

Combining CMB datasets with consistent foreground modelling

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ABSTRACT

We present a joint cosmological analysis combining data from the *Planck* satellite, the Atacama Cosmology Telescope, and the South Pole Telescope, constructing a unified likelihood that reproduces the measured temperature and polarisation power spectra by jointly modelling the cosmic microwave background (CMB) signal, galactic and extragalactic foregrounds, and instrumental systematics across all datasets. This approach reduces reliance on external priors and improves the robustness of parameter estimation. Within this joint analysis, Λ CDM parameters exhibit remarkable stability with respect to variations in foreground modelling. Extended cosmological parameters are more sensitive to these assumptions, with uncertainties increasing by up to 35%. Despite this, the combined constraints show no significant deviation from Λ CDM expectations, and several previously reported tensions – such as the preference for non-zero curvature or the excess of lensing amplitude A_L – are significantly reduced or resolved. In contrast, the determination of foreground parameters more severely depends on the assumptions made about the underlying models. Overall, this work demonstrates the feasibility and reliability of a fully joint analysis of current CMB experiments, and emphasizes the importance of consistent and accurate foreground modelling for the scientific goals of next-generation, high-sensitivity CMB surveys.

Key words. cosmology: observations – cosmic background radiation – cosmological parameters – methods: data analysis

1. Introduction

The cosmic microwave background (CMB) remains one of the most powerful probes of the early Universe, providing precise measurements of the fundamental parameters that govern cosmology. Over the past decade, multiple ground-based and space-borne experiments – most notably the *Planck* satellite, the Atacama Cosmology Telescope (ACT), and the South Pole Telescope (SPT) – have independently mapped the CMB sky with increasing sensitivity, resolution, and frequency coverage.

Each of these experiments contributes complementary strengths: *Planck* offers full-sky coverage and excellent sensitivity at large angular scales; ACT and SPT provide high-resolution measurements on smaller scales with deeper observations over limited regions of the sky. To illustrate the relative sensitivity of each experiment across angular scales, we show in Fig. 1 the uncertainties on the TT , TE , and EE power spectra as a function of multipole for *Planck* PR4 (Tristram et al. 2024), ACT DR6 (Louis et al. 2025), and SPT-3G D1 (Camphuis et al. 2025). This comparison highlights the complementarity of the datasets: *Planck* dominates at low multipoles, while ACT and SPT significantly improve constraints at smaller angular scales, particularly in polarisation.

However, these differences in sky coverage, angular resolution, and frequency channels introduce critical challenges to a consistent joint analysis. Historically, residual foregrounds – including Galactic dust, thermal and kinetic Sunyaev–Zel’dovich effects, the cosmic infrared background, and extragalactic sources – have been modelled independently by each collaboration, limiting the consistency of the combined analysis and the reliability of the derived constraints. Recently, Beringue et al.

(2025) explored the combination of these three experiments using the ACT foreground model applied to *Planck* and SPT.

In this work, we present a coherent combination of the *Planck*, ACT, and SPT datasets, jointly analysing their temperature and polarisation power spectra within a common model for both cosmological and foreground contributions. This approach leverages the full constraining power of each experiment accounting for differences in frequency coverage and resolution while maintaining inter-dataset consistency. By fitting a unified model to all three datasets simultaneously, we are able to minimise reliance on external priors, tighten constraints on the standard Λ CDM parameters and gain new insights into the robustness of foreground treatments across experiments.

This paper is organised as follows: we describe the likelihood construction in Sect. 2 and the datasets in Sect. 3; in Sect. 4, we detail the instrumental effects and in Sect. 5 the sky model used in the joint fit; we present the results of the combined analysis for Λ CDM parameters in Sect. 6; and in Sect. 7, we discuss the impact of the foreground modelling. We derive constraints on some extensions to Λ CDM in Sect. 8. Finally, we conclude in Sect. 9.

2. Likelihood

Information at large angular scales ($\ell < 30$) is derived from CMB maps obtained after component separation of the *Planck* data and is treated separately from the smaller scales.

At high multipoles ($\ell \gtrsim 30$), current CMB likelihoods are based on frequency cross-spectra and we adopt a Gaussian approximation, as in the original implementations of the *Planck*, SPT, and ACT likelihoods. Although the exact distribution of auto- and cross-spectra is non-Gaussian (scaled- χ^2 and more

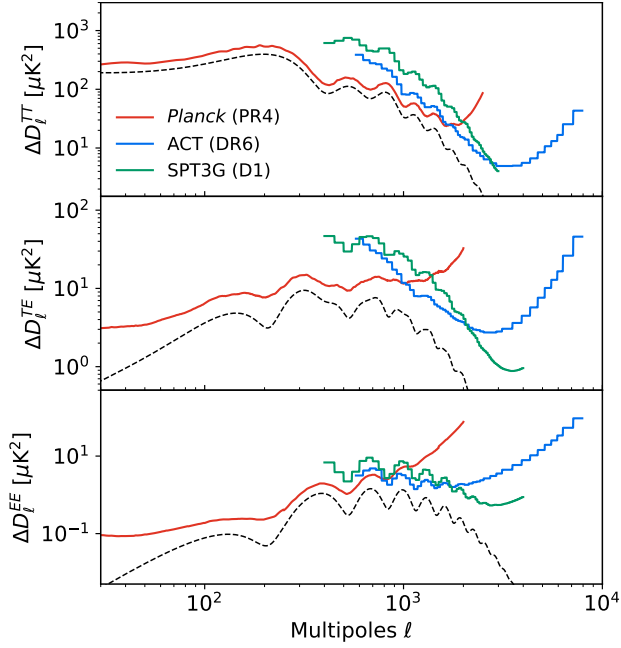


Fig. 1: Uncertainties on angular power spectra TT , TE , and EE for the *Planck* PR4 (red, Tristram et al. 2024), ACT DR6 (blue, Louis et al. 2025), and SPT-3G D1 (green, Camphuis et al. 2025) datasets compared to full-sky cosmic variance (dashed line). Uncertainties are averaged from all cross-frequency spectra according to their respective covariance matrix.

complex forms; see Mangilli et al. 2015), the large number of modes at high- ℓ makes the Gaussian approximation sufficiently accurate (Carron 2013). However, cutting the sky to reduce galactic contamination induces correlations between the C_ℓ values, necessitating the estimation of a full covariance matrix. The cross-spectra from the frequency maps C_ℓ are then compared with the data model $\hat{C}_\ell$, in the likelihood:

$$-2 \ln \mathcal{L} = (\mathbf{C} - \hat{\mathbf{C}})^T \Sigma^{-1} (\mathbf{C} - \hat{\mathbf{C}}) \quad (1)$$

The covariance matrix Σ includes both instrumental and cosmic variance and is, in principle, model-dependent. Correlations between multipoles and cross-frequency spectra are included in the likelihood. However, in this work, we do not consider any correlation between the datasets. As a consequence, to avoid overlap in multipoles, we restrict the multipole range of the ACT spectra when combined with *Planck*. Specifically, we use *Planck* data for multipoles from $\ell = 2$ up to 2000 for TT , 1500 for TE , and 1000 for EE , while ACT data are used for multipoles above these thresholds. The correlations between *Planck* and SPT are neglected, as the two experiments overlap over less than 5% of the sky.

We analyse each dataset (described in Sect. 3) both independently and in combination. The model includes the CMB signal, together with residuals of diffuse and unresolved foreground emissions in power spectra (Sect. 5) and instrumental systematic effects (Sect. 4).

3. Data sets

3.1. Planck

The *Planck* sky measurements used in this analysis are the PR4 maps. They have been produced with the NPIPE processing

pipeline, which creates calibrated frequency maps in temperature and polarisation from the *Planck* Low Frequency Instrument (LFI) and High Frequency Instrument (HFI) data. As described in Planck Collaboration Int. LVII (2020), NPIPE processing includes several improvements, resulting in lower levels of noise and systematics in both frequency and component-separated maps at essentially all angular scales, as well as notable improvements in internal consistency between the various frequencies.

To cover the very low multipoles ($\ell < 30$), we make use of the Commander TT likelihood (Planck Collaboration IV 2020) and for polarisation, we add the low- ℓ likelihood LoLLiPoP introduced in Tristram et al. (2021, 2022).

At high multipoles, the data are made of 15 cross-spectra computed from the two detector-set (*detset*) maps at three frequencies (100, 143 and 217 GHz). Those are further combined into six cross-frequency spectra applying a calibration associated to each *detset* map. The resulting *Planck* likelihood is a binned version of HiLLiPoP presented in Tristram et al. (2024) (hereafter T24).

3.2. Atacama Cosmology Telescope

The DR6 data release is based on the 90, 150 and 220 GHz data from the 2017-2022 observing seasons of the AdvancedACT camera. The survey consists of 19 000 square degrees of the sky, with a median combined depth of $10 \mu\text{K}\cdot\text{arcmin}$. The description of the frequency maps in temperature and polarisation and the data reduction pipeline can be found in Naess et al. (2025).

In this study, we adopt a likelihood implementation closely aligned with the MFLike software developed for the Simons Observatory and used in Louis et al. (2025), hereafter L25, retaining the same set of instrumental parameters. The data include five different sets of maps from three different wafers: *pa5* (90 and 150 GHz), *pa6* (90 and 150 GHz), and *pa4* (at 220 GHz), the latter being considered only in temperature. The main modification concerns the foreground treatment, which we adapt to allow for a unified modelling across all datasets. We have verified that, under identical model assumptions and priors, our implementation reproduces the results of L25 up to machine precision.

3.3. South Pole Telescope

The SPT data used in this analysis correspond to the SPT-3G D1 data set, which includes observations from 2019 and 2020 of the SPT-3G Main field, covering the 95, 150, and 220 GHz frequency bands. The data released include TT band powers that span the angular-multipole range $400 < \ell < 3000$, and TE/EE bandpowers that cover $400 < \ell < 4000$. The cross-frequency spectra are binned into bandpowers of width $\Delta\ell = 50$ (Camphuis et al. 2025, hereafter C25). The likelihood implementation used in this work has been validated against the official *candl* likelihood up to machine precision.

4. Instrumental effects

In the combined *Planck*, SPT, and ACT likelihood, we implement the same systematic corrections as those used in the individual likelihoods. These corrections account for a variety of well-understood instrumental and astrophysical effects:

Calibration: For each dataset, we sample map inter-calibration coefficients relative to a chosen reference. For *Planck*, the reference is the first *detset* map at 143 GHz and we sample coefficients for the five remaining *detset* maps. In the

SPT likelihood, the 150 GHz channel is taken as the reference, and we vary the coefficients for the 90 and 220 GHz channel maps. For the ACT likelihood, we sample five inter-calibration parameters to rescale the wafer maps relative to *pa5* at 150 GHz. Gaussian priors from the original analyses are applied in each case (see Table C.1).

In addition, we include absolute calibration parameters for each dataset, using *Planck* as the reference. Its calibration, A_{Planck} , is constrained with a strong prior ($A_{\text{Planck}} = 1.00 \pm 0.0025$) based on the orbital dipole uncertainty, while the ACT and SPT calibrations are determined relative to it.

Polar efficiency: The polarisation efficiencies ρ are sampled independently for each map with flat priors.

Super-sample lensing amplitude: The super-sample lensing amplitude in SPT is parameterised by an additional parameter κ constrained by a Gaussian prior ($\kappa = 0.00000 \pm 0.00045$). For ACT and *Planck*, the bias on the angular power spectra is negligible. However, ACT included a second-order correction in the C_ℓ covariance matrix to account for the lensing super-sample covariance arising from fluctuations in the mean convergence across the observed area.

Aberration: For SPT, the aberration is corrected in the likelihood with a fixed parameter, while for ACT the power spectra are already corrected using simulations. In the case of *Planck*, the aberration effect is negligible.

Bandpass integration: To compute the foreground amplitudes for a given spectral energy density (SED), it is necessary to integrate over the bandpass of each channel. This is commonly approximated by assigning an effective frequency to each component and channel, as done in the SPT and *Planck* likelihoods. However, L25 showed that bandpass uncertainties in ACT-DR6 contribute significantly to the overall error budget in the recovery of foreground parameters. To account for this, we sample six additional bandpass shift parameters, Δ_{bp} , constrained by strong priors derived from instrumental calibration. In addition, the ACT team incorporates chromatic beam window functions to model the frequency dependence of the beam, an approach we adopt here following Giardiello et al. (2025).

Beam uncertainties: Beam uncertainties are negligible for the *Planck* dataset (see Planck Collaboration XV 2014). For ACT, they are included as an additional contribution to the covariance matrix (L25). In the case of SPT, beam uncertainties are propagated by fitting the amplitude β_i of the first nine eigenmodes of the beam covariance matrix. Furthermore, the SPT likelihood includes additional freedom to rescale the polarisation amplitude of the sidelobes at each frequency (C25).

Temperature-to-Polarisation Leakage: For *Planck*, the spectra are already corrected and the associated uncertainty is negligible. In the case of ACT, it is included as an additional term in the covariance matrix, while the SPT likelihood propagates the uncertainty by fitting a contribution from quadrupolar beam leakage, modelled analytically for each frequency (C25). Associated Gaussian priors are given in Table C.1.

The nuisance parameters and their associated priors are listed in Table C.1. When combining all datasets, a total of 48 nuisance parameters are sampled. Although the large number of parameters can pose challenges for MCMC sampling – particularly due to degeneracies and long correlation lengths – they are essential for accurately propagating instrumental uncertainties that affect the angular power spectra. Including these parameters ensures

that the cosmological constraints properly account for residual systematics and modelling uncertainties across all experiments.

5. Sky model

We now present the sky model ($\hat{\mathbf{C}}_\ell$) used in the CMB likelihood (Eq. 1). Foreground emissions are first mitigated by masking high-signal regions and selecting an appropriate multipole range. Residual diffuse and unresolved components are then modelled directly in the likelihood, with contributions in temperature (T) and polarisation (P) treated as follows:

- galactic emissions (T, P),
- the cosmic infrared background (T),
- the Sunyaev-Zel’dovich effect: thermal, kinetic, and correlation between tSZ and infrared galaxies (T),
- radio sources (T, P).

The cosmic infrared background (CIB) and the Sunyaev-Zel’dovich effects (tSZ, kSZ) as well as their correlation (tSZ×CIB) are assumed to be unpolarised. Emission from extragalactic carbon monoxide (Maniyar et al. 2023; Kokron et al. 2024) is not included in our foreground model, as it remains difficult to model reliably.

In the original implementations of the CMB likelihoods, extragalactic foregrounds are typically modelled using fixed angular power spectrum templates, which amplitudes are fitted simultaneously with the CMB signal. These templates are often derived from external observations (e.g., SZ or CIB measurements), computed from simulations, or based on simplified assumptions (power-law).

In this study, we ensure that foregrounds are modelled consistently for all datasets. Foreground templates are shared by all CMB likelihoods and extra-galactic emissions (this includes the CIB, tSZ, kSZ and tSZ×CIB) are fitted consistently using the same parameters across the datasets. The summary of all parameters related to the foreground modelling is given in Table B.1.

Crucially, combining multiple experiments with complementary frequency coverage, angular resolution, and noise properties significantly enhances our ability to disentangle foreground contributions. *Planck* is particularly effective for galactic components and the CIB, while ACT and SPT with higher resolution and small-scale sensitivity, better constrain tSZ, kSZ, and point-source signals. This complementarity breaks degeneracies that persist in single-dataset analyses, allowing many of the informative priors usually imposed on foreground parameters to be relaxed or removed, resulting in a more robust, data-driven sky model (see Table B.1).

5.1. Cosmic Microwave Background (CMB)

The CMB signal, $\hat{\mathbf{C}}_\ell^{\text{CMB}}$, is computed by numerically solving the background and perturbation equations for a given cosmological model. In this paper, we consider a Λ CDM model with six free parameters: the present-day baryon density ($\Omega_b h^2$), cold dark matter density ($\Omega_c h^2$), the angular size of the sound horizon at recombination (θ_s), the reionisation optical depth (τ), and the amplitude (A_s) and the spectral index (n_s) of the primordial scalar power spectrum. We open the model to some single-parameter extensions in Sect. 8.

To compute the CMB power spectra, we rely on the Boltzmann solver CAMB (Lewis et al. 2000), as well as the CosmoPower emulator (Spurio Mancini et al. 2022). The emulator is primarily used to accelerate the initial MCMC explorations, while final results are validated with the full Boltzmann

codes to ensure accuracy. To meet the precision requirements of high-resolution surveys, we adopt the accuracy settings of the ACT-DR6 analysis (L25).

Unlike the original analyses, we do not impose any prior on the reionisation optical depth τ even when datasets covering only high multipoles (ACT or SPT) are considered individually.

5.2. Galactic emissions

For galactic emissions, CMB likelihoods consider only Galactic dust, neglecting synchrotron or free-free emissions in the frequency range of our datasets (Planck Collaboration IV 2020).

At first order, the frequency dependence of Galactic dust grains can be modelled as a greybody $a_{\nu}^{\text{dust}} = \nu^{\beta_d} B_{\nu}(T_d)$. Throughout the frequency range of our datasets, the Rayleigh–Jeans approximation applies, leading to a strong degeneracy between β_d and T_d . Consequently, the greybody temperature is commonly fixed to $T_d = 19.6$ K. The spectral indices can be fixed or marginalised over with Gaussian priors.

The power spectrum of the Galactic dust emission has been measured from *Planck* full-sky maps at high-frequency channels (353, 545 and 857 GHz) for temperature and for polarisation (Planck Collaboration Int. XXX 2016; Planck Collaboration XI 2020). The dust spectrum is well fitted with a red index power law α_d , which means the dominant signal is at low multipoles. In the ACT likelihood (L25), the slope of the power-law is fixed to $\alpha_d^{TT} = -2.6$ and $\alpha_d^{TE/EE} = -2.4$. For SPT, C25 marginalise over the index with strong priors $\alpha_d^{TT} = \mathcal{N}(-2.53, 0.05)$ and $\alpha_d^{TE/EE} = \mathcal{N}(-2.42, 0.04)$, based on an analysis of *Planck* data over the SPT-3G survey patch. For the *Planck* likelihoods, the masking procedure is adapted to each frequency, so the dust model can involve three different indices. In T24, for PR4, the indices are -2.65 , -2.55 , -2.45 for sky fractions of 80, 70 and 60%, respectively. In Planck Collaboration V (2020), the PR3 likelihood used an ad-hoc effective analytical model, $C_{\ell} \propto (1 + h\ell^k \exp(-\ell/t) \times (\ell/\ell_p)^{\alpha_d})^{\alpha_d}$, with free parameters h , k , t , and α_d fitted to *Planck* 545 GHz data and adapted to each mask. The recovered indices are -2.65 , -2.57 , and -2.55 at 70%, 60% and 50% sky fractions, respectively. For *TE/EE*, the index is fixed to $\alpha_d = -2.4$.

Although the index can vary slightly across the sky, dust emissions are well described by a global $\alpha_d = -2.6$ in temperature and -2.4 in polarisation (Planck Collaboration V 2020; Planck Collaboration XI 2020). In this study, we use a common power-law spectrum to describe the dust power spectra:

$$\hat{C}_{\ell}^{\text{dust}}(\nu \times \nu') = A^{\text{dust}} \frac{a_{\nu}^{\text{dust}} a_{\nu'}^{\text{dust}}}{a_{\nu_0}^{\text{dust}} a_{\nu_0'}^{\text{dust}}} \left(\frac{\ell}{\ell_0} \right)^{\alpha_d}, \quad (2)$$

where α_d is free in temperature and fixed to $\alpha_d = -2.4$ for *TE/EE*. The frequency dependence follows a greybody spectrum with priors on β_d from *Planck* measurements, $\beta_d^{TT} = \mathcal{N}(1.51, 0.01)$ and $\beta_d^{TE/EE} = \mathcal{N}(1.59, 0.02)$, and a fixed temperature $T_{\text{dust}} = 19.6$ K (Planck Collaboration Int. XXII 2015). The amplitude A^{dust} , expressed at the reference frequency $\nu_0 = 150$ GHz and multipole $\ell_0 = 3000$, is adapted to each sky survey (incl. the three surveys for *Planck*) and each polarisation mode (*TT*, *TE*, *EE*). In the high-multipole CMB datasets, ACT and SPT, the residual Galactic dust contribution is subdominant, and we therefore impose informative priors on A^{dust} . Conversely, in the case of *Planck*, where Galactic dust dominates the low multipoles across all frequencies, we adopt flat priors on the corresponding amplitudes.

5.3. Cosmic infrared background (CIB)

The CIB consists of emission from unresolved dusty star-forming galaxies (Puget et al. 1996; Gispert et al. 2000; Lagache et al. 2005). The angular power spectrum from these galaxies can be modelled as the sum of a Poisson noise term and a clustered component related to the formation of large-scale structures.

The frequency dependence of the CIB can be described as a greybody emission $a_{\nu}^{\text{CIB}} = \nu^{\beta_{\text{CIB}}} B_{\nu}(T_{\text{CIB}})$. However, the temperature adopted in the likelihoods of different CMB datasets vary, from $T_{\text{CIB}} = 9.7$ K in ACT (from Addison et al. 2012a) to $T_{\text{CIB}} = 25$ K for *Planck* (from Planck Collaboration XXX 2014). In its last data release, SPT chose not to model the CIB emission law explicitly, instead using one amplitude for each cross-spectrum including frequencies above 150 GHz, while neglecting the CIB for 90×90 , 90×150 , and 90×220 cross-spectra.

The angular power spectrum of the clustered CIB was initially described with a power-law $\ell^{\alpha_{\text{CIB}}}$ (in C_{ℓ}). Here again, the index α_{CIB} varies from -1.25 (Addison et al. 2012a) to -1.47 (Mak et al. 2017). Since 2015, *Planck* likelihoods (Planck Collaboration XIII 2016; Planck Collaboration VI 2020; Tristram et al. 2024), as well as the recent ACT likelihood (L25), have used a more complex model, including 1-halo and 2-halo terms, which provides an accurate description of the *Planck* and IRAS CIB spectra from 217 GHz up to 3000 GHz (Planck Collaboration XXX 2014). This model was later updated by Lenz et al. (2019). For SPT, by contrast, C25 continue to use a power-law with $\alpha_{\text{CIB}} = -1.47$ from Mak et al. (2017). Several codes are now emerging to compute the angular power spectrum from a halo model (see for instance Viero et al. 2013; Maniyar et al. 2021; Zagatti et al. 2024).

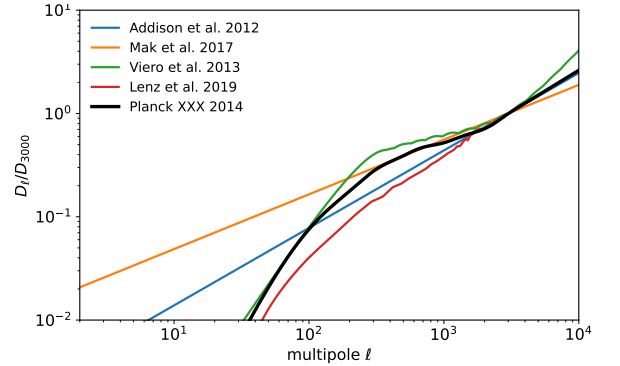


Fig. 2: Templates of CIB power spectrum normalised at $\ell_0 = 3000$ (black line: baseline template).

In this study, we adopt the power spectrum from Planck Collaboration XXX (2014) as a baseline and examine the impact of alternative C_{ℓ}^{CIB} shapes (Fig. 2). In the likelihoods, the C_{ℓ}^{CIB} template is rescaled by the amplitude of the clustered component, A^{CIB} , and we add the Poisson term, A^{IR} , both expressed at the reference frequency $\nu_0 = 150$ GHz and multipole $\ell_0 = 3000$.

$$\hat{C}_{\ell}^{\text{CIB}}(\nu \times \nu') = \frac{a_{\nu}^{\text{CIB}} a_{\nu'}^{\text{CIB}}}{a_{\nu_0}^{\text{CIB}} a_{\nu_0'}^{\text{CIB}}} (A^{\text{CIB}} C_{\ell}^{\text{CIB}} + A^{\text{IR}}) \quad (3)$$

We assume perfect correlation of the emission across the frequency range considered (95–220 GHz) and adopt a flat prior on β_{CIB} with $T_{\text{CIB}} = 25$ K, except for analyses of individual

datasets, where we use a Gaussian prior based on *Planck* measurements, $\beta_{\text{CIB}} = \mathcal{N}(1.75, 0.06)$ (Planck Collaboration XXX 2014).

5.4. Thermal Sunyaev–Zeldovich (tSZ)

The tSZ effect is a spectral distortion of the CMB caused by inverse Compton scattering of photons off the hot gas of massive halos, groups and clusters of galaxies (Sunyaev & Zeldovich 1972). After masking the resolved clusters in the maps, the CMB likelihoods include a component accounting for residual unresolved tSZ emission.

Neglecting the relativistic correction, the frequency dependence of the tSZ effect is given by $a_\nu^{\text{tSZ}} = x(e^x + 1)/(e^x - 1) - 4$ with $x = h\nu/(k_B T_{\text{CMB}})$.

The *Planck* PR3 likelihoods adopt an empirical template for the tSZ power spectrum, as described in Efstathiou & Migliaccio (2012). The ACT team uses models from Battaglia et al. (2012), while the SPT team initially relied on a template from Shaw et al. (2010) and later adopted one derived from the Agora simulations (Omori 2024). HiLLiPoP *Planck* PR4 used the tSZ power spectrum modelled using a halo-based approach described in Douspis et al. (2022), and measured by *Planck* in Planck Collaboration XXII (2016) and Tanimura et al. (2022). In addition, L25 proposed the introduction of additional flexibility by allowing a scale dependence in the tSZ template, parameterised as $C_\ell^{\text{tSZ}} \rightarrow \ell^{\alpha_{\text{tSZ}}} C_\ell^{\text{tSZ}}$.

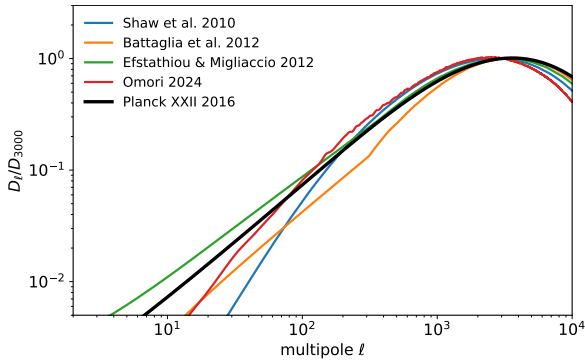


Fig. 3: Templates of tSZ power spectrum normalised at $\ell_0 = 3000$ (black line: baseline template).

In this study, we adopt the power spectrum from Planck Collaboration XXII (2016) as a baseline and assess the impact of alternative C_ℓ^{tSZ} shapes (Fig. 3). The model is parameterised by a single amplitude A^{tSZ} , corresponding to the tSZ emission at the reference frequency ($\nu_0 = 150$ GHz) and multipole $\ell = 3000$:

$$\hat{C}_\ell^{\text{tSZ}}(\nu \times \nu') = A^{\text{tSZ}} a_\nu^{\text{tSZ}} a_{\nu'}^{\text{tSZ}} C_\ell^{\text{tSZ}} \quad (4)$$

5.5. Kinetic Sunyaev–Zeldovich (kSZ)

Due to the motion of galaxy clusters with respect to the CMB rest frame, an additional spectral distortion arises from the Doppler effect of their bulk velocity on the scattered CMB photons. This distortion is known as the kinetic Sunyaev–Zeldovich (kSZ) effect (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1980), with amplitude proportional to the line-of-sight velocity v_c , $\Delta T_{\text{kSZ}}/T_{\text{CMB}} = -\tau_e v_c/c$, where τ_e is the optical depth of the cluster. The kSZ signal shares the same blackbody spectrum

as the primary CMB, making it indistinguishable in frequency space and thus requiring template-based modelling.

The kSZ power spectrum is typically derived from simulations or halo models and is often decomposed into (i) a late-time, homogeneous component from scattering by free electrons in the ionised IGM after reionisation, and (ii) a patchy component from scattering off ionised bubbles during reionisation.

The ACT likelihood uses a model based on hydrodynamic simulations with AGN feedback (Battaglia et al. 2010), while SPT adopts an Agora-based template (Omori 2024), both including only the late-time signal. *Planck* PR3 similarly used a late-time model (Trac et al. 2011), whereas HiLLiPoP PR4 combined late-time (Shaw et al. 2012) and patchy (Battaglia et al. 2013) spectra. More recently, models describing the power spectrum of the free-electron density contrast in terms of the history and morphology of reionisation have been proposed in Gorce et al. (2020), enabling a natural combination of both contributions.

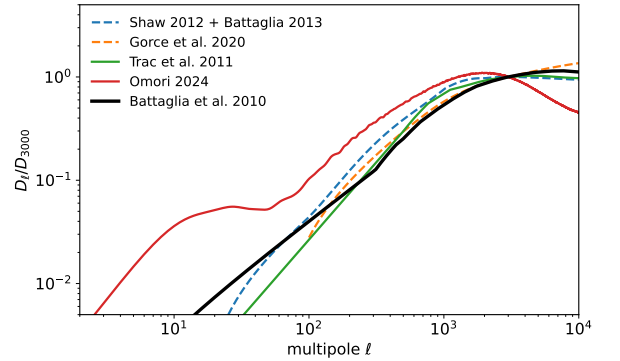


Fig. 4: Templates of kSZ power spectrum normalised at $\ell_0 = 3000$ (black line: baseline template). The late-time kSZ signal is modelled in solid lines while dashed lines represent the combination of late-time and reionisation kSZ signals.

In this study, we adopt the late-time kSZ of Battaglia et al. (2010) as a baseline and will assess the impact of including a reionisation kSZ contribution and alternative template shapes (see Fig. 4). The kSZ emission is parameterised by an amplitude A^{kSZ} , defined at $\ell = 3000$, which rescales the template:

$$\hat{C}_\ell^{\text{kSZ}}(\nu \times \nu') = A^{\text{kSZ}} C_\ell^{\text{kSZ}} \quad (5)$$

5.6. Thermal SZ×CIB correlation

Given our frequency range, we expect an anti-correlation between the tSZ signal from galaxy clusters and the CIB, since dark-matter overdensities host both hot ionised gas (producing negative tSZ) and dusty star-forming galaxies (positive CIB).

The ℓ -dependence of this correlation has been modelled in different ways: as the geometric mean of the tSZ and CIB power spectra (as for the SPT likelihood), through empirical descriptions calibrated to CIB simulations (Zahn et al. 2012; Shang et al. 2012), or using halo-model approaches (Addison et al. 2012b; Maniyar et al. 2021), the latter adopted in the *Planck* and ACT likelihoods.

In this study, we adopt the Addison et al. (2012b) template as a baseline and test alternative shapes (Fig. 5). Given the amplitudes of the CIB and tSZ signals (A^{CIB} , A^{tSZ}), the correlation tSZ×CIB is parameterised by

$$\hat{C}_\ell^{\text{tSZ} \times \text{CIB}}(\nu \times \nu') = -\xi \sqrt{A^{\text{tSZ}} A^{\text{CIB}}} \times (a_\nu^{\text{tSZ}} a_{\nu'}^{\text{CIB}} + a_\nu^{\text{CIB}} a_{\nu'}^{\text{tSZ}}) C_\ell^{\text{tSZ} \times \text{CIB}} \quad (6)$$

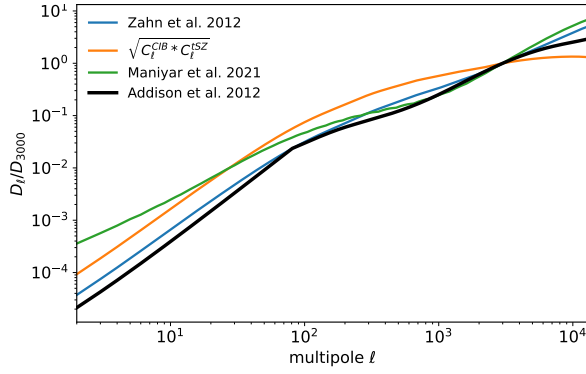


Fig. 5: Templates of tSZ×CIB power spectrum normalised at $\ell_0 = 3000$ (black line: baseline template).

with ξ the correlation coefficient scaling the template $C_\ell^{\text{tSZ} \times \text{CIB}}$.

5.7. Radio sources

Radio sources are modelled with a flat Poisson power spectrum and a power-law spectral energy distribution, $a_\nu^{\text{radio}} = \nu^{-\beta_{\text{radio}}}$ (Tucci et al. 2011). The SED index β_{radio} is not well known and depends on the source population.

In this study, we fit β_{radio} and scale the model with a free amplitude A^{radio} , normalised at $\nu_0 = 150$ GHz:

$$C_\ell^{\text{radio}}(\nu \times \nu') = A^{\text{radio}} \frac{a_\nu^{\text{radio}} a_{\nu'}^{\text{radio}}}{a_{\nu_0}^{\text{radio}} a_{\nu'_0}^{\text{radio}}}. \quad (7)$$

We sample the amplitude independently for each survey in TT and EE . The contribution in TE is assumed to be zero, as galaxy polarisation angles are uncorrelated. When individual datasets are analysed separately, we constrain β_{radio} with a Gaussian prior $\mathcal{N}(-0.8, 0.1)$ (see Tucci et al. 2011).

6. Constraints on Λ CDM

In this section, we describe the constraints on the parameters of the Λ CDM cosmological model with the likelihood described in Sect. 2 using the three datasets: *Planck*, ACT, and SPT. We first present the results obtained using the baseline models, examining the consistency between the different datasets and between the spectra. We then discuss parameter correlations and explore the impact of foreground modelling on Λ CDM parameters by varying the templates and model assumptions.

6.1. Λ CDM

Figure 6 presents a comparison of the posterior distributions for the Λ CDM parameters derived from *Planck*, ACT, SPT and their combination, using the coherent likelihood described in Sect. 2 and the baseline foreground templates (Sect. 5). Each dataset incorporates measurements from the TT , TE and EE power spectra. Constraints on Λ CDM cosmological parameters are summarised in Table 1.

Overall, the three datasets show very good agreement on the Λ CDM parameters. A slight deviation is observed for $\Omega_b h^2$ between ACT and the other datasets, although the difference remains only at the 1.8σ level. We also note the slightly lower value of the scalar index n_s for SPT though it is also below 1.5σ . Due to the absence of low-multipole data in ACT and SPT, the degeneracy between τ and A_s remains unresolved, resulting in

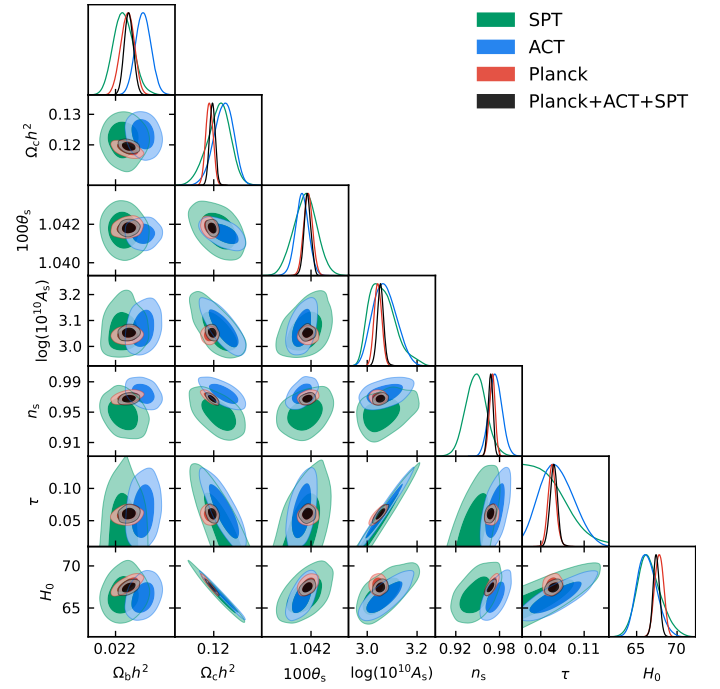


Fig. 6: Posterior distributions for the Λ CDM cosmological parameters derived from *Planck*, ACT, or SPT, and their combination using the baseline foreground templates.

large uncertainties for these two parameters. Nevertheless, the overall level of consistency is sufficient to justify combining the datasets.

The constraints on Λ CDM parameters are dominated by *Planck*, although ACT is approaching *Planck*'s sensitivity for $\Omega_b h^2$. Relative to *Planck* alone, the joint analysis of the three CMB datasets yields substantial reductions in the uncertainties in $\Omega_b h^2$, $\Omega_c h^2$ and θ_s by 50%, 17% and 22%, respectively (see Table 1). In contrast, the improvements for τ , A_s , and n_s are marginal. When all datasets are combined, the uncertainty on the amplitude of scalar perturbations, $\log(10^{10} A_s)$, is slightly larger than that reported in C25 ($\sigma_{\log(10^{10} A_s)} = 0.0099$), but is compatible with the value in L25 ($\sigma_{\log(10^{10} A_s)} = 0.013$). This increase arises because we relax some constraints on the absolute calibration and allow the polarisation efficiencies to vary freely.

Correlations between foreground and cosmological parameters are generally weak, supporting the robustness of the cosmological results with respect to the foreground modelling (see Fig. A.1). Although small, these correlations are not strictly zero, motivating a closer investigation of the impact of foreground modelling on the Λ CDM parameters (Sect. 6.3). In contrast, we observe strong correlations among the extragalactic foreground parameters, whose impact is further explored in Sect. 7.

We note that we do not expect the results presented here to exactly match the results obtained by each respective team. The main reason is that we have changed the foreground modelling, and we do not use the same priors. For cosmology, we did not apply any prior (even on the reionisation optical depth τ). For instrumental systematics, we adopted the priors specific to each experiment. For the foreground components, no priors were used, except for the dust amplitudes in the ACT and SPT datasets. The only additional exceptions are for β_{CIB} and β_{radio} , for which Gaussian priors are applied when analysing the datasets individually. We ensured that, using the same assump-

Table 1: Parameter constraints in Λ CDM for each dataset and the combination of CMB data. The last column shows the results of the chain marginalised over the foreground models. We report mean values and symmetrical 68% confidence intervals.

Parameter	SPT (baseline)	ACT (baseline)	<i>Planck</i> (baseline)	<i>Planck</i> +ACT+SPT (baseline)	<i>Planck</i> +ACT+SPT (fg marg.)
$\Omega_b h^2$	0.02216 ± 0.00021	0.02258 ± 0.00016	0.02224 ± 0.00014	0.02228 ± 0.00010	0.02228 ± 0.00010
$\Omega_c h^2$	0.1217 ± 0.0038	0.1234 ± 0.0034	0.1185 ± 0.0012	0.1196 ± 0.0010	0.1195 ± 0.0010
$100\theta_s$	1.04167 ± 0.00063	1.04151 ± 0.00037	1.04184 ± 0.00024	1.04181 ± 0.00018	1.04182 ± 0.00019
$\log(10^{10}A_s)$	3.060 ± 0.052	3.068 ± 0.047	3.042 ± 0.014	3.053 ± 0.013	3.054 ± 0.013
n_s	0.9475 ± 0.0138	0.9741 ± 0.0089	0.9701 ± 0.0039	0.9672 ± 0.0034	0.9669 ± 0.0037
τ	0.0527 ± 0.0297	0.0651 ± 0.0271	0.0585 ± 0.0063	0.0611 ± 0.0060	0.0610 ± 0.0060
H_0	66.55 ± 1.44	66.29 ± 1.24	67.77 ± 0.54	67.42 ± 0.39	67.46 ± 0.41
Ω_m	0.3396 ± 0.0229	0.3469 ± 0.0204	0.3201 ± 0.0075	0.3258 ± 0.0058	0.3252 ± 0.0061
σ_8	0.8178 ± 0.0172	0.8330 ± 0.0140	0.8073 ± 0.0068	0.8142 ± 0.0055	0.8139 ± 0.0056
S_8	0.869 ± 0.031	0.895 ± 0.023	0.834 ± 0.015	0.848 ± 0.010	0.847 ± 0.011

tions, we recovered the same results as those published in T24, L25 and C25 for *Planck*, ACT, and SPT respectively.

6.2. Consistency between temperature and polarisation

We check the agreement of Λ CDM parameter constraints from the temperature and polarisation data. This allows us to assess the ability of Λ CDM to jointly describe temperature and polarisation data. We find consistency (at the $\sim 1\sigma$ level) between the constraints of all individual and joint TT , TE , and EE likelihoods (Fig. 7). For EE and TE , we include the LoLLiPoP low- ℓ EE likelihood from *Planck*. For TT , we include the Commander low- ℓ TT likelihood. We find very good consistency between the constraints from TT and TE , while those from EE are broader and show a mild shift of the amplitude A_s and the index n_s of the scalar fluctuations towards higher values. In general, the TT and TE spectra provide comparable constraining power, but the relative weight depends on the parameter: for example, the combined result for $\Omega_c h^2$ is primarily driven by TE , whereas the constraint on n_s is dominated by TT .

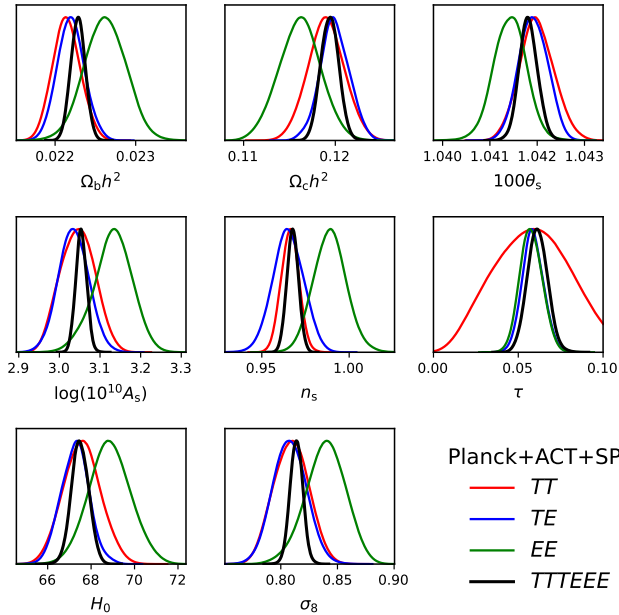


Fig. 7: Posterior distributions for the Λ CDM cosmological parameters derived from the combination of *Planck*, ACT, and SPT, using TT , TE , EE power spectra and their combination.

6.3. Impact of foregrounds models

Figure 8 illustrates how the posterior distributions of the Λ CDM parameters change when different foreground templates are considered. We run several chains corresponding to different combinations of foreground spectra in order to assess how parameter estimates vary with the details of the foreground modelling. In practice, we vary only one foreground model at a time and then concatenate the resulting MCMC chains into a pseudo-chain from which we draw the marginal posteriors. This procedure approximates Bayesian model averaging under the assumption that the models considered are equally probable and yield nearly identical Bayesian evidence (which we checked to be the case). In this regime, the pseudo-chain posteriors provide a good proxy for full marginalisation over foreground models.

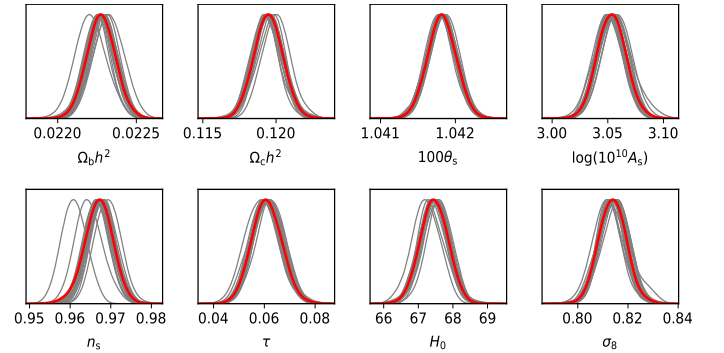


Fig. 8: Posterior distributions for the Λ CDM parameters varying templates for the foreground models as described in Sect. 7 (in gray) as compared to the posteriors after marginalisation over the foreground models (in red).

Overall, the choice of template has only a minor impact, confirming that the cosmological results are robust against reasonable variations in foreground modelling. As reported in Table 1, marginalisation over the foreground templates increases parameter uncertainties by no more than 5%, with the exception of n_s , for which the uncertainty increases by 9%. The mean value of n_s is slightly affected by assumptions on the dust and point-source models, reflecting the mild but non-negligible correlation observed between n_s and the clustered CIB amplitude, A_{CIB} , in the parameter correlation matrix. In particular, the main outlier in the n_s posterior distributions corresponds to the case where the greybody assumption for the dust SED is relaxed, fitting in-

dependent dust amplitudes for each frequency and mode, which drives A_{CIB} higher and n_s lower.

7. Discussions on the foregrounds

An important aspect of this work is the adoption of a coherent foreground model across all likelihoods, ensuring that the different datasets can be compared and combined on equal footing. Figure 9 presents the posterior distributions for the amplitudes of the CIB, tSZ, and kSZ components, together with the correlation coefficient $\xi_{\text{tSZ} \times \text{CIB}}$ and the SED indices for the CIB and radio sources, shown both for each experiment individually and for the combined analysis. Compared to the tight constraints on the ΛCDM parameters, *Planck* alone provides weaker constraints on the foreground sector, while ACT and SPT add significant constraining power.

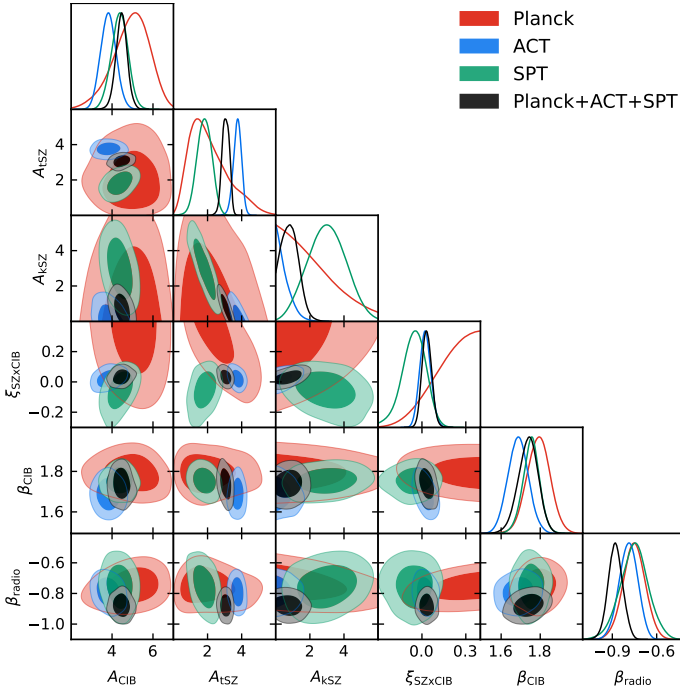


Fig. 9: Posterior distributions for the extragalactic parameters from *Planck*, ACT, or SPT.

In this section, we further exploit the flexibility of our likelihood framework to explore the impact of foreground modelling, both to assess the robustness of the cosmological constraints and to investigate how the recovered foreground parameters depend on the choice of the model. For each extragalactic foreground amplitude, we compute the average of the mean posterior across all foreground model combinations and report two types of uncertainties: the average of the recovered uncertainties (labeled *stat*) and the dispersion of the mean arising from variations in the full set of foreground models (labeled “fg”).

7.1. Galactic foregrounds

We explored different assumptions for modelling the dust residual component. First, we remove the prior on the dust SED index β_d . Then we open the priors on the dust amplitude A_{dust} associated with ACT and SPT. Finally, we consider a parametrisation without SED, fitting one amplitude for each cross-spectrum.

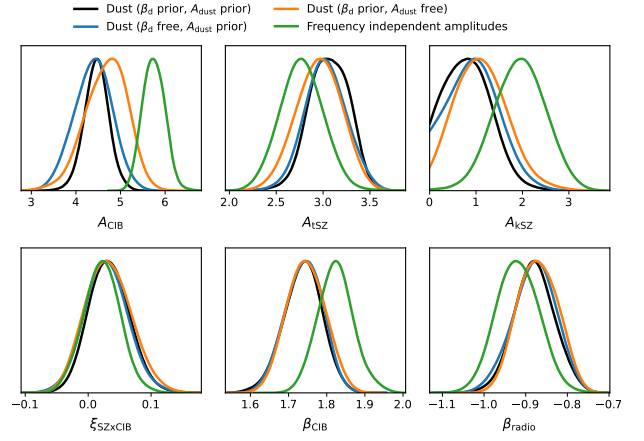


Fig. 10: Posterior distributions for extragalactic foregrounds with various hypothesis on the dust model for the combination *Planck*+ACT+SPT.

The constraint on the dust SED spectral index is driven by the priors, $\beta_d^{TT} = \mathcal{N}(1.51, 0.01)$ and $\beta_d^{TE/EE} = \mathcal{N}(1.59, 0.02)$, as the data lack the sensitivity to measure this parameter directly. We verified this by removing the priors, in which case we find the same results for all extragalactic foregrounds, with $\beta_d^{TT} = 1.44 \pm 0.17$ and posteriors for $\beta_d^{TE/EE}$ essentially flat, confirming that the datasets themselves provide no meaningful constraint.

The recovered constraint on the power-law index of the dust template in temperature is

$$\alpha_d^{TT} = -2.620 \pm 0.054(\text{stat}) \pm 0.037(\text{fg}),$$

in agreement with the *Planck* measurement, and is primarily driven by the *Planck* data. In polarisation, the sensitivity is lower than in temperature, so we adopt the *Planck* fit and fix $\alpha_d = -2.4$ for *TE* and *EE*. Allowing these indices to vary yields consistent results, with $\alpha_d^{TE} = -2.50 \pm 0.09$ and $\alpha_d^{EE} = -2.40 \pm 0.06$.

Removing the priors on the dust amplitude (orange curve in Fig. 10) yields posteriors that are very similar for all the extragalactic foregrounds. However, it significantly slows down the MCMC convergence because the two ground-based observatories provide little direct information on the dust parameters.

Finally, avoiding a specific dust model requires fitting the amplitudes of the *T* and *E* signals independently at each frequency. This introduces additional degeneracies with other foregrounds and translates into a shift in the extragalactic parameters with a noticeable increase of the CIB amplitude (green curve in Fig. 10).

Overall, the impact of dust modelling is most significant for the reconstruction of the CIB signal. This is because the spectral energy distributions of Galactic dust and CIB are very similar, leading to correlations between the two components. As a result, uncertainties in the dust model propagate directly into the inferred CIB amplitude.

7.2. Cosmic Infrared Background (CIB)

We find a clear detection of the CIB, with an amplitude at $\ell = 3000$ well measured by the three datasets. However, the amplitude of the CIB signal depends significantly on the model considered as shown in Fig. 11.

The recovered value for the amplitude of the CIB is

$$A_{\text{CIB}} = 4.32 \pm 0.33(\text{stat}) \pm 0.60(\text{fg}) \mu\text{K}^2,$$

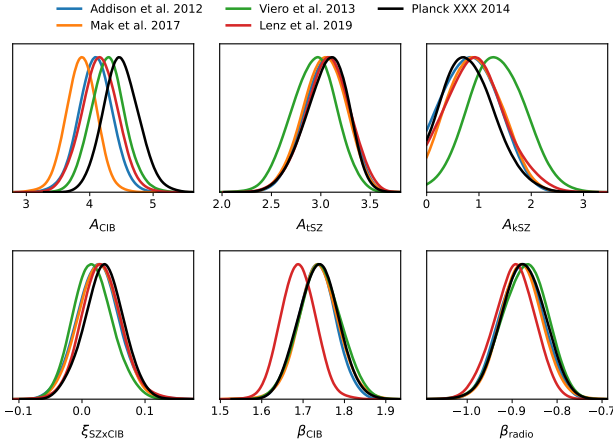


Fig. 11: Posterior distributions for extragalactic foregrounds with the CIB templates presented in Fig. 2 for the combination *Planck*+ACT+SPT.

where the dispersion of the mean due to foreground modelling is about 2 times larger than the statistical uncertainty. This detection is compatible with the results of ACT, $3.69 \pm 0.47 \mu\text{K}^2$ (L25), but significantly higher than SPT which finds $1.88 \pm 0.80 \mu\text{K}^2$ for the cross-spectrum 150×150 (C25).

For the index of the CIB spectral energy density, we find

$$\beta_{\text{CIB}} = 1.79 \pm 0.07(\text{stat}) \pm 0.16(\text{fg}),$$

in good agreement with independent measurements from *Planck* CIB reconstructed maps, $\beta_{\text{CIB}} = 1.75 \pm 0.06$ (Planck Collaboration XXX 2014). Once again, uncertainty associated with the choice of foreground templates dominates over the statistical error.

7.3. Thermal Sunyaev–Zeldovich (tSZ)

The thermal Sunyaev–Zeldovich (tSZ) signal is detected with high significance. However, we observe a substantial dispersion in the recovered posterior distributions across datasets (see Fig. 9), with amplitudes ranging from ~ 2 to $4 \mu\text{K}^2$.

Varying the tSZ template induces changes in the amplitude from 2.7 to $3.3 \mu\text{K}^2$ (see Fig. 12). Note that both the amplitude of the kSZ component and the tSZ $\times$ CIB correlation are also affected by the choice of the tSZ template. Combining the three datasets, we find

$$A_{\text{tSZ}} = 2.98 \pm 0.22(\text{stat}) \pm 0.10(\text{fg}) \mu\text{K}^2,$$

where the dispersion of the mean due to foreground modelling corresponds to approximately 50% of the statistical uncertainty.

This result is consistent with the value reported by L25 ($3.35 \pm 0.35 \mu\text{K}^2$), but less compatible with that of C25, $0.93 \pm 0.5 \mu\text{K}^2$. The lower amplitude recovered in the SPT dataset may be due to the masking of the brightest clusters for the estimation of the angular power spectra. This suggests that both the amplitude and shape of the tSZ contribution may need to be adjusted depending on the cluster masking strategy.

Allowing for an additional scaling parameter, with all datasets together, we find no evidence for a deviation from the baseline shape, obtaining $\alpha_{\text{tSZ}} = -0.11 \pm 0.13$.

7.4. Kinetic Sunyaev–Zeldovich (kSZ)

The kinetic Sunyaev–Zeldovich is not detected by *Planck* and ACT, but we find some signal in the SPT data around $3 \mu\text{K}^2$

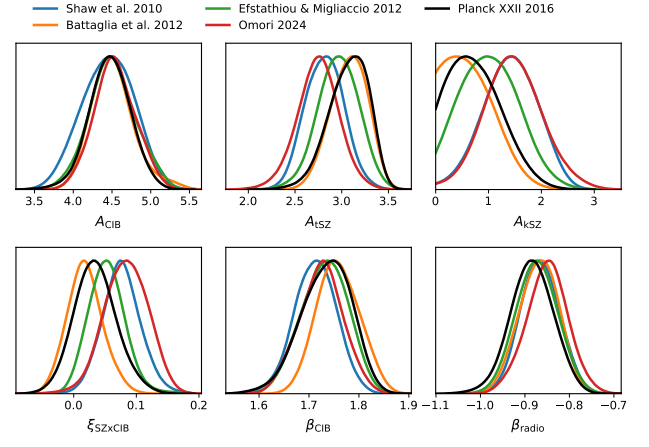


Fig. 12: Posterior distributions for extragalactic foregrounds with the tSZ templates presented in Fig. 3 for the combination *Planck*+ACT+SPT.

(Fig. 9). With the combination of the three datasets, our result remains compatible with zero and stable across different kSZ power-spectrum templates (Fig. 13). However, the recovered amplitude of the kinetic SZ signal is found to vary significantly with the assumptions adopted for other foreground components, particularly the tSZ (Fig. 12), the tSZ $\times$ CIB correlation (Fig. 14), and the point sources (Fig. 15).

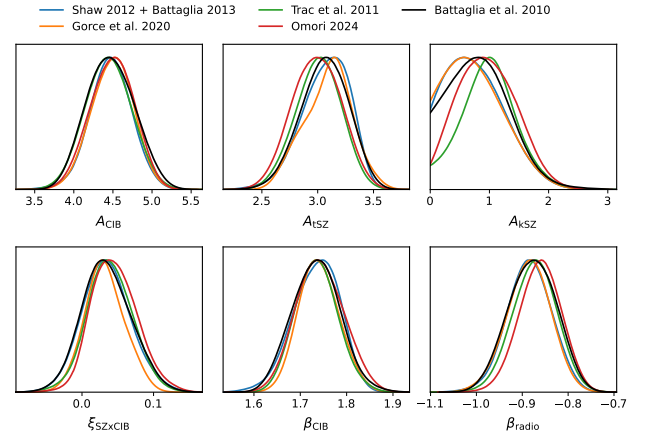


Fig. 13: Posterior distributions for extragalactic foregrounds with the kSZ templates presented in Fig. 4 for the combination *Planck*+ACT+SPT.

Across all templates, we found upper limits at 95% confidence limit ranging from $A_{\text{kSZ}} < 1.43$ to $5.36 \mu\text{K}^2$. This is to be compared to the results of ACT which reported an upper limit at 95% CL of $A_{\text{kSZ}} < 3.7 \mu\text{K}^2$ (L25) and from SPT for which C25 find $A_{\text{kSZ}} < 2.90 \mu\text{K}^2$. Averaging over all foreground models, we find a kSZ amplitude of

$$A_{\text{kSZ}} = 1.20 \pm 0.52(\text{stat}) \pm 0.64(\text{fg}) \mu\text{K}^2,$$

where the uncertainty arising from foreground templates exceeds the statistical one. When marginalising over the foreground modelling, the result is consistent with zero at the 1.4σ level, with an upper-limit of $A_{\text{kSZ}} < 2.79 \mu\text{K}^2$ (95% CL).

7.5. tSZ×CIB

We find that the cross-correlation between tSZ and CIB is consistent with zero for all datasets (Fig. 9). The correlation coefficient ξ is affected by the shape of the tSZ signal (Fig. 12) and the point source model (Fig. 15), but we found only a weak dependence with respect to the shape of the tSZ×CIB model (Fig. 14).

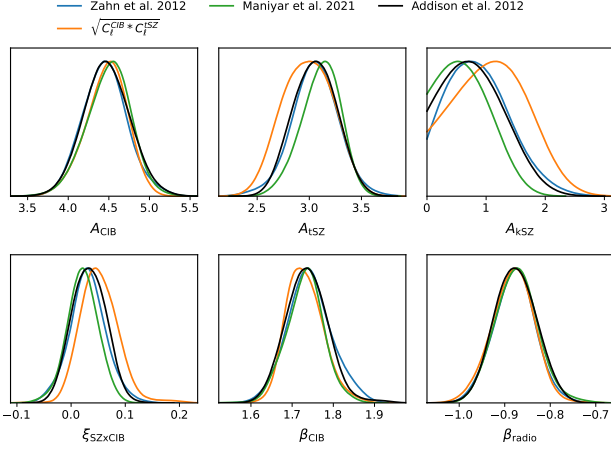


Fig. 14: Posterior distributions for extragalactic foregrounds with the tSZ×CIB templates presented in Fig. 5 for the combination *Planck*+ACT+SPT.

Across all foreground models, the amplitude of the tSZ×CIB correlation coefficient is

$$\xi_{\text{tSZ}\times\text{CIB}} = 0.044 \pm 0.036(\text{stat}) \pm 0.045(\text{fg}).$$

The constraint is compatible with that of ACT, $\xi = 0.088^{+0.036}_{-0.075}$ (L25) and is consistent with the analysis of SPT that neglects this signal.

7.6. Point Sources

The amplitudes of radio sources are fitted independently for each dataset to account for the different flux cuts of each survey, while a common spectral index is assumed for the SED. We recover

$$\beta_{\text{radio}} = -0.877 \pm 0.048(\text{stat}) \pm 0.013(\text{fg}),$$

consistent with expectations from the source studies (Tucci et al. 2011; Lagache et al. 2020).

When fitted separately from the CIB, the index of the Poisson term of the CIB, so-called dusty star-forming galaxies (DSFGs), is found to be $\beta_{\text{DSFG}} = 1.68 \pm 0.06$ with an associated clustered CIB index of $\beta_{\text{CIB}} = 2.06 \pm 0.16$. For comparison, when the same index is used for both Poisson and clustered terms, we find $\beta_{\text{CIB}} = 1.79 \pm 0.19$ after marginalisation (Sect. 7.2).

We also test how the recovered foreground parameters change when point sources are not jointly modelled but instead fitted independently for each cross-spectrum of each dataset (see the orange curve in Fig. 15). Because the number of free parameters is much larger (18 instead of 7 in the point-source sector), the resulting uncertainties are correspondingly larger. Nevertheless, the impact of point-sources modelling is significant: they can suppress the CIB amplitude by more than a factor of two while simultaneously enhancing the kSZ signal and the tSZ×CIB correlation.

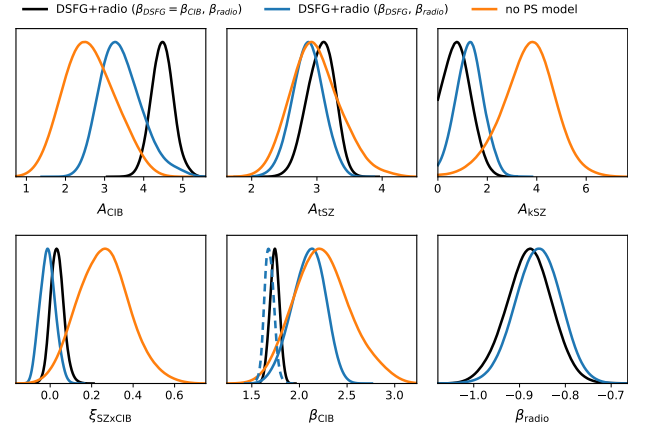


Fig. 15: Posterior distributions for extragalactic foregrounds with various point sources models. The dashed line represent the index for the DSFG (β_{DSFG}).

8. Extensions to Λ CDM

We explore several extensions to the baseline Λ CDM model – namely the phenomenological lensing amplitude A_L , the effective number of relativistic species N_{eff} , the total neutrino mass $\sum m_\nu$, and the spatial curvature Ω_K – which have been at the center of key CMB tensions in recent years. Testing these parameters within our joint framework is particularly informative, as combining *Planck* with high-resolution, small-scale measurements from ACT and SPT helps break degeneracies (see Annex D) and assess whether these anomalies persist when the datasets are modelled consistently.

8.1. Amplitude of the gravitational lensing, A_L

The phenomenological parameter A_L rescales the amplitude of the gravitational lensing potential in the CMB power spectra, effectively modulating the amount of peak smoothing induced by lensing. Previous analyses of *Planck* data have reported values of $A_L > 1$, suggesting an apparent excess of lensing compared to Λ CDM predictions and motivating further investigation with independent datasets.

Combining the three CMB datasets and averaging over the foreground models, we obtain

$$A_L = 1.063 \pm 0.033(\text{stat}) \pm 0.003(\text{fg}) \quad (\text{Planck+ACT+SPT}).$$

This result is slightly lower and more precise than the values reported in C25 for the same dataset combination ($A_L = 1.083 \pm 0.037$) or in the P-ACT combination of L25 ($A_L = 1.081 \pm 0.043$).

Overall we show a limited impact of the foreground modelling on the estimation of A_L , and no evidence for anomalous lensing relative to Λ CDM expectations.

8.2. Sum of the neutrino masses, $\sum m_\nu$

Figure 16 shows the posterior distributions for the sum of the neutrino masses, $\sum m_\nu$. Averaging over the different foreground models, we obtain

$$\sum m_\nu < 0.181 \text{ eV} \quad (95\% \text{ CL, Planck+ACT+SPT}),$$

with a standard deviation of 0.019 eV across foreground models. This dispersion represents 35% of the $1-\sigma$ statistical sensitivity

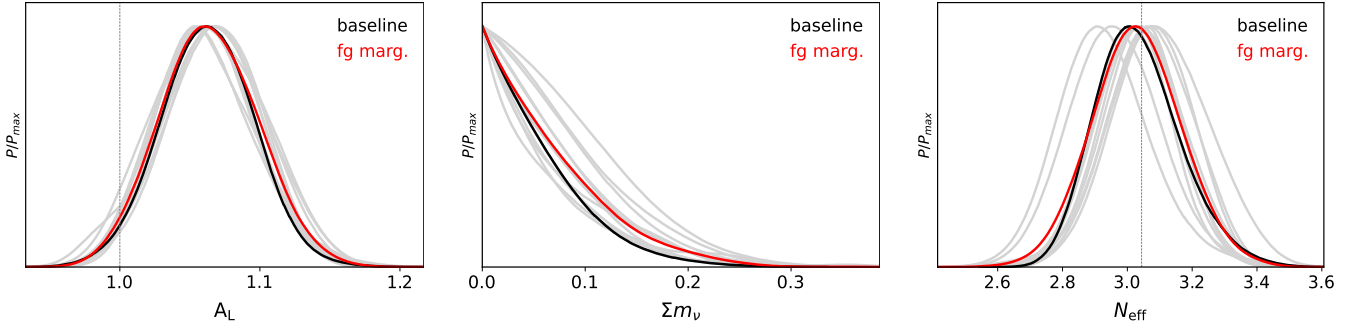


Fig. 16: Posterior distributions for A_L (left), Σm_ν (center), and N_{eff} (right) combining *Planck*, ACT, and SPT, varying foreground templates as described in Sect. 7 (in gray) as compared to the posteriors after marginalisation over the foreground models (in red).

(0.055 eV), indicating that the combined constraint is sensitive to the details of the foreground modelling.

Compared to the *Planck*-only limit (see Annexe D), the combined analysis improves the upper bound by more than a factor of two. The improvement is primarily driven by the small-scale information provided by ACT and SPT, which are sensitive to the modification of the smoothing due to gravitational lensing in the TT and EE power spectra.

8.3. Effective number of relativistic species, N_{eff}

The effective number of relativistic species, N_{eff} , quantifies the total radiation energy density in the early Universe and is sensitive to any additional light relics beyond the three active neutrino species predicted by the Standard Model.

Figure 16 shows the posterior distributions for the combination of *Planck*, ACT, SPT for the various foreground templates considered in this paper. Averaging over foreground models, we obtain

$$N_{\text{eff}} = 3.031 \pm 0.130 (\text{stat}) \pm 0.045 (\text{fg}) \quad (\text{Planck+ACT+SPT}).$$

This result is in excellent agreement with the Standard Model expectation of $N_{\text{eff}} = 3.044$ for three neutrino species undergoing non-instantaneous decoupling from the primordial plasma (Froustey et al. 2020; Akita & Yamaguchi 2020).

This combined result is more precise than any individual dataset and highlights the importance of both the improved *Planck* PR4 data release and the use of a coherent joint likelihood. In contrast, earlier analyses based on PR3 data and obtained by only multiplying likelihoods reported lower values, such as $N_{\text{eff}} = 2.81 \pm 0.12$ in C25 and $N_{\text{eff}} = 2.73 \pm 0.14$ in Calabrese et al. (2025). With PR4 and the present methodology, these apparent downward shifts are no longer observed: our result is fully consistent with the standard model expectation and robust against the foreground modelling.

8.4. Spatial curvature, Ω_K

The curvature parameter, Ω_K , measures the deviation of the Universe from spatial flatness. Analyses based on *Planck* PR3 data have shown a mild preference for a closed Universe ($\Omega_K < 0$), hinting at a small but notable tension with the flat Λ CDM prediction.

Figure 17 shows the posterior for Ω_K together with its geometric degeneracy with H_0 . As discussed in T24, with *Planck* PR4, the tail of the 2-d posterior in the H_0 - Ω_K plane at low H_0 and negative Ω_K is significantly less preferred. Indeed, using the combined CMB datasets, averaging over foreground models, we

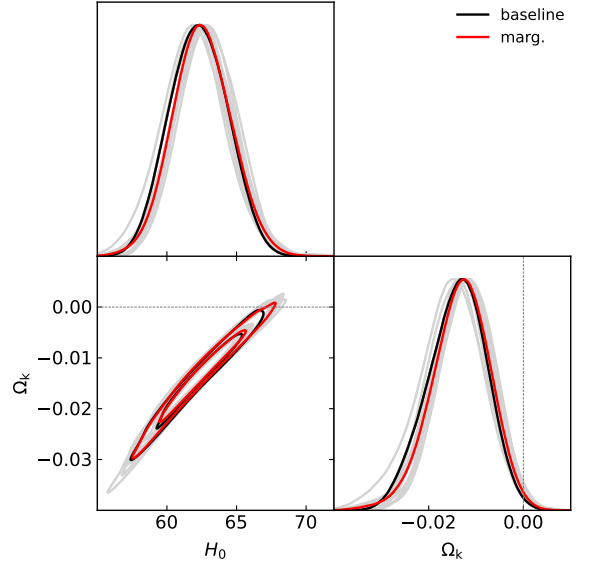


Fig. 17: Posterior distributions for Ω_K combining *Planck*, ACT, and SPT, varying foreground templates (in gray) as compared to the posteriors after marginalisation over the foreground models (in red).

obtain

$$\Omega_K = -0.0132^{+0.0066}_{-0.0056} (\text{stat}) \pm 0.0009 (\text{fg}) \quad (\text{Planck+ACT+SPT}),$$

which is consistent with a flat Universe ($\Omega_K = 0$) at the 2σ level. This combined result reduces the preference for a closed Universe that appeared in the *Planck* PR3 analysis with *Plik*, showing that the apparent deviation was driven by dataset- and likelihood-specific systematics rather than new physics.

9. Conclusion

In this work, we have constructed a CMB likelihood that combines the latest data from *Planck*, ACT, and SPT within a single, coherent framework. Particular care was taken to implement a consistent treatment of both foreground modelling and instrumental systematics, ensuring that the different datasets can be robustly analysed together. Compared to a simple multiplication of the individual likelihoods, our approach explicitly enforces consistency in the modelling assumptions and parameterisations across experiments. This is essential for disentangling true cosmological signals from experiment-specific systematics or foreground treatments, and it allows us to quantify uncertainties in

a uniform way. Crucially, we can also then marginalise over the choice of foreground templates, ensuring that the resulting cosmological constraints reflect not only statistical precision but also the systematic uncertainty associated with different modelling assumptions.

We first examined the impact of this joint likelihood on the Λ CDM parameters. The combined constraints are consistent with those obtained from the each experiments, and the parameter means show negligible dependence on the details of the foreground modelling. Marginalising over the foreground templates increases the uncertainties by less than 5%, except for n_s (9%). Relative to *Planck* alone, the joint analysis improves parameter precision by up to 20%, demonstrating the complementarity of the datasets and the benefit of a coherent and combined analysis.

Turning to foreground parameters constraints, we observe a stronger dependence on modelling choices. In particular, the reconstruction of the CIB amplitude, the kSZ signal and the tSZ×CIB correlation show significant sensitivity to the adopted templates, while the tSZ is relatively more stable. The uncertainties on A_{CIB} (and, to a lesser extent, on A_{kSZ} and $\xi_{\text{tSZ} \times \text{CIB}}$) are dominated by variations in the foreground modelling rather than by statistical noise. The treatment of point-source and dust residuals emerge as the most important factors influencing the inferred amplitudes of extragalactic foregrounds. This underlines the necessity of a unified multi-dataset framework to achieve reliable astrophysical constraints.

Finally, we investigate the impact of the joint likelihood on several simple extensions to Λ CDM. For the lensing amplitude A_L , the sum of neutrino masses $\sum m_\nu$, the curvature parameter Ω_k , and the effective number of relativistic species N_{eff} , the combination of datasets provides substantial gains in precision, with no evidence for departures from Λ CDM expectations. Several previously reported tensions – such as hints of non-zero curvature, or values of $A_L > 1$ – are significantly reduced or even resolved when using PR4 data and the combined likelihood. We further find that uncertainties in foreground modelling have a non-negligible effect, increasing the error bars by up to 35%.

Overall, this work demonstrates that a coherent combination of *Planck*, ACT, and SPT data is feasible and robust. While cosmological constraints remain stable against foreground modelling, the treatment of foregrounds emerges as an important source of uncertainty for Λ CDM extensions. The framework developed here provides a solid basis for future analyses, particularly those aiming to incorporate more sophisticated physical models, such as halo-based descriptions of extragalactic components (as done for instance in Douspis et al. 2022; Gorce et al. 2022; Bolliet et al. 2025), that can propagate astrophysical uncertainties when fitting CMB data in a controlled manner. Such developments will be crucial for the next generation of high-sensitivity surveys – including the Simons Observatory and the South Pole Observatory – where the precision of cosmological inference will increasingly depend on the accuracy of foreground modelling. In this context, the combination with *Planck* will continue to play a central role, both as a calibration anchor and as a unique source of large-scale information.

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Appendix A: Correlations

Figure A.1 shows the correlation matrix for the cosmological and foreground parameters. The upper triangular half of the matrix displays the correlation coefficients, while the lower triangular half visualises them using the color scale.

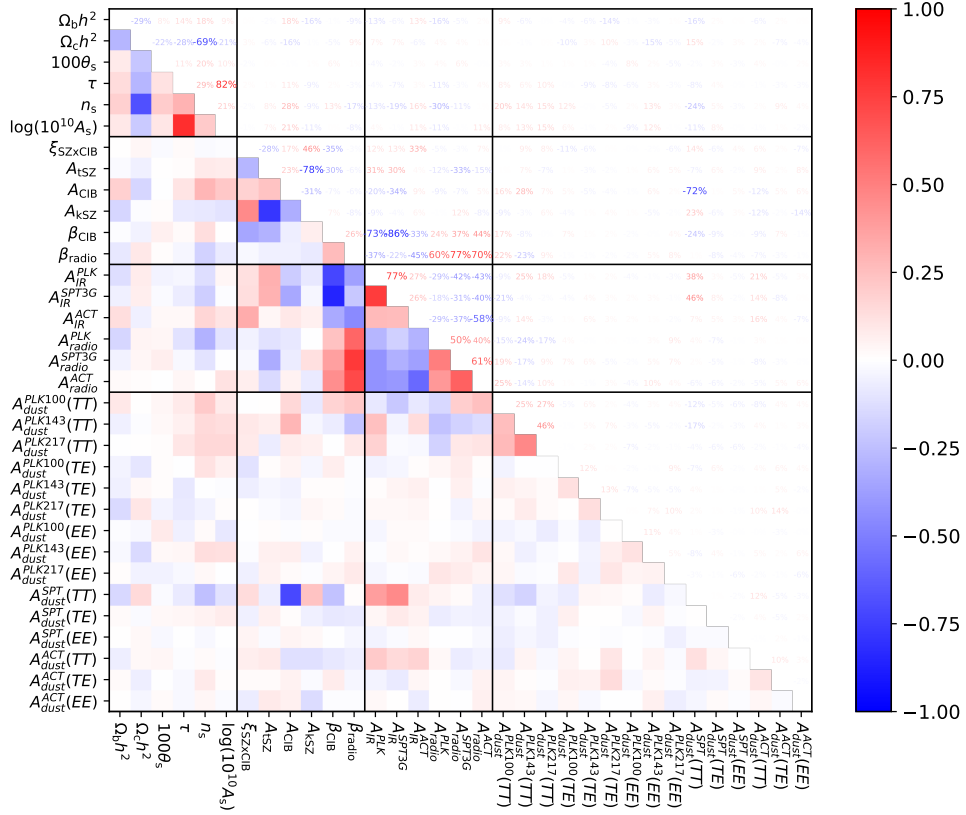


Fig. A.1: Correlation matrix for the fitted parameters of the combined likelihood TTTEEE. The first block corresponds to cosmological parameters from the Λ CDM model, the second block gathers the foreground parameters common to all datasets, the third and fourth blocks show, for each dataset, the amplitude of the point sources and the dust amplitudes respectively.

Appendix B: Foreground parameters

Likelihood	Parameter	Prior	Posterior	Spectrum	Description
Common	A_{CIB}	[0, 50]	4.32 ± 0.73	TT	Clustered CIB amplitude
	A_{tSZ}	[0, 50]	2.98 ± 0.25	TT	tSZ amplitude
	A_{kSZ}	[0, 50]	1.20 ± 0.87	TT	kSZ amplitude
	$\xi_{\text{SZ} \times \text{CIB}}$	[-1, 1]	0.044 ± 0.064	TT	SZ $\times$ CIB amplitude
	β_{CIB}	[1, 3]	1.79 ± 0.19	TT	CIB spectral index
	β_{radio}	[-1.5, 0.0]	-0.88 ± 0.05	TT	Radio sources spectral index
	β_{d}^{TT}	$\mathcal{N}(1.51, 0.01)$	1.510 ± 0.009	TT	Dust TT spectral index
	β_{d}^{TE}	$\mathcal{N}(1.59, 0.02)$	1.588 ± 0.021	TE	Dust TE spectral index
	β_{d}^{EE}	$\mathcal{N}(1.59, 0.02)$	1.612 ± 0.020	EE	Dust EE spectral index
	α_{d}^{TT}	[-3, -2]	-2.62 ± 0.07	TT	Power-law index of the TT dust spectrum
	α_{d}^{TE}	-2.4	-2.4	TE	Power-law index of the TE dust spectrum
	α_{d}^{EE}	-2.4	-2.4	EE	Power-law index of the EE dust spectrum
Planck	$A_{\text{dust}}^{\text{PLK100}}(TT)$	[0, 100]	24.2 ± 9.0	TT	Dust amplitude in TT in 100 GHz maps
	$A_{\text{dust}}^{\text{PLK143}}(TT)$	[0, 100]	21.0 ± 1.8	TT	Dust amplitude in TT in 143 GHz maps
	$A_{\text{dust}}^{\text{PLK217}}(TT)$	[0, 100]	9.71 ± 0.72	TT	Dust amplitude in TT in 217 GHz maps
	$A_{\text{dust}}^{\text{PLK100}}(TE)$	[0, 10]	1.477 ± 0.320	TE	Dust amplitude in TE in 100 GHz maps
	$A_{\text{dust}}^{\text{PLK143}}(TE)$	[0, 10]	0.792 ± 0.151	TE	Dust amplitude in TE in 143 GHz maps
	$A_{\text{dust}}^{\text{PLK217}}(TE)$	[0, 10]	0.434 ± 0.044	TE	Dust amplitude in TE in 217 GHz maps
	$A_{\text{dust}}^{\text{PLK100}}(EE)$	[0, 10]	0.628 ± 0.201	EE	Dust amplitude in EE in 100 GHz maps
	$A_{\text{dust}}^{\text{PLK143}}(EE)$	[0, 10]	0.293 ± 0.012	EE	Dust amplitude in EE in 143 GHz maps
	$A_{\text{dust}}^{\text{PLK217}}(EE)$	[0, 10]	0.174 ± 0.017	EE	Dust amplitude in EE in 217 GHz maps
	$A_{\text{radio}}^{\text{PLK}}$	[0, 150]	58.06 ± 11.41	TT	Unresolved radio sources
ACT	$A_{\text{dust}}^{\text{ACT}}(TT)$	$\mathcal{N}(7.95, 0.32)$	7.81 ± 0.53	TT	Dust amplitude in TT
	$A_{\text{dust}}^{\text{ACT}}(TE)$	$\mathcal{N}(0.423, 0.030)$	0.413 ± 0.051	TE	Dust amplitude in TE
	$A_{\text{dust}}^{\text{ACT}}(EE)$	$\mathcal{N}(0.168, 0.017)$	0.185 ± 0.080	EE	Dust amplitude in EE
	$A_{\text{radio}}^{\text{ACT}}$	[0, 20]	3.01 ± 2.21	TT	Unresolved radio sources in TT
	$A_{\text{IR}}^{\text{ACT}}$	[0, 20]	7.45 ± 1.52	TT	Unresolved infrared sources
SPT	$A_{\text{dust}}^{\text{SPT}}(TT)$	$\mathcal{N}(1.88, 0.960)$	2.30 ± 1.03	TT	Dust amplitude in TT
	$A_{\text{dust}}^{\text{SPT}}(TE)$	$\mathcal{N}(0.12, 0.051)$	0.105 ± 0.033	TE	Dust amplitude in TE
	$A_{\text{dust}}^{\text{SPT}}(EE)$	$\mathcal{N}(0.05, 0.022)$	0.057 ± 0.013	EE	Dust amplitude in EE
	$A_{\text{radio}}^{\text{SPT3G}}$	[0, 10]	0.68 ± 0.06	TT	Unresolved radio sources in TT
	$A_{\text{IR}}^{\text{SPT3G}}$	[0, 10]	7.33 ± 0.64	TT	Unresolved infrared sources

Table B.1: Foreground parameters for each dataset, along with their associated priors and constraints after marginalisation over the foreground templates (mean and standard deviation). In total, this amounts to 31 parameters: 10 parameters common to all datasets, 11 parameters specific to *Planck*, 5 to ACT, and 5 to SPT. The datasets share the same set of foreground parameters, except for the dust and point sources amplitudes. The dust amplitudes depend on the sky fraction: *Planck* includes three amplitudes, while ACT and SPT each have only one, corresponding to their single mask survey.

Appendix C: Nuisance parameters

Table C.1 presents all nuisance parameters together with their constraint for the combination of *Planck*, ACT, and SPT data after marginalising over the foreground templates.

While *Planck* is calibrated using the orbital dipole (through the satellite velocity and the CMB monopole), ground-based experiments usually rely on measurements of planet fluxes to derive their map calibration. In a second step, their spectra can be re-calibrated by cross-correlating with *Planck* maps. As mentioned in Sect. 4, we allow the datasets to recalibrate on *Planck*. This is done by introducing calibration factors applied to SPT, and ACT independently.

We find a good consistency between the datasets

$$c_{\text{SPT}} = 1.0011 \pm 0.0043,$$

$$c_{\text{ACT}} = 0.9868 \pm 0.0049,$$

although, for ACT, we find a deviation of -1.3%, significantly greater than the absolute calibration accuracy of 0.3% reported by L25. In contrast, for SPT our analysis yields an overall recalibration of 0.11%, consistent with the absolute calibration uncertainty of 0.36% reported by C25.

All polar efficiencies are found to be compatible with unity at better than 1σ . The uncertainties depend on the dataset with $\sim 3\%$, 2.5%, and 4% for *Planck* at 100, 143 and 217 GHz respectively, and $\sim 0.4\%$ for SPT or ACT.

Likelihood	Parameter	Prior	Posterior	Description
<i>Planck</i>	A_{planck}	$\mathcal{N}(1.00, 0.0025)$	1.001 ± 0.002	Absolute calibration uncertainty
	c_{100A}^{PLK}	$\mathcal{N}(1.00, 0.01)$	1.0007 ± 0.0074	Intercalibration map 100A
	c_{100B}^{PLK}	$\mathcal{N}(1.00, 0.01)$	0.9961 ± 0.0076	Intercalibration map 100B
	c_{143A}^{PLK}	1.00(fixed)	1.00	Calibration reference map 143A
	c_{143B}^{PLK}	$\mathcal{N}(1.00, 0.01)$	1.0002 ± 0.0072	Intercalibration map 143B
	c_{217A}^{PLK}	$\mathcal{N}(1.00, 0.01)$	1.0026 ± 0.0080	Intercalibration map 217A
	c_{217B}^{PLK}	$\mathcal{N}(1.00, 0.01)$	1.0001 ± 0.0079	Intercalibration map 217B
	ρ_{100A}^{PLK}	[0.8, 1.2]	1.007 ± 0.032	Polarisation efficiency map 100A
	ρ_{100B}^{PLK}	[0.8, 1.2]	1.013 ± 0.030	Polarisation efficiency map 100B
	ρ_{143A}^{PLK}	[0.8, 1.2]	0.962 ± 0.025	Polarisation efficiency map 143A
	ρ_{143B}^{PLK}	[0.8, 1.2]	1.022 ± 0.026	Polarisation efficiency map 143B
	ρ_{217A}^{PLK}	[0.8, 1.2]	1.035 ± 0.041	Polarisation efficiency map 217A
	ρ_{217B}^{PLK}	[0.8, 1.2]	1.021 ± 0.041	Polarisation efficiency map 217B
ACT	A^{ACT}	[0.7, 1.3]	0.9868 ± 0.0052	ACT calibration
	c_{090pa5}^{ACT}	$\mathcal{N}(1.00, 0.0016)$	0.9981 ± 0.0013	Intercalibration pa5 90 GHz
	c_{090pa6}^{ACT}	$\mathcal{N}(1.00, 0.0018)$	0.9990 ± 0.0014	Intercalibration pa6 90 GHz
	c_{150pa5}^{ACT}	1.00(fixed)	1.00	Calibration reference (pa5 150 GHz)
	c_{150pa6}^{ACT}	$\mathcal{N}(1.00, 0.0024)$	1.0038 ± 0.0016	Intercalibration pa6 150 GHz
	c_{220pa4}^{ACT}	$\mathcal{N}(1.00, 0.013)$	0.9908 ± 0.0079	Intercalibration pa4 220 GHz
	$\rho_{090pa5}^{\text{ACT}}$	[0.8, 1.2]	1.0015 ± 0.0042	Polarisation efficiency pa5 90 GHz
	$\rho_{090pa6}^{\text{ACT}}$	[0.8, 1.2]	1.0109 ± 0.0044	Polarisation efficiency pa6 90 GHz
	$\rho_{150pa5}^{\text{ACT}}$	[0.8, 1.2]	1.0067 ± 0.0043	Polarisation efficiency pa5 150 GHz
	$\rho_{150pa6}^{\text{ACT}}$	[0.8, 1.2]	1.0085 ± 0.0048	Polarisation efficiency pa6 150 GHz
	$\Delta_{090pa5}^{\text{ACT}}$	$\mathcal{N}(0.0, 1.0)$	0.46 ± 0.63	Bandpass uncertainty pa5 90 GHz
	$\Delta_{090pa6}^{\text{ACT}}$	$\mathcal{N}(0.0, 1.2)$	0.92 ± 0.79	Bandpass uncertainty pa6 90 GHz
	$\Delta_{150pa5}^{\text{ACT}}$	$\mathcal{N}(0.0, 1, 3)$	-1.64 ± 0.83	Bandpass uncertainty pa5 150 GHz
	$\Delta_{150pa6}^{\text{ACT}}$	$\mathcal{N}(0.0, 1.1)$	-1.00 ± 0.76	Bandpass uncertainty pa6 150 GHz
	$\Delta_{220pa4}^{\text{ACT}}$	$\mathcal{N}(0.0, 3.6)$	0.06 ± 1.68	Bandpass uncertainty pa4 220 GHz
SPT	A^{SPT}	[0.7, 1.3]	1.0013 ± 0.0050	SPT calibration
	c_{90}^{SPT}	$\mathcal{N}(1.00, 0.0003)$	1.0004 ± 0.0004	Intercalibration 90 GHz
	c_{150}^{SPT}	1.00(fixed)	1.00	Calibration reference 150 GHz
	c_{220}^{SPT}	$\mathcal{N}(1.00, 0.001)$	1.0092 ± 0.0011	Intercalibration 220 GHz
	ρ_{90}^{SPT}	[0.8, 1.2]	1.0014 ± 0.0039	Polarisation efficiency 90 GHz
	ρ_{150}^{SPT}	[0.8, 1.2]	1.0030 ± 0.0039	Polarisation efficiency 150 GHz
	ρ_{220}^{SPT}	[0.8, 1.2]	0.9981 ± 0.0049	Polarisation efficiency 220 GHz
	β_1^{SPT}	$\mathcal{N}(0.0, 1.0)$	-1.15 ± 0.94	Main beam eigenmodes
	β_2^{SPT}	$\mathcal{N}(0.0, 1.0)$	-0.50 ± 0.88	Main beam eigenmodes
	β_3^{SPT}	$\mathcal{N}(0.0, 1.0)$	0.03 ± 0.92	Main beam eigenmodes
	β_4^{SPT}	$\mathcal{N}(0.0, 1.0)$	-1.85 ± 0.63	Main beam eigenmodes
	β_5^{SPT}	$\mathcal{N}(0.0, 1.0)$	0.24 ± 0.92	Main beam eigenmodes
	β_6^{SPT}	$\mathcal{N}(0.0, 1.0)$	-1.40 ± 0.90	Main beam eigenmodes
	β_7^{SPT}	$\mathcal{N}(0.0, 1.0)$	0.15 ± 0.72	Main beam eigenmodes
	β_8^{SPT}	$\mathcal{N}(0.0, 1.0)$	-0.20 ± 0.98	Main beam eigenmodes
	β_9^{SPT}	$\mathcal{N}(0.0, 1.0)$	0.42 ± 0.97	Main beam eigenmodes
	β_{90}^{SPT}	[0.0, 1.0]	0.555 ± 0.079	Beam sidelobe polarisation fraction 90 GHz
	β_{150}^{SPT}	[0.0, 1.0]	0.716 ± 0.100	Beam sidelobe polarisation fraction 150 GHz
	β_{220}^{SPT}	[0.0, 1.0]	0.681 ± 0.124	Beam sidelobe polarisation fraction 220 GHz
	$\epsilon_{90}^{\text{SPT}}$	$\mathcal{N}(-0.0065, 0.0011)$	-0.0073 ± 0.0008	Temperature-to-polarisation leakage 90 GHz
	$\epsilon_{150}^{\text{SPT}}$	$\mathcal{N}(-0.012, 0.0021)$	-0.0154 ± 0.0014	Temperature-to-polarisation leakage 150 GHz
	$\epsilon_{220}^{\text{SPT}}$	$\mathcal{N}(-0.023, 0.0066)$	-0.0291 ± 0.0036	Temperature-to-polarisation leakage 220 GHz
	κ^{SPT}	$\mathcal{N}(0.00000, 0.00045)$	-0.00008 ± 0.00029	Super-sample lensing

Table C.1: Nuisance parameters for instrumental effects for each dataset, along with their associated priors and constraints after marginalisation over the foreground templates (mean and standard deviation). This results in 48 parameters: 12 parameters for *Planck*, 14 parameters for ACT, and 22 parameters for SPT.

Appendix D: Extensions

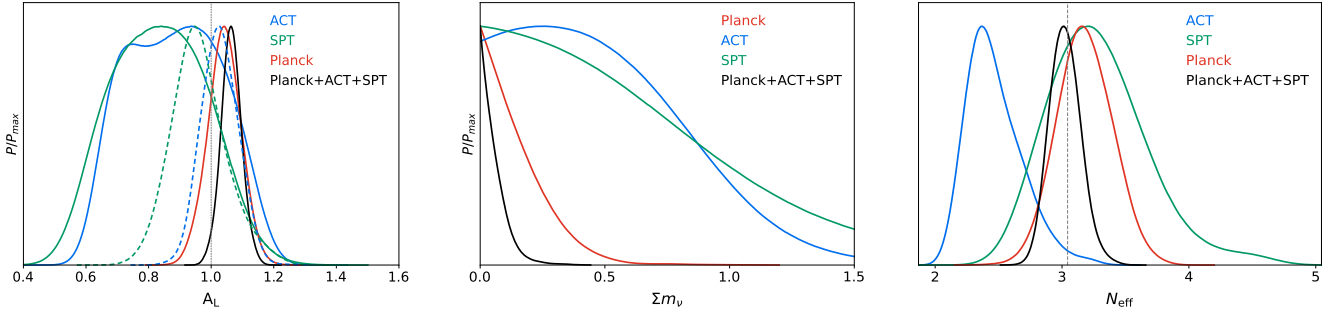


Fig. D.1: Posterior distributions for A_L (left), $\sum m_\nu$ (center), and N_{eff} (right) using *Planck*, ACT, or SPT, and their combination. The dashed lines on the left plot represent the posterior distribution with a prior on the reionisation optical depth, $\tau = \mathcal{N}(0.057, 0.006)$.

D.1. Amplitude of the gravitational lensing, A_L

We vary A_L for the three CMB datasets and derived the corresponding marginal posteriors, shown in Fig. D.1. For all datasets, the recovered values of A_L are consistent with unity, indicating that the smoothing of the acoustic peaks in the CMB spectra is compatible with Λ CDM expectations:

$$\begin{aligned} A_L &= 1.042 \pm 0.052 \quad (\text{Planck}) \\ A_L &= 0.881 \pm 0.175 \quad (\text{ACT}) \\ A_L &= 0.833 \pm 0.174 \quad (\text{SPT}) \end{aligned}$$

For *Planck*, the result is fully consistent with T24, which report $A_L = 1.039 \pm 0.052$. The uncertainties for ACT and SPT are larger, primarily due to the degeneracy with A_s and τ , which remains unresolved without additional constraints on the reionisation optical depth. When a prior on τ is applied, $\tau = \mathcal{N}(0.057, 0.006)$, we obtain $A_L = 1.027 \pm 0.058$ for ACT and $A_L = 0.961 \pm 0.089$ for SPT, values that are closer to those reported in L25 and C25, respectively.

D.2. Sum of the neutrino masses, $\sum m_\nu$

Figure D.1 shows the posterior distributions for the sum of the neutrino masses, $\sum m_\nu$. No evidence for a non-zero neutrino mass is found, and we report the following upper limits:

$$\begin{aligned} \sum m_\nu &< 0.402 \text{ eV} \quad (95\% \text{ CL, Planck}) \\ \sum m_\nu &< 1.120 \text{ eV} \quad (95\% \text{ CL, ACT}) \\ \sum m_\nu &< 1.447 \text{ eV} \quad (95\% \text{ CL, SPT}) \end{aligned}$$

D.3. Effective number of relativistic species, N_{eff}

Figure D.1 shows the posteriors for *Planck*, ACT, SPT, and their combination when we consider the N_{eff} extension. The individual datasets yield:

$$\begin{aligned} N_{\text{eff}} &= 3.20^{+0.21}_{-0.23} \quad (\text{Planck}) \\ N_{\text{eff}} &= 2.45^{+0.17}_{-0.25} \quad (\text{ACT}) \\ N_{\text{eff}} &= 3.23^{+0.32}_{-0.38} \quad (\text{SPT}) \end{aligned}$$

These results are consistent with those obtained in the original analyses, which quote $N_{\text{eff}} = 2.60^{+0.21}_{-0.29}$ for ACT (Calabrese et al. 2025), $N_{\text{eff}} = 3.17^{+0.29}_{-0.33}$ for SPT (C25), and $N_{\text{eff}} = 3.08 \pm 0.17$ for *Planck* PR4 (T24). In particular, we reproduce the tendency of ACT to shift N_{eff} toward low values.

D.4. Spatial curvature, Ω_K

We find that CMB data are compatible with a flat Universe, with Ω_K consistent with zero for all datasets:

$$\begin{aligned} \Omega_K &= -0.015^{+0.011}_{-0.009} \quad (\text{Planck}) \\ \Omega_K &= -0.006^{+0.020}_{-0.016} \quad (\text{ACT}) \\ \Omega_K &= 0.003^{+0.019}_{-0.014} \quad (\text{SPT}) \end{aligned}$$

These results are compatible with those of [L25](#) and [C25](#), although with larger error bars due to the absence of the τ -prior. For *Planck*, our constraint is comparable to the HiLLiPoP PR4 result ($\Omega_K = -0.012 \pm 0.010$, [T24](#)) and should be contrasted with $\Omega_K = -0.025^{+0.013}_{-0.010}$ from Camspec PR4 ([Rosenberg et al. 2022](#)) and $\Omega_K = -0.044^{+0.018}_{-0.015}$ from *Planck* PR3 ([Planck Collaboration VI 2020](#)).

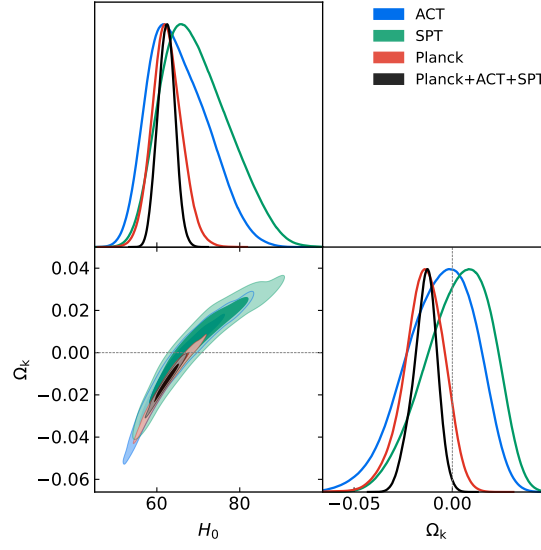


Fig. D.2: Posterior distributions for Ω_K using *Planck*, ACT, or SPT, and their combination.

Figure [D.2](#) shows the posterior for Ω_K together with its geometric degeneracy with H_0 . As discussed in [T24](#), with *Planck* PR4, the tail of the 2-d posterior in the H_0 - Ω_K plane at low H_0 and negative Ω_K is less preferred compared to *Planck* PR3.