

Correlation-Powered Work: Equivalence in Peak Yield, Differences in Robustness

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Initial system-environment correlations are a thermodynamic resource, enabling work extraction via their erasure. We compare the work potential of classical, quantum, and hypothetical stronger-than-quantum correlations as a function of measurement misalignment. While all models can yield a peak extractable work of $k_B T \ln 2$, corresponding to a mutual information of $\ln 2$, their value as a resource differs critically in its robustness. The classical resource is fragile, decaying linearly with misalignment, whereas the quantum resource is robust, decaying only quadratically. Thus, the degree of nonlocality maps not to the maximum energetic value of a correlation, but to its operational robustness as a thermodynamic fuel.

I. INTRODUCTION

In classical thermodynamics, an *isolated* system is defined by boundaries impermeable to matter and energy. For macroscopic, uncorrelated systems, this definition is adequate, and the second law dictates an irreversible evolution towards maximal entropy.

This picture is challenged by microscopic systems prepared with *initial* correlations to their environment. Even when a boundary blocks energy exchange, the system's local state remains dependent on environmental degrees of freedom. Consequently, operations on the environment can alter the system's conditional state. When an agent possesses side information about these correlations, they can be consumed to perform work, a process governed by generalized thermodynamic laws that incorporate information-theoretic balances [1–3].

This paper aims to (i) formalize a correlation-aware definition of thermodynamic isolation, (ii) review the generalized free-energy and work bounds that account for mutual information, and (iii) compute how the nonlocal character of different correlation laws relates to their value as a thermodynamic resource. We demonstrate that regardless of whether a correlation is classical, quantum, or “stronger-than-quantum,” the extractable work per binary system is fundamentally bounded by its mutual information, consistent with Landauer's principle [4–10].

II. GENERALIZED THERMODYNAMIC LAWS

Consider a system A interacting with an environment E, which is modeled as an ideal thermal bath at a fixed temperature T . The initial joint state is described by ρ_{AE} . The von Neumann entropies $S(\cdot)$ of the subsystems and the total system are related through the mutual information, $I(A : E) = S(\rho_A) + S(\rho_E) - S(\rho_{AE})$, which quantifies the amount of shared information between the two systems: the average information gained about one system by measuring the other. Mutual information is

also commonly denoted with a semicolon, $I(A; E)$. Likewise, the joint entropy $S(\rho_{AE})$ is often written simply as $S(A, E)$.

For any entropy-preserving global evolution of the combined system, the total entropy change is zero, $\Delta S(\rho_{AE}) = 0$. From the definition of mutual information, it follows that for such a process, the changes in subsystem entropies are related to the change in mutual information by the identity:

$$\Delta S(\rho_A) + \Delta S(\rho_E) = \Delta I(A : E). \quad (1)$$

The first law for system A is $\Delta E_A = Q_E + W$, where W is the work done *on* A and Q_E is the heat it absorbs from the bath. The standard non-equilibrium (Helmholtz) free energy—the energy available in a system to do work at a constant temperature T —of A is $F_T(\rho_A) := \text{Tr}[H_A \rho_A] - k_B T S(\rho_A)$. For an ideal bath, the heat transfer is related to its entropy change by $Q_E = -k_B T \Delta S(\rho_E)$.

Combining these relations yields the generalized second law. We start with the free energy change and substitute the first law and the heat relation:

$$\begin{aligned} \Delta F_T(\rho_A) &= \Delta E_A - k_B T \Delta S(\rho_A) \\ &= (Q_E + W) - k_B T \Delta S(\rho_A) \\ &= [-k_B T \Delta S(\rho_E) + W] - k_B T \Delta S(\rho_A) \\ &= W - k_B T [\Delta S(\rho_A) + \Delta S(\rho_E)]. \end{aligned} \quad (2)$$

Using the identity from Eq. (1), this becomes:

$$\Delta F_T(\rho_A) = W - k_B T \Delta I(A : E). \quad (3)$$

This can be rearranged for the work extracted, $W_{\text{ext}} := -W$, to give the bound:

$$W_{\text{ext}} \leq -\Delta F_T(\rho_A) - k_B T \Delta I(A : E). \quad (4)$$

This central result shows that correlations act as a thermodynamic resource. As noted by Bera et al. [1], if the initial state is correlated, the change in mutual information during a process that consumes this correlation is negative, $\Delta I(A : E) \leq 0$, where $\Delta I(A : E) \equiv I_f(A : E) - I_i(A : E)$. This negative change makes the final term in Eq. (4) positive, allowing for work extraction beyond the conventional change in free energy.

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For a cyclic process on A ($\Delta F_T(\rho_A) = 0$) that begins with correlation $I_i(A : E)$ and ends in an uncorrelated state ($I_f(A : E) = 0$), we have $\Delta I(A : E) = -I_i(A : E)$. The bound on extractable work then simplifies to:

$$W_{\text{ext}} \leq k_B T I_i(A : E). \quad (5)$$

This leads to a refined definition of isolation: a system is truly isolated only if it is both energetically sealed and initially *uncorrelated* with its surroundings.

III. A HIERARCHY OF CORRELATION LAWS AND NONLOCALITY

We now analyze three bipartite $\{\pm 1\}$ -outcome correlation laws $E(\theta)$ as a function of the relative angle $\theta \in [0, \pi]$ between measurement settings. For the sake of differentiating various types of correlations (classical, quantum, stronger-than-quantum) we define the “optimal” Clauser-Horne-Shimony-Holt (CHSH) parameter $\mathcal{S}_{\text{CHSH}} := |E(\mathbf{a}, \mathbf{b}) + E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) - E(\mathbf{a}', \mathbf{b}')|$ derived from hull calculations of a classical correlation polytope [11, 12]. Local realism demands $\mathcal{S}_{\text{CHSH}} \leq 2$, quantum mechanics permits $\mathcal{S}_{\text{CHSH}} \leq 2\sqrt{2}$ (the Tsirelson bound) [13], while the algebraic limit is 4. For any $E(\theta) \in [-1, 1]$, a valid no-signaling probability distribution with uniform marginals is $P(x, y|\theta) = \frac{1}{4}[1 + xyE(\theta)]$, and $x, y \in \{\pm 1\}$ are the measurement outcomes for the two parties. The functional forms of these three laws are illustrated in Fig. 1.

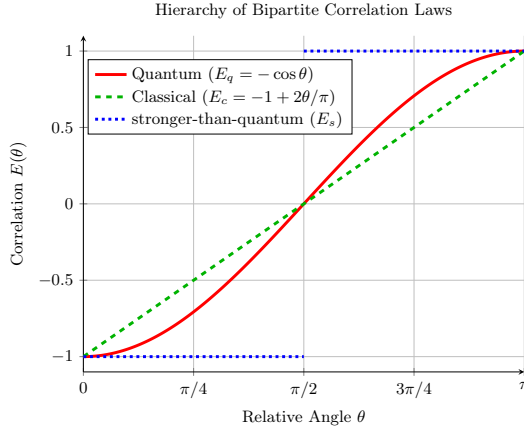


FIG. 1. A comparison of the three correlation functions $E(\theta)$ versus the relative measurement angle θ . The quantum (solid red) and classical (dashed green) correlations only achieve perfect anti-correlation ($E = -1$) at $\theta = 0$. The hypothetical stronger-than-quantum correlation (dotted blue) is perfect for any angle $\theta \neq \pi/2$.

A. Classical (Linear) Correlation

A simple local-hidden-variable model [14] gives the linear correlation $E_c(\theta) = -1 + 2\theta/\pi$. To test this against the CHSH inequality, we adopt the standard angles used to find the quantum bound: $\phi_a = 0$, $\phi_{a'} = \pi/2$, $\phi_b = \pi/4$, and $\phi_{b'} = -\pi/4$. This choice results in three instances of the relative angle $\theta_1 = \pi/4$ and one instance of $\theta_2 = 3\pi/4$.

The correlations are $E_c(\pi/4) = -1 + 2(\pi/4)/\pi = -0.5$ and $E_c(3\pi/4) = -1 + 2(3\pi/4)/\pi = 0.5$. Substituting these values into the CHSH parameter yields: $\mathcal{S}_{\text{CHSH}}^{(c)} = |3E_c(\pi/4) - E_c(3\pi/4)| = |3(-0.5) - 0.5| = |-2| = 2$. This result correctly saturates the classical bound predicted by local realism.

B. Quantum (Cosine) Correlation

For a two-qubit singlet state, measurements yield $E_q(\theta) = -\cos \theta$. With the standard CHSH angles $\phi_a = 0$, $\phi_{a'} = \pi/2$, $\phi_b = \pi/4$, and $\phi_{b'} = -\pi/4$, the relative angles are $\pi/4$ and $3\pi/4$. The correlations are $E_q(\pi/4) = -\sqrt{2}/2$ and $E_q(3\pi/4) = \sqrt{2}/2$, giving $\mathcal{S}_{\text{CHSH}}^{(q)} = 2\sqrt{2}$.

This value, $2\sqrt{2}$, is the maximal allowed in quantum theory. To see this, define the CHSH operator $\mathcal{B} = A \otimes (B + B') + A' \otimes (B - B')$, where A, A', B, B' are Hermitian operators with eigenvalues ± 1 . Since $A^2 = I$, etc., the square of the operator is $\mathcal{B}^2 = 4I \otimes I + [A, A'] \otimes [B, B']$. The operator norm is bounded by $\|[X, Y]\| \leq 2\|X\|\|Y\|$, so $\|[A, A']\| \leq 2$ and $\|[B, B']\| \leq 2$. Thus, $\|\mathcal{B}^2\| \leq 4 + 4 = 8$. It follows that $\|\mathcal{B}\| \leq 2\sqrt{2}$, which bounds its expectation value $\mathcal{S}_{\text{CHSH}} = |\langle \mathcal{B} \rangle|$ for any quantum state.

C. stronger-than-quantum (Step-Function) Correlation

A hypothetical no-signaling correlation, stronger than that allowed by quantum mechanics, is the step function [15, 16]:

$$E_s(\theta) = \text{sgn}\left(\frac{2\theta}{\pi} - 1\right) = \begin{cases} -1, & 0 \leq \theta < \pi/2 \\ +1, & \pi/2 < \theta \leq \pi \end{cases} \quad (6)$$

Using the same set of relative angles as before, $\theta_1 = \pi/4 < \pi/2$ and $\theta_2 = 3\pi/4 > \pi/2$, we find the correlations to be $E_s(\pi/4) = -1$ and $E_s(3\pi/4) = +1$. The CHSH parameter thus becomes: $\mathcal{S}_{\text{CHSH}}^{(s)} = |3(-1) - (+1)| = 4$. This value saturates the algebraic maximum of 4, representing the strongest possible correlation consistent with the no-signaling principle.

IV. MUTUAL INFORMATION AS THERMODYNAMIC FUEL

The thermodynamic value of these correlations is not given by their degree of nonlocality but by their mutual information. For a joint law $P(x, y|\theta)$, the mutual information is

$$I_\theta(A : E) = \ln 2 - H(A|E) = \ln 2 - h_2\left(\frac{1 + E(\theta)}{2}\right), \quad (7)$$

where $h_2(p) = -p \ln p - (1-p) \ln(1-p)$ is the binary entropy in nats. This formula is a direct expression of the definition of mutual information, $I(A : E) = H(A) - H(A|E)$. The leading term, $\ln 2$, is the maximal Shannon entropy $H(A)$ for a single unbiased binary system, representing the total initial uncertainty. The binary entropy term, $h_2(\cdot)$, quantifies the conditional entropy $H(A|E)$: the average uncertainty about system A's outcome that *remains* even after the outcome of system E is known. The mutual information is thus the total initial uncertainty reduced by this residual uncertainty, which directly measures the imperfection of the correlation. For a perfect correlation ($E(\theta) = \pm 1$), the conditional entropy is $h_2(0) = h_2(1) = 0$, yielding the maximal mutual information $I = \ln 2$. For no correlation ($E(\theta) = 0$), the conditional entropy is maximal at $h_2(1/2) = \ln 2$, yielding zero mutual information. The information content for our three models is:

$$I_c(\theta) = \ln 2 - h_2(\theta/\pi) \quad (8)$$

$$I_q(\theta) = \ln 2 - h_2(\sin^2(\theta/2)) \quad (9)$$

$$I_s(\theta) = \begin{cases} \ln 2, & \theta \neq \pi/2 \\ 0, & \theta = \pi/2 \end{cases} \quad (10)$$

In all cases, as depicted in Fig. 2, perfect correlation or anti-correlation ($E = \pm 1$) yields the maximal information $I = \ln 2$. Consequently, the maximum work extractable per correlated binary pair, according to Eq. (5), is universally bounded by $k_B T \ln 2$. While stronger-than-quantum correlations provide maximal mutual information over a wider range of angles, they offer no advantage in the peak extractable work per instance.

V. THERMODYNAMIC EQUIVALENCE VERSUS OPERATIONAL ROBUSTNESS

The preceding analysis established that the maximum extractable work from a single bipartite correlation is universally bounded by $k_B T \ln 2$, irrespective of whether the underlying correlation law is classical, quantum, or stronger-than-quantum. This thermodynamic equivalence stems from the fact that all three models can, under ideal conditions, achieve a perfect correlation ($|E(\theta)| = 1$) and thus a maximal mutual information of $I(A : E) = \ln 2$.

This perspective, however, obscures significant operational differences in how readily this informational resource can be accessed. The practical advantage of non-

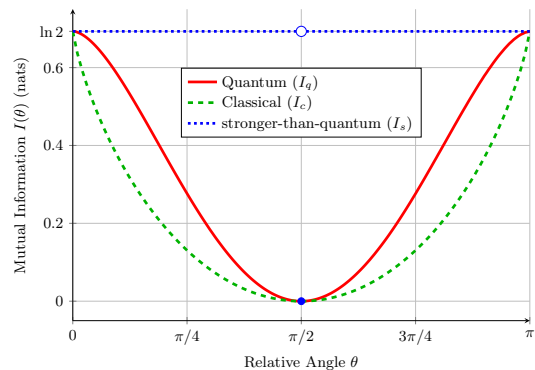


FIG. 2. Mutual information as a function of relative measurement angle θ for the three correlation laws discussed in the text: Quantum (I_q , solid red), Classical (I_c , dashed green), and stronger-than-quantum (I_s , dotted blue). While classical and quantum correlations only provide the maximal information resource of $\ln 2$ nats at perfect alignment ($\theta = 0$) or anti-alignment ($\theta = \pi$), the hypothetical stronger-than-quantum correlation yields this maximum for all angles except the single point $\theta = \pi/2$.

classical correlations becomes apparent when one considers the conditions required to achieve this maximum.

- Classical Brittleness:** For the classical linear model, $E_c(\theta) = -1 + 2\theta/\pi$, the maximal information is available only at the sharp points $\theta = 0$ and $\theta = \pi$. Any deviation from perfect alignment or anti-alignment of the measurement bases results in a linear degradation of the correlation and a corresponding loss of potential work. Accessing the full thermodynamic potential therefore requires a perfectly shared and stable reference frame between the two parties, making the resource *brittle* with respect to experimental noise or misalignment.
- Quantum Robustness:** The quantum correlation $E_q(\theta) = -\cos \theta$ offers a distinct advantage. While it also requires alignment ($\theta = 0$ or $\theta = \pi$) for maximal information, the function's derivative is zero at these points. Consequently, for a small misalignment $\delta\theta$, the deviation from perfect correlation is of order $\mathcal{O}(\delta\theta^2)$. The mutual information near $\theta = 0$ is approximately $I_q(\delta\theta) \approx \ln 2 - \mathcal{O}(\delta\theta^2)$, showing a quadratic, rather than linear, decrease. This property makes the quantum resource significantly more *robust* against small experimental imperfections in establishing a shared reference frame.
- Stronger-Than-Quantum Flexibility:** The hypothetical step-function correlation $E_s(\theta)$ represents the ultimate operational advantage. It yields maximal mutual information ($I_s(\theta) = \ln 2$) for *any* relative angle $\theta \in [0, \pi]$ except for the single measurement-zero point at $\theta = \pi/2$. This makes the resource maximally *flexible*. The two parties would not need to coordinate their measurement settings or share

a reference frame at all to access the full thermodynamic potential. As long as their chosen bases are not perfectly orthogonal, the correlation is perfect and the maximum work can be extracted.

Thus, a clear operational hierarchy emerges. While the peak thermodynamic value of a single, perfectly established correlation is universal, the practical utility and resilience of that resource are dictated by its non-local character. Quantum correlations are operationally superior to classical ones due to their robustness, and hypothetical stronger-than-quantum correlations would be superior still, offering near-total freedom from the physical constraint of a shared reference frame.

VI. AN ENERGETIC FORMULATION OF THE CHSH PARAMETER

The analysis in Sec. V established an operational hierarchy based on the robustness of different correlation laws against misalignment. While the CHSH parameter $\mathcal{S}_{\text{CHSH}}$ quantifies this from the perspective of non-locality, we can construct an analogous "energetic" parameter that captures the same hierarchy from a thermodynamic resource perspective. This is achieved by substituting the correlation function $E(\mathbf{a}, \mathbf{b})$ in the CHSH expression with the corresponding extractable work potential, $W(\mathbf{a}, \mathbf{b}) = k_B T I(A : E)_{\mathbf{a}, \mathbf{b}}$.

We thus define the energetic CHSH parameter, $\mathcal{S}_W$, as:

$$\mathcal{S}_W := |W(\mathbf{a}, \mathbf{b}) + W(\mathbf{a}, \mathbf{b}') + W(\mathbf{a}', \mathbf{b}) - W(\mathbf{a}', \mathbf{b}')|. \quad (11)$$

By factoring out the constant term $k_B T$, this becomes a direct combination of the mutual information values accessible across the four measurement settings of a Bell test:

$$\mathcal{S}_W = k_B T |I_{\mathbf{a}, \mathbf{b}} + I_{\mathbf{a}, \mathbf{b}'} + I_{\mathbf{a}', \mathbf{b}} - I_{\mathbf{a}', \mathbf{b}'}|. \quad (12)$$

This parameter does not represent the work from a single process, but rather quantifies the total thermodynamic resource available across the specific set of operations that constitute a CHSH test. A larger value of $\mathcal{S}_W$ signifies a greater and more resilient thermodynamic potential when subjected to the misaligned measurements inherent to demonstrating non-locality.

To compare the three correlation laws, we again adopt the standard CHSH angles that maximize the quantum value: $\phi_a = 0$, $\phi_{a'} = \pi/2$, $\phi_b = \pi/4$, and $\phi_{b'} = -\pi/4$. This choice results in three instances of the relative angle $\theta_1 = \pi/4$ and one instance of $\theta_2 = 3\pi/4$. The energetic parameter for this specific configuration simplifies to:

$$\mathcal{S}_W = k_B T |3I(\pi/4) - I(3\pi/4)|. \quad (13)$$

We now evaluate this expression for our three models using their analytic forms.

A. Classical Energetic CHSH Term

The classical mutual information is $I_c(\theta) = \ln 2 - h_2(\theta/\pi)$. The two required values are $I_c(\pi/4) = \ln 2 - h_2(1/4)$ and $I_c(3\pi/4) = \ln 2 - h_2(3/4)$. Given the symmetry of the binary entropy function, $h_2(p) = h_2(1-p)$, it is clear that $h_2(1/4) = h_2(3/4)$, and thus $I_c(\pi/4) = I_c(3\pi/4)$. Substituting this into the expression for $\mathcal{S}_W$ yields:

$$\begin{aligned} \mathcal{S}_W^{(c)} &= k_B T |3I_c(\pi/4) - I_c(\pi/4)| = 2k_B T I_c(\pi/4) \\ &= 2k_B T [\ln 2 - h_2(1/4)]. \end{aligned} \quad (14)$$

B. Quantum Energetic CHSH Term

The quantum mutual information is $I_q(\theta) = \ln 2 - h_2(\sin^2(\theta/2))$. The two values are $I_q(\pi/4) = \ln 2 - h_2(\sin^2(\pi/8))$ and $I_q(3\pi/4) = \ln 2 - h_2(\sin^2(3\pi/8))$. Using the identity $\sin^2(x) + \cos^2(x) = 1$ and noting that $\sin(3\pi/8) = \cos(\pi/8)$, the arguments of the entropy function are $p = \sin^2(\pi/8)$ and $1-p = \cos^2(\pi/8) = \sin^2(3\pi/8)$. The symmetry of $h_2(p)$ again implies that $I_q(\pi/4) = I_q(3\pi/4)$. The energetic parameter is therefore:

$$\begin{aligned} \mathcal{S}_W^{(q)} &= k_B T |3I_q(\pi/4) - I_q(\pi/4)| = 2k_B T I_q(\pi/4) \\ &= 2k_B T [\ln 2 - h_2(\sin^2(\pi/8))]. \end{aligned} \quad (15)$$

C. Stronger-than-Quantum Energetic CHSH Term

The stronger-than-quantum mutual information is $I_s(\theta) = \ln 2$ for all $\theta \neq \pi/2$. As both $\pi/4$ and $3\pi/4$ satisfy this condition, we have $I_s(\pi/4) = I_s(3\pi/4) = \ln 2$. The calculation is straightforward:

$$\mathcal{S}_W^{(s)} = k_B T |3\ln 2 - \ln 2| = 2k_B T \ln 2. \quad (16)$$

D. Comparison

To compare the three energetic CHSH values, we must compare the magnitude of the binary entropy terms being subtracted from $\ln 2$. The binary entropy $h_2(p)$ is a monotonically increasing function for $p \in [0, 1/2]$. We compare the arguments for the classical and quantum cases: The classical argument is $p_c = 1/4$, whereas the quantum argument is $p_q = \sin^2(\pi/8) = [1 - \cos(\pi/4)]/2 = (2 - \sqrt{2})/4$. Since $\sqrt{2} > 1$, we have $2 - \sqrt{2} < 1$, which implies that $(2 - \sqrt{2})/4 < 1/4$. Therefore, $p_q < p_c$. Because both arguments are less than $1/2$, it follows that $h_2(p_q) < h_2(p_c)$. This establishes a clear hierarchy for the energetic CHSH parameters:

$$\mathcal{S}_W^{(c)} < \mathcal{S}_W^{(q)} < \mathcal{S}_W^{(s)}. \quad (17)$$

This result demonstrates that the operational advantage of robustness is captured quantitatively by the energetic

CHSH parameter. The hierarchy of classical, quantum, and stronger-than-quantum is mirrored perfectly by a hierarchy of the collective thermodynamic potential accessible in a Bell-test scenario. Stronger non-locality translates directly into a more valuable and resilient thermodynamic resource under operational constraints.

VII. CONCRETE REALIZATION: A CORRELATION-POWERED ENGINE

We can illustrate this principle with a modified Szilard engine. The system (A) is a single particle in a box of volume V at temperature T . The environment (E) is a distant two-state memory bit. They are prepared in a perfectly correlated state: if E is ‘0’, the particle is in the left half of the box; if E is ‘1’, the particle is in the right half. The initial mutual information is $I_i(A : E) = \ln 2$.

The work extraction cycle is as follows:

1. **Information Acquisition:** An agent measures the memory bit E and finds it in state ‘0’. This reveals the particle’s location without interacting with it.
2. **Feedback:** Knowing the particle is on the left, the agent inserts a partition in the middle of the box and places a piston against it. This costs no work.
3. **Isothermal Expansion:** The particle expands against the piston, moving it to the right end of the box. The system draws heat from the reservoir to keep its temperature constant, converting it into work.
4. **Work Calculation:** The work extracted from this isothermal expansion from $V/2$ to V is:

$$W_{\text{ext}} = \int_{V/2}^V \frac{k_B T}{v} dv = k_B T \ln 2. \quad (18)$$

At the cycle’s end, the system A has returned to its initial state ($\Delta F_T(\rho_A) = 0$), but the correlation has been consumed ($I_f(A : E) = 0$). The process has converted information worth $k_B T \ln 2$ entirely into work, saturating the bound in Eq. (5).

VIII. DISCUSSION AND CONCLUSION

Correlations between a system and its environment challenge the classical notion of thermodynamic isolation.

Even when energy and matter exchanges are blocked, pre-existing correlations allow conditional control and work extraction. These correlations—whether classical, quantum, or stronger-than-quantum—constitute a genuine thermodynamic resource governed by generalized second laws that include information flow.

Our analysis shows that the thermodynamic value of any correlation is universally limited by its mutual information: at most $k_B T \ln 2$ of work can be extracted per perfectly correlated bit, in agreement with Landauer’s principle. The apparent hierarchy among classical, quantum and hypothetical superquantum correlations therefore lies not in the magnitude of extractable work but in how robustly that resource can be accessed. Quantum correlations offer a distinct operational advantage, maintaining near-maximal efficiency under small misalignments, whereas classical correlations degrade linearly and superquantum models would be fully alignment-independent.

Recognizing mutual information as the unifying currency of work reconciles thermodynamics and information theory on microscopic scales. Engines that consume correlations do not violate the second law; they reveal that information, too, must be treated as a physical fuel. A truly isolated system must thus be sealed not only against exchanges of energy and matter, but also against preexisting informational ties to its surroundings.

By linking nonlocal correlations to thermodynamic performance, this work clarifies the operational meaning of information in physics and suggests a path toward exploiting quantum correlations as resilient, high-fidelity energy resources in the microscopic domain.

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