

A new multiprobe analysis of modified gravity and evolving dark energy

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We study the (w_0, w_a) parametrization of the dark energy (DE) equation of state, with and without the effective field theory of dark energy (EFTofDE) framework to describe the DE perturbations, parametrized here by the braiding parameter α_B and the running of the Planck mass α_M . We combine the EFTofLSS full-shape analysis of the power spectrum and bispectrum of BOSS data with the tomographic angular power spectra C_ℓ^{gg} , $C_\ell^{\kappa g}$, C_ℓ^{Tg} and $C_\ell^{T\kappa}$, where g , κ and T stand for the DESI luminous red galaxy map, *Planck* PR4 lensing map and *Planck* PR4 temperature map, respectively. To analyze these angular power spectra, we go beyond the Limber approximation, allowing us to include large-scales data in C_ℓ^{gg} . The combination of all these probes with *Planck* PR4, DESI DR2 BAO and DES Y5 improves the constraint on the 2D posterior distribution of $\{w_0, w_a\}$ by $\sim 50\%$ and increases the preference for evolving dark energy over Λ from 3.8σ to 4.6σ . When we remove BAO and supernovae data, we obtain a hint for evolving dark energy at 2.3σ . Regarding the EFTofDE parameters, we improve the constraints on α_B and α_M by $\sim 40\%$ and 50% respectively, finding results compatible with general relativity at $\sim 2\sigma$. We show that these constraints do not depend on the choice of the BAO and supernovae likelihoods.

I. INTRODUCTION

Since the discovery of dark energy (DE) in 1998 [1], cosmological data seemed to favor the existence of a cosmological constant Λ , a key element of the standard paradigm of cosmology, the Λ CDM model. Cosmological data from cosmic microwave background (CMB) [2], large-scale structure (LSS) [3] or supernovae [4] appeared to favor a dark energy equation of state parameter close to $w = -1$, with no compelling evidence for evolving dark energy. However, since the beginning of 2024, this picture has been challenged by new measurements from the Dark Energy Spectroscopic Instrument (DESI) [5] and supernovae compilations [6–8]. The first DESI data release suggests a mild preference for evolving dark energy when combined with CMB and supernovae data, deviating from a cosmological constant by $2.5\sigma - 3.9\sigma$ [9, 10] (depending on the supernova data used). This raises concerns about the consistency of supernova data – for instance, Ref. [11] pointed out that a systematic offset in the magnitude between low and high redshift supernovae could mimic an evolving dark energy signal. However, with the second DESI data release, a preference for evolving dark energy at 3.1σ was observed without supernovae measurements [12], along with a 2.3σ discrepancy between DESI DR2 BAO and *Planck* (assuming the Λ CDM model) in the 2D posterior distribution of $\{\Omega_m, r_d \cdot h\}$, where r_d is the sound horizon at baryon drag. This discrepancy is exacerbated to 3.7σ in the SPT + ACT anal-

ysis [13]. We note that this inconsistency can also be mitigated if we vary the sum of the neutrino mass [14], increase the value of the optical depth τ [15], or by some early-time dynamics [16–18].

Interestingly, the reconstructed dark energy equation of state crosses $w = -1$ around $z \sim 0.5$, implying a violation of the null energy condition at high redshift. Various models have been proposed to explain this behaviour and address this challenge. For example, the so-called phantom mirage mechanism – where a coupling between quintessence and dark matter produces an apparent evolution in w – can mimic dynamical dark energy [19, 20]. We can also invoke axion-like scalar field models [21], scalar field with a non-canonical kinetic term [22] or non-minimally coupled scalar field models [23]. An alternative explanation involves the quintom scenario [24–31], in which two scalar fields (one of quintessence type and the other of phantom type) combine to allow a crossing of the $w = -1$ limit without instability.

On the other hand, the potential preference for evolving dark energy is a good motivation for studying modified gravity (MG) models: is the evolving dark energy in fact the manifestation of an extension or deviation from general relativity [32–35]? In the case of single scalar degree of freedom, the Effective Theory of Dark Energy (EFTofDE) [36–42] provides a unified description of gravity beyond general relativity, separating background expansion from linear perturbations. The expansion history can be parametrized by an arbitrary equation of state $w(z)$, while the dynamics of perturbations are described by the general time dependent functions α_K , α_B , α_M , and α_T . Though no significant derivation from general relativity has been reported so far, a well-known

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challenge is the correlation between the braiding parameter α_B and the lensing amplitude A_L . The so-called “lensing anomaly” in CMB data can shift α_B away from zero by up to 2σ [43–46], although the significance of this anomaly is reduced in the *Planck* 2020 analysis compared to *Planck* 2018 [47].

Modified gravity alters the evolution of metric perturbations at late times through the Poisson equation and the Weyl potential, which can be directly probed by cosmological observations that are sensitive to the late-time perturbed universe. For instance, it will affect the matter overdensity field (and therefore the galaxy overdensity field), the CMB gravitational lensing, as well as the CMB temperature power spectrum at large angular scales [48] (through the integrated Sachs-Wolfe (ISW) effect). In order to constrain the EFTofDE parameters, we consider in this paper cross-correlations between these three observables, namely $C_\ell^{\kappa g}$, C_ℓ^{Tg} , and $C_\ell^{T\kappa}$, corresponding respectively to the cross-angular power spectra between CMB lensing and galaxy clustering, between CMB temperature and galaxy clustering, as well as between CMB temperature and CMB lensing. Ref. [49] has already shown that the cross-angular power spectrum between the *Planck* PR4 temperature and lensing maps ($C_\ell^{T\kappa}$) imposes a strong constraint on the braiding parameter α_B . In addition, it has already been demonstrated that C_ℓ^{Tg} provides an independent test of general relativity and allows strong constraints to be set on modified gravity scenarios [50–53]. In particular, using pre-DESI and *Planck* 2015 data [50], Ref. [54] imposed significant constraints on the EFTofDE parameters. Concerning C_ℓ^{gg} and $C_\ell^{\kappa g}$, they are also highly sensitive to α_B and α_M [55–58], and can also put strong constraints on the background parameters, especially Ω_m , h , w_0 , w_a , and σ_8 [59–64]. A recent study [65] has also shown that analyses based on C_ℓ^{gg} and $C_\ell^{\kappa g}$ can exhibit a preference for evolving dark energy, even without incorporating primary CMB data.

In parallel, the Effective Field Theory of Large-Scale Structure (EFTofLSS) [66–76] provides a framework for extracting maximum information from the galaxy overdensity field. It provides an accurate modelling of the 3D galaxy power spectrum and bispectrum [77], incorporating the effects of redshift space distortions [72, 78], galaxy bias [73, 74, 79], and IR resummation [80–82], thereby allowing robust cosmological parameter inference [83–85]. In particular, the one-loop EFTofLSS prediction of the galaxy power spectrum have made possible the determination of the Λ CDM parameters from the full-shape analysis of (e)BOSS data [3] at precision higher than that from conventional BAO and redshift space distortions ($\text{BAO}/f\sigma_8$) analyses (see Ref. [86]), and even comparable to that of CMB experiments. Additionally, the inclusion of the one-loop bispectrum yields an additional $\sim 30\%$ gain in constraining power on the dark energy equation of state [43, 87]. Based on the equivalence principle [88–91], the EFTofLSS can further consistently capture the impact of modify gravity [92, 93] and dark energy [43, 94–97] from the full shape of the galaxy power spectrum and

bispectrum.

The main goal of this work is therefore to comprehensively assess the constraining power of the current measurements of tomographic angular power spectra (derived from photometric surveys and CMB data) and the 3D galaxy power spectrum and bispectrum on the CPL parametrization of the DE equation of state, with and without the EFTofDE framework to describe the DE perturbations. To do so, we use (i) the EFTofLSS full-shape analysis of the power spectrum and bispectrum of BOSS Luminous Red Galaxies, (ii) the angular power spectrum $C_\ell^{T\kappa}$ from the cross correlation between the *Planck* PR4 temperature and lensing maps, and (iii) the auto angular galaxy power spectra C_ℓ^{gg} from DESI Luminous Red Galaxies, together with the cross angular power spectra $C_\ell^{\kappa g}$ and C_ℓ^{Tg} from the cross correlation with the *Planck* PR4 lensing and temperature maps.

This paper is organized as follows. In Sec. II, we describe the theoretical framework by introducing the EFTofDE setup used in this work, while in Sec. III we detail the analysis setup and the modelling of the 3D galaxy power spectrum and angular power spectra mentioned above. In Sec. IV, we present and discuss the cosmological constraints obtained on the CPL parametrization and on the EFTofDE parameters, before concluding in Sec. V.

II. THEORETICAL FRAMEWORK: DARK ENERGY MODELS

At the background level, DE can be phenomenologically described as an effective fluid characterized by an equation of state $w(a) \equiv p_{\text{DE}}(a)/\rho_{\text{DE}}(a)$. In this work, we adopt the commonly used Chevallier–Polarski–Linder (CPL) parametrization [98, 99], where $w(a) = w_0 + w_a(1 - a)$. To specify the DE perturbations, we consider two different frameworks: (i) a phenomenological fluid parametrization called *Parametrized Post-Friedmann*, and (ii) a physically motivated effective field theory (EFT) approach that captures broad classes of single scalar field and modified gravity models (in this work we focus on Horndeski’s theories).

A. Dark Energy as a fluid

The *Parametrized Post-Friedmann* (PPF) framework [100–102] is a phenomenological description of the DE perturbations which is commonly adopted to safely allow a phantom crossing. Since dark energy is not directly coupled to matter, its perturbations affect structure formation only through the metric potentials in the Poisson equation. In the conformal Newtonian gauge (de-

noted “N”), it reads

$$\begin{aligned} k^2\phi &= -4\pi G a^2 \left(\delta\rho_{\text{tot}}^N - 3\mathcal{H}(\rho_{\text{tot}} + p_{\text{tot}}) \frac{\theta_{\text{tot}}^N}{k^2} \right) \\ &= k^2\Gamma - 4\pi G a^2 \left(\rho_t^N - 3\mathcal{H}(\rho_t + p_t) \frac{\theta_t^N}{k^2} \right). \end{aligned} \quad (1)$$

In the second line, we isolate the contribution of DE perturbations (expressed in the DE rest frame) as $k^2\Gamma \equiv -4\pi G a^2 \delta\rho_{\text{DE}}^{\text{rest}}$, while the subscript “t” denotes all species other than dark energy.

The evolution equation of Γ is obtained from an interpolation between the super-horizon and sub-horizon limits:

$$\dot{\Gamma} = H \left[S \left(1 + \frac{c_\Gamma^2 k^2}{\mathcal{H}^2} \right)^{-1} - \Gamma \left(1 + \frac{c_\Gamma^2 k^2}{\mathcal{H}^2} \right) \right], \quad (2)$$

where c_Γ is an effective sound speed controlling the transition between large and small scales, and S is defined as

$$S \equiv \frac{4\pi G a^2}{\mathcal{H}^2} (\rho_{\text{DE}} + p_{\text{DE}}) \frac{\theta_t^N}{k^2}. \quad (3)$$

The CPL + PPF parametrization therefore provides a flexible and simple phenomenological framework for studying deviations from a cosmological constant at both the background and linear perturbation levels.

B. EFTofDE framework

EFTofDE provides a physically motivated model-independent description of the DE perturbations around an FLRW background for theories with a single scalar degree of freedom [103, 104]. Working in unitary gauge and including all operators that are invariant under time-dependent spatial diffeomorphisms, the quadratic action for linear scalar and tensor perturbations can be parametrized by a small set of time-dependent functions. Restricting to Horndeski-type theories [104–108], whose equations of motion contain at most second-order derivatives, it is customary to introduce four dimensionless EFT functions: α_K , α_B , α_M and α_T , which control the kinetic term of the scalar field, the braiding between the kinetic terms of the scalar and the metric, the running of the Planck mass, and tensor speed excess, respectively. We note that the parameter α_M is related to the effective Planck mass via

$$\alpha_M \equiv \frac{d \ln M_*^2}{d \ln a}. \quad (4)$$

In principle, the EFT parameters can have arbitrary time dependence. In this work, we consider that $\alpha_i(a) = c_i \cdot \Omega_{\text{DE}}(a)$ [39, 93, 109, 110], allowing us to recover general relativity at high redshifts. We note that this choice is not unique and that our cosmological results depend on

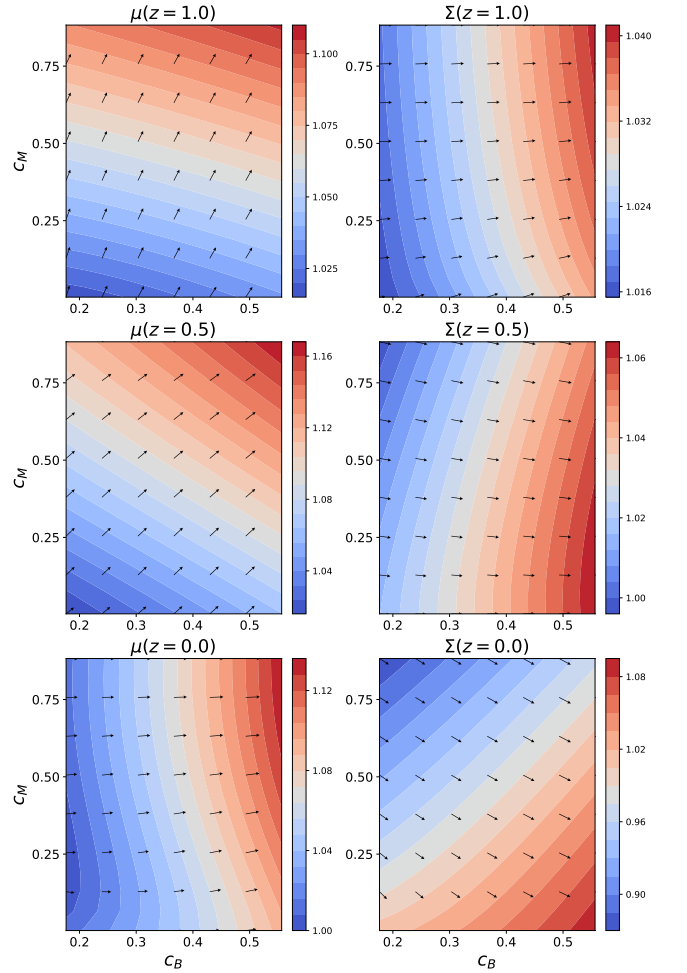


FIG. 1: The response of μ and Σ to variations in c_M and c_B at redshifts $z = 1.0, 0.5$, and 0.0 . The arrows indicate the gradient direction: μ is more sensitive to c_M at high redshift and to c_B at low redshift, while Σ shows the opposite trend. The cosmology is fixed to the best-fit values of the “All” analysis obtained in Sec. IV B 4

this particular parametrization. However, this choice is able to capture a wide variety of Horndeski models [111–113].

A minimal set of stability conditions for linear scalar perturbations are the absence of ghost and gradient instabilities. In terms of the EFT functions, we can respectively expressed those conditions as [104, 114]

$$D_{\text{kin}} \equiv \alpha_K + \frac{3}{2}\alpha_B^2 > 0, \quad c_s^2 > 0, \quad (5)$$

where c_s corresponds to the scalar propagation speed, which depends on the background expansion history, the α -functions, and the matter content as [44]

$$\begin{aligned} c_s^2 = \frac{1}{D_{\text{kin}}} & \left[(2 - \alpha_B) \left(-\frac{H'}{aH^2} + \frac{1}{2}\alpha_B + \alpha_M \right) \right. \\ & \left. - \frac{3(\rho_t + p_t)}{H^2 M_*^2} + \frac{a'_B}{aH} \right], \end{aligned} \quad (6)$$

where $' = d/d\tau$ denotes derivative with respect to the conformal time. Let us note that once the α -functions are set, some parts of the $\{w_0, w_a\}$ parameter space allowed in the CPL + PPF framework become theoretically forbidden due to the EFTofDE stability conditions [104, 114].

Restricting to the quasi-static approximation, where we can neglect the time evolution of the scalar degree of freedom [94, 115, 116], the EFT functions modify linear perturbations through two effective functions, namely $\mu(k, a)$, which rescales the Poisson equation and modifies the matter clustering, and $\Sigma(k, a)$, which rescales the Weyl potential and controls the photon trajectories [117, 118]:

$$k^2\Psi = -4\pi G a^2 \mu(k, a) \sum_i \rho_i \Delta_i, \quad (7)$$

$$k^2(\Psi + \Phi) = -8\pi G a^2 \Sigma(k, a) \sum_i \rho_i \Delta_i, \quad (8)$$

where ρ_i and Δ_i denote the density and the gauge-invariant overdensity fields of the species i , respectively. Within the EFTofDE framework, the functions $\mu(k, a)$ and $\Sigma(k, a)$ can be written in terms of $\{M_*, \alpha_i, c_s^2\}$ as [39, 44, 104]

$$\mu(a) = \frac{1}{M_*^2} \left[1 + \frac{2(\alpha_M + \frac{1}{2}\alpha_B)^2}{c_s^2(\alpha_k + \frac{3}{2}\alpha_B^2)} \right], \quad (9)$$

$$\Sigma(a) = \frac{1}{M_*^2} \left[1 + \frac{(\alpha_M + \frac{1}{2}\alpha_B)(\alpha_M + \alpha_B)}{c_s^2(\alpha_k + \frac{3}{2}\alpha_B^2)} \right]. \quad (10)$$

These functions characterize deviations from General Relativity (GR) by modifying the relations between the metric potentials and the matter perturbation field, and are therefore directly constrained by large-scale structure and CMB observations. Note that in this work μ and Σ are only functions of time. In addition, we set $c_K = 1$ because within the quasi-static limit studied in this work, the cosmological data are insensitive to this parameter [104, 119–123]. We also set $c_T = 0$ since the simultaneous detection of GW170817 and its electromagnetic counterpart [124] set a strong constraint on this parameter with $\alpha_T \lesssim 10^{-15}$ [119, 125–128]. Therefore, in this paper, we aim to constrain the theoretical parameter space $\{c_B, c_M\}$. Fig. 1 illustrates the dependence of $\mu(a)$ and $\Sigma(a)$ on c_B and c_M at different redshifts. In particular, at high redshifts $\mu(a)$ mainly responds to variations in c_M , whereas at low redshifts it is more sensitive to c_B , while $\Sigma(a)$ exhibits the opposite trend. Therefore, the dependence of $\mu(a)$ and $\Sigma(a)$ on c_B and c_M depends strongly on the redshift, which is a key feature that motivates a multiprobe analysis spanning a wide redshift range.

III. ANALYSIS SETUP

A. Datasets and inference framework

In this paper, we perform Markov chain Monte Carlo (MCMC) analyses using the Metropolis-Hastings algorithm from `MontePython-v3`¹ code [129, 130] interfaced with `hi-class`,² a modified version of `CLASS`³ [131] including EFTofDE cosmologies (see Refs. [132, 133] for alternative codes). In the following, we describe the various likelihoods used in our analysis.

1. Datasets

Our baseline dataset is made up of the following likelihoods:

- **Planck PR4:** The high- ℓ TT, TE, EE and low- ℓ EE CMB power spectrum data (dubbed C_ℓ^{TT} , C_ℓ^{EE} and C_ℓ^{TE} in the following) from the *Planck* Public Release 4 (PR4) analyzed with the NPIPE processing pipeline [134], respectively described using the Hillipop and LoLLiPoP likelihoods [47, 135]. Similar to *Planck* Public Release 3 (PR3), we also include the low- ℓ TT likelihood using `Commander` [136].
- **Lensing:** The CMB gravitational lensing spectrum $C_\ell^{\kappa\kappa}$ from *Planck* PR4 [137] analyzed in the multipole range $8 < L < 400$. Note that we have adapted the public `Cobaya` likelihood⁴ to `Montepython-v3`.
- **DESI DR2 BAO:** The DESI DR2 BAO data from Refs. [12, 138]. These data include BAO from bright galaxies ($0.1 < z < 0.4$), luminous red galaxies ($0.4 < z < 1.1$), emission line galaxies ($0.8 < z < 1.6$), quasars ($0.8 < z < 2.1$), and Lyman- α forest quasars ($1.8 < z < 4.2$).
- **DES Y5:** DES year 5⁵ catalog of uncalibrated luminosity distance of SNeIa in the range $0.10 < z < 1.13$, combined with an external sample of SNeIa at low redshifts from $0.024 < z < 0.10$ [140]. We have adapted the public `Cobaya` [141] likelihood to `Montepython-v3` in Ref. [43].

The main purpose of this work is to combine this baseline dataset with the following likelihoods in order to assess their impact on the constraints on the background expansion and gravitational properties:

¹ https://github.com/brinckmann/montepython_public

² https://migueltzuma.github.io/hi-class_public/

³ https://lesgourg.github.io/class_public/class.html

⁴ https://github.com/carronj/planck_PR4_lensing

⁵ In the recent update of the DES supernova analysis [139], the evidence for evolving dark energy has been reduced from 4.2σ to 3.2σ .

- **ISWL:** The angular power spectrum $C_\ell^{T\kappa}$ from the cross correlation between the *Planck* PR4 temperature map [134] and the *Planck* PR4 lensing map [137] from Ref. [142] analyzed in the multipole range $2 < L < 100$. Note that we have adapted the public *Cobaya* likelihood⁶ to *Montepython-v3*.⁷
- **EFTBOSS:** The full-shape analysis of the power spectrum and bispectrum of BOSS Luminous Red Galaxies (LRG) [3]. To model those observables, we use the effective field theory of large-scale structure (EFTofLSS) up to one loop [66–76] (see below for more details). The power spectrum and bispectrum are measured in Refs. [77, 85, 87, 143] from the BOSS catalogs DR12 (v5) combined CMASS-LOWZ⁸ [144], and are divided in two redshift bins (and four sky-cuts): low- z , $0.2 < z < 0.43$ ($z_{\text{eff}} = 0.32$), and high- z , $0.43 < z < 0.7$ ($z_{\text{eff}} = 0.57$), with north and south galactic skies for each. The covariance, including the correlation between power spectrum and bispectrum, is measured through 2048 Patchy mocks [145], while the window function is measured from *fkpwin*⁹ [146].
- **DESIcross:** The auto-angular galaxy power spectra C_ℓ^{gg} from a photometric sample of Luminous Red Galaxies (LRG) of the DESI Legacy Imaging Survey DR9 [147, 148], together with the cross-angular power spectra $C_\ell^{\kappa g}$ from the cross correlation between the same LRG photometric sample and the *Planck* PR4 lensing map [137]. The data and covariances, analyzed in the multipole range $20 \leq L \leq 243$ and separated into 4 photometric redshift bins (with the effective redshifts $z_{\text{eff}} = \{0.470, 0.625, 0.785, 0.914\}$), are determined in Refs. [63, 149] using the *NaMaster* code¹⁰ [150]. Our *MontePython-v3* likelihood – inspired from the *MaPar* likelihood¹¹ written in *Cobaya* [63, 149] – do not consider the Limber approximation as in Refs. [63, 149], but uses *SwiftC_l*¹² [151] instead, which is an accurate and differentiable JAX-based code computing the angular galaxy power spectrum beyond the Limber approximation (see below). In addition, we also measure the cross-angular power spectra C_ℓ^{Tg} from the cross-correlation between the DESI DR9 LRG photometric samples and the *Planck* PR4 temperature map [134], where we use *NaMaster* to determine the data and covariances

and *SwiftC_l* to compute the theoretical prediction. We refer the reader to App. A for more details on the determination of the C_ℓ^{Tg} data as well as on the difference between the Limber approximation and *SwiftC_l*. The data used in this likelihood (C_ℓ^{gg} , $C_\ell^{\kappa g}$ and C_ℓ^{Tg}) and the theoretical predictions are shown in Fig. 8 of this appendix. The validation of our implementation is detailed in Sec. A

2. Correlations between datasets

In our analyses, we use the C_ℓ^{TT} , C_ℓ^{EE} , $C_\ell^{\kappa\kappa}$ and C_ℓ^{gg} auto-angular power spectra, while we use the C_ℓ^{TE} (from *Planck* PR4), $C_\ell^{T\kappa}$ (from ISWL), C_ℓ^{Tg} (from DESIcross), and $C_\ell^{\kappa g}$ (from DESIcross) cross-angular power spectra. The objective of this paper is therefore to perform a multiprobe analysis to exploit as much information as possible from several cosmological probes and their cross-correlations. However, caution must be taken when performing such an analysis to avoid double counting information. Below, we list how we take into account the various correlations in our analysis:

- The correlations between the *Planck* (C_ℓ^{TT} , C_ℓ^{EE} and C_ℓ^{TE}), lensing ($C_\ell^{\kappa\kappa}$) and ISWL ($C_\ell^{T\kappa}$) likelihoods are included in our analysis following Refs. [137, 142].
- There is a non-negligible covariance between $C_\ell^{\kappa\kappa}$ (from the lensing likelihood) and $C_\ell^{\kappa g}$ (from the DESIcross likelihood). For simplicity, when combining $C_\ell^{\kappa g}$ with $C_\ell^{\kappa\kappa}$, we remove the overlapping multipoles by excluding $\ell < 243$ for $C_\ell^{\kappa\kappa}$ (see *e.g.* Ref. [59] for an estimation of such a covariance).
- There are also correlations between $C_\ell^{\kappa g}$ (DESIcross) and $C_\ell^{\kappa T}$ (ISWL), as well as between C_ℓ^{Tg} (DESIcross) and $C_\ell^{TT-TE}/C_\ell^{T\kappa}$ (*Planck*/ISWL). However, using a Gaussian approximation to compute the full covariance [152, 153],

$$\text{Cov}_{LL'}^{\text{AB,CD}} = \frac{\delta_{LL'}}{(2\ell_{L'} + 1) \Delta\ell f_{\text{sky}}} \times \left[C_L^{\text{AC}} C_L^{\text{BD}} + C_L^{\text{AD}} C_L^{\text{BC}} \right], \quad (11)$$

the correlation coefficient $\rho(\ell) = \frac{\text{Cov}(C_\ell^{Tx}, C_\ell^{AB})}{\sigma(C_\ell^{Tx})\sigma(C_\ell^{AB})}$ is found to be below 8% for these cross-correlations and is therefore neglected in this work.

- As in Ref. [154], we neglect the correlation between photometric and spectroscopic data given that they probe different cosmological volumes (see Ref. [153] for more details), implying that we can safely combine the DESIcross likelihood with the DESI DR2 BAO and EFTBOSS likelihoods.

⁶ https://github.com/carronj/planck_PR4_lensing

⁷ We have checked that our *Montepython-v3* likelihood is consistent with the original *Cobaya* one up to 0.2σ .

⁸ Publicly available at <https://data.sdss.org/sas/dr12/boos/lss/>

⁹ <https://github.com/pierrexyz/fkpwin>

¹⁰ <https://github.com/LSTDESC/NaMaster>

¹¹ <https://github.com/NoahSailer/MaPar/tree/main>

¹² https://cosmo-gitlab.phys.ethz.ch/cosmo_public/swiftcl

- Finally, in our analysis, we combine DESI DR2 BAO with EFTBOSS. To mitigate the correlation between these two likelihoods, we use the sound horizon r_s information from the BAO analysis of DESI, while we marginalize it in the EFTBOSS full-shape analysis, as done in Ref. [43]. To do so, we follow Refs. [155, 156], splitting the linear power spectrum into its broadband shape and its wiggle component, before marginalizing over a new scaling parameter, dubbed α_{r_s} . In DESI DR1, approximately 27% of BGS and LRG1 galaxies and 9% of LRG2 galaxies were already observed by SDSS [157], which induces a $\sim 10\%$ correlation between the two datasets, reduced to less than 5% when marginalizing over the sound horizon r_s [43]. This overlap increases in DESI DR2 [12], and we evaluate that the correlation is at most 10%¹³ after the marginalization over the sound horizon. To further support our results, each time we combine DESI DR2 BAO with EFTBOSS, we reproduce the same analysis by replacing DESI DR2 BAO with ext-BAO, a likelihood – that can be safely combine with BOSS – including BAO measurements from 6dFGS at $z = 0.106$ [158], SDSS DR7 at $z = 0.15$ [159], as well as eBOSS DR14 Ly- α absorption auto-correlation at $z = 2.34$ and cross-correlation with quasars at $z = 2.35$ [160, 161].

3. Analysis settings

For all runs performed, we impose large flat priors on the Λ CDM parameters $\{\omega_b, \omega_{\text{cdm}}, H_0, A_s, n_s, \tau_{\text{reio}}\}$, respectively corresponding to the dimensionless baryon energy density, the dimensionless CDM energy density, the current Hubble parameter, the variance of the primordial power spectrum at $k = 0.05 \text{ Mpc}^{-1}$, the primordial power spectrum tilt, and the optical depth to reionization. We also impose large flat priors on the DE background parameters $\{w_0, w_a\}$ and on the EFTofDE perturbation parameters $\{c_B, c_B\}$.

Note that throughout this paper we use the *Planck* neutrino treatment convention, including two massless and one massive species with $m_\nu = 0.06 \text{ eV}$ [2]. We consider that our chains have converged when the Gelman-Rubin criterion $R - 1 < 0.05$. We acknowledge the use of

Procolli [162] to extract the bestfit parameters and the bestfit χ^2 as well as the use of *GetDist* [163] to extract the probability density functions and produce our plots.

B. Modelling and theoretical predictions

In this section, we describe the modelling and the theoretical predictions associated with the three likelihoods we are adding to our baseline dataset, namely EFTBOSS (in Sec. III B 1), ISWL and DESIcross (in Sec. III B 2).

1. Galaxy power spectrum and the EFTofLSS

Modelling the non-linear evolution of large-scale structure and connecting the galaxy overdensity field to the underlying dark matter overdensity field is challenging. In this paper, we adopt the effective field theory of large-scale structure (EFTofLSS) framework allowing us to organize the expansions in fluctuations and derivatives of the density and velocity fields of the observed tracers at long wavelengths, allowing us to compute the galaxy power spectrum and bispectrum in redshift space up to one-loop.¹⁴ Note that following previous studies (see *e.g.* Ref. [43]) showing that the Einstein-de Sitter (EdS) approximation is sufficient for BOSS data, we adopt it here, with the time dependence of density perturbations given by $\delta^{(n)}(a) \propto D_+^n(a)$.

To determine the full-modelling information of BOSS data (denoted “EFTBOSS”), we adopt the PyBird EFT likelihood¹⁵ [83], for which we specify all the important details below:

- **Renormalization scales:** In the redshift space galaxy power spectrum, there are three renormalization scales : the non-linear scale renormalizing the dark matter field k_{NL} , the spatial extension of the observed objects renormalizing the spatial derivative expansion k_{M} , and the “dispersion” scale renormalizing the velocity products appearing in the redshift-space expansion k_{R} . According to Refs. [171, 172], we set $k_{\text{M}} = k_{\text{NL}} = 0.7 h \text{ Mpc}^{-1}$ and $k_{\text{R}} = 0.25 h \text{ Mpc}^{-1}$. In addition, we set the mean galaxy number density to $\bar{n}_g = 4 \cdot 10^{-4} (\text{Mpc}/h)^3$.
- **Priors:** For all the EFT parameters, we impose a Gaussian prior centered on 0 with a width of 2,

¹³ We note that the DESI BGS and LRG samples together constitute approximately 28% of the total DESI BAO volume (see Tab. III in Ref. [12]). As a conservative estimate, we assume that roughly half of the galaxies in the DESI BGS and LRG samples were already observed by BOSS. Assuming further that BOSS covers half the data volume of DESI, and noting that the r_s -marginalization procedure removes 55% of the correlation [43], we obtain an upper bound on the residual correlation of $\rho = \frac{0.5 \times V_{\text{BGS,LRG}}}{\sqrt{V_{\text{DESI}} V_{\text{BOSS}}}} \times 45\% \sim 10\%$, where $0.5 \times V_{\text{BGS,LRG}}$ corresponds to the overlapping volume between BOSS and DESI.

¹⁴ The first formulation of the EFTofLSS was carried out in Eulerian space in Refs. [66, 69] and in Lagrangian space in [164]. Once this theoretical framework was established, many efforts were made to improve this theory and make it predictive, such as the understanding of renormalization [165, 166], the IR-resummation of the long displacement fields [72, 80–82, 167, 168], and the computation of the two-loop matter power spectrum [169, 170]

¹⁵ <https://github.com/pierrexyz/pybird>

except for the linear bias parameter b_1 where we impose a log-normal prior, $\log b_1 \sim \mathcal{N}(0.8, 0.8944)$ (see Refs. [77, 85] for all the details on the EFT parameters and the associated priors). Following Ref. [83], we analytically marginalize all the EFT parameters that enter linearly in the theory, while we vary the other parameters, namely $\{b_1, b_2, b_5\}$ (see Ref. [77]), allowing us to considerably reduce the parameter space probed by the MCMC, going from 41 EFT parameters per skycut to only 3. Finally, we consider correlations between the skycuts – thanks to a multivariate Gaussian prior – by imposing the EFT parameters to vary by at most 10% between north and south hemispheres, and by at most 20% between the low- z and high- z samples (see Refs. [85, 87]).

- **Cutoff scales:** For both the power spectrum and the bispectrum, we consider $k \in [0.01, 0.20] h\text{Mpc}^{-1}$ for the low- z sample and $k \in [0.01, 0.23] h\text{Mpc}^{-1}$ for the high- z sample (see Refs. [83, 85, 173]).
- **Observational effects:** Our analysis includes several observational modellings (see *e.g.*, Ref. [171]), such as the window functions from Ref. [146] (see App. A of Ref. [174] for more details), the binning effect [85] or the Alcock-Paszynski effect [175].

2. Angular power spectra

We now turn to the description of the modelling of the ISWL and DESIcross likelihoods. The cross-angular power spectrum between two tracers i, j on the 2D full sky is computed from the following line-of-sight integral (see *e.g.*, Ref. [176])

$$C_{\ell}^{ij} = \frac{2}{\pi} \int dk k^2 \Delta_{\ell}^i(k) \Delta_{\ell}^j(k) P(k, \chi, \chi'), \quad (12)$$

where $P(k, \chi, \chi')$ corresponds to the unequal-time matter power spectrum, and where $\Delta_{\ell}^i(k)$ denotes the source function for the tracer i . The latter takes the form of a time dependent window function $W^i(\chi)$ which is line-of-sight integrated over the spherical Bessel functions, together with a ℓ -dependent prefactor $c(\ell)$:

$$\Delta_{\ell}^i(k) = c(\ell) \int_0^{\infty} d\chi W^i(\chi) j_{\ell}^{(n)}(k\chi), \quad (13)$$

where (n) corresponds the n -th derivative. In this paper, we do not consider the commonly used Limber approximation [177–179] – reducing the triple integral in Eq. (12) into a single integral over the comoving distance χ – which is valid only at small scales, but we perform the full integral in Eq. (12) using **SwiftC_ℓ** [151] (see also Ref. [180] for an alternative code). In particular, this code uses an FFTlog decomposition of the χ -dependent part of the integrand, in order to analytically compute

the integrals over χ and χ' before numerically integrating Eq. (12) over k . We refer the interested reader to Ref. [151] for further detail.

In the ISWL and DESIcross likelihoods, we consider the angular power spectra for the following pairs of fields: $(g, g), (g, \kappa), (g, T), (\kappa, T)$. Therefore, we need to determine the kernels for the galaxy clustering Δ_{ℓ}^g , the CMB lensing Δ_{ℓ}^{κ} , and the CMB temperature Δ_{ℓ}^T in order to obtain the angular power spectra from Eq. 12. In the following, we describe the theoretical predictions for these observables within the EFTofDE framework depicted in Sec. II.

Galaxy clustering kernel. The galaxy clustering source function Δ_{ℓ}^g implemented in **SwiftC_ℓ** [151] includes contributions from galaxy fluctuations $\Delta_{\ell}^{g,D}$ (see *e.g.*, Ref. [176]), magnification bias $\Delta_{\ell}^{g,\mu}$ [181–183], and redshift-space distortions $\Delta_{\ell}^{g,\text{RSD}}$ [183], such that

$$\Delta_{\ell}^g = \Delta_{\ell}^{g,D} + \Delta_{\ell}^{g,\mu} + \Delta_{\ell}^{g,\text{RSD}}, \quad (14)$$

where

$$\begin{aligned} \Delta_{\ell}^{g,D} &= b_1(z) \int_0^{\infty} d\chi W^{\delta,g}(\chi) D_+(k, z(\chi)) j_{\ell}(k\chi), \\ \Delta_{\ell}^{g,\mu} &= \frac{\ell(\ell+1)}{k^2} (5s_{\mu} - 2) \int_0^{\infty} \frac{d\chi}{\chi^2} W^{\kappa,g}(\chi) D_+(k, z(\chi)) j_{\ell}(k\chi), \\ \Delta_{\ell}^{g,\text{RSD}} &= - \int_0^{\infty} f_+(z(\chi)) W^{\delta,g}(\chi) D_+(k, z(\chi)) j_{\ell}^{(2)}(k\chi). \end{aligned} \quad (15)$$

The full galaxy clustering kernel depends on the galaxy fluctuation ($W^{\delta,g}$) and magnification bias ($W^{\kappa,g}$) window functions, which are respectively given by

$$\begin{aligned} W^{\delta,g}(\chi) &= n(z(\chi)), \\ W^{\kappa,g}(\chi) &= \frac{3\Omega_m H_0^2}{2c^2} \int_{\chi}^{\infty} d\chi' n(\chi') \frac{\chi}{a(\chi)} \frac{\chi' - \chi}{\chi}, \end{aligned} \quad (16)$$

where $n(z(\chi))$ corresponds the normalized galaxy redshift distribution. In this set of equations, $D_+(k, z(\chi)) = \sqrt{P(k, z(\chi))/P(k)}$ corresponds to the growth factor and $f_+ = d \ln D_+ / d \ln a$ corresponds to the growth rate, which are directly extracted from **hiclass**. Unlike the EFTBOSS likelihood, we consider a linear galaxy bias model to determine the galaxy overdensity field in the DESIcross likelihood (see Refs. [184, 185]), where $b_1(z)$ – defined as $\delta_g = b_1(z) \delta_m$ – corresponds to the linear bias parameter. We vary b_1 for each LRG redshift bin. We also include a shot noise contribution SN_i in $C_{\ell,i}^{gg}$ for each redshift bin i (see Ref. [149]), in such way that $C_{\ell,i}^{gg} \rightarrow C_{\ell,i}^{gg} + \text{SN}_i$. Additionally, the magnification bias kernel depends on one free parameter, s_{μ} , corresponding to the number count slope. We note that this contribution is subdominant in our analysis, and that the reconstructed posteriors of s_{μ} is largely dominated by the priors on the (linear) scales we are considering. Finally, unlike Ref. [149], we also include a contribution from the

redshift space distortion effect, which becomes important at large scale, as we show in App. A (see also Ref. [149]).

CMB lensing kernel. The source function for the CMB lensing is given by

$$\Delta_\ell^\kappa = \ell(\ell+1) \frac{3\Omega_m H_0^2}{2c^2 k^2} \int_0^{\chi_*} \frac{d\chi}{a(\chi)} \frac{\chi_* - \chi}{\chi \chi_*} D_+(k, z(\chi)) j_\ell(k\chi), \quad (17)$$

where χ_* corresponds to the comoving distance to the last scattering surface. In this equation, we have incorporated the CMB lensing kernel which is analog to the magnification bias kernel (where we replace the galaxy redshift distribution by the lens redshift distribution).

CMB temperature kernel. The CMB gravitational lensing field and the galaxy overdensity field are correlated with the integrated Sachs-Wolfe effect contribution of the CMB temperature field since the same gravitational potentials are responsible for these three effects. The source function for the CMB temperature anisotropies is given by [186]:

$$\Delta_\ell^T = 2T_{\text{CMB}} \frac{3\Omega_m H_0^2}{2c^3 k^2} \times \int_0^{\chi_*} (1 - \tilde{f}_+(z(\chi))) D_+(z(\chi)) H(z(\chi)) j_\ell(k\chi), \quad (18)$$

where T_{CMB} is the current CMB temperature, and $\tilde{f}_+ = d \ln(D_+ \Sigma) / d \ln a$ corresponds to the modified growth rate (see Ref. [54]). The Σ factor, defined in Eq. 10, arises from the Weyl potential $\Phi + \Psi$, which governs the photon geodesics. Note that the growth factor D_+ here inherently includes modifications to gravity through the μ parameter.

In this work, we adopt the following settings for the DESIcross likelihood:

- **Cutoff scales:** We consider the following scale cuts for the angular power spectra across the four redshift bins:

$$C_\ell^{gg} : [20, 124], [20, 178], [20, 243], [20, 243],$$

$$C_\ell^{\kappa g} : [20, 178], [20, 178], [20, 243], [20, 243],$$

$$C_\ell^{Tg} : [20, 178], [20, 178], [20, 178], [20, 178].$$

We discuss in App. A how we determine these scale cut.

- **Priors:** We impose, for each redshift bin, an uniform prior on b_1 and Gaussian priors on s_μ and SN_i (see Refs. [147, 149]):

$$b_1 \sim \mathcal{U}(1, 3),$$

$$s_\mu^i \sim \mathcal{N}(1, 0.1),$$

$$\text{SN}_i \sim \mathcal{N}(\text{SN}_{i,\text{fid}}, 0.3 \text{SN}_{i,\text{fid}}),$$

where $\text{SN}_{i,\text{fid}}$ corresponds to the shot noise fiducial values for the four redshift bins $\text{SN}_{i,\text{fid}} = \{4.07 \cdot 10^{-6}, 2.25 \cdot 10^{-6}, 2.05 \cdot 10^{-6}, 2.25 \cdot 10^{-6}\}$.

For the details (and validations) about the modelling of the DESIcross likelihood, we refer the interested reader to App. A.

IV. RESULTS

In this section, we discuss the cosmological constraints on the CPL parametrization and on the EFTofDE configuration considered in this work (see Sec. II). Our main results are presented in Fig. 2, where we show the 2D posterior distributions of $\{w_0, w_a\}$ and $\{c_B, c_M\}$ for all data combinations presented in Sec. III. The associated cosmological constraints, the bestfit χ^2 and the preferences over Λ CDM are displayed in Tabs. I and II of App. B. Finally, in Fig. 12 of this appendix, we also plot the 1D and 2D posterior distributions for all cosmological parameters.

A. Baseline analysis and CMB likelihoods

For the baseline dataset, corresponding to *Planck* PR4 + Lensing + DESI DR2 BAO + DES Y5, we obtain $w_0 = -0.769 \pm 0.056$ and $w_a = -0.75 \pm 0.22$ at 68% CL (with PPF), and a 3.8σ preference over Λ (see App. B). When we extend this analysis to an EFT description of the DE perturbations, the constraint on $\{w_0, w_a\}$ is shifted by 1.2σ towards its Λ CDM limit. For the EFTofDE parameters, we obtain $c_B = 0.65_{-0.37}^{+0.24}$ and $c_M = 0.75_{-1.0}^{+0.63}$, leading to a 3.4σ preference over Λ CDM (see App. B). While c_M is fully consistent with 0, we observe that c_B is shifted by 1.7σ compared to its GR limit, which is related to the lensing anomaly [47, 187, 188], in line with previous results [44, 45, 189].¹⁶ Finally, we note that the correlations between $\{w_0, w_a\}$ and $\{c_B, c_M\}$ is weak, as shown in Fig. 12.

Compared with the equivalent CPL analysis (without EFTofDE) from the DESI DR2 cosmological results [12], the best-fit value of w_a shows a mild shift towards Λ CDM, while the evidence for evolving dark energy decreases from 4.2σ to 3.8σ . This difference arises from the choice of the CMB likelihoods, for both the lensing and the primary spectra. In Fig. 3, we compare some variations of our baseline analysis with different CMB likelihoods, namely Hillipop [47] vs Camspec [190] for the high- ℓ TTTEEE likelihood, Lollipop [47] vs Simall [136] for the low- ℓ TT likelihood, and *Planck* PR4 [137] vs ACT DR6 [191, 192] for the lensing likelihood. We summarize our results in the following:

- **Hillipop vs Camspec:** We observe a shift up to 0.2σ on the 1D posteriors of $\{w_0, w_a, c_B, c_M\}$ (see

¹⁶ Although this anomaly has been reduced from 2.8σ in PR3 to 1.7σ in Camspec [47, 135] and to 0.7σ in Hillipop [190].

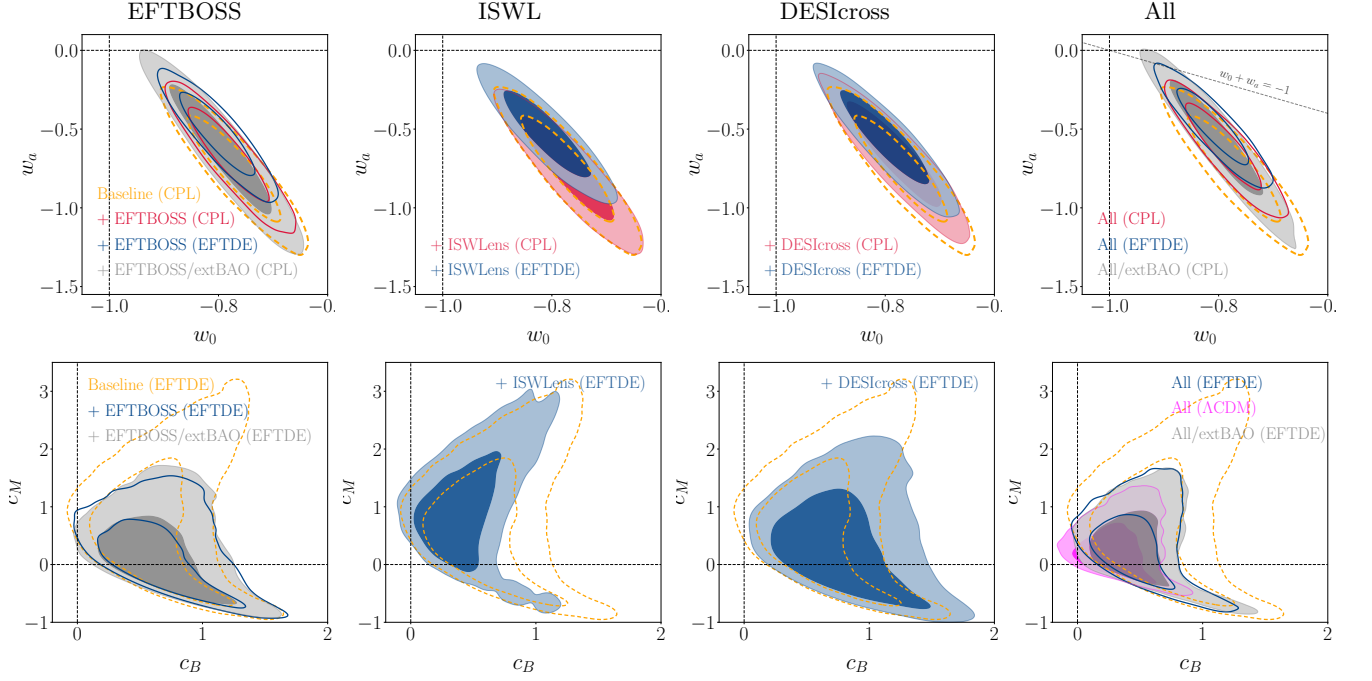


FIG. 2: 2D posterior distributions of $\{w_0, w_a\}$ and $\{c_B, c_M\}$ reconstructed from all data combinations considered in this work, namely Baseline, Baseline + EFTBOSS, Baseline + ISWL, Baseline + DESIcross and the combination of all these datasets. The black dashed lines correspond to the Λ CDM limits, while the baseline analysis is shown in yellow dotted line.

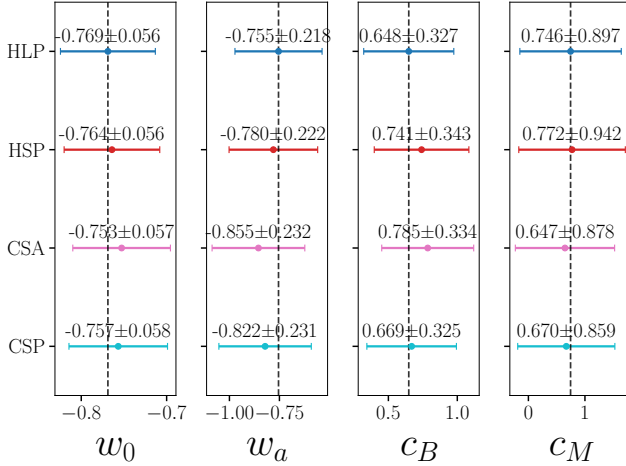


FIG. 3: Constraints on $\{w_0, w_a, c_B, c_M\}$ for several CMB likelihood combinations, namely Hillipop (H) vs Camspec (C) for the high- ℓ TTTEEE likelihood, Lollipop (L) vs Simall (S) for the low- ℓ TT likelihood, and *Planck* PR4 (P) vs ACT DR6 (A) for the lensing likelihood. Note that for all the data combinations, we use the low- ℓ TT likelihood from Commander, together with DESI DR2 BAO and DES Y5. In the first two columns, we assume the CPL parametrization without EFTofDE, while in the last two columns we consider our EFTofDE configuration (with a CPL background). The black dashed lines represent the mean values of our baseline analysis.

HSP vs CSP), and a change in the error bars up to 10% (for c_M).

- **Lollipop vs Simall:** Lollipop gives slightly tighter constraints than Simall and shifts the posterior up to 0.3σ for c_B (see HLP vs HSP).
- **Planck PR4 vs ACT DR6 lensing:** The constraints are similar between the two likelihoods with a shift up to 0.35σ for c_B (see CSA vs CSP).

In the end, we manage to reproduce the results of Ref. [12], and the main difference between our baseline analysis (HLP) and this reference (CSA) is a shift in w_a of 0.44σ . Note that in our baseline analysis, the constraining power on $\{c_B, c_M\}$ mostly comes from the late integrated Sachs-Wolfe effect and the CMB lensing [46]. In the following, we perform a multiprobe analysis to improve the constraints on these parameters.

B. Multiprobe analysis

1. EFTBOSS

We first extend the baseline dataset by incorporating both the power spectrum and bispectrum measurements from BOSS thanks to an EFTofLSS description of the dark matter overdensity field (see Sec. III). Unlike Ref. [43], which employed the exact time dependence

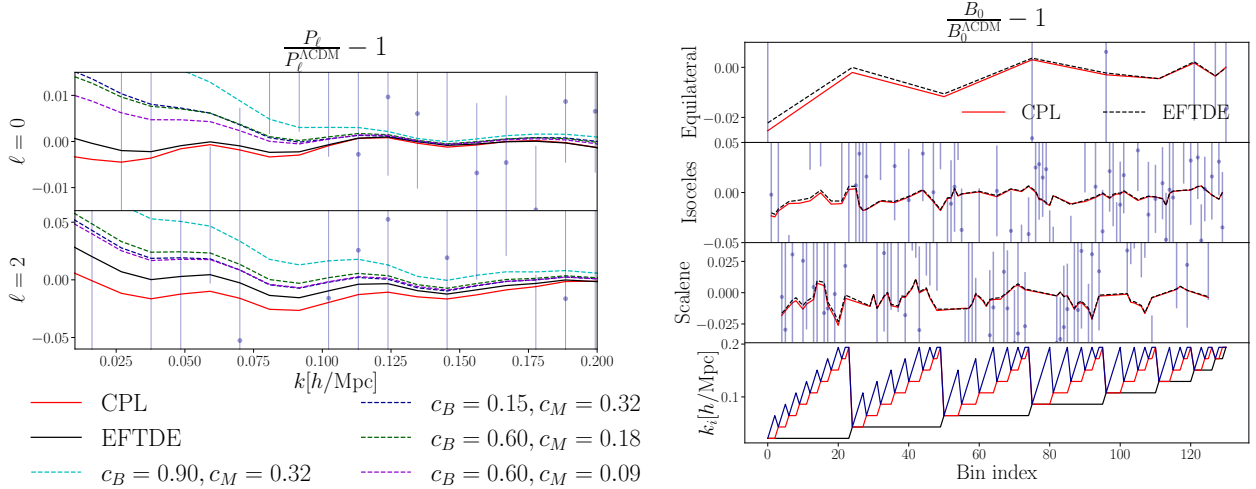


FIG. 4: *Left*: Residuals of the monopole and quadrupole of the galaxy power spectrum for the CPL model with and without EFTofDE, normalized to the Λ CDM model (for the baseline + EFTBOSS analysis). We also show the impact of varying c_B and c_M on this observable, setting the other cosmological parameters to the best-fit. Note that we show here the predictions and the data for the low- z NGC sample of BOSS. *Right*: Same for the bispectrum monopole for different triangle configurations.

description of the EFTofLSS operators, we adopt here the EdS approximation where the time evolution of all the operators is only encoded in the linear growth factor $D_+(a)$, since, as demonstrated in Ref. [43], the difference between the exact-time and EdS treatments is negligible for BOSS sensitivity. We further restrict our analysis to the smooth quintessence case, as the smooth and clustering scenarios [193, 194] studied in Ref. [43] exhibit similar deviations. Our results are presented in the first column of Fig. 2.

The sensitivity of EFTBOSS to c_B and c_M in the quasi-static limit arises from the fact that they act as additional sources in the Poisson equation [115], as can be seen in Eq. (7). This implies a modification of the growth factor $D_+(a)$ (and therefore of the growth rate $f_+(a)$) through the continuity and Euler equations, which will modify the time dependence of the EFTofLSS operators [70, 195, 196]. These parameters also affect the overall amplitude of the linear power spectrum within the scales of interest, namely $0.01 < k/[h/\text{Mpc}] < 0.23$, which propagate into the one-loop corrections. In the top panel of Fig. 5, we display the ratios of the linear matter power spectrum for several values of c_B and c_M , showing that variations in these parameters lead to an almost scale independent modification of the linear matter power spectrum amplitude. We further note that EFTBOSS is more affected by a change in c_M than a change in c_B , coming from the fact that μ is more sensitive to a variation in the former parameter at high redshift, while a variation in the latter parameter becomes

significant only at low redshift (see Fig. 1).¹⁷ In the left panel of Fig. 4, we display the residuals of the monopole and quadrupole of the galaxy power spectrum for several values of c_B and c_M , allowing us to stress that they affect the galaxy power spectrum in the same way, namely that an increase in these parameters involves an enhancement of this observable. This suggests that there is a negative correlation between these two parameters in the EFTBOSS likelihood, and that a slight increase in c_M can be compensated for by a larger decrease in c_B .

When we add EFTBOSS to the baseline analysis (using the r_s -marg procedure described in Sec. III), the constraint on the 2D plane $\{w_0, w_a\}$ is improved by 27%,¹⁸ and the preference over Λ increases from 3.7σ to 4.7σ , consistent with previous findings [43]. Within the EFTofDE framework, we obtain an 10.3% (84.7%) improvement for the $\{w_0, w_a\}$ ($\{c_B, c_M\}$) plane, increasing the preference over Λ from 3.4σ to 4.3σ . As shown in Fig. 12, EFTBOSS is able to break the degeneracy between σ_8 and c_M , leading to a 38% improvement in the constraint of c_M (compared to 2% for c_B), allowing us to exclude large positive values of the running of the

¹⁷ Let us note that since the growth factor is coupled to μ through a differential equation, the effect of c_M on the matter density perturbations accumulates over cosmic time, implying that EFTBOSS is more sensitive to this parameter than c_B .

¹⁸ Throughout the paper, in order to quantify an improvement in the $\{w_0, w_a\}$ or $\{c_B, c_M\}$ 2D planes, we use the ratio of the Figure of Merit (FOM) [10, 197], defined as $\text{FOM} \propto |\det C|^{-1/2}$, where C is the covariance matrix of the parameter posteriors. This metric should be taken with a grain of salt for the $\{c_B, c_M\}$ 2D posterior distributions, given that they are not Gaussian.

Planck mass, namely $c_M \gtrsim 1.5$ (see Fig. 2). Finally, and as expected, EFTBOSS induces a negative correlation between c_B and c_M (see Fig. 2) due to the exclusion of high values of c_M (for which the galaxy power spectrum is more sensitive).

To further investigate the constraining power of EFTBOSS, we show, in Fig. 4, the power spectrum and bispectrum residuals for the best-fit of CPL with and without EFTofDE, normalized to the equivalent Λ CDM best-fit. For the CPL model with PPF, we obtain $\delta\chi^2_{\min}(\text{EFTBOSS}) = -7.4$ with respect to Λ CDM (within the baseline + EFTBOSS analysis), corresponding to a non-negligible improvement of the fit to the BOSS data. Within the EFTofDE framework, we do not further improve this fit (as can be seen in Fig. 4), with $\delta\chi^2_{\min} = -7.5$.

Using the extBAO dataset (instead of DESI BAO DR2), and without considering the r_s -marg procedure for the EFTBOSS likelihood, we observe a very similar trend, as shown in Fig. 2. The extBAO likelihood provides weaker constraints on the late-time expansion than the DESI DR2 BAO likelihood, leading to a smaller constraint on the $\{w_0, w_a\}$ plane, although consistent with previous analyses. However, the constraints on c_B and c_M remain comparable, resulting from their weak correlation with the background parameters, especially Ω_m , w_0 and w_a (see Fig. 12). We note that the statistical significance for evolving dark energy decreases from 4.7σ to 3.6σ with PPF and from 4.3σ to 3.3σ with EFTofDE.

2. ISWL

In Fig. 5, we show the ratios of $C_\ell^{T\kappa}$ for several values of c_B and c_M , allowing us to highlight that this observable is highly sensitive to these parameters. In particular, the more we increase the value of the braiding parameter c_B , the more the $C_\ell^{T\kappa}$ power spectrum is suppressed, which is explained by the fact that this parameter reduces the integrated Sachs-Wolfe effect (and can even make it negative for large positive values of c_B) [49, 54, 104, 198]. For the running of Planck mass c_M , we observe the opposite behavior at large scales, implying that the ISWL likelihood introduces a positive correlation between these two EFT parameters (see Fig. 2).

When we add the ISWL likelihood to the baseline analysis, we find that the constraints on c_B and c_M are improved by 25% and 16%, respectively, while the background dark energy parameters remain essentially unchanged (see Fig. 2). As expected, we can see in Fig. 2 that this likelihood induces a positive correlation in the $\{c_B, c_M\}$ plane, which is significantly reduced by 51.6%. This positive correlation is accompanied by the exclusion of large positive values of the braiding parameter, namely $c_B \gtrsim 1$, which is consistent with Ref. [49]. In addition, we note that the inclusion of the ISWL likelihood does not impact the preference for evolving dark energy, which remains at 3.8σ (3.4σ) for the analysis without (with) the

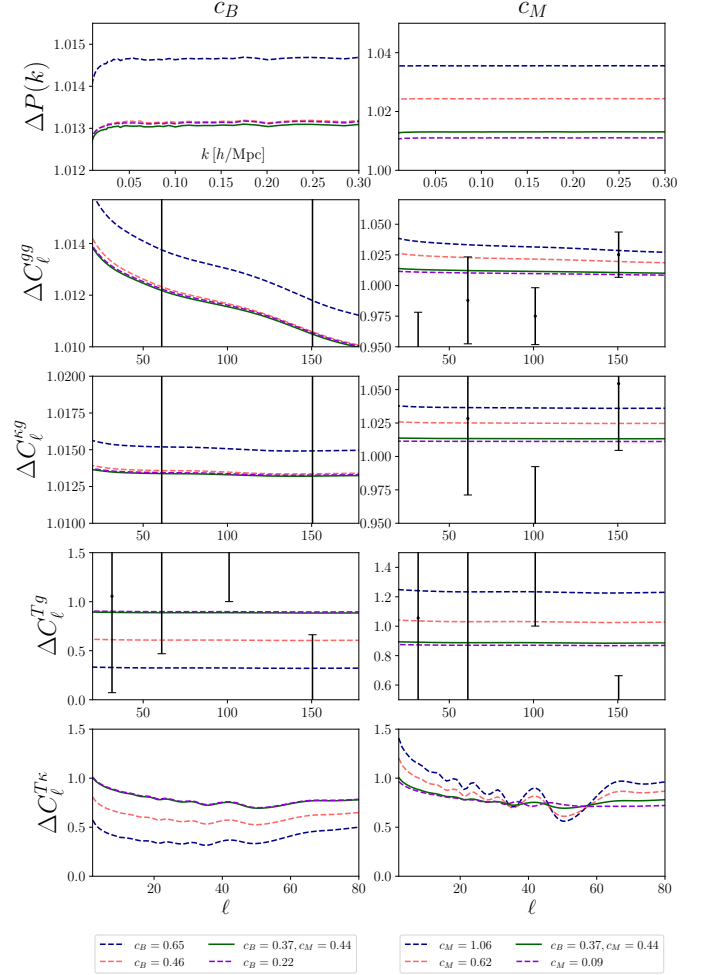


FIG. 5: Residuals (with respect to Λ CDM) of the linear matter power spectrum, C_ℓ^{gg} , C_ℓ^{kg} , C_ℓ^{Tg} and $C_\ell^{T\kappa}$ for several values of c_B (left) and c_M (right). The residuals are computed at $z = 0.625$, corresponding to the effective redshift of the second redshift range of the DESIcross likelihood. All cosmological parameters are fixed to their best-fit values (reconstructed from the “All” analysis), while the solid green curves correspond to the best-fit values of c_B and c_M . For the sake of clarity, we divide the error bars of C_ℓ^{kg} and C_ℓ^{Tg} by 2 (see Fig. 8 for a real representation of these data points).

EFT description of the DE perturbations.

3. DESIcross

In Fig. 5, we also show the ratios of C_ℓ^{gg} , C_ℓ^{kg} and C_ℓ^{Tg} for several values of c_B and c_M . Interestingly, a variation of these parameters induces a scale independent modification of the amplitude of these three observables, with different correlations between c_B and c_M . In particular, C_ℓ^{Tg} exhibits the same behavior as $C_\ell^{T\kappa}$ (see

Ref. [54]) since both are directly sensitive to Σ , as shown in Eq. (18), therefore introducing a positive correlation between the two EFT parameters. However, we observe a different trend for C_ℓ^{gg} and $C_\ell^{\kappa g}$, where an increase in both c_B or c_M leads to an enhancement of these observables, implying a negative correlation between these parameters. Interestingly, we expect that combining C_ℓ^{gg} , $C_\ell^{\kappa g}$, C_ℓ^{Tg} , and $C_\ell^{T\kappa}$ will allow us to break the various degeneracies described above.

When we add the DESIcross likelihood to the baseline dataset (where we have removed the low- ℓ CMB lensing data as explained in Sec. III A 2), the precision on the $\{w_0, w_a\}$ plane remains equivalent to the baseline analysis (regardless of the DE perturbation parametrization), though the mean values are slightly shifted towards Λ CDM: $w_0 = -0.769 \pm 0.056 \rightarrow -0.780 \pm 0.056$ and $w_a = -0.75 \pm 0.22 \rightarrow -0.67 \pm 0.22$ (see Fig. 2). Regarding the EFTofDE parameters, the constraint on c_M is improved by 17%, while the constraint on c_B remains the same. Indeed, as shown in Fig. 2, the DESIcross likelihood is able to set a stronger constraint on the upper limit of c_M compared to the baseline analysis, excluding the region $c_M \gtrsim 2$, consistent with Ref. [54]. As a result, we can clearly see a negative correlation between the two EFT parameters, coming from the fact that the main constraining power comes from C_ℓ^{gg} . This is expected given the small error bars of this observable compared to C_ℓ^{Tg} and $C_\ell^{\kappa g}$ (see Fig. 8 of App. A). Finally, we note that the preference for evolving dark energy remains similar to the baseline analysis.

Our analysis can be improved by including the low- ℓ data points of the CMB lensing, after determining the covariance between $C_\ell^{\kappa\kappa}$ and $C_\ell^{\kappa g}$. We further note that including smaller scales in C_ℓ^{gg} would help to improve the constraints on the EFTofDE parameters (see Fig. 5). We leave this exploration for future work, using a non-linear galaxy bias expansion, as done in Ref. [149].

4. All

In the previous sections, we saw that EFTBOSS and DESIcross introduce a negative correlation between c_B and c_M , by imposing a stronger constraint on the upper limit of c_M compared to the baseline analysis (see Fig. 2). Conversely, the ISWL likelihood introduce a positive correlation between these two parameters, by imposing a stronger constraint on the upper limit of c_B compared to the baseline analysis (see Fig. 2). Therefore, we expect that combining all these likelihoods will further break these degeneracies, and then improve the constraints on c_B and c_M .

When we perform such an analysis (always by removing the low- ℓ CMB lensing data), we improve the constraint on the $\{w_0, w_a\}$ plane by 46% (52%) for the analysis without (with) EFTofDE compared to the baseline analysis. This is accompanied by a 33% (35%) reduction of the $\{h, \Omega_m\}$ 2D posterior distribution. Re-

garding the EFTofDE parameters, the $\{c_B, c_M\}$ plane is reduced by 177% compared to the baseline analysis, and we obtain $c_B = 0.46^{+0.16}_{-0.22}$ and $c_M = 0.31^{+0.39}_{-0.49}$, respectively corresponding to an improvement of 37% and 46%. As explained above, this strong constraint stems from the different degeneracies between the EFTofDE parameters that are inherent in the different likelihoods considered in our work. We finally obtain a preference for evolving dark energy of 4.6σ (4.2σ) for the analysis with PPF (EFTofDE), compared to 3.8σ (3.4σ) for the baseline analysis, while we note that the 2D posterior distribution of $\{c_B, c_M\}$ is compatible with its GR limit at $\sim 2\sigma$. We note that the posterior of $\{w_0, w_a\}$ lies at 3.5σ (3.1σ) from the $w_0 + w_a = -1$ line, corresponding to a transition from $w < -1$ in the past to $w > -1$ today (see quintom models [24–31]). To understand the origin of this preference, we display in Tab. III the bestfit χ^2 for each likelihood for Λ CDM, CPL with PPF and CPL with EFTofDE. We obtain a total $\Delta\chi^2_{\min} = -24.6$ ($\Delta\chi^2_{\min} = -26.2$) for the CPL model with PPF (EFTofDE), mostly coming from the EFTBOSS ($\delta\chi^2_{\min} \sim -7$), DESI BAO ($\delta\chi^2_{\min} \sim -5$) and DES Y5 ($\delta\chi^2_{\min} \sim -12$) likelihoods.

To investigate the dependence of the $\{c_B, c_M\}$ 2D posterior distribution on the background cosmology, we perform the same analysis fixing the DE background evolution to Λ , as shown in Fig. 2. We obtain $c_B = 0.32^{+0.18}_{-0.21}$ and $c_M = 0.25^{+0.25}_{-0.43}$, respectively corresponding to a better constraint of 10% and 22% compared to the analysis with the CPL background evolution. We further note that the $\{c_B, c_M\}$ plane is slightly shifted towards its GR limit, corresponding to a preference over Λ CDM of 0.68σ (and a χ^2 improvement of -1.4).

C. Impact of BAO and SN data

In this section, we study the impact of the BAO and supernovae data in our analysis. First, given that the preference for evolving dark energy depends on the choice of the supernovae and BAO likelihoods (see *e.g.*, [12]), we carry out the “All” analysis by replacing DESI DR2 BAO with ext-BAO and DES Y5 with Pantheon+ [6], as shown in the top panel of Fig. 6. We observe that the preference for evolving dark energy decreases from 4.2σ (with DESI DR2 BAO and DES Y5) to 3.5σ (with ext-BAO and DES Y5), then to 3.0σ (with ext-BAO and Pantheon+). Regarding the EFTofDE parameters, we obtain very similar constraints when considering the ext-BAO and Pantheon+ likelihoods, with $c_B = 0.46^{+0.37}_{-0.51}$ and $c_M = 0.29^{+0.37}_{-0.51}$, due to the low correlations between the EFTofDE parameters and the background parameters (see Fig. 12).

Second, to gauge the impact of the BAO and supernovae likelihoods on our conclusions, we remove those datasets in our All analysis, as shown in Fig. 6. For *Planck* PR4 + Lensing + EFTBOSS + ISWL + DESIcross, we obtain a 2.3σ preference for the CPL model with

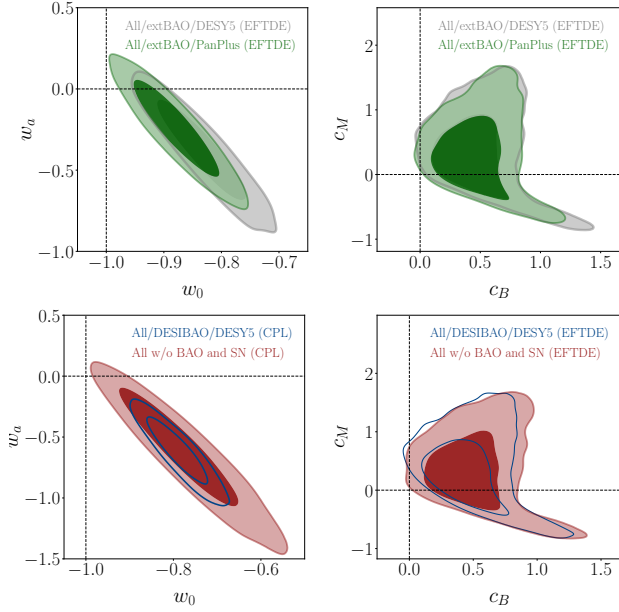


FIG. 6: 2D posterior distributions of $\{w_0, w_a\}$ and $\{c_B, c_M\}$ for several variations of BAO and supernovae data. *Top:* We compare the All dataset (with ext-BAO) with the All dataset in which DES Y5 has been replaced by Pantheon+. *Bottom:* We compare the All dataset and the All dataset in which DESI BAO DR2 and supernovae data have been removed.

PPF. We also note that when we remove the BAO and supernovae data, the constraints on c_B and c_M remain unchanged (see Fig. 6). In Fig. 7, we present the DESI DR2 BAO data alongside the prediction from our analysis which does not include BAO and supernova data. When we fit this prediction to the DESI DR2 BAO measurements, we obtain $\chi^2_{\min}(\text{BAO}) = 8.06$, which is significantly better than the *Planck* ΛCDM best-fit (with $\chi^2_{\min}(\text{BAO}) = 21.74$) or even to the ΛCDM fit to the DESI DR2 BAO + BBN dataset (with $\chi^2_{\min}(\text{BAO}) = 10.28$), as shown in Fig. 7. We conclude that our All analysis without BAO and supernovae data shows a slight preference for evolving dark energy and is able to better accommodate the DESI DR2 BAO measurements than ΛCDM .

V. CONCLUSION

In this work, we constrain the CPL parametrization of the DE equation of state, considering either the parametrized Post-Friedmann framework or the EFTofDE framework to describe the DE perturbations. Within the EFTofDE framework, we vary α_B , corresponding to the braiding parameter, and α_M , corresponding to the running of the Planck mass, both parametrized by $\alpha_i(a) = c_i \cdot \Omega_{\text{DE}}(a)$. The main goal of this paper is to constrain the parameter space of

$\{w_0, w_a, c_B, c_M\}$ with (i) the EFTofLSS full-shape analysis of the power spectrum and bispectrum of BOSS Luminous Red Galaxies (dubbed “EFTBOSS”), (ii) the angular power spectrum $C_\ell^{T\kappa}$ from the cross correlation between the *Planck* PR4 temperature and lensing maps (dubbed “ISWL”), and (iii) the auto angular galaxy power spectra C_ℓ^{gg} from DESI Luminous Red Galaxies, together with the cross angular power spectra $C_\ell^{\kappa g}$ and C_ℓ^{Tg} from the cross correlation with the *Planck* PR4 lensing and temperature maps (dubbed “DESIcross”).

One of the main novelties of this work is the measurement of C_ℓ^{Tg} (in the DESIcross likelihood), detected with a signal-to-noise ratio of 2.04, and the inclusion of the scales $20 < \ell < 79$ in C_ℓ^{gg} , thanks to the `SwiftCℓ` code allowing us to go beyond the Limber approximation.

We summarize our main results in the following:

- For the baseline analysis – including the primary power spectra and CMB gravitational lensing spectrum from *Planck* PR4, the BAO measurements from DESI DR2 and the supernovae compilation from DES Y5 – we obtain a preference for evolving dark energy over Λ at 3.8σ (3.4σ) when considering the PPF (EFTofDE) framework. We also show that the choice of the CMB likelihood, for both the lensing and the primary spectra, can lead to shifts up to 0.4σ in the reconstructed DE parameters.
- When we include the EFTBOSS likelihood in addition to the baseline dataset, we increase the preference for evolving dark energy to 4.7σ (4.3σ) for the PPF (EFTofDE) parametrization. The constraint on the 2D plane $\{w_0, w_a\}$ is improved by $\sim 30\%$. Moreover, the EFTBOSS likelihood introduces a negative correlation between the EFTofDE parameters with a strong constraint on the upper bound of c_M , excluding $c_M \gtrsim 1.5$ at 98% CL.
- When we include the ISWL likelihood in addition to the baseline dataset, the preference for evolving dark energy and the constraint on the $\{w_0, w_a\}$ plane remain unchanged. However, the constraint on the $\{c_B, c_M\}$ plane is significantly improved by $\sim 50\%$, while the ISWL likelihood induces a positive correlation between the EFT parameters with a strong constraint on the upper bound of c_B , excluding $c_B \gtrsim 1$ at 98% CL.
- When we include the DESIcross likelihood in addition to the baseline dataset, the preference for evolving dark energy and the constraint on the $\{w_0, w_a\}$ plane remain unchanged as well. However, the DESIcross likelihood leads to a negative correlation between the EFTofDE parameters (coming from C_ℓ^{gg}), while improving the upper bound on c_M , excluding $c_M \gtrsim 2$ at 98% CL.
- When we combine all these likelihoods, we reduce the uncertainty on the $\{w_0, w_a\}$ plane by $\sim 50\%$, and we increase the preference for evolving dark

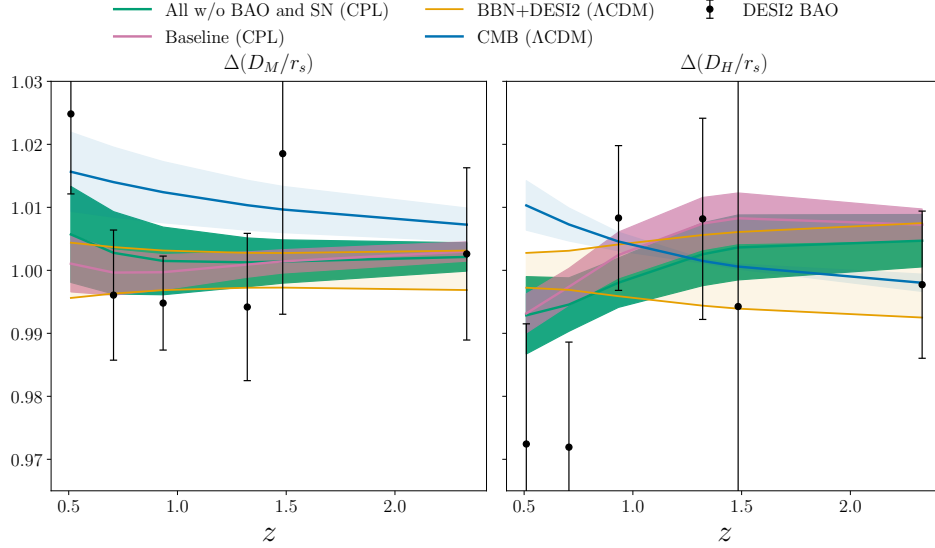


FIG. 7: Residuals of the DESI DR2 BAO transverse D_M/r_s (left) and radial D_H/r_s (right) modes, together with theoretical predictions from different data and models, namely DESI DR2 BAO (with BBN) under Λ CDM, *Planck* PR4 under Λ CDM, Baseline under CPL (with PPF), and All without BAO and supernovae data under CPL (with PPF). The data and theoretical predictions are normalized to the prediction from DESI DR2 BAO + BBN (under Λ CDM). The shaded bands represent the 68% CL reconstructed from the full posterior distribution.

energy to 4.6σ (4.2σ) for the PPF (EFTofDE) framework. Regarding the EFTofDE parameters, we can exploit the various correlations presented above to significantly improve the constraint on the $\{c_B, c_M\}$ parameter space by 177%, with $c_B = 0.46^{+0.16}_{-0.22}$ and $c_M = 0.31^{+0.39}_{-0.49}$. Those constraints are compatible with GR at 2σ , and this conclusion does not change when we consider a Λ CDM background instead.

- Finally, when we remove the BAO and supernovae data, we obtain a hint for evolving dark energy at 2.3σ . We also note that the constraints on the EFTofDE parameters do not depend on the BAO and supernovae likelihoods (due to the low correlation between the background and EFTofDE parameters).

There are several ways in which our work can be improved and extended. First, our analysis considers a linear galaxy bias expansion in the DESIcross likelihood, restricting our analysis of C_ℓ^{gg} , $C_\ell^{\kappa g}$ and C_ℓ^{Tg} to linear scales. Incorporating a consistent EFTofLSS modelling for the galaxy overdensity field, as in Ref. [60], or a hybrid-EFT approach, as in Ref. [154], would allow the inclusion of smaller scales and further enhance the constraining power of the DESIcross likelihood. Second, developing a self-consistent covariance model between $C_\ell^{\kappa\kappa}$ and $C_\ell^{\kappa g}$ would allow us to include the low- ℓ CMB lensing data in the combined analysis and would help to further break parameter degeneracies. Finally, our analysis can be reproduced for future high-precision galaxy maps, coming from DESI [199], *Euclid* [200] or LSST [201], to

improve the signal-to-noise ratio of the EFTBOSS and DESIcross likelihoods, and to enhance the measurement of the ISW signal, which is currently limited by observational noise.

Around the completion of this work, we note that Ref. [59] performed a similar 5×2 pt analysis. Their study focuses on early and late dark energy models, without considering the EFTofDE framework. Our results are consistent with, and independent of, theirs.

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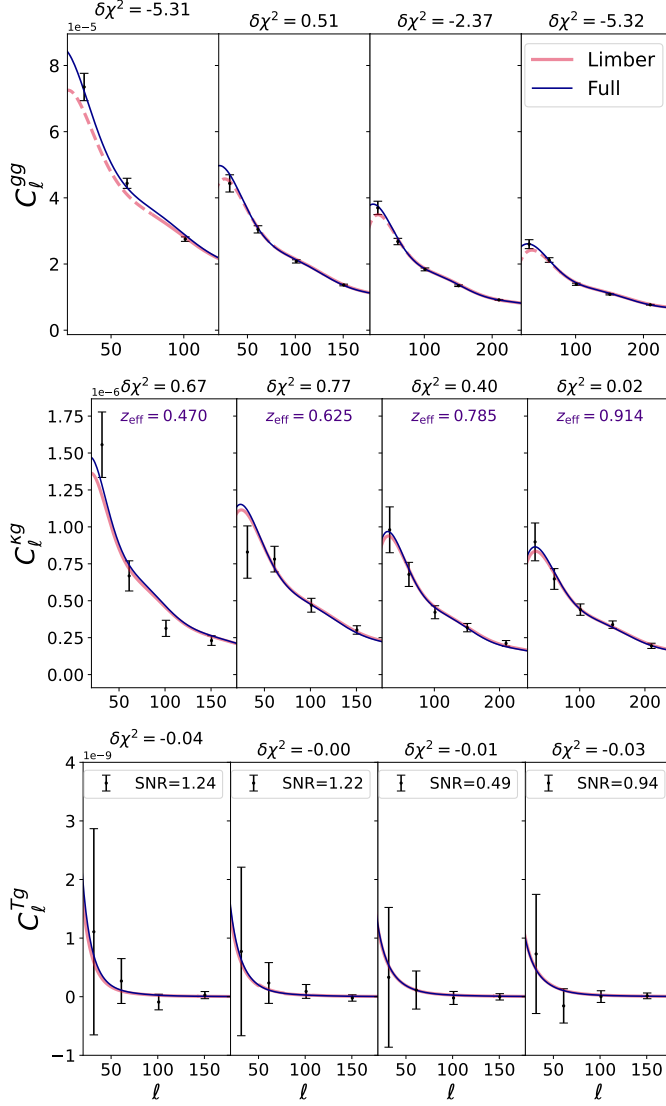


FIG. 8: The C_ℓ^{gg} , C_ℓ^{kg} and C_ℓ^{Tg} data points considered in the DESIcross likelihood, together with the best-fit prediction from the full integral (darkblue) or from the Limber approximation (crimson). The dashed lines corresponds to the regimes where the Limber approximation fails. We also display the individual $\delta\chi^2 = \chi^2(\text{Limber}) - \chi^2(\text{full integral})$ for each angular power spectra and each redshift bin.

resources from the LUPM’s cloud computing infrastructure founded by Ocevu labex and France-Grilles. We finally acknowledge the use the standard python packages, such as `numpy` [202], `scipy` [203], and `matplotlib` [204].

Appendix A: Details and validation of the DESIcross likelihood

1. Details on the DESIcross likelihood

This appendix documents the implementation of the DESIcross likelihood used in `MontePython` and highlights the differences with `Mapar` [149], a publicly available likelihood designed to extract cosmological information from the auto- and cross-correlations between CMB lensing and luminous red galaxies (LRGs) maps. This likelihood makes use of the `NaMaster` code [150] to measure the angular power spectra $C_\ell^{\kappa g}$ and C_ℓ^{gg} as well as their covariance matrices under the Gaussian approximation. For the theoretical prediction, Ref. [149] numerically integrates the angular power spectra using the Limber approximation. In addition, the `Mapar` likelihood considers a Hybrid Effective Field Theory (HEFT) model [205] to determine the galaxy overdensity field from the underlying dark matter field. In our `MontePython` likelihood (DESIcross), we use the data and covariances from `Mapar` for both $C_\ell^{\kappa g}$ and C_ℓ^{gg} . However, we make use of `SwiftCl` for the theoretical prediction, allowing us to go beyond the Limber approximation. This enables us to include additional data in the range $20 \leq \ell \leq 79$ for C_ℓ^{gg} , thus improving the constraints. In addition, in our work, we focus on linear scales and adopt a linear galaxy bias model, meaning that we do not consider the small scales data included in Ref. [149]. In particular, this reference includes data up to $l_{\text{max}} = 600$ for C_ℓ^{gg} and $C_\ell^{\kappa g}$, while we perform our analysis up to $l_{\text{max}} = 124 - 243$ (see Sec. III B 2), as determined in Ref. [149] for the linear bias galaxy model. These scales correspond to the maximum scales such that the linear prediction does not deviate significantly from the non-linear prediction given the data covariance [46, 206, 207].¹⁹ To do so, the χ^2 obtained using the linear perturbation theory prediction is compared to the one obtained using the `Halofit` prediction [208, 209] for several ℓ_{max} , and we impose that $\chi^2 = \Delta C_\ell^T \mathbf{C}^{-1} \Delta C_\ell < 1.0$, where $\Delta C_\ell = C_\ell^{\text{lin}} - C_\ell^{\text{nl}}$. We further adopt the more conservative choice compared to Table 2 of Ref. [149].

We adopt the same pixel and bandpower window functions as those used in the `Mapar` likelihood. The theoretical angular spectra are multiplied by the pixel window function to correct for the pixelization introduced when constructing a `Healpix` [210, 211] overdensity map from the galaxy catalog, and then convolved with the bandpower window function to match the mask-deconvolved measurements from `NaMaster`. Finally, we note that the

¹⁹ Note that Ref. [149] did not include the first redshift bin for C_ℓ^{gg} and $C_\ell^{\kappa g}$ in their linear bias analysis due to the limited scale range probed by the linear perturbation theory at low redshift. Given that in our analysis we go to higher scales, the constraining power of the first redshift bin becomes relevant, and we determine ℓ_{max} using the procedure describe in the main text.

shot-noise parameters are not analytically marginalized, but are instead varied as free parameters in the MCMC sampling.

One of the main novelty of our work is that we additionally include the cross-correlation between LRGs and *Planck* PR4 temperature maps (SEVEM) as shown in Fig. 8. To measure C_ℓ^{Tg} , we use **NaMaster**, which provides a pseudo- C_ℓ estimator that accounts for survey masks. To cross-check our results, we also measure C_ℓ^{Tg} with the **Polspice** code [212], and obtain a good consistency up to 0.2σ for all data points. To determine the maximum scale $\ell_{\max}$, we use the procedure detailed before, allowing us to determine $\ell_{\max} = 2212$. However, due to the limited statistical power of the small-scale modes, we fix $\ell_{\max} = 178$ for C_ℓ^{Tg} across the four redshift bins. Finally, the measurements of C_ℓ^{Tg} yields a cumulative signal-to-noise ratio [63, 149] of $\text{SNR} = \sqrt{\sum_i \chi^2(C_\ell^{Tg,i})} = 2.04$.

In Fig. 8, we display the data points considered in this work for C_ℓ^{gg} , $C_\ell^{\kappa g}$ and C_ℓ^{Tg} , together with the best-fit predictions either from the Limber approximation or from the full integral (computed with **SwiftC_ℓ**).²⁰ We also show the individual $\delta\chi^2 = \chi^2(\text{full integral}) - \chi^2(\text{Limber})$ for each angular power spectra and each redshift bin. The largest improvements are found in the first and fourth redshift bins of C_ℓ^{gg} , where the χ^2 differences reach -5.31 and -5.32 , respectively. For $C_\ell^{\kappa g}$ and C_ℓ^{Tg} , the difference between the full integral and the Limber approximation is negligible, even at large scales. This is consistent with Ref. [149] which applies the Limber approximation until the same $\ell_{\min}$ as in our analysis for C_ℓ^{gg} but not for C_ℓ^{gg} .

2. Validation of our pipeline

To validate our pipeline, we compare in Fig. 9 the cosmological posteriors from our DESIcross analysis, using **MontePython** and **SwiftC_ℓ**, with those from **Mapar**, using **Cobaya** and the Limber approximation. For comparison purposes, we include only C_ℓ^{gg} and $C_\ell^{\kappa g}$, while we consider the scale cuts, determined in Ref. [149], which are valid for the Limber approximation and the linear bias model across the four redshift bins. We further note that the shot noise terms are fixed to their fiducial values. The excellent agreement between the two approaches confirms both the validity of our **MontePython** likelihood implementation and the validity of the **SwiftC_ℓ** calculation.

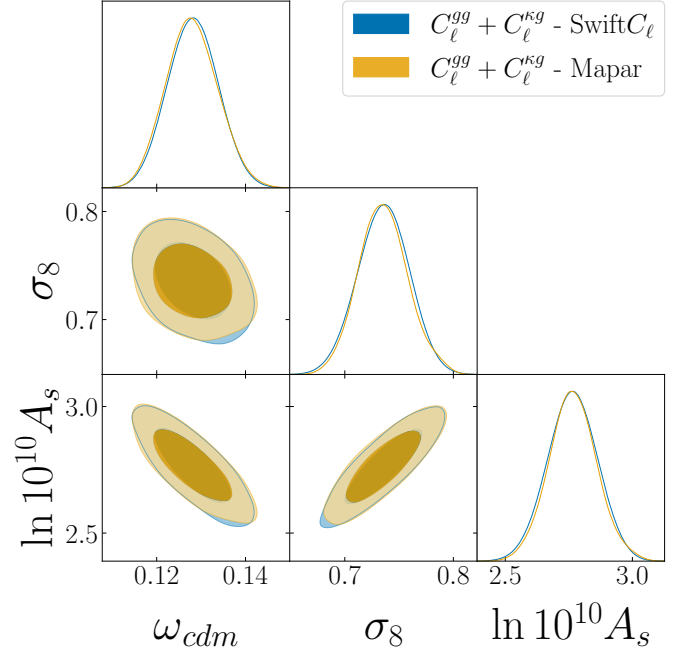


FIG. 9: 1D and 2D posterior distributions reconstructed from our DESIcross likelihood (using **MontePython** and **SwiftC_ℓ**) and from the **Mapar** likelihood (using **Cobaya** and the Limber approximation) for the Λ CDM model.

3. Assessment of the constraining power of the DESIcross likelihood

In Fig. 10, we compare the cosmological constraints obtained for the Λ CDM model with different analysis settings within our DESIcross likelihood. We vary here two cosmological parameters, namely $\{\omega_{\text{cdm}}, \ln(10^{10} A_s)\}$, and twelve nuisance parameters, namely $\{b_i, s_{\mu,i}, \text{SN}_i\}$, where $i = 1, \dots, 4$ corresponds to the four redshift bins. In Fig. 11, we show the equivalent analysis for our EFTofDE configuration.

First, we evaluate the impact of the inclusion of the additional data points in the range $20 < \ell < 79$ for C_ℓ^{gg} (see Limber vs full). With the full integral analysis, we obtain $\Omega_m = 0.3063 \pm 0.0090$, $\sigma_8 = 0.750 \pm 0.025$, $\ln 10^{10} A_s = 2.908 \pm 0.098$ and $S_8 = 0.7576 \pm 0.022$. This respectively corresponds to an improvement in the constraining power of 43.8%, 3.8%, 21.6%, 5.5%, and a shift of 1.7σ , 0.4σ , 1.1σ , 0.6σ over the Limber approximation analysis. However, the inclusion of these additional data points does not lead to a noticeable improvement in the constraints on the DE parameters (see Fig. 11).

Second, we evaluate the impact of the inclusion of the C_ℓ^{Tg} data in the DESIcross likelihood, an important novelty compared to the **Mapar** likelihood of Ref. [149]. Although the inclusion of C_ℓ^{Tg} does not improve the constraints on the standard Λ CDM parameters (see Fig. 10), it does provide additional constraining power on the EFTofDE parameters c_B and c_M , as illustrated in Fig. 11.

²⁰ We note that the Limber prediction for the low- ℓ part of the galaxy auto-angular power spectrum is extrapolated from the small scale prediction (in dashed line in Fig. 8).

The constraints on c_B and c_M are improved by 55% and 21%, from $c_B = 1.2^{+2.0}_{-1.5}$ and $c_M = 1.4^{+2.3}_{-1.5}$ (without C_ℓ^{Tg}) to $c_B = 0.94 \pm 0.78$ and $c_M = 1.5^{+1.7}_{-1.3}$ (with C_ℓ^{Tg}). This is attributed to the improved sensitivity to the Weyl potential thanks to the temperature-galaxy cross-correlation. Concerning the constraints on the dark energy equation of state parameters w_0 and w_a , we do not get much improvement.

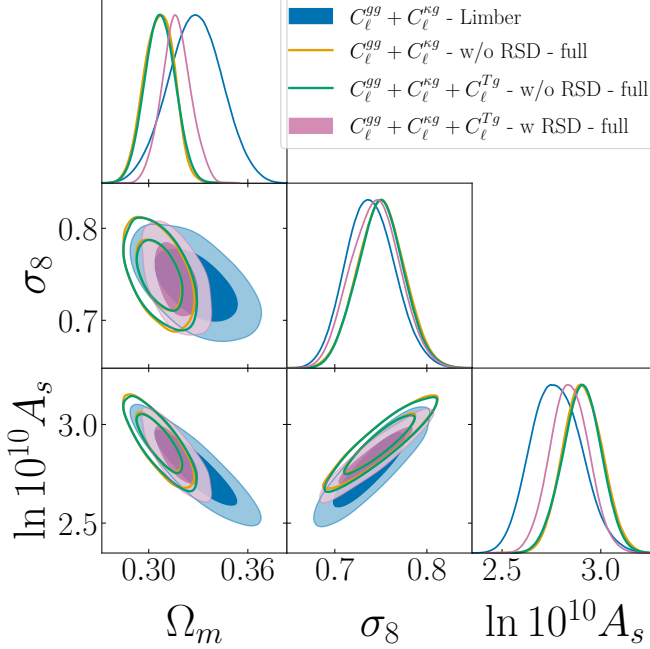


FIG. 10: 1D and 2D posterior distributions reconstructed from our DESIcross likelihood for the Λ CDM model and for several analysis settings. In particular, we evaluate the impact of (i) the full integral prediction compared to the Limber approximation, (ii) the RSD correction and (iii) the inclusion of the C_ℓ^{Tg} data.

Third, always in Fig. 10, we gauge the impact of adding the redshift-space distortion (RSD) contribution, which is significant only on the largest angular scales ($\ell \lesssim 79$ [149]). We observe a noticeable shift in the parameter posterior of 1.1σ , 0.2σ and 0.6σ for Ω_m , σ_8 and $\ln(10^{10}A_s)$. We then obtain as a final constraint $\Omega_m = 0.3167 \pm 0.0088$, $\sigma_8 = 0.744 \pm 0.026$, $\ln 10^{10}A_s = 2.842 \pm 0.097$ and $S_8 = 0.76 \pm 0.024$ for $C_\ell^{gg} + C_\ell^{kg} + C_\ell^{Tg}$ (with the full integral and the RSD contribution). For the DE parameters, we do not observe any significant shift and improvement, and we obtain as a final constraint $w_0 = -0.79 \pm 0.32$, $w_a = -1.46^{+0.61}_{-1.4}$, $c_B = 0.82 \pm 0.78$, and $c_M = 1.7 \pm 1.3$.

Appendix B: Supplementary analysis products

In this appendix, we report the cosmological constraints, the $\Delta\chi^2_{\min}$ with respect to Λ CDM, and the preference over Λ CDM, for the CPL model with PPF in Tab. I and for the CPL model with EFTofDE in Tab. II. In addition, in Tab. III, we display the best-fit χ^2 per experiment reconstructed from our All analysis for the Λ CDM model, the CPL model with PPF, and the CPL model with EFTofDE. Finally, in Fig. 12, we display the 1D and 2D posterior distributions reconstructed from our Baseline and All analyses for the CPL model with and without EFTofDE.

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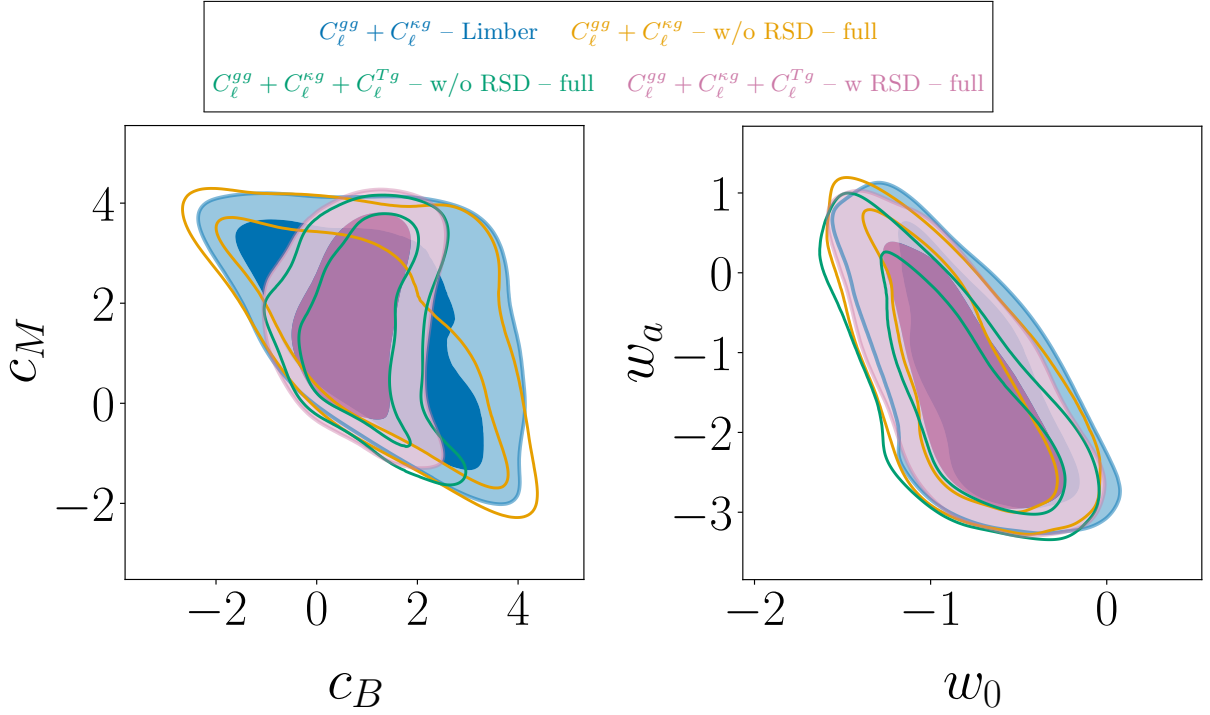


FIG. 11: 2D posterior distributions of $\{c_B, c_M\}$ and $\{w_0, w_a\}$ reconstructed from our DESIcross likelihood for our EFTofDE configuration and for several analysis settings. In particular, we evaluate the impact of (i) the `Swift` C_ℓ prediction compared to the Limber approximation, (ii) the RSD correction and (iii) the inclusion of the C_ℓ^{Tg} data.

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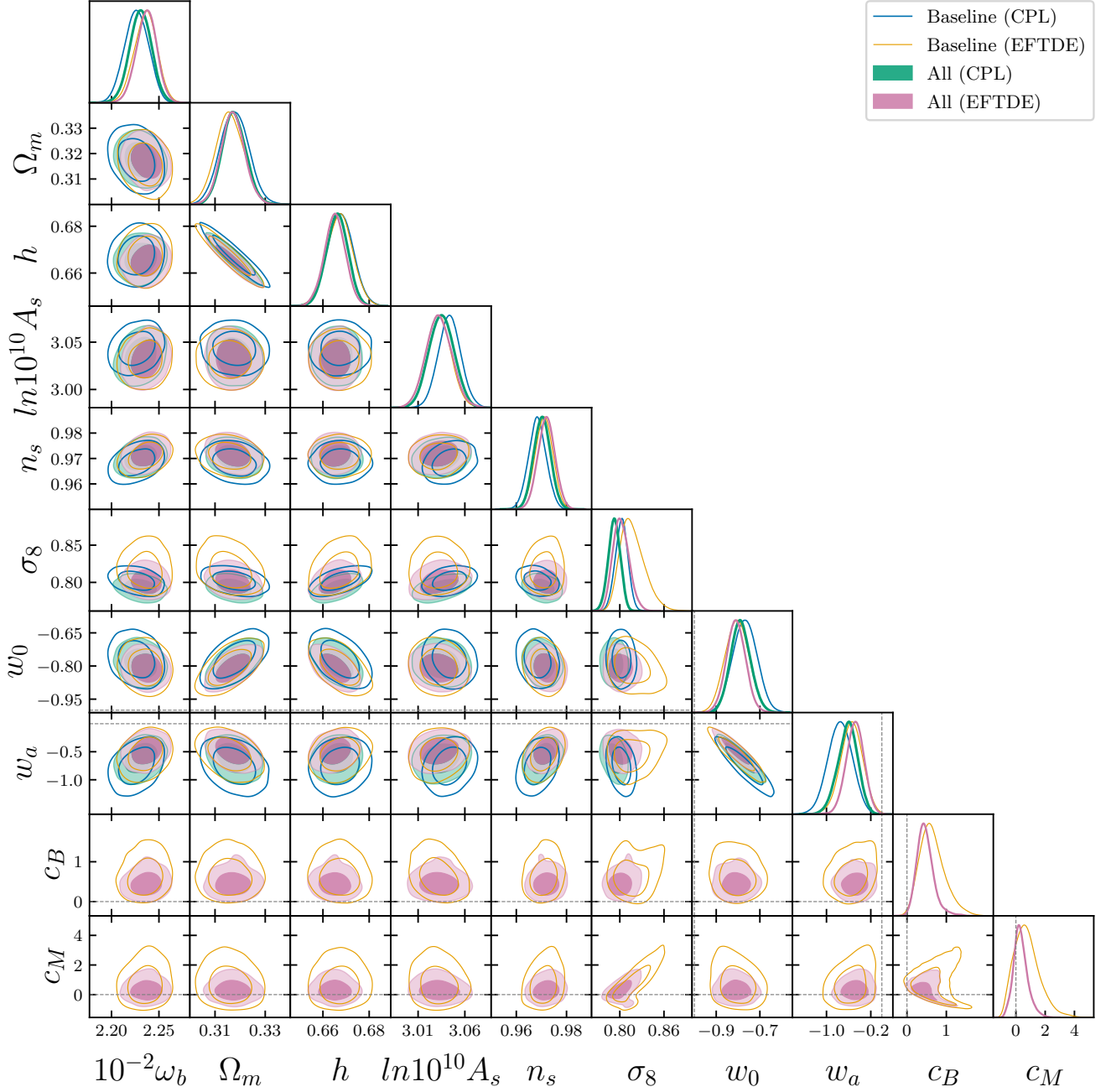


FIG. 12: 1D and 2D posterior distributions reconstructed from our Baseline and All analyses for the CPL model with and without EFTofDE.

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CPL	Baseline	Baseline + ISWL	Baseline + EFTBOSS	Baseline + DESICross	All	All w/o BAO and SN
$10^2 \omega_b$	2.233 2.226 ± 0.012	2.229 2.226 ± 0.012	2.226 $2.229^{+0.013}_{-0.012}$	2.228 2.230 ± 0.012	2.230 2.231 ± 0.011	2.230 $2.230^{+0.013}_{-0.011}$
Ω_m	0.3173 0.3178 ± 0.0056	0.3181 0.3176 ± 0.0055	0.3177 0.3183 ± 0.0049	0.3170 0.3172 ± 0.0055	0.3174 0.3176 ± 0.0047	0.3168 $0.3218^{+0.0080}_{-0.0091}$
h	0.6673 0.6675 ± 0.0056	0.6664 0.6676 ± 0.0055	0.6675 0.6661 ± 0.0047	0.6671 0.6670 ± 0.0055	0.6666 0.6661 ± 0.0046	0.6680 0.6621 ± 0.0084
$\ln 10^{10} A_s$	3.046 3.044 ± 0.012	3.044 3.043 ± 0.012	3.044 3.042 ± 0.012	3.041 $3.039^{+0.013}_{-0.014}$	3.042 3.035 ± 0.013	3.039 3.035 ± 0.013
n_s	0.9692 0.9684 ± 0.0035	0.9693 0.9684 ± 0.0035	0.9700 0.9693 ± 0.0036	0.9712 $0.9698^{+0.0038}_{-0.0033}$	0.9707 0.9703 ± 0.0033	0.9697 0.9698 ± 0.0037
σ_8	0.8003 0.8023 ± 0.0085	0.7993 0.8022 ± 0.0083	0.8033 0.7975 ± 0.0077	0.7981 0.7964 ± 0.0090	0.7971 0.7925 ± 0.0080	0.7993 0.789 ± 0.010
w_0	-0.788 -0.769 ± 0.056	-0.779 -0.768 ± 0.056	-0.778 -0.780 ± 0.049	-0.784 -0.780 ± 0.057	-0.778 -0.788 ± 0.046	-0.774 $-0.777^{+0.078}_{-0.10}$
w_a	-0.65 -0.75 ± 0.22	-0.68 $-0.76^{+0.23}_{-0.21}$	-0.72 $-0.67^{+0.21}_{-0.18}$	-0.66 $-0.67^{+0.23}_{-0.20}$	-0.67 $-0.62^{+0.19}_{-0.16}$	-0.72 $-0.62^{+0.37}_{-0.27}$
$\chi^2_{\min}$	32218.2	32220.2	32863.6	32247.6	32895.2	31249.6
$\Delta\chi^2_{\min}$	-17.8	-18.0	-25.4	-17.0	-24.6	-7.6
n- σ	3.8	3.8	4.7	3.7	4.6	2.3

TABLE I: best-fit values (first row) and Mean $\pm 1\sigma$ (second row) of the reconstructed parameters for the CPL model with PPF. We also display the total best-fit $\chi^2_{\min}$, the $\Delta\chi^2_{\min}$ with respect to Λ CDM, and the preference over Λ CDM.

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EFTofDE	Baseline	Baseline + ISWL	Baseline + EFTBOSS	Baseline + DESICross	All	All w/o BAO and SN
$10^2 \omega_b$	2.234 2.236 ± 0.012	2.235 2.235 ± 0.012	2.233 $2.236^{+0.013}_{-0.011}$	2.236 2.237 ± 0.011	2.236 $2.237^{+0.011}_{-0.010}$	2.236 2.235 ± 0.012
Ω_m	0.3165 0.3159 ± 0.0056	0.3166 0.3158 ± 0.0056	0.3171 0.3173 ± 0.0047	0.3167 0.3159 ± 0.0054	0.3162 0.3172 ± 0.0046	0.3173 0.3234 ± 0.0091
h	0.6675 0.6673 ± 0.0056	0.6671 0.6674 ± 0.0056	0.6672 0.6655 ± 0.0047	0.6666 0.6670 ± 0.0055	0.6673 0.6652 ± 0.0045	0.6661 0.6591 ± 0.0089
$\ln 10^{10} A_s$	3.037 3.032 ± 0.013	3.038 3.036 ± 0.013	3.037 3.029 ± 0.013	3.037 3.032 ± 0.014	3.040 3.033 ± 0.014	3.039 3.033 ± 0.014
n_s	0.9706 0.9710 ± 0.0035	0.9711 0.9710 ± 0.0035	0.9703 0.9710 ± 0.0035	0.9715 0.9715 ± 0.0033	0.9720 0.9720 ± 0.0034	0.9725 0.9716 ± 0.0036
σ_8	0.805 $0.817^{+0.013}_{-0.019}$	0.807 $0.821^{+0.014}_{-0.016}$	0.804 $0.8033^{+0.0087}_{-0.011}$	0.801 $0.812^{+0.011}_{-0.017}$	0.805 $0.8016^{+0.0097}_{-0.011}$	0.803 0.796 ± 0.013
w_0	-0.788 -0.805 ± 0.054	-0.792 -0.809 ± 0.052	-0.789 -0.801 ± 0.045	-0.792 -0.798 ± 0.054	-0.798 -0.809 ± 0.043	-0.805 $-0.820^{+0.078}_{-0.088}$
w_a	-0.64 -0.55 ± 0.20	-0.61 -0.53 ± 0.18	-0.64 -0.53 ± 0.17	-0.59 $-0.56^{+0.21}_{-0.18}$	-0.58 -0.49 ± 0.16	-0.54 -0.38 ± 0.28
c_B	0.36 $0.65^{+0.24}_{-0.37}$	0.20 $0.45^{+0.16}_{-0.30}$	0.60 $0.67^{+0.23}_{-0.37}$	0.52 $0.81^{+0.28}_{-0.41}$	0.27 $0.46^{+0.16}_{-0.22}$	0.28 $0.50^{+0.18}_{-0.26}$
c_M	0.10 $0.75^{+0.63}_{-1.0}$	0.26 $0.98^{+0.56}_{-0.82}$	-0.33 $0.16^{+0.41}_{-0.60}$	-0.03 $0.38^{+0.53}_{-0.83}$	0.18 $0.31^{+0.39}_{-0.49}$	0.19 $0.35^{+0.39}_{-0.54}$
$\chi^2_{\min}$	32216.4	32218.8	32862.0	32245.2	32893.6	31247.8
$\Delta\chi^2_{\min}$	-19.6	-19.4	-27.0	-19.4	-26.2	-9.4
n- σ	3.4	3.4	4.3	3.4	4.2	1.9

TABLE II: best-fit values (first row) and Mean $\pm 1\sigma$ (second row) of the reconstructed parameters of the CPL model with EFTofDE. We also display the total best-fit $\chi^2_{\min}$, the $\Delta\chi^2_{\min}$ with respect to Λ CDM, and the preference over Λ CDM.

$\chi^2_{\min}$	high- ℓ TTTEEE	low- ℓ TT	low- ℓ EE	Lensing	ISWL	DESICross	EFTBOSS	DESI BAO	DES Y5	Total
Λ CDM	30510.69	21.77	32.97	4.87	2.08	31.90	653.00	12.77	1649.18	32919.8
CPL	30509.83	22.02	32.83	4.16	1.92	32.30	645.86	8.01	1637.65	32895.2
EFTofDE	30508.91	22.08	32.87	3.54	2.08	32.38	645.57	8.00	1637.61	32893.6

TABLE III: best-fit χ^2 of the various likelihoods included in our full data combination analysis (*i.e.*, the “All” analysis) for Λ CDM, CPL with PPF and CPL with EFTofDE.

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