

An anomaly-free leptophilic $U(1)'_\ell$ completion of the Standard Model

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Abstract

We develop a minimal and fully anomaly-free realization of a universal leptophilic $U(1)'_\ell$ gauge symmetry, under which all Standard-Model leptons carry a common charge while quarks remain neutral. Gauge consistency is restored by introducing one vectorlike lepton family and singlet scalars whose vacuum expectation values both break $U(1)'_\ell$ and generate masses for the new gauge boson Z_ℓ and the heavy leptons.

We derive compact anomaly-cancellation conditions, closed-form Z_ℓ couplings and decay widths, and the structure of gauge-kinetic mixing, including the trace condition $\text{Tr}(YQ') = 0$ that suppresses radiative mixing. The setup naturally embeds a type-I seesaw for neutrino masses and provides a stable singlet field as a minimal dark-matter candidate. Collider and precision constraints are expressed analytically over the full parameter space, covering pure leptophilic production, small-mixing hadron colliders, and the contact-interaction limit $M_{Z_\ell} \gg \sqrt{s}$.

The model thus provides a single, internally consistent benchmark that unifies gauge, scalar, Yukawa, neutrino, and dark-matter sectors, and is directly suited for global data recasts and targeted searches at current and future lepton and hadron colliders

I. INTRODUCTION

Neutrino oscillations [1] have established that individual lepton flavors are not conserved, implying physics beyond the Standard Model (SM). A minimal way to address this is to gauge an additional Abelian symmetry that acts on leptons. In this spirit, we consider a general leptophilic $U(1)'_\ell$ extension in which the three SM lepton families carry arbitrary charges (q_e, q_μ, q_τ) while quarks are neutral. Prominent special cases include the universal assignment $q_e = q_\mu = q_\tau$ and the well-known $L_\mu - L_\tau$ scenario [2]. The associated gauge boson, denoted Z'_ℓ (or Z_ℓ henceforth), couples to leptons at tree level, while any quark coupling arises only via kinetic mixing.

Extensions of the SM by extra $U(1)$ factors remain among the simplest and most predictive portals for new interactions [3]. A fully consistent framework, however, requires a UV-complete realization in which gauge, mixed, and gravitational anomalies are canceled and the symmetry is spontaneously broken. Here we construct such a realization for a generic

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leptophilic $U(1)'_\ell$ by introducing one vectorlike lepton family and SM-singlet scalars whose vacuum expectation values generate masses for both Z_ℓ and the heavy leptons. We work in a left-handed Weyl basis and derive compact anomaly-cancellation relations applicable to both universal and nonuniversal charge assignments.

Kinetic mixing between $U(1)_Y$ and $U(1)'_\ell$ [4] is radiatively induced unless $\text{Tr}(YQ') = 0$, a condition naturally satisfied for several anomaly-free solutions. We present analytic Z_ℓ couplings and widths, including kinetic-mixing effects, and discuss useful limits such as the contact-interaction regime $M_{Z_\ell}^2 \gg s$ [5]. The setup accommodates type-I seesaw realizations of neutrino masses and admits minimal dark-matter candidates in the form of stable singlet fermions annihilating through Z_ℓ exchange [6], thereby tying collider, precision, and cosmological probes to a single gauge-parameter space.

The goal of this paper is analytic and model-building-oriented: to provide a closed-form benchmark that links gauge, scalar, and Yukawa sectors and enables transparent phenomenological mappings. We present the field content, charge assignments, anomaly checks, symmetry breaking, mass spectra, kinetic-mixing structure, and resulting parametric dependences for collider, precision, and dark-sector observables. A full global analysis (including likelihood fits and public code) is left to future work.

Models gauging total lepton number have been discussed in various contexts [7–12]. The present work differs by providing a fully anomaly-free realization with a single vectorlike family, connecting the $U(1)'_\ell$ breaking scale to neutrino masses and dark matter, and by embedding all phenomenological predictions in a compact $(M_{Z_\ell}, g_{\text{eff}})$ parameter space.

The paper is organized as follows. Section II details the gauge structure and anomaly cancellation; Sec. III describes the scalar sector and symmetry breaking; Sec. IV covers Yukawa couplings and heavy-lepton masses; Sec. V analyzes kinetic mixing; Sec. VI outlines neutrino-mass realizations; Sec. VII discusses dark-matter candidates; Sec. VIII summarizes collider, precision, and cosmological implications; Sec. IX concludes.

II. GAUGE STRUCTURE AND ANOMALY CANCELLATION

Conventions. We use $Q = T_3 + Y$ with $Y_{\text{PDG}} = 2Y$. The covariant derivative is $D_\mu = \partial_\mu - igW_\mu^a T^a - ig_Y Y B_\mu - ig_\ell Q' B'_\mu$, and kinetic mixing ε is introduced in Sec. V.

We enlarge the SM gauge symmetry to

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'_\ell,$$

where $U(1)'_\ell$ acts on leptons but not on quarks. We adopt the standard hypercharge normalization $Q = T_3 + Y$, implying $Y = Y_{\text{PDG}}/2$.

Before turning to the most general flavor-dependent case, it is instructive to begin with a universal charge assignment and examine the resulting anomaly pattern.

A. Universal assignment and anomaly constraints

A genuinely leptophilic setup follows from assigning a common charge a_ℓ to all lepton multiplets,

$$Q'(L_\ell) = Q'(e_{R,\ell}) = a_\ell, \quad Q'(\text{quarks}) = 0,$$

while keeping the Higgs uncharged. Unlike the special choice $L_\mu - L_\tau$ [13, 14], this universal assignment induces mixed anomalies with $SU(2)_L$ and $U(1)_Y$. A consistent $U(1)'$ gauge theory requires these anomalies to vanish; otherwise gauge invariance would be violated at the quantum level.

Anomaly structure and cancellation strategy. We evaluate anomalies in a left-handed Weyl basis, replacing right-handed fields by their conjugates with opposite $U(1)'_\ell$ and $U(1)_Y$ charges. Including three right-handed neutrinos, the SM leptons contribute

$$[SU(2)_L]^2 U(1)'_\ell : \quad \frac{3}{2} a_\ell \neq 0, \tag{1}$$

$$[U(1)_Y]^2 U(1)'_\ell : \quad -\frac{3}{2} a_\ell \neq 0, \tag{2}$$

$$[U(1)'_\ell]^2 U(1)_Y : \quad 0, \tag{3}$$

$$[\text{grav}]^2 U(1)'_\ell : \quad 0, \tag{4}$$

$$[U(1)'_\ell]^3 : \quad 0. \tag{5}$$

The mixed anomalies must therefore be canceled by new fermions. Guided by the principle of minimality—keeping the exotic content to a single vectorlike family while retaining anomaly freedom—we introduce one vectorlike lepton doublet (L'_L, L'_R) and one vectorlike charged singlet (E'_L, E'_R) , vectorlike under the SM but chiral under $U(1)'_\ell$.¹

¹ See Appendix A for charge conventions and explicit derivations of the anomaly conditions.

B. Complete anomaly cancellation for general (q_e, q_μ, q_τ)

It is useful to record the general solution valid for arbitrary lepton charges (q_e, q_μ, q_τ) , recovering the universal case by setting $q_\alpha = a_\ell$. Working in a left-handed basis with $Q'(L_\alpha) = Q'(e_\alpha^c) = q_\alpha$ and defining

$$\Sigma q = q_e + q_\mu + q_\tau, \quad \Sigma q^3 = q_e^3 + q_\mu^3 + q_\tau^3,$$

the anomaly-free assignments for the exotics are

$$\boxed{\Delta_L = -\Sigma q, \quad \Delta_E = +\Sigma q, \quad \text{Tr}(YQ') = 0 \Rightarrow s_E = -\Sigma q},$$

where $\Delta_{L,E} \equiv Q'(L'_{L,E}) - Q'(L'_{R,E})$ and $s_{L,E} \equiv Q'(L'_{L,E}) + Q'(L'_{R,E})$. These relations cancel $[SU(2)_L]^2 U(1)'_\ell$, $[U(1)_Y]^2 U(1)'_\ell$, and gravitational anomalies, while the cubic condition fixes s_L :

$$\boxed{s_L = -\frac{4}{3} \frac{\Sigma q^3}{(\Sigma q)^2} + \Sigma q \quad (\Sigma q \neq 0)}.$$

Corollary ($\Sigma q = 0$). — If $\Sigma q = 0$, as in $L_\mu - L_\tau$ or $L_e - L_\mu$, all anomalies cancel within the SM (augmented by ν_R), and no exotic fermions are required.

This construction shows that anomaly-free leptophilic $U(1)'$ models are highly constrained yet sufficiently flexible to realize both universal and flavor-dependent scenarios. Together with the scalar and Yukawa sectors discussed in the following sections, it completes the anomaly-free backbone promised in Sec. I, establishing the theoretical closure that will underpin the phenomenological analysis to come.

Explicit benchmark example. For illustration, consider the nonuniversal choice

$$(q_e, q_\mu, q_\tau) = (1, 1, -2),$$

for which $\Sigma q = 0$ and $\Sigma q^3 = 1 + 1 - 8 = -6$. In this case all gauge and gravitational anomalies cancel within the SM (augmented by ν_i^c) and no exotic fermions are required, as anticipated by the corollary below Eq. (12). By contrast, for a nonvanishing trace, e.g.

$$(q_e, q_\mu, q_\tau) = (1, 0, 0),$$

one finds $\Sigma q = 1$ and the anomaly conditions fix the chiral differences $\Delta_{L,E}$ and the sums $s_{L,E}$ as in Eq. (11), together with the cubic constraint on s_L . The corresponding Yukawa

structure follows from the same charge bookkeeping: for each lepton family the admissible Φ -Yukawa couplings are obtained by solving $Q'(F_L)+Q'(F_R)+Q'(\Phi) = 0$, so that mass terms for the exotic states are again directly tied to the $U(1)'_\ell$ breaking pattern. This makes explicit how the general framework smoothly interpolates between universal and family-dependent leptophilic scenarios.

III. SCALAR SECTOR AND SYMMETRY BREAKING

Spontaneous breaking of $U(1)'_\ell$ generates the Z_ℓ mass and induces a possible mixing between the SM Higgs and a new singlet scalar. We extend the scalar sector by adding an SM-singlet field Φ of charge q_Φ under $U(1)'_\ell$. The renormalizable potential reads

$$V(H, \Phi) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \lambda_{H\Phi} |H|^2 |\Phi|^2, \quad (6)$$

with vacuum expectation values (vevs)

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Phi \rangle = \frac{v_\Phi}{\sqrt{2}}.$$

The $U(1)'_\ell$ gauge boson and scalar mass spectrum follow from the covariant derivative and the H - Φ mixing:

$$M_{Z_\ell} = g_\ell |q_\Phi| v_\Phi, \quad \tan 2\theta = \frac{2\lambda_{H\Phi} v v_\Phi}{2\lambda_\Phi v_\Phi^2 - 2\lambda_H v^2}. \quad (7)$$

The observed Higgs boson is

$$h = h_{\text{SM}} \cos \theta + \phi \sin \theta,$$

so that the Higgs signal strengths scale as $\mu_i = \cos^2 \theta$ at leading order, while the singlet-like state ϕ inherits suppressed SM couplings $\propto \sin \theta$. The angle θ therefore controls both current Higgs-coupling constraints and the visibility of ϕ at colliders. Stability of the potential follows the usual bounded-from-below conditions,² and explicit tadpole and mass expressions are collected in Appendix B.

Relevant interactions include the vertices $\phi Z_\ell Z_\ell$ (from the scalar kinetic term) and $\phi h h$ (from the Higgs-portal coupling). A possible gauge-kinetic mixing term of the form

$$\mathcal{L} \supset -\frac{\varepsilon}{2} B_{\mu\nu} F'^{\mu\nu}$$

² Namely $\lambda_H, \lambda_\Phi > 0$ and $\lambda_{H\Phi} > -2\sqrt{\lambda_H \lambda_\Phi}$.

is allowed by symmetry and is treated as a small independent parameter.

Phenomenological regimes. If $m_\phi > 2M_{Z_\ell}$, the decay $\phi \rightarrow Z_\ell Z_\ell$ dominates via the gauge interaction; for $m_\phi < 2M_{Z_\ell}$, decays into SM final states proceed through the Higgs portal. It is therefore convenient to express predictions in the $(M_{Z_\ell}, g_{\text{eff}})$ plane, where $g_{\text{eff}} = g_\ell a_\ell$ denotes the effective leptonic coupling of Z_ℓ , and

$$v_\Phi = \frac{M_{Z_\ell}}{|q_\Phi|g_\ell}$$

fixes the singlet vev in terms of observables. Combined constraints from Higgs signal strengths ($\theta \lesssim 0.1$) and dilepton searches at colliders then restrict viable regions in this plane.

While a strong first-order $U(1)'_\ell$ transition may yield gravitational-wave signatures, such effects are model-dependent and will not be pursued here.

IV. YUKAWA SECTOR AND LEPTON MASSES

The anomaly-cancelling fermions are vectorlike under the SM gauge group but chiral under $U(1)'_\ell$, and hence their Dirac masses arise from Yukawa couplings to the scalar(s) responsible for $U(1)'_\ell$ breaking. For the minimal anomaly-free charge assignment (explicitly given in Appendix A),

$$q(L') = q(L'^c) = -\frac{3}{2}a_\ell, \quad q(E') = q(E'^c) = q(N') = q(N'^c) = +\frac{3}{2}a_\ell,$$

which uniquely determines the allowed scalar charges entering the Yukawa operators.

A. Minimal single-singlet implementation

With a single SM-singlet scalar Φ of charge $q_\Phi = +3a_\ell$, the renormalizable Yukawa Lagrangian takes the form

$$\mathcal{L}_Y \supset y_L \Phi L' L'^c + y_E \Phi^\dagger E' E'^c + y_N \Phi^\dagger N' N'^c + \text{h.c.}, \quad (8)$$

yielding the mass terms

$$m_{L'} = \frac{y_L v_\Phi}{\sqrt{2}}, \quad m_{E'} = \frac{y_E v_\Phi}{\sqrt{2}}, \quad m_{N'} = \frac{y_N v_\Phi}{\sqrt{2}}, \quad (9)$$

after $\langle \Phi \rangle = v_\Phi / \sqrt{2}$. Making use of $M_{Z_\ell} = g_\ell |q_\Phi| v_\Phi$ and $g_{\text{eff}} = g_\ell a_\ell$, the exotic masses can be compactly expressed as

$$m_{L', E', N'} = \frac{y_{L, E, N}}{3\sqrt{2}} \frac{M_{Z_\ell}}{|g_{\text{eff}}|}, \quad (10)$$

which highlights their proportionality to the $U(1)'_\ell$ -breaking scale. This form is especially convenient when scanning over the $(M_{Z_\ell}, g_{\text{eff}})$ parameter space in collider studies.

B. Two-singlet generalization

If two scalars $\Phi_{1,2}$ with charges $q_{1,2}$ acquire vevs $v_{1,2}$, gauge invariance in the universal benchmark requires

$$q_1 = -(q(L') + q(L'^c)) = +3a_\ell, \quad q_2 = +(q(E') + q(E'^c)) = +3a_\ell,$$

so that $q_1 = q_2$ in this specific realization. In other words, both Φ_1 and Φ_2 are *allowed* to couple to (L', E', N') at the renormalizable level. In our minimal texture we choose a basis in which L' masses arise dominantly from Φ_1 while E' and N' receive masses from $\Phi_2^\dagger$,

$$\mathcal{L}_Y \supset y_L \Phi_1 L' L'^c + y_E \Phi_2^\dagger E' E'^c + y_N \Phi_2^\dagger N' N'^c + \text{h.c.}, \quad (11)$$

so that

$$m_{L'} = \frac{y_L v_1}{\sqrt{2}}, \quad m_{E'} = \frac{y_E v_2}{\sqrt{2}}, \quad m_{N'} = \frac{y_N v_2}{\sqrt{2}}. \quad (12)$$

Additional mixed couplings (e.g. $\Phi_1^\dagger E' E'^c$ or $\Phi_2 L' L'^c$) are technically natural to set to zero: they are absent at tree level and only regenerated radiatively at higher order, and including them does not affect anomaly cancellation. We therefore retain the simpler texture above as our benchmark without loss of generality.

C. Mixing structure and flavor protection

For multiple charged singlets E'_i , the mass matrix takes the block form

$$\mathcal{L}_m = \begin{pmatrix} \overline{L'_L} & \overline{E'_{L,1}} & \overline{E'_{L,2}} \end{pmatrix} \begin{pmatrix} m_{L'} & 0 & 0 \\ 0 & M_{11} & M_{12} \\ 0 & M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} L'_R \\ E'_{R,1} \\ E'_{R,2} \end{pmatrix} + \text{h.c.}, \quad (13)$$

where $M_{ij} = \frac{y_{ij}^E v_2}{\sqrt{2}}$. The absence of $L'-E'$ mixing at the renormalizable level follows directly from the $U(1)'_\ell$ charge assignments, providing a built-in protection against charged-lepton flavor violation (CLFV). Operators such as $\bar{L}HE'_R$ or $\bar{L}'_L H e_R$ are forbidden in the minimal setup, and any higher-dimensional corrections (e.g. $(\bar{L}HE'_R)(\Phi/\Lambda)^n$) are strongly constrained by lepton-universality tests [15, 16].

Theoretical necessity. The additional vectorlike leptons introduced above are not an ad hoc embellishment but a *minimal and unavoidable* ingredient of the construction: they are required to ensure full gauge and gravitational anomaly *cancellation* in a leptophilic $U(1)'_\ell$ extension. While such states have not been observed experimentally, their presence is a generic prediction of anomaly-free frameworks of this type and can be probed through multilepton and (potentially) long-lived signatures at colliders. This theoretical necessity underpins the phenomenological study in Sec. VIII, where we map the $(M_{Z_\ell}, g_{\text{eff}})$ plane to the exotic-lepton spectrum and search channels.

D. Couplings and collider outlook

The exotic fermions couple to Z_ℓ with strength $g_\ell q_{F'}$ ($F' = L', E', N'$), and the interactions are purely vectorlike when $q_{L'} = q_{L'^c}$, etc. Current LHC bounds on vectorlike leptons lie in the $\mathcal{O}(10^2\text{--}10^3)$ GeV range, with ATLAS excluding masses between 130–900 GeV in third-generation doublet scenarios and CMS excluding up to about 790 GeV in multilepton final states [17, 18]. In regions where mixing with SM leptons is suppressed, long-lived signatures may arise; otherwise, prompt decays yield $Z_\ell + \ell$ or $Z_\ell + \cancel{E}_T$ final states. The Yukawa sector thus provides a predictive link between $(M_{Z_\ell}, g_{\text{eff}})$, the exotic-lepton spectrum, and their collider phenomenology.

Together with the anomaly-free gauge and scalar sectors discussed above, the Yukawa structure completes the renormalizable foundation of the model, linking the symmetry-breaking scale to the observable lepton spectrum and collider signatures.

V. KINETIC MIXING AND PRECISION CONSTRAINTS

Two Abelian gauge factors generically allow a renormalizable kinetic-mixing term,

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\varepsilon}{2}B_{\mu\nu}F'^{\mu\nu}, \quad (14)$$

parametrized by the dimensionless coefficient ε [4]. Even if set to zero at tree level, loops of fermions carrying both $U(1)_Y$ and $U(1)'_\ell$ charges radiatively induce

$$\varepsilon(\mu) \simeq \frac{g_Y g_\ell}{16\pi^2} \text{Tr}(YQ') \ln \frac{\Lambda^2}{\mu^2} + \text{finite thresholds}, \quad (15)$$

where $\text{Tr}(YQ') \equiv \sum_f Y_f Q'_f$ is over chiral fermions. Hence the leading logarithm vanishes if

$$\boxed{\text{Tr}(YQ') = 0} \quad (16)$$

and ε remains threshold-suppressed and naturally small.³ Throughout we adopt the convention $Q = T_3 + Y$.

A. Mass basis and induced couplings

After electroweak symmetry breaking and canonical normalization to $\mathcal{O}(\varepsilon)$, the shift $B_\mu \rightarrow B_\mu + \varepsilon B'_\mu$ removes the mixed kinetic term and induces a small interaction between Z_ℓ and the hypercharge current,

$$\mathcal{L} \supset \varepsilon g_Y J_Y^\mu B'_\mu \Rightarrow g_{Z_\ell f} \rightarrow g_{Z_\ell f} + \varepsilon g_Y Y_f,$$

so that quarks acquire effective Z_ℓ couplings proportional to their hypercharge. Exact leptophilia is therefore spoiled unless Eq. (16) holds, in which case ε is loop-suppressed and model-dependent.

Example with $\text{Tr}(YQ') = 0$. A consistent assignment that satisfies the anomaly-cancellation relations (cf. Secs. II and IV) and enforces $\text{Tr}(YQ') = 0$ is

$$q(L') = 0, \quad q(L'^c) = +3a_\ell, \quad q(E') = +3a_\ell, \quad q(E'^c) = 0, \quad q(N') \text{ arbitrary}, \quad q(N'^c) \text{ arbitrary}, \quad (17)$$

which yields chiral differences $\Delta_L \equiv q(L') - q(L'^c) = -3a_\ell$, $\Delta_E \equiv q(E') - q(E'^c) = +3a_\ell$ (as required for the universal case with $\Sigma q = 3a_\ell$). The hypercharge trace from the exotic sector then obeys

$$\text{Tr}(YQ')_{\text{new}} = \underbrace{(2Y_L) \Delta_L}_{-\Delta_L} + \underbrace{Y_E \Delta_E}_{-\Delta_E} + \underbrace{Y_N \Delta_N}_0 = -(\Delta_L + \Delta_E) = 0,$$

³ For vectorlike pairs with masses $m_1 \neq m_2$, finite threshold pieces scale as $\Delta\varepsilon \sim (g_Y g_\ell / 16\pi^2) (Y \Delta Q') \ln(m_1^2/m_2^2)$, yielding $\varepsilon \sim 10^{-4}$ – 10^{-3} for $\mathcal{O}(1)$ splittings.

since $2Y_L = -1$, $Y_E = -1$, and $Y_N = 0$ in our convention. For the SM leptons with universal a_ℓ (and including ν_R), one has $\text{Tr}(YQ')_{\text{SM}} = 0$, so the total trace vanishes. Consequently, ε is generated only by finite threshold effects from mass splittings within vectorlike pairs and can naturally lie in the 10^{-4} – 10^{-3} range.

B. Precision bounds

A nonzero ε induces Z_ℓ –quark couplings and shifts in leptonic observables. The resulting limits arise from (i) LEP electroweak precision tests, (ii) low-energy parity violation, and (iii) high-mass dilepton searches at the LHC. For $M_{Z_\ell} \lesssim \text{TeV}$, representative bounds require

$$|\varepsilon| \lesssim \mathcal{O}(10^{-2}),$$

with the precise limit depending on M_{Z_ℓ} and decay modes [19–21]. In what follows we either enforce $\text{Tr}(YQ') = 0$ or treat ε as an independent loop-suppressed parameter and display contours in the $(M_{Z_\ell}, g_{\text{eff}})$ plane.

C. Small-angle Z – Z_ℓ mixing

Starting from the kinetic-mixed Lagrangian, we first normalize away the B – B' term via $B_\mu \rightarrow B_\mu + \varepsilon B'_\mu + \mathcal{O}(\varepsilon^2)$. After the usual Weinberg rotation $(B_\mu, W_\mu^3) \rightarrow (A_\mu, Z_\mu^0)$, one obtains a 2×2 mass submatrix in the (Z^0, B') sector:

$$\mathcal{M}_{(Z^0, B')}^2 = \begin{pmatrix} M_Z^2 & \delta \\ \delta & M_{Z_\ell}^2 \end{pmatrix} + \mathcal{O}(\varepsilon^2),$$

with $M_Z^2 = \frac{1}{4}(g^2 + g_Y^2)v^2$ and M_{Z_ℓ} set by $U(1)'_\ell$ breaking. For an uncharged Higgs, $\delta \simeq \varepsilon s_W M_Z^2$ at leading order.⁴ A small mixing angle,

$$\xi \simeq \frac{\delta}{M_{Z_\ell}^2 - M_Z^2},$$

diagonalizes the system, inducing $\mathcal{O}(\xi, \varepsilon)$ shifts in Z couplings, and

$$g_{Z_\ell f} \rightarrow g_\ell Q'_f + \varepsilon g_Y Y_f + \mathcal{O}(\xi).$$

A compact derivation, including the full 3×3 neutral-gauge-boson mass matrix and the normalization of gauge eigenstates, is provided in Appendix C.

⁴ $s_W \equiv \sin \theta_W$ and $c_W \equiv \cos \theta_W$.

VI. NEUTRINO MASSES

A leptophilic $U(1)'_\ell$ naturally connects the gauge sector to the origin of neutrino masses. In the left-handed Weyl basis (Appendix A), the SM Dirac operator

$$\mathcal{L} \supset y_\nu L H \nu^c + \text{h.c.}, \quad Q'(L) = +a_\ell, \quad Q'(\nu^c) = -a_\ell, \quad Q'(H) = 0, \quad (18)$$

yields $m_D = y_\nu v / \sqrt{2}$ after electroweak symmetry breaking. Purely Dirac neutrinos would then require $y_\nu \sim 10^{-11}$, making the seesaw mechanism the more compelling alternative.

A. Type-I seesaw from $U(1)'_\ell$ breaking

A Majorana mass for the right-handed neutrinos can be generated by a singlet S with

$$Q'(S) = +2a_\ell,$$

via

$$\mathcal{L} \supset \frac{1}{2} y_N S \nu^c \nu^c + \text{h.c.}, \quad (19)$$

so that $\langle S \rangle = v_S / \sqrt{2}$ induces $M_N = y_N v_S / \sqrt{2}$ while the Dirac term remains $m_D = y_\nu v / \sqrt{2}$. In the (ν, ν^c) basis,

$$\boxed{\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}, \quad m_\nu \simeq -m_D^T M_N^{-1} m_D} \quad (20)$$

so that the light-neutrino scale is set by the same $U(1)'_\ell$ -breaking dynamics that generates M_{Z_ℓ} .

Since S is charged under $U(1)'_\ell$, its vev also contributes to the Z_ℓ mass. In the presence of all singlet vevs, the gauge boson mass is

$$M_{Z_\ell}^2 = g_\ell^2 (q_\Phi^2 v_\Phi^2 + q_S^2 v_S^2 + \dots), \quad (21)$$

which reduces to Eq. (7) when $v_S \ll v_\Phi$ or when the S contribution is absorbed into an effective v_Φ . In our phenomenological benchmarks we are effectively working in this limit, so that M_{Z_ℓ} can be parameterized in terms of a single breaking scale while the seesaw scale is controlled by v_S and y_N .

In the minimal benchmark where M_{Z_ℓ} arises from a scalar Φ with $Q'(\Phi) = 3a_\ell$, the seesaw requires S to be distinct from Φ (since $Q'(S) = 2a_\ell$). This is fully compatible with anomaly

cancellation and fits naturally within the two-singlet scalar structure of Appendix B, where the CP-odd mode is lifted by the soft term μ_{12}^2 .

B. Phenomenology and cosmology

For $v_S \sim \text{TeV}$, the heavy neutrinos may be accessible at colliders through displaced-vertex or multilepton signatures [22–24]. For larger v_S , the seesaw scale decouples but still permits thermal leptogenesis [25, 26]. Because ν^c carries $Y = 0$, sterile states do not contribute to $\text{Tr}(YQ')$ and thus do not upset the kinetic-mixing structure of Sec. V. The neutrino sector therefore provides a second, independent probe of the $U(1)'_\ell$ breaking scale, linking collider, precision, and cosmological signatures. Taken together, the anomaly-free gauge completion, scalar breaking, and neutrino-mass generation establish a coherent leptophilic framework linking particle masses to the same $U(1)'_\ell$ origin.

VII. DARK MATTER CANDIDATE

A leptophilic $U(1)'_\ell$ framework admits a stable dark-matter (DM) candidate without imposing any extra Z_2 symmetry. An SM-singlet field charged under $U(1)'_\ell$ is automatically stable whenever no renormalizable operator permits its decay, and in many cases a residual discrete symmetry survives after $U(1)'_\ell$ breaking. We consider two minimal realizations: (i) a Dirac fermion χ with charge q_χ and coupling $g_\chi \equiv g_\ell q_\chi$, and (ii) a complex scalar X with the same charge. In both cases, annihilation is dominated by s -channel Z_ℓ exchange into leptons.

A. Thermal relic density

The standard freeze-out estimate,

$$\Omega_{\text{DM}} h^2 \simeq \frac{1.07 \times 10^9 x_f}{\sqrt{g_*} M_{\text{Pl}} \langle \sigma v \rangle}, \quad (22)$$

is controlled by $\langle \sigma v \rangle_{\text{ann}}$ into $\ell^+ \ell^-$ and $\nu \bar{\nu}$. For universal $U(1)'_\ell$ charges and massless final leptons,

$$\langle \sigma v \rangle_{\chi \bar{\chi} \rightarrow \ell \ell} \simeq \frac{N_\ell}{2\pi} \frac{g_\chi^2 g_{\text{eff}}^2 m_\chi^2}{(M_{Z_\ell}^2 - 4m_\chi^2)^2 + M_{Z_\ell}^2 \Gamma_{Z_\ell}^2}, \quad (23)$$

$$\langle \sigma v \rangle_{XX^* \rightarrow \ell\ell} \simeq \frac{N_\ell}{6\pi} \frac{g_\chi^2 g_{\text{eff}}^2 m_X^2 v_{\text{rel}}^2}{(M_{Z_\ell}^2 - 4m_X^2)^2 + M_{Z_\ell}^2 \Gamma_{Z_\ell}^2}, \quad (24)$$

where $g_{\text{eff}} = g_\ell a_\ell$ and N_ℓ counts accessible leptonic species ($N_\ell = 6$ for three charged and three Dirac neutrinos). Dirac DM is s -wave, whereas complex scalar DM is p -wave suppressed.

In the contact limit $M_{Z_\ell} \gg m_{\text{DM}}$,

$$\langle \sigma v \rangle_{\text{cont}}^X \simeq \frac{N_\ell}{2\pi} \frac{g_\chi^2 g_{\text{eff}}^2 m_{\text{DM}}^2}{M_{Z_\ell}^4}, \quad \langle \sigma v \rangle_{\text{cont}}^X \simeq \frac{N_\ell}{6\pi} \frac{g_\chi^2 g_{\text{eff}}^2 m_{\text{DM}}^2 v_{\text{rel}}^2}{M_{Z_\ell}^4}. \quad (25)$$

B. Invisible width and collider complementarity

If $M_{Z_\ell} > 2m_{\text{DM}}$, the DM channel contributes to the total width,

$$\Gamma(Z_\ell \rightarrow \chi\bar{\chi}) = \frac{g_\chi^2}{12\pi} M_{Z_\ell} \left(1 + \frac{2m_\chi^2}{M_{Z_\ell}^2} \right) \sqrt{1 - \frac{4m_\chi^2}{M_{Z_\ell}^2}}, \quad (26)$$

$$\Gamma(Z_\ell \rightarrow XX^*) = \frac{g_\chi^2}{48\pi} M_{Z_\ell} \left(1 - \frac{4m_X^2}{M_{Z_\ell}^2} \right)^{3/2}, \quad (27)$$

reducing dilepton branching ratios and altering collider bounds. Accordingly, all constraints are displayed in the $(M_{Z_\ell}, g_{\text{eff}})$ plane with invisible modes profiled over m_{DM} .

C. Direct and indirect searches

Because quarks are neutral under $U(1)'_\ell$, spin-independent nuclear scattering is highly suppressed; the leading contribution arises from kinetic mixing ε (Sec. V) and is negligible for $|\varepsilon| \ll 10^{-2}$. Thus, XENON1T, PandaX-4T, and LZ place only weak bounds. Electron-recoil experiments probe

$$\sigma_e^{\text{SI}} \simeq \frac{g_\chi^2 g_{\text{eff}}^2 \mu_{e\text{DM}}^2}{\pi M_{Z_\ell}^4}, \quad (28)$$

but current sensitivities leave wide viable regions for $M_{Z_\ell} \gtrsim 100$ GeV. Indirect probes (Fermi-LAT, AMS, IceCube) are most relevant for the s -wave fermionic case and remain compatible with a thermal relic away from the resonance region.

Benchmark point and scalar portal. To illustrate viability, consider a representative fermionic benchmark

$$M_{Z_\ell} = 1 \text{ TeV}, \quad |g_{\text{eff}}| = 0.3, \quad g_\chi = 0.5, \quad m_\chi = 400 \text{ GeV}.$$

Away from the s -channel resonance, Eq. (23) yields an annihilation cross section of order $\langle\sigma v\rangle \sim 10^{-26} \text{ cm}^3/\text{s}$, which is compatible with the observed relic density while satisfying collider and indirect limits for leptonic final states. This explicitly demonstrates that large portions of the parameter space shown in Fig. 1 remain viable once dark-matter constraints are correctly restricted to leptophilic scenarios.

For scalar dark matter, Higgs-portal interactions of the form $\lambda_{XH} |X|^2 |H|^2$ are tightly constrained by direct detection. In our benchmarks we set $\lambda_{XH} = 0$ at tree level, so that annihilation proceeds dominantly via Z_ℓ exchange as in Eq. (24), and radiatively induced portal couplings remain well below current experimental limits. This choice is technically natural and consistent with the leptophilic focus of the model.

D. Outlook

Future e^+e^- colliders (ILC, FCC-ee) can simultaneously test the freeze-out strip and the Z_ℓ properties through line-shape scans and invisible-width measurements, providing a unified cosmology-collider probe. Overall, the DM sector emerges as an intrinsic prediction of the anomaly-free leptophilic model: the same parameters $(M_{Z_\ell}, g_{\text{eff}}, g_\chi)$ that control dilepton signals also determine Ω_{DM} and search visibility.

VIII. PHENOMENOLOGY

The anomaly-free leptophilic $U(1)_\ell'$ model yields correlated signatures across colliders, precision observables, and cosmology. The new gauge boson Z_ℓ couples universally to leptons, while quarks remain neutral up to small effects induced by kinetic mixing (Sec. V). The scalar singlet sector controls M_{Z_ℓ} and provides additional resonances, and anomaly cancellation predicts new vectorlike leptons with characteristic collider signals.

Related work. Complementary phenomenological studies of leptophilic Z' scenarios have been performed in global analyses [27], where collider and flavour data were combined to constrain the effective couplings. In contrast, the present work provides a fully anomaly-free and ultraviolet-complete realization of a leptophilic $U(1)_\ell'$ symmetry, linking the gauge, scalar, and Yukawa sectors in a single predictive framework that can naturally reproduce and extend such phenomenological bounds.

Lepton colliders. At e^+e^- machines, Z_ℓ exchange modifies $e^+e^- \rightarrow \ell^+\ell^-$ cross sections and angular distributions through interference, even when M_{Z_ℓ} lies above threshold. In the contact regime the effect is captured by

$$\mathcal{L}_{\text{eff}} = \frac{g_{\text{eff}}^2}{M_{Z_\ell}^2} (\bar{\ell}\gamma^\mu\ell)(\bar{\ell}\gamma_\mu\ell),$$

so sensitivities are conveniently shown in the $(M_{Z_\ell}, g_{\text{eff}})$ plane as contours of constant $\Lambda = M_{Z_\ell}/g_{\text{eff}}$ [28–30]. If kinematically open, on-shell Z_ℓ production leads to narrow dilepton peaks; invisible decays to DM (Sec. VII) reduce visible branching ratios but can be inferred from lineshapes and missing-energy spectra.

Hadron colliders. Drell–Yan production arises from kinetic mixing ε , which induces small Z_ℓ –quark couplings. For $\varepsilon \ll 1$ the reach is weaker than at lepton colliders, yet high luminosity and broad mass coverage make LHC dilepton searches complementary. Heavy vectorlike leptons predicted by anomaly cancellation can be pair-produced electroweakly and decay to SM leptons plus Z_ℓ , yielding clean multilepton final states [17, 18].

Precision tests. Lepton–universality ratios and low-energy observables such as neutrino–electron scattering and atomic parity violation constrain $M_{Z_\ell}/g_{\text{eff}}$ to the multi-TeV range, in agreement with LEP fits [19, 31, 32]. Upcoming facilities like Belle II [15], MOLLER [33], and FCC–ee are expected to strengthen these limits and probe regions consistent with thermal freeze-out.

Cosmology and indirect searches. For leptonic final states, DM annihilation may leave imprints on the CMB and generate gamma rays, cosmic $e^\pm$, and neutrinos [34–37]. Fermionic DM exhibits s –wave annihilation (stronger low-mass constraints), while complex–scalar DM is p –wave suppressed at late times (Sec. VII). If the $U(1)'_\ell$ breaking proceeds via a strong first-order phase transition, a stochastic gravitational-wave background may emerge in the LISA/DECIGO bands [38, 39].

Synthesis. Collider signatures, precision data, indirect searches, and possible gravitational waves probe complementary regions of the same parameter set $(M_{Z_\ell}, g_{\text{eff}})$ together with the scalar and DM sectors. Our analysis embeds these observables in a fully consistent, anomaly-free framework, yielding a unified and testable picture that can be mapped onto the standard $(M_{Z_\ell}, g_{\text{eff}})$ plane, with invisible channels and kinetic mixing profiled where relevant, forming the basis for future dedicated studies.

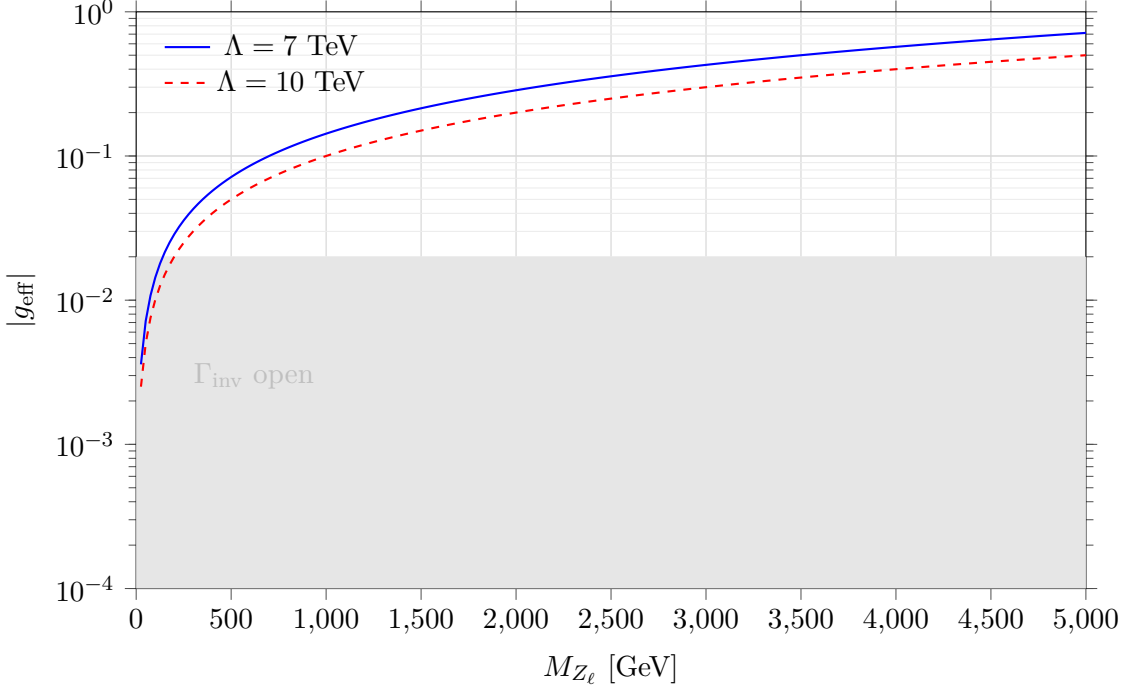


FIG. 1. **Global sensitivity map** in the $(M_{Z_\ell}, |g_{\text{eff}}|)$ plane. Diagonal lines illustrate representative contact scales $\Lambda = M_{Z_\ell}/|g_{\text{eff}}|$; moving along a line preserves the LEP contact reach. The shaded band schematically marks the regime where invisible decays dominate, in which visible dilepton searches weaken while missing-energy and precision handles become comparatively stronger.

Global sensitivity map. Combining the collider, precision, and dark-sector bounds discussed above, Fig. 1 summarizes the viable parameter space in the $(M_{Z_\ell}, |g_{\text{eff}}|)$ plane by projecting all leading constraints onto the effective LEP contact scale $\Lambda \equiv M_{Z_\ell}/|g_{\text{eff}}|$. The diagonal guide lines correspond to representative constant Λ contours; moving along a line preserves the contact-interaction reach, i.e. collider sensitivities remain approximately constant. Conversely, moving vertically or horizontally changes the mediator interpretation. The shaded band (schematic; actual boundary depends on m_{DM} and Γ_{inv}) indicates where invisible decays dominate, in which case visible dilepton searches weaken while missing-energy probes and precision observables become comparatively stronger. This single-panel view makes explicit the complementarity among these bounds: LEP-like contact limits restrict Λ almost uniformly, whereas the line shape and branching structure control the transition between visible and invisible final states.

IX. CONCLUSIONS

We have developed a fully consistent and anomaly-free extension of the Standard Model based on a universal leptophilic gauge symmetry $U(1)'_\ell$. Motivated by neutrino oscillation data, all leptons carry a common charge under $U(1)'_\ell$ while quarks remain neutral, yielding a genuinely leptophilic gauge boson Z_ℓ at tree level. Quantum consistency is restored by introducing a minimal set of vectorlike leptons and singlet scalars, with the explicit charge assignments and anomaly conditions summarized in Appendix A.

The scalar sector breaks $U(1)'_\ell$, determines M_{Z_ℓ} , and can mix with the Standard Model Higgs. It also enables a type-I seesaw via sterile states, linking the new symmetry to the origin of neutrino masses and potentially to baryogenesis through leptogenesis. The anomaly-cancelling leptons acquire masses via singlet Yukawa couplings and do not perturb the SM charged-lepton mass matrix at renormalizable level, thereby suppressing flavor violation. Gauge-kinetic mixing with hypercharge, while generically present, can be loop-suppressed by enforcing $\text{Tr}(YQ') = 0$ (Sec. V), preserving the leptophilic structure and satisfying precision constraints.

The same ingredients naturally accommodate a stable dark-matter candidate, either fermionic or scalar, whose relic abundance and signatures are governed by Z_ℓ -mediated annihilation channels (Sec. VII). Collider searches, precision data, cosmology, and indirect probes thus provide complementary handles on a single parameter set $(M_{Z_\ell}, g_{\text{eff}})$, with invisible channels and kinetic mixing profiled where relevant.

In summary, gauging a universal lepton number yields a compact, internally consistent framework that simultaneously addresses anomaly cancellation, neutrino mass generation, dark matter, and collider signatures through a unifying symmetry principle. The model therefore stands as a minimal benchmark for upcoming high-luminosity and precision programs.

Outlook—The closed-form relations presented here, together with the global sensitivity map of Fig. 1, provide a direct foundation for future global recasts, numerical scans, and public likelihoods targeting high-precision lepton-collider and dark-sector probes.

Field (LH Weyl)	Rep. $(SU(2)_L, Y)$	Q'_ℓ	Role in anomaly cancellation
L_i (lepton doublet)	$(2, -\frac{1}{2})$	$+a_\ell$	Generates $[SU(2)_L]^2 U(1)'_\ell$, $[U(1)_Y]^2 U(1)'_\ell$, and gravitational terms.
e_i^c (RH electron)	$(1, +1)$	$-a_\ell$	Partially cancels the hypercharge piece of L_i .
ν_i^c (RH neutrino)	$(1, 0)$	$-a_\ell$	Removes the residual gravitational term from L_i .
L'	$(2, -\frac{1}{2})$	$q(L')$	Cancels SM $[SU(2)_L]^2 U(1)'_\ell$.
L'^c	$(2, +\frac{1}{2})$	$q(L'^c)$	Balances the hypercharge part of L' .
E'	$(1, -1)$	$q(E')$	Cancels $[U(1)_Y]^2 U(1)'_\ell$ and gravitational anomalies.
E'^c	$(1, +1)$	$q(E'^c)$	Balances E' contribution.
N'	$(1, 0)$	$q(N')$	Contributes to gravitational and cubic anomalies.
N'^c	$(1, 0)$	$q(N'^c)$	Balances N' contribution.

TABLE I. Minimal field content in the left-handed Weyl basis with SM representations, $U(1)'_\ell$ charges, and schematic roles in anomaly cancellation.

Appendix A: Technical details of the $U(1)'_\ell$ model

We provide here the explicit checks of anomaly cancellation and the charge relations underlying our benchmark setup. Although not required for the phenomenological discussion in the main text, this appendix demonstrates that the leptophilic construction is fully consistent at the quantum level.

Charge assignments and anomaly conditions. We work in a basis of left-handed Weyl fermions. The SM leptons are

$$L_i \sim (2, -\tfrac{1}{2}), \quad e_i^c \sim (1, +1), \quad \nu_i^c \sim (1, 0),$$

each assigned $U(1)'_\ell$ charges $(+a_\ell, -a_\ell, -a_\ell)$ respectively, while quarks are neutral. The new leptons are vectorlike under the SM but chiral under $U(1)'_\ell$:

$$L', L'^c \sim (2, -\tfrac{1}{2}), (2, +\tfrac{1}{2}), \quad E', E'^c \sim (1, -1), (1, +1), \quad N', N'^c \sim (1, 0), (1, 0).$$

A minimal anomaly-free assignment consistent with universal lepton charges is

$$q(L') = q(L'^c) = -\tfrac{3}{2}a_\ell, \quad q(E') = q(E'^c) = +\tfrac{3}{2}a_\ell, \quad q(N') = q(N'^c) = +\tfrac{3}{2}a_\ell. \quad (\text{A.1})$$

Explicit cancellation. With these assignments all relevant anomaly coefficients vanish:

$$[SU(2)_L]^2 U(1)'_\ell : \quad \tfrac{1}{2}(3a_\ell) + \tfrac{1}{2}(q(L') + q(L'^c)) = 0, \quad (\text{A.2})$$

$$[U(1)_Y]^2 U(1)'_\ell : \quad -\tfrac{3}{2}a_\ell + \tfrac{1}{2}(q(L') + q(L'^c)) + (q(E') + q(E'^c)) = 0, \quad (\text{A.3})$$

$$[U(1)'_\ell]^2 U(1)_Y : \quad -q(L')^2 + q(L'^c)^2 - q(E')^2 + q(E'^c)^2 = 0, \quad (\text{A.4})$$

$$[\text{grav}]^2 U(1)'_\ell : \quad \dots = 0, \quad (\text{A.5})$$

$$[U(1)'_\ell]^3 : \quad \dots = 0. \quad (\text{A.6})$$

General solution. For generic charges

$$x = q(L'), \quad x_c = q(L'^c), \quad y = q(E'), \quad y_c = q(E'^c), \quad z = q(N'), \quad z_c = q(N'^c),$$

the anomaly equations reduce to

$$x + x_c = -3a_\ell, \quad y + y_c = +3a_\ell, \quad z + z_c = +3a_\ell, \quad (\text{A.7})$$

$$(y_c - y) = (x_c - x), \quad (z_c - z) = \pm(x_c - x). \quad (\text{A.8})$$

Equation (A.1) corresponds to one representative solution within this family (see also Refs. [40, 41]).

Mass and coupling relations. A scalar Φ carrying $q_\Phi = 3a_\ell$ allows Yukawa couplings:

$$\mathcal{L} \supset y_L \Phi L' L'^c + y_E \Phi^\dagger E' E'^c + y_N \Phi^\dagger N' N'^c + \text{h.c.}, \quad (\text{A.9})$$

yielding Dirac masses $m_{L',E',N'} = \frac{y}{\sqrt{2}} v_\Phi$ after Φ acquires v_Φ . The $U(1)_\ell'$ gauge boson mass follows as

$$M_{Z_\ell} = g_\ell |q_\Phi| v_\Phi, \quad g_{\text{eff}} = g_\ell a_\ell,$$

implying

$$m_{L',E',N'} = \frac{y_{L,E,N}}{3\sqrt{2}} \frac{M_{Z_\ell}}{|g_{\text{eff}}|} \quad \text{for } q_\Phi = 3a_\ell.$$

Useful reference formulas. For convenience:

- **Partial width to a massless Dirac fermion f :**

$$\Gamma(Z_\ell \rightarrow f \bar{f}) = N_c^f \frac{g_\ell^2 q_f^2}{12\pi} M_{Z_\ell}.$$

- **Total leptonic width (three families):**

$$\Gamma_{\text{lept}}^{(\text{SM})} = \frac{g_{\text{eff}}^2}{2\pi} M_{Z_\ell}.$$

- **Contact–interaction limit ($M_{Z_\ell} \gg \sqrt{s}$):**

$$\mathcal{L}_{\text{eff}} = \frac{g_{\text{eff}}^2}{M_{Z_\ell}^2} (\bar{\ell} \gamma^\mu \ell) (\bar{\ell} \gamma_\mu \ell), \quad \Lambda = \frac{M_{Z_\ell}}{|g_{\text{eff}}|}.$$

Summary. The anomaly analysis confirms that the universal leptophilic $U(1)_\ell'$ is internally consistent and predictive: all anomalies cancel within one vectorlike generation, and all relevant masses and couplings can be expressed in terms of $(M_{Z_\ell}, g_{\text{eff}})$ and the singlet Yukawas $y_{L,E,N}$. This appendix thereby completes the quantum–field–theoretic consistency of the leptophilic framework.

Appendix B: Extended scalar sector and mixing

For completeness, we provide details of the two–singlet realization of the scalar sector. The SM Higgs doublet H and two $U(1)_\ell'$ –charged singlets $\Phi_{1,2}$ acquire vacuum expectation values (vevs)

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{2}}.$$

Renormalizable potential and soft term. The most general renormalizable potential is

$$V(H, \Phi_1, \Phi_2) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_1^2 |\Phi_1|^2 + \lambda_1 |\Phi_1|^4 - \mu_2^2 |\Phi_2|^2 + \lambda_2 |\Phi_2|^4 \\ + \lambda_{H1} |H|^2 |\Phi_1|^2 + \lambda_{H2} |H|^2 |\Phi_2|^2 + \lambda_{12} |\Phi_1|^2 |\Phi_2|^2 + V_{\text{soft}}, \quad (\text{B.1})$$

where the optional soft term is

$$V_{\text{soft}} \equiv -\mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}), \quad \text{allowed only when } q_1 = q_2. \quad (\text{B.2})$$

Optional $U(1)_\ell'$ D-term quartics. In UV completions with a non-decoupling $U(1)_\ell'$ D-term (e.g. supersymmetry), the quartic

$$V_D = \frac{g_\ell^2}{2} (q_1 |\Phi_1|^2 + q_2 |\Phi_2|^2)^2 \quad (\text{B.3})$$

induces

$$\lambda_1^{\text{eff}} = \lambda_1 + \frac{g_\ell^2}{2} q_1^2, \quad \lambda_2^{\text{eff}} = \lambda_2 + \frac{g_\ell^2}{2} q_2^2, \quad \lambda_{12}^{\text{eff}} = \lambda_{12} + g_\ell^2 q_1 q_2. \quad (\text{B.4})$$

These “effective” quartics are used below; if no D-terms are present, simply read $\lambda_i^{\text{eff}} \rightarrow \lambda_i$.

Tadpole conditions. Extremizing the potential gives

$$0 = -\mu_H^2 v + \lambda_H v^3 + \frac{1}{2} \lambda_{H1} v v_1^2 + \frac{1}{2} \lambda_{H2} v v_2^2, \quad (\text{B.5})$$

$$0 = -\mu_1^2 v_1 + \lambda_1^{\text{eff}} v_1^3 + \frac{1}{2} \lambda_{H1} v^2 v_1 + \frac{1}{2} \lambda_{12}^{\text{eff}} v_1 v_2^2 - \mu_{12}^2 v_2, \quad (\text{B.6})$$

$$0 = -\mu_2^2 v_2 + \lambda_2^{\text{eff}} v_2^3 + \frac{1}{2} \lambda_{H2} v^2 v_2 + \frac{1}{2} \lambda_{12}^{\text{eff}} v_2 v_1^2 - \mu_{12}^2 v_1. \quad (\text{B.7})$$

Gauge boson mass and Goldstone mode.

$$M_{Z_\ell}^2 = g_\ell^2 (q_1^2 v_1^2 + q_2^2 v_2^2), \quad G' \propto q_1 v_1 \text{Im } \Phi_1 + q_2 v_2 \text{Im } \Phi_2, \quad (\text{B.8})$$

where G' is the would-be Goldstone boson eaten by Z_ℓ .

CP-even mass matrix. Expanding $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$ and $\Phi_i = \frac{1}{\sqrt{2}}(v_i + \phi_i + i a_i)$, the (h, ϕ_1, ϕ_2) mass matrix reads

$$\mathcal{M}_{\text{even}}^2 = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H1} v v_1 & \lambda_{H2} v v_2 \\ \lambda_{H1} v v_1 & 2\lambda_1^{\text{eff}} v_1^2 & \lambda_{12}^{\text{eff}} v_1 v_2 - \mu_{12}^2 \\ \lambda_{H2} v v_2 & \lambda_{12}^{\text{eff}} v_1 v_2 - \mu_{12}^2 & 2\lambda_2^{\text{eff}} v_2^2 \end{pmatrix}. \quad (\text{B.9})$$

In the decoupling regime ($m_{\phi_i}^2 \gg m_h^2$), the lightest state $h_1 \simeq h$ is SM-like, and the mixing angles reduce to

$$\sin \theta_{h\phi_1} \simeq \frac{\lambda_{H1} v v_1}{m_{\phi_1}^2 - m_h^2}, \quad \sin \theta_{h\phi_2} \simeq \frac{\lambda_{H2} v v_2}{m_{\phi_2}^2 - m_h^2}, \quad m_{\phi_i}^2 \simeq 2\lambda_i^{\text{eff}} v_i^2. \quad (\text{B.10})$$

Hence the Higgs signal strength scales as $\mu_h \simeq 1 - (\sin^2 \theta_{h\phi_1} + \sin^2 \theta_{h\phi_2})$.

CP-odd sector. Writing $\Phi_i = \frac{1}{\sqrt{2}}(v_i + \phi_i + ia_i)$, the CP-odd fields (a_1, a_2) span one would-be Goldstone boson G' and one physical pseudoscalar A . In the (a_1, a_2) basis, the quadratic terms from V_{soft} give

$$\mathcal{M}_{\text{odd}}^2 = \mu_{12}^2 \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}. \quad (\text{B.11})$$

The eigenstates are

$$G' \propto v_1 a_1 + v_2 a_2, \quad (\text{B.12})$$

$$A \propto -v_2 a_1 + v_1 a_2, \quad (\text{B.13})$$

with

$$m_A^2 = \mu_{12}^2 \left(\frac{v_1}{v_2} + \frac{v_2}{v_1} \right), \quad m_{G'}^2 = 0. \quad (\text{B.14})$$

In the limit $\mu_{12}^2 \rightarrow 0$, the CP-odd scalar A becomes massless, signaling the restoration of a global $U(1)$ in the singlet sector. Conversely, a nonzero μ_{12}^2 both lifts A and ensures that only one Goldstone boson is eaten by Z_ℓ .

Appendix C: Gauge-boson mixing: A - Z - Z_ℓ diagonalization with kinetic mixing

We start from the kinetic and mass terms

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu} - \frac{\varepsilon}{2}B_{\mu\nu}B'^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} \\ & + \frac{1}{2}M_Z^2(Z_\mu^0)^2 + \frac{1}{2}M_{Z_\ell,0}^2(B'_\mu)^2 + (\text{currents}), \end{aligned} \quad (\text{C.1})$$

with the usual Weinberg rotation $Z_\mu^0 \equiv c_W W_\mu^3 - s_W B_\mu$, $A_\mu \equiv s_W W_\mu^3 + c_W B_\mu$, $s_W \equiv g_Y/\sqrt{g^2 + g_Y^2}$, and $M_Z^2 = \frac{1}{4}(g^2 + g_Y^2)v^2$. Canonical normalization to $\mathcal{O}(\varepsilon)$ via $B_\mu \rightarrow B_\mu + \varepsilon B'_\mu$ removes the mixed kinetic term and leaves the photon strictly massless. In the (A_μ, Z_μ^0, B'_μ) basis the mass matrix becomes block-diagonal:

$$\mathcal{M}_{(A,Z^0,B')}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_Z^2 & \delta \\ 0 & \delta & M_{Z_\ell}^2 \end{pmatrix} + \mathcal{O}(\varepsilon^2), \quad \delta \equiv \varepsilon s_W M_Z^2, \quad (\text{C.2})$$

where $M_{Z_\ell}^2 = M_{Z_\ell,0}^2 + \mathcal{O}(\varepsilon^2)$ if the SM Higgs is uncharged under $U(1)'_\ell$.

Diagonalization reduces to a rotation in the (Z^0, B') plane:

$$\begin{pmatrix} Z \\ Z_\ell \end{pmatrix} = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} Z^0 \\ B' \end{pmatrix}, \quad \tan 2\xi = \frac{2\delta}{M_{Z_\ell}^2 - M_Z^2}, \quad (\text{C.3})$$

so that for small ε (and hence ξ)

$$M_{Z,\text{phys}}^2 \simeq M_Z^2 - \frac{\delta^2}{M_{Z_\ell}^2 - M_Z^2}, \quad M_{Z_\ell,\text{phys}}^2 \simeq M_{Z_\ell}^2 + \frac{\delta^2}{M_{Z_\ell}^2 - M_Z^2}, \quad (\text{C.4})$$

$$g_{Zf}^{\text{phys}} \simeq g_{Zf} + \xi g_\ell Q'_f + \varepsilon g_Y Y_f s_W, \quad g_{Z_\ell f}^{\text{phys}} \simeq g_\ell Q'_f + \varepsilon g_Y Y_f - \xi g_{Zf}. \quad (\text{C.5})$$

Here g_{Zf} denotes the SM Z coupling before kinetic/mass mixing, and Q'_f (Y_f) is the $U(1)'_\ell$ (hypercharge) of f . The $\mathcal{O}(\varepsilon^2)$ terms are omitted.

Decoupling limit and useful approximations. In the limit $M_{Z_\ell}^2 \gg M_Z^2$ one has

$$\xi \simeq \frac{\delta}{M_{Z_\ell}^2} \simeq \varepsilon s_W \frac{M_Z^2}{M_{Z_\ell}^2}, \quad g_{Z_\ell q} \simeq \varepsilon g_Y Y_q, \quad g_{Z_\ell \ell} \simeq g_\ell Q'_\ell + \varepsilon g_Y Y_\ell, \quad (\text{C.6})$$

so Drell–Yan production is controlled by ε , while the tree–level leptophilic piece remains $g_\ell Q'_\ell$. Corrections to the Z pole scale as $\mathcal{O}(\xi, \varepsilon)$.

Higgs-charged case. If the Higgs doublet carries a small $U(1)'_\ell$ charge, additional $\mathcal{O}(v^2)$ entries appear in the (Z^0, B') submatrix. Equations (C.3)–(C.5) remain valid upon the replacements $\delta \rightarrow \delta + \delta_{\text{Higgs}}$ and $M_{Z_\ell}^2 \rightarrow M_{Z_\ell}^2 + \Delta_{\text{Higgs}}$, with $\delta_{\text{Higgs}}, \Delta_{\text{Higgs}}$ determined by the $U(1)'_\ell$ charge of H .

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