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Evolution of the radio source position uncertainties in radio astrometric catalogs over the past three decades

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Abstract In this paper, progress in improving the coordinates uncertainty of extragalactic radio sources derived from astrometric and geodetic VLBI observations is investigated. For this purpose, 30 catalogs of radio source positions computed in 1997–2025 were analyzed. Over these years, the median source position uncertainty represented by the semi-major axis of the error ellipse for 467 sources in common to all catalogs improved by one order of magnitude: from 0.156 mas to 0.015 mas. It was also found that the improvement in the position uncertainty over time follows a power law with high accuracy. The number of observations used for determination of the coordinates of the common sources in the input catalogs increased over time from 1.3 to 14.4 million, also following a power law. A discussion of the results led us to the conclusion that the number of observations is the primary factor in improving the source positions uncertainty compared to the number of sessions in which the source was observed.

Key words: astrometry – reference systems – techniques: interferometric

1 INTRODUCTION

Establishing and maintenance of celestial reference frame (CRF) based on accurate positions of extragalactic radio sources (radio astrometry) is one of the key tasks solved by the astrometric and geodetic VLBI from the beginning of its history (M. H. Cohen & D. B. Shaffer 1971). The formal error (uncertainty) of these first determinations of the extragalactic radio source coordinates was at a level of $1 - 3''$. However, already by the middle of the 1970s the uncertainty of the source positions approaching the accuracy of the best (fundamental) optical catalogs of $\approx 0.1''$, and in the 1980s the accuracy of radio astrometric catalogs reached a level of several mas (H. G. Walter 1989). As the result, the International Astronomical Union (IAU) at its XXIIIrd General Assembly approved the resolution B2, which recommended that effective 1 January 1998, VLBI-based radio source catalogs will be realizations of the newly established International Celestial Reference Frame (ICRF).

Since then, the VLBI technique has been constantly developed, and the number of observations collected in the framework of numerous observing programs increased manifold, which has led to significant improvement in the accuracy of the radio astrometric catalogs. In this paper we analyzed 30 catalogs of radio source positions derived from astrometric and geodetic VLBI observing programs since 1997 with main purpose to assess the evolution of the uncertainties in the radio source positions over past 28 years, since 1997 till 2025.

The paper is organized as follows. Section 2 describes the data used in this study, Section 3 is devoted to analysis of the source position uncertainties in the input catalogs, and the last Section sums up the results obtained in this work.

2 DATA AND ANALYSIS

For this analysis, we used 30 catalogs of radio source positions computed in 1997–2025 using the Calc/Solve VLBI analysis software developed at the Goddard Space Flight Center, NASA, and described in C. Ma et al. (1986, 1998); A. L. Fey et al. (2015); P. Charlot et al. (2020) and NASA technical publications referenced therein. All the catalogs were computed using the same software and close methodology, albeit with gradual updates over the years. To compute the source coordinates in these catalogs VLBI observations at X (7.7–8.8 GHz) and S (2.2–2.3 GHz) bands were processed. Observations at S band were only used to compute the ionosphere correction, source positions in the catalogs are related to X band.

The source position uncertainties in the input catalogs were computed in two modes. Most of catalogs contain the position uncertainties directly coming from a global solution using the least square adjustment. They will be called hereafter catalogs with original uncertainties. In some cases the original position uncertainties were additionally inflated to obtain more realistic position errors. The inflated uncertainties were computed using the following expressions (A. L. Fey et al. 2015):

$$\begin{aligned} \sigma'^2_{\alpha^*} &= (f_\alpha\, \sigma_{\alpha^*})^2 + \sigma^2_{\alpha,0}\,, \\ \sigma'^2_{\delta} &= (f_\delta\, \sigma'^2_{\delta}) + \sigma^2_{\delta,0}\,, \end{aligned} \qquad (1)$$

where σ_{α^*} and σ_δ are original uncertainties in right ascension multiplied by $\cos\delta$ and declination, respectively, σ_{α^*} and σ_δ are inflated uncertainties, f_α and f_δ are scaling factors, and $\sigma_{\alpha,0}$ and $\sigma_{\delta,0}$ are error floors. The scaling factors and error floors for α and δ are determined using special procedures, and are arbitrary to some extent. For example, for the ICRF catalog (C. Ma et al. 1998) $f_\alpha = f_\delta$ = 1.5, $\sigma_{\alpha,0} = \sigma_{\delta,0}$ = 0.25 mas, for the ICRF2 catalog (A. L. Fey et al. 2015) $f_\alpha = f_\delta$ = 1.5, $\sigma_{\alpha,0} = \sigma_{\delta,0}$ = 0.04 mas, and for the ICRF3-SX catalog (P. Charlot et al. 2020) $f_\alpha = f_\delta$ = 1.5, $\sigma_{\alpha,0} = \sigma_{\delta,0}$ = 0.03 mas.

In this work, input catalogs with the original uncertainties were used. If the catalog is only available in a version with inflated uncertainties, it was recomputed using the information provided in the original catalog and inverse expression Eq. (1).

General information and basic statistics related to the catalogs processed for this study is presented in Table 1 and Fig. 1.

The number of observations of the common sources is about 70% of the total number of observations. See also Fig. 2 with observation statistics for the latest catalog used in this study, usn2025a.

Unlike the plot with the number of sources, there is no visible jump in the plots of the number of observations. The main reason is that although a large number of sources were added to the radio astrometric catalogs after 2008, the number of observations of these new sources is relatively small as compared with "old" sources regularly observed in the VLBI geodetic observing programs.

3 SOURCE POSITION UNCERTAINTIES

Figure 3 shows the evolution median position (the semi-major axis of the error ellipse, SMA) over 28 years. The top plot presents uncertainties for all catalogs including all included sources. As was mentioned above, abrupt increasing of the number of source after 2009 caused the increase in the median position uncertainty because most new sources have less accurate positions due to more poor observation history. The lower plot in Fig. 3 shows only

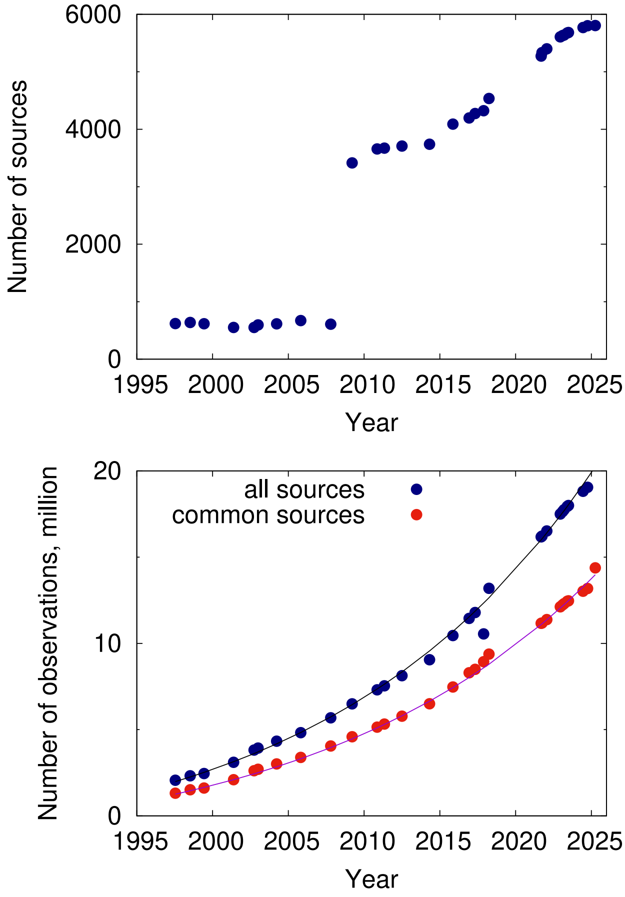

Fig. 1 Observation statistics for 30 catalogs used in this work: the number of sources in each catalog (top panel) and the number of observations (bottom panel) for all sources and for 467 sources in common for all catalogs. The line in the bottom panel corresponds to a power law approximation of the number of observations of common sources.

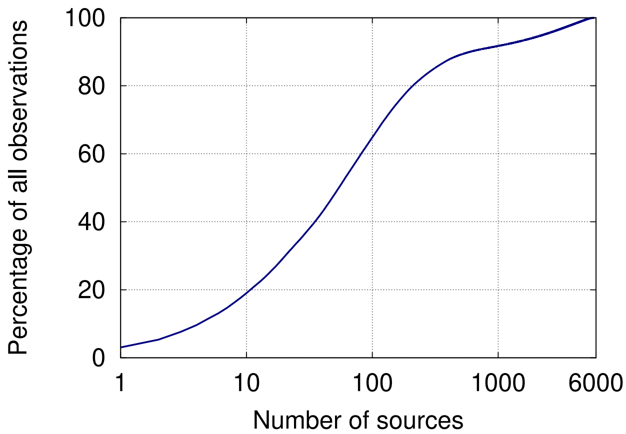

Fig. 2 Cumulative number of observations depending on the number of sources for the usn2025a catalog. Summations starts from sources with the maximum number of observations.

Table 1 Basic information and statistics for the 30 input catalogs.

Catalog	Last epoch		All sources					467 Common sources			
	MJD	Year	Nsou	Nobs	Median uncertainty, mas			Nobs	Median uncertainty, mas		
					σ_{α^*}	σ_δ	SMA		σ_{α^*}	σ_δ	SMA
gsf1997c	50646	1997.54	620	2063551	0.1362	0.1817	0.1881	1314749	0.1103	0.1461	0.1557
gsf1998g	51003	1998.52	638	2322661	0.0973	0.1432	0.1468	1513593	0.0828	0.1199	0.1208
gsf19995	51339	1999.44	617	2451068	0.0888	0.1240	0.1300	1610531	0.0767	0.1118	0.1137
gsf2001c	52053	2001.39	552	3103286	0.0580	0.0865	0.0882	2096415	0.0580	0.0866	0.0885
gsf2002c	52545	2002.74	552	3815486	0.0474	0.0724	0.0731	2614614	0.0470	0.0713	0.0726
gsf2003a	52640	2003.00	596	3928397	0.0494	0.0762	0.0778	2699910	0.0453	0.0707	0.0712
gsf2004b	53091	2004.23	615	4331451	0.0469	0.0723	0.0727	3011868	0.0423	0.0648	0.0657
gsf2005e	53672	2005.82	671	4824569	0.0426	0.0649	0.0657	3390553	0.0359	0.0541	0.0544
gsf2007d	54393	2007.80	609	5685251	0.0333	0.0497	0.0497	4048831	0.0308	0.0450	0.0452
ICRF2	54908	2009.21	3414	6491828	0.2632	0.4919	0.5098	4585559	0.0271	0.0380	0.0384
gsf2010a	55511	2010.86	3658	7309370	0.2717	0.5155	0.5401	5143723	0.0244	0.0340	0.0350
gsf2011a	55686	2011.34	3671	7540393	0.2685	0.5110	0.5326	5321900	0.0228	0.0330	0.0337
gsf2012a	56110	2012.50	3708	8128449	0.2389	0.4540	0.4735	5781338	0.0201	0.0290	0.0293
gsf2014a	56776	2014.32	3740	9049465	0.2102	0.3985	0.4123	6498120	0.0186	0.0260	0.0267
gsf2015b	57337	2015.86	4089	10453527	0.1228	0.2100	0.2181	7469980	0.0170	0.0226	0.0228
gsf2016b	57729	2016.93	4196	11447130	0.1286	0.2131	0.2230	8297258	0.0166	0.0223	0.0226
gsf2017a	57873	2017.33	4273	11791279	0.1066	0.1835	0.1905	8493225	0.0157	0.0214	0.0216
gsf2018c	58080	2017.89	4323	10547496	0.0838	0.1460	0.1488	8935897	0.0150	0.0209	0.0211
ICRF3-SX	58206	2018.24	4536	13190228	0.0821	0.1439	0.1471	9379123	0.0144	0.0201	0.0202
usn2021a	59466	2021.69	5274	16173976	0.0678	0.1203	0.1229	11163384	0.0139	0.0185	0.0186
usn2021b	59486	2021.74	5333	16219604	0.0684	0.1216	0.1240	11179607	0.0139	0.0185	0.0186
usn2022a	59601	2022.06	5399	16514351	0.0688	0.1218	0.1242	11381237	0.0139	0.0183	0.0183
usn2022d	59927	2022.95	5608	17497505	0.0696	0.1245	0.1271	12122013	0.0135	0.0173	0.0174
usn2023a	59983	2023.10	5629	17643510	0.0696	0.1249	0.1277	12230629	0.0135	0.0172	0.0173
usn2023b	60020	2023.20	5639	17750815	0.0694	0.1247	0.1271	12306145	0.0135	0.0171	0.0173
usn2023c	60097	2023.41	5676	17940636	0.0693	0.1245	0.1269	12440866	0.0133	0.0170	0.0172
usn2023d	60123	2023.49	5683	17994850	0.0691	0.1241	0.1267	12466656	0.0133	0.0170	0.0172
usn2024a	60476	2024.45	5770	18819817	0.0684	0.1244	0.1267	13024993	0.0134	0.0170	0.0171
usn2024b	60585	2024.75	5802	19047642	0.0690	0.1255	0.1279	13179020	0.0134	0.0170	0.0170
usn2025a	60770	2025.26	5804	20570840	0.0606	0.1117	0.1137	14379983	0.0117	0.0149	0.0149

Notes. Nsou is the number of sources in the catalog, Nobs is the number of observations processed to compute the catalog, σ_{α^*} is the uncertainty in right ascension multiplied by $\cos\delta$, σ_δ is the uncertainty in declination, SMA is the semi-major axis of the error ellipse.

data for 467 sources in common for all 30 catalogs, and it can be used to assess the evolution of the source position uncertainties over time.

Overall improvement in the position uncertainty is about one order of magnitude. Uncertainty in δ is greater than uncertainty in α^* in all catalogs because of stronger distribution of latitude-oriented baselines, as is well known from previous analysis. For this reason, the SMA value is dominated by the uncertainty in δ.

Comparing Fig. 3 (top panel) with Fig. 1 (top panel) one can conclude that massive addition of new sources caused substantial degradation of the median position uncertainties. The main reason of this is that the additional sources have worse observation history. It is interesting to estimate how the source position uncertainties depend on the number of sessions in which the source was observed and the total number of observations used for computation of the source coordinates. Figure 4 presented corresponding data for the latest catalog usn2025a.

It can be concluded from comparison of the scatter of the data points on the top and bottom panels of Fig. 4 that the dependence of the uncertainties in the source position on the observation history is better described by the number of observations rather than by the number of sessions. Moreover, in the latter case, such a dependence appears very weak for sources observed in fewer than about 10 sessions. In contrast, the source position uncertainty improves more rapidly with increasing the number of observations for observations less than about 1000 than with further increases in the number of observations. The dependence of the SMA on the number of observations (Nobs) can be approximated by a quadratic function, while the dependence of the SMA on the number of sessions (Nsess) can be approximated by a linear function, as follows:

$$\begin{aligned} \log \mathrm{SMA} &= 2.34 - 1.57 \log N\mathrm{obs} + 0.124 \log^2 N\mathrm{obs}\,, \\ &= -0.36 - 0.60 \log N\mathrm{sess}\,, \end{aligned} \tag{2}$$

where SMA is expressed in mas. Red lines in Fig. 4 correspond these approximations.

Comparison of Fig. 1 and Fig. 3 also shows that both increasing of the number of observations of the common sources over time and decreasing of the median position uncertainty follow a power law.

Figure 5 presents the distribution of the source position uncertainties (SMA) in the usn2025a catalog. It shows the percentage of sources with the position uncertainty less than the given value shown on the abscissa axis. In particular, more than 95% sources has position uncertainty less

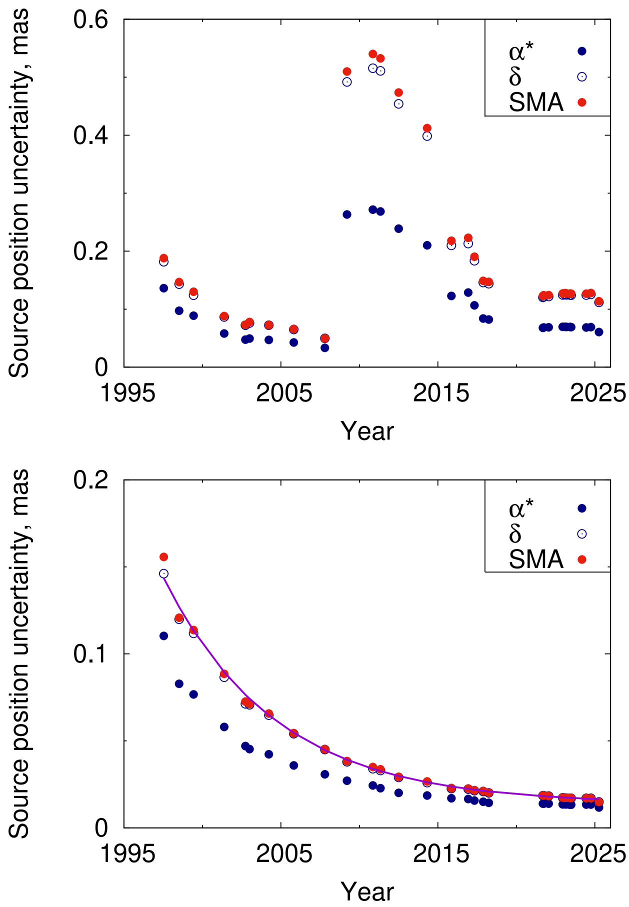

Fig. 3 Median position uncertainties in α^* and δ, and the semi-major axis of the error ellipse (SMA) for 30 catalogs used in this work for all sources (top panel) and 467 common sources (bottom panel). The line in the bottom panel corresponds to a power law approximation of the SMA data.

than 1 mas, and about 40% sources has position uncertainty less than 0.1 mas. The whole range of the position uncertainties is from 3 μas to 141 mas (31 mas without one source).

4 CONCLUSIONS

An interesting fact was noticed that both the increase in the number of observations used to derive the source coordinates and the decrease in the median source position uncertainties follow a power law with high accuracy. Of course, an increase in the number of observations may be not the only factor in improving the source position uncertainties over time. Improving the VLBI technology, extending the VLBI networks, and developings in the data analysis should also play a role in improving the accuracy of catalogs over time. However, it is difficult, if possible at all, to quantify the contribution of these factors because the sources were observed in different sessions of different de-

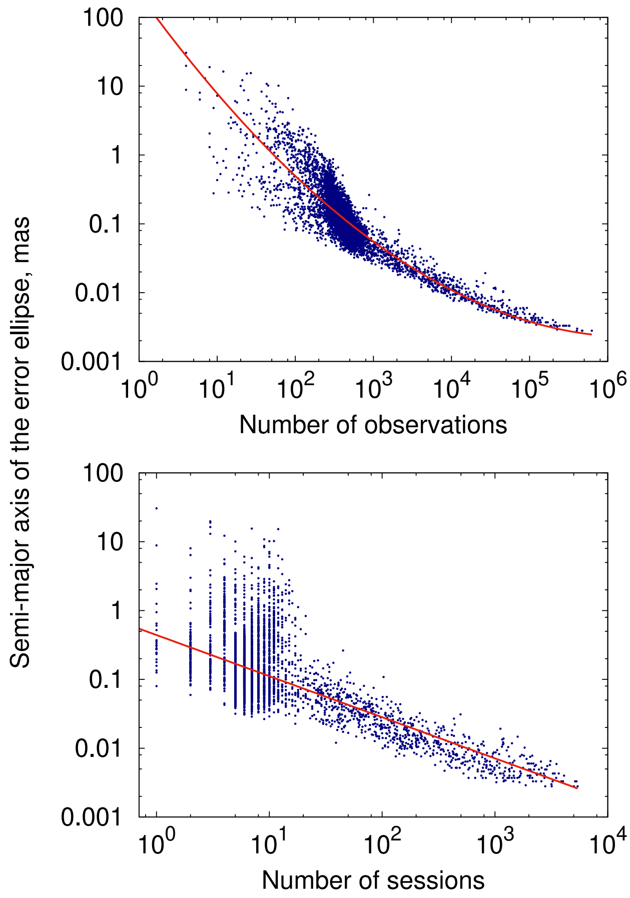

Fig. 4 Dependence of the source position uncertainty represented by the semi-major axis of the error ellipse on the number of observations (top panel) and the number of sessions (bottom panel) for the usn2025a catalog. One source with the position uncertainty of 141 mas is not shown.

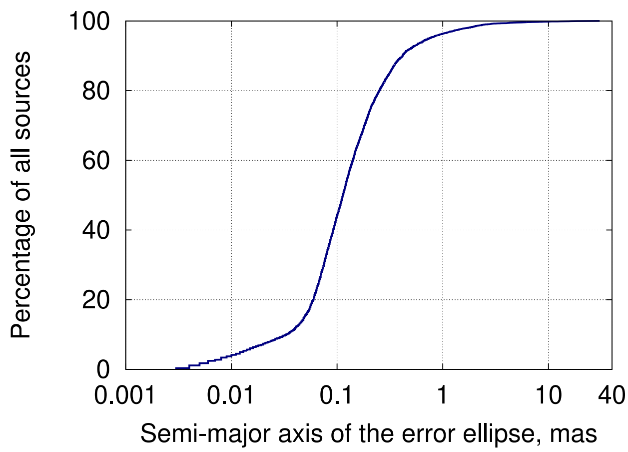

Fig. 5 Cumulative number of sources with the position uncertainty represented by the semi-major axis of the error ellipse is less than the value given on the x axis (usn2025a catalog). One source with the position uncertainty of 141 mas is not shown.

sign and not simultaneously. On the other hand, it can be expected that these improvements in the VLBI technique should manifest themselves in improved catalogs of radio source position. As was shown by Z. M. Malkin (2020), the accuracy of the baseline length determinations has only slightly improved after the late 1900s, and the accuracy of the Earth orientation parameters, although has improved after the late 1900s, but to a much lesser extent compared with the improvement in the accuracy of source positions. This allows as to suggest that the number of observations is the primary factor determining the accuracy of the VLBI-based radio source catalogs.

Generally speaking, it can be noticed that the power law can often serve as a means of describing various features of astrometric and geodetic VLBI observations. For example, in addition to the results of the present study, it was shown in Z. Malkin (2009) that both precision and accuracy of the EOP obtained from VLBI observations can be well described by a power function of the geometric volume of the network.

It was also shown that the dependence of the position uncertainties on the source observation history is better described by the number of observations rather than by the number of sessions, which can be taken into account for optimal scheduling of the astrometric VLBI observations. As follows from this result, observing fewer sessions with more observations appears to be preferable to observing more sessions with a smaller total number of observations. The former strategy is also preferable for better source imaging.

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