle energy ϵ_0 depends on $p = |\boldsymbol{p}|$ only. Explicitly (in the absence of V_μ), the action reads

$$I_{\mathrm{U},a} = \int_{x,\boldsymbol{p}} (-n_0'(\epsilon_0)) \Big\{ (\partial_t \phi(\boldsymbol{v})) \partial_t \phi^a(\boldsymbol{v}) + \left[(\boldsymbol{v} \cdot \boldsymbol{\partial}) \phi(\boldsymbol{v}) \, \partial_t \phi^a(\boldsymbol{v}) + (r \leftrightarrow a) \right] \Big\} \,. \tag{5}$$

The E.o.M from Eq. (5) for $\phi(\boldsymbol{v})$ yields the standard linearized kinetic equation with the identification $\delta n \equiv n - n_0(\epsilon_0) = (-n'(\epsilon_0)) \, \partial_t \phi(\boldsymbol{v})$, see L.H.S. of Eq. (13a). The $\boldsymbol{v}$-dependent shift symmetry reflects that particles with different velocities choose their phases independently. The gauge invariant form in the presence of V_μ can be found in Refs. [33, 34].

Broken Hidden U(1).– We now examine scenario b) where the symmetry group $\mathrm{U}(1)_1 \times \dots \mathrm{U}(1)_K \times \mathrm{U}(1)_V$ is spontaneously broken into a diagonal U(1). The relevant phase fields are φ_n with $n = 1, \dots, K$ and φ_V. Unlike scenario a), we can gauge φ_n by introducing dynamical "photon" fields $A_{\mu,n}$. This does not lead to unwanted gapless excitations since those gauge fields acquire mass through the Higgs mechanism. The gauge invariant dynamical variables include $\mathcal{A}_{n,\mu} \equiv (\partial_\mu \varphi_n + A_{\mu,n})$ and $\mathcal{A}_{\mu,K+1} \equiv (\partial_\mu \varphi_V + V_\mu)$.

For the "broken" scenario, we propose the shift symmetry under the following transformation

$$\delta_{\rm s}\varphi_n = \lambda(t,\boldsymbol{x}) \qquad \text{so} \qquad \delta_s\mathcal{A}_{n,\mu} = \partial_\mu\lambda(t,\boldsymbol{x})\,. \quad (6)$$

Since $\mathrm{U}(1)_n$s are broken, we can no longer require the shift function to depend on index n. Compared to Eq. (4), the shift function $\lambda(t,\boldsymbol{x})$ varies in time, and the transformation (6) also applies on the gauge invariant combination $\mathcal{A}$. To satisfy this shift symmetry, the action will depend on the phase fields only through shift-invariant combinations $\sum_n c_n\varphi_n$ with K independent choices of c_n such that $\sum_n c_n = 0$. We can therefore decompose them into the basis of Goldstone bosons $\Delta\varphi_n \equiv \varphi_{n+1} - \varphi_n$.

To leading order in derivatives, the action, expressed in terms of $\Delta\mathcal{A}_{\mu,n} \equiv \mathcal{A}_{\mu,n+1} - \mathcal{A}_{\mu,n}$ and "EM" fields $\boldsymbol{E}_n = \boldsymbol{\nabla}\boldsymbol{A}_{t,n} - \partial_t\boldsymbol{A}_n$ and $\boldsymbol{B}_n = \boldsymbol{\nabla}\times\boldsymbol{A}_n$, reads

$$I_{\mathrm{B},a}[\varphi_V,\mathcal{A}_n] = \int_x \sum_{n=1}^{K} \left(f_n^2\,\Delta\mathcal{A}_{t,n}\,\Delta\mathcal{A}^a_{t,n} - g_n^2\Delta\boldsymbol{\mathcal{A}}_n\Delta\boldsymbol{\mathcal{A}}^a_n + \epsilon_n\boldsymbol{E}_n\,\boldsymbol{E}^a_n - \kappa_n\boldsymbol{B}_n\boldsymbol{B}^a_n + \sigma_n\boldsymbol{\mathcal{A}}^a_n\cdot\boldsymbol{E}_n\right)\,, \quad (7)$$

where we count $\Delta\mathcal{A}$ the same order as $\partial\mathcal{A}$ and focus on situations where $\mathcal{A}_n$ can only couple to its neighbors $\mathcal{A}_{n\pm1}$. By employing field redefinition, we remove terms of the form $\Delta\boldsymbol{\mathcal{A}}\cdot\boldsymbol{E}$. The above action contains the dissipative part $I_{a,\mathrm{diss}}$ that is proportional to σ_n. It acts as a conductivity that eventually damps the gauge fields. The remaining part is non-dissipative and can be factorized as the difference between the action on the forward and backward leg of the SK contour [23]. This part depends on the "decay constant" f_n, the gauge boson mass g_n, and the electric and magnetic constants, ϵ_n, κ_n, of each hidden photon sector.

The E.o.M for φ_V expresses charge conservation

$$\partial_\mu J^\mu_{\mathrm{B}} = 0\,, \qquad J^\mu_{\mathrm{B}} = (f_K^2\Delta\mathcal{A}_{K,t}, -g_K^2\Delta\boldsymbol{\mathcal{A}}_K)\,, \quad (8)$$

where the charge current for the broken hidden symmetry scenario J^μ_B is obtained by varying the action w.r.t. V_μ. The $\mathcal{A}_{\mu,n}$ obey Maxwell-like equations in the medium:

$$\epsilon_n\boldsymbol{\partial}\cdot\boldsymbol{E}_n = -f_n^2\Delta\mathcal{A}_{n,t} + f_{n-1}^2\Delta\mathcal{A}_{n-1,t}\,, \quad (9a)$$
$$\epsilon_n\partial_t\boldsymbol{E}_n + \kappa_n\boldsymbol{\partial}\times\boldsymbol{B}_n = g_n^2\Delta\boldsymbol{A}_n - g_{n-1}^2\Delta\boldsymbol{A}_{n-1} + \sigma_n\partial_t\boldsymbol{\mathcal{A}}_n. \quad (9b)$$

To allow for a finite background charge density, we set the (boundary) condition $\mathcal{A}_{1,t} = 0$ when solving them [32].

We focus on excitations in the longitudinal channel and express Eq. (8), at $V_\mu = 0$, as $R(\omega,\boldsymbol{q})(\omega\varphi_V) = 0$ where $R(\omega,\boldsymbol{q})$ is determined by solving E.o.M for $\mathcal{A}$. The excitations are then determined by $R(\omega,\boldsymbol{q}) = 0$. Below, we survey various specific examples.

The $K=1$ and Maxwell-Cattaneo theory.– When $K=1$, the dispersion relation is given by (omitting the subscript $n=1$)

$$R(\omega,\boldsymbol{q}) = f^2\left(\omega + \frac{u\,q^2(\epsilon\,\omega + ig^2\tau_{\mathrm{B}})}{g^2 - \omega(\epsilon\,\omega + ig^2\,\tau_{\mathrm{B}})}\right) = 0\,, \quad (10)$$

where $\tau_{\mathrm{B}} \equiv \sigma/g^2, u \equiv g/f$. For small q, Eq. (10) yields a diffusive mode (1) with diffusive constant $D_{\mathrm{B}} = u^2\tau_{\mathrm{B}}$ and a pair of nonhydrodynamic modes $\omega_\pm = \pm\omega_R - i\omega_I$ determined by $\epsilon\omega^2 + ig^2\tau_{\mathrm{B}}\,\omega - g^2 = 0$, where the magnitude of ω_R is set by the gauge boson mass g. We refer to such excitations with non-zero $\mathrm{Re}(\omega)$ at $q=0$ as *damped oscillatory* (DO) *modes*, which resemble holographic QNMs. Setting $g = \sqrt{2\epsilon}/\tau_{\mathrm{B}}$ gives $\omega_\pm = (\pm1 - i)\tau_{\mathrm{B}}^{-1}$. With $\tau_{\mathrm{B}} = 1/(2\pi T)$ and $u=1$, we recover the first pair of QNMs $\omega_\pm = (\pm1-i)2\pi T$ and the diffusion constant $D_{\mathrm{SYM}} = 1/2\pi T$ of strongly coupled $\mathcal{N}=4$ SYM theory [8, 35], indicating the model's capability and flexibility to describe holographic excitations. This discussion is reminiscent of linear regime of relativistic hydrodynamics equations of motion considered in [36, 37] with the key difference being that the present discussion is driven by the symmetry-principle.

The special case $\epsilon = \kappa = 0$ is also interesting. The Lagrangian density, in the absence of external fields, reduces to

$$\mathcal{L}_{\mathrm{B},K=1} = f^2\Big[(\partial_t\varphi_V)\partial_t\varphi^a_V - u^2\,(\boldsymbol{\partial}\varphi_V - \boldsymbol{\mathcal{A}})\cdot(\boldsymbol{\partial}\varphi^a_V - \boldsymbol{\mathcal{A}}^a) - u^2\,\tau_{\mathrm{B}}\,\boldsymbol{\mathcal{A}}^a\cdot\partial_t\boldsymbol{\mathcal{A}}\Big]\,. \quad (11)$$

At small q, there is only one nonhydrodynamic mode which is purely imaginary, while the diffusive one is unchanged. At large q, the excitations are a pair of DO modes with phase velocity u. This behavior is characteristic of Maxwell-Cattaneo (MC) theory [38–40]. Indeed, the E.o.M for $\boldsymbol{\mathcal{A}}$ from Eq. (11) can be recast into the equation for $\boldsymbol{J}_{\mathrm{B}} = -f^2u^2\,(\boldsymbol{\partial}\varphi_V - \boldsymbol{\mathcal{A}})$, which reads $\tau_{\mathrm{B}}\partial_t\boldsymbol{J}_{\mathrm{B}} = -(\boldsymbol{J}_{\mathrm{B}} + D_{\mathrm{B}}\boldsymbol{\partial}J^t_{\mathrm{B}})$. This is precisely the MC equation for the spatial current, which relaxes to Fick's law at the timescale τ_{B}, and is analogous to Müller-Isreal-Stewart theory in the energy-momentum sector [3]. This demonstrates that Eq. (11) provides an action formulation of MC theory.

General K and holography.– For general K, there are K-pairs of DO modes in nonhydrodynamic sectors, corresponding to the propagation and dissipation of "massive photons". The continuum limit $K \to \infty$ is of particular interest. Introducing an extra-dimensional coordinate ρ by $\rho_n = n\,\Delta\rho$ (with $\Delta\rho$ infinitesimal) and viewing $\mathcal{A}_n = \mathcal{A}(\rho_n)$, the action (7) becomes that of Maxwell theory in $4+1$ dimensional curved background. Thus, holographic models can be viewed as a special class of action (7) in the continuum limit (see Ref. [32] for details). This relationship is analogous to that between hidden local symmetry models and holographic QCD [41].

Previous works have coupled holographic-like QNMs to hydrodynamic sectors [36] and used MC-type equations to ensure causality. The action (11) was studied by Nickel and Son in the context of holographic liquids [42]. However, the unified framework presented here, which describes diverse nonhydrodynamic excitations, is new in the literature.

Shift symmetry and structural difference in excitations.– Our framework offers new insights into the differences in the analytic structures of kinetic and holographic theories. This distinction becomes evident when we consider the homogeneous limit $q \to 0$, where holographic QNMs behave as DO modes, while kinetic excitations are purely imaginary. This difference is often attributed to variations in interaction strength.

However, our analysis reveals that this distinction is closely tied to the underlying symmetry structure. In scenario b), the "broken case", where holographic theories belong, the n-independent shift symmetry allows for a mass term $\Delta\boldsymbol{\mathcal{A}}_n \cdot \Delta\boldsymbol{\mathcal{A}}^a_n$ in $I_{\rm B}$. This term generates the real part in the dispersion relation. Conversely, in scenario a), the "unbroken case", the transformation prohibits the analogous term $\Delta\phi_\alpha \Delta\phi^a_\alpha$ in $I_{\rm U}$, thereby resulting in a purely imaginary spectrum at $q = 0$.

The hybridization of kinetic theory and QNMs.– Let us consider the mixed scenario. One motivation is to explore the properties of QGP. It is anticipated that QGP behaves like a holographic liquid at long distances while admitting a kinetic description with quasi-particles at shorter scales, making hybrid approaches particularly relevant [24–26, 43].

A naive combination of the kinetic action $I_{\rm U}$ with $I_{\rm B}$ conserves charge separately in each sector, yielding two diffusive modes rather than one. This doubling of hydrodynamic modes when hybridizing holographic and kinetic sectors has already been observed in previous models [26]. Instead, we couple those sectors through φ_V as $I = I_{\rm U}[\phi] + I_{\rm B}[\varphi_V, \mathcal{A}] + \Delta I[\phi, \varphi_V]$ where

$$\Delta\mathcal{L} = -\frac{f_{\rm U}^2}{\tau_{\rm R}} \int_\Omega (\partial_t\phi(\boldsymbol{v}) - \partial_t\varphi_V)(\phi^a(\boldsymbol{v}) - \varphi^a_V)\,. \quad (12)$$

Here $\int_\Omega = \int d\Omega/4\pi$ represents the solid angle integration in phase space and $f_{\rm U}^2 = \int \frac{dp}{2\pi^2} p^2(-n'(\epsilon_0))$ is the kinetic sector susceptibility. The coupling preserves gauge invariance and both shift symmetries.

From this action the E.o.M for $\phi(\boldsymbol{v}), \varphi_V$ read

$$(\partial_t + \boldsymbol{v}\cdot\partial)(\partial_t\phi(\boldsymbol{v})) = -\frac{1}{\tau_{\rm R}}(\partial_t\phi(\boldsymbol{v}) - \partial_t\varphi_V)\,, \quad (13a)$$

$$\partial_\mu J^\mu_{\rm B} = -\frac{f_{\rm U}^2}{\tau_{\rm R}} \int_\Omega (\partial_t\varphi_V - \partial_t\phi(\boldsymbol{v}))\,. \quad (13b)$$

It follows that the total charge is conserved $\partial_\mu(J_{\rm B} + J_{\rm U})^\mu = 0$ where the charge current for the kinetic sector is given by $J^\mu_{\rm U} = f_{\rm U}^2 \int_\Omega (1, \boldsymbol{v})\,\partial_t\phi(\boldsymbol{v})$. The charge density will redistribute between the two sectors until they reach a chemical equilibrium where $\mu_{\rm B} \equiv \partial_t\varphi_V = \int_\Omega \partial_t\phi(\boldsymbol{v}) \equiv \mu_{\rm U}$ such that R.H.S. of Eq. (13b) vanishes. The charge ratio in equilibrium $r \equiv J^t_{\rm U}/J^t_{\rm B} = \chi_{\rm U}/\chi_{\rm B}$, with χ the susceptibility, determines the relative importance of the two sectors. Because of this chemical equilibration process, we find one diffusive mode $\omega_{\rm D}$ and one damping mode $\omega_{\rm C}$ instead of two diffusive modes. In particular, the damping mode is given by $\omega_{\rm C} = -i(1+r)/\tau_{\rm R}$, indicating that the chemical equilibration between two sectors is faster than the thermalization of quasi-particles.

Explicitly solving Eq. (13) along with Eq. (9), the condition for the dispersion relations reads (assuming relativistic particle in the kinetic sector)

$$R(\omega, \boldsymbol{q}) + \frac{if_{\rm U}^2}{\tau_{\rm R}}\left(1 - \frac{1}{2iq\tau_{\rm R}}\log\left(\frac{\omega - q + i\tau_{\rm R}^{-1}}{\omega + q + i\tau_{\rm R}^{-1}}\right)\right) = 0\,. \quad (14)$$

Without R, it reduces to the condition for excitation in RTA kinetic theory [14]. For definiteness, we now take $R(\omega, \boldsymbol{q})$ for $K = 1$ from Eq. (10) and illustrate the structure of excitations in Fig. 1. The branch points $\pm q - i\tau_{\rm R}^{-1}$ originated from the branch-cut associated with quasi-particle excitations, while the two DO modes, $\omega_\pm$, come from the broken sector. At small q, the diffusive mode is

$$\omega_{\rm D} = -\frac{i}{1+r}\left(u^2\tau_{\rm B} + \frac{r}{3}\tau_{\rm R}\right)q^2\,, \quad (15)$$

so the diffusivity interpolates between $D_{\rm B} = u^2\tau_{\rm B}$ (small r) and kinetic theory's $\tau_{\rm R}/3$ (large r).

By matching the broken sector to strongly coupled gauge theories, we find $\mathrm{Im}\,\omega_\pm \sim \tau_{\rm B}^{-1} \sim T$, while the kinetic description of QGP gives $\tau_{\rm R} \sim 1/(g^4_{\rm QCD}T)$. This naturally establishes the hierarchy $\tau_{\rm R}^{-1} \ll \tau_{\rm B}^{-1}$, implying that quasi-particles dominate at time scale longer than $\tau_{\rm R}$. Nevertheless, for small (but not exponentially small) r, the diffusive constant (14) can still be comparable to $D_{\rm SYM}$. Despite its simplicity, the model presents a novel scenario to resolve how QGP can exhibit both quasi-particle excitation and strongly coupled transport.

Fluctuation and shift symmetry.– The action formalism is advantageous for describing fluctuations, which are captured in terms beyond the linear order of the a-field.

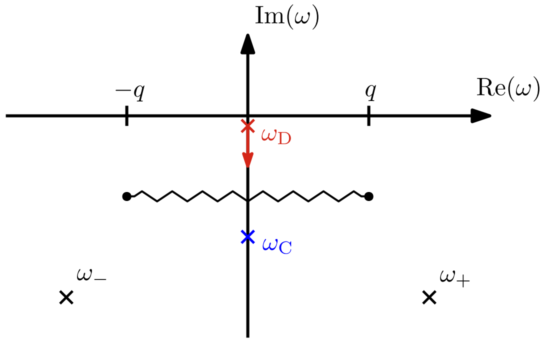

FIG. 1. Mode structure of the retarded correlation function for the hybrid model at small values of $q\tau_{\mathrm{B}}$, showing the hierarchy between the excitations. The arrow indicates the motion of the diffusive mode when increasing q.

We now present the construction of I_{aa}. The fluctuation-dissipation (FD) relation (or Kubo–Martin–Schwinger (KMS) condition) fixes I_{aa}. This can be implemented by requiring invariance of $I_{\mathrm{FD}} \equiv I_{a,\mathrm{diss}} + I_{aa}$ under the KMS transformation $\chi_\pm(t) \to \Theta\chi_\pm(t \pm i/2T)$, where $\chi_\pm = \chi \pm \chi_a/2$ represent forward$(-)$/backward $(+)$ contour fields. However, this transformation is nonlocal, indicating I_{aa} thus obtained is also nonlocal. The locality of KMS transformation is only restored at low-frequency $\omega/T \ll 1$ where $\chi(t) \to \Theta\chi(t), \chi^a(t) \to \Theta\chi^a(t) + iT^{-1}\partial_t\Theta\chi(-t)$, but nonhydrodynamic modes are generally important when ω/T is not small.

Inspired by the successful description of the tower of QNMs through inclusion of "hidden" fields, we introduce a continuum of fields $\tilde{\chi}(\theta, x)$ with an extra coordinate $\theta \in (0, 2\pi)$ for the construction of I_{aa}. Including $\tilde{\chi}$ is also motivated by the general situation in field theory, where recovering locality requires relevant modes to be "integrated-in." Those fields $\tilde{\chi}(\theta, x)$ are assumed to be related to SK fields via

$$\tilde{\chi}(\theta = 0) = \chi_-\,, \qquad \tilde{\chi}(\theta = 2\pi) = \chi_+\,. \tag{16}$$

We seek a local action $\tilde{I}[\tilde{\chi}]$ such that

$$\exp\left(i\, I_{\mathrm{FD}}[\chi, \chi_a]\right) = \int \mathcal{D}\tilde{\chi}\, \exp\left(i\tilde{I}[\tilde{\chi}]\right)\,. \tag{17}$$

The key idea underlying the construction is again the shift symmetry of $\tilde{I}$ under the transformation for $\tilde{\chi}$:

$$\delta_s\tilde{\chi}(\theta, x) = \tilde{\lambda}(x)\,. \tag{18}$$

This symmetry implies that $\tilde{I}$ should be proportional to $\partial_\theta\tilde{\chi}$. This guarantees that the configuration $\tilde{\chi}(x, \theta) = \chi_r(x)$ results in $I_{\mathrm{FD}} = 0$, which is required by $I[\chi_r, 0] = 0$ in Eq. (3). Under this symmetry constraint, the most general action, up to second order in derivatives, reads

$$I_{\mathrm{FD}} = \int_x \int_0^{2\pi} d\theta \frac{1}{2}\gamma \left[i(\partial_\theta\tilde{\chi})^2 - \frac{1}{2\pi T'}\left(\partial_\theta\tilde{\chi}\right)\partial_t\tilde{\chi}\right], \tag{19}$$

where γ, T' are underdetermined parameters, and the i-factor in front of ∂_θ^2 is required by Eq. (3). Evaluating the path-integral (17) at the saddle point under the condition (16) gives

$$I_{\mathrm{FD}} = \int_{\boldsymbol{x},\omega} \omega\,\gamma\,\chi^a(-\omega)\left[i\chi(\omega) + \coth\left(\frac{\omega}{2T'}\right)\chi^a(\omega)\right]. \tag{20}$$

The action (20) is invariant under the KMS transformation with $T' = T$. Note that the first term in real space reads $\int d^4x(-\gamma\chi\partial_t\chi^a)$, representing damping, but the second one, corresponding to I_{aa} is nonlocal. This concludes our construction, where the nonlocal I_{FD} is represented by the local action (19) in terms of the hidden fields $\tilde{\chi}$ obeying shift-symmetry and condition (16). To our knowledge, both the result and the shift-symmetry-based approach we employ are new in the literature. Notably, the derivation of the action (19) is solely based on symmetry, making it potentially useful in far-from-equilibrium situations.

Our general approach to fluctuations applies to various cases (see Ref. [32] for details). For the action (7), identifying $\boldsymbol{\mathcal{A}}, \sigma$ with χ, γ respectively yields a complete description of excitations and fluctuations. It also applies RTA-typed kinetic theory [32]. Recent efforts [44, 45] have incorporated fluctuations in MC-type theories using local KMS symmetry, while fluctuating RTA kinetic theory was similarly formulated in [46]. Our approach goes beyond these developments by remaining valid for general frequencies. In the continuum limit, our approach matches with the SK action for holographic theories, which are based on a specific prescription to select the in-falling wave boundary condition near the black hole horizon [47–52]. However, our findings arise from imposing the shift symmetry rather than specific traits of black holes.

Summary and outlook.–"Hydrodynamic modes are all alike; every nonhydrodynamic sector is different in its own way."

Contrary to this common lore of thermalization in quantum field theory, we have shown that diverse nonhydrodynamic excitations—from holographic quasi-normal modes to kinetic quasi-particles and their fluctuations—can be unified in a local EFT framework by generalizing the concept of shift symmetry. These symmetries underlie the distinct analytic structures of different theories and enable the construction of hybrid models relevant to systems like the asymptotically free QGP liquid. The restrictive shift-symmetry constraint notwithstanding, our framework covers almost all interesting situations discussed in the literature regarding charge current.

Our work is motivated by the apparent importance of nonhydrodynamnic modes in characterizing the thermalization in various physical systems [53–60]. The framework presented here provides the basis for the action formulation to describe these modes, which is currently missing.

Future directions include extending our approach by incorporating diffeomorphisms symmetry for the energy-momentum sector [23], multiple non-Abelian symmetries (e.g. SU(2) in the context of going beyond spin hydrodynamics [61]) and symmetries for higher spin degrees of freedom [62]. All of these aspects are relevant for a direct application to QCD. Our hybrid model can be further developed in more complex scenarios by taking into account multiple hidden layers as well as more degrees of freedom that couple the broken and unbroken sectors, which in turn leads to a richer landscape of nonhydrodynamic excitations [63–65]. Finally, it would be interesting to explore the applications of our framework to non-Gaussian fluctuations and noises far from equilibrium [66–69].

Acknowledgments.– This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant number: 101089093 / project acronym: High-TheQ). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them. This work was partially supported by the Priority Research Area Digiworld under the program Excellence Initiative - Research University at the Jagiellonian University in Krakow. YY acknowledges the support from NSFC under Grant No. 12575150 and by CUHK-Shenzhen University Development Fund under the Grant No. UDF01003791 . We thank M. Baggioli, J.P. Blaizot, A. Jain, M. Stephanov, D. Teaney, B. Withers for helpful discussion. We also thank the support and hospitality of *Galileo Galilei Institute* and *European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT*)* where some part of the work was completed.

* xin.an@ugent.be
† robbe.brants@ugent.be
‡ michal.p.heller@ugent.be
§ yiyin@cuhk.edu.cn

[1] L. Landau and E. Lifshitz, *Fluid Mechanics*, vol. 6 of *Course of Theoretical Physics*. Elsevier Science, 2013.

[2] S. Jeon and U. Heinz, *Introduction to Hydrodynamics*, Int. J. Mod. Phys. E **24** (2015), no. 10 1530010 [arXiv:1503.03931].

[3] W. Florkowski, M. P. Heller and M. Spalinski, *New theories of relativistic hydrodynamics in the LHC era*, Rept. Prog. Phys. **81** (2018), no. 4 046001 [arXiv:1707.02282].

[4] P. Romatschke and U. Romatschke, *Relativistic Fluid Dynamics In and Out of Equilibrium*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5, 2019.

[5] J. Berges, M. P. Heller, A. Mazeliauskas and R. Venugopalan, *QCD thermalization: Ab initio approaches and interdisciplinary connections*, Rev. Mod. Phys. **93** (2021), no. 3 035003 [arXiv:2005.12299].

[6] G. T. Horowitz and V. E. Hubeny, *Quasinormal modes of AdS black holes and the approach to thermal equilibrium*, Phys. Rev. D **62** (2000) 024027 [arXiv:hep-th/9909056].

[7] D. Birmingham, I. Sachs and S. N. Solodukhin, *Conformal field theory interpretation of black hole quasinormal modes*, Phys. Rev. Lett. **88** (2002) 151301 [arXiv:hep-th/0112055].

[8] P. K. Kovtun and A. O. Starinets, *Quasinormal modes and holography*, Phys. Rev. D **72** (2005) 086009 [arXiv:hep-th/0506184].

[9] A. Buchel, M. P. Heller and R. C. Myers, *Equilibration rates in a strongly coupled nonconformal quark-gluon plasma*, Phys. Rev. Lett. **114** (2015), no. 25 251601 [arXiv:1503.07114].

[10] R. A. Janik, G. Plewa, H. Soltanpanahi and M. Spalinski, *Linearized nonequilibrium dynamics in nonconformal plasma*, Phys. Rev. D **91** (2015), no. 12 126013 [arXiv:1503.07149].

[11] R. A. Janik, J. Jankowski and H. Soltanpanahi, *Quasinormal modes and the phase structure of strongly coupled matter*, JHEP **06** (2016) 047 [arXiv:1603.05950].

[12] B. Withers, *Short-lived modes from hydrodynamic dispersion relations*, JHEP **06** (2018) 059 [arXiv:1803.08058].

[13] S. Grozdanov, P. K. Kovtun, A. O. Starinets and P. Tadić, *Convergence of the Gradient Expansion in Hydrodynamics*, Phys. Rev. Lett. **122** (2019), no. 25 251601 [arXiv:1904.01018].

[14] P. Romatschke, *Retarded correlators in kinetic theory: branch cuts, poles and hydrodynamic onset transitions*, Eur. Phys. J. C **76** (2016), no. 6 352 [arXiv:1512.02641].

[15] A. Kurkela and U. A. Wiedemann, *Analytic structure of nonhydrodynamic modes in kinetic theory*, Eur. Phys. J. C **79** (2019), no. 9 776 [arXiv:1712.04376].

[16] G. D. Moore, *Stress-stress correlator in ϕ^4 theory: poles or a cut?*, JHEP **05** (2018) 084 [arXiv:1803.00736].

[17] S. Ochsenfeld and S. Schlichting, *Hydrodynamic and non-hydrodynamic excitations in kinetic theory — a numerical analysis in scalar field theory*, JHEP **09** (2023) 186 [arXiv:2308.04491].

[18] M. Bajec, S. Grozdanov and A. Soloviev, *Spectra of correlators in the relaxation time approximation of kinetic theory*, JHEP **08** (2024) 065 [arXiv:2403.17769].

[19] R. Brants, *New insights into the analytic structure of correlation functions via kinetic theory*, Phys. Rev. D **110** (2024), no. 11 116027 [arXiv:2409.09022].

[20] S. Bhattacharyya, V. E. Hubeny, S. Minwalla and M. Rangamani, *Nonlinear Fluid Dynamics from Gravity*, JHEP **02** (2008) 045 [arXiv:0712.2456].

[21] M. Crossley, P. Glorioso and H. Liu, *Effective field theory of dissipative fluids*, JHEP **09** (2017) 095 [arXiv:1511.03646].

[22] M. Crossley, P. Glorioso and H. Liu, *Effective field theory of dissipative fluids (II): classical limit, dynamical KMS symmetry and entropy current*, JHEP **1709** (2017) 096 [arXiv:1701.07817].

[23] H. Liu and P. Glorioso, *Lectures on non-equilibrium effective field theories and fluctuating hydrodynamics*, PoS **TASI2017** (2018) 008 [arXiv:1805.09331].

[24] E. Iancu and A. Mukhopadhyay, *A semi-holographic model for heavy-ion collisions*, JHEP **06** (2015) 003 [arXiv:1410.6448].

[25] A. Mukhopadhyay, F. Preis, A. Rebhan and S. A. Stricker, *Semi-Holography for Heavy Ion Collisions: Self-Consistency and First Numerical Tests*, JHEP **05** (2016) 141 [arXiv:1512.06445].

[26] A. Kurkela, A. Mukhopadhyay, F. Preis, A. Rebhan and A. Soloviev, *Hybrid Fluid Models from Mutual Effective Metric Couplings*, JHEP **08** (2018) 054 [arXiv:1805.05213].

[27] C. O. Akyuz, G. Goon and R. Penco, *The Schwinger-Keldysh coset construction*, JHEP **06** (2024) 004 [arXiv:2306.17232].

[28] L. V. Keldysh, *Diagram Technique for Nonequilibrium Processes*, Sov. Phys. JETP **20** (1965) 1018–1026.

[29] F. M. Haehl, R. Loganayagam and M. Rangamani, *The Fluid Manifesto: Emergent symmetries, hydrodynamics, and black holes*, JHEP **01** (2016) 184 [arXiv:1510.02494].

[30] K. Jensen, N. Pinzani-Fokeeva and A. Yarom, *Dissipative hydrodynamics in superspace*, JHEP **09** (2018) 127 [arXiv:1701.07436].

[31] X. Chen-Lin, L. V. Delacrétaz and S. A. Hartnoll, *Theory of diffusive fluctuations*, Phys. Rev. Lett. **122** (2019), no. 9 091602 [arXiv:1811.12540].

[32] X. An, R. Brants, M. P. Heller and Y. Yin, in progress.

[33] X. Huang, A. Lucas, U. Mehta and M. Qi, *Effective field theory for ersatz Fermi liquids*, Phys. Rev. B **110** (2024), no. 3 035102 [arXiv:2402.14066].

[34] L. V. Delacretaz, Y.-H. Du, U. Mehta and D. T. Son, *Nonlinear bosonization of Fermi surfaces: The method of coadjoint orbits*, Phys. Rev. Res. **4** (2022), no. 3 033131 [arXiv:2203.05004].

[35] Y. Bu, M. Lublinsky and A. Sharon, *$U(1)$ current from the AdS/CFT: diffusion, conductivity and causality*, JHEP **04** (2016) 136 [arXiv:1511.08789].

[36] M. P. Heller, R. A. Janik, M. Spaliński and P. Witaszczyk, *Coupling hydrodynamics to nonequilibrium degrees of freedom in strongly interacting quark-gluon plasma*, Phys. Rev. Lett. **113** (2014), no. 26 261601 [arXiv:1409.5087].

[37] M. P. Heller, A. Serantes, M. Spaliński, V. Svensson and B. Withers, *Relativistic Hydrodynamics: A Singulant Perspective*, Phys. Rev. X **12** (2022), no. 4 041010 [arXiv:2112.12794].

[38] J. C. Maxwell, *Iv. on the dynamical theory of gases*, Philosophical transactions of the Royal Society of London (1867), no. 157 49–88.

[39] C. Cattaneo, *Sulla conduzione del calore*, Atti Sem. Mat. Fis. Univ. Modena **3** (1948) 83–101.

[40] Y. Ahn, M. Baggioli, Y. Bu, M. Matsumoto and X. Sun, *Simple holographic dual of the Maxwell-Cattaneo model and the fate of KMS symmetry for nonhydrodynamic modes*, Phys. Rev. D **112** (2025), no. 8 086013 [arXiv:2506.00926].

[41] D. T. Son and M. A. Stephanov, *QCD and dimensional deconstruction*, Phys. Rev. D **69** (2004) 065020 [arXiv:hep-ph/0304182].

[42] D. Nickel and D. T. Son, *Deconstructing holographic liquids*, New J. Phys. **13** (2011) 075010 [arXiv:1009.3094].

[43] J. Casalderrey-Solana, D. C. Gulhan, J. G. Milhano, D. Pablos and K. Rajagopal, *A Hybrid Strong/Weak Coupling Approach to Jet Quenching*, JHEP **10** (2014) 019 [arXiv:1405.3864]. [Erratum: JHEP 09, 175 (2015)].

[44] A. Jain and P. Kovtun, *Schwinger-Keldysh effective field theory for stable and causal relativistic hydrodynamics*, JHEP **01** (2024) 162 [arXiv:2309.00511].

[45] N. Mullins, M. Hippert and J. Noronha, *Stochastic fluctuations in relativistic fluids: Causality, stability, and the information current*, Phys. Rev. D **108** (2023), no. 7 076013 [arXiv:2306.08635].

[46] N. Abbasi, M. Kaminski and D. H. Rischke, *Comparison between Causal and Acausal Diffusion: a Schwinger-Keldysh Effective Field Theory Perspective*, arXiv:2506.20500.

[47] C. P. Herzog and D. T. Son, *Schwinger-Keldysh propagators from AdS/CFT correspondence*, JHEP **03** (2003) 046 [arXiv:hep-th/0212072].

[48] M. Crossley, P. Glorioso, H. Liu and Y. Wang, *Off-shell hydrodynamics from holography*, JHEP **02** (2016) 124 [arXiv:1504.07611].

[49] P. Glorioso, M. Crossley and H. Liu, *A prescription for holographic Schwinger-Keldysh contour in non-equilibrium systems*, arXiv:1812.08785.

[50] J. de Boer, M. P. Heller and N. Pinzani-Fokeeva, *Holographic Schwinger-Keldysh effective field theories*, JHEP **05** (2019) 188 [arXiv:1812.06093].

[51] B. Chakrabarty, J. Chakravarty, S. Chaudhuri, C. Jana, R. Loganayagam and A. Sivakumar, *Nonlinear Langevin dynamics via holography*, JHEP **01** (2020) 165 [arXiv:1906.07762].

[52] M. Baggioli, Y. Bu and V. Ziogas, *U(1) quasi-hydrodynamics: Schwinger-Keldysh effective field theory and holography*, JHEP **09** (2023) 019 [arXiv:2304.14173].

[53] P. Romatschke, *Relativistic Fluid Dynamics Far From Local Equilibrium*, Phys. Rev. Lett. **120** (2018), no. 1 012301 [arXiv:1704.08699].

[54] A. Kurkela, W. van der Schee, U. A. Wiedemann and B. Wu, *Early- and Late-Time Behavior of Attractors in Heavy-Ion Collisions*, Phys. Rev. Lett. **124** (2020), no. 10 102301 [arXiv:1907.08101].

[55] J. Brewer, L. Yan and Y. Yin, *Adiabatic hydrodynamization in rapidly-expanding quark–gluon plasma*, Phys. Lett. B **816** (2021) 136189 [arXiv:1910.00021].

[56] J. Brewer, B. Scheihing-Hitschfeld and Y. Yin, *Scaling and adiabaticity in a rapidly expanding gluon plasma*, arXiv:2203.02427.

[57] W. Ke and Y. Yin, *Does a Quark-Gluon Plasma Feature an Extended Hydrodynamic Regime?*, Phys. Rev. Lett. **130** (2023), no. 21 212303 [arXiv:2208.01046].

[58] X. An and M. Spaliński, *QGP physics from attractor perturbations*, Phys. Rev. D **110** (2024), no. 11 114043 [arXiv:2312.17237].

[59] M. De Lescluze, M. P. Heller, A. Mazeliauskas, B. Scheihing-Hitschfeld and C. Werthmann, *Adiabatic hydrodynamization and quasinormal modes of nonthermal attractors*, arXiv:2510.15016.

[60] M. De Lescluze and M. P. Heller, *Quasinormal Modes of Nonthermal Fixed Points*, Phys. Rev. Lett. **135** (2025), no. 9 091601 [arXiv:2502.01622].

[61] W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, *Relativistic fluid dynamics with spin*, Phys. Rev. C **97** (2018), no. 4 041901 [arXiv:1705.00587].

[62] S. Giombi and X. Yin, *The Higher Spin/Vector Model Duality*, J. Phys. A **46** (2013) 214003 [arXiv:1208.4036].

[63] S. A. Hartnoll and S. P. Kumar, *AdS black holes and*

thermal Yang-Mills correlators, JHEP **12** (2005) 036 [arXiv:hep-th/0508092].

[64] S. Grozdanov, N. Kaplis and A. O. Starinets, *From strong to weak coupling in holographic models of thermalization*, JHEP **07** (2016) 151 [arXiv:1605.02173].

[65] S. Grozdanov and A. Soloviev, *Towards the BBGKY hierarchy: a scheme beyond the Boltzmann equation and application to a weakly confined QCD gas*, arXiv:2501.00099.

[66] A. Jain and P. Kovtun, *Late Time Correlations in Hydrodynamics: Beyond Constitutive Relations*, Phys. Rev. Lett. **128** (2022), no. 7 071601 [arXiv:2009.01356].

[67] X. An, G. Basar, M. Stephanov and H.-U. Yee, *Evolution of Non-Gaussian Hydrodynamic Fluctuations*, Phys. Rev. Lett. **127** (2021), no. 7 072301 [arXiv:2009.10742].

[68] N. Sogabe and Y. Yin, *Off-equilibrium non-Gaussian fluctuations near the QCD critical point: an effective field theory perspective*, JHEP **03** (2022) 124 [arXiv:2111.14667].

[69] N. Abbasi and D. H. Rischke, *Three-point functions from a Schwinger-Keldysh effective action, resummed in derivatives*, JHEP **05** (2025) 241 [arXiv:2410.07929].