

Electroweak phase transition enhanced by a CP-violating dark sector

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ABSTRACT: Within a well-motivated 3-Higgs doublet model, in which the extended dark sector accommodates CP violation, we analyse the electroweak phase transition (EWPT) at one- and two-loop order. We show the importance of higher loop calculations in EWPT analyses and identify the regions of the parameter space of our model where EWPT is of first order while in agreement with all theoretical and experimental bounds, including Dark Matter relic density and direct and indirect searches.

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Contents

1	Introduction	1
2	The extended scalar sector	4
2.1	Explicit CP-violation	6
2.2	The mass spectrum	6
2.3	The span of θ_{CPV}	8
2.4	Input parameters of the model	8
2.4.1	Renormalization and input parameters	9
3	Constraints on the parameter space	10
3.1	DM abundance and the selection of benchmarks	12
4	Thermal corrections	14
4.1	High- T effective theory for 3HDMs	15
4.2	The effective potential	17
5	EWPT dynamics	17
5.1	Setup	17
5.2	Numerical results	19
5.2.1	Towards electroweak baryogenesis	19
5.2.2	Two-step phase transitions	19
6	Conclusion and outlook	25

1 Introduction

The Standard Model (SM) of particle physics has undergone extensive experimental verification and remains in remarkable agreement with observations. The discovery of its last missing component, the Higgs boson, at the LHC in 2012 [1, 2] solidified its predictive power. Despite ongoing searches, no substantial deviations from the SM have been observed at the LHC, and the properties of the discovered scalar particle align well with those of the SM Higgs boson [3, 4].

However, the SM does not account for several fundamental phenomena, including the observed baryon asymmetry of the Universe and the existence of a viable Dark Matter (DM) candidate. Various astrophysical observations suggest the necessity of a DM particle that is cosmologically stable, cold, non-baryonic, electrically neutral, and weakly interacting. No such particle exists within the SM framework. Furthermore, the amount of CP violation

predicted by the SM is several orders of magnitude smaller than what is required to generate the observed matter-antimatter asymmetry [5–7].

These shortcomings strongly indicate the need for beyond Standard Model (BSM) physics in the search for a more complete theory of Nature. Among the simplest BSM extensions addressing the issues mentioned above, are non-minimal Higgs sectors, predicting the Higgs boson discovered at the LHC is part of a broader scalar sector. Extending the scalar sector can introduce new sources of CP violation and provide viable DM candidates. The scalar potential remains one of the least constrained sectors of the SM, making it a promising avenue for such extensions. Additionally, non-minimal Higgs sectors incorporating discrete symmetries naturally, allowing for Weakly Interacting Massive Particles (WIMPs) [8–11], which are among the most widely studied DM candidates. The stability of these WIMPs is ensured by conserved discrete symmetries, allowing them to achieve a relic abundance consistent with observation [12] via the freeze-out mechanism.

Extensive research has been conducted on one-singlet and one-doublet scalar extensions of the SM, particularly in the form of Higgs portal models and 2-Higgs-doublet models (2HDMs) (see, e.g., [13, 14] and references therein). However, these models, by design, offer only partial solutions to the unresolved problems of the SM; notably, the well-established $\mathbb{Z}_2$ -symmetric Higgs portal model [15] and the Inert Doublet Model (IDM) [16], are consistent with both direct and indirect DM search constraints. However, they face stringent limitations from LHC data, particularly from measurements of the Higgs invisible decay width and Higgs signal strength. Moreover, the scalar potential in these models is inherently CP conserving. If one disregards the requirement of a DM candidate, the 2HDM scalar potential can, in principle, accommodate CP violation. However, the introduction of new CP-violating interactions modifies SM Higgs couplings and contributes to the electric dipole moments (EDMs) of the neutron, electron, and certain atomic nuclei [17], leading to very strong experimental constraints [18–21]. On the other hand, purely singlet scalar extensions of the SM remain CP conserving regardless of apparent phases in the vevs or potential parameters.¹ This points to the fact that for the scalar sector to yield CP violation and a viable DM candidate, inevitably, it needs to be extended beyond the simple one singlet or one doublet setup.

In contrast to the simple scenarios mentioned before, more elaborate scalar extensions, such as 3-Higgs-doublet models (3HDMs), provide viable DM candidates, introduce new sources of CP violation, enable a strongly first-order phase transition necessary for baryogenesis, incorporate inflaton candidates relevant for the early universe inflationary epoch, and address the fermion mass hierarchy problem, all in a single framework. These features arise due to various symmetries and symmetry-breaking patterns within the scalar potential, which dictate the number of *active* (developing a vacuum expectation value (vev)) and *inert* (lacking a vev) scalar doublets [23–39]. If CP violation is introduced in the active scalar sector while a separate inert sector provides the DM candidate, the model faces similar constraints

¹Due to the absence of gauge interactions, there is significant arbitrariness in defining charge conjugation for singlet scalars, making these models inherently CP conserving [22].

to those found in 2HDMs, IDMs, and Higgs portal models [40, 41]. However, extending the inert sector to host both DM and CP violation alleviates these constraints.

The concept of dark or inert CP violation was first introduced in [28] and further developed in [29–32, 42] within the framework of a 3HDM. In this scenario, CP-mixed dark scalars interact with the SM particles via Higgs and gauge boson interactions. These studies have demonstrated that dark-sector CP violation allows for more flexibility in gauge-scalar couplings and significantly expands the viable parameter space for a CP-violating DM candidate that aligns with cosmological and collider constraints. Additionally, since the inert sector does not directly couple to the SM fermions, its CP-violating effects evade the stringent bounds imposed by EDM measurements.² As a result, one can construct a CP-violating DM model with unbounded dark CP violation. In fact, the CP-violating dark particles need not have a Higgs-DM coupling and can interact with the SM merely through the gauge bosons, ridding the model of all current (in)direct detection and LHC bounds, while yielding relic abundance in agreement with observation through the freeze-out or freeze-in mechanisms [31].

Given the success of 3HDMs as minimal BSM frameworks accommodating both CP violation and DM, it is natural to ask whether these models can also support a strong first-order electroweak phase transition (EWPT), as required by the mechanism of electroweak baryogenesis [44–47]. This mechanism is particularly appealing due to its testability, in contrast to alternatives such as leptogenesis or GUT- and Planck-scale baryogenesis, which may remain experimentally inaccessible (see [48] for a comprehensive review). Electroweak baryogenesis assumes no initial baryon asymmetry in the symmetric phase of the early universe. As the temperature drops below the electroweak scale (~ 100 GeV), the Higgs field develops a non-zero vev, spontaneously breaking electroweak symmetry. If the resulting phase transition is first order, bubbles of the broken phase nucleate within the symmetric phase and coalesce until they fill the entire space. The strength of the EWPT is thus a key component in the viability of the electroweak baryogenesis mechanism in a given model.

In the SM, perturbative analyses of the finite-temperature effective potential indicate a smooth crossover for the observed Higgs mass of 125 GeV [49, 50]. Non-perturbative lattice studies confirm this, ruling out a first-order EWPT for Higgs masses above 80 GeV [51–53]. Extensions of the SM with additional scalar fields modify the Higgs potential, potentially strengthening the EWPT at finite temperature. Such studies typically rely on perturbative methods, as non-perturbative simulations are computationally expensive. However, perturbative approaches to finite-temperature field theory are hindered by infrared divergences, gauge and renormalisation ambiguities, slow convergence, and difficulties in computing quantities such as the surface tension, which critically impact the determination of transition strength [54–56]. For instance, in the SM, perturbative methods significantly underestimated the transition temperature compared to non-perturbative results [49, 50, 57]. Perturbative

²It is important to note that if the extended inert sector consists of a singlet and a doublet rather than two doublets, the available CP violation is inherently reduced due to the presence of the singlet [43]. Moreover, the number of possible co-annihilation channels for the DM candidate is smaller, leading to a more constrained and less favourable model compared to the 3HDM.

approaches in 2HDMs and singlet-extended models have naively demonstrated the possibility of achieving a strong first-order transition, provided the additional scalars introduce significant modifications to the Higgs thermal potential [58–65]. However, non-perturbative lattice simulations have shown that such transitions, while possible, are typically confined to narrow and often fine-tuned regions of parameter space, and that perturbative analyses tend to significantly overestimate both the strength and the critical temperature of the transition [66, 67].

In more elaborate scenarios such as 3HDMs and multi-scalar extensions, naive one-loop perturbative studies indicate an increased likelihood of strong first-order phase transitions [68–70]. However, owing to computational challenges, systematic perturbative analyses and non-perturbative studies of the EWPT in these models remain limited, leaving their phase structure poorly characterised. We aim to bridge this gap by developing precise perturbative tools that can later be extended to non-perturbative lattice simulations.

Given the computational difficulty of fully non-perturbative treatments, it is highly advantageous to first constrain the viable parameter space using rigorous perturbative tools. As a first step in this direction, we compute the EWPT at two-loop level in a well-motivated 3HDM featuring a CP-violating dark sector. Our analysis follows the methodology of existing lattice simulations [66, 71–74] which are typically based on dimensionally reduced high-temperature effective theories. These effective theories, constructed at two-loop order by integrating out heavy modes, improve upon one-loop analyses and simplify both perturbative and non-perturbative treatments. In many cases, they retain the SM field content, enabling direct use of established lattice frameworks.

The remainder of the paper is organised as follows. Section 2 introduces the model and parameter inputs. Theoretical and experimental constraints, together with the chosen benchmark scenarios, are discussed in Section 3. Section 4 outlines the thermal corrections and construction of the effective potential while Section 5 presents our numerical setup and results. We conclude and discuss future directions in Section 6.

2 The extended scalar sector

In models with N -Higgs doublets, the scalar potential invariant under a discrete group $\mathbb{G}$ of phase rotations can be written as the sum of two parts: a generic V_0 part, invariant under global continuous phase rotations, and a symmetry-specific $V_{\mathbb{G}}$ part, invariant only under the discrete group $\mathbb{G}$ [34, 75]. Accordingly, one can write the scalar potential for a $\mathbb{Z}_2$ -symmetric 3HDM, as follows:

$$\begin{aligned}
 V_{\text{3HDM}} &= V_0 + V_{\mathbb{Z}_2}, \\
 V_0 &= -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - \mu_3^2(\phi_3^\dagger\phi_3) \\
 &\quad + \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{22}(\phi_2^\dagger\phi_2)^2 + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\
 &\quad + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_{31}(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1) \\
 &\quad + \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) + \lambda'_{31}(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3), \\
 V_{\mathbb{Z}_2} &= -\mu_{12}^2(\phi_1^\dagger\phi_2) + \lambda_1(\phi_1^\dagger\phi_2)^2 + \lambda_2(\phi_2^\dagger\phi_3)^2 + \lambda_3(\phi_3^\dagger\phi_1)^2 + h.c.,
 \end{aligned} \tag{2.1}$$

where the three Higgs doublets, ϕ_1, ϕ_2 and ϕ_3 , transform under the $\mathbb{Z}_2$ group, respectively, as

$$g_{\mathbb{Z}_2} = \text{diag}(-1, -1, +1). \quad (2.2)$$

The four explicitly $\mathbb{Z}_2$ -symmetric terms included in $V_{\mathbb{Z}_2}$ ensure that the scalar potential of the model is invariant only under the $\mathbb{Z}_2$ group without any accidental enhancement to a larger symmetry [75]. While additional $\mathbb{Z}_2$ -invariant operators, such as $(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_1)$, $(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_2)$, $(\phi_3^\dagger \phi_1)(\phi_2^\dagger \phi_3)$, and $(\phi_1^\dagger \phi_2)(\phi_3^\dagger \phi_3)$, are allowed by symmetry, they do not qualitatively affect the phenomenology of the model.³ To simplify the analysis, we, therefore, omit these operators by setting their coefficients to zero.

The three scalar doublets are defined as

$$\phi_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(H_1 + i A_1) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H_2 + i A_2) \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + i G^0) \end{pmatrix}, \quad (2.3)$$

where ϕ_1 and ϕ_2 are the $\mathbb{Z}_2$ -odd *inert* doublets with vanishing vevs, $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$, while ϕ_3 is the $\mathbb{Z}_2$ -even *active* doublet developing a non-zero vev, $\langle \phi_3 \rangle = \frac{v}{\sqrt{2}}$. The field h is identified with the physical Higgs boson of the SM, while $G^\pm$ and G^0 are the corresponding would-be Goldstone bosons.

The $\mathbb{Z}_2$ -charge assignment follows the generator defined in Eq. (2.2), with the inert doublets ϕ_1 and ϕ_2 assigned odd parity and the active doublet ϕ_3 assigned even parity. Consequently, the vacuum configuration $(0, 0, \frac{v}{\sqrt{2}})$ respects the $\mathbb{Z}_2$ symmetry of the scalar potential.

The CP-even scalar h , originating from the active doublet ϕ_3 , has tree-level couplings identical to those of the SM Higgs boson. As a result, all CP-violating effects are confined to the inert sector, which remains decoupled from the active sector due to the exact $\mathbb{Z}_2$ symmetry. This separation permits arbitrarily large CP violation in the inert sector without violating experimental bounds on EDMs. This phenomena, known as *dark CP violation*, was first proposed in [28]. The DM candidate in this model is the lightest neutral state arising from CP mixing in the inert doublets,⁴ and its stability is ensured by the unbroken $\mathbb{Z}_2$ symmetry.

As noted earlier, the $\mathbb{Z}_2$ symmetry is extended to the full Lagrangian by assigning even $\mathbb{Z}_2$ -parity to all SM gauge bosons and fermions. The Yukawa sector is then implemented in a Type-I configuration, in which only the third doublet ϕ_3 couples to fermions:

$$\mathcal{L}_Y = \Gamma_{mn}^u \bar{q}_{m,L} \tilde{\phi}_3 u_{n,R} + \Gamma_{mn}^d \bar{q}_{m,L} \phi_3 d_{n,R} + \Gamma_{mn}^e \bar{l}_{m,L} \phi_3 e_{n,R} + \Gamma_{mn}^\nu \bar{l}_{m,L} \tilde{\phi}_3 \nu_{n,R} + \text{h.c.} \quad (2.4)$$

This structure eliminates tree-level flavour-changing neutral currents (FCNCs). Moreover, since ϕ_1 and ϕ_2 do not acquire vevs, they remain completely decoupled from the fermionic sector.

³The terms $(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_1)$, $(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_2)$ appear only in inert scalar self-interactions and are irrelevant to our analysis. The terms $(\phi_3^\dagger \phi_1)(\phi_2^\dagger \phi_3)$, $(\phi_1^\dagger \phi_2)(\phi_3^\dagger \phi_3)$ effectively shift the values of the $\lambda_{2,3}$ parameters. Our numerical study confirms that excluding these terms does not reduce the generality of the model.

⁴We consider only regions of parameter space where the lightest inert state is neutral, excluding scenarios in which a charged inert scalar is the lightest.

2.1 Explicit CP-violation

The parameters of the phase-invariant part of the potential, V_0 , are real by construction. Explicit CP violation is introduced via complex parameters $\mu_{12}^2, \lambda_1, \lambda_2, \lambda_3$ in the potential defined in Eq. (2.1). The following notation will be employed consistently throughout this paper:

$$\lambda_j = |\lambda_j| e^{i\theta_j} \quad (j = 1, 2, 3), \quad \text{and} \quad \mu_{12}^2 = |\mu_{12}^2| e^{i\theta_{12}}. \quad (2.5)$$

It is important to note that λ_1 , along with other dark-sector parameters $\lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$, governs only self-interactions among inert scalars and does not affect the tree-level DM or collider phenomenology of the model. These parameters are constrained solely by perturbative unitarity and boundedness-from-below conditions on the scalar potential. As they do not enter our tree-level analysis, their values are fixed to 0.1.⁵

The phenomenologically relevant parameters are μ_3^2 and λ_{33} , which are fixed by the Higgs mass, and $\mu_1^2, \mu_2^2, \mu_{12}^2, \lambda_{31}, \lambda_{23}, \lambda'_{31}, \lambda'_{23}, \lambda_2, \lambda_3$, which determine the inert scalar masses and their couplings. While the latter nine parameters are a priori independent, we simplify the analysis by adopting the *dark democracy* limit [25, 27, 28, 30], in which

$$\mu_1^2 = \mu_2^2, \quad \lambda_3 = \lambda_2, \quad \lambda_{31} = \lambda_{23}, \quad \lambda'_{31} = \lambda'_{23}. \quad (2.6)$$

The model remains explicitly CP violating in this limit, provided

$$(\lambda_{22} - \lambda_{11}) \left[\lambda_1 (\mu_{12}^{2*})^2 - \lambda_1^* (\mu_{12}^2)^2 \right] \neq 0, \quad (2.7)$$

as shown in [76, 77]. This condition ensures that at least one CP-odd invariant is non-zero.⁶ By imposing the dark democracy limit, the only two parameters that remain complex are μ_{12}^2 and λ_2 . Note, however, that one can redefine the doublets as

$$\begin{cases} \phi_1 \rightarrow \phi_1 e^{i\theta_{12}/2} \\ \phi_2 \rightarrow \phi_2 e^{-i\theta_{12}/2} \\ \phi_3 \rightarrow \phi_3 \end{cases} \implies \begin{cases} |\mu_{12}^2| e^{i\theta_{12}} \rightarrow |\mu_{12}^2| \\ |\lambda_2| e^{i\theta_2} \rightarrow |\lambda_2| e^{i(\theta_2 + \theta_{12})} \end{cases} \quad (2.8)$$

to nominally remove the phase of the μ_{12}^2 parameter. As a result, the only relevant CP-violating parameter in the dark democracy limit is the *shifted* phase of the λ_2 coupling, $\theta_2 + \theta_{12}$, which we denote by θ_{CPV} throughout the paper.

2.2 The mass spectrum

The vacuum configuration $(0, 0, \frac{v}{\sqrt{2}})$ minimises the potential when $v^2 = \mu_3^2 / \lambda_{33}$. The active doublet ϕ_3 plays the role of the SM Higgs doublet, yielding the massless Goldstone bosons G^0 and $G^\pm$, and the SM-like Higgs boson h with

$$m_h^2 = 2\mu_3^2 = 2\lambda_{33}v^2 = (125 \text{ GeV})^2. \quad (2.9)$$

⁵The subtleties of the loop-level analysis will be discussed in Sec. 5.

⁶For a 3HDM to violate CP explicitly, at least one CP-odd invariant must be non-vanishing. The condition above is sufficient but not necessary, as other non-zero invariants can also lead to CP violation.

The charged inert states: The charged mass-squared matrix in the $(H_1^\pm, H_2^\pm)$ basis is given by

$$\mathcal{M}_C^2 = \begin{pmatrix} -\mu_2^2 + \frac{1}{2}\lambda_{23}v^2 & -\mu_{12}^2 \\ -\mu_{12}^2 & -\mu_2^2 + \frac{1}{2}\lambda_{23}v^2 \end{pmatrix}. \quad (2.10)$$

The physical charged eigenstates are

$$S_1^\pm = \frac{H_1^\pm + H_2^\pm}{\sqrt{2}}, \quad S_2^\pm = \frac{H_1^\pm - H_2^\pm}{\sqrt{2}}. \quad (2.11)$$

The corresponding masses are

$$m_{S_1^\pm}^2 = -\mu_2^2 - \mu_{12}^2 + \frac{1}{2}\lambda_{23}v^2, \quad m_{S_2^\pm}^2 = -\mu_2^2 + \mu_{12}^2 + \frac{1}{2}\lambda_{23}v^2, \quad (2.12)$$

where we take $\mu_{12}^2 > 0$, such that $m_{S_1^\pm} < m_{S_2^\pm}$.

The neutral inert states: The neutral mass-squared matrix in the (H_1, H_2, A_1, A_2) basis is

$$\mathcal{M}_N^2 = \frac{1}{4} \begin{pmatrix} \Lambda_c^+ & -2\mu_{12}^2 & -\Lambda_s & 0 \\ -2\mu_{12}^2 & \Lambda_c^+ & 0 & \Lambda_s \\ -\Lambda_s & 0 & \Lambda_c^- & -2\mu_{12}^2 \\ 0 & \Lambda_s & -2\mu_{12}^2 & \Lambda_c^- \end{pmatrix}, \quad (2.13)$$

where

$$\Lambda_s = 2|\lambda_2|v^2 \sin \theta_{\text{CPV}}, \quad \Lambda_c^\pm = -2\mu_2^2 + v^2(\lambda_{23} + \lambda'_{23} \pm 2|\lambda_2| \cos \theta_{\text{CPV}}). \quad (2.14)$$

The physical CP-mixed neutral eigenstates $S_{1,2,3,4}$ are given by

$$S_1 = \frac{\alpha H_1 - A_1 + \alpha H_2 + A_2}{\sqrt{2(\alpha^2 + 1)}}, \quad S_2 = \frac{H_1 + \alpha A_1 + H_2 - \alpha A_2}{\sqrt{2(\alpha^2 + 1)}}, \quad (2.15)$$

$$S_3 = \frac{\beta H_1 + A_1 - \beta H_2 + A_2}{\sqrt{2(\beta^2 + 1)}}, \quad S_4 = \frac{-H_1 + \beta A_1 + H_2 + \beta A_2}{\sqrt{2(\beta^2 + 1)}}, \quad (2.16)$$

with mixing parameters

$$\alpha = \frac{-\mu_{12}^2 + v^2|\lambda_2| \cos \theta_{\text{CPV}} - \Lambda^-}{v^2|\lambda_2| \sin \theta_{\text{CPV}}}, \quad \beta = \frac{-\mu_{12}^2 - v^2|\lambda_2| \cos \theta_{\text{CPV}} + \Lambda^+}{v^2|\lambda_2| \sin \theta_{\text{CPV}}}, \quad (2.17)$$

and

$$\Lambda^\mp = \sqrt{(\mu_{12}^2)^2 + v^4|\lambda_2|^2 \mp 2v^2\mu_{12}^2|\lambda_2| \cos \theta_{\text{CPV}}}. \quad (2.18)$$

The corresponding masses are

$$m_{S_{1,2}}^2 = -\mu_2^2 + \frac{v^2}{2}(\lambda'_{23} + \lambda_{23}) \mp \Lambda^-, \quad (2.19)$$

$$m_{S_{3,4}}^2 = -\mu_2^2 + \frac{v^2}{2}(\lambda'_{23} + \lambda_{23}) \mp \Lambda^+. \quad (2.20)$$

We identify S_1 as the lightest inert scalar and the DM candidate. Throughout the paper, we use the notations S_1 and DM interchangeably.

2.3 The span of θ_{CPV}

- To reproduce the results of [25, 27] in which $\lambda_2 < 0$ and S_1 is the DM candidate, we require⁷

$$\frac{\pi}{2} < \theta_{\text{CPV}} < \frac{3\pi}{2} \quad \Rightarrow \quad m_{S_1} < m_{S_2}, m_{S_3}, m_{S_4}. \quad (2.21)$$

In contrast, for θ_{CPV} in the first or fourth quadrants, S_3 becomes the lightest neutral inert particle:

$$\left. \begin{array}{l} 0 < \theta_{\text{CPV}} < \frac{\pi}{2} \\ \frac{3\pi}{2} < \theta_{\text{CPV}} < 2\pi \end{array} \right\} \Rightarrow m_{S_3} < m_{S_1}, m_{S_2}, m_{S_4}. \quad (2.22)$$

- At $\theta_{\text{CPV}} = \frac{\pi}{2}, \frac{3\pi}{2}$ points where $\Lambda^+ = \Lambda^-$, a mass degeneracy occurs:

$$\theta_{\text{CPV}} = \frac{\pi}{2}, \frac{3\pi}{2} \quad \Rightarrow \quad \begin{cases} m_{S_1} = m_{S_3}, \\ m_{S_2} = m_{S_4}. \end{cases} \quad (2.23)$$

- The model reduces to the CP-conserving limit for $\theta_{\text{CPV}} = 0, \pi$, where $S_{1,3}$ and $S_{2,4}$ become CP eigenstates:

$$\theta_{\text{CPV}} = 0, \pi \quad \Rightarrow \quad \text{CP-conserving limit : } \begin{cases} S_{1,3} = \frac{H_1 \pm H_2}{\sqrt{2}}, \\ S_{2,4} = \frac{A_1 \pm A_2}{\sqrt{2}}. \end{cases} \quad (2.24)$$

2.4 Input parameters of the model

We choose the following set of physical observables as independent input parameters:

$$m_{S_1}, \quad m_{S_2}, \quad m_{S_1^\pm}, \quad m_{S_2^\pm}, \quad \theta_{\text{CPV}}, \quad g_{h\text{DM}}, \quad (2.25)$$

where $g_{h\text{DM}} \equiv g_{S_1 S_1 h}$ is the Higgs-DM coupling. The relevant interaction terms in the Lagrangian are

$$\mathcal{L} \supset g_{Z S_i S_j} Z_\mu (S_i \partial^\mu S_j - S_j \partial^\mu S_i) + \frac{v}{2} g_{S_i S_i h} h S_i^2 + v g_{S_i S_j h} h S_i S_j + v g_{S_i^\pm S_j^\mp h} h S_i^\pm S_j^\mp. \quad (2.26)$$

The parameters of the scalar potential can be expressed in terms of the input set in Eq. (2.25) as:

$$\begin{aligned} \mu_{12}^2 &= \frac{1}{2}(m_{S_2^\pm}^2 - m_{S_1^\pm}^2), \\ \lambda_{23} &= \frac{1}{v^2} (2\mu_2^2 + m_{S_2^\pm}^2 + m_{S_1^\pm}^2), \\ \lambda'_{23} &= \frac{1}{v^2} (m_{S_2}^2 + m_{S_1}^2 - m_{S_2^\pm}^2 - m_{S_1^\pm}^2), \\ |\lambda_2| &= \frac{1}{v^2} \left[\mu_{12}^2 \cos \theta_{\text{CPV}} + \frac{1}{4} \sqrt{(2\mu_{12}^2 \cos \theta_{\text{CPV}})^2 + (m_{S_2}^2 - m_{S_1}^2)^2 - (m_{S_2^\pm}^2 - m_{S_1^\pm}^2)^2} \right], \\ \mu_2^2 &= \frac{v^2}{2} g_{h\text{DM}} - \frac{v^2}{\alpha^2 + 1} (2\alpha \sin \theta_{\text{CPV}} + (\alpha^2 - 1) \cos \theta_{\text{CPV}}) |\lambda_2| - \frac{1}{2}(m_{S_2}^2 + m_{S_1}^2). \end{aligned} \quad (2.27)$$

⁷For $\lambda_2 > 0$, the same statements hold upon relabelling $S_1 \leftrightarrow S_3$ and $S_2 \leftrightarrow S_4$.

We impose the condition that the extracted value of $|\lambda_2|$ must be real and positive, ensuring all input parameters correspond to physically meaningful configurations. Using the above relations, all other masses and couplings can be derived. For instance, the masses of S_3 and S_4 are

$$m_{S_{3,4}}^2 = m_{S_1}^2 + \Lambda^- \mp \Lambda^+, \quad \text{with} \quad (\Lambda^+)^2 = (\Lambda^-)^2 + 4v^2\mu_{12}^2|\lambda_2|\cos\theta_{\text{CPV}}. \quad (2.28)$$

The neutral scalar-gauge couplings are given by

$$|g_{ZS_1S_3}| = |g_{ZS_2S_4}| = g_Z^{\text{CPC}} \left(\frac{\alpha + \beta}{\sqrt{\alpha^2 + 1}\sqrt{\beta^2 + 1}} \right), \quad (2.29)$$

$$|g_{ZS_1S_4}| = |g_{ZS_2S_3}| = g_Z^{\text{CPC}} \left(\frac{\alpha\beta - 1}{\sqrt{\alpha^2 + 1}\sqrt{\beta^2 + 1}} \right), \quad (2.30)$$

where $g_Z^{\text{CPC}} = \frac{e}{2c_{\theta_W}s_{\theta_W}}$ is the $|g_{ZS_iS_j}|$ value in the CP-conserving limit, with e the electric charge and c_{θ_W} , s_{θ_W} the cosine and sine of the weak mixing angle. Note that

$$g_{ZS_1S_3}^2 + g_{ZS_1S_4}^2 = (g_Z^{\text{CPC}})^2, \quad g_{ZS_2S_3}^2 + g_{ZS_2S_4}^2 = (g_Z^{\text{CPC}})^2, \quad (2.31)$$

while $g_{ZS_1S_2} = g_{ZS_3S_4} = 0$ in the dark democracy limit.

The charged scalar-gauge couplings are similarly obtained:

$$\begin{aligned} |g_{W^\pm S_1^\mp S_1}| &= |g_{W^\pm S_2^\mp S_2}| = g_W^{\text{CPC}} \left(\frac{\alpha}{\sqrt{\alpha^2 + 1}} \right), \\ |g_{W^\pm S_1^\mp S_2}| &= |g_{W^\pm S_2^\mp S_1}| = g_W^{\text{CPC}} \left(\frac{1}{\sqrt{\alpha^2 + 1}} \right), \\ |g_{W^\pm S_1^\mp S_3}| &= |g_{W^\pm S_2^\mp S_4}| = g_W^{\text{CPC}} \left(\frac{1}{\sqrt{\beta^2 + 1}} \right), \\ |g_{W^\pm S_1^\mp S_4}| &= |g_{W^\pm S_2^\mp S_3}| = g_W^{\text{CPC}} \left(\frac{\beta}{\sqrt{\beta^2 + 1}} \right), \end{aligned} \quad (2.32)$$

where $g_W^{\text{CPC}} = \frac{e}{s_{\theta_W}}$ is the $|g_{W^\pm S_i^\mp S_j}|$ value in the CP-conserving limit. It is important to note that, unlike the CP-conserving case, the gauge-scalar interactions now depend on the mixing parameters α and β , defined in Eq. (2.17), and therefore on the scalar masses in the CP-violating scenario.

2.4.1 Renormalization and input parameters

In this work, thermal corrections to masses, couplings, and the effective potential V_{eff} are computed at parametric order g^4 or λ^2 . In contrast, the $T = 0$ input parameters are only accurate to $\mathcal{O}(g^2)$, as we do not include loop corrections to pole masses or the full matching of renormalised action parameters to physical observables at $\mathcal{O}(g^4)$. This discrepancy should be borne in mind when interpreting results.

Our focus here is primarily on the thermal dynamics of the model, and not on the precision matching of $T = 0$ quantities. A similar treatment, with analogous justifications, was adopted in the context of singlet-extended models in [67], where more details can be found.

3 Constraints on the parameter space

The parameter space of the model is constrained by theoretical, observational and experimental bounds which are satisfied in all our benchmark scenarios to follow:

1. For the potential to have a stable vacuum (i.e. for the potential to be bounded from below), the following conditions are required [40]:

$$\lambda_{ii} > 0, \quad i = 1, 2, 3, \quad (3.1)$$

$$\lambda_x > -2\sqrt{\lambda_{11}\lambda_{22}}, \quad \lambda_y > -2\sqrt{\lambda_{11}\lambda_{33}}, \quad \lambda_z > -2\sqrt{\lambda_{22}\lambda_{33}}, \quad (3.2)$$

$$\begin{cases} \sqrt{\lambda_{33}}\lambda_x + \sqrt{\lambda_{11}}\lambda_z + \sqrt{\lambda_{22}}\lambda_y \geq 0 \\ \text{or} \\ \lambda_{33}\lambda_x^2 + \lambda_{11}\lambda_z^2 + \lambda_{22}\lambda_y^2 - \lambda_{11}\lambda_{22}\lambda_{33} - 2\lambda_x\lambda_y\lambda_z < 0. \end{cases} \quad (3.3)$$

where

$$\lambda_x = \lambda_{12} + \min(0, \lambda'_{12} - 2|\lambda_1|), \quad (3.4)$$

$$\lambda_y = \lambda_{31} + \min(0, \lambda'_{31} - 2|\lambda_3|), \quad (3.5)$$

$$\lambda_z = \lambda_{23} + \min(0, \lambda'_{23} - 2|\lambda_2|). \quad (3.6)$$

As noted in [78], these conditions are sufficient but not necessary, as it is possible to construct examples of this model in which the potential is bounded from below, but which violate conditions Eqs. (3.1)-(3.3).

2. We take all couplings to be $|\lambda_i| \leq 4\pi$ in accordance with perturbative unitarity limits.
3. Parametrised by the electroweak oblique parameters S, T, U [79–82], inert particles $S_i, S_i^\pm$ may introduce important radiative corrections to gauge boson propagators. We impose a 2σ agreement with electroweak precision observables at 95% confidence level [83],

$$S = 0.05 \pm 0.11, \quad T = 0.09 \pm 0.13, \quad U = 0.01 \pm 0.11. \quad (3.7)$$

Similar to the 2HDM, this condition requires each charged state to be close in mass with a neutral state, in the dark sector [84, 85].

4. The contribution of the inert scalars to the total decay width of the electroweak gauge bosons constrains the masses of the inert scalars to be [86]

$$m_{S_i} + m_{S_{1,2}^\pm} \geq m_{W^\pm}, \quad m_{S_i} + m_{S_j} \geq m_Z, \quad 2m_{S_{1,2}^\pm} \geq m_Z, \quad i, j = 1, 2, 3, 4. \quad (3.8)$$

5. Non-observation of charged scalars puts a model-independent lower bound on their mass [87–89] and an upper bound on their lifetime [90] to be

$$m_{S_{1,2}^\pm} \geq 70 \text{ GeV}, \quad \tau_{S_{1,2}^\pm} \leq 10^{-7} \text{ s} \Rightarrow \Gamma_{S_{1,2}^\pm}^{\text{tot}} \geq 6.58 \times 10^{-18} \text{ GeV}, \quad (3.9)$$

to guarantee their decay within the detector. In all our benchmark scenarios, the mass of both charged scalars is above 95 GeV and their decay width, primarily to $S_i^\pm \rightarrow S_j W^\pm$, is of the order of 10^{-1} GeV, which is well within limits.

- Any model introducing new decay channels for the SM-Higgs boson is constrained by an upper limit on the Higgs total decay width, $\Gamma_{\text{tot}}^h \leq 9$ MeV [91], and Higgs signal strengths [92–94]. In our model, the SM-like Higgs could decay to a pair of inert scalars, provided $m_{S_i} + m_{S_j} < m_h$ and $S_{i,j}$ are long-lived enough ($\tau \geq 10^{-7}$ s). As a result, $S_{i,j}$ will not decay inside the detector and therefore contribute to the Higgs invisible decay, $h \rightarrow S_i S_j$, with a branching ratio of

$$\text{BR}(h \rightarrow S_i S_j) = \frac{\sum_{i,j} \Gamma(h \rightarrow S_i S_j)}{\Gamma_h^{\text{SM}} + \sum_{i,j} \Gamma(h \rightarrow S_i S_j)}, \quad (3.10)$$

with

$$\Gamma(h \rightarrow S_i S_j) = \frac{g_{hS_i S_j}^2 v^2}{32\pi m_h^3} \left((m_h^2 - (m_{S_i} + m_{S_j})^2) (m_h^2 - (m_{S_i} - m_{S_j})^2) \right)^{1/2}, \quad (3.11)$$

which sets strong limits on the Higgs-inert couplings. Moreover, the partial decay $\Gamma(h \rightarrow \gamma\gamma)$ receives contributions from the inert charged scalars. The combined ATLAS and CMS Run I results for Higgs to $\gamma\gamma$ signal strength require $\mu_{\gamma\gamma} = 1.14^{+0.38}_{-0.36}$ [92]. In Run II, ATLAS reports $\mu_{\gamma\gamma} = 0.99^{+0.14}_{-0.14}$ [93], and CMS reports $\mu_{\gamma\gamma} = 1.18^{+0.17}_{-0.14}$ [94] with both of which we are in 2σ agreement.

- Reinterpretation of LEP 2 and LHC Run I searches for Supersymmetric (SUSY) particles (mainly sneutrinos and sleptons) for the IDM [87, 95] excludes the region of parameter space where the following conditions are simultaneously satisfied ($i = 2, 3, 4$):

$$m_{S_1} \leq 80 \text{ GeV}, \quad m_{S_i} \leq 100 \text{ GeV}, \quad \Delta m(S_1, S_i) \geq 8 \text{ GeV}. \quad (3.12)$$

We take these limits into account for our DM candidate paired with any other neutral scalar. We also check the validity of our benchmark scenarios against LHC searches for new particles in accordance with the analysis for the IDM [96].

- DM relic density measurements from the Planck experiment [12],

$$\Omega_{\text{DM}} h^2 = 0.1197 \pm 0.0022, \quad (3.13)$$

require the relic abundance of the DM candidate to lie within these bounds if it constitutes 100% of DM in the universe.

A DM candidate with $\Omega_{\text{DM}} h^2$ smaller than the observed value is allowed; however, an additional DM candidate is needed to complement the missing relic density. Regions of the parameter space corresponding to values of $\Omega_{\text{DM}} h^2$ larger than the Planck upper limit are excluded.

We impose a 3σ agreement with the observation on the relic abundance of our DM candidate, S_1 .

9. The latest XENON1T results for DM direct detection experiments [97] and FermiLAT results for indirect detection searches [98] do not constrain the model any further. Having set the Higgs portal couplings to zero in our benchmark scenarios, the largest direct detection cross section is $\sigma_{DM-N} \approx 10^{-14}$ pb and the largest indirect detection cross section is $\langle v\sigma \rangle \approx 10^{-32}$ cm³/s, both of which are well below the limits [99].

3.1 DM abundance and the selection of benchmarks

The relic abundance of the DM candidate, S_1 , after freeze-out is given by the solution of the Boltzmann equation,

$$\frac{dn_{S_1}}{dt} = -3H n_{S_1} - \langle \sigma_{eff} v \rangle \left[(n_{S_1})^2 - (n_{S_1}^{eq})^2 \right], \quad (3.14)$$

where n_{S_1} is the number density of the S_1 particle, H is the Hubble parameter, and $n_{S_1}^{eq}$ is the number density of S_1 at equilibrium. The thermally averaged effective (co)annihilation cross section, $\langle \sigma_{eff} v \rangle$, receives contributions from all relevant annihilation processes of any $S_i S_j$ pair into SM particles, so that

$$\langle \sigma_{eff} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{S_i}^{eq}}{n_{S_1}^{eq}} \frac{n_{S_j}^{eq}}{n_{S_1}^{eq}}, \quad \text{where} \quad \frac{n_{S_i}^{eq}}{n_{S_1}^{eq}} \sim \exp\left(-\frac{m_{S_i} - m_{S_1}}{T}\right). \quad (3.15)$$

However, only processes with the $S_i - S_1$ mass splitting comparable to the thermal bath temperature T provide a sizeable contribution.

A common feature of non-minimal Higgs DM models is that in a large region of the parameter space the most important process for DM annihilation is through the $S_1 S_1 \rightarrow h_{SM} \rightarrow f \bar{f}$ channel whose efficiency depends on both the DM mass and the Higgs-DM coupling. In the region where $m_{DM} < m_h/2$, generally one requires a large Higgs-DM coupling in order to produce relic density in agreement with Eq. (3.13). However, such large Higgs-DM coupling leads to large direct detection and indirect detection cross sections and significant deviations from SM-Higgs coupling measurements, which are ruled out by experimental and observational data. On the other hand, a small Higgs-DM coupling fails to annihilate the DM candidate effectively and leads to the over-closure of the universe. This is where co-annihilation processes play an important role as they can contribute to changes in the DM relic density.

In models with extended dark sectors, in addition to the standard Higgs mediated annihilation channels of DM, there exists the possibility of co-annihilation with heavier states, provided they are close in mass [25, 27–30]. The relevance of this effect depends not only on the DM mass and the mass splittings but also on the strength of the standard DM annihilation channel.

It is worth pointing out that in the IDM, where by construction CP violation is not allowed, the only co-annihilation process is through the Z -mediated $H A \rightarrow Z \rightarrow f \bar{f}$ channel whose sub-dominant effect fails to revive the model in the low mass region. Extending the inert sector, as shown in [25, 27, 29] in the CP-conserving limit, opens up several co-annihilation

channels, both Higgs-mediated $H_1 H_2 \rightarrow h \rightarrow f \bar{f}$ and Z -mediated $H_1 A_{1,2} \rightarrow Z \rightarrow f \bar{f}$. However, their collective contribution to DM co-annihilation is not sufficient and one still needs a non-zero Higgs-DM coupling to satisfy relic density bounds. Introducing CP violation in the extended dark sector [28, 30, 100] opens up many co-annihilation channels through the Higgs and Z bosons, $S_i S_j \rightarrow h/Z \rightarrow f \bar{f}$, which can significantly affect the DM phenomenology. In fact, the Z -mediated co-annihilations can be strong enough to relieve the model of the need for any Higgs-mediated (co)annihilation processes.

To show the effect of Z portal CP violation on the abundance of DM, we set the Higgs-DM coupling to zero, $g_{h\text{DM}} = 0$, thereby removing the main DM annihilation process, $S_1 S_1 \rightarrow h \rightarrow f \bar{f}$. All other $S_i S_j h$ vertex coefficients are also reduced to a point where their resulting co-annihilation processes have negligible contributions to the DM relic density. So, the only communication between the dark sector and the visible sector is through the gauge bosons $W^\pm$ and Z .

In the region of the parameter space where Higgs portal interactions are negligible ($g_{h\text{DM}} \approx 10^{-4}$), the total DM annihilation cross section receives contributions from the following:

- **DM annihilation processes:**

$$S_1 S_1 \rightarrow VV, \quad S_1 S_1 \rightarrow VV^* \rightarrow V f f', \quad S_1 S_1 \rightarrow V^* V^* \rightarrow f f' f f', \quad (3.16)$$

where V is any of the SM gauge bosons. In the $m_{DM} < m_{W^\pm}$ region, the processes with off-shell gauge bosons dominate over the ones with on-shell gauge bosons.

- **DM co-annihilation processes:**

$$S_1 S_{2,3,4} \rightarrow Z^* \rightarrow f \bar{f}, \quad S_1 S_{1,2}^\pm \rightarrow W^{\pm*} \rightarrow f f', \quad (3.17)$$

where the co-annihilating dark scalars are up to 20% heavier than the DM candidate.

- **(co)annihilation of other dark states:**

$$S_i S_i \rightarrow VV, \quad S_i S_i \rightarrow VV^* \rightarrow V f f', \quad S_i S_i \rightarrow V^* V^* \rightarrow f f' f f', \quad S_i S_j \rightarrow V^* \rightarrow f f', \quad (3.18)$$

where $S_i \neq S_j$ are any of the dark scalars $S_{2,3,4}, S_{1,2}^\pm$ which are all close in mass.

Taking all such processes into account, we define our benchmark scenarios in Sec. 5 with distinct DM phenomenology. It is convenient to introduce the mass splittings between the DM candidate and other inert scalars as

$$\delta_{12} = m_{S_2} - m_{S_1}, \quad \delta_c = m_{S_2^\pm} - m_{S_1^\pm}, \quad \delta_{1c} = m_{S_1^\pm} - m_{S_1}. \quad (3.19)$$

4 Thermal corrections

To study the phase structure of the model at finite-temperature we construct the effective potential, V_{eff} , using the *dimensional reduction* formalism briefly outlined as follows. Dimensional reduction starts with the imaginary time formalism of finite-temperature field theory and where the (imaginary) time integral in the action spans from 0 to $1/T$ in order to marry the partition functions of quantum field theory (QFT) and thermal and statistical physics (TSP),

$$Z_{QFT} = \text{Tr} \left[e^{-it\hat{H}} \right] \quad \text{and} \quad Z_{TSP} = \text{Tr} \left[e^{-\hat{H}/T} \right]. \quad (4.1)$$

This compactification leads to Matsubara modes ω_n [101] which are discrete modes that can be easily seen in the propagator:

$$\sum_{n=-\infty}^{\infty} \int d^3p \left(\frac{1}{\vec{p}^2 + m^2 + w_n^2} \right), \quad \begin{cases} \omega_n = 2n\pi T & \text{for bosons} \\ \omega_n = (2n+1)\pi T & \text{for fermions} \end{cases} \quad (4.2)$$

In the high temperature limit, $\pi T \gg m$, where m denotes the particle mass of the particle the non-zero Matsubara modes, otherwise known as *hard modes*, are much heavier than the zero modes, i.e. *soft modes*, and so can be integrated out. This leaves just the soft modes of the bosonic sector to govern the infrared (IR) behaviour of the effective theory. These soft modes have no (imaginary) time dependence, leading to a trivial time integration which reduces the theory from a four-dimensional (4D) to a three-dimensional (3D) effective field theory (EFT),

$$S = \int_0^{\frac{1}{T}} d\tau \int d^3x \mathcal{L}_E^{4D}(\tau, \vec{x}) \xrightarrow{\text{DR}} \frac{1}{T} \int d^3x \mathcal{L}_E^{3D}(\vec{x}). \quad (4.3)$$

As a result, the soft modes can be used to describe equilibrium dynamics and are well suited to be used for finding the critical temperature of the phase transition.⁸

The light fields that are left in the effective theory, receive large IR contributions from so the so-called *daisy* or *ring* diagrams which are a class of diagrams all of the same order, $\mathcal{O}(g^3 T^4)$ or equivalently $\mathcal{O}(\lambda^{3/2} T^4)$. These diagrams require resummation [49, 103] which is automatically handled in the dimensional reduction treatment.

These IR contributions are particularly important for EWPTs which typically involve light scalar excitations near the critical temperature (the temperature at which the symmetry preserving and broken minima are degenerate). A more severe issue arises in the gauge sector; in the symmetric phase, gauge bosons remain perturbatively massless, and their spatial components, corresponding to the zero Matsubara modes, become strongly coupled at the scale $g^2 T$. This is the origin of the *Linde problem* [104] which states that in non-Abelian gauge theories, while the temporal components of the gauge fields acquire a thermal Debye mass of order gT , the spatial components remain massless to all orders in perturbation theory. As a result, perturbative expansions for thermodynamic quantities such as the free energy

⁸Once the bubbles of the new phase start nucleating and releasing energy into the thermal bath, local equilibrium can be lost. Calling into question the reliability of this approach (see [102] for further details).

suffer from severe infrared divergences. In particular, contributions to the free energy at order $g^6 T^4$ and beyond become non-perturbative, rendering standard resummation techniques insufficient. This infrared sensitivity necessitates the use of non-perturbative methods, such as dimensional reduction, for an effective 3D theory followed by lattice simulations.

Despite this breakdown at high orders, the lower-order terms in V_{eff} can be computed reliably within perturbation theory, provided that IR divergences are under control. The commonly used one-loop effective potential with daisy resummation, as implemented in codes such as `CosmoTransitions` [105], corresponds to a partial $\mathcal{O}(g^3)$ computation (including some $\mathcal{O}(g^4)$ terms from $T = 0$ corrections).⁹ While qualitatively useful, this approximation is known to substantially overestimate the strength of the EWPT and yield an incorrect value for the critical temperature compared to non-perturbative results [66, 108–110]. More accurate predictions are obtained at full $\mathcal{O}(g^4)$, which requires two-loop computations and a consistent resummation beyond the daisy level [49, 111].

This can be achieved systematically through the dimensional reduction formalism [49, 112–114]. This is done by integrating out heavy modes (including non-zero Matsubara frequencies), hence, mapping the 4D theory onto a 3D EFT valid for scales $\lesssim T$. This procedure resums thermal contributions for light fields and organises corrections in powers of m/T and coupling constants. The resulting EFT simplifies both perturbative and non-perturbative treatments of the phase transition. In particular, the thermodynamics of the 3D EFT can be studied non-perturbatively on the lattice. This approach has been successfully applied to the SM and its extensions [50, 53, 57, 66, 67, 108–110, 115], and has recently been applied to BSM scenarios using automated tools such as the `DRalgo` package [116]. In this work, we employ dimensional reduction to construct the high- T EFT for a 3HDM with a CP-violating dark sector, and compute the two-loop V_{eff} to determine the nature of the EWPT.

4.1 High- T effective theory for 3HDMs

For the 3HDM potential defined in Eq. (2.1), the high- T EFT has the form

$$S_{\text{3D}} = \frac{1}{T} \int d^3x \left\{ \frac{1}{2} \text{Tr} F_{ij} F_{ij} + \frac{1}{4} f_{ij} f_{ij} + \sum_{n=1}^3 (D_i \phi_n)^\dagger (D_i \phi_n) + \bar{V}_{\text{3HDM}} \right\}, \quad (4.4)$$

where F_{ij} and f_{ij} are the SU(2) and U(1) field strengths, D_i is the 3D covariant derivative, and $\bar{V}_{\text{3HDM}}$ has the same form as the 4D potential but with temperature-dependent parameters. Fermions are integrated out during dimensional reduction, so their effects are encoded in the EFT parameters. The explicit $1/T$ prefactor can be absorbed via field rescaling [67], but is kept here for clarity.

We assume all physical masses are parametrically small compared to the temperature, including the scalar mixing parameter μ_{12}^2 . Without this assumption, diagonalising the ϕ_1 – ϕ_2 subspace could yield heavy eigenstates with $m \gg T$, invalidating the high- T assumption

⁹Note that the power counting described here is schematic, and that for scalars undergoing a phase transition the relevant perturbative expansion can have more a intricate structure [106, 107].

required for constructing the EFT. We therefore adopt the power counting $\mu_{12}^2 \sim g^2 T^2$ (or alternatively $\mu_{12}^2 \sim \lambda T^2$), allowing the μ_{12}^2 mixing term to be treated as a perturbative two-point interaction in Feynman diagrams. This treatment is consistent with Refs. [73, 117–120] and is implemented in `DRalgo`.

Several comments on the structure and truncation of the EFT are in order:

- As mentioned before, the symmetry of the potential in Eq. (2.1) allows for operators of the form $(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_1)$, $(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_2)$, $(\phi_3^\dagger \phi_1)(\phi_2^\dagger \phi_3)$, $(\phi_1^\dagger \phi_2)(\phi_3^\dagger \phi_3)$ and their conjugates. Since we do not expect these terms to qualitatively change the phenomenology of the model, we have set their coefficients to zero at tree level. In principle, these operators will be generated by thermal loops and should be included in the EFT. However, they can only arise from loops that contain the $\mu_{12}^2 \phi_1^\dagger \phi_2$ interaction vertex and are thus suppressed in the high- T approximation. The lowest order at which these operators must be included is $\lambda^2 \mu_{12}^2 \sim \mathcal{O}(\lambda^2 g^2)$ which goes beyond our g^4 accuracy goal. We therefore neglect these operators from our EFT.
- Mixed kinetic terms such as $(D_i \phi_1)^\dagger (D_i \phi_2)$ could similarly be generated by loop corrections arising from momentum dependence of two-point correlators of the doublets in a quadratic $\lambda^2 \mu_{12}^2 \sim \mathcal{O}(\lambda^2 g^2)$ vertex. The simplest contributing diagram is of the order $g^2 \mu_{12}^2 p^2$, where p is an external momentum at the EFT scale. Since the relevant momentum scale for the EFT is $p \leq gT$, kinetic mixing operators do not contribute at order g^4 .
- As with any local EFT, the integration over heavy modes generates operators at all possible field dimensions. However, our constructed EFT in Eq. (4.4) is truncated at field dimension four. In this setup, the first corrections to the EFT arise at dimension six and include operators such as $(\phi_3^\dagger \phi_3)^3$. The Wilson coefficients associated with dimension-six operators are formally of order g^6 and can again be neglected in our g^4 analysis. It is important to note that this counting breaks down if the high- T assumption, $m \ll \pi T$, is not strictly satisfied. Violating this assumption could lead to higher-dimensional operators becoming important if there are large mass hierarchies in the scalar sector. Effects of thermal dimension-six operators in the context of EWPT, have been studied in [67, 74, 114, 121, 122].
- In constructing the EFT in Eq. (4.4), we have also integrated out the time component of gauge fields (A_0, B_0 , and the corresponding SU(3) field), since in the imaginary time formalism, they acquire thermal Debye masses $\sim gT$ and behave like adjoint-representation scalars. We perform the integration over these fields in a separate step after the actual $4D \rightarrow 3D$ matching giving a theory at the ultrasoft scale ($g^2 T$). This modifies the EFT parameters somewhat, but the numerical impact of this process is known to be sub-leading compared to the top-quark loop effects [67, 114, 118].

4.2 The effective potential

The two-loop effective potential is computed from the EFT in Eq. (4.4), in the R_ξ Landau gauge. We consider homogeneous background fields in the neutral component of each doublet:

$$\phi_i \rightarrow \phi_i + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_i \end{pmatrix}, \quad (4.5)$$

where $\bar{v}_i$ are real background fields. The effective potential is a function of these fields, $V_{\text{eff}} = V_{\text{eff}}(\bar{v}_1, \bar{v}_2, \bar{v}_3)$, and is calculated from one- and two-loop vacuum diagrams (i.e. no external legs) with propagators and vertices depending on $\bar{v}_i$. For details, see e.g. [108, 123].

The three-field parametrization used here is not the most general background-field configuration possible in the 3HDM, which includes nine real degrees of freedom after gauge fixing. In particular, configurations with non-vanishing charged or CP-violating background fields are not considered here. Such configurations are not expected to dominate the thermal evolution in the regions we study. The current paper is intended to be a proof of concept that strong first order EWPT is viable in a 3HDM with a CP-violating dark sector. A comprehensive analysis of the 3HDM phase structure, including non-vanishing charged and CP-violating background fields, will be presented in a forthcoming publication.

The full two-loop V_{eff} can be obtained, in symbolic form, using **DRalgo**. A complication here is that the loop calculation needs to be done in field-basis where the mass matrix is diagonal, i.e. there are no quadratic mixing terms.¹⁰ In practice, we must give **DRalgo** rotation matrices that diagonalize both the scalar and gauge sectors, and this diagonalization now depends on the background fields $\bar{v}_i$. While for gauge fields this is straightforward and proceeds much like in the minimal SM, for our scalars the diagonalizing rotation cannot be found analytically. Specifically, the 3HDM has $3 \times 4 = 12$ real scalar fields so the mass matrix is 12×12 . For the background-field configuration defined in Eq. (4.5), it is possible to permute the fields such that the mass matrix becomes block diagonal, reducing the problem to diagonalization of two 6×6 symmetric matrices which still needs to be done numerically. Details of our implementation, including the treatment of background-dependent rotation matrices and the **DRalgo** model files, are available in an open source code that we constructed to numerically evaluate the effective potential, called **BL00P** (**B**eyond **o**ne **L**00p **P**hase **t**ransition)¹¹.

5 EWPT dynamics

5.1 Setup

We now outline the numerical setup used to compute the effective potential and extract the phase structure, closely following the methodology of [122]. As discussed, reliable predictions

¹⁰**DRalgo** has an option for computing the V_{eff} using undiagonalized propagators, i.e. all off-diagonal elements in the mass matrix are treated as perturbations. We have not used this approximation in the present paper.

¹¹The code is available via <https://github.com/BL00P-JTC/BL00P> with the manual upcoming.

for the critical temperature of the EWPT require going beyond the standard one-loop approximation. We therefore compute the thermal effective potential at two-loop level using **DRalgo v1.2** in combination with **GroupMath v3** [124] and **BLOOP v0.1**.¹² Matching relations, β -functions, and tree-level mass matrices are also generated via **DRalgo**. A fully consistent loop-level mass matrix is left for future work.

Matching from the hard-to-soft scale is performed at $\mu = 4\pi e^{-\gamma}T$, where γ is the Euler–Mascheroni constant, and the soft-to-ultrasoft scale is matched at $\mu = T$. We minimise the potential with respect to three real CP-even background fields,

$$\phi_i = \begin{pmatrix} 0 \\ \frac{\bar{v}_i + 0}{\sqrt{2}} \end{pmatrix}, \quad (5.1)$$

restricting to CP- and charge-conserving field configurations. This simplification allows the 12×12 scalar mass matrix to be permuted into two 6×6 blocks for neutral and charged states, respectively.

We scan the field space over

$$v_1 \in (-60, 60) \text{ GeV}^{1/2}, \quad v_{2,3} \in (10^{-4}, 60) \text{ GeV}^{1/2}, \quad (5.2)$$

exploiting gauge and $\mathbb{Z}_2$ symmetry to fix $v_2, v_3 > 0$.

Minimisation is performed using the **BOBYQA** and **DIRECT** routines from the **NLOPT** library [125]. We treat V_{eff} as a complex-valued function and minimise its real part, using the imaginary part as a consistency check. All global minima are found to be real, indicating absence of spurious solutions. The global minimum is tracked over a temperature range $T \in [50, 400] \text{ GeV}$ with step size $\Delta T = 0.1 \text{ GeV}$.

We perform a random scan over one million benchmark points with input parameters sampled uniformly in the following ranges:

$$\begin{aligned} m_{S_1} &\in [63, 100] \text{ GeV} & \delta_{12} &\in [5, 100] \text{ GeV} \\ \delta_{1c} &\in [5, 100] \text{ GeV} & \delta_c &\in [5, 100] \text{ GeV} \\ g_{h\text{DM}} &\in [0, 1] & \theta_{\text{CPV}} &\in [\pi/2, 3\pi/2] \end{aligned} \quad (5.3)$$

Each benchmark is required to satisfy the theoretical and experimental constraints listed in Sec. 3 in addition to ensuring that the global minimum at $T = 0$ is at $(0, 0, 246 \text{ GeV})$. It is important to note that for most of our benchmark points, specially for points with a relatively large $g_{h\text{DM}}$, the relic density of the DM candidate is only a fraction of the observed relic abundance, which subsequently relaxes the (in)direct detection bounds considerably.

These constraints are conservative; e.g. we impose absolute vacuum stability, whereas metastability would suffice. We also reject points where the correct vacuum is not recovered at $T = 50 \text{ GeV}$, to avoid scenarios where the transition temperature lies below our scan window.

¹²Only scalar self-energies are computed at two-loop; cubic and quartic couplings are computed at one-loop.

5.2 Numerical results

We characterise the strength of the phase transition by the dimensionless jump in the Higgs VEV at T_c :

$$\frac{\Delta v}{\sqrt{T_c}} = \frac{\Delta \sqrt{v_1^2 + v_2^2 + v_3^2}}{\sqrt{T_c}}. \quad (5.4)$$

This quantity is used throughout as a proxy for the order parameter. Figs. 1-2 shows the dependence of transition strength on model parameters. White regions correspond to benchmarks with no strong first-order phase transition, either due to insufficient strength or exclusion by constraints. The majority of transitions proceed from the symmetric vacuum to $(0, 0, v_3)$; the subset involving two-step transitions is discussed in Sec. 5.2.2.

We observe that the strength of the transition increases with m_{S_1} and $g_{h\text{DM}}$, and is largely insensitive to θ_{CPV} . This trend is consistent with other SM-like models: a strong transition typically requires heavy states coupled to the Higgs to enhance thermal corrections [126]. At two-loop level, the transition strength is reduced on average by 27%, and the critical temperature shifts downward by 7.2%. Out of one million points, 10,423 exhibit a strong first-order phase transition at one-loop, but only 2,335 remain strong at two-loop.

5.2.1 Towards electroweak baryogenesis

For electroweak baryogenesis, a strong first-order transition must be accompanied by sizeable CP violation, i.e. $\theta_{\text{CPV}} \approx \pi/2$. The best candidate from our scan is:

$$\begin{aligned} \theta_{\text{CPV}} &= 1.64, & \delta_{12} &= 99.9 \text{ GeV}, & m_{S_1} &= 98.0 \text{ GeV}, \\ g_{h\text{DM}} &= 0.639, & \delta_{1c} &= 90.0 \text{ GeV}, & \delta_c &= 7.47 \text{ GeV}. \end{aligned} \quad (5.5)$$

This point exhibits an increase in transition strength from 0.78 (one-loop) to 0.93 (two-loop). The corresponding potential profiles are shown in Fig. 4. A dedicated analysis of baryogenesis dynamics in this model will be presented in a forthcoming publication.

5.2.2 Two-step phase transitions

Approximately 20% of benchmarks exhibit a two-step phase transition at one-loop level. These proceed via an intermediate phase $(v_1, v_2, 0)$, followed by a transition to the electroweak vacuum $(0, 0, v_3)$. However, since the first step is typically very weak, sphaleron suppression is insufficient, and such transitions are not viable for baryogenesis. They may, however, lead to interesting gravitational wave signals.

Figs. 5-6 show the strength of the second transition. While qualitative behaviour is similar to the one-step case, the range of T_c is broader. As most two-step transitions are weak, perturbative results are unreliable and require confirmation by lattice methods. In the few cases where we performed two-loop calculations, we again find that the strongest two-step transition at one-loop corresponds to a one-step strong first-order phase transition at two-loop level. This is illustrated in Fig. 7 for the benchmark in Eq. (5.5).

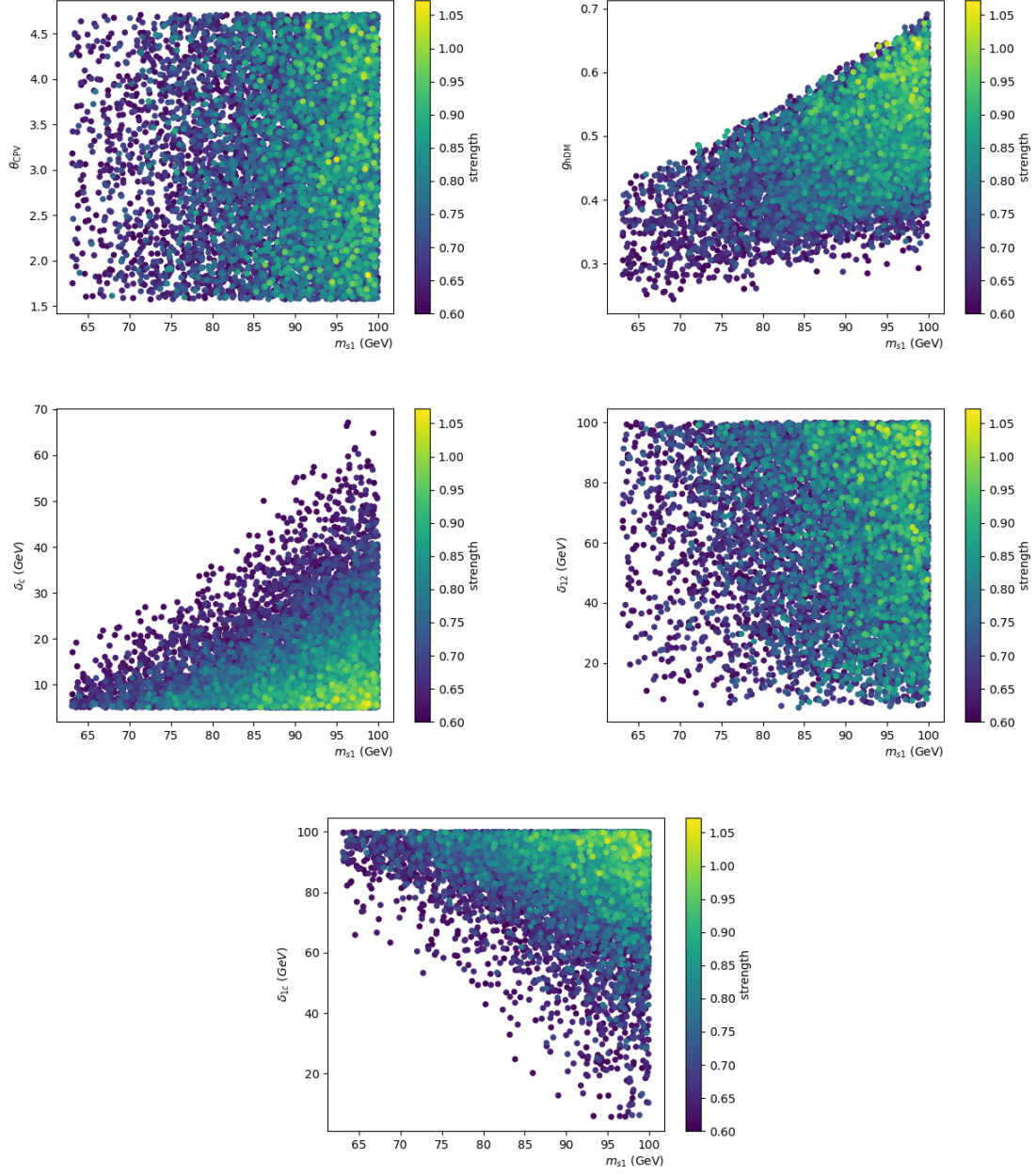


Figure 1. Heat maps of the phase transition strength at one-loop as a function of scan parameters. The step size is 0.1 GeV.

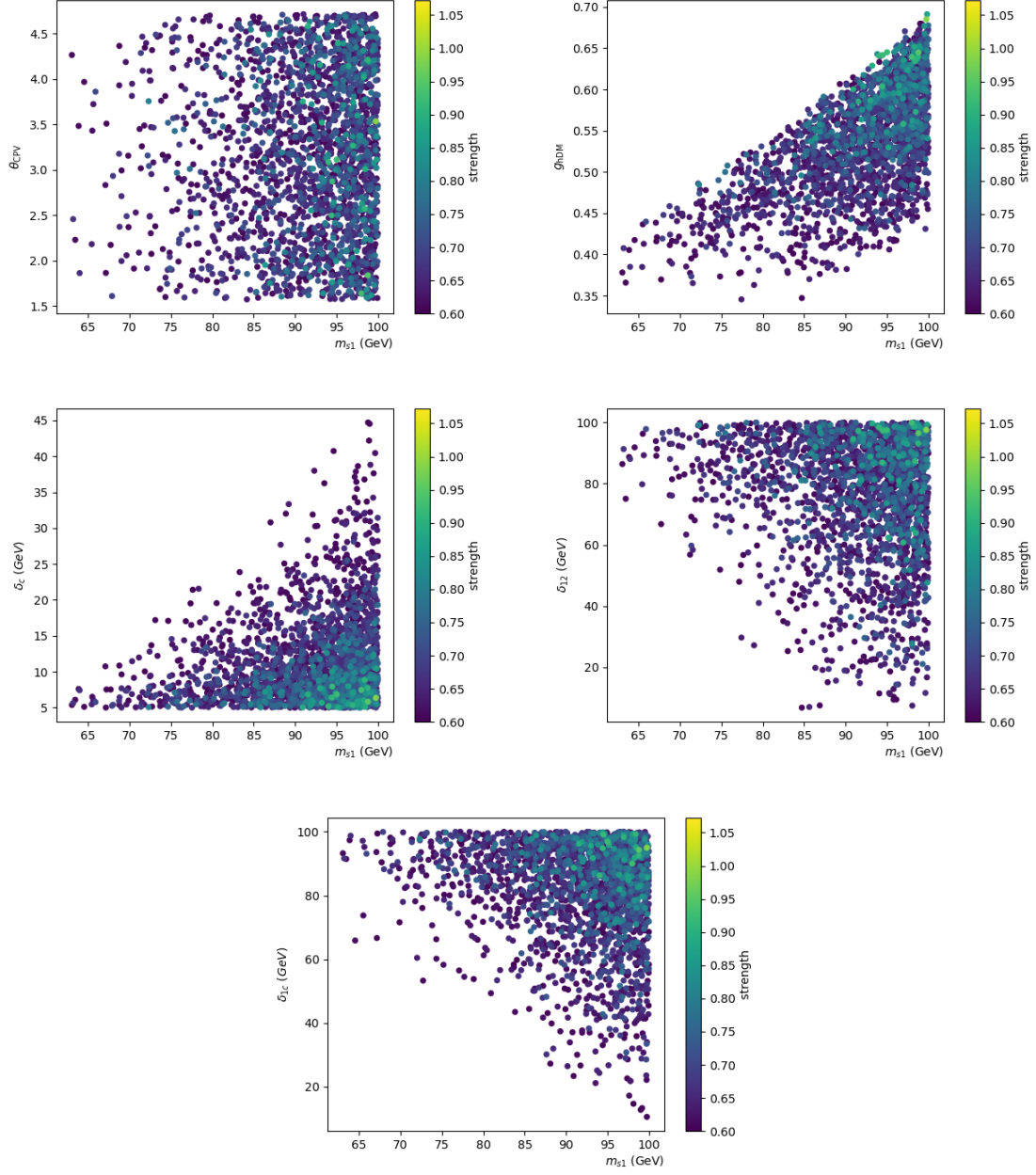


Figure 2. Heat maps of the phase transition strength at two-loop as a function of scan parameters. The step size is 0.1 GeV.

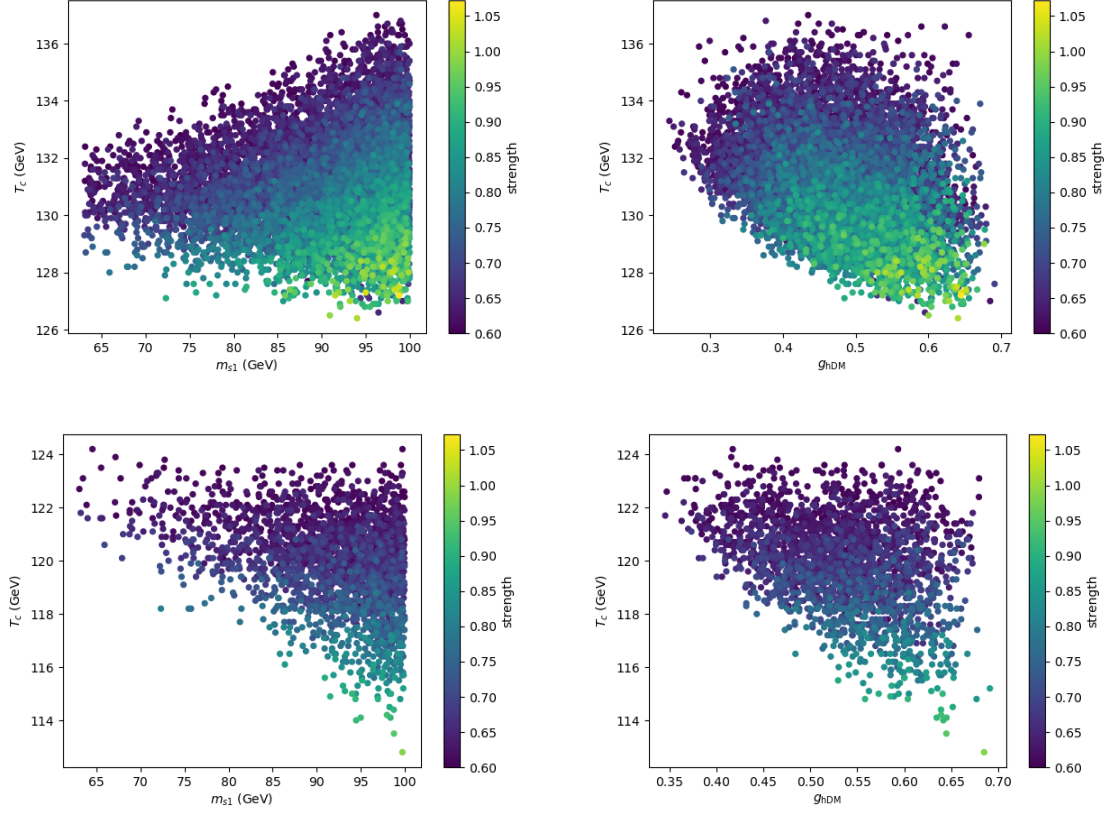


Figure 3. Heat maps of critical temperature T_c as a function of m_{S_1} and g_{hDM} at one-loop (top) and two-loop (bottom).

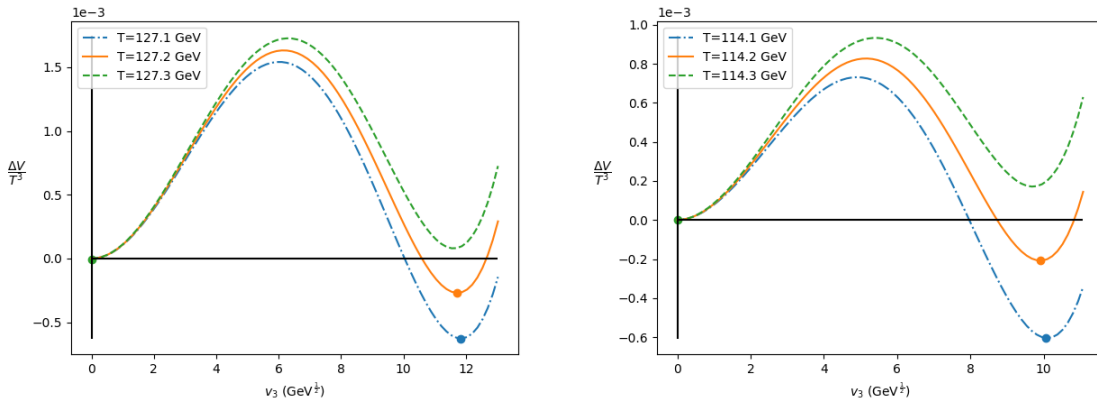


Figure 4. Temperature-normalised effective potential at one-loop (left) and two-loop (right) for the benchmark in Eq. (5.5). Filled circles indicate the global minimum.

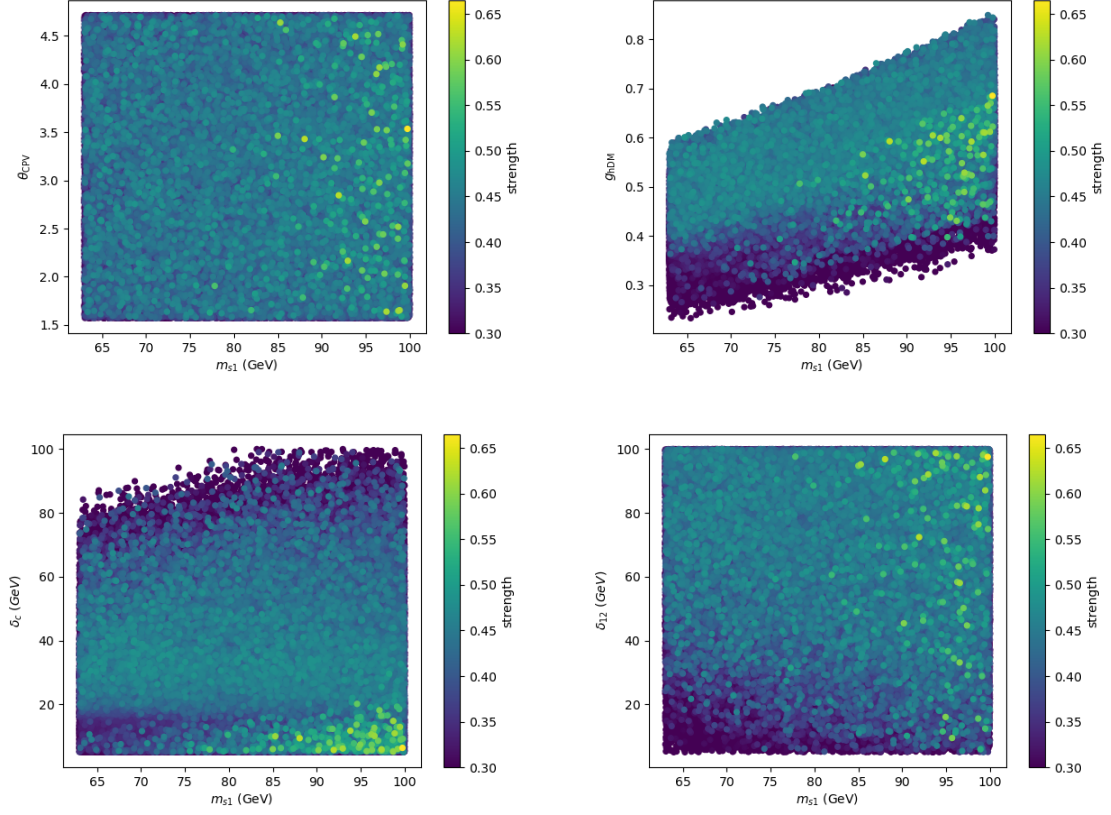


Figure 5. Heat maps of the strength of the $(v_1, v_2, 0) \rightarrow (0, 0, v_3)$ transition at one-loop. Step size: 1 GeV.

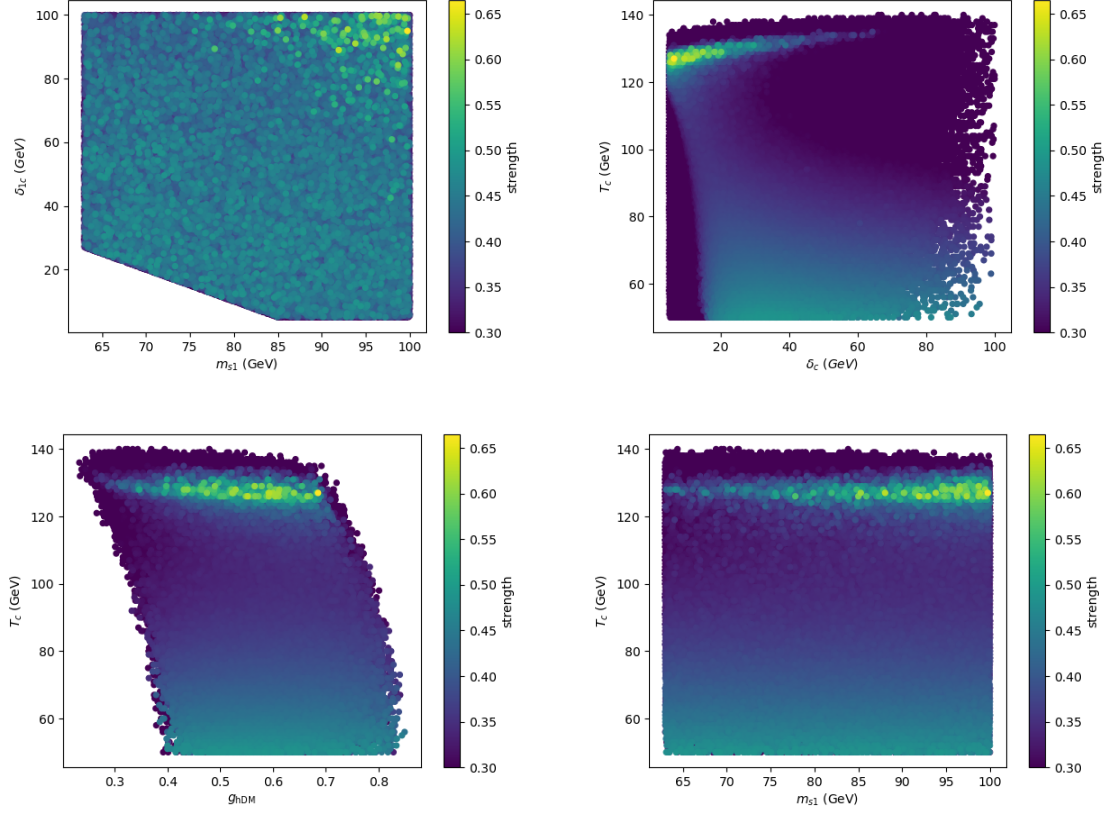


Figure 6. Heat maps of the strength of the $(v_1, v_2, 0) \rightarrow (0, 0, v_3)$ transition at one-loop. Step size: 1 GeV.

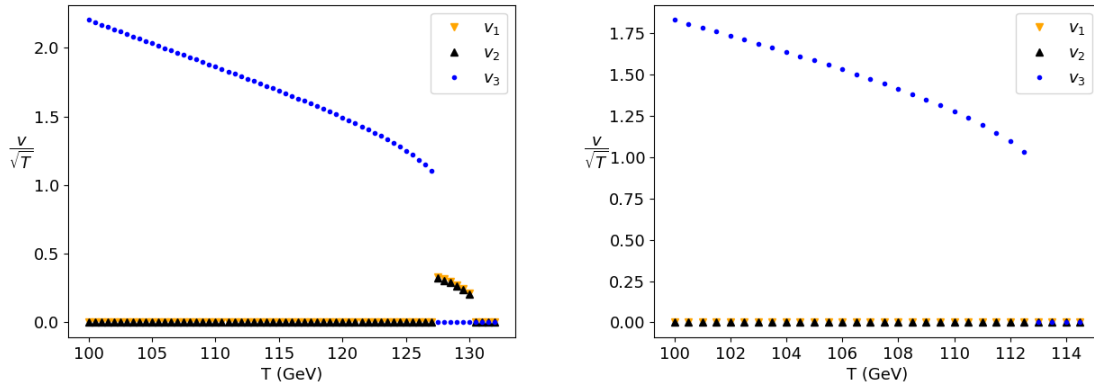


Figure 7. Evolution of the vacuum location with temperature for the benchmark in Eq. (5.5). one-loop (left), two-loop (right).

6 Conclusion and outlook

We have analysed the electroweak phase transition in a 3-Higgs-doublet model with a CP-violating dark sector, motivated by the simultaneous presence of dark matter and electroweak baryogenesis. Employing high-temperature dimensional reduction and the `DRa1go` framework, we computed the two-loop thermal effective potential and performed a systematic scan over the parameter space of the model.

Our results show that strong first-order phase transitions are readily achievable in sizeable regions of parameter space. We found that two-loop corrections significantly affect the strength and critical temperature of the transition, with a typical $\sim 27\%$ reduction in $\Delta v/\sqrt{T_c}$ compared to one-loop estimates. While perturbative predictions overestimate the transition strength, the two-loop calculation provides more accurate guidance for identifying viable benchmarks.

We identified a benchmark point with both a strong first-order phase transition and maximal CP-violating phase, making it a promising candidate for electroweak baryogenesis. Additionally, we observed a subset of scenarios exhibiting two-step transitions, which, though not suitable for baryogenesis due to insufficient sphaleron suppression, may yield interesting gravitational wave signatures.

Our analysis illustrates the importance of going beyond the one-loop approximation in BSM scenarios with extended scalar sectors. The methodology presented here, based on dimensional reduction, symbolic matching, and numerical evaluation, can be readily applied to other multi-scalar models of baryogenesis and dark matter. A dedicated study of more elaborate and exotic vacua the baryon asymmetry generation, including transport dynamics and sphaleron rate calculations, is left for future work.

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