

Thermal Leptogenesis in $SO(10) \times U(1)_A$ SUSY GUT

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Abstract

We investigate thermal leptogenesis within a supersymmetric grand unified theory (SUSY GUT) based on the $SO(10) \times U(1)_A$ symmetry, where both the doublet–triplet splitting problem and the unrealistic Yukawa relations are resolved under the natural assumption that all symmetry-allowed interactions appear with $\mathcal{O}(1)$ coefficients. In this framework, the structures of the Dirac neutrino Yukawa couplings and right-handed neutrino masses are determined entirely by the symmetry. The baryon asymmetry of the Universe is computed taking into account the flavor effects, Higgs asymmetry contributions, and the impact of the second-lightest right-handed neutrino. While the predicted asymmetry is too small when all $\mathcal{O}(1)$ coefficients of the Dirac neutrino Yukawa couplings are set to unity, a moderate enhancement factor $r_1 \sim 5.4$ for the lightest right-handed neutrino mass reproduces the observed baryon asymmetry without spoiling low-energy neutrino data. This corresponds to a suppressed lightest neutrino mass, $m_{\nu_1} \sim (1/r_1) \times (\text{symmetry-determined value})$, typically $m_{\nu_1} \sim 4.5 \times 10^{-4}$ eV. We further explore cases where the $\mathcal{O}(1)$ coefficients of the Dirac neutrino Yukawa couplings are ± 1 , and find that roughly half of them successfully generate the observed baryon asymmetry for $r_1 \leq 11$. Moreover, the others yield results of the correct order of magnitude, although they do not generate the observed Baryon asymmetry. These findings demonstrate that thermal leptogenesis is realized in this $SO(10) \times U(1)_A$ SUSY GUT, establishing a link between the observed baryon asymmetry and predictions for the lightest neutrino mass.

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I. INTRODUCTION

The Standard Model (SM), completed with the discovery of the Higgs boson in 2012, is a highly successful theory that explains most known phenomena. Nevertheless, several phenomena remain beyond its scope, such as neutrino oscillations, the baryon asymmetry of the universe, dark matter, and inflation. Neutrino oscillations, however, can be accounted for by endowing neutrinos with tiny masses. The simplest and most natural way to generate neutrino masses is to introduce right-handed (RH) neutrinos in addition to the left-handed (LH) ones. Since all quarks and leptons in the SM except neutrinos already possess both LH and RH components, the inclusion of RH neutrinos can be regarded as a natural extension of the SM. Moreover, introducing RH neutrinos is particularly compelling not only because it explains why LH neutrinos are much lighter than the other quarks and leptons [1–4], but also because it can account for the baryon asymmetry of the universe via the leptogenesis mechanism [5].

If we estimate the mass scale M_R of RH neutrinos using the seesaw formula $m_\nu = y_{\nu_D}^2 v^2 / M_R$, with the Dirac neutrino Yukawa coupling $y_{\nu_D} \sim 1$, Higgs vacuum expectation value $v \sim 175$ GeV, and neutrino mass $m_\nu \sim 0.05$ eV, we obtain $M_R = 6 \times 10^{14}$ GeV $:= \Lambda_R$, which is close to the supersymmetric grand unification scale $\Lambda_G = 2 \times 10^{16}$ GeV. This proximity strongly suggests a class of grand unified theories (GUTs) in which RH neutrinos acquire mass through the spontaneous breaking of the unified group. An $SO(10)$ GUT is precisely such a theory [6]: it unifies one generation of quarks and leptons (15 components) together with one RH neutrino (1 component) into a single 16-dimensional spinor representation, where the RH neutrino cannot acquire mass unless $SO(10)$ is broken.

Unfortunately, the simplest $SO(10)$ GUT suffers from a problem of Yukawa unification, $y_u = y_d = y_e = y_{\nu_D}$, arising from quark–lepton unification. Here, y_u , y_d , y_e , and y_{ν_D} are the Yukawa matrices of up-type quarks, down-type quarks, charged leptons, and Dirac neutrinos, respectively. The observed quark and lepton masses and mixing angles clearly contradict this unrealistic relation. Since the 1990s, when neutrino oscillations were first reported, $SO(10)$ GUTs have been extensively studied, including attempts to resolve this Yukawa unification problem.

On the other hand, difficulties also arise in implementing thermal leptogenesis within $SO(10)$ GUTs, particularly when attempting to reproduce neutrino masses and large mixing

angles. The observed neutrino masses and mixing angles suggest a weak mass hierarchy among LH neutrinos. In contrast, simple $SO(10)$ GUTs typically predict that the Dirac neutrino Yukawa hierarchy

mirrors that of the up-type quarks, forcing the RH neutrino mass hierarchy to be the square of the up-type quark hierarchy:

$$M_{\nu_R} \sim \left(\left(\frac{m_u}{m_t} \right)^2, \left(\frac{m_c}{m_t} \right)^2, 1 \right) M_R \sim (10^5 \text{GeV}, 10^{11} \text{GeV}, 10^{15} \text{GeV}).$$

however, requires the lightest RH neutrino mass to exceed $\sim 10^9 \text{GeV}$ in order to account for the observed baryon asymmetry[7]. The above prediction clearly violates this Davidson–Ibarra bound.

One possible way to circumvent this problem is to incorporate the effects of the second-lightest RH neutrino [8–16]. At temperatures above 10^{13}GeV , where the τ Yukawa interaction is not yet thermalized, the lepton asymmetry generated by the second-lightest RH neutrino is typically washed out by the thermal interactions of the lightest RH neutrino, so the lightest one dominates. However, at temperatures below 10^{13}GeV , where the τ Yukawa coupling becomes thermalized, each lepton flavor asymmetry must be treated separately [17–19]. In such cases, the effects of the second-lightest RH neutrino may not be negligible. Since its mass satisfies the Davidson–Ibarra bound, it can potentially generate the observed baryon asymmetry, motivating many studies in this direction. Still, a fundamental question remains: why does the RH neutrino mass hierarchy become so much stronger than other Yukawa hierarchies? This poses a significant obstacle when one attempts to explain the Yukawa structure through flavor symmetries.

One of the simplest ways to relax the unrealistic Yukawa unification in $SO(10)$ GUTs is to introduce a **10**-dimensional matter representation in addition to the three **16**’s [20–22]. When the three spinors **16**_{*i*} (*i* = 1, 2, 3) and one **10** are decomposed into $SU(5)$ multiplets, we obtain **16**_{*i*} = **10**_{*i*} + $\bar{\mathbf{5}}_i + 1_i$ and **10** = **5**’ + $\bar{\mathbf{5}}$ ’, which yield four $\bar{\mathbf{5}}$ ’s in total. Upon breaking $SO(10)$ to $SU(5)$, one pair of **5** + $\bar{\mathbf{5}}$ becomes superheavy. Assuming that $\bar{\mathbf{5}}_3$, which has larger Yukawa couplings, is the superheavy field, the SM fermions are then composed of $\bar{\mathbf{5}}_1$, **5**’, and $\bar{\mathbf{5}}_2$. If the first, second, and third generations of quarks and leptons are built from $\bar{\mathbf{5}}_1$, **5**’, and $\bar{\mathbf{5}}_2$, respectively, the hierarchy among $\bar{\mathbf{5}}$ ’s becomes very mild, almost flat. Consequently, compared with the **10**_{*i*}’s of $SU(5)$ that retain the original strong generational hierarchy, the $\bar{\mathbf{5}}$ hierarchy is much weaker. This construction explains not only the differing

mass hierarchies of quarks and leptons, but also why quark mixing angles are smaller than lepton mixing angles.

The resulting Yukawa relations are $y_u \neq y_d = y_e^T \sim y_{\nu_D}^T$. If the $SU(5)$ relation $y_d = y_e^T$ is avoided, realistic Yukawa matrices can be obtained. Importantly, realistic neutrino masses can also be realized with a much milder RH neutrino hierarchy, requiring only the first power of the up-quark hierarchy:

$$M_{\nu_R} = \left(\frac{m_u}{m_t}, \frac{m_c}{m_t}, 1 \right) (y_{\nu_D})_{33} M_R \simeq (10^8 \text{GeV}, 10^{11} \text{GeV}, 10^{13} \text{GeV}),$$

where $(y_{\nu_D})_{33}$, the (33) element of the Dirac neutrino Yukawa matrix, is significantly smaller than unity (taken here as 0.04) because $\bar{\mathbf{5}}_3$ became superheavy.

Notably, this framework naturally reproduces the various quark and lepton mass hierarchies and mixing angles within $SO(10)$ GUTs when combined with family symmetries such as anomalous $U(1)_A$. Nevertheless, even in this case, the lightest RH neutrino mass is 10^8GeV an improvement, but still below the Davidson–Ibarra bound. It has therefore been argued that non-thermal leptogenesis mechanisms may be required to reproduce the observed baryon asymmetry [12].

In this paper, we numerically calculate the baryon asymmetry via thermal leptogenesis, taking into account supersymmetric effects, flavor effects, and the role of the second-lightest RH neutrino in a natural $SO(10)$ GUT, where the above Yukawa structures are realized but the $SU(5)$ relations are avoided. In this GUT scenario, the doublet–triplet splitting problem, one of the most serious issues in SUSY GUTs, can also be solved under natural assumptions. The baryon asymmetry computed in this model is much smaller than the observed value when all $\mathcal{O}(1)$ coefficients are taken as unity. However, increasing the $\mathcal{O}(1)$ coefficient r_1 associated with the lightest RH neutrino mass yields a sufficiently large baryon asymmetry. Note that raising the lightest RH neutrino mass does not affect neutrino observables, except for the lightest active neutrino mass, which has not been unmeasured yet. We therefore analyze how large the lightest RH neutrino mass must be to reproduce the observed baryon asymmetry. We find that for $r_1 \sim 5\text{--}6$, the observed baryon asymmetry can indeed be obtained. In this case, the mass of the lightest LH neutrino is predicted to be $1/r_1$ of the symmetry-determined value.

For comparison, previous numerical studies considering supersymmetric and flavor effects have been performed in E_6 GUTs, where about 16 times larger lightest RH neutrino mass

than the value fixed by the symmetry was required. In E_6 GUTs, the presence of six RH neutrinos allows for a separation between those responsible for LH neutrino masses and those relevant for leptogenesis. Thus, even when the lightest RH neutrino mass is increased, no predictions arise for low-energy observables. Furthermore, those studies did not include the effects of the second-lightest RH neutrino.

The structure of this paper is as follows. In section II, we review the natural $SO(10)$ GUT. Section III provides a review of leptogenesis in the natural $SO(10)$ GUT, while section IV presents our numerical results. In Section V we compare our results in natural $SO(10)$ GUT with those in natural E_6 GUT. Finally Sec. VI is devoted to the summary.

II. NATURAL $SO(10)$ GUT

In this section, we briefly review the natural $SO(10)$ GUT[20–22]. Table I lists typical quantum numbers for a natural $SO(10)$ GUT. In simple terms, a “natural” GUT is one in which the two issues present in SUSY GUTs—the doublet–triplet splitting problem and the unrealistic Yukawa relations arising from matter unification—are resolved under the natural assumption that all interactions allowed by the symmetry are present with $\mathcal{O}(1)$ coefficients. An essential role is played by an anomalous $U(1)_A$ gauge symmetry. Although anomalous at first sight, the $U(1)_A$ anomalies are canceled by the Green–Schwarz mechanism[23], i.e., by a dilaton shift. One may regard this as a $U(1)$ gauge symmetry with a Fayet–Iliopoulos (FI) term,

$$\xi^2 \int d^2\theta d^2\bar{\theta} V_A,$$

where ξ is the FI parameter and V_A is the vector supermultiplet of $U(1)_A$.

In many constructions, the anomalous $U(1)_A$ also serves as a flavor symmetry. For example, retaining only the matter fields Ψ_i ($i = 1, 2, 3$), the Higgs field H , and the flavon Θ , the $U(1)_A$ -invariant superpotential is

$$W_Y = y_{ij} \left(\frac{\Theta}{\Lambda} \right)^{\psi_i + \psi_j + h} \Psi_i \Psi_j H, \quad (1)$$

where Λ is the cutoff scale, y_{ij} are $\mathcal{O}(1)$ coefficients, and ψ_i and h are the $U(1)_A$ charges of Ψ_i and H , respectively. We take the $U(1)_A$ charge of the flavon Θ to be -1 . Imposing D -flatness fixes the VEV as $\langle \Theta \rangle = \xi = \lambda \Lambda$, and in this paper we use $\lambda \simeq 0.22$. Thus,

$$W_Y = y_{ij} \lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j H. \quad (2)$$

TABLE I: Field content of a natural $SO(10)$ GUT with $U(1)_A$ charges which are denoted by small characters. The $\pm$ labels denote the Z_2 parity.

$SO(10)$	negatively charged fields	positively charged fields	matter fields
45	$A (a = -1, -)$	$A' (a' = 3, -)$	
16	$C (c = -4, +)$	$C' (c' = 3, -)$	$\Psi_i (\psi_1 = \frac{9}{2}, \psi_2 = \frac{7}{2}, \psi_3 = \frac{3}{2}, +)$
$\bar{16}$	$\bar{C} (\bar{c} = -1, +)$	$\bar{C}' (\bar{c}' = 7, -)$	
10	$H (h = -3, +)$	$H' (h' = 4, -)$	$T (t = \frac{5}{2}, +)$
1	$\Theta (\theta = -1, +)$ $Z (z = -2, -), \bar{Z} (\bar{z} = -2, -)$	$S (s = 5, +)$	

The crucial point is that, under the natural assumption that all symmetry-allowed operators—including higher-dimensional ones— appear with $\mathcal{O}(1)$ coefficients, hierarchical Yukawa structures are reproduced automatically[24]. When this assumption is extended to the whole theory, including the Higgs sector responsible for breaking the unified gauge group, the resulting framework is referred to as a natural GUT. Apart from the $\mathcal{O}(1)$ coefficients, the theory is then essentially fixed by symmetry.

The VEV of a field F is determined by its $U(1)_A$ charge f as[20–22]

$$\langle F \rangle \sim \begin{cases} \lambda^{-f} \Lambda & (f \leq 0), \\ 0 & (f > 0). \end{cases} \quad (3)$$

For example, the positively charged fields A' , C' , $\bar{C}'$, H' , and S in Table I have vanishing VEVs, while the negatively charged fields A , C , $\bar{C}$, Θ , Z , and $\bar{Z}$ can develop nonzero VEVs. The VEV $\langle A \rangle \sim \lambda^{-a} \Lambda$ breaks $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and the VEVs $\langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-\frac{(c+\bar{c})}{2}} \Lambda$ ¹ break $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$.

As discussed in Ref. [21, 22], gauge coupling unification in a natural GUT requires the cutoff Λ to coincide with the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV. An interesting prediction is enhanced proton decay, since the adjoint VEV $\langle A \rangle \sim \lambda^{-a} \Lambda_G$ is smaller than Λ_G itself. It is

¹ The gauge-invariant VEV $\langle \bar{C}C \rangle$ scales as $\lambda^{-(c+\bar{c})} \Lambda^2$. $SO(10)$ D -flatness requires $\langle C \rangle = \langle \bar{C} \rangle$, hence $\langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-\frac{c+\bar{c}}{2}} \Lambda$.

particularly important that positively charged fields have vanishing VEVs²; consequently, operators with negative total $U(1)_A$ charge are forbidden by symmetry. This “supersymmetric zero mechanism” is crucial for resolving the doublet–triplet splitting problem.

Under this natural assumption, both issues of SUSY GUTs can be addressed. For instance, since $\langle A \rangle \sim \lambda^{-a} \Lambda$, the contribution from the higher-dimensional operator $\lambda^{\psi_i+\psi_j+a+h} \Psi_i (A/\Lambda) \Psi_j H$ is of the same order as that from $\lambda^{\psi_i+\psi_j+h} \Psi_i \Psi_j H$. Thus, the unrealistic Yukawa relations from matter unification are naturally avoided. We do not elaborate on the solution to the doublet–triplet splitting problem here, as it is not directly related to the subject of this paper.

A. Quark and Lepton Mass Matrices

For leptogenesis in the natural GUT, it is important that both the Dirac neutrino Yukawa couplings and the RH-neutrino masses are symmetry-determined. Their explicit values are summarized in Table II.

Note that the Dirac neutrino Yukawas are not as large as those predicted by the naive $SO(10)$ relation $y_u = y_{\nu_D}$. This is because, in addition to the three spinor matter fields Ψ_i (each in the **16**, $i = 1, 2, 3$), one introduces a vector T (in the **10**). Upon decomposition under $SU(5)$,

$$\mathbf{16}_i = \mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i, \quad (4)$$

$$\mathbf{10}_T = \mathbf{5}_T + \bar{\mathbf{5}}_T, \quad (5)$$

so that four $\bar{\mathbf{5}}$ representations appear in total. One of these, together with $\mathbf{5}_T$, becomes superheavy after developing the VEVs $\langle C \rangle = \langle \bar{C} \rangle$ which break $SO(10)$ into $SU(5)$, and the SM fermions correspond to the remaining three $\bar{\mathbf{5}}$ ’s. It is reasonable that the $\bar{\mathbf{5}}_3$ with the largest Yukawas becomes the heavy mode $\bar{\mathbf{5}}_M$, so that the three generations are mainly

$$(\bar{\mathbf{5}}_1^0, \bar{\mathbf{5}}_2^0, \bar{\mathbf{5}}_3^0) \sim (\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_T, \bar{\mathbf{5}}_2). \quad (6)$$

As a result, the Dirac neutrino Yukawas are suppressed. Indeed, since

$$\bar{\mathbf{5}}_3 \sim \bar{\mathbf{5}}_M + \lambda^\Delta \bar{\mathbf{5}}_2^0 + \dots, \quad (7)$$

² Strictly speaking, we assume all positively charged fields have vanishing VEVs. Because $U(1)_A$ D -flatness imposes $\xi^2 + \sum_i z_i |Z_i|^2 = 0$ with $\xi^2 \ll \Lambda^2$, negatively charged fields are forced to acquire small VEVs. Such vacua can have interesting low-energy implications.

TABLE II: Three RH-neutrino masses and Dirac neutrino Yukawa entries fixed by symmetry in this scenario, up to $\mathcal{O}(1)$ coefficients.

parameter		value
Wolfenstein parameter λ		0.22
GUT scale	Λ_G	$2.0 \times 10^{16} \text{ GeV}$
N_1 mass	$\overline{M}_1 = \lambda^{12} \Lambda_G$	$2.6 \times 10^8 \text{ GeV}$
N_2 mass	$M_2 = \lambda^{10} \Lambda_G$	$5.3 \times 10^9 \text{ GeV}$
N_3 mass	$M_3 = \lambda^6 \Lambda_G$	$2.7 \times 10^{12} \text{ GeV}$
($e1$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{e1} = \lambda^6$	1.1×10^{-4}
($\mu 1$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\mu 1} = \lambda^{5.5}$	2.4×10^{-4}
($\tau 1$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\tau 1} = \lambda^5$	5.2×10^{-4}
($e2$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{e2} = \lambda^5$	5.2×10^{-4}
($\mu 2$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\mu 2} = \lambda^{4.5}$	1.1×10^{-3}
($\tau 2$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\tau 2} = \lambda^4$	2.3×10^{-3}
($e3$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{e3} = \lambda^3$	1.1×10^{-2}
($\mu 3$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\mu 3} = \lambda^{2.5}$	2.3×10^{-2}
($\tau 3$) of $(y_{\nu_D})_{\alpha i}$	$(y_{\nu_D})_{\tau 3} = \lambda^2$	4.8×10^{-2}

the state $\bar{5}_2^0$ inherits Yukawa couplings. In the model of Table I, one finds $\Delta = t - \psi_3 + \frac{1}{2}(\bar{c} - c) = 2.5$, leading to

$$y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad y_d \sim y_e^T \sim y_{\nu_D}^T \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}, \quad (8)$$

which yield realistic hierarchies. The $SU(5)$ relation $y_d = y_e^T$ can be avoided by contributions involving the adjoint VEV $\langle A \rangle \sim \lambda^{-a} \Lambda$, because the higher-dimensional operator $\lambda^{\psi_i + \psi_j + a + h} \Psi_i A \Psi_j H$ is of the same order as $\lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j H$ and does not respect exact $SU(5)$ relations. Consequently, realistic Yukawa matrices are obtained.

The RH-neutrino mass arises from

$$\lambda^{\psi_i + \psi_j + 2\bar{c}} \Psi_i \Psi_j \bar{C}^2 \Rightarrow (M_{\nu_R})_{ij} \sim \lambda^{\psi_i + \psi_j + \bar{c} - c} \Lambda, \quad (9)$$

where $\langle \bar{C} \rangle = \langle C \rangle = \lambda^{-\frac{1}{2}(c+\bar{c})} \Lambda$ is fixed by the charges. Without loss of generality, we may take (M_{ν_R}) diagonal, $(M_{\nu_R})_{ij} = \lambda^{\psi_i + \psi_j + \bar{c} - c} \Lambda \delta_{ij}$. Then the LH-neutrino masses are generated via the seesaw:

$$(M_\nu) = (M_{\nu_D})(M_{\nu_R})^{-1}(M_{\nu_D})^T \sim \lambda^{4+h+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \frac{v^2}{\Lambda}, \quad (10)$$

with $M_{\nu_D} := y_{\nu_D} v$ and v the SM Higgs VEV, yielding the observed neutrino masses. The quark and lepton mixing matrices are

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \quad (11)$$

It is important that M_ν is also fixed by the $U(1)_A$ charges. For example,

$$(M_\nu)_{33} = \lambda^{2\psi_2 + 2h - \bar{c} + c} \frac{v^2}{\Lambda}, \quad (12)$$

so that, even after summing over RH generations, the dependence on the RH charge ψ_i cancels. This indicates that when RH-neutrino masses are symmetry-fixed, the individual RH contributions to LH masses are of comparable order.

For later use, we parametrize an $\mathcal{O}(1)$ coefficient r_1 for the lightest RH-neutrino mass, $M_1 := r_1 \bar{M}_1$, since the generated baryon asymmetry is highly sensitive to this parameter (all other $\mathcal{O}(1)$ coefficients are set to unity). Here $\bar{M}_1$ denotes the mass for $r_1 = 1$. The observed neutrino masses and mixings remain unchanged even for $r_1 \gg 1$. Because the three RH states contribute comparably to M_ν , the heavier two LH eigenvalues are essentially unchanged. However, using

$$m_{\nu 1} m_{\nu 2} m_{\nu 3} = \det(M_\nu) = \frac{[\det(M_{\nu_D})]^2}{\det(M_{\nu_R})} = \frac{[\det(M_{\nu_D})]^2}{M_1 M_2 M_3}, \quad (13)$$

we see that the lightest LH-neutrino mass is reduced by a factor $1/r_1$ relative to its symmetry value. Since the mixing angles are unaffected, increasing M_1 by r_1 lowers $m_{\nu 1}$ by $1/r_1$, while other low-energy neutrino observables remain essentially unchanged.

III. LEPTOGENESIS IN $SO(10) \times U(1)_A$ MODEL

A $B - L$ asymmetry is dynamically generated through CP-violating (inverse) decays and scatterings of RH neutrinos. The baryon (B) and lepton (L) asymmetries are defined as

the net differences between baryons and antibaryons, and between leptons and antileptons, respectively. Part of the generated $B - L$ asymmetry is converted into a baryon asymmetry via sphaleron processes [25],

$$Y_B(t_0) = \frac{8}{23} Y_{B-L}(t_f), \quad (14)$$

where t_f denotes the freeze-out time of Y_{B-L} and t_0 the present epoch. To account for the observed baryon asymmetry, one requires $Y_{B-L}^{\text{obs}} \simeq 2.5 \times 10^{-10}$ [26]. The final freeze-out value $Y_{B-L}(t_f)$ is obtained by solving a coupled network of Boltzmann equations (see Appendix A). The generated $B - L$ asymmetry depends on both the neutrino Yukawa couplings and the RH-neutrino mass spectrum, which are dictated by the $SO(10) \times U(1)_A$ GUT structure. Thus, the prediction for baryon asymmetry directly reflects the underlying GUT model. To highlight this connection with the model parameters, we first outline the basic mechanism of unflavored (“vanilla”) leptogenesis, and then discuss the flavor-dependent dynamics essential for evaluating $B - L$ asymmetry within $SO(10) \times U(1)_A$.

At temperature $T \gtrsim 10^{13}$ GeV, charged-lepton Yukawa interactions are out of thermal equilibrium.³ In this case, the generated lepton asymmetry is mainly controlled by the CP-asymmetry parameters ϵ_i and decay parameters K_i . The total CP-asymmetry can be expressed as [27]

$$\begin{aligned} \epsilon_i &:= \epsilon_{N_i}^{\ell_L} + \epsilon_{N_i}^{\tilde{\ell}_L} + \epsilon_{N_i}^{\ell_L} + \epsilon_{N_i}^{\tilde{\ell}_L} = 4\epsilon_{N_i}^{\ell_L} \\ &\simeq -\frac{1}{2\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{ii}} \sum_j^3 \text{Im} [(y_{\nu_D}^\dagger y_{\nu_D})_{ij}^2] f(M_j^2/M_i^2), \end{aligned} \quad (15)$$

where

$$\begin{aligned} \epsilon_{N_i}^{\ell_L} &:= \frac{\Gamma(N_i \rightarrow \ell_L + H_u) - \Gamma(N_i \rightarrow \bar{\ell}_L + H_u^\dagger)}{\Gamma(N_i \rightarrow \ell_L + H_u) + \Gamma(N_i \rightarrow \bar{\ell}_L + H_u^\dagger)} \\ &\simeq -\frac{1}{8\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{ii}} \sum_j^3 \text{Im} [(y_{\nu_D}^\dagger y_{\nu_D})_{ij}^2] f(M_j^2/M_i^2), \end{aligned} \quad (16)$$

$$f(x) = \sqrt{x} \left(\frac{2}{x-1} + \ln \left[\frac{1+x}{x} \right] \right). \quad (17)$$

The other CP asymmetries $\epsilon_{N_i}^{\tilde{\ell}_L}$, $\epsilon_{N_i}^{\ell_L}$, and $\epsilon_{N_i}^{\tilde{\ell}_L}$ are defined analogously. Here ℓ_L , $\tilde{\ell}_L$, and $\tilde{N}_i$ denote lepton doublets, slepton doublets, and RH sneutrinos, respectively. The resulting

³ The precise scale depends on $\tan \beta := \langle H_u \rangle / \langle H_d \rangle$, since charged-lepton Yukawa couplings scale with $\tan \beta$.

Here H_u and H_d denote the up- and down-type Higgs doublets, respectively.

$B - L$ asymmetry is essentially proportional to ϵ_i (up to washout effects). The efficiency of washout is governed by the decay parameter

$$K_i := \frac{\Gamma_{N_i}(T=0)}{H(T=M_i)} \simeq \sqrt{\frac{45}{4\pi^3 g_*}} \frac{(y_{\nu_D}^\dagger y_{\nu_D})_{ii}}{8\pi} \frac{M_{pl}}{M_i}, \quad (18)$$

where g_* counts the effective number of relativistic degrees of freedom.

If $K_1 \lesssim 3$, the contribution from N_2 must be taken into account. For $K_1 \gtrsim 3$, washout processes erase any dependence on initial conditions and suppress contributions from heavier RH neutrinos. In contrast, for $K_1 \lesssim 3$, washout is weak, allowing asymmetries generated by heavier states to survive.

When the temperature falls below 10^{13} GeV, the τ Yukawa interaction equilibrates, marking the onset of the flavored regime of leptogenesis. As temperature decreases further, additional Yukawa interactions equilibrate, causing the lepton asymmetry to decohere into independent flavor components. Accounting for these flavor effects is crucial, as they can qualitatively change the efficiency and even the sign of the final baryon asymmetry.

In what follows, we focus on the interplay between flavored CP asymmetries $\epsilon_{\alpha i}$ and flavored decay parameters $K_{\alpha i}$ ($\alpha = e, \mu, \tau$) within the $SO(10) \times U(1)_A$ framework. This allows us to quantify how the flavor dynamics amplify the baryon asymmetry compared to the unflavored case.

In our setup, we track the asymmetries generated by N_1 and N_2 , while neglecting N_3 because of its much higher mass scale ($M_3 \gg M_{1,2}$) and the strong washout expected from $K_2 \gg 1$. Numerically, the relevant scales are $\overline{M}_1 \sim 2.6 \times 10^8$ GeV and $M_2 \sim 5.3 \times 10^9$ GeV, placing the thermal window in the range $10^7 \lesssim T \lesssim 10^{11}$ GeV. In this regime, the τ and μ Yukawa interactions are in thermal equilibrium, whereas the electron Yukawa is not. As a result, the flavor coherence of the $SU(2)_L$ doublet leptons is broken, and the asymmetry evolves separately in the τ , μ , and e sectors [17–19, 28]. This phenomenon is known as the flavor effect in leptogenesis.

Earlier studies have shown that flavor effects can significantly alter the final baryon asymmetry, especially in the strong washout regime ($K \gg 1$) [17, 19]. The enhancement is particularly pronounced for hierarchical neutrino Yukawa couplings, which is exactly the case in the $SO(10) \times U(1)_A$ models where most $\mathcal{O}(1)$ coefficients are taken to be unity except for the one controlling the mass of the lightest RH neutrino. In this setting, the flavored

TABLE III: Flavored CP asymmetries and decay parameters for $r_1 = 1$ in $SO(10) \times U(1)_A$ GUT scenario.

	ϵ_i	ϵ_{ei}	$\epsilon_{\mu i}$	$\epsilon_{\tau i}$	K_i	K_{ei}	$K_{\mu i}$	$K_{\tau i}$
N_1	-1.6×10^{-7}	-6.1×10^{-9}	-2.8×10^{-8}	-1.3×10^{-7}	51	1.9	8.7	40
N_2	-1.7×10^{-6}	-6.1×10^{-8}	-2.8×10^{-7}	-1.3×10^{-6}	51	1.9	8.7	40

decay parameters, $K_{\alpha i} := \Gamma_{N_i \rightarrow \ell_\alpha H}(T=0)/H(T=M_i)$, read

$$K_{\alpha 1} = \bar{K}_\alpha \left(\frac{2.6 \times 10^8 \text{ GeV}}{M_1} \right) = \frac{\bar{K}_\alpha}{r_1}, \quad K_{\alpha 2} = \bar{K}_\alpha \quad (19)$$

with $\bar{K}_e = 1.9$, $\bar{K}_\mu = 8.7$, and $\bar{K}_\tau = 40$. These values confirm that the τ flavor asymmetry is strongly washed out, while the e and μ channels are comparatively weaker. Since r_1 parameterizes the variation in the lightest RH neutrino mass, the washout of N_1 is directly governed by this coefficient. Thus, the $SO(10) \times U(1)_A$ structure tightly correlates flavor hierarchies with the flavored lepton asymmetries.

The CP asymmetries are also flavor dependent. They quantify the net number of each lepton flavor generated per N_i decay, and are given by [27]

$$\begin{aligned} \epsilon_{\alpha i} &:= 4 \times \frac{\Gamma(N_i \rightarrow \ell_{L_\alpha} + H_u) - \Gamma(N_i \rightarrow \bar{\ell}_{L_\alpha} + H_u^\dagger)}{\Gamma(N_i \rightarrow \ell_L + H_u) + \Gamma(N_i \rightarrow \bar{\ell}_L + H_u^\dagger)} \\ &= -\frac{1}{2\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{ii}} \sum_{j \neq i} \text{Im} [(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}] f(M_j^2/M_i^2). \end{aligned} \quad (20)$$

In the hierarchical limit $M_1 \ll M_2, M_3$, one finds

$$\epsilon_{N_1}^{\ell_L} \simeq -\frac{3}{8\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \sum_j^3 \text{Im} [(y_{\nu_D}^\dagger y_{\nu_D})_{1j}^2] \frac{M_1}{M_j} \propto r_1. \quad (21)$$

demonstrating that the N_1 contribution scales linearly with r_1 . Table III summarizes the flavored CP asymmetries $\epsilon_{\alpha i}$ and the decay parameters $K_{\alpha i}$ for $r_1 = 1$. The evaluation of $\epsilon_{\alpha i}$ proceeds in three steps. First, we compute $(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}$ using the real Yukawa couplings determined by the underlying symmetry (Table II). Second, since all Yukawa couplings are assumed to be complex, we replace the imaginary part with the corresponding real part, $\text{Im} [(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}] = \text{Re} [(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}]$. Finally, substituting these expressions yields the explicit values of the flavored CP asymmetries.

The evolution of the lepton asymmetry is further constrained by equilibrated SM interactions. Sphalerons and Yukawa scatterings preserve the charges $\Delta_\alpha = B/3 - L_\alpha$ ($\alpha = e, \mu, \tau$), with $B - L = \sum_\alpha \Delta_\alpha$. The Boltzmann equations governing N_i and Δ_α thus include redistribution terms encoding how lepton and Higgs asymmetries are mapped into Δ_α through conversion coefficients $C_{\alpha\beta}^L$ and C_β^H : $Y_{\ell_{L\alpha}} - Y_{\bar{\ell}_{L\alpha}} = -\sum_\beta C_{\alpha\beta}^L Y_{\Delta_\beta}$, $Y_H - Y_{\bar{H}} = -\sum_\beta C_\beta^H Y_{\Delta_\beta}$. The effect of Higgs asymmetry is so-called the spectator effect, which could affect the final amount of $B - L$ asymmetry by a factor of $\mathcal{O}(1)$ [29]. For $10^8 \lesssim T \lesssim 10^{11}$ GeV, the relevant coefficients are [30]

$$C^H = \frac{1}{358} \begin{pmatrix} 37 & 52 & 52 \end{pmatrix}, \quad C^L = \frac{1}{2148} \begin{pmatrix} 906 & -120 & -120 \\ -75 & 688 & -28 \\ -75 & -28 & 688 \end{pmatrix}. \quad (22)$$

At lower temperatures, additional Yukawa interactions (e.g. down quark) equilibrate. For $\tan\beta \simeq 7-8$ as favored in $SO(10) \times U(1)_A$, this may require a density matrix approach to fully account for coherence effects. In the present work, however, we adopt the Boltzmann framework, which suffices for extracting the leading dependence of the baryon asymmetry on the underlying GUT parameters.

IV. NUMERICAL RESULTS

In this section, we present a numerical analysis of the baryon asymmetry generated in the $SO(10) \times U(1)_A$ model. We begin by examining the dependence of the asymmetry on M_1 with a fixed neutrino Yukawa matrix. This serves as a first step toward identifying the favored parameter space in this framework. To further clarify the role of M_1 within the model structure, we then analyze three key aspects individually: (a) the contribution of the second-lightest RH neutrino N_2 , (b) the impact of flavor effects and spectator effect on leptogenesis, and (c) alternative patterns of neutrino Yukawa matrices. Finally, we compare the results with those obtained in the $E_6 \times U(1)_A$ scenario, highlighting the dependence of baryon asymmetry generation on the underlying GUT structure.

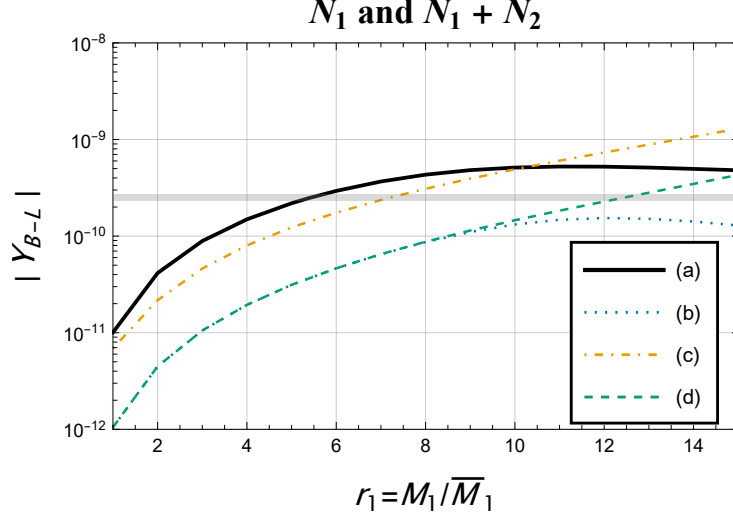


FIG. 1: r_1 dependence of Y_{B-L} for (a) $N_1 + N_2$ with the flavor effect, (b) $N_1 + N_2$ without the flavor effect, (c) N_1 only with the flavor effect, (d) N_1 only without the flavor effect.

A. M_1 dependence

Fig. 1 shows the r_1 dependence of the $B-L$ asymmetry for (a) $N_1 + N_2$ with flavor effects, (b) $N_1 + N_2$ without flavor effects, (c) N_1 only with flavor effects, and (d) N_1 only without flavor effects. The gray band indicates the required $B-L$ asymmetry that reproduces the observed baryon asymmetry. In the most realistic setup, namely $N_1 + N_2$ with flavor effects, this corresponds to $r_1 = 5.4$. Such an enhancement is naturally achievable through $\mathcal{O}(1)$ variations in the model parameters.

We first consider the case without flavor effects. As r_1 increases, the generated $B-L$ asymmetry grows because the CP asymmetry scales as $\epsilon_1 \propto r_1$, while the decay parameter decreases as $K_1 \simeq 50/r_1$ and approaches unity. Since K_1 is typically large, the N_2 contribution is negligible for $M_1 \ll M_2$, and becomes relevant only when $r_1 \gtrsim 10$ where M_1 approaches M_2 .

Turning to the flavored case, a qualitatively different picture emerges. The key point is that the flavored decay parameters, in particular K_{e1} and $K_{\mu 1}$, can be of order unity or smaller, so the N_2 contribution cannot be neglected. As shown in Table III, the flavored decay parameters at $r_1 = 1$ are already smaller than the unflavored K_1 , and since $K_{\alpha 1} \propto 1/r_1$, these become even smaller when $r_1 > 1$. Indeed, Fig. 1 demonstrates that the N_2 effect remains significant even for $r_1 = 1$, largely because the e -flavor asymmetry retains sensitivity

to its initial conditions because $K_{e1} = 1.9 < 3$.

From this analysis, the favored value of the lightest RH-neutrino mass is determined as $M_1 = 5.4 \times (2.6 \times 10^8 \text{ GeV}) = 1.4 \times 10^9 \text{ GeV}$, consistent with the $SO(10) \times U(1)_A$ symmetry structure and the observed baryon asymmetry. This further implies a prediction for the lightest LH-neutrino mass of $m_{\nu 1} \sim m_{\nu 1}(r_1 = 1)/r_1 \sim \lambda^2 m_{\nu 3}/r_1 \sim 4.5 \times 10^{-4} \text{ eV}$. Here, we have used the relation between LH neutrino masses fixed by the symmetry as $m_{\nu 1}(r_1 = 1) \sim \lambda^2 m_{\nu 3}$ [20, 22], and the measured value, $m_{\nu 3} \sim 0.05 \text{ eV}$.

B. Evolution of $B - L$ asymmetry with N_2 (without flavor effect)

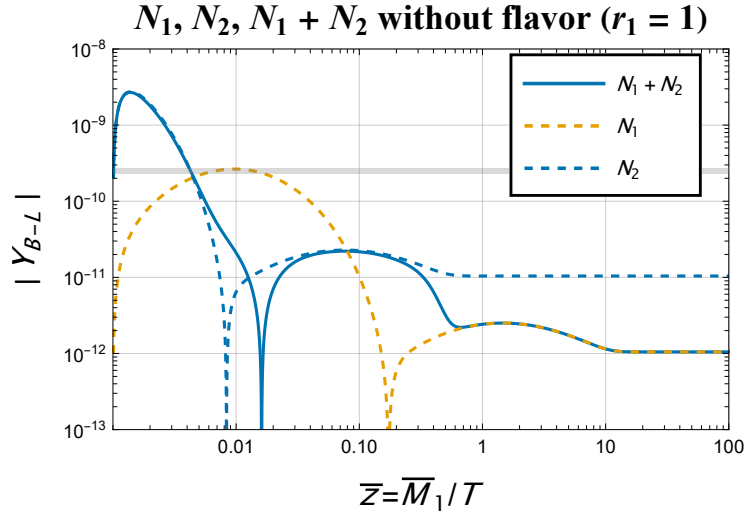


FIG. 2: Y_{B-L} evolutions for $N_1 + N_2$ (blue solid), for N_2 only (blue dashed), and for N_1 only (yellow dashed). This calculation does not take the flavor effect into account. Gray band indicates the range of $B - L$ asymmetry required to reproduce the observed baryon asymmetry.

In order to understand the results of the flavored leptogenesis, we demonstrate the evolution of $B - L$ asymmetry to study the effects of N_2 in this subsection. Fig. 2 shows the evolution of the $B - L$ asymmetry as a function of $\bar{z} := \bar{M}_1/T$ for $r_1 = 1$ in three cases: $[N_1 + N_2]$, $[N_1 \text{ only}]$, and $[N_2 \text{ only}]$. Although the evolution differs clearly among these cases, the final $B - L$ asymmetry coincides in the $[N_1 + N_2]$ and $[N_1 \text{ only}]$ cases. This behavior reflects the strong washout regime, where $K_1 \simeq 50 \gg 1$. At early times ($\bar{z} \lesssim 0.5$), the $B - L$ asymmetry is mainly washed out by N_2 interactions. For $\bar{z} \gtrsim 0.5$, however, washout induced

by N_1 becomes dominant, driving the total $B - L$ asymmetry toward that of the $[N_1 \text{ only}]$ case. To understand the result, we examine each stage of the evolution step by step.

$[N_1 \text{ only (yellow dashed)}]$

1. At the early stage ($\bar{z} \simeq 0.001$), a $B - L$ asymmetry with the same sign as the CP asymmetry is generated during the thermal production of N_1 .
2. Once N_1 approaches equilibrium ($0.01 \lesssim \bar{z} \lesssim 0.2$), the initially produced $B - L$ asymmetry is partially washed out by CP-conserving processes, such as $\ell_L \bar{N}_1 \rightarrow Q_L \bar{t}$, where Q_L and t denote the quark doublet and the singlet top quark, respectively. Concurrently, CP-violating decays of N_1 generate a $B - L$ asymmetry with the opposite sign. As the evolution progresses, the positive contribution dominates, and from this point onward, the total $B - L$ asymmetry remains positive.
3. After N_1 decays ($\bar{z} \gtrsim 1$), the $B - L$ asymmetry experiences a slight reduction due to ongoing washout processes. Eventually ($\bar{z} \gtrsim 10$), the asymmetry reaches a constant value, as the rates of $B - L$ -violating reactions drop below the Hubble expansion rate. This “freeze-out” value is a conserved quantity in the SM.

$[N_1 + N_2 \text{ (blue solid)}]$

1. For $\bar{z} \lesssim 0.2$, the $B - L$ asymmetry is primarily determined by N_2 decays. Since ε_2 is roughly an order of magnitude larger than ε_1 for similar decay parameters ($K_2 \simeq K_1$), the generated $B - L$ asymmetry exceeds that of the $[N_1 \text{ only}]$ scenario by about one order of magnitude.
2. For $\bar{z} \gtrsim 0.2$, the washout induced by N_1 becomes dominant ($K_1 \sim 50$), and the total $B - L$ asymmetry gradually converges toward the $[N_1 \text{ only}]$ result. It should be noted that if $K_1 \lesssim 3$, the asymmetry in the $[N_1 + N_2]$ context would freeze out before fully reaching the $[N_1 \text{ only}]$ value, due to the weaker washout effect.

C. Flavor effect on leptogenesis

In this section, we numerically illustrate the impact of flavor effects and spectator effect within our model parameters. Comparing the black solid line in the left panel of Fig. 3 with

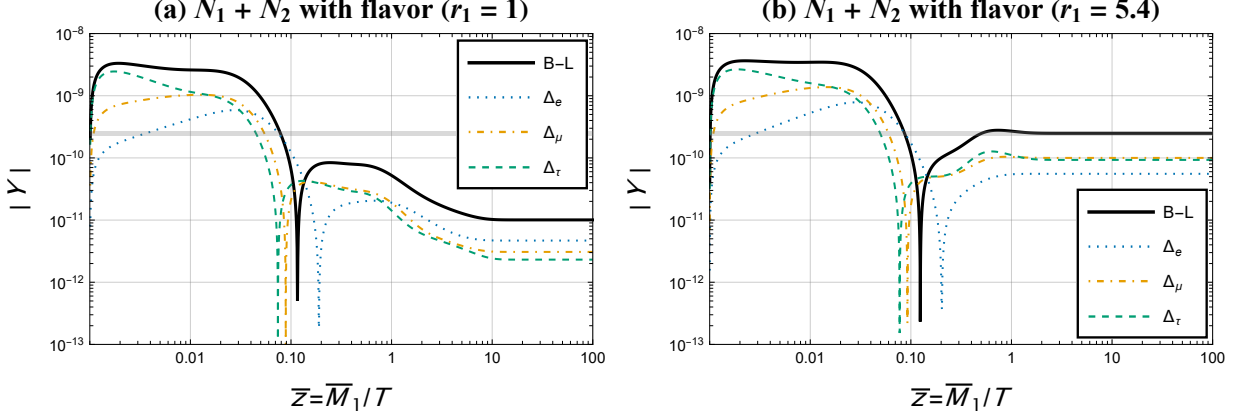


FIG. 3: Evolutions of Y_{B-L} and each flavor contribution for $r_1 = 1$ (a) and 5.4 (b). Gray band indicates the range of $B - L$ asymmetry required to reproduce the observed baryon asymmetry.

the blue solid line in Fig. 2, we observe that the final $B - L$ asymmetry is enhanced by roughly an order of magnitude. This increase is mainly due to the reduced washout arising from the smaller flavored decay parameters, $1 < K_e < K_\mu < K_\tau < K_1$. Consequently, the individual flavored asymmetries satisfy $Y_{\Delta_e} > Y_{\Delta_\mu} > Y_{\Delta_\tau} > Y_{B-L}^{\text{without flavor}}$, even though the corresponding CP asymmetries follow $\epsilon_e < \epsilon_\mu < \epsilon_\tau < \epsilon_1$, as summarized in Table III.

The right panel in Fig. 3 shows the same quantities as the left panel but for $r_1 = 5.4$. Compared to $r_1 = 1$, the flavored decay parameters $K_{\alpha 1}$ are reduced as $K_{e1} < 1 \sim K_{\mu 1} < K_{\tau 1} \sim 8$, and the CP asymmetries $\epsilon_{\alpha 1}$ are larger with the relations $\epsilon_{e1} < \epsilon_{\mu 1} < \epsilon_{\tau 1}$. As a result, the muon and tau asymmetries are enhanced, while the electron asymmetry is mainly determined by N_2 decays, where $\epsilon_{e2} > \epsilon_{e1}$ and $K_{e2} \sim 2$, since the washout from N_1 is weak for this flavor. These effects together allow the total $B - L$ asymmetry to reach a level consistent with the observed baryon number.

Fig. 3 also illustrates the $\bar{z} := \bar{M}_1/T$ evolution of the flavored asymmetries for $N_1 + N_2$ with flavor effects. For $r_1 = 1$, the strong washout regime leads to a marked suppression of the muon and tau asymmetries, yielding a total $B - L$ below the observed value. In contrast, $r_1 = 5.4$ enhances $\epsilon_{\alpha 1}$ and reduces K_μ and K_τ toward unity, resulting in the correct baryon asymmetry. Values of r_1 above 5.4 further increase the tau contribution, causing the total $B - L$ to exceed the required value. Thus, M_1 is a crucial parameter controlling both the flavor effects and the magnitude of the flavored CP asymmetries, as reflected in eq.(19) and

eq.(21).

D. Other patterns of neutrino Yukawa matrices

We so far have computed the flavored CP asymmetries given in Eq. (20) under the following assumptions. Using the real Yukawa couplings fixed by symmetry (Table II) with all $\mathcal{O}(1)$ coefficients set to unity, we have evaluated $(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}$. Since the couplings are generically complex, we have taken $\text{Im} [(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}] = \text{Re} [(y_{\nu_D})_{\alpha i}^* (y_{\nu_D})_{\alpha j} (y_{\nu_D}^\dagger y_{\nu_D})_{ij}]$. This yields maximal CP asymmetries, because contributions from virtual RH neutrinos add constructively. In general, cancellations among complex coefficients can reduce the asymmetries. To explore this, we also consider scenarios where some Yukawa entries have -1 as their $\mathcal{O}(1)$ coefficient.

Flipping signs in the neutrino Yukawa matrix changes the flavored CP asymmetries $\epsilon_{i\alpha}$ in magnitude and sign, while the decay parameters remain unchanged. Table IV summarizes all patterns, which can be classified into 15 patterns (see Appendix B). The leftmost column indicates which Yukawa components are flipped. Columns two and three show the CP asymmetry signs for each flavor, including $r_1 = 15$ when they differ from $r_1 = 1$. Column four gives $R^{\text{max}} = Y_{B-L}(r_1^{\text{max}})/Y_{B-L}^{\text{obs.}}$, where the $B - L$ asymmetry is maximal, and column five lists the corresponding r_1 required to reproduce the observed baryon asymmetry.

All patterns with negative $\epsilon_{1\alpha}$ can achieve the observed baryon asymmetry except for pattern 6. In these cases, all flavored contributions Δ_α add constructively (Fig. 4(a) and (d)), although two of them (patterns 4 and 5) change the sign of $\epsilon_{1\alpha}$ between $r_1 = 1$ and 15. In pattern 5, Δ_μ flips sign near $r_1 \sim 5$ due to the sign-change of $\epsilon_{1\mu}$. Pattern 6 fails because positive ϵ_{2e} and $\epsilon_{2\mu}$ generate $B - L$ from N_2 decay with opposite sign, and small $K_{e1}, K_{\mu1} \lesssim 1$ enhances N_2 sensitivity, preventing the observed asymmetry.

Patterns with mixed-sign $\epsilon_{1\alpha}$ generally cannot reach the observed asymmetry, except for pattern 1, due to cancellations among flavored contributions (Fig. 4(b) and (c)). In pattern 3, for example, the total $B - L$ asymmetry changes sign around $r_1 \sim 6$. Overall, seven of fifteen patterns can achieve the observed asymmetry with certain $r_1 < 15$; the others yield $B - L$ of similar order but slightly below the observed value.

Our calculations may underestimate $B - L$, since ± 1 coefficients enhance cancellations.

TABLE IV: The correlation between the sign pattern of each component of the Yukawa matrix y_{ν_D} and resulting sign of the flavored CP asymmetry $\epsilon_{\alpha i}$, and the corresponding maximum value of the $B - L$ asymmetry Y_{B-L} . From left to right column, (1) the components whose signs are flipped, (2) the signs of $\epsilon_{e1}, \epsilon_{\mu 1}, \epsilon_{\tau 1}, \epsilon_{e2}, \epsilon_{\mu 2}, \epsilon_{\tau 2}$ for $r_1 = 1$, (3) the signs of $\epsilon_{e1}, \epsilon_{\mu 1}, \epsilon_{\tau 1}, \epsilon_{e2}, \epsilon_{\mu 2}, \epsilon_{\tau 2}$ for $r_1 = 15$, (4) the maximum value of $R(r_1) := Y_{B-L}(r_1)/Y_{B-L}^{\text{obs}}$, and (5) r_1 accounting for Y_{B-L}^{obs} .

Flipped component(s)	$\epsilon_{1\alpha}, \epsilon_{2\alpha}(r_1 = 1)$	$\epsilon_{1\alpha}, \epsilon_{2\alpha}(r_1 = 15)$	$R^{\text{max}} (r_1^{\text{max}})$	r_1^{obs}
0.(all positive)	---, ---		2.1 (11)	5.4
1. $[(y_{\nu_D})_{e1}], \dots$	+--, ---		1.4 (11)	7 ~ 8
2. $[(y_{\nu_D})_{\mu 1}], \dots$	-+-, ---		0.7 (12)	
3. $[(y_{\nu_D})_{\tau 1}], \dots$	++-, ---		0.2 (12)	
4. $[(y_{\nu_D})_{e2}], \dots$	---, +--	+--, +--	1.5 (11)	6 ~ 7
5. $[(y_{\nu_D})_{\mu 2}], \dots$	---, -+-	-+-, -+-	1.2 (11)	8 ~ 9
6. $[(y_{\nu_D})_{\tau 2}], \dots$	---, ++-		0.8 (10)	
7. $[(y_{\nu_D})_{e3}], \dots$	---, +--		1.6 (11)	6 ~ 7
8. $[(y_{\nu_D})_{\mu 3}], \dots$	---, -+-		1.3 (12)	7 ~ 8
9. $[(y_{\nu_D})_{\tau 3}], \dots$	---, ++-		1.0 (12)	10 ~ 11
10. $[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu 2}], \dots$	+--, -+-	++-, -+-	0.6 (10)	
11. $[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\tau 2}], \dots$	+--, ++-	-+-, ++-	0.7 (11)	
12. $[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu 3}], \dots$	+--, -+-		0.7 (11)	
13. $[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\tau 3}], \dots$	+--, ++-		0.8 (11)	
14. $[(y_{\nu_D})_{\mu 1}, (y_{\nu_D})_{e2}], \dots$	-+-, +--	++-, +--	0.2 (11)	
15. $[(y_{\nu_D})_{\mu 1}, (y_{\nu_D})_{e3}], \dots$	-+-, +--		0.3 (12)	

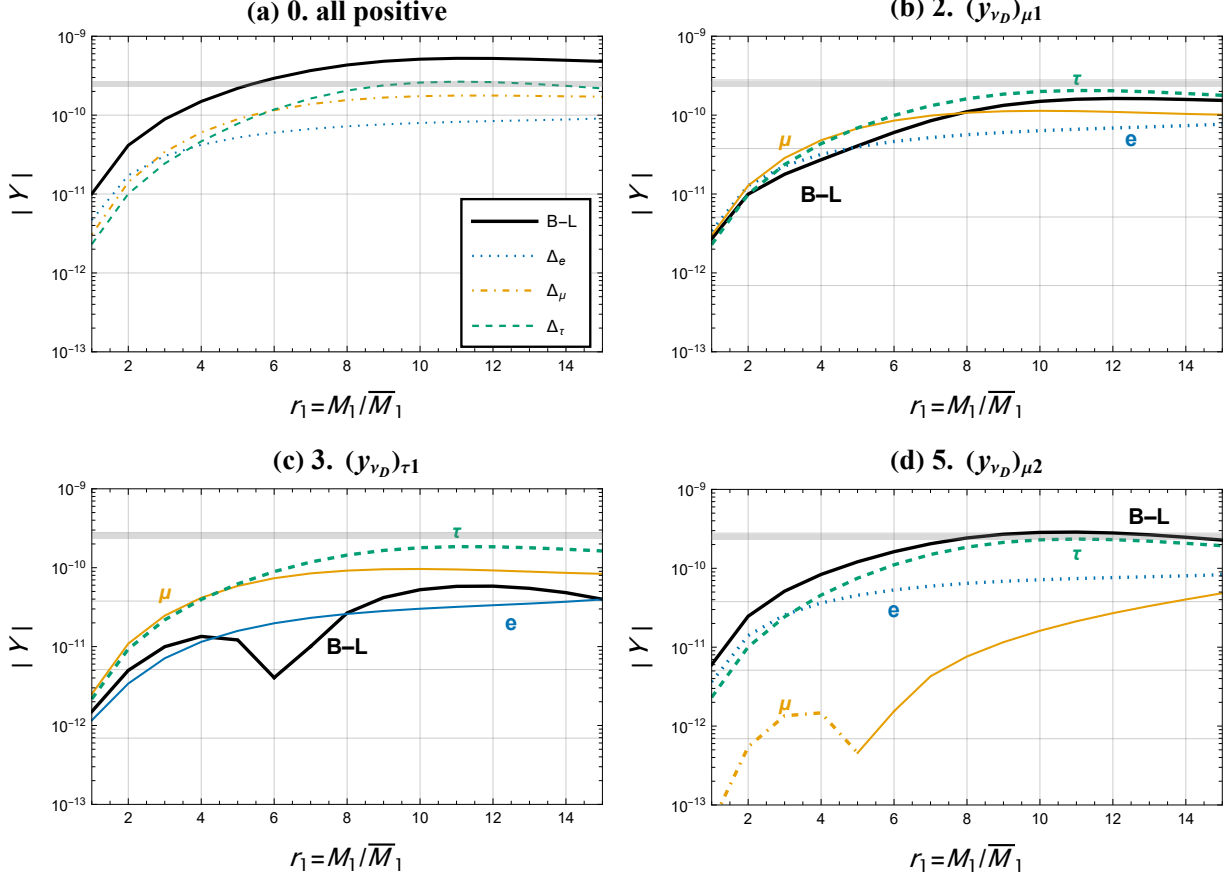


FIG. 4: The r_1 dependence of flavored Y_{B-L} in four cases: (a) pattern 0, (b) pattern 1, (c) pattern 3, and (d) pattern 5. The absolute value of total Y_{B-L} represents thick solid line. The thin solid lines denote negative Y_{Δ_α} in Figures (b), (c), and (d), while the dotted, dot-dashed, and dashed lines denote the positive Y_{Δ_α} .

For instance, in pattern 5 with $[(y_{\nu_D})_{\mu 2}]$ flipped,

$$\begin{aligned} \epsilon_{\mu 1} \propto & -\text{Im} \left[(y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 2} \left\{ (y_{\nu_D})_{e 1}^* (y_{\nu_D})_{e 2} - (y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 2} + (y_{\nu_D})_{\tau 1}^* (y_{\nu_D})_{\tau 2} \right\} \right] f \left(\frac{M_2^2}{M_1^2} \right) \\ & + \text{Im} \left[(y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 3} \left\{ (y_{\nu_D})_{e 1}^* (y_{\nu_D})_{e 3} + (y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 3} + (y_{\nu_D})_{\tau 1}^* (y_{\nu_D})_{\tau 3} \right\} \right] f \left(\frac{M_3^2}{M_1^2} \right) \end{aligned} \quad (23)$$

$$\begin{aligned} = & -\text{Im} \left[(y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 2} (y_{\nu_D}^\dagger y_{\nu_D})_{12} \right] f \left(\frac{M_2^2}{M_1^2} \right) + \text{Im} \left[(y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 3} (y_{\nu_D}^\dagger y_{\nu_D})_{13} \right] f \left(\frac{M_3^2}{M_1^2} \right) \\ & + 2 \text{Im} \left[\left\{ (y_{\nu_D})_{\mu 1}^* (y_{\nu_D})_{\mu 2} \right\}^2 \right] f \left(\frac{M_2^2}{M_1^2} \right), \end{aligned} \quad (24)$$

where all $(y_{\nu_D})_{\alpha i}$ are given in Table II with positive signs. When $M_2, M_3 \gg M_1$, the functions $f \left(\frac{M_i^2}{M_1^2} \right)$ ($i = 2, 3$) become approximately M_1/M_i . Thus, in eq.(24) the first two

terms ($\sim \lambda^{22}$) completely cancel, leaving the positive third term ($\sim \lambda^{24}$) to dominate. Since this cancellation arises from the $\mathcal{O}(1)$ coefficients taking values of ± 1 , the CP asymmetry may be underestimated owing to a stronger cancellation compared with that for general complex Yukawa couplings. Moreover, the above consideration can explain the sign-flips of several $\epsilon_{\alpha i}$ between $r_1 = 1$ and 15. This is because the accuracy of the above approximation $M_1 \ll M_2$ decreases with larger r_1 . As a result, the first term in eq.(24) dominates, and the overall sign of $\epsilon_{\mu 1}$ can flip from positive to negative for larger r_1 , explaining the sign flip of Y_{Δ_μ} around $r_1 \sim 5$ in Fig.4(d). Similar reasoning applies to other $\epsilon_{\alpha i}$ flips between $r_1 = 1$ and $r_1 = 15$.

Fig. 4 also shows representative $B - L$ evolutions. Across patterns, $B - L$ increases with r_1 and saturates around $r_1 \sim 10$. At large r_1 , the tau flavor often dominates due to larger ϵ_τ and K_τ approaching unity. Saturation is likely caused by quasi-degeneracy of N_1 and N_2 , consistent with Ref. [31], which notes significant effects for $M_2 \lesssim 3M_1$, that corresponds to $r_1 \gtrsim 7$.

V. COMPARISON WITH $E_6 \times U(1)_A$ MODEL

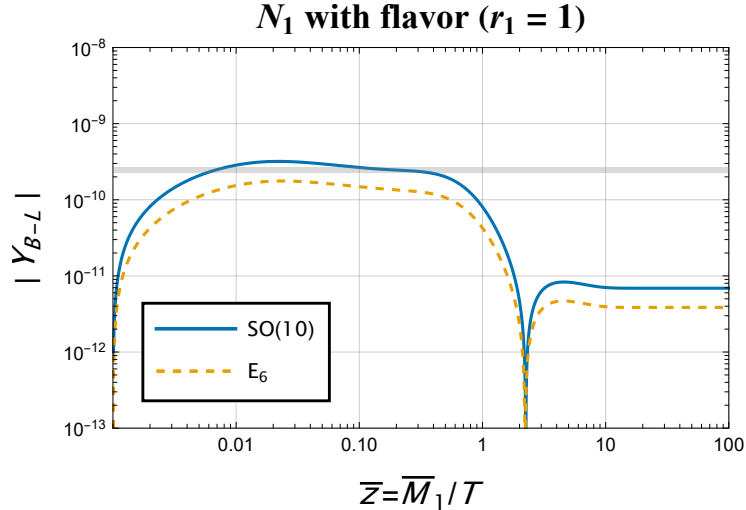


FIG. 5: Evolution of Y_{B-L} in the $SO(10) \times U(1)_A$ and $E_6 \times U(1)_A$ scenarios. We set $r_1 = 1$ in the $N_1 + N_2$ framework including flavor effects. The grey band denotes the required $B - L$ asymmetry to account for the observed baryon asymmetry.

GUT models based on the E_6 gauge group are also attractive, with the $E_6 \times U(1)_A$

scenario being particularly successful from a phenomenological perspective [32, 33]. Before summarizing this work, we compare the dependence of baryon asymmetry generation on the choice of GUT scenario, namely $SO(10) \times U(1)_A$ versus $E_6 \times U(1)_A$ [34].

TABLE V: Masses of the lightest and heaviest RH neutrinos, and ij components of $y_{\nu_D}^\dagger y_{\nu_D}$. For simplicity, all $\mathcal{O}(1)$ coefficients are set to 1.

	lightest N_i mass	heaviest N_i mass	$(y_{\nu_D}^\dagger y_{\nu_D})_{11}$	$(y_{\nu_D}^\dagger y_{\nu_D})_{13}$	$(y_{\nu_D}^\dagger y_{\nu_D})_{16}$
$SO(10)$	$M_1 \sim \lambda^{12} \Lambda_G$	$M_3 \sim \lambda^6 \Lambda_G$	λ^{10}	λ^7	
E_6	$M_1 \sim \lambda^{13} \Lambda_G$	$M_6 \sim \lambda^6 \Lambda_G$	λ^{11}	λ^{10}	$\lambda^{7.5}$

The evolution of the $B - L$ asymmetry for $r_1 = 1$ in these scenarios is shown in Fig. 5, where flavor effects are included. For simplicity, all $\mathcal{O}(1)$ coefficients are set to 1. Within this setup, the $SO(10) \times U(1)_A$ scenario produces approximately twice the $B - L$ asymmetry of the $E_6 \times U(1)_A$. This conclusion holds whether or not flavor effects are included.

The $E_6 \times U(1)_A$ model has six RH neutrinos, and the mass of the lightest RH neutrino is smaller than that in the $SO(10) \times U(1)_A$ case, as shown in Table V. However, the flavored decay parameters in the $E_6 \times U(1)_A$ model are of the same order in λ as those in the $SO(10) \times U(1)_A$ model (see Table III), because the Dirac neutrino Yukawa couplings of the lightest RH neutrino are smaller.

The key ingredient in understanding the factor-of-two difference between the scenarios is the CP asymmetry, which follows from the symmetries in each models as

$$\begin{aligned}
|\epsilon_1^{SO(10)}| &\sim \frac{3}{8\pi} \times 2 \times \frac{\text{Im}[(y_{\nu_D}^\dagger y_{\nu_D})_{13}^2]}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \frac{M_1}{M_3} \sim \frac{3}{4\pi} \lambda^{10}, \\
|\epsilon_1^{E_6}| &\sim \frac{3}{8\pi} \times 5 \times \frac{\text{Im}[(y_{\nu_D}^\dagger y_{\nu_D})_{16}^2]}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \frac{M_1}{M_6} \sim \frac{15}{8\pi} \lambda^{11} \sim 0.55 \epsilon_1^{SO(10)},
\end{aligned} \tag{25}$$

where the relevant RH-neutrino masses and Yukawa combinations are summarized in Table V, and detailed parameter values in $E_6 \times U(1)_A$ GUT are given in Ref. [34].

Note that the $U(1)_A$ symmetry ensures the universality of λ dependence in the combinations of couplings and masses involving virtual RH neutrinos, i.e.,

$$\frac{\text{Im}[(y_{\nu_D}^\dagger y_{\nu_D})_{1i}^2]}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \cdot \frac{M_1}{M_i} \propto \lambda^{10} \text{ in } SO(10), \quad \propto \lambda^{11} \text{ in } E_6,$$

for any generation i . Accordingly, we first estimate the CP asymmetry considering only the heaviest RH neutrino, and then multiply by a factor counting the number of generations of virtual RH neutrinos: 2 in $\epsilon_1^{SO(10)}$ and 5 in $\epsilon_1^{E_6}$, respectively ⁴.

Even when flavor effects are included, this reasoning holds, since each flavor component of the neutrino Yukawa matrices scales with λ in the same way in both scenarios. Consequently, we indicate that the $SO(10) \times U(1)_A$ scenario tends to yield a larger baryon asymmetry than the $E_6 \times U(1)_A$ case with fixed r_1 , owing to the milder suppression of the CP asymmetry with respect to λ .

VI. CONCLUSION

We have studied leptogenesis in the natural $SO(10)$ GUT ($SO(10) \times U(1)_A$ GUT), in which the doublet–triplet splitting problem is solved and the observed quark and lepton masses and mixings can be reproduced under the natural assumption that all interactions, including higher-dimensional ones, appear with $\mathcal{O}(1)$ coefficients. In the natural GUT, this assumption implies that everything is determined by the symmetry except for the $\mathcal{O}(1)$ coefficients.

To obtain the predicted value of the baryon asymmetry in the natural $SO(10)$ GUT, we have to fix the $\mathcal{O}(1)$ coefficients. In this paper, to account for the observed baryon asymmetry, we have used the $\mathcal{O}(1)$ coefficient for M_1 while all $\mathcal{O}(1)$ coefficients of the Dirac neutrino Yukawa couplings are taken to be unity. We showed that the observed baryon asymmetry can be obtained when the coefficient r_1 is taken to be 5.4, corresponding to $M_1 \sim 1.4 \times 10^9$ GeV. This result suggests that the mass of the lightest LH neutrino, m_{ν_1} , is reduced to $1/5.4$ of its symmetry-determined value, yielding $m_{\nu_1} \sim 4.5 \times 10^{-4}$ eV.

We included flavor effects and the contribution of the second-lightest RH neutrino N_2 in our calculation. As a result, we reconfirmed the importance of N_2 in leptogenesis with flavor effects. In particular, the N_2 contribution becomes significant when some of the flavored decay parameters of the lightest RH neutrino are smaller than 3, or when $3M_1 > M_2$, which is consistent with the results of Ref. [31].

In our calculation, we estimated the magnitudes of the flavored CP asymmetries using

⁴ In the literature on E_6 leptogenesis, a counting factor $\sqrt{4} = 2$ is sometimes used instead of 5. Here the factor 4 counts the internal-loop RH neutrinos N_3, N_4, N_5 , and N_6 , and the square root reflects the randomness of signs in the neutrino Yukawa couplings [34].

real Yukawa couplings determined by the symmetry, with $\mathcal{O}(1)$ coefficients set to unity, rather than directly evaluating the imaginary part of the expressions with complex Yukawa couplings. In this case, the flavored CP asymmetries are relatively large due to multiple additive contributions. However, with complex $\mathcal{O}(1)$ coefficients, cancellations may occur in the flavored CP asymmetries. We examined this cancellation effect by flipping the signs of the Yukawa couplings and showed that in half of the 15 possible sign patterns, the observed baryon asymmetry can be realized. Even in the remaining patterns, the predicted baryon asymmetry, which does not reach to the observed value, becomes the same order of the observed baryon asymmetry by taking a larger $r_1 < 15$.

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Appendix A: Boltzmann Equations

In this appendix, we write down the Boltzmann equations used in our calculation of the $B - L$ asymmetry.

The evolution of the $B - L$ asymmetry is governed by the Boltzmann equations for the right-handed neutrinos N_1, N_2 and the flavored $B - L$ asymmetries Δ_α . The scattering terms include the (inverse-) decays and scattering processes of these species. Diagrams containing multiple Dirac neutrino Yukawa couplings are neglected, since these couplings are at most of order $\mathcal{O}(\lambda^2) \sim 0.048$ in the natural $SO(10)$ GUT.

Introducing the notation $\dot{Y} := sH z dY/dz$, the Boltzmann equations can be written as

$$-\dot{Y}_{N_i} = \left(\frac{Y_{N_i}}{Y_{N_i}^{eq}} - 1 \right) \left(\gamma_{N_i} + 4\gamma_{t_i}^{eq(0)} + 4\gamma_{t_i}^{eq(1)} + 4\gamma_{t_i}^{eq(2)} + 3\gamma_{t_i}^{eq(3)} + 4\gamma_{t_i}^{eq(4)} \right), \quad (\text{A1})$$

$$-\dot{Y}_{+i} = \left(\frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} - 1 \right) \left(\gamma_{\tilde{N}_i} + 3\gamma_{22_i} + 2\gamma_{t_i}^{eq(5)} + 2\gamma_{t_i}^{eq(6)} + 2\gamma_{t_i}^{eq(7)} + \gamma_{t_i}^{eq(8)} + 2\gamma_{t_i}^{eq(9)} \right), \quad (\text{A2})$$

$$\begin{aligned} -\dot{Y}_{-i} = & \frac{Y_{-i}}{Y_{\tilde{N}_i^c}^{eq}} \left(\gamma_{\tilde{N}_i} + 3\gamma_{22_i} + 2\gamma_{t_i}^{eq(5)} + 2\gamma_{t_i}^{eq(6)} + 2\gamma_{t_i}^{eq(7)} + \gamma_{t_i}^{eq(8)} + 2\gamma_{t_i}^{eq(9)} \right) \\ & - \frac{Y_{\Delta\ell}}{Y_{\ell}^{eq}} \left(\frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \gamma_{t_i}^{eq(5)} + 2\gamma_{t_i}^{eq(6)} + 2\gamma_{t_i}^{eq(7)} \right) \\ & - \frac{Y_{\Delta\tilde{\ell}}}{Y_{\tilde{\ell}}^{eq}} \left[\left(2 + \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \right) \gamma_{22_i} - \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \gamma_{t_i}^{eq(8)} - 2\gamma_{t_i}^{eq(9)} \right] \\ & - 2 \frac{Y_{\Delta\tilde{H}_u}}{Y_{\tilde{H}_u}^{eq}} \left[\left(2 + \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \right) \gamma_{22_i} + \gamma_{t_i}^{eq(5)} + \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \gamma_{t_i}^{eq(6)} + \gamma_{t_i}^{eq(7)} \right] \\ & - \frac{Y_{\Delta H_u}}{Y_H^{eq}} \left[\left(2 + \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \right) \gamma_{22_i} + 2\gamma_{t_i}^{eq(5)} + \left(1 + \frac{1}{2} \frac{Y_{+i}}{Y_{\tilde{N}_i}^{eq}} \right) \left(\gamma_{t_i}^{eq(6)} + \gamma_{t_i}^{eq(7)} \right) - \gamma_{t_i}^{eq(8)} - 2\gamma_{t_i}^{eq(9)} \right], \end{aligned} \quad (\text{A3})$$

$$\begin{aligned}
-\dot{Y}_{\Delta\alpha} = & \sum_i \left[\epsilon_{\alpha i} \left(\frac{Y_{N_i}}{Y_{N_i}^{eq}} - 1 \right) \left(\gamma_{N_i} + 4\gamma_{t_i}^{eq(0)} + 4\gamma_{t_i}^{eq(1)} + 4\gamma_{t_i}^{eq(2)} + 3\gamma_{t_i}^{eq(3)} + 4\gamma_{t_i}^{eq(4)} \right) \right. \\
& + \epsilon_{\alpha i} \left(\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} - 1 \right) \left(\gamma_{\widetilde{N}_i} + 3\gamma_{22_i} + 2\gamma_{t_i}^{eq(5)} + 2\gamma_{t_i}^{eq(6)} + 2\gamma_{t_i}^{eq(7)} + \gamma_{t_i}^{eq(8)} + 2\gamma_{t_i}^{eq(9)} \right) \\
& - \left(\frac{Y_{\Delta\ell}}{Y_{\ell}^{eq}} + \frac{Y_{\Delta\widetilde{\ell}}}{Y_{\widetilde{\ell}}^{eq}} + \frac{Y_{\Delta H_u}}{Y_{H_u}^{eq}} + \frac{Y_{\Delta\widetilde{H}_u}}{Y_{\widetilde{H}_u}^{eq}} \right) \left(\gamma_{\widetilde{N}_i\alpha}^{eq} + \frac{1}{2}\gamma_{N_i\alpha}^{eq} \right) \\
& - \left(\frac{Y_{\Delta\widetilde{\ell}}}{Y_{\widetilde{\ell}}^{eq}} + 2\frac{Y_{\Delta\widetilde{H}_u}}{Y_{\widetilde{H}_u}^{eq}} - \frac{Y_{\Delta H_u}}{Y_{H_u}^{eq}} \right) \left(2 + \frac{1}{2}\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} \right) \gamma_{22_i\alpha}^{eq} \\
& - \frac{Y_{\Delta\ell}}{Y_{\ell}^{eq}} \left(\frac{Y_{N_i}}{Y_{N_i}^{eq}} \gamma_{t_i\alpha}^{eq(3)} + \frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} \gamma_{t_i\alpha}^{eq(5)} + 2\gamma_{t_i\alpha}^{eq(4)} + 2\gamma_{t_i\alpha}^{eq(6)} + 2\gamma_{t_i\alpha}^{eq(7)} \right) \\
& - \frac{Y_{\Delta\widetilde{\ell}}}{Y_{\widetilde{\ell}}^{eq}} \left(2\frac{Y_{N_i}}{Y_{N_i}^{eq}} \gamma_{t_i\alpha}^{eq(0)} + 2\gamma_{t_i\alpha}^{eq(1)} + 2\gamma_{t_i\alpha}^{eq(2)} + \frac{1}{2}\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} \gamma_{t_i\alpha}^{eq(8)} + 2\gamma_{t_i\alpha}^{eq(9)} \right) \\
& - \frac{Y_{\Delta H_u}}{Y_{H_u}^{eq}} \left\{ \frac{1}{2} \left(\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} - 2 \right) \left(-\gamma_{t_i}^{eq(6)} + \gamma_{t_i}^{eq(7)} \right) + \left(1 + \frac{1}{2}\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} \right) \gamma_{t_i}^{eq(9)} + \gamma_{t_i}^{eq(8)} \right\} \\
& - \frac{Y_{\Delta H_u}}{Y_{H_u}^{eq}} \left\{ \left(\frac{Y_{N_i}}{Y_{N_i}^{eq}} - 1 \right) \gamma_{t_i\alpha}^{eq(1)} - \frac{Y_{N_i}}{Y_{N_i}^{eq}} \gamma_{t_i\alpha}^{eq(2)} + \gamma_{t_i\alpha}^{eq(3)} + \left(\frac{Y_{N_i}}{Y_{N_i}^{eq}} + 1 \right) \gamma_{t_i\alpha}^{eq(4)} \right\} \\
& - 2\frac{Y_{\Delta\widetilde{H}_u}}{Y_{\widetilde{H}_u}^{eq}} \left(\gamma_{t_i}^{eq(0)} + \gamma_{t_i\alpha}^{eq(1)} + \frac{Y_{N_i}}{Y_{N_i}^{eq}} \gamma_{t_i\alpha}^{eq(2)} + \gamma_{t_i\alpha}^{eq(5)} + \frac{1}{2}\frac{Y_{+i}}{Y_{\widetilde{N}^c_i}^{eq}} \gamma_{t_i\alpha}^{eq(6)} + \gamma_{t_i\alpha}^{eq(7)} \right) \\
& \left. + \frac{Y_{-i}}{Y_{\widetilde{N}^c_i}^{eq}} \left(2\gamma_{t_i\alpha}^{eq(5)} + 2\gamma_{t_i\alpha}^{eq(6)} + 2\gamma_{t_i\alpha}^{eq(7)} - \gamma_{t_i\alpha}^{eq(8)} - 2\gamma_{t_i\alpha}^{eq(9)} \right) \right], \tag{A4}
\end{aligned}$$

where,

$$Y_{\Delta b, \Delta f} := \frac{1}{g_{b,f}} \frac{\Delta n_{b,f}}{s} \quad (\text{boson, fermion}), \tag{A5}$$

$$Y_{\pm i} := Y_{\widetilde{N}_i} \pm Y_{\widetilde{N}_i^\dagger} \quad (i = 1, 2), \tag{A6}$$

and equilibrium density

$$Y_{\ell}^{eq} = \frac{1}{2}Y_{\widetilde{\ell}}^{eq} = \frac{1}{2}Y_{H_u}^{eq} = Y_{\widetilde{H}_u}^{eq} = \frac{15}{8\pi^2 g_*}. \tag{A7}$$

When the effects of interactions are incorporated into the Boltzmann equations, thermally averaged reaction rates are used. In particular, the reaction densities for the decay of RH

(s)neutrinos and for scatterings involving (s)tops are given by

$$\begin{aligned}
\gamma_{N_i\alpha}^{eq} &:= \gamma \left(N_i \leftrightarrow \tilde{\ell}_\alpha \tilde{H}_u \right) + \gamma \left(N_i \leftrightarrow \tilde{\ell}_\alpha^* \overline{\tilde{H}_u} \right) + \gamma \left(N_i \leftrightarrow \ell_\alpha H_u \right) + \gamma \left(N_i \leftrightarrow \bar{\ell}_\alpha H_u^* \right), \\
\gamma_{\tilde{N}_i\alpha}^{eq} &:= \gamma \left(\tilde{N}_i \leftrightarrow \tilde{\ell}_\alpha H_u \right) + \gamma \left(\tilde{N}_i \leftrightarrow \bar{\ell}_\alpha \overline{\tilde{H}_u} \right) + \gamma \left(\tilde{N}_i^* \leftrightarrow \tilde{\ell}_\alpha^* H_u^* \right) + \gamma \left(\tilde{N}_i \leftrightarrow \ell_\alpha \tilde{H}_u \right), \\
\gamma_{22_i\alpha} &:= \gamma \left(\tilde{N}_i \tilde{\ell}_\alpha \leftrightarrow \tilde{Q} \tilde{u}^* \right) = \gamma \left(\tilde{N}_i \tilde{Q}^* \leftrightarrow \tilde{\ell}_\alpha^* \tilde{u}^* \right) = \gamma \left(\tilde{N}_i \tilde{u} \leftrightarrow \tilde{\ell}_\alpha^* \tilde{Q} \right), \\
\gamma_{t_i\alpha}^{eq(0)} &:= \gamma \left(\tilde{N}_i \tilde{\ell}_\alpha \leftrightarrow Q \tilde{u}^* \right) = \gamma \left(\tilde{N}_i \tilde{\ell}_\alpha \leftrightarrow \tilde{Q} \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(1)} &:= \gamma \left(\tilde{N}_i \bar{Q} \leftrightarrow \tilde{\ell}_\alpha^* \tilde{u}^* \right) = \gamma \left(\tilde{N}_i u \leftrightarrow \tilde{\ell}_\alpha^* \tilde{Q} \right), \\
\gamma_{t_i\alpha}^{eq(2)} &:= \gamma \left(N_i \tilde{u} \leftrightarrow \tilde{\ell}_\alpha^* Q \right) = \gamma \left(N_i \tilde{Q}^* \leftrightarrow \tilde{\ell}_\alpha^* \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(3)} &:= \gamma \left(N_i \ell_\alpha \leftrightarrow Q \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(4)} &:= \gamma \left(N_i u \leftrightarrow \bar{\ell}_\alpha Q \right) = \gamma \left(N_i \bar{Q} \leftrightarrow \bar{\ell}_\alpha \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(5)} &:= \gamma \left(\tilde{N}_i \ell_\alpha \leftrightarrow Q \tilde{u}^* \right) = \gamma \left(\tilde{N}_i \ell_\alpha \leftrightarrow \tilde{Q} \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(6)} &:= \gamma \left(\tilde{N}_i \tilde{u} \leftrightarrow \bar{\ell}_\alpha Q \right) = \gamma \left(\tilde{N}_i \tilde{Q}^* \leftrightarrow \bar{\ell}_\alpha \bar{u} \right), \\
\gamma_{t_i\alpha}^{eq(7)} &:= \gamma \left(\tilde{N}_i \bar{Q} \leftrightarrow \bar{\ell}_\alpha \tilde{u}^* \right) = \gamma \left(\tilde{N}_i u \leftrightarrow \bar{\ell}_\alpha \tilde{Q} \right), \\
\gamma_{t_i\alpha}^{eq(8)} &:= \gamma \left(\tilde{N} \tilde{\ell}_\alpha^* \leftrightarrow \bar{Q} u \right), \\
\gamma_{t_i\alpha}^{eq(9)} &:= \gamma \left(\tilde{N}_i Q \leftrightarrow \tilde{\ell}_\alpha u \right) = \gamma \left(\tilde{N}_i \bar{u} \leftrightarrow \tilde{\ell}_\alpha \bar{Q} \right),
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{N_i}^{eq} &:= \sum_{\alpha} \gamma_{N_i\alpha}^{eq}, \quad \gamma_{\tilde{N}_i}^{eq} := \sum_{\alpha} \gamma_{\tilde{N}_i\alpha}^{eq}, \quad \gamma_{22_i}^{eq} := \sum_{\alpha} \gamma_{22_i\alpha}^{eq}, \\
\gamma_{t_i}^{eq(n)} &:= \sum_{\alpha} \gamma_{t_i\alpha}^{eq(n)} \quad (n = 1, 2, 3, \dots, 9).
\end{aligned}$$

The details of the above reaction densities are given in [35].

Appendix B: Sign-flipped Dirac neutrino Yukawa and flavored CP asymmetries

In this appendix, we examine the consequences of flipping the signs of individual entries in the Dirac neutrino Yukawa matrix. The purpose is to show how to classify the sign-flipped patterns of Dirac neutrino Yukawa couplings so that each group gives the same prediction of the Baryon asymmetry.

First, the flavored decay parameters do not change at all by changing the sign of the Dirac neutrino Yukawa couplings. Therefore, we focus on the flavored CP asymmetries.

To begin, we introduce a notation for denoting sign-flip operations of Dirac neutrino Yukawa components. Specifically, we write $[(y_{\nu_D})_{\alpha i}, (y_{\nu_D})_{\beta j}, \dots]$ to indicate a procedure in which the signs of the listed Yukawa components are flipped, i.e., $(y_{\nu_D})_{\alpha i} \rightarrow -(y_{\nu_D})_{\alpha i}$, $(y_{\nu_D})_{\beta j} \rightarrow -(y_{\nu_D})_{\beta j}$, $\dots$

Among these operations, some of them leave the flavored CP asymmetries unchanged. For example, the operations $[(y_{\nu_D})_{ei}, (y_{\nu_D})_{\mu i}, (y_{\nu_D})_{\tau i}]$, $(i = 1, 2, 3)$ do not change the flavored CP asymmetries. As an illustration, the asymmetry ϵ_{e1} under the flip $[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu 1}, (y_{\nu_D})_{\tau 1}]$ becomes

$$\epsilon_{e1} = -\frac{3}{4\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \left[\text{Im} [(y_{\nu_D})_{e1}^* (y_{\nu_D})_{e2} (y_{\nu_D}^\dagger y_{\nu_D})_{12}] f\left(\frac{M_2^2}{M_1^2}\right) + \text{Im} [(y_{\nu_D})_{e1}^* (y_{\nu_D})_{e3} (y_{\nu_D}^\dagger y_{\nu_D})_{13}] f\left(\frac{M_3^2}{M_1^2}\right) \right] \quad (\text{B1})$$

$$\begin{aligned} &\rightarrow -\frac{3}{4\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \left[\text{Im} [(-y_{\nu_D})_{e1}^* (y_{\nu_D})_{e2} (-y_{\nu_D}^\dagger y_{\nu_D})_{12}] f\left(\frac{M_2^2}{M_1^2}\right) + \text{Im} [(-y_{\nu_D})_{e1}^* (y_{\nu_D})_{e3} (-y_{\nu_D}^\dagger y_{\nu_D})_{13}] f\left(\frac{M_3^2}{M_1^2}\right) \right] \\ &= -\frac{3}{4\pi} \frac{1}{(y_{\nu_D}^\dagger y_{\nu_D})_{11}} \left[\text{Im} [(y_{\nu_D})_{e1}^* (y_{\nu_D})_{e2} (y_{\nu_D}^\dagger y_{\nu_D})_{12}] f\left(\frac{M_2^2}{M_1^2}\right) + \text{Im} [(y_{\nu_D})_{e1}^* (y_{\nu_D})_{e3} (y_{\nu_D}^\dagger y_{\nu_D})_{13}] f\left(\frac{M_3^2}{M_1^2}\right) \right] = \epsilon_{e1}. \quad (\text{B2}) \end{aligned}$$

It is easily shown that all the other $\epsilon_{\alpha i}$ are invariant under this type of flip.

Similarly, it is shown that the operations $[(y_{\nu_D})_{\alpha 1}, (y_{\nu_D})_{\alpha 2}, (y_{\nu_D})_{\alpha 3}]$, $(\alpha = e, \mu, \tau)$ do not change the flavored CP asymmetries.

By exploiting the above two types of operations, flipping an entire column or an entire row, it follows that any arbitrary sign-flip configuration involving three or more entries can always be reduced to a configuration involving only one or two entries. As a result, the full

TABLE VI: 15 Groups among all operations for flipping the signs of Dirac neutrino Yukawa couplings. The Yukawa models which are given by some sign flip operations in each group give the same prediction on the Baryon asymmetry. 45 operations which involve only one or two entries are explicitly shown below.

group	flipped components
1.	$[(y_{\nu_D})_{e1}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{e3}], [(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{\tau1}], \dots$
2.	$[(y_{\nu_D})_{\mu1}], [(y_{\nu_D})_{\mu2}, (y_{\nu_D})_{\mu3}], [(y_{\nu_D})_{e1}, (y_{\nu_D})_{\tau1}], \dots$
3.	$[(y_{\nu_D})_{\tau1}], [(y_{\nu_D})_{\tau2}, (y_{\nu_D})_{\tau3}], [(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu1}], \dots$
4.	$[(y_{\nu_D})_{e2}], [(y_{\nu_D})_{e1}, (y_{\nu_D})_{e3}], [(y_{\nu_D})_{\mu2}, (y_{\nu_D})_{\tau2}], \dots$
5.	$[(y_{\nu_D})_{\mu2}], [(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{\mu3}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{\tau2}], \dots$
6.	$[(y_{\nu_D})_{\tau2}], [(y_{\nu_D})_{\tau1}, (y_{\nu_D})_{\tau3}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{\mu2}], \dots$
7.	$[(y_{\nu_D})_{e3}], [(y_{\nu_D})_{e1}, (y_{\nu_D})_{e2}], [(y_{\nu_D})_{\mu3}, (y_{\nu_D})_{\tau3}], \dots$
8.	$[(y_{\nu_D})_{\mu3}], [(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{\mu2}], [(y_{\nu_D})_{e3}, (y_{\nu_D})_{\tau3}], \dots$
9.	$[(y_{\nu_D})_{\tau3}], [(y_{\nu_D})_{\tau1}, (y_{\nu_D})_{\tau2}], [(y_{\nu_D})_{e3}, (y_{\nu_D})_{\mu3}], \dots$
10.	$[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu2}], [(y_{\nu_D})_{e3}, (y_{\nu_D})_{\tau2}], [(y_{\nu_D})_{\mu3}, (y_{\nu_D})_{\tau1}], \dots$
11.	$[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\tau2}], [(y_{\nu_D})_{e3}, (y_{\nu_D})_{\mu2}], [(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{\tau3}], \dots$
12.	$[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\mu3}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{\tau3}], [(y_{\nu_D})_{\mu2}, (y_{\nu_D})_{\tau1}], \dots$
13.	$[(y_{\nu_D})_{e1}, (y_{\nu_D})_{\tau3}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{\mu3}], [(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{\tau2}], \dots$
14.	$[(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{e2}], [(y_{\nu_D})_{e3}, (y_{\nu_D})_{\tau1}], [(y_{\nu_D})_{\mu3}, (y_{\nu_D})_{\tau2}], \dots$
15.	$[(y_{\nu_D})_{\mu1}, (y_{\nu_D})_{e3}], [(y_{\nu_D})_{e2}, (y_{\nu_D})_{\tau1}], [(y_{\nu_D})_{\mu2}, (y_{\nu_D})_{\tau3}], \dots$

set of possible sign-flip patterns can be classified into 15 groups as in Table VI.

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