JACOBI IDENTITIES FOR WRONSKIAN DETERMINANTS OVER MULTIDIMENSION

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ABSTRACT. The generalised Wronskian of differential order $k \geqslant 1$ for N functions $f_1, \ldots, f_N$ in $d \geqslant 1$ independent variables $x^1, \ldots, x^d$ is the determinant of the matrix with these functions' derivatives $\partial^{|\sigma_i|} f_j / \partial(x^1)^{\sigma_i^1} \cdots \partial(x^d)^{\sigma_i^d}$ (of orders $0 \leqslant |\sigma_i| \leqslant k$), where the multi-indices σ_i mark (all or part of) fibre variables u_{σ_i} in the kth jet space $J^k\big(\mathbb{R}^d \to \mathbb{R}\big)$. We prove that these (in)complete Wronskians – provided that their lowest-order parts are complete at differential orders $\ell \leqslant 1$ – over the d-dimensional base satisfy the table of bi-linear, Jacobi-type identities for Schlessinger–Stasheff's strongly homotopy Lie algebras.

1. INTRODUCTION

The Wronskian determinants are used to inspect linear (in)dependence of functions[1] $f_1, \ldots, f_N$ (differentiable enough many times on an interval in $\mathbb{R}$): if their Wronskian,

$$W^{0,1,\ldots,N-1}(f_1, \ldots, f_N) = \det\big(\partial^{i-1} f_j / \partial x^{i-1}\big)$$

is not identically zero, then they are linearly independent.

Example 1. $W^{0,1}(x, x^2) = \left|\begin{smallmatrix} x & x^2 \\ 1 & 2x \end{smallmatrix}\right| = x^2 \not\equiv 0$ on $[-1, 1] \ni x$. Still the vanishing of the Wronskian on an interval does not yet imply linear independence; here is Peano's counterexample: $W^{0,1}(x^2, x|x|) \equiv 0$ on $[-1, 1] \ni x$, but the functions x^2 and $x|x|$ are linearly independent on any open neighbourhood of the origin.

For differentiable functions $f_j \in C^k\big(\mathbb{R}^{d\geqslant 1} \to \mathbb{R}\big)$ in many variables $x^1, \ldots, x^d$, the concept of Wronskian was re-discovered over decades by many authors from various disciplines (see [1–3], also [4] referring to 2002–3 or A. G. Khovanskii in 2003–4 (private communication)).

To generalise the Wronskian determinants to spaces of functions on $\mathbb{R}^d$ of dimensions $d \geqslant 1$, fix the differential order $k \geqslant 1$ (and work with arguments $f_j \in C^k(\mathbb{R}^d \to \mathbb{R})$). List

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[1] The Wronskian of N scalar functions has conformal weight $N(N-1)/2$, so itself is not a scalar function if $N > 1$. Likewise, the arguments f_j can be not scalar functions (of conformal weight 0) but coefficients of positive-order differential operators on $\mathbb{R}$, hence themselves behave under coordinate changes on the base $\mathbb{R} \ni x$.

all the (multi-indices of) derivatives[2] u_{σ_i} of intermediate orders: $0 \leqslant |\sigma_i| \leqslant k$. We say that in a fixed system of coordinates on the affine $\mathbb{R}^d$, the *complete* kth differential order Wronskian $W^{d\geqslant 1}_{k\leqslant 1}$ of $N = \binom{d+k}{d}$ arguments f_j viewed as functions from $C^k(\mathbb{R}^d \to \mathbb{R})$ is defined by the formula

$$W^{d\geqslant 1}_{k\geqslant 1}\bigl(f_1,\ldots,f_N\bigr) = \det\Bigl(\partial^{|\sigma_i|} f_j / \partial(x^1)^{\sigma_i^1} \ldots \partial(x^d)^{\sigma_i^d}\Bigr), \tag{1}$$

where $\sigma_i = (\sigma_i^1,\ldots,\sigma_i^d) = (\#x^1,\ldots,\#x^d)$ runs over the multi-indices of kth jet's fibre coordinates u_{σ_i}; the index i enumerates rows and j counts columns ($1 \leqslant i,j \leqslant N$).

Example 2 (cf. [4])**.** Over the Cartesian plane $\mathbb{R}^{d=2} \ni (x,y)$ and for the order bound $k=1$, the complete Wronskian is $W^{d=2}_{k=1} = \mathbf{1} \wedge \partial/\partial x \wedge \partial/\partial y$, that is

$$W^{d=2}_{k=1}\bigl(f,g,h\bigr) = \begin{vmatrix} f & g & h \\ f_x & g_x & h_x \\ f_y & g_y & h_y \end{vmatrix}, \qquad f,g,h \in C^1(\mathbb{R}^2 \to \mathbb{R}). \tag{2}$$

This ternary operator is tri-linear and totally antisymmetric w.r.t. its arguments:

$$W^{d=2}_{k=1}\bigl(\pi(f),\pi(g),\pi(h)\bigr) = (-)^{\pi} W^{d=2}_{k=1}\bigl(f,g,h\bigr)$$

for any permutation $\pi \in \mathbb{S}_3$.

Remark 1. The (in)complete generalised Wronskians over dimensions $d \geqslant 1$, which we presently describe, are subject to the same reservations – about their (in)sufficience to show the linear (in)dependence of functions – as in the classical case of $d=1$. For example, the three functions $f(x,y) = x^2y^2$, $g(x,y) = x|x|\cdot y^2$, and $h(x,y) = x^2 \cdot y|y|$ are linearly independent on the square $[-1,1]\times[-1,1] \ni (x,y)$, yet their complete first-order generalised Wronskian from Eq. (2), see above, vanishes identically on the entire domain of definition: $W^{d=2}_{k=1}\bigl(x^2y^2, x|x|\cdot y^2, x^2\cdot y|y|\bigr) \equiv 0$. Indeed, for $x \geqslant 0$ (and any $y \in \mathbb{R}$) determinant's 1st and 2nd columns coincide; for $y \geqslant 0$ (and any $x \in \mathbb{R}$) the 1st and 3rd columns coincide, whereas on $x<0$ and $y<0$, the 2nd and 3rd columns equal minus the first.

Definition 1. The generalised Wronskian determinant $'W^{d\geqslant 1}_{\underline{k\geqslant 1}}$ is *incomplete* if its list $\{\sigma_i\}$ lacks certain multi-indices; exclusion is allowed only for the *highest*-order derivatives (with $|\sigma_i| = k$).

Example 3. For dimension $d=2$ and order $k=2$, by excluding the last multi-index $\sigma_6 = yy = (0,2)$ of top order $|\sigma_6| = 2$ from their full list $\{\varnothing, x, y, xx, xy, yy\}$ at all orders $0 \leqslant \ell \leqslant k = 2$, we obtain the incomplete Wronskian dererminant of size 5×5. Clearly, if this determinant already is not identically zero for five given functions, they are linearly independent (irrespective of the values of their second partial derivatives in y).

Preliminaries: strongly homotopy Lie algebras. Let us recall that the usual Wronskians (over dimension $d=1$, see [5]) and complete generalised Wronskians (over $d>1$ and of differential orders $k,\ell \geqslant 1$, see [4]) satisfy the two-parametric (as $k,\ell \in \mathbb{N}$) table of Jacobi-type identities, bilinear w.r.t. the N-ary structures of orders k and ℓ, for strongly homotopy Lie algebras with zero differential.[3] Namely, denote by $A := C^{r\gg 1}(\mathbb{R}^d \to \mathbb{R})$

[2] **Lemma.** The dimension of kth jet fibre in the jet bundle $J^k(\mathbb{R}^d \to \mathbb{R})$, counting $\sigma = \varnothing$ as well, equals $\binom{d+k}{d}$ under the natural assumption that $u_{xy} = u_{yx}$, etc., for all u_σ.

[3] The reader is addressed to the notes [6] for definitions and physical context: how homotopy Lie algebras appear in various models, see literature references therein.

the algebra of 'good' functions; let $\Delta \in \mathrm{Hom}_{\Bbbk}\bigl(\bigwedge^{N_{\text{out}}} A, A\bigr)$ and $\nabla \in \mathrm{Hom}_{\Bbbk}\bigl(\bigwedge^{N_{\text{in}}} A, A\bigr)$ be $\Bbbk$-linear totally antisymmetric operators on A. By definition, the *action* of Δ on ∇ is $\Delta[\nabla]\bigl(a_1,\ldots,a_{N_{\text{out}}+N_{\text{in}}-1}\bigr) = \bigl[N_{\text{in}}!(N_{\text{out}}-1)!\bigr]^{-1}\cdot$

$$\sum_{\tau\in\mathbb{S}_{N_{\text{in}}+N_{\text{out}}-1}} (-)^{\tau}\Delta\Bigl(\nabla\bigl(a_{\tau(1)},\ldots,a_{\tau(N_{\text{in}})}\bigr),a_{\tau(N_{\text{in}}+1)},\ldots,a_{\tau(N_{\text{in}}+N_{\text{out}}-1)}\Bigr);$$

the $(N_{\text{in}}+N_{\text{out}}-1)$-ary operator $\Delta[\nabla]$ is totally antisymmetric in $a_m \in A$.

Dzhumadil'daev proved in [5] for $d=1$ (cf. [4] with any $d \geqslant 1$) that Wronskian determinants of arbitrary orders $k_{\text{out}}, \ell_{\text{in}}$ satisfy the table of Jacobi identities,

$$W^{d=1}_{k_{\text{out}}\geqslant 1}\bigl[W^{d=1}_{\ell_{\text{in}}\geqslant 1}\bigr] = 0. \tag{3}$$

In the recent work [7] we recall in which way the Jacobi identities of this specific type for N-ary structures, given on A by the Wronskians, appear in the course of homotopy deformation of the Lie algebra $\mathfrak{X}^1(\mathbb{R}^{d=1})$ of vector fields on a one-dimensional base manifold.

2. Jacobi identities for (in)complete Wronskians

We now strengthen the result in [4], extending the table of Jacobi-type identities (3) (over $d=1$ and over $d>1$ for complete sets of top-order derivatives in either Wronskian) to the case of *incomplete* Wronskians: they may lack subsets of derivatives in the highest orders $k,\ell>1$ over dimension $d>1$.

Condition 1. In what follows (and in contrast with Counterexample 6 on p. 5 below), the (in)complete Wronskians $'W^{d\geqslant 1}_{\underline{s\geqslant 1}}$ are admissible only if their set of *first*-order derivatives is complete,i.e. every $\partial/\partial x^a$ shows up in $'W^{d\geqslant 1}_{\underline{s\geqslant 1}} = \mathbf{1}\wedge\partial_{x^1}\wedge\ldots\wedge\partial_{x^d}\wedge\ldots$; omission of multi-indice(s) can occur only in the highest order $s>1$.

Example 4. Consider again the example ($d=2$,$k=2$) in footnote 3 on p. 2: admissible are, for instance, the incomplete Wronskians $\mathbf{1}\wedge\partial_x\wedge\partial_y\wedge\partial_{xx}$ or $\mathbf{1}\wedge\partial_x\wedge\partial_y\wedge\partial_{xx}\wedge\partial_{yy}$, etc., but not $\mathbf{1}\wedge\partial_y\wedge\partial_{xx}\wedge\partial_{xy}\wedge\partial_{yy}$ which lacks $\partial_x=\partial/\partial x$ in order 1.

We recall from Lemma in footnote 2 on p. 2 that $N=\binom{d+k}{d}$ is the number of different (modulo $u_{xy}=u_{yx}$, etc.) partial derivatives of all orders $0,\ldots,k \geqslant 1$ w.r.t. the $d\geqslant 1$ independent variables $x^1,\ldots,x^d$. The *complete* generalised Wronskians $W^{d\geqslant 1}_{k\geqslant 1} = \mathbf{1}\wedge\partial_{x^1}\wedge\ldots\wedge\partial_{x^d}\wedge\ldots\wedge\partial^k_{x^d}$ contain all the multi-indices of these derivatives, starting from $\varnothing$ in the leading wedge factor $\mathbf{1}$ till all of the derivations $\partial^{|\sigma|}/\partial\boldsymbol{x}^\sigma$ with $|\sigma|=k$. In this case of complete sets, we proved that Jacobi identities (3) extend from $d=1$ to arbitrary dimensions $d\geqslant 1$.

Example 5 (cf. [4])**.** Over $d=2$ at order $k=1$, ternary bracket (2) satisfies the ternary Jacobi identity, $\mathbf{1}\wedge\partial_x\wedge\partial_y\bigl[\mathbf{1}\wedge\partial_x\wedge\partial_y\bigr]=0$, which is verified by direct calculation.

Theorem 1 ([4])**.** *Over $d\geqslant 1$, the complete generalised Wronskians satisfy the Jacobi identities $W^{d\geqslant 1}_{k_{\text{out}}\geqslant 1}\bigl[W^{d\geqslant 1}_{\ell_{\text{in}}\geqslant 1}\bigr]=0$ for all differential orders $k_{\text{out}},\ell_{\text{in}}\in\mathbb{N}$.*

Proof scheme (cf. [7, Prop. 5] for $d=1$ and [4] for $d\geqslant 1$). By construction, the operator $W^d_k\bigl[W^d_\ell\bigr]$ is totally antisymmetric w.r.t. its $N_{\text{in}}+N_{\text{out}}-1$ arguments; hence, to be nonzero, this Jacobiator must act by pairwise non-coinciding differentiations $\partial^{|\sigma|}/\partial\boldsymbol{x}^\sigma$ on all of its arguments. The key idea is to estimate the overall sum of their differential orders (in other words, count all $\partial/\partial x^a$ at hand for whatever $a \in \{1,\ldots,d\}$). Without loss of generality suppose $\ell_{\text{in}} = \max(k_{\text{out}},\ell_{\text{in}}) \geqslant k_{\text{out}}$; the complete Wronskian $W^{d\geqslant1}_{\ell\geqslant1} = \mathbf{1} \wedge \partial/\partial x^1 \wedge \ldots \wedge \partial^\ell/\partial(x^d)^\ell$ contains all the differentiations of all orders $0,\ldots,\ell_{\text{in}}$. To have more derivatives that would not repeat the previously considered ones, higher-order operators (of orders $> \ell_{\text{in}}$) are needed for the second, ..., last arguments of the other Wronskian. The other Wronskian, $\mathbf{1} \wedge \langle$terms of order $\leqslant \ell_{\text{in}}\rangle$, contains the right number of positive-order terms, but each of those differential orders does not exceed $\ell_{\text{in}} < \ell_{\text{in}} + 1$, contrary to the required. Therefore, at least one differentiation repeats twice, and the antisymmetrisation cancels out the entire operator's action. $\square$

Remark 2. From the proof it is readily seen that Wronskians can be pre-multiplied by an arbitrary factor $\varrho(\boldsymbol{x})$ – which, via the Leibniz rule, can absorb part of the derivatives acting on the inner Wronskian when it becomes the first argument of the outer structure, – still preserving the statement of Theorem 1: all the Jacobiators vanish.

The flaw of assertion self in Theorem 1 is that over big dimension $d > 1$, the matrix size of either Wronskian determinant leaps from $N(d,r) = \binom{d+r}{d}$ to $N(d,r+1) = \binom{d+r+1}{d} > N(d,r)+1$ as the order r increments by $+1$. We claim that whenever $k,\ell > 1$, the conclusion (with basically the same proof) can be strengthened: having completed the Wronskians $W^{d\geqslant1}_{k_{\text{out}}\geqslant1}$ and $W^{d\geqslant1}_{\ell_{\text{in}}\geqslant1}$ at the preceding differential orders, we can *gradually* accumulate either Wronskian in the next order $k_{\text{out}}+1$ and $\ell_{\text{in}}+1$ by incorporating new derivatives one by one. Along many intermediate scenarios (for choosing the subsets of multi-indices in the next, not yet complete differential order), the *complete* Wronskians $W^{d\geqslant1}_{k_{\text{out}}+1>1}$ and $W^{d\geqslant1}_{\ell_{\text{in}}+1>1}$ are attained.

In what follows we assume again that, without loss of generality, $\ell_{\text{in}} \geqslant k_{\text{out}}$ (otherwise, swap ‘in’ $\rightleftarrows$ ‘out’). We stress that under Condition 1, both the (in)complete Wronskians ${}'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ and ${}'W^{d\geqslant1}_{\underline{\ell_{\text{in}}\geqslant1}}$ must contain the complete sets of *first*-order derivations $\partial_{x^1} \wedge \ldots \wedge \partial_{x^d}$.

Theorem 2. *Suppose that in the senior order $\ell_{\text{in}} \geqslant k_{\text{out}} \geqslant 1$, the Wronskian $W^{d\geqslant1}_{\ell_{\text{in}}\geqslant1}$ is complete; the other Wronskian ${}'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ can be either incomplete in its highest order $1 < k_{\text{out}} \leqslant \ell_{\text{in}}$ or complete of order $k_{\text{out}} = 1$, $W^{d\geqslant1}_{k_{\text{out}}=1}$ without any derivatives of order $\geqslant 2$. Then the Jacobi identity holds: ${}'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}\big[W^{d\geqslant1}_{\ell_{\text{in}}\geqslant1}\big] = 0$.*

Proof. Here, the proof repeats – word by word – that of Theorem 1. $\square$

Theorem 3. *Suppose that in its senior order $\ell_{\text{in}} > 1$ the Wronskian ${}'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}$ is incomplete, still the (in)complete other Wronskian determinant ${}'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ of size $N_{\text{out}} \times N_{\text{out}}$ with $k_{\text{out}} \leqslant \ell_{\text{in}}$ is such that $N_{\text{out}} - 1 > \big($the number of highest, ℓ_{in}th-order derivatives missing in the top of the incomplete senior-order Wronskian ${}'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}\big)$. Then the Jacobi identity holds: ${}'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}\big[{}'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}\big] = 0$.*

Proof. We only need to bound the (sum of) orders $|\sigma|$ of $\partial^{|\sigma|}/\partial\boldsymbol{x}^\sigma$. Do ‘complete’ the senior-order Wronskian by fictitiously moving the lacking number of derivatives from the lower-order Wronskian — neglecting any repetitions of multi-indices σ and pretending that all the carried derivatives are senior order, ℓ_{in}. One Wronskian now completed, the other again does not attain the required differential order $\ell_{\text{in}}+1$ in each of its second, ..., last remaining wedge factor. $\square$

Theorem 4. *Suppose that in its senior order $\ell_{\text{in}} > 1$ the Wronskian $'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}$ is incomplete, and the (in)complete other Wronskian determinant $'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ of size $N_{\text{out}} \times N_{\text{out}}$ with $k_{\text{out}} \leqslant \ell_{\text{in}}$ is such that $N_{\text{out}} - 1 \leqslant \big($the number of highest, ℓ_{in}th-order derivatives missing in the top of the incomplete senior-order Wronskian $'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}\big)$. Then the Jacobi identity holds: $'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}\big[{}'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}\big] = 0$.*

Proof. Indeed, the outer Wronskian $'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ cannot supply $N_{\text{out}} - 1$ derivatives of order $\ell_{\text{in}} > 1$ – to let the Jacobiator act by non-coinciding derivations on all of its arguments – because $'W^{d\geqslant1}_{\underline{k_{\text{out}}\geqslant1}}$ contains at least one lowest-order derivation $\partial/\partial x^a$, yet their full set is already present in $'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}$ of higher order. □

Only the last case, Theorem 4 explicitly relies on the assumption $\ell_{\text{in}} > 1$ and Condition 1 that the set of first-order multi-indices is full in $'W^{d\geqslant1}_{\underline{\ell_{\text{in}}>1}}$. In fact, the outer Wronskian can then be incomplete of order 1 !

Counterexample 6. But this is what happens when the above assumption ($\ell_{\text{in}} > 1$) and Condition 1 are ignored: over $d = 2$, we have

$$\mathbf{1} \wedge \partial/\partial y\big[\mathbf{1} \wedge \partial/\partial x\big] = 2 \cdot W^{d=2}_{r=1} \not\equiv 0,$$

i.e. the action of one incomplete low-order Wronskian on the other of same type recovers ternary bracket (2).

3. Conclusion

We established the 'no-gaps' set of Jacobi identities (from the entire table of identities for the strongly homotopy Lie algebra with zero differential): we are now free to increment the size of either Wronskian determinant by $+1$, that is without huge leaps to the dimension of the next, higher-order jet fibre. Through Condition 1 (contrasted by Counterexample 6), the Wronskians over multidimension $d > 1$ – participating in the homotopy of *unknown* Lie-algebraic object – still reproduce the fact that over $d = 1$, the original structure to-deform was the Lie algebra $\mathfrak{X}^1(\mathbb{R}^1)$ of vector fields on the line, whence the wedge $\mathbf{1} \wedge \partial/\partial x$ (from the commutator of vector fields on $\mathbb{R}^1$) was seen in every Wronskian $W^{0,1,\ldots,N-1} = \mathbf{1} \wedge \partial_x \wedge \ldots \wedge \partial_x^{N-1}$. The deformation of $\mathfrak{X}^1(\mathbb{R}^1)$ ran over higher-order differential operators $w_j(x)\,\partial_x^{N/2}$ for even $N = 2p \in \mathbb{N}$ and over still-unrecognised objects for N odd. The nature of deformation's higher-order terms over $d > 1$ is not yet identified. The contrast of three new Theorems 2–4 with Counterexample 6 (when Condition 1 is violated) indicates the $(d+1)$-arity of the first-order 'commutator' $W^{d>1}_{r=1}$ – for the objects on $\mathbb{R}^{d>1}$ – which undergoes the homotopy deformation. An open problem is to describe the integral object for the algebra with the bracket built of $W^{d>1}_{r=1}$ and its homotopy by $'W^{d>1}_{\underline{s>1}}$.[4]

[4] Over $d = 1$, the Lie algebra $\mathfrak{X}^1(M^1)$ integrated to $\mathrm{Diffeo}(M^1)$ with associative composition $\circ$; the L_∞-structure from [4–6] integrated to an A_∞-deformation of $\mathrm{Diffeo}(M^1)$ (note that $\circ$ is binary as $2 = d + 1$ for M^1). What is the $(d + 1)$-ary analogue of the binary composition of diffeomorphisms ?

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