Duality-based Mode Operations and Pyramid Multilayer Mapping for Rhetorical Modes

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Abstract

Rhetorical modes are useful in both academic and non-academic writing, and can be subjects to be studied within linguistic research and computational modeling. Establishing a conceptual bridge among these domains could enable each to benefit from the others. This paper proposes duality-based mode operations (split-unite, forward-backward, expansion-reduction and orthogonal dualities) to expand the set of rhetorical modes, introducing generated modes like combination and generalization, thereby enhancing epistemic diversity across multiple applications. It further presents a pyramid multilayer mapping framework (e.g., three layers from the rhetorical model layer, to cognitive layer, and to epistemic layers) that reduces the resulting cognitive complexity. The degrees of expressive diversity and complexity reduction are quantified through binomial combinatorics and Shannon entropy analysis. A Marginal Rhetorical Bit (MRB) is identified, permitting the definition of a rhetorical-scalable parameter that measures expressive growth speed in bits per stage. A direct entropy measure shows that hierarchical selection over smaller subsets markedly reduces choice uncertainty compared with flat selection across all modes. These considerations appear to transform static and non-measurable rhetorical taxonomies into more dynamic and more measurable systems for discourse design. From this work, it would be possible to identify a pathway for future AI systems to operate not only on language tokens but on layered rhetorical reasoning structures, bridging linguistic, pedagogical, academic, and computational research.

Keywords: rhetorical modes; duality-based operations; generated rhetorical mode; pyramid multilayer mapping; Marginal Rhetorical Bit (MRB); rhetorical-scalable parameter; cognitive entropy; epistemic diversity

1 Introduction

Rhetorical modes—also known as patterns of development—have long served as fundamental templates for organizing discourse. From Aristotle's *On Rhetoric* (2007) to the modern pedagogical tradition established by Bain (1866) and Hill (1895), rhetorical instruction evolved from oratorical training toward written composition organized by recognizable textual forms. Connors (1997) provides the most detailed account of this transformation, showing how nineteenth- and twentieth-century composition pedagogy shifted from persuasive appeals (ethos, pathos, logos) to structured "modes of discourse."

Several major composition textbooks (e.g., Corbett & Connors 1999; Kirszner & Mandell 1986; Nadell, Langan & Coxwell-Teague; Lunsford 2015, 2021; Hacker & Sommers 2020) promote an overlapping cluster of rhetorical modes—narration, description, exemplification-illustration, definition, classification-division, comparison-contrast, cause-effect, process analysis, argument-persuasion, and exposition. Subsequent works (e.g., Oshima & Hogue 2007; Smalley, Ruetten & Kozyrev 2011) expand this cluster to include additional modes such as analysis, synthesis, evaluation, and problem–solution.

Because rhetorical modes retain pedagogical importance (for example, they continue to underpin college-level writing instruction), one might reasonably expect that scholars would have examined them as interrelated cognitive operations and, ultimately, as contributing to epistemic ends. Figure 1 sketches an expected developmental pathway. Because rhetorical modes remain central to pedagogy, scholars have long called for theory-building that links inventional heuristics to composing processes (Young, 1978; Lauer, 2004; Bawarshi, 2003). Cognitive-process and metacognitive

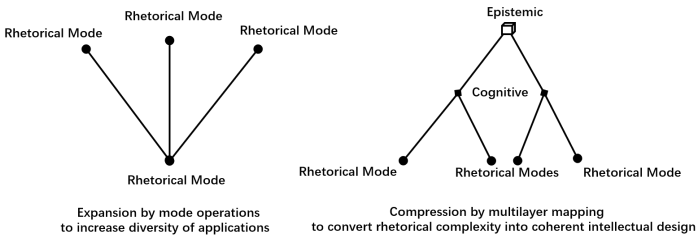

Fig. 1 Expected pathway in rhetorical mode development. Left: the growth and recombination of modes from existing families to increase epistemic diversity. Right: the mapping of rhetorical modes to epistemic purposes via an intermediate cognitive layer.

research shows composing as recursive planning, translating, and monitoring (Flower & Hayes, 1981; Flavell, 1979), while discourse- and genre-based work emphasizes purpose, audience, and contextual framing (Bazerman, 1988; Beaufort, 2007; Hoey, 1983). At the same time, quantitative and information-theoretic traditions (Shannon, 1948; Zipf, 1949; Lotman, 1990; Simonton, 2004) reveal exponential/combinatorial patterns in language and cognition, but, according to the knowledge of the present author, these insights have not yet been widely extended to model rhetorical structure; together these literatures suggest a promising basis for an integrated, measurable account of modes → cognition → epistemic ends.

Rhetorical modes are recurrent, rule-governed patterns of discourse—stable configurations of moves, constraints, and affordances that writers deploy to achieve communicative ends (McQuade & Atwan, 1998). Because a mode is a pattern rather than a unitary string, it has formal properties (shape: sequence of moves; size: scope; orientation: foregrounding vs. backgrounding; extension: span across a text; edges: limits), which make it amenable to systematic operations such as splitting, combining, reversing, contracting/expanding, or bifurcating. Crucially, modes are typically oriented toward epistemic ends (e.g., to communicate, to evaluate, to teach, to discover), and those ends show patterned regularities; therefore modes can be mapped to epistemic purposes. Introducing an intermediate functional layer between surface form and epistemic goal reduces representational complexity and makes a three-layer architecture (rhetorical modes → cognitive or functional layer → epistemic purpose) both tractable and pedagogically useful.

In Section 2 we explore extending the set of base rhetorical modes by generating new modes through mode operations (e.g., split-unite, forward-backward), motivated by demands from academic writing, learning analytics, AI-assisted discourse modeling, and multimodal communication. Building on a canonical set of fourteen modes, we propose duality operations: many modes are inherently diatomic (e.g., comparison–contrast, classification–division) and can be decomposed into atomic components or recomposed into new hybrids; other modes can be paired with functional counterparts to form a more symmetrical repertoire. We formalize diversity and combination growth with simple combinatorics (binomial coefficients) and introduce a tentative rhetorical-scalable parameter that captures combinative capacity and measurable growth rates. A pedagogical "rhetorical cone" links this expansion to educational stages (from preschool to graduate study), showing how repertoire grows with instruction.

Because unconstrained expansion risks producing non-linguistic or cognitively unstable operations, Section 3 develops a three-layer cognitive mapping in which surface modes project through an intermediate functional layer (cognitive function layer) to epistemic purposes; this mapping constrains redundancy, reduces cognitive entropy, and preserves epistemic coherence. Together, these elements provide a tractable, operational framework with possible pedagogical and computational applications.

Table 1 lists the commonly seen rhetorical modes (in alphabetical order), that will be considered as the base rhetorical modes for mode operations and multilayer mapping.

Table 1 Concise Summary of Base Rhetorical Modes (Rm)

Code	Name	Core Function / Scope
Rm(1)	Analysis–Synthesis	Decomposing concepts or systems and integrating parts into coherent understanding.
Rm(2)	Analogy	Explaining ideas through structural or functional similarity.
Rm(3)	Argument–Persuasion	Reasoning to demonstrate validity and influence belief or action.
Rm(4)	Cause–Effect	Linking antecedent factors with resulting outcomes or consequences.
Rm(5)	Classification–Division	Grouping entities by shared traits or partitioning wholes into component parts.
Rm(6)	Comparison–Contrast	Analyzing similarities and differences among entities or ideas.
Rm(7)	Definition	Specifying the meaning or essential boundaries of a concept.
Rm(8)	Description	Depicting observable or sensory features to create vivid representation.
Rm(9)	Evaluation	Applying explicit criteria to assess quality, value, or effectiveness.
Rm(10)	Exemplification–Illustration	Supporting general claims with specific instances or evidence.
Rm(11)	Exposition	Systematically clarifying information or ideas for transparency.
Rm(12)	Narration	Presenting events or experiences in temporal or logical sequence.
Rm(13)	Problem–Solution	Identifying an issue and proposing or justifying a means of resolution.
Rm(14)	Process Analysis	Explaining ordered steps or stages that accomplish a task or outcome.

2 Rhetorical Mode Expansion through Duality-based Mode Operation

The extension of the base rhetorical modes is realized through properly selected duality-based mode operations. The key point for mode operation is to find the duality between two rhetorical modes (of which one is old, one is new, or both are old).

2.1 Duality-based Mode Operations

2.1.1 Introduction of mode operations

Let $\mathcal{R}_m = \{R_m(1), R_m(2), \ldots, R_m(K_0)\}$ denote the set of base or canonical rhetorical modes (e.g., narration, description, definition, classification-division shown in Table 1, where $K_0 = 14$), which can be divided into 7 atomic rhetorical modes ($R_m^{(a)}(k)$) (like narattive, description, definition) and 7 diatomic rhetorical modes or compound rhetorical modes ($R_m^{(c)}(k)$) (like classification-division, cause-effect, exemplification-illustration, argumentation-persuation, problem-solution, comparision-constrast).

Figure 2 is an illustration of mode generation from operations done on base rhetorical modes. The modes thus built are called generated modes or extended modes, which may become rhetorical modes to be added to the existing list of rhetorical modes (Table 1), or remain as primitive congnitive modes that complement the functional spreading of rhetorical modes in its proper sence.

Below we introduce four mode operations, based on split-unite duality, forward-backward duality, expansion-contraction duality and orthogonal duality. These four

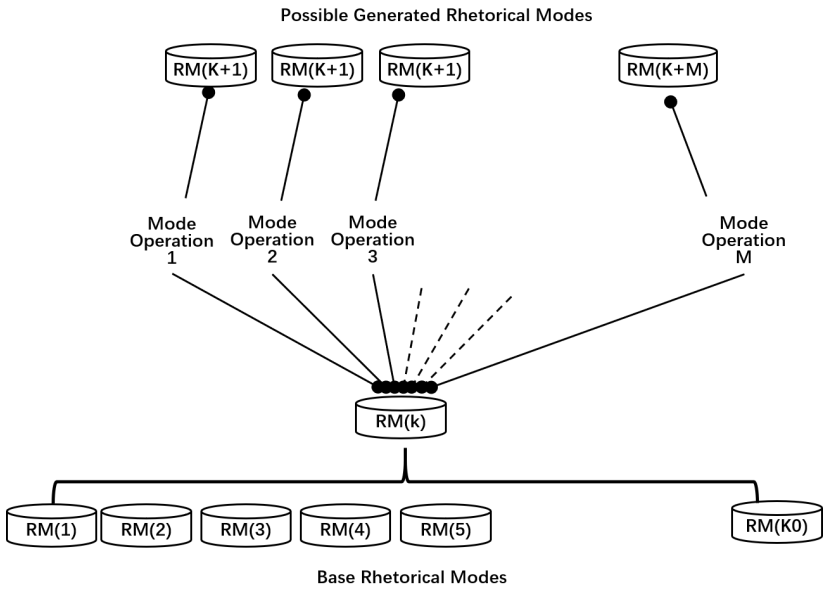

Fig. 2 Schematic display of rhetorical mode operations, generating generated rhetorical modes from base rhetorical modes

ones are not exclusive, but in this paper we do not consider other possible mode operations.

2.1.2 Split-Unite Duality

The split-unite duality suggests a split operation $\mathcal{O}_{\text{split}}$ which splits a diatomic $R_m^{(c)}(k)$ into two atomic rhetorical modes ($R_m^{(a)}$):

$$\mathcal{O}_{\text{split}} : \; R_m^{(c)}(k) \mapsto \{R_m^{(a)}(i), R_m^{(a)}(j)\},$$

as illustrated in Figure 3. In Figure 3 we also illustrates unite operation ($\mathcal{O}_{\text{unite}}$):

$$\mathcal{O}_{\text{unite}} : \; \{R_m^{(a)}(i), R_m^{(a)}(j)\} \mapsto R_m^{(c)}(k)\}$$

which produces a diatomic rhetorical mode from any two atomic rhetorical modes.

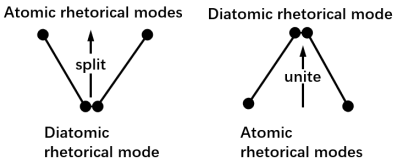

Fig. 3 Schematic display of split-unite operation

The application of $\mathcal{O}_{\text{split}}$ to all $R_m^{(c)}(k)$ in Table 1 leads to 14 novel atomic rhetorical modes, as shown in Table 2. For instance, apply $\mathcal{O}_{\text{split}}$ to **Classification-Division** leads to **Classification** and **Dvision**.

One may also apply the unite operation $\mathcal{O}_{\text{unite}}$ to the atomic rhetorical modes ($R_m^{(a)}(k)$) in Table 1 and Table 2, producing for instance new compound mode like "**narrative-description**", see Figure 4.

The **narration–description** mode is generated through the Unite operation of the Split–Unite Duality, combining the atomic modes **narration** and **description** into a single diatomic form. While narration organizes events in temporal sequence and description depicts the concrete qualities of each event or scene, their unification produces a discourse structure in which actions unfold over time while each stage is vividly rendered in detail. This mode enables readers to perceive both process and texture—to follow a sequence while simultaneously visualizing its internal states. Cognitively, it integrates sequential reasoning with perceptual grounding, facilitating

Table 2 Concise Summary of Decomposed Rhetorical Modes (from Compound Modes)

Code	Name	Core Function / Scope
Rm(15)	Classification	Grouping entities or ideas by shared traits or organizing principles.
Rm(16)	Division	Partitioning a whole into constituent parts to clarify structure or function.
Rm(17)	Cause	Identifying antecedent factors or conditions producing a specific result.
Rm(18)	Effect	Describing outcomes or consequences emerging from identified causes.
Rm(19)	Exemplification	Supporting general claims with concrete examples or representative cases.
Rm(20)	Illustration (Evidence)	Providing factual proof or documentation that substantiates a claim.
Rm(21)	Argument	Advancing a logically supported claim based on reasons and evidence.
Rm(22)	Persuasion	Influencing attitudes or actions through emotional, ethical, or value-based appeal.
Rm(23)	Problem	Identifying a question, issue, or gap requiring interpretation or resolution.
Rm(24)	Solution	Proposing and justifying a method or means to resolve an identified problem.
Rm(25)	Comparison	Highlighting similarities among multiple entities, ideas, or phenomena.
Rm(26)	Contrast	Highlighting and interpreting key differences among entities, ideas, or phenomena.
Rm(27)	Analysis	Breaking a concept or system into parts to reveal its logic or internal relationships.
Rm(28)	Synthesis	Integrating diverse perspectives or arguments into a unified understanding.

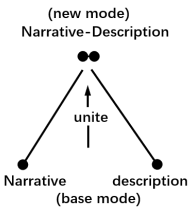

Fig. 4 Schematic display of model operation, using split-unite duality, leading to new mode "narrative-description".

stronger memory encoding and conceptual understanding. In practical application, the narration–description mode is particularly effective in teaching demonstrations, experimental reporting, and case-based explanations, where dynamic processes must be conveyed with both chronological clarity and descriptive precision.

If all choices are possible among the 7 atomic rhetorical modes in Table 1, then we would have

$$\binom{7}{2} = \frac{7!}{2!(7-2)!} = 21$$

new possibilities for diatomic $R_m^{(c)}(k)$.

If further, we consider all the atomic rhetorical modes, including the seven ones in Table 1 and the 14 novel atomic rhetorical modes, as shown in Table 2, then we

have in total 21 atomic rhetorical modes, and the possibiliy to have diatomic modes ($R_m^{(c)}$) by unite operation is

$$\binom{21}{2} = 210$$

A triatomic mode may be defined as the composition of three atomic modes. We do not develop this case separately since it is a direct extension of the diatomic generation rule. Section 3 formalizes academic functions that couple one core mode with supplementary modes; triatomic compositions are therefore covered by that formulation.

2.1.3 Forward-Backward Duality

This operation reverses the temporal or logical direction of a mode:

$$\mathcal{F}_R(R_m(k)) = R_m^{-1}(k)$$

For instance, apply this duality operation generates from **Cause** to **Effect**, **Problem** to **Solution**, and vice versa. These are still among the atomic rheotrical modes presented in Table 2.

However, if this operation is applied to **Exemplification**, we would have **Generalization**, which seems to be a new rhetorical mode. In fact, a special case for Forward-Backward Duality is Deduction-induction Duality, which means reasoning from **Generalization** to **Exemplification** (deduction) and vice versa. Furthermore, if this operation is applied to **Division**, we would have **Combination** as a generated mode. See Figure 5 for this kind of operation leading to generated modes.

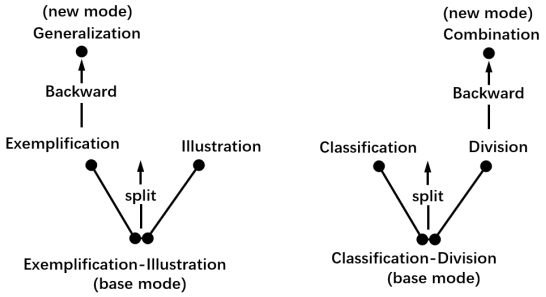

Fig. 5 Schematic display of model operation, using split-unite duality and forward-backward duality, leading to generated mode **generalization** and **combination**.

We do not discuss whether this operation generates more generated modes if applied to other atomic modes (the reader may use this operation to do more).

2.1.4 Expansion-Reduction Duality

This operator adjusts the scale or granularity of a mode:

$$\mathcal{E}_{\text{expand}}(R_i) = R_i^+, \qquad \mathcal{E}_{\text{reduce}}(R_i) = R_i^-,$$

where R_i^+ provides elaboration or amplification, while R_i^- compresses or summarizes the discourse. For example, **Exposition** corresponds to expansion, by Expansion-Reduction Duality, we obtain from **Exposition** the generated mode **Summary**.

2.1.5 Orthogonal Duality ($\mathcal{O}_\perp$)

Two modes are orthogonal if they operate in distinct cognitive dimensions, e.g., horizontal (temporal) versus vertical (structural):

$$\mathcal{O}_\perp(\text{Narration}) = \text{Description}, \quad \mathcal{O}_\perp(\text{Description}) = \text{Narration}.$$

This duality models complementary perceptual axes sequence versus depth.

2.1.6 Rhetorical completeness and the need to have operator calculus

The four mode operations, while not exclusive, explain both the familiar dualities of classical rhetorical modes and how generated modes can be derived toward rhetorical completeness. Some of the generated modes coincide with categories already used in rhetorical practice; others fall outside the classical sense but nevertheless exhibit coherent patterns and useful applications. Such departures from classical form do not preclude these modes from having well-defined structure or practical value.

Together, the four and possible other mode operations could provide a mechanism for discovering, pairing, and extending rhetorical modes. It is hopeful that, by iterative use, mode operations may help to generate a balanced and potentially open-ended rhetorical system, in which expressive and cognitive capacities increase through structural symmetry and logical reversibility.

However, to achieve the above purpose more fomally, it would be useful to develop, in the future, operator calculus, like in comptional modelling, that can abstract the logical and functional relations among rhetorical modes. More precisely, the full specification of four or even more mode operations—each expressed as a symbolic operator with definable algebraic behavior—could be assigned symbolic operators that are programmable for computer aided searching of more rhetorical modes, and then put rhetorical modes as patterns of language that help to increase the efficiency of large Language Models.

2.2 Rhetorical Scalable Parameter Measuring the Relative Rhetorical Capacity

Rhetorical modes are not merely names in a textbook; they are patterned, reusable operations for developing ideas in discourse. Treating modes as measurable objects makes it possible to ask quantitative questions that are pedagogically and scientifically useful: How many distinct rhetorical choices does a text offer? How quickly do new modes increase the expressive possibilities of a course or curriculum? At what pace does rhetorical growth risk overwhelming learners? The rhetorical-scalable parameter discussed here formalizes these questions.

2.2.1 Rhetorical combination capacity and marginal rhetorical bit (MRB)

A rhetorical-scalable parameter measures the relative rhetorical growth speed, so we need an inherent parameter that remains to be a constant.

The number K of rhetorical modes ($\mathcal{R}_m$), available in a system (an academic paper, a course, a genre), is now considered as a variable and called the rhetorical width. This width represents the expressive dimensionality or *linguistic bandwidth* of the discourse universe. A system with higher rhetorical width allows more expressive directions and functional differentiation, analogous to degrees of freedom in mechanics or dimensions in geometry.

At the level of an individual discourse unit (a sentence, paragraph, or slide), one typically use only a small subset (k) of the K modes. The number of distinct ways to choose exactly k modes from $\mathcal{R}_m$ is the binomial coefficient

$$K_C(k, K) = \binom{K}{k} = \frac{K!}{k!(K-k)!} \tag{1}$$

This quantity measures local expressive richness: small k indicates single-mode or narrowly focused discourse; larger k indicates dense, multimodal units. This counts how many unique combinations of modes can occur at each structural level of discourse and measures the local expressive richness or compositional order of a text. Small k corresponds to near single-mode writing; large k represents complex, multimodal discourse.

The maximum number $K_{\max}(K) = K_C(k, K)$ of k-combinations occurs at $k = k_m$ with

$$k_m = \left\lfloor \frac{K}{2} \right\rfloor \approx \frac{K}{2}$$

and this peak value ($K_{\max}(K) = K_C(k_m, K)$) is a simple proxy for rhetorical complexity. The parameter $K_{\max}(K)$ measures the maximum combinational potentional since it corresponds to the greatest number of distinct multimodal configurations that can coexist at equilibrium.

The total number of (non-empty) rhetorical combinations (NRC) that can be formed from those K modes is

$$K_{NRC}(K) = \sum_{k=1}^{K} \binom{K}{k} = 2^K - 1. \tag{2}$$

The quantity of $K_{NRC}(K)$ shows rhetorical capacity grows exponentially with the number of modes: every added mode doubles the set of possible combinations (approximately — the "-1" is negligible for moderate-to-large K). Because exponential growth is hard to reason about intuitively, we work on the log scale. Using base-2 logarithms, we define the rhetorical capacity $K_{RC}(K)$ as

$$K_{RC}(K) := \ln_2 K_{NRC}(K) = K \tag{3}$$

so each additional mode contributes one bit of new combinatorial expressive capacity. This gives an immediate, interpretable metric: adding one new mode doubles the number of possible rhetorical configurations. Thus, $K_{RC}(K)$ quantifies the log-scale expressive potential of a system: one extra mode one extra bit of rhetorical capacity.

Now we are ready to identify an inherent parameter that remains to be a constant with variable K. This inherent parameter is the Marginal Rhetorical Bit (MRB), defined as the derivative of $K_{RC}(K)$ with respect to K

$$\text{MRC}:=\frac{dK_{RC}(K)}{dK}$$

Because of (3) we have

$$\text{MRC} = 1 \text{ (bit per mode)} \tag{4}$$

so this parameter is indeed a constant.

The parameter MRB measures how many bits of additional rhetorical capacity are gained by adding one mode. Because MRB ≈ 1, each new mode contributes roughly one bit (i.e., doubles the space of configurations).

2.2.2 Rate of rhetorical introduction (RRI)

Rhetorical modes can be introduced gradually through progress of education, writing and reasoning. A stage of education, like elementary school, may introduce two modes over six years; a writing course may introduce one or two new rhetorical modes per week; a writing program may add three modes per year.

To quantify the progress, it is convient to introduce the rate of rhetorical introduction (RRI), to be denoted by L_n. The rate of rhetorical introduction (L_n) is an empirical, directly observable quantity. It refers to the average number of new rhetorical modes introduced per stage (stage = paragraph, hour, semester, year, etc. — choose the unit that fits your application)

$$L_n = \frac{\text{new rhetorical modes introduced}}{\text{stage}} \text{ (mode/stage)}$$

Table 3 shows possible values of L_n during each educational stage.

2.2.3 Rhetorical-scalable parameter: RRI divided by MRB

Define the rhetorical-scalable parameter R_{scale} as L_n (mode per stage) multiplied by MRB (bit per mode),

$$R_{scale} := L_n \times \text{MRB (bit per stage)}$$

Because of MRB=1 (bit per mode), it follows algebraically that

$$R_{scale} = L_n$$

Thus the rhetorical-scalable parameter equals the rate of rhetorical introduction (RRI) and can be read directly as bits of rhetorical capacity added per stage. The rhetorical-scalable parameter is therefore an immediate, interpretable measure: if $R_C = 0.5$, you add half a bit per stage (one new mode every two stages); if $R_C = 2$, you add two bits per stage (two new modes every stage).

Table 3 Typical cumulative acquisition path of rhetorical modes across educational levels

Educational Stage	K	**New Modes Introduced**	L_n **(Mode/Year)**
KG (Preschool)	1–2	Narration, Description	$L_n < 0.33$
Elementary School	4–5	Exemplification, Definition, Classification	$L_n \approx 0.33$
Middle School	7–8	Comparison, Contrast, Cause–Effect, Process, Argument	$L_n \approx 0.66$
High School	10–12	Analogy, Illustration, Evaluation, Problem–Solution, Persuasion	$L_n \approx 0.66$
Undergraduate (Y1–Y4)	14–16	Analysis, Synthesis, Exposition, Division–Classification (dual use)	$L_n \approx 1$
Graduate (Master / PhD)	> 20	All canonical + dual expansions (Combination, Generalization, etc.)	$L_n > 1$

2.2.4 Practical implications and normalization.

Reporting R_{scale} is operational: instructors or designers can prescribe or estimate L_n and so immediately quantify rhetorical growth in bits/stage. Because MRB is constant, comparisons across curricula or interventions are straightforward.

For cognitive-load diagnosis, normalize R_{scale} against an empirically determined processing capacity C_0 (bits/stage a learner can assimilate). Define normalized load $R^*_{scale} = R_{sclae}/C_0$. The values

$$R^*_{scale} > 1$$

indicate potential overload. Subcritical load means $R^*_{scale} < 1$, critical load means $R^*_{scale} = 1$, and supercritical load means $R^*_{scale} > 1$.

2.3 Rhetorical mode cone: scalable pedagogy that aligns with levels of cognitive development

Table 4 gives the maximum number of combination $K_{\max}(K)$ (and the corresponding k_m) and the total number of rhetorical combinations (K_{NRC}) for rhetorical mode width K up to 30.

Table 4 Rhetorical parameters $(k_m, K_{\max}, K_{NRC})$ for rhetorical width up to $K = 1$–30

K	k_m	$K_{max}(K)$	$K_{NRC}(K)$	K	k_m	$K_{max}(K)$	$K_{NRC}(K)$
1	0	1	1	16	8	12,870	65,535
2	1	2	3	17	8	24,310	131,071
3	1	3	7	18	9	48,620	262,143
4	2	6	15	19	9	92,378	524,287
5	2	10	31	20	10	184,756	1,048,575
6	3	20	63	21	10	352,716	2,097,151
7	3	35	127	22	11	705,432	4,194,303
8	4	70	255	23	11	1,352,078	8,388,607
9	4	126	511	24	12	2,704,156	16,777,215
10	5	252	1,023	25	12	5,200,300	33,554,431
11	5	462	2,047	26	13	10,400,600	67,108,863
12	6	924	4,095	27	13	20,058,300	134,217,727
13	6	1,716	8,191	28	14	40,116,600	268,435,455
14	7	3,432	16,383	29	14	77,558,760	536,870,911
15	7	6,435	32,767	30	15	155,117,520	1,073,741,823

For $K = 10$, we have $k_m = 5$, $K_{\max} = 252$ and $K_{NRC} = 2^{10} - 1 = 1{,}023$. If the rhetorical width is increased to $K = 16$, then $k_m = 8$, $K_{\max} = 12,870$ and

$K_{NRC} = 65{,}535$. This gives a ratio $\frac{K_{NRC}(16)}{K_{NRC}(10)} \approx 64$, showing the role of adding the extensions.

If the rhetorical width is further increased to $K = 20$, then

$$k_m = 10,\ K_{\max} = 184,756,\ K_{NRC} = 1{,}048{,}575.$$

This gives a ratio $\frac{K_{NRC}(20)}{K_{NRC}(10)} \approx 1{,}025$.

If we want K_{NRC} to be just more than 10 millions, then $K > \frac{\ln(10^8+1)}{\ln 2} \simeq 27$.

Thus, as K grows, the number of possible non-empty configurations rises exponentially—from 1,023 at $K = 10$ to 1,048,575 at $K = 20$ to 1.34×10^8 at $K = 27$. This formalizes the intuition that adding modes does not merely add options linearly; it *multiplies* the expressive space. Thus, the total cognitive intensity grows exponentially with the rhetorical width K, each additional mode disproportionately expands the expressive potential of discourse. This quantifies how extending and balancing the system of modes greatly increases the functional capacity of language.

The the proportion of available rhetorical modes actually employed in a text could be evaluated through identifying the Rhetorical Mode Coverage

$$C_m = \frac{K_u}{K}.$$

When $C_m \to 1$, the text demonstrates high rhetorical diversity, integrating nearly all available modes (typical of advanced academic or pedagogical writing). When $C_m \approx 0.5$, the text has moderate rhetorical variety, common in undergraduate or professional prose. When $C_m < 0.3$, the text has limited rhetorical deployment, typical of early educational writing or narrowly technical genres. Rhetorical Mode Coverage provides a direct, quantifiable index of rhetorical maturity across developmental stages. For instance, a middle-school essay employing narration, description, and exemplification may have $C_m = 0.15$, whereas a graduate thesis integrating definition, classification, comparison, cause–effect, evaluation, and synthesis may reach $C_m = 0.8$. Thus, C_m allows teachers and researchers to map rhetorical development longitudinally across education levels.

Thus $K, K_{\max}$ and K_{NRC} offer a *scalable pedagogy* that aligns with levels of cognitive development.

Figure 6 defines a rhetorical mode cone, showing how rhetorical widths (K) and other combinative parameters increase with education stage.

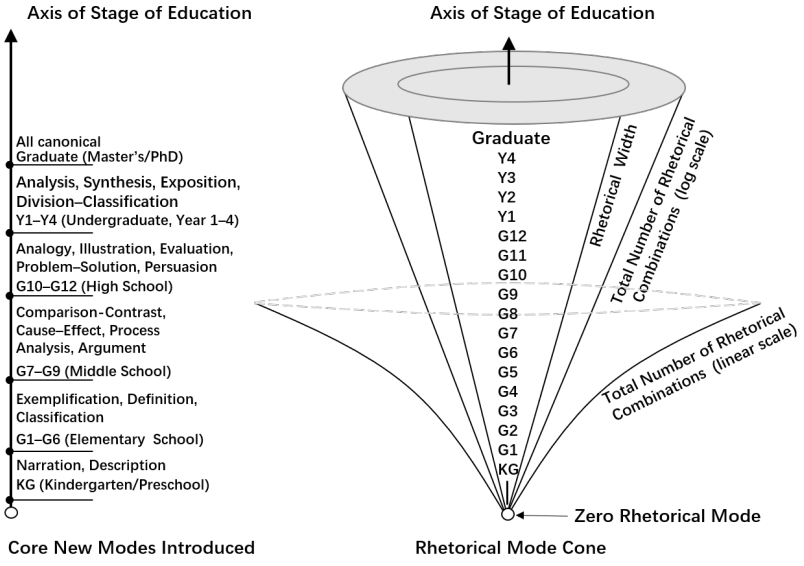

Fig. 6 Rhetorical Mode Cone, displaying educational related growth in Rhetorical Width (K) and the number of (non-empty) rhetorical combinations (K_{NRC}) (log compressed and unlogged).

Here we have provided some quantities using binomial coefficient to show how the expressive potential increases exponentially with the number of modes, demonstrating

that the richer the system, the greater the cognitive and expressive space it opens. Yet, such expansion also implies higher entropy—that is, greater uncertainty and cognitive demand when selecting or interpreting rhetorical forms.

3 Pyramid Multilayer Mapping and Academic Applications

To counterbalance the growth of complexity with extended rhetorical modes, we attempt to introduce the concept of multilayer mapping, showing that structured mappings between rhetorical, cognitive, and epistemic layers can reduce entropy—the informational uncertainty that burdens both writers and readers. Since each element of an upper layer corresponds to several elements of an adjacent lower layer. the shape is like a pyramid, so we call this Pyramid Shape Multilayer Mapping (Pyramid Multilayer Mapping for short).

3.1 Multilayer Mapping

The expansion of rhetorical modes through duality operations presented in Section 2 reveals a vast expressive landscape whose combinatorial richness grows exponentially. Yet this same richness also multiplies the number of possible rhetorical configurations, producing higher cognitive and informational entropy (uncertainty). Without an organizing mechanism, writers and readers must navigate an almost unbounded field of choices, increasing the mental effort required to select, structure, and interpret discourse.

To address this challenge, an efficient way is to introduce a pyramid multilayer mapping framework that connects rhetorical modes to final applications. Here we consider such a framework that connects three layers: rhetorical, cognitive, and epistemic layers.

It is knowen from information-theoretic principles, that hierarchical mapping acts as an entropy-reduction process. Here, such a mapping constrains rhetorical variability by aligning linguistic operations with reasoning functions and knowledge purposes. In this view, the act of academic writing becomes a progressive transformation from disorder to order—from expressive diversity to purposeful structure.

Figure 7 illustrates a three-layer mapping framework, mapping from rhetorical modes (R) to cognitive functions (C), and laterly to epistemic purposes (E),.

Formally, let $R = \{r(1), r(2), \cdots, r(K_R)\}$, $C = \{c(1), c(2), \cdots, c(K_C)\}$, and $E = \{e(1), e(2), \cdots, e(K_E)\}$. The three-layer mapping is realized through $R \to C$, $C \to E$, and then $R \to E$.

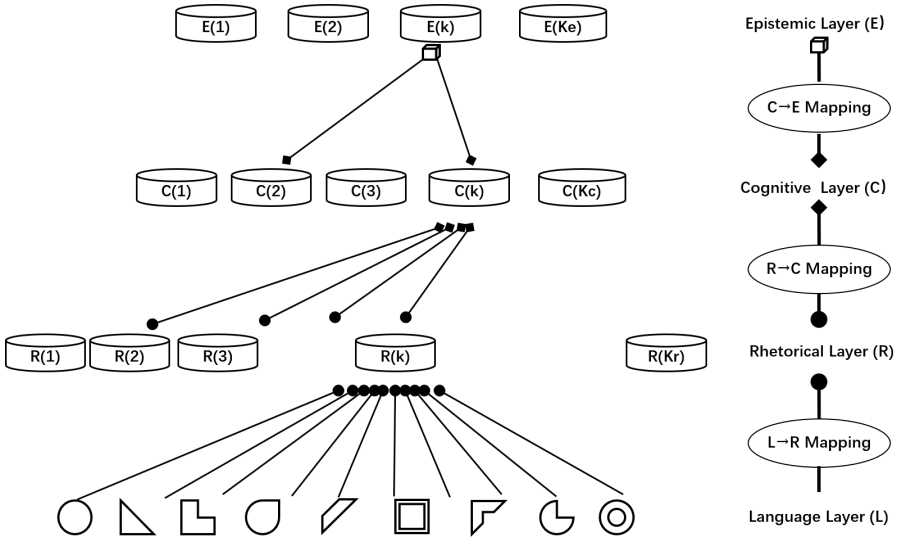

Fig. 7 Schematic display of three layer mapping in general pyramid multilayer mapping

Mapping between rhetorical, cognitive, and epistemic layers reduces cognitive burden, and this reduction can be quantified through information entropy, if mapping can be specified. Entropy provides a natural measure of *uncertainty* or *choice space*—that is, how many possible configurations a writer or reader must navigate.

Consider a rhetorical system with K available modes. If each mode occurs with probability p_i, the entropy of rhetorical choice is

$$H_R(K) = -\sum_{i=1}^{K} p_i \log_2 p_i.$$

When all modes are equally likely, this simplifies to $H_R = \log_2 K$. For instance, with 18 canonical modes, we have $H_R \approx 4.17$ bits, indicating that a writer effectively chooses among about $2^{4.17} \approx 18$ rhetorical alternatives per unit of discourse.

If we instead consider all possible subsets of these modes, the number of choices of size k is given by (1), and the total number of nonempty subsets is given by (2). The corresponding probability mass function is therefore $p_k = \frac{K_C(k,K)}{K_{NRC}(K)}$, and the entropy becomes

$$H_R(K) = -\sum_{k=1}^{K} \frac{\binom{K}{k}}{2^K - 1} \log_2 \left(\frac{\binom{K}{k}}{2^K - 1} \right). \tag{5}$$

This expression represents the Shannon entropy (in bits) of the distribution of subset sizes when each nonempty subset of K elements is equally probable.

According to Cover & Thomas (2006) and Knessl (1998), the right-hand-side of (2) has an asymptotic value $\frac{1}{2}\log_2\left(\frac{\pi e K}{2}\right)$,

$$H_R(K) \approx \frac{1}{2} \log_2 \left(\frac{\pi e K}{2} \right) + o(1).$$

Numerically, for $K = 2$, $H_R \approx 0.92$ bits; for $K = 7$, $H_R \approx 2.39$ bits; for $K = 10$, $H_R \approx 2.70$ bits; for $K = 14$, $H_R \approx 2.95$ bits; for $K = 18$, $H_R \approx 3.13$ bits; for $K = 20$, $H_R \approx 3.21$ bits; and for $K = 100$, $H_R \approx 4.37$ bits.
The reduction of entropy with smaller K implies that introducing a multilayer mapping here each layer involves a smaller number of choices, which reduces the overall uncertainty of rhetorical selection.

For example, if the rhetorical-mode layer has $K = 20$, then $H_R(20) \approx 3.21$ bits. At the next layer, each cognitive function selects among at most four rhetorical modes, giving $H_R(4) \approx 1.81$ bits. Similarly, each epistemic function selects from at most four cognitive functions, again yielding $H_R(4) \approx 1.81$ bits. The combined entropy of these two layers is therefore considerably lower than a direct single-layer selection from all $K = 20$ modes, demonstrating the efficiency of hierarchical or layered rhetorical organization. These numbers demonstrate why multilayer mapping is valuable, though real entropy reduction depends also how we organize the connection between different layers.

Rhetorical diversity expands expressive potential but also inflates cognitive entropy. Cognitive and epistemic mappings act as structured constraints that transform this diversity into intelligible order. The quantitative reduction in entropy thus represents not information loss but *knowledge organization*—a measurable indicator of how academic discourse converts linguistic variety into purposeful understanding.

3.2 The Three Layer Mapping and Applications

3.2.1 Rhetorical Layer to Cognitive Layer

The cognitive layer is supposed to contain K_C cognitive functions, denoted $C(k), k = 1, 2, \cdots, K_C$. It could be imagined that **observe** is one such function, which means to register phenomena and qualitative data, with the help of rhetorical modes (like description, narration, exemplification and evidence) in the R layer. Table 5 listed $K_C = 14$ tyical cognitive functions, including the code, the function name, meaning or operation, and typical rhetorical modes in mapping, i.e., typical rhetorical modes realizing the cognitive functions. The 14 cognitive functions listed in Table 5 are not exclusive. For some specific applications we may have different functions as discussed in section 3.2.4, where we also discuss how rhetorical modes can be refreshed to have more academic oriented meanings.

Table 5 Mapping between Cognitive Functions (C-layer) and Typical Rhetorical Modes (R-layer)

Code	Cognitive Function	Meaning / Operation	Typical Rhetorical Modes Realizing the Function
C(1)	Observe	Register phenomena and qualitative data.	Description, Narration, Exemplification, **Evidence**
C(2)	Identify	Distinguish entities or patterns.	Definition, Contrast, Classification, **Evidence**
C(3)	Compare	Relate attributes or outcomes.	Comparison, Analogy, Evaluation
C(4)	Classify	Categorize into structured groups.	Classification, Division, Definition
C(5)	Abstract	Extract general properties or patterns.	Exemplification, Exposition, Analogy
C(6)	Hypothesize	Formulate possible explanations.	Problem, Cause, Argumentation
C(7)	Model	Represent systems or relations symbolically.	**Process Analysis**, Analogy, Exposition
C(8)	Infer	Deduce implications or rules.	Cause, Effect, Argumentation
C(9)	Test / Validate	Evaluate hypotheses or models against evidence.	**Evidence**, Illustration, Evaluation
C(10)	Explain	Provide causal or functional accounts.	Cause and Effect, Exposition, **Process Analysis**
C(11)	Evaluate	Judge validity or relevance of claims.	Evaluation, Comparison, Argumentation, **Persuasion**
C(12)	Predict	Anticipate outcomes or states.	Analogy, Cause, Process Analysis
C(13)	Integrate / Synthesize	Combine diverse reasoning into a coherent whole.	Exposition, Analogy, Synthesis (from Analysis ynthesis pair)
C(14)	Reflect / Meta-cognitive Assessment	Assess one's reasoning or knowledge limits.	Evaluation, Definition, Problem-solution, Exposition, **Persuasion**

3.2.2 Cognitive Layer to Epistemic Layer

The epistemic layer is supposed to contain K_E epistemic functions, denoted $E(k), k = 1, 2, \cdots, K_E$. It is evident that **Teaching / Learning** is one such function, which means pedagogical transmission of knowledge, with the help of cognitive functions, such as **Observe**, **Identify**, **Model**, **Explain**, **Evaluate**, and **Reflect / Meta-cognitive Assessment**, from the C layer. Table 6 lists all $K_E = 8$ epistemic functions, including the code, function name, meaning or purpose, and typical cognitive functions in mapping, i.e., cognitive operations realizing each epistemic purpose.

Again, the 8 epistemic functions listed in Table 6 are not exclusive. For some specific applications we may have more functions.

3.2.3 An Example in Teaching/Learning

As illustration, we take the example of Wu (2025a), which hyperbolized in a classroom lesson entitled "The Nature of Memory", the teacher uses the method of rhetorical-mode-driven instruction with three layers. The example demonstrates how strategic sequencing of modes can realize distinct pedagogical functions through introducing a cognitive layer. Figure 8 shows the details of the lesson, and Figure 9 is the correspongding three layer mapping.

The lesson "The Nature of Memory" corresponds to **Teaching / Learning** in the epistemic layer.

Part I (Introduction and Cognitive Evocation) corresponds to the cognitive functions **observe** and **identify**, and uses four rhetorical modes (narrative, definition, exemplification and evidence) to stimulate interest and establish topic background.

Table 6 Mapping between Epistemic Functions (E-layer) and Cognitive Functions (C-layer)

Code	Epistemic Function	Meaning / Purpose	Typical Cognitive Functions Realizing the Epistemic Purpose
E(1)	Knowledge Formation	Learning and conceptual development.	Observe, Identify, Classify, Abstract
E(2)	Scientific Discovery	Forming and testing new hypotheses.	Hypothesize, Model, Test / Validate, Infer
E(3)	Communication	Dissemination of knowledge through text or discourse.	Compare, Explain, Integrate / Synthesize, Evaluate
E(4)	Teaching / Learning	Pedagogical transmission of knowledge.	Observe, Identify, Model, Explain, Evaluate, Reflect / Meta-cognitive Assessment
E(5)	Problem Solving	Practical or scientific application of knowledge.	Classify, Hypothesize, Infer, Test / Validate, Evaluate
E(6)	Innovation / Design	Creative generation of new systems or methods.	Model, Abstract, Integrate / Synthesize, Predict
E(7)	Evaluation / Decision-Making	Policy or strategic assessment based on evidence.	Evaluate, Reflect / Meta-cognitive Assessment, Infer
E(8)	Policy / Action Implementation	Applying knowledge in real contexts.	Predict, Explain, Integrate / Synthesize, Evaluate

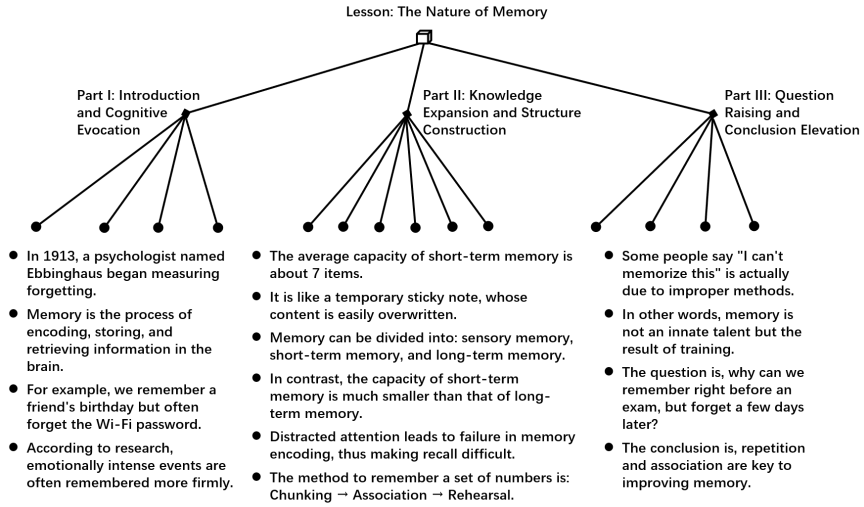

Fig. 8 Composition of a contents in a lesson

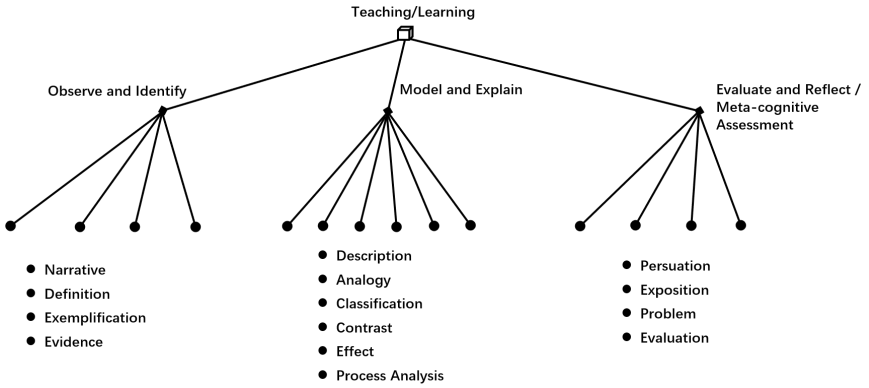

Fig. 9 Correspondence in three layer mapping from the composition of a contents in a lesson shown in Figure 8

Part II (Knowledge Expansion and Structure Construction) corresponds to the cognitive functions **model** and **explain**, and uses six rhetorical modes (description, analogy, classification, contrast, effect, process analysis) to accomplish quantitative description → deconstruct structure→ establish relationships → guide operational path.

Part III (Question Raising and Conclusion Elevation) corresponds to the cognitive functions **evaluate** and **reflect / meta-cognitive assessment**, and uses four rhetorical modes (persuation, exposition problem, evaluation) to summarize causes → eliminate cognitive misconceptions → raise common questions → elevate into teaching conclusions.

The lesson exemplifies a three-tiered orchestration:

1. Evocation complex (narrative + definition + example + evidence)
2. Construction complex (description + classification + comparison + cause–effect + process)
3. Reflection complex (argument + clarification + question + evaluation)

Through deliberate sequencing, the instructor activates cognitive curiosity, builds conceptual structure, and consolidates understanding. The case confirms that effective pedagogy inherently employs cognitive mapping to bridge information and cognition. This educational example demonstrates several theoretical implications.

- Rhetorical layering as cognitive scaffolding: Each layer of the lesson corresponds to a cognitive stage—activation, construction, and reflection—showing that rhetorical orchestration parallels the learning cycle proposed in educational psychology.
- Transferability across disciplines: The same compositional logic governs a physics explanation, a history lecture, or a literature analysis, in the way that knowledge is built by combining definitional, exemplificatory, causal, and evaluative operations.

3.2.4 Alternative modes or functions in each layer and refreshed rhetorical modes for academic function

The three-layer mapping describes a canonical path - from rhetorical modes (R) to cognitive operations (C) and then to epistemic purposes (E)- based on abstract functional classification. However, the rhetorical modes and cognitive-layer functions (Tables 5) and epistemic-layer functions (Table 6) are **not exclusive**. For any specific applications, one may have different modes or functions.

For instance, in some type of academic writing, the rhetorical modes may be refreshed to generate academic functions through combining a core rhetorical mode and some supplementart modes (Wu 2025b). The cognitive functions may be replaced by the following seven ones, each is mapped into some refreshed rhetorical modes, here called academic functions, in the R-layer.

1. R- layer: information presentation function

This function organizes and presents foundational information of the research subject - its background, features, structure, and hierarchy. The typical academic functions are:narration, description, definition, classification, decomposition, grading, summarization, delineation. Cognitive operation: perceptual organization ⇒ Epistemic purpose: establishing shared conceptual ground.

2. R- layer: relational reasoning function

This function analyzes relationships among entities to construct explanatory or theoretical frameworks. The typical academic functions are: *comparison, contrast, analogy, relational analysis, causal analysis, induction, synthesis.* Cognitive operation: *analytical linking* ⇒ Epistemic purpose: *constructing relational understanding.*

3. R- layer: process construction function

This function explains sequences, developments, or operational steps, common in methods and procedures. The typical academic functions are:: *process analysis, procedural description.* Cognitive operation: *sequential reasoning* ⇒ Epistemic purpose: *revealing operational logic.*

4. R- layer: argumentation support function

This builds and supports authorial claims and reasoning chains. Typical academic functions are: *exemplification, evidence, argumentation, persuasion, elaboration, claim-making.* Cognitive operation: *inferential justification* ⇒ Epistemic purpose: *validating propositions.*

5. R- layer: understanding construction function

This deepens conceptual clarity and connects known with unknown. Typical academic functions are: *clarification, explanation.* Cognitive operation: *integrative comprehension* ⇒ Epistemic purpose: *achieving conceptual coherence.*

6. R- layer: interaction construction function

This duides the reader for thinking and invites engagement. Typical academic functions are: *questioning, answering.* Cognitive operation: *dialogic coordination* ⇒ Epistemic purpose: *co-constructing understanding.*

7. R- layer: evaluation and reflection function

This assesses value or validity of ideas, methods, or results. Typical academic functions are: *evaluation, verification, validation.* Cognitive operation: *critical judgment* ⇒ Epistemic purpose: *determining reliability and significance.*

3.2.5 Mapping through code mode strenthened by supplementary modes

As mentioned above, academic functions (R_{academic}) are still in the R- layer, and represent refreshed modes, each is obtained through combining a core rhetorical mode (also called dominant mode) and some supplementart modes (also called auxiliary mode). These refreshed modes are more closer to cognitive functions.

Formally, each academic function emerges through a special mapping that combines a core rhetorical mode with several supplementary modes:

$$R_{\text{core}} + \sum_{i=1}^{m} R_{\text{sup},i} \Rightarrow R_{\text{academic}}$$

An academic function may have the same name of a rhetorical mode, such as definition, or different name, such as claim.

1. Example one: definition as academic function

When definition is strengthened to be an academic function, then the core mode is Definition and the supplementary ones are Comparison, Exemplification, Classification. The *definition complex* expands a simple defining act into a multidimensional explanation in the following way

- Core mode: Definition - states the essential meaning (e.g. “Memory is the mental process by which information is encoded, stored, and retrieved.”)
- Supplementary 1: Comparison - clarifies by opposition or similarity.(e.g.“Unlike perception, which involves immediate sensory input, memory preserves information beyond the present moment.”)
- Supplementary 2: Exemplification - grounds meaning in instance.(e.g.“For instance, recalling a friend’s birthday or the melody of a song illustrates long-term memory.”)
- Supplementary 3: Classification - locates the term within a category system.(e.g.“Memory can be divided into sensory, short-term, and long-term types, each serving distinct cognitive functions.”)

Together, these four operations form a higher-order definition mode that realizes the academic function (Definition), performing *conceptual structuring* more closer to the cognitive layer and *establishing academic meaning* at the epistemic layer: 2. Example two: claim as academic function

For the claim function, the core rhetorical mode is Description or Cause–Effect, and the supplementary ones are Argumentation or Process Analysis, Evaluation, Comparison and Evidence. The *claim complex* thus represents an academic function distinct from its core rhetorical base. It transforms a mere assertion into a reasoned, validated proposition, in the following way

- Core mode: Description / Cause–Effect -illustates a relation (e.g. “Theorem 1. In any right triangle, the hypotenuse is longer than either leg.”)

- Supplementary 1: Argumentation or Process Analysis - explains why it holds. (e.g."Because the square of the hypotenuse equals the sum of the squares of the other two sides, its measure must exceed each leg.")
- Supplementary 2: Evaluation- assesses disciplinary importance. (e.g."This theorem forms the foundation of Euclidean geometry and underpins trigonometric calculation.")
- Supplementary 3: Comparison - differentiates it from alternative claims. (e.g."Unlike empirical generalizations, this theorem is derived deductively rather than from observation.")
- Supplementary 4: Evidence - anchors the claim in verification. (e.g."This result is confirmed by Euclid - Elements, Book I, Proposition 47.")

When integrated, these operations create a higher-order claim mode realizing Claim, which performs *inferential justification* cognitively and *knowledge validation* epistemically.

4 Conclusion

This study has sought to reinterpret rhetorical modes as measurable, generative elements rather than static pedagogical categories. By introducing non-exclusive four duality-based mode operations (split-unite duality, forward-backward duality, expansion-reduction duality, orthogonal duality), the work demonstrates how new rhetorical forms can be derived from canonical ones, yielding an extended, more symmetrical system of expressive operations. Quantitative modeling through binomial combinatorics shows that rhetorical capacity expands exponentially with the number of available modes, and the notion of a Marginal Rhetorical Bit (MRB) offers a simple measure of expressive gain per added mode. The rate of rhetorical introduction further enables the definition of a rhetorical-scalable parameter, connecting rhetorical growth with cognitive development across educational stages.

To reduce the cognitive complexity in using a growing number of rhetorical modes, this study also proposed a pyramid multilayer mapping framework that links rhetorical, cognitive, and epistemic layers. The proposed mappings provide a conceptual bridge between linguistic diversity and knowledge structure, suggesting that rhetorical systems can be both expansive and cognitively economical.

While the *pyramid multilayer mapping* defines a conceptual taxonomy linking rhetorical, cognitive, and epistemic layers in a hierarchical manner and offers a structural ontology of functions (e.g., $R \to C \to E$), however, the modes or functions in each layer should not be considered as fixed. For instance, for academic writing, the R layer could use refreshed rhetorical mode (academic function) through through compositional orchestration, i.e., through combining a core mode and some supplementary modes. The same may be true for cognitive layer and epistemic layer.

Several limitations should be acknowledged. For instance, there lacks a formal way to evaluate whether a generated rhetorical mode can be used as a rhetorical mode. The mappings among layers, while conceptually clear, could vary across disciplines and languages, requiring further validation. Moreover, the operator calculus for mode generation is presented schematically and would benefit from formalization in symbolic or computational terms.

Even with these constraints, the results indicate that rhetorical modes can be studied quantitatively and modeled systematically. Such an approach could inform writing pedagogy, academic writing, and computational language models. Future work could explore how AI systems might apply layered rhetorical reasoning in discourse generation. In this sense, the present study offers an initial step toward integrating rhetorical mode theory with quantitative and computational perspectives in a careful, incremental manner.

Acknowledgements This work draws upon insights accumulated over six years of teaching academic writing to undergraduate and graduate students, whose thoughtful questions and reflections continually inspired the development of this study. The author made limited use of AI-powered language tools to assist with linguistic refinement and the formulation of illustrative examples. Post-doc student Wang MM and PhD student Wang XY helped to prepare the latex file and some figures.

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