Continuum limit of gauged tensor network states

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It is well known that all physically relevant states of gauge theories lie in the sectors of the Hilbert space which satisfy the Gauss law. On the lattice, the manifeslty gauge invariant subspace is known to be exactly spanned by gauged tensor networks. In this work, we demonstrate that the continuum limit of certain types of gauged tensor networks is well defined and leads to a new class of states that may be helpful for the non-perturbative study of gauge theories directly in the continuum.

I. INTRODUCTION

Gauge theories form one of the backbones of modern physics. Most notably, in the standard model they are responsible for the fundamental interactions between matter [1] and more recently, it has been demonstrated that they play a key role in describing higher form symmetries [2, 3] as well as dualities on the lattice [4, 5] and in the continuum [6, 7]. Despite their huge importance, gauge theories are notoriously hard to study due to their rich phase diagram that contains many phases that cannot be understood perturbatively [8, 9].

Indeed, the gauge invariant objects that constitute relevant states are non-local Wilson loops and lines that are not well described within perturbation theory when the coupling is large [10]. To better understand strongly interacting gauge theories it is therefore desirable to have a framework that contains only these gauge invariant objects from the start, on the lattice it has been demonstrated that gauged projected entangled pair states (gPEPS) [11–13] provide such a framework.

For one discrete spatial dimension, pure-matter tensor networks exist in the form of matrix product states (MPS) [14], their continuumlimit leads to continuous matrix product states (CMPS) [15, 16] that have proven to be a valuable numerical ansatz [17–24]. In higher dimensions MPS generalize to projected entangled pair states (PEPS) [25], their continuumlimit are continuous projected entangled pair states (CPEPS) [26–28] that have also been used as a numerical ansatz [29].

In this work, we take the continuum limit of gauged projected entangled pair states which leads to gauged continuous projected entangled pair states (gCPEPS) that are the natural generalization of pure matter continuous tensor networks. In one spatial dimension we additionally demonstrate that these states reduce to a gauge invariant generalization of continuous matrix product states [15]. Note that the existence of this continuum-limit is particularly relevant in light of the recent proofs that gauged PEPS and MPS are the most general gauge invariant states on the lattice [30, 31].

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The outline of the paper is as follows. In section 2 we start by defining gauged continuous tensor networks, demonstrate their gauge invariance explicitly and demonstrate their relation to the continuous projected entangled pair states from [26–28]. In section 3 we focus on the one-dimensional scenario and introduce gauge invariant CMPS. Finally, in section 4 we demonstrate that gauged continuous PEPS are really the continuum limit of sequences of discrete gauged tensor networks. To do this we start in subsection 4.1 with a brief review of the relevant pure matter tensor networks on the lattice. In subsection 4.2 we demonstrate how to gauge those and finally in subsection 4.3 we take the continuum limit.

II. GAUGED CONTINUOUS TENSOR NETWORK STATES

Given a gauge group G with irrep labels j and color labels α, we wish to construct manifestly gauge invariant states on a spatial manifold $\mathcal{M}$ with boundary $\partial\mathcal{M}$. Here we focus on gauge theories with matter fields that are complex scalars $\hat{\phi}_\alpha(x)$ in the fundamental representation $\bar{j}_f$ of G and gauge fields $\hat{A}^a_\mu(x)$ that are valued in the lie-algebra of G. Apart from these, our construction depends on a set of auxiliary complex scalar fields $\hat{\chi}^j_\alpha(x)$. As usual, each of these fields has a canonically conjugate momentum, in particular for the gauge fields we denote $\hat{\Pi}_{A^a_\mu}(x) = \hat{E}^a_\mu(x)$ so that $[\hat{A}^a_\mu(x), \hat{E}^b_\nu(y)] = i\delta(x-y)\delta^{ab}\delta_{\mu\nu}$. Finally, we define a basis of *field eigenstates* $|\phi\rangle$ so that

$$\hat{\phi}_\alpha(x)\,|\phi\rangle = \phi_\alpha(x)\,|\phi\rangle \tag{1}$$

and

$$\hat{\phi}^\dagger_\alpha(x)\,|\phi\rangle = \bar{\phi}_\alpha(x)\,|\phi\rangle \tag{2}$$

and the *electric singlet state* $|s_E\rangle$ so that $\hat{E}^a_\mu(x)\,|s_E\rangle = 0$.

Using all the above definitions we now introduce the variational parameters of the state in the form of two bulk functionals $V[\chi, \partial_\mu\chi, \bar{\chi}, \partial_\mu\bar{\chi}]$, $J[\chi, \partial_\mu\chi, \bar{\chi}, \partial_\mu\bar{\chi}, \phi, \bar{\phi}]$ and one boundary functional $B[\chi_{\partial\mathcal{M}}, \partial_\mu\chi_{\partial\mathcal{M}}, \bar{\chi}_{\partial\mathcal{M}}, \partial_\mu\bar{\chi}_{\partial\mathcal{M}}]$ that depend on the fields, their derivatives as well as the complex conjugates thereof. Using those, the gauge invariant CPEPS is defined as:

$$|\psi_{B,V,J}\rangle = \int \mathcal{D}^2\phi\, \mathcal{D}^2\chi\, B[\chi_{\partial M}] e^{-\int_{\mathcal{M}} d^D x \hat{\mathcal{L}}[\hat{A}^a_\mu, \chi, \phi]}\, |\phi\rangle\, |s_E\rangle \tag{3}$$

where $\mathcal{D}^2\phi = \mathcal{D}\phi\mathcal{D}\bar{\phi}$ and similar for the auxiliary fields. The generating Lagrangian is defined as:

$$\hat{\mathcal{L}}[\hat{A}^a_\mu, \chi, \phi] = \left(\hat{D}_\mu \chi^j_\alpha(x)\right)^\dagger \hat{D}^\mu \chi^j_\alpha(x) + V[\chi] + J[\chi, \phi] \tag{4}$$

and contains the covariant derivative $\hat{D}_\mu$ that is defined as:

$$\hat{D}_\mu \chi^j_\alpha(x) = \partial_\mu \chi^j_\alpha(x) - ig\hat{A}^a_\mu(x)\ (T^{j\,a})_{\alpha\beta}\ \chi^j_\beta(x) \tag{5}$$

where g is the dimensionful coupling constant of the relevant gauge theory and $T^{j\,a}$ are the generators of the group acting on the irrep labeled by j. As we will soon see, the above described state is gauge invariant if the generating Lagrangian $\hat{\mathcal{L}}[\hat{A}^a_\mu, \chi, \phi]$ is that of an interacting gauge theory for gauge group G.

As mentioned in [27], natural choices of the boundary functional are $B[\chi_{\partial\mathcal{M}}] \propto \delta(\chi_{\partial\mathcal{M}})$ or $B[\chi_{\partial\mathcal{M}}] \propto \delta(n_\mu \partial_\mu \chi_{\partial\mathcal{M}})$ with n_μ normal to $\partial\mathcal{M}$. Other, non-local, choices are also possible and discussed in [27].

A. Explicit verification of gauge invariance and Gauss law

Here we wish to explicitly check that $|\psi_{B,V,J}\rangle$ has a local symmetry

$$\hat{U}[g]\ |\psi_{B,V,J}\rangle = e^{i\int d^D x\, \theta^a(g(x))\hat{G}^a(x)}\ |\psi_{B,V,J}\rangle = |\psi_{B,V,J}\rangle \tag{6}$$

for all $g(x)$ and for all choices of the parameters B, V, J. The generators $\hat{G}^a(x)$ are the Gauss law operators, they are given by

$$\hat{G}^a(x) = \hat{Q}^a(x) - \frac{1}{g}\hat{D}_\mu \hat{E}^a_\mu(x) \tag{7}$$

where

$$\hat{Q}^a(x) = -i\,\hat{\pi}_\alpha(x)\,(T^{j_f\,a})_{\alpha\beta}\,\hat{\phi}_\beta(x) + h.c. \tag{8}$$

is the charge density of the scalar matter fields,

$$\hat{D}_\mu \hat{E}^a_\mu(x) = \partial_\mu \hat{E}^a_\mu(x) - ig\hat{A}^b_\mu(if^{abc})\hat{E}^c_\mu \tag{9}$$

the covariant derivative of the electric field and $\theta_a(g(x))$ the dimensionless coordinates *on the group manifold* of the group element $g(x)$. To get the covariant derivative for the electric field from 5 we used the fact that the electric field $E^a_\mu(x)$ is valued in the adjoint representation of G so that the relevant generators are

$$(T^b_{\text{adjoint}})_{ac} = if^{abc} \tag{10}$$

with f^{abc} the structure constants of the group. As is well known 6 is equivalent to stating that the state satisfies the Gauss law constraints:

$$\left(\hat{Q}^a(x) - \frac{1}{g}\hat{D}_\mu \hat{E}^a_\mu(x)\right)|\psi_{B,V,J}\rangle = 0 \qquad \forall\ a,\ x \tag{11}$$

for all choices of the parameters B, V and J.

To prove that gCPEPS has such a symmetry, let us first study the action of the symmetry on the physical fields $\hat{\phi}_\alpha(x)$ gauge fields $\hat{A}^a_\mu(x)$. First, it is easy to see that $\hat{\phi}^\alpha(x)$ transforms with respect to anti-fundamental representation of $\hat{U}[g]$ so that

$$\hat{U}[g]\,\hat{\phi}_\alpha(x)\,\hat{U}^\dagger[g] = D^{j_f}_{\alpha\beta}(g^{-1}(x))\,\hat{\phi}_\beta(x) \tag{12}$$

where $D^j_{\alpha\beta}(g(x)) = e^{i\theta_a(g(x))T^{j\,a}}|_{\alpha\beta}$ are the Wigner matrices for a representation j. For the coherent states $|\phi\rangle$ one easily derive the transformation rule

$$\hat{U}[g]\,|\phi\rangle = \prod_\alpha |D_{\alpha\beta}(g(x))\,\phi^\beta(x)\rangle \equiv |D[g]\,\phi\rangle \tag{13}$$

where we defined the shorthand notation $D[g]\,\phi$ for the transformed field.

Similarly, since $\hat{A}^a_\mu(x)$ lives in the Lie algebra (i.e. adjoint representation) of the group we find:

$$\hat{U}[g]\,\hat{A}^a_\mu(x)\,\hat{U}^\dagger[g] \tag{14}$$

$$= \hat{A}^a_\mu(x) + \frac{1}{g}\hat{D}_\mu \theta^a(g(x)) \tag{15}$$

$$= \hat{A}^a_\mu(x) + \frac{1}{g}\partial_\mu \theta^a(g(x)) + f^{abc}\hat{A}^b_\mu(x)\theta^c(g(x)) \tag{16}$$

where we used the fact that the group manifold coordinates $\theta^a(g(x))$ transform with respect to the adjoint representation as well. Finally, $|s_E\rangle$ has an eigenvalue zero for all $E^a_\mu(x)$ so that

$$\hat{U}[g]\,|s_E\rangle = |s_E\rangle \quad . \tag{17}$$

With all this, we get

$$\hat{U}[g]\,|\psi_{B,V,J}\rangle \tag{18}$$

$$= \int \mathcal{D}^2\phi\,\mathcal{D}^2\chi\,B[\chi_{\partial\mathcal{M}}]e^{-\int_{\mathcal{M}} d^D x\ \hat{\mathcal{L}}[\hat{\tilde{A}}^a_\mu, \chi, \phi]}\ |D[g]\phi\rangle\,|s_E\rangle$$

where $\hat{\tilde{A}}^a_\mu(x)$ is the right-hand side of 16. Finally, a change of integration variables into $\tilde{\phi}_\alpha(x) = D^{j_f}(g(x))_{\alpha\beta}\phi_\beta(x)$ and $\tilde{\chi}^j_\alpha(x) = D^j(g^{-1}(x))_{\alpha\beta}\chi^j_\beta(x)$ leads to:

$$\hat{U}[g]\,|\psi_{B,V,J}\rangle = \int \mathcal{D}^2\tilde{\phi}\,\mathcal{D}^2\tilde{\chi}\,B[D[g^{-1}]\chi_{\partial\mathcal{M}}]\exp\left\{-\int_{\mathcal{M}} d^Dx\ \hat{\mathcal{L}}[\tilde{\hat{A}}^a_\mu, D[g^{-1}]\tilde{\chi}, D[g^{-1}]\tilde{\phi}]\right\}|\tilde{\phi}\rangle\,|s_E\rangle$$

which is the original state if:

$$\hat{\mathcal{L}}[\hat{A}^a_\mu,\chi,\phi] = \hat{\mathcal{L}}\left[\hat{A}^a_\mu + \frac{1}{g}\hat{D}_\mu\theta^a[g]\,,\ D[g]\chi,\ D[g^{-1}]\phi\right] \tag{19}$$

and this is true if the generating Lagrangian of our tensor network is gauge invariant.

B. Connection to conventional continuous tensor networks

Clearly, when the coupling constant $g = 0$, the proposed state reduces to the conventional CPEPS introduced in [27]. Expanding this for small nonzero g gives the first order correction

$$|\psi_{B,V,J}\rangle = \left(1+\mathcal{O}(g^2)\right)\int \mathcal{D}^2\phi\,\mathcal{D}^2\chi\,B[\chi_{\partial\mathcal{M}}]\exp\left\{-ig\int_{\mathcal{M}} d^Dx\,\hat{A}^a_\mu(x)J^a_\mu(x)\right\}\exp\left\{-\int_{\mathcal{M}} d^Dx\ \mathcal{L}_0[\chi,\phi]\right\}|\phi\rangle\,|s_E\rangle \tag{20}$$

where $\mathcal{L}_0[\chi,\phi]$ is obtained from $\hat{\mathcal{L}}[\hat{A}^a_\mu,\chi,\phi]$ by replacing all covariant derivatives with conventional partial derivatives and

$$J^a_\mu(x) = -i\,\partial_\mu(\chi^j_\alpha(x))\,(T^{j\,a})_{\alpha\beta}\chi^j_\beta(x) \tag{21}$$

is the current of the virtual degrees of freedom. Upon closer inspection this first order correction is simply the creator of Wilson lines

$$\hat{W}^a(a\to_P b) = \exp\left(ig\int_P dl_\mu \hat{A}^a_\mu(l)\right) \tag{22}$$

along paths P along which there is a flux of virtual particles. The ignored higher order terms can be interpreted as a back-reaction from the physical gauge fields onto the virtual degrees of freedom.

III. ONE DIMENSIONAL SCENARIO: GAUGED CMPS

It is well known that one-dimensional tensor networks in one dimension greatly simplify due to the possibility to treat the auxiliary space in first quantization. Here we will see that gauged continuous matrix product states (gCMPS) are of the form:

$$|\psi\rangle = tr_{aux}\left(B\,\mathcal{P}\exp\int dx\left(F - ig\hat{A}^a_x(x)Q^a_{virt} + R^\phi_\alpha\,\hat{\phi}_\alpha(x) + R^{\bar{\phi}}_\alpha\,\hat{\phi}^\dagger_\alpha(x)\right)\right)|s_E\rangle\,|0_{fock}\rangle \tag{23}$$

where F, Q^a_{virt}, R^ϕ_α, $R^{\bar{\phi}}_\alpha$ and B are finite dimensional matrices acting on an auxiliary vector space that is charged with respect to the symmetry group G, Q^a_{virt} is the charge operator. To ensure gauge symmetry, these matrices are block-diagonal with respect to the irreps of the gauge group G. In particular $F_{jj'}\propto\delta_{jj'}$, $R^\phi_{\alpha\,jj'}\propto\delta_{j\,j'+j_f}$ and $R^{\bar{\phi}}_{\alpha\,jj'}\propto\delta_{j\,j'-j_f}$ where j_f is the irrep of the matter fields. With this in mind one can already convince themselves that this state is gauge invariant due to the Wilson lines taht are created by the $-ig\hat{A}^a_x(x)Q^a_{virt}$ term. In what follows we will explicitly prove that this state is indeed the one-dimensional version of 3 and that it therefore indeed gauge invariant.

To start, we eliminate the integral over physical field eigenstates by recalling

$$\langle\phi|0_{fock}\rangle = e^{\int dx\,\bar{\phi}_\alpha(x)\phi_\alpha(x)} \tag{24}$$

and

$$\mathbb{1} = \int \mathcal{D}^2\phi\,|\phi\rangle\,\langle\phi| \tag{25}$$

where $|0_{fock}\rangle$ is the Fock vacuum. With this we find

$$|\psi_{B,V,J}\rangle = \int \mathcal{D}^2\chi\, B[\chi_{\partial M}]\, \exp\left\{ - \int_{\mathcal{M}} d^D x\, \left(\hat{\mathcal{L}}[\hat{A}^a_\mu(x), \chi, \hat{\phi}(x)] + \hat{\phi}^\dagger(x)\hat{\phi}(x) \right) \right\} |s_E\rangle\, |0_{fock}\rangle \tag{26}$$

and to simplify the notation we will ignore the $\hat{\phi}^\dagger(x)\hat{\phi}(x)$ term since it be adsorbed into a suitable redefinition of $\hat{\mathcal{L}}$.

To get rid of the remaining integral over virtual field configurations we discretize space into points x_i with spacing a and define an auxiliary Hilbert space with thereto associated operators $\hat{\chi}^j_\alpha$, $\hat{\chi}^{\dagger\, j}_\alpha$ and respective canonical conjugates $\hat{\pi}^j_\alpha$, $\hat{\pi}^{\dagger\, j}_\alpha$. On this space, we define transfer matrices $\hat{T}_i = e^{-a\hat{H}_i}$ so that:

$$\langle \chi_{i+1}|\, \hat{T}_i\, |\chi_i\rangle = e^{-a\hat{\mathcal{L}}\left[\hat{A}^a_\mu(x_i), \chi_i, \frac{\chi_{i+1}-\chi_i}{a}, \hat{\phi}(x_i)\right]} \tag{27}$$

where $|\chi\rangle$ are the mutual eigenstates of $\hat{\chi}^j_\alpha$ and $\hat{\chi}^{\dagger\, j}_\alpha$. Using these transfer matrices we can write 26 with boundary conditions $B[\chi_{\partial\mathcal{M}}] = \delta^2(\chi^j_\alpha(0) - \chi_0)\, \delta^2(\chi^j_\alpha(L) - \chi_L)$ as:

$$|\psi_{B,V,J}\rangle = tr_{aux}\left(|\chi_L\rangle \langle\chi_0|\, \mathcal{P} \exp \int dx \hat{H}(x) \right) |s_E\rangle\, |0_{fock}\rangle \tag{28}$$

which is already very close to the desired form. Let us now assume that the potential and sources of the generating Lagrangian depend only on χ and $\bar{\chi}$ but not on their derivatives. In this case one can check that

$$\begin{aligned}\hat{H}_i = \hat{\pi}^j_\alpha \hat{\pi}^{\dagger\, j}_\alpha - ig\hat{A}^a_x(x_i)(-i)\left(\hat{\pi}^j_\alpha (T^{j\, a})_{\alpha\beta}\hat{\chi}^j_\beta - h.c.\right) \\ + V(\hat{\chi}^j_\alpha, \hat{\chi}^{\dagger\, j}_\alpha) + J(\hat{\chi}^j_\alpha, \hat{\chi}^{\dagger\, j}_\alpha, \hat{\phi}_\alpha, \hat{\phi}^\dagger_\alpha)\end{aligned} \tag{29}$$

satisfies 27 by explicitly computing the left hand side through the introduction of complete sets of $|\pi\rangle$ eigenstates and performing the resulting Gaussian integrals. In fact, this remains true for potentials and sources depending on the derivatives of the auxiliary fields as long as they appear in a Weyl ordered fashion[PS]. Note that the term proportional to g^2 is not valid in this formulation.

At this point the corresponding state is already of the desired form with e.g. $F = \hat{\pi}^j_\alpha \hat{\pi}^{\dagger\, j}_\alpha + V(\hat{\chi}^j_\alpha, \hat{\chi}^{\dagger\, j}_\alpha, \pi, \pi^\dagger)$ represented as infinite dimensional matrices. However, when we introduce bosonic creation and annihilation operators $\hat{a}$, $\hat{a}^\dagger, \hat{b}$, $\hat{b}^\dagger$ so that:

$$\hat{\chi}^j_\alpha = \frac{1}{\sqrt{2}}\left(\hat{a}^j_\alpha + \hat{b}^{\dagger\, j}_\alpha\right) \tag{30}$$

$$\hat{\pi}^j_\alpha = \frac{-i}{\sqrt{2}}\left(\hat{a}^j_\alpha - \hat{b}^{\dagger\, j}_\alpha\right) \tag{31}$$

we get :

$$\begin{aligned}\hat{H}(x) = \frac{1}{2}\hat{a}^{\dagger\, j}_\alpha \hat{a}^j_\alpha + \frac{1}{2}\hat{b}^{\dagger\, j}_\alpha \hat{b}^j_\alpha \\ - ig\hat{A}^a_x(x)\left(\hat{a}^{\dagger\, j}_\alpha T^{j\, a}_{\alpha\beta}\hat{a}^j_\beta - \hat{b}^{\dagger\, j}_\alpha T^{j\, a}_{\alpha\beta}\hat{b}^j_\beta\right) \\ + \tilde{V}(a, a^\dagger, b, b^\dagger)\end{aligned} \tag{32}$$

where $\tilde{V} = V - \chi^j_\alpha \chi^{j\,\dagger}_\alpha$. Finally, focussing on the scenario where $\hat{H}(x)$ commutes with the total number density allows us to constrain the auxiliary Fock space to its single particle sector by picking suitable boundary conditions [32]. Upon denoting $v = (\{a^{\dagger\, j}_\alpha\}\{b^{\dagger\, j}_\alpha\})$ as a vector of with creation operators, this leads to the proposed form 23 with

$$v\, F\, v^\dagger = \frac{1}{2}\hat{a}^{\dagger\, j}_\alpha \hat{a}^j_\alpha + \frac{1}{2}\hat{b}^{\dagger j}_\alpha \hat{b}^j_\alpha + \tilde{V}(a, a^\dagger, b, b^\dagger) \tag{33}$$

$$v\, Q^a_{virt}\, v^\dagger = \hat{a}^{\dagger\, j}_\alpha T^{j\, a}_{\alpha\beta}\hat{a}^j_\beta - \hat{b}^{\dagger\, j}_\alpha T^{j\, a}_{\alpha\beta}\hat{b}^j_\beta \tag{34}$$

$$v\, R^\phi_\alpha\, v^\dagger = \text{the terms in } J(\hat{\chi}^j_\alpha, \hat{\chi}^{\dagger\, j}_\alpha, \hat{\phi}_\alpha, \hat{\phi}^\dagger_\alpha) \text{ that are proportional to } \hat{\phi}_\alpha \tag{35}$$

$$v\, R^{\bar{\phi}}_\alpha\, v^\dagger = \text{the terms in } J(\hat{\chi}^j_\alpha, \hat{\chi}^{\dagger\, j}_\alpha, \hat{\phi}_\alpha, \hat{\phi}^\dagger_\alpha) \text{ that are proportional to } \hat{\phi}^\dagger_\alpha \tag{36}$$

which concludes the proof.

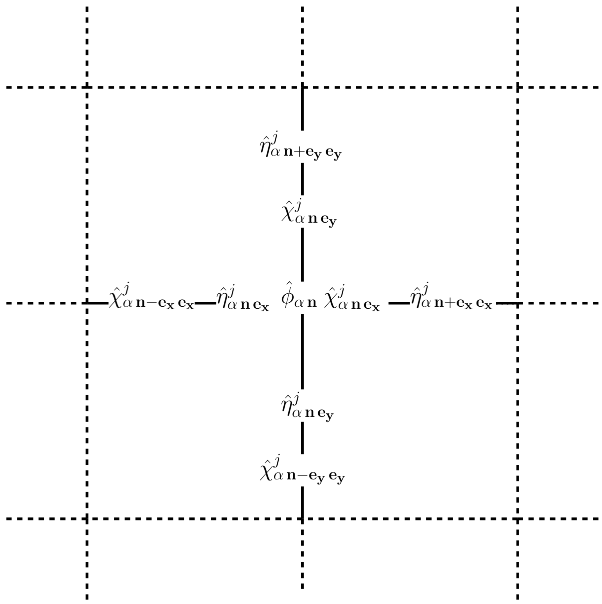
IV. GAUGED CPEPS AS CONTINUUM LIMITS OF GAUGED PEPS

Let us now demonstrate that gauged CPEPS are the continuum limit of suitable sequences of gauged PEPS. This is particular relevant in light of the current proof that gauged PEPS are the most general gauge invariant states on the lattice [30, 31]. Indeed, with this in mind it seems to be so that gauged CPEPS are the most general gauge invariant states in the continuum. To achieve this goal we will first set the stage be reviewing the usual construction in a language that suits our needs.

First we introduce PEPS with global symmetries in second quantization rather than the usual first quantized approach. Second, we will review the gauging procedure in second quantization and write down the resulting PEPS with local gauge symmetry. Finally, we take the continuum limit to recover the above proposed gauged CPEPS.

A. PEPS with a global symmetry

It is well known that the definition of PEPS requires the introduction of some auxiliary Hilbert spaces on the bonds of the lattice [25]. Since we eventually plan on taking a continuum limit, the dimension of each of these spaces ought to be infinite. The reason behind this is explained in detail in [27], a less rigorous argument is that finite dimensionful correlation lengths



$\chi_{corr}[m] = \chi_{latt}a[m]$ imply infinite dimensionless correlation as the lattice spacing a approaches that of the continuum. Therefore, the underlying PEPS should have infinite bond dimension as it approaches the continuum PEPS. With this in mind, it is natural to consider them to be associated to virtual fields in second quantization rather than Hilbert spaces in first quantization. Furthermore, since we will want to PEPS to have a global symmetry with respect to the group G, it is natural that these virtual fields carry irrep labels j and color labels α.

With all this in mind we define operators $\hat{\chi}^j_{\alpha\,\mathbf{n}\,\mathbf{e_i}}$ and $\hat{\pi}^j_{\chi\ \alpha\,\mathbf{n}\,\mathbf{e_i}}$ for each position $\mathbf{n}$ that live on the links pointing away from $\mathbf{n}$ in the positive $\mathbf{e_i}$ direction. Similarly, we define $\hat{\eta}^j_{\alpha\,\mathbf{n}\,\mathbf{e_i}}$ and $\hat{\pi}^j_{\eta\ \alpha\,\mathbf{n}\,\mathbf{e_i}}$ for the links pointing towards negative $\mathbf{e_i}$ direction. Finally, we define the discrete matter field $\hat{\phi}_{\alpha\,\mathbf{n}}$ and its canonically conjugate momentum $\hat{\pi}_{\phi\ \alpha\,\mathbf{n}}$. As before we introduce bases $|\phi_{\mathbf{n}}\rangle = \prod_\alpha |\phi^\alpha_{\mathbf{n}}\rangle$, $|\chi_{\mathbf{n}\,\mathbf{e_i}}\rangle = \prod_{j\ \alpha} |\chi^{j\ \alpha}_{\mathbf{n}\ \mathbf{e_i}}\rangle$ and $|\eta_{\mathbf{n}\,\mathbf{e_i}}\rangle = \prod_{j\ \alpha} |\eta^{j\ \alpha}_{\mathbf{n}\ \mathbf{e_i}}\rangle$ of eigenvectors for the respective field operators. For clarity, we depict the various fields operators and their locations on the lattice below.

On these spaces, we define the group elements

$$\hat{\theta}(g)_{\mathbf{n}} = e^{i\phi_a(g)\hat{Q}^a_{\phi\,\mathbf{n}}} \tag{37}$$

where

$$Q^a_{\phi\,\mathbf{n}} = -i\left(\hat{\pi}_{\phi\ \alpha\,\mathbf{n}}\, T^a_{\alpha\beta}\, \hat{\phi}^\beta_{\beta\,\mathbf{n}\,\mathbf{e_i}} + h.c.\right) \quad , \tag{38}$$

the action of these group elements on the field operators and their eigenstates is similar to what we discussed before i.e. anti-fundamental.

Now, define the PEPS

$$|\psi\rangle = \otimes_{\mathbf{n}\,\mathbf{e_i}} \langle L_{\mathbf{n}\,\mathbf{e_i}}| \otimes_{\mathbf{n}} |A_{\mathbf{n}}\rangle \tag{39}$$

from the on site tensors

$$|A_{\mathbf{n}}\rangle = \int \mathcal{D}^2\phi_{\mathbf{n}}\mathcal{D}^2\eta_{\mathbf{n}}\mathcal{D}^2\chi_{\mathbf{n}}\ A(\phi_{\mathbf{n}},\eta_{\mathbf{n}},\chi_{\mathbf{n}})\,|\phi_{\mathbf{n}}\rangle\,|\eta_{\mathbf{n}}\rangle\,|\chi_{\mathbf{n}}\rangle \quad , \tag{40}$$

and bond tensors

$$|L_{\mathbf{n}\ \mathbf{e_i}}\rangle = \int \mathcal{D}^2\chi\mathcal{D}^2\eta\ \delta^2(\chi_{\mathbf{n}\ \mathbf{e_i}} - \bar{\eta}_{\mathbf{n+e_i}\ \mathbf{-e_i}}) \quad . \tag{41}$$

With these, the contracted PEPS is:

$$|\psi\rangle = \int \mathcal{D}^2\phi\mathcal{D}^2\chi \prod_{\mathbf{n}} A(\phi_{\mathbf{n}}, \bar{\chi}_{\mathbf{n-e_i}\,\mathbf{e_i}}, \chi_{\mathbf{n}\,\mathbf{e_i}})\,|\phi_{\mathbf{n}}\rangle \tag{42}$$

and it easy to see that this is symmetric with respect to the action of the the global symmetry $\hat{U}(g) = \otimes_{\mathbf{n}}\hat{\theta}(g)_{\mathbf{n}}$ if

$$A\left(\phi_{\mathbf{n}},\eta_{\mathbf{n}},\chi_{\mathbf{n}}\right) = A(D(g^{-1})\phi_{\mathbf{n}}, \eta_{\mathbf{n}}D(g^{-1}), D(g)\chi_{\mathbf{n}}) \quad , \tag{43}$$

indeed, to see this one simply has to do a change of integration variables as before.

Note that 43 also implies that

$$\theta(g)_{\mathbf{n}} \prod_{\mathbf{e_i}} \theta_\eta(g)_{\mathbf{ne_i}}\,\theta_\chi(g)_{\mathbf{ne_i}}\,|A_{\mathbf{n}}\rangle = |A_{\mathbf{n}}\rangle \tag{44}$$

and similarly, due to their definition, the bond states satisfy

$$\theta_\chi(g)_{\mathbf{n}\,\mathbf{e_i}}\theta_\eta(g)_{\mathbf{n+e_i}\,\mathbf{e_i}}\,|L_{\mathbf{n}\,\mathbf{e_i}}\rangle = |L_{\mathbf{n}\,\mathbf{e_i}}\rangle \tag{45}$$

where we defined the group elements acting on the virtual spaces as

$$\hat{\theta}_\eta(g)_{\mathbf{n}\,\mathbf{e_i}} = e^{i\phi_a(g)\hat{Q}^a_{\eta\,\mathbf{n}\,\mathbf{e_i}}} \tag{46}$$

$$\hat{\theta}_\chi(g)_{\mathbf{n}\,\mathbf{e_i}} = e^{-i\phi_a(g)\hat{Q}^a_{\chi\,\mathbf{n}\,\mathbf{e_i}}} \tag{47}$$

where

$$\hat{Q}^a_{\eta\,\mathbf{n}\,\mathbf{e_i}} = -i\left(\hat{\pi}^j_{\eta\ \alpha\,\mathbf{ne_i}} T^{j\ a}_{\beta\alpha}\hat{\eta}^j_{\beta\,\mathbf{ne_i}} + h.c.\right) \tag{48}$$

$$\hat{Q}^a_{\chi\,\mathbf{n}\,\mathbf{e_i}} = -i\left(\hat{\pi}^j_{\chi\ \alpha\mathbf{n}\,\mathbf{e_i}} T^{j\ a}_{\alpha\beta}\hat{\chi}^j_{\beta\,\mathbf{ne_i}} + h.c.\right) \tag{49}$$

so that

$$\hat{\theta}_\eta(g)_{\mathbf{ne_i}}\hat{\eta}^j_{\alpha\mathbf{ne_i}}\hat{\theta}^\dagger_\eta(g)_{\mathbf{ne_i}} = \eta^j_{\beta\mathbf{ne_i}}D(g^{-1})_{\beta\alpha} \tag{50}$$

and

$$\hat{\theta}_\chi(g)_{\mathbf{ne_i}}\hat{\chi}^j_{\alpha\mathbf{ne_i}}\hat{\theta}^\dagger_\chi(g)_{\mathbf{ne_i}} = D(g)_{\alpha\beta}\chi^j_{\beta\mathbf{ne_i}} \quad . \tag{51}$$

B. From global symmetry to local symmetry

One can lift this global symmetry to a local one by minimally coupling the PEPS to gauge degrees of freedom that reside on the links of the lattice. To start, we define bosonic operators $\hat{L}^a_{\mathbf{ne_i}}$ and $\hat{R}^a_{\mathbf{ne_i}}$ that act on the link pointing away from site $\mathbf{n}$ in the $\mathbf{e_i}$ direction. These commute across different links and on the same link they satisfy

$$[\hat{L}^a_{\mathbf{ne_i}}, \hat{L}^b_{\mathbf{ne_i}}] = -if^{abc}L^c_{\mathbf{ne_i}} \tag{52}$$
$$[\hat{R}^a_{\mathbf{ne_i}}, \hat{R}^b_{\mathbf{ne_i}}] = \quad if^{abc}R^c_{\mathbf{ne_i}} \quad . \tag{53}$$

Using these, we define left and right group elements

$$\hat{\theta}_L(g)_{\mathbf{ne_i}} = e^{-i\phi_a(g)\hat{L}^a_{\mathbf{ne_i}}} \tag{54}$$
$$\theta_R(g)_{\mathbf{ne_i}} = e^{+i\phi_a(g)\hat{R}^a_{\mathbf{ne_i}}} \tag{55}$$

and the basis $|g_{\mathbf{ne_i}}\rangle\ \forall g \in G$ so that

$$\hat{\theta}_L(g_L)_{\mathbf{ne_i}}\hat{\theta}_R(g_R)_{\mathbf{ne_i}} |g\rangle_{\mathbf{ne_i}} = |g_L g g_R^{-1}\rangle_{\mathbf{ne_i}} \quad . \tag{56}$$

Another useful basis is the electric field basis

$$|jmn\rangle_{\mathbf{ne_i}} = \int dg \sqrt{\frac{|G|}{dim(j)}} D^j_{mn}(g)\, |g\rangle_{\mathbf{ne_i}} \tag{57}$$

where

$$\hat{\theta}_L(g_L)\hat{\theta}_R(g_R)\, |jnm\rangle_{\mathbf{ne_i}} = D^j_{mm'}(g_L^{-1})\, |jm'n'\rangle_{\mathbf{ne_i}}\, D^j_{n'n}(g_R) \quad . \tag{58}$$

The singlet state $|j=0\rangle$ corresponds to the zero electric field ket and will be denoted as $|s_E\rangle$. An in depth review of the relevant Hilbert space can be found in [33].

Using these bases we are now ready to define

$$\hat{U}_{\mathrm{Gauge}\,\mathbf{ne_i}} = \int dg\; e^{i\theta_a(g)\hat{Q}^a_{\eta\mathbf{ne_i}}}\, |g\rangle_{\mathbf{ne_i}} \langle g|_{\mathbf{ne_i}} \tag{59}$$

which is a controlled raising operator for the electrical field on the link $\mathbf{n\,e_i}$. Note, we can also interpret this operator as a controlled left group action on the η variables, this will be helpful later. Crucially, using change of integration variables, one can show that this operator satisfies:

$$\begin{aligned}\theta_L(g)_{\mathbf{n\,e_i}}\, U_{\mathrm{Gauge}\,\mathbf{ne_i}}\, |s_E\rangle &= \theta_\eta(g^{-1})_{\mathbf{ne_i}}\, U_{\mathrm{Gauge}\,\mathbf{ne_i}}\, |s_E\rangle \\ \theta_R(g)_{\mathbf{n\,e_i}}\, U_{\mathrm{Gauge}\,\mathbf{ne_i}}\, |s_E\rangle &= U_{\mathrm{Gauge}\,\mathbf{ne_i}}\, \theta_\eta(g)_{\mathbf{ne_i}}\, |s_E\rangle\end{aligned} \tag{60}$$

and with that knowledge one can check that the gauged PEPS

$$|\psi_{\mathrm{gauged}}\rangle = \otimes_{\mathbf{n\,e_i}} \langle L_{\mathbf{n\,e_i}}| \otimes_{\mathbf{n\,e_i}} U_{\mathrm{gauge}\,\mathbf{ne_i}} \otimes_{\mathbf{n}} |A_{\mathbf{n}}\rangle \tag{61}$$

has a local symmetry with respect to the action of $\theta(g)_{\mathbf{n}} \prod_{\mathbf{e_i}} \theta_L(g)_{\mathbf{ne_i}}\theta_R(g)_{\mathbf{n-e_i}}$. The full derivation can be found in [34], here we simply note that 60 together with 45 lift the local symmetry of the tensors 44 to a physical symmetry of the state.

Finally, explicitly contracting the gauged PEPS leads to :

$$|\psi_{\mathrm{gauged}}\rangle = \int \mathcal{D}^2\phi\mathcal{D}^2\chi dg \prod_{\mathbf{n}} A(\phi_{\mathbf{n}}, \bar{\chi}^j_{\alpha\ \mathbf{n-e_i\,e_i}} D(g^{-1}_{\mathbf{n-e_i\,e_i}})^j_{\alpha\beta}, \chi^j_{\beta\ \mathbf{n\,e_i}})\, |g\rangle_{\mathbf{n\,e_i}} |\phi_{\mathbf{n}}\rangle \tag{62}$$

where we used the fact that U_{Gauge} can be interpreted as a controlled rotation on the virtual degrees of freedom.

C. Taking the continuum limit

To take the continuum limit, let us first assume that

$$A(\phi,\chi,\eta) = e^{\mathcal{A}(\phi,\chi,\eta)} \tag{63}$$

so that the contracted state becomes:

$$|\psi_{\mathrm{gauged}}\rangle = \int \mathcal{D}^2\phi\mathcal{D}^2\chi dg e^{\sum_{\mathbf{n}} \mathcal{A}\left(\phi_{\mathbf{n}}, \bar{\chi}^j_{\alpha\ \mathbf{n-e_i\,e_i}} D(g^{-1}_{\mathbf{n-e_i\,e_i}})^j_{\alpha\beta}, \chi^j_{\beta\ \mathbf{n\,e_i}}\right)} \prod_{\mathbf{ne_i}} |g\rangle_{\mathbf{n\,e_i}} |\phi_{\mathbf{n}}\rangle \tag{64}$$

where without loss of generality we split $\mathcal{A}$ into its purely virtual part and the rest i.e.: $\mathcal{A}(\phi,\eta,\chi) = K(\eta,\chi) + V(\phi,\eta,\chi)$. Following [27] we assume:

$$K(\chi,\eta) = \frac{-Z_0}{2} \sum_{\mathbf{e_i}} \left(\bar{\chi}_{\mathbf{n\,e_i}} - \eta_{\mathbf{n\,e_i}}\right)\left(\chi_{\mathbf{n\,e_i}} - \bar{\eta}_{\mathbf{n\,e_i}}\right) \quad . \tag{65}$$

To go to the continuum, we must replace dimensionless lattice fields with dimensionful continuum fields. In particular, we define $\phi(x = \mathbf{n}a) = a^{[\phi(x)]}\phi_{\mathbf{n}}$ and $\chi(x = (\mathbf{n}+\mathbf{e_i}/2)a) = a^{[\chi(x)]}\chi_{\mathbf{n}\,\mathbf{e_i}}$ where $[\phi(x)] = [\chi(x)] = \frac{D-1}{2}$ for the scalar fields and $A_i^a(x = (\mathbf{n}+\mathbf{e_i}/2)a) = \frac{1}{ag_{QFT}}\phi^a(g_{\mathbf{n}\,\mathbf{e_i}})$ for the gauge manifold coordinates. With this we get:

$$\chi^j_{\alpha\ \mathbf{n}\,\mathbf{e_i}} - D(g_{\mathbf{n}-\mathbf{e_i}\,\mathbf{e_i}})_{\alpha\beta}\chi^j_{\beta\ \mathbf{n}-\mathbf{e_i}\,\mathbf{e_i}} = \frac{a}{a^{[\chi(x)]}}D_i\chi^j_\beta(\mathbf{n}) \tag{66}$$

where we assumed $a \to 0$ and remembered the definition of the covariant derivative 5. From this we immediately get:

$$K(\bar{\chi}^j_{\alpha\ \mathbf{n}-\mathbf{e_i}\,\mathbf{e_i}} D(g^{-1}_{\mathbf{n}-\mathbf{e_i}\,\mathbf{e_i}})^j_{\alpha\beta}, \chi^j_{\beta\ \mathbf{n}\,\mathbf{e_i}}) = \frac{-Z_0}{2}\frac{a^2}{a^{2[\chi]}}\bar{D}_i\bar{\chi}^j_\beta(\mathbf{n})D_i\chi^j_\beta(\mathbf{n}) \tag{67}$$

so that the continuum limit of the gauged PEPS will be :

$$|\psi_{\text{gauged}}\rangle = \int \mathcal{D}^2\phi\mathcal{D}^2\chi dA e^{-\int d^Dx\bar{D}_i\bar{\chi}(x)D^i\chi(x)+V(\phi(x),\chi(x))}\,|A(x)\rangle\,|\phi(x)\rangle\ . \tag{68}$$

which equals the proposed gauged CPEPS 3 upon realization that $\langle A(x)|s_E\rangle = 1\forall\,|A(x)\rangle$. Note that the continuumlimit of a gauged PEPS satisfying 42 will automatically lead to gauged CPEPS with a generating Lagrangian that satisfies 19.

V. OUTLOOK AND CONCLUSION

In this work, we proposed a new class of manifestly gauge invariant states in the continuum and demonstrated that they are the continuum limit of sequences of gauged PEPS. Furthermore, we demonstrated that in one dimension these states reduce to gauged CMPS. The intuition behind these states is that they take conventional globally symmetric continuous projected entangled pair states and lift the global symmetry to a local one. This is possible because tensor networks with global symmetries already respect a virtual Gauss law.

Since it was recently proven that gauged PEPS are the most general gauge invariant states on the lattice we suspect that our states are the most general gauge invariant states in the continuum. A rigorous proof of this statement is left for future work.

Another possible outlook is to use these new states as a numerical ansatz for the analysis of gauge theories directly in the continuum. In particular in one dimension we expect this to be a feasible approach due to the absence of loops in one dimensional tensor networks. In higher dimensions the contraction of the presented gauged PEPS is expected to be very costly but here one might consider the subset of gauged Gaussian continuous PEPS.

Furthermore, recently it has also been understood that it is possible to exactly contract pure-matter continuous tensor networks if the Hamiltonian of the transfer matrix is an integrable model. In future works we wish to investigate how this situation is changed with the inclusion of gauge degrees of freedom.

Finally, the presented approach can also be used to take continuum limits of gauged matrix product operators. Since gauged matrix operators are related to dualities on the lattice, this opens a window towards studying dualities in the continuum. A first step in this direction has already been taken in [6].

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[1] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory* (Addison-Wesley, Reading, USA, 1995).

[2] P. R. S. Gomes, An introduction to higher-form symmetries, SciPost Physics Lecture Notes 10.21468/scipostphyslectnotes.74 (2023).

[3] D. Gaiotto, A. Kapustin, N. Seiberg, and B. Willett, Generalized global symmetries, Journal of High Energy Physics **2015**, 10.1007/jhep02(2015)172 (2015).

[4] B. V.-D. Cuiper, J. Garre-Rubio, F. Verstraete, K. Vervoort, D. J. Williamson, and L. Lootens, From gauging to duality in one-dimensional quantum lattice models (2025), arXiv:2509.22051 [cond-mat.str-el].

[5] S. Ashkenazi and E. Zohar, Duality as a feasible physical transformation for quantum simulation, Physical Review A **105**, 10.1103/physreva.105.022431 (2022).

[6] G. Roose and E. Zohar, T-duality and bosonization as examples of continuum gauging and disentangling (2025), arXiv:2509.24630 [hep-th].

[7] T. Buscher, Path-integral derivation of quantum duality in nonlinear sigma-models, Physics Letters B **201**, 466 (1988).

[8] S. Reddy, Novel phases at high density and their roles in the structure and evolution of neutron stars (2002), arXiv:nucl-th/0211045 [nucl-th].

[9] K. RAJAGOPAL and F. WILCZEK, The condensed matter physics of qcd, in *At The Frontier of Particle Physics* (WORLD SCIENTIFIC, 2001) p. 2061–2151.

[10] D. J. Gross and F. Wilczek, Asymptotically Free Gauge Theories - I, Phys. Rev. D **8**, 3633 (1973).

[11] E. Zohar and J. I. Cirac, Combining tensor networks with monte carlo methods for lattice gauge theories, Physical Review D **97**, 10.1103/physrevd.97.034510 (2018).

[12] E. Zohar and M. Burrello, Building projected entangled pair states with a local gauge symmetry, New Journal of Physics **18**, 043008 (2016).

[13] J. Haegeman, K. Van Acoleyen, N. Schuch, J. I. Cirac, and F. Verstraete, Gauging quantum states: From global to local symmetries in many-body systems, Physical Review X **5**, 10.1103/physrevx.5.011024 (2015).

[14] F. Verstraete, V. Murg, and J. Cirac, Matrix product states, projected entangled pair states, and variational renormalization group methods for quantum spin systems, Advances in Physics **57**, 143 (2008), https://doi.org/10.1080/14789940801912366.

[15] F. Verstraete and J. I. Cirac, Continuous matrix product states for quantum fields, Physical Review Letters **104**, 10.1103/physrevlett.104.190405 (2010).

[16] J. Haegeman, J. I. Cirac, T. J. Osborne, and F. Verstraete, Calculus of continuous matrix product states, Physical Review B **88**, 10.1103/physrevb.88.085118 (2013).

[17] B. Tuybens, J. De Nardis, J. Haegeman, and F. Verstraete, Variational optimization of continuous matrix product states, Phys. Rev. Lett. **128**, 020501 (2022).

[18] J. Haegeman, J. I. Cirac, T. J. Osborne, H. Verschelde, and F. Verstraete, Applying the variational principle to (1+1)-dimensional quantum field theories, Physical Review Letters **105**, 10.1103/physrevlett.105.251601 (2010).

[19] M. Ganahl, J. Rincón, and G. Vidal, Continuous matrix product states for quantum fields: An energy minimization algorithm, Physical Review Letters **118**, 10.1103/physrevlett.118.220402 (2017).

[20] D. Draxler, J. Haegeman, F. Verstraete, and M. Rizzi, Continuous matrix product states with periodic boundary conditions and an application to atomtronics, Phys. Rev. B **95**, 045145 (2017).

[21] F. Quijandría, J. J. García-Ripoll, and D. Zueco, Continuous matrix product states for coupled fields: Application to luttinger liquids and quantum simulators, Phys. Rev. B **90**, 235142 (2014).

[22] F. Quijandría and D. Zueco, Continuous-matrix-product-state solution for the mixing-demixing transition in one-dimensional quantum fields, Phys. Rev. A **92**, 043629 (2015).

[23] K. Tiwana, E. Lauria, and A. Tilloy, A relativistic continuous matrix product state study of field theories with defects, Journal of High Energy Physics **2025**, 97 (2025).

[24] A. Tilloy, A study of the quantum sinh-gordon model with relativistic continuous matrix product states (2022), arXiv:2209.05341 [hep-th].

[25] J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete, Matrix product states and projected entangled pair states: Concepts, symmetries, theorems, Reviews of Modern Physics **93**, 10.1103/revmodphys.93.045003 (2021).

[26] T. Shachar and E. Zohar, Approximating relativistic quantum field theories with continuous tensor networks, Phys. Rev. D **105**, 045016 (2022), arXiv:2110.01603 [quant-ph].

[27] A. Tilloy and J. I. Cirac, Continuous tensor network states for quantum fields, Physical Review X **9**, 10.1103/physrevx.9.021040 (2019).

[28] C. Brockt, J. Haegeman, D. Jennings, T. J. Osborne, and F. Verstraete, The continuum limit of a tensor network: a path integral representation (2012), arXiv:1210.5401 [quant-ph].

[29] T. D. Karanikolaou, P. Emonts, and A. Tilloy, Gaussian continuous tensor network states for simple bosonic field theories, Physical Review Research **3**, 10.1103/physrevresearch.3.023059 (2021).

[30] I. Kull, A. Molnar, E. Zohar, and J. I. Cirac, Classification of matrix product states with a local (gauge) symmetry, Annals of Physics **386**, 199 (2017).

[31] D. Blanik, J. Garre-Rubio, A. Molnár, and E. Zohar, Internal structure of gauge-invariant projected entangled pair states, Journal of Physics A: Mathematical and Theoretical **58**, 065301 (2025).

[32] D. Jennings, C. Brockt, J. Haegeman, T. J. Osborne, and F. Verstraete, Continuum tensor network field states, path integral representations and spatial symmetries, New Journal of Physics **17**, 063039 (2015).

[33] G. Roose and E. Zohar, Gauging a superposition of fermionic gaussian projected entangled pair states to get lattice gauge theory eigenstates (2025), arXiv:2412.01737 [hep-lat].

[34] A. Kelman, U. Borla, I. Gomelski, J. Elyovich, G. Roose, P. Emonts, and E. Zohar, Gauged gaussian projected entangled pair states: A high dimensional tensor network formulation for lattice gauge theories, Physical Review D **110**, 10.1103/physrevd.110.054511 (2024).