

Test of the GENIE neutrino event generator against $^{16}\text{O}(e, e'p)$ data: Reduced cross section

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The reduced cross section of the semiexclusive $(l, l'p)$ lepton scattering process irrespective of the type of interaction is determined mainly by bound nucleon momentum distribution in target and nucleon final state interaction with residual nucleus. These cross sections can be identified with distorted nuclear spectral functions and therefore are similar up to Coulomb corrections for neutrino and electron scattering on nuclei. In this article we exploit this similarity and use data with precise kinematics and large statistics for semiexclusive electron scattering on oxygen target to test models employed in the GENIE neutrino event generator. We find that these models can not reproduce well the measured reduced cross sections in all allowed kinematic region and the GENIE event generator needs to better describe both the nuclear ground states and nucleon final state interaction. The approach presented in this paper provides a great opportunity to test better the accuracy of nuclear models of quasielastic neutrino-nucleus scattering, employed in neutrino event generators.

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I. INTRODUCTION

In current [1, 2] and future [3–5] accelerator-based neutrino experiments the probabilities of neutrino oscillations as functions of neutrino energy are measured to evaluate the oscillation parameters and testing the three-flavor paradigm. The accuracy to which neutrino oscillation parameters can be extracted depends on the ability of experiments to determine the individual energy of detected neutrino. These experiments rely on neutrino event generator codes, simulations of neutrino-nucleus interactions used to estimate efficiencies, backgrounds, and systematic uncertainties for obtained results.

The neutrino beams are not monoenergetic and have broad distributions that range from tens of MeVs to a few GeVs. At the GeV-scale neutrino energies the neutrino can interact with a nucleus through a wide range of reaction channels. These include the charged-current (CC) quasielastic (QE) scattering, two-body meson exchange current (MEC) channels, resonance production and deep inelastic scattering. The incident neutrino energy is reconstructed using kinematic or calorimetric methods. At energy about 1 GeV, where the CCQE scattering is dominant, the incoming neutrino energy can be derived from lepton or proton kinematics alone and as sum of lepton and proton energies. In calorimetric method a model-dependent fit obtained from simulation is used to estimate total hadronic energy. Thus, this method relies not only on the visible energy measured in the detector, but also on the models of the neutrino-nucleus interactions that are implemented in neutrino event generators. In addition the neutrino-nucleus scattering model is critical for obtaining background estimates, in analysis aimed at determining the neutrino oscillation parameters. The further progress in reducing systematic errors in neutrino oscillation experiments requires a more extensive use of measurements of final-states protons. It is needed for neutrino energy reconstruction because more exclusive final-state measurement allows us to better estimate the neutrino energy and provide information about multinucleon contribution to the inclusive cross section. Inclusive reactions are relatively insensitive to the details of the final nuclear states.

Neutrino- and electron-nucleus interactions are similar due to their shared origin in electroweak theory. Electrons interact via a vector current and neutrinos interact via vector and axial-vector currents. The nuclear structure and final state interactions of the outgoing hadrons are expected to be identical for both leptons. These similarities can be used to test

models of neutrino-nucleus interaction. The semiexclusive $(l, l'p)$ lepton scattering process involves the specific asymptotic states and allows us to test more in detail the nuclear model. The reduced cross section, obtained from the measured differential semiexclusive electron scattering cross section dividing by the kinematic factor and the off-shell electron-proton cross section, can be identified as the distorted spectral function. Thus, irrespective of the type of interaction (electromagnetic or weak), the distorted spectral function is determined mainly by the intrinsic properties of the target and the ejected nucleon interaction with residual nucleus and depends upon the initial and ejectile nucleon's momenta and angle between them. Microscopic and unfactorized the relativistic distorted wave impulse approximation (RDWIA), initially designed for description of exclusive $(e, e'p)$ data [6–8] was used [9–12] for calculation of the charged-current (CC) quasielastic (QE) neutrino scattering reduced cross sections as function of the missing nucleon momentum. The results were compared with ones obtained from measurements of $(e, e'p)$ scattering on oxygen, carbon, calcium, and argon targets. It was shown that these cross sections are similar to those of electron scattering data and are in agreement with electron scattering data. This approach provides novel constraints on nuclear models of the CCQE scattering and can be applied to test spectral functions and final-state interaction (FSI) employed in neutrino event generators. A systematic comparison of the CCQE reduced cross sections calculated within the models employed in the GENIE event generator [13, 14] with ones measured in electron scattering on carbon targets was carried out in Ref. [15]. The goal of the present paper is to test of the GENIE's models against to semiexclusive electron scattering data on oxygen. This target is the main detector component in water Cherenkov neutrino detectors and for oxygen there are high-precision data sets with different monochromatic electron beams.

The outline of this article is as follows. In Sec.II we present briefly the formalism for the CCQE semiexclusive scattering process, calculation of the reduced cross sections with neutrino event generator, and regard basic aspects of comprehensive model configurations employed in the GENIE version 3 simulation framework. Results are presented and discussed in Sec.III. The conclusions are summarized in Sec.IV.

II. FORMALISM OF QUASIELASTIC SCATTERING AND NEUTRINO EVENT GENERATOR AS A TOOL IN REDUCED CROSS SECTION STUDY

A. CCQE lepton-nucleus cross sections

In the laboratory frame the differential cross section for exclusive electron (σ^{el}) and (anti)neutrino (σ^{cc}) CC scattering can be written as

$$\frac{d^6\sigma^{el}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\mathbf{p}_x| \varepsilon_x \varepsilon_f \alpha^2}{(2\pi)^3 \varepsilon_i Q^4} L_{\mu\nu}^{(el)} \mathcal{W}^{\mu\nu(el)} \quad (1a)$$

$$\frac{d^6\sigma^{cc}}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{|\mathbf{p}_x| \varepsilon_x |\mathbf{k}_f| G^2 \cos^2 \theta_c}{(2\pi)^5 \varepsilon_i 2} L_{\mu\nu}^{(cc)} \mathcal{W}^{\mu\nu(cc)}, \quad (1b)$$

where $k_i = (\varepsilon_i, \mathbf{k}_i)$ and $k_f = (\varepsilon_f, \mathbf{k}_f)$ are initial and final lepton 4-momenta, $p_A = (\varepsilon_A, \mathbf{p}_A)$, and $p_B = (\varepsilon_B, \mathbf{p}_B)$ are the initial and final target 4-momenta, $p_x = (\varepsilon_x, \mathbf{p}_x)$ is the ejectile nucleon 4-momentum, $q = (\omega, \mathbf{q})$ is the 4-momentum transfer carried by the virtual photon (W-boson), and $Q^2 = -q^2 = \mathbf{q}^2 - \omega^2$ is the photon (W-boson) virtuality. In Eqs. (1a) and (1b) Ω_f is the solid angle for the lepton momentum, Ω_x is the solid angle for the ejectile nucleon momentum, $\alpha \simeq 1/137$ is the fine-structure constant, $G \simeq 1.16639 \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi constant, θ_C is the Cabbibo angle ($\cos \theta_C \approx 0.9749$), $L^{\mu\nu}$ is the lepton tensor, and $\mathcal{W}_{\mu\nu}^{(el)}$ and $\mathcal{W}_{\mu\nu}^{(cc)}$ are correspondingly the electromagnetic and weak CC nuclear tensors.

For exclusive reactions in which only a single discrete state or a narrow resonance of the target is excited, it is possible to integrate over the peak in missing energy and obtain a fivefold differential cross section of the form

$$\frac{d^5\sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x \varepsilon_f \alpha^2}{(2\pi)^3 \varepsilon_i Q^4} L_{\mu\nu}^{(el)} \mathcal{W}^{\mu\nu(el)} \quad (2a)$$

$$\frac{d^5\sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{|\mathbf{p}_x| \tilde{\varepsilon}_x |\mathbf{k}_f| G^2 \cos^2 \theta_c}{(2\pi)^5 \varepsilon_i 2} L_{\mu\nu}^{(cc)} \mathcal{W}^{\mu\nu(cc)}, \quad (2b)$$

where R is a recoil factor

$$R = \int d\varepsilon_x \delta(\varepsilon_x + \varepsilon_B - \omega - m_A) = \left| 1 - \frac{\tilde{\varepsilon}_x \mathbf{p}_x \cdot \mathbf{p}_B}{\varepsilon_B \mathbf{p}_x \cdot \mathbf{p}_x} \right|^{-1}, \quad (3)$$

where $\tilde{\varepsilon}_x$ is the solution to the equation $\varepsilon_x + \varepsilon_B - m_A - \omega = 0$, where $\varepsilon_B = \sqrt{m_B^2 + \mathbf{p}_B^2}$, $\mathbf{p}_B = \mathbf{q} - \mathbf{p}_x$ and m_A and m_B are masses of the target and recoil nucleus, respectively. Note that missing momentum is $\mathbf{p}_m = \mathbf{p}_x - \mathbf{q}$ and missing energy ε_m are defined by $\varepsilon_m =$

$m + m_B - m_A$. The invariant electromagnetic (weak CC) matrix element is represented by the contraction of electron (neutrino) and nuclear response tensors of the form

$$L_{\mu\nu}^{(el)(cc)} = \langle j_{\mu}^{(el)(cc)} j_{\nu}^{+(el)(cc)} \rangle \quad (4)$$

$$W_{\mu\nu}^{(el)(cc)} = \langle J_{\mu}^{(el)(cc)} J_{\nu}^{+(el)(cc)} \rangle, \quad (5)$$

where j^{μ} is the electron or neutrino current, $J_{\mu}^{(el)(cc)}$ is a matrix element of the nuclear electromagnetic or CC current, and the angled brackets denote averages over initial states and sums over final states. The electromagnetic and the weak CC hadronic tensors, $\mathcal{W}_{\mu\nu}^{(el)}$ and $\mathcal{W}_{\mu\nu}^{(cc)}$, are given by bilinear products of the transition matrix elements of the nuclear electromagnetic or CC operator $J_{\mu}^{(el)(cc)}$ between the initial nucleus state $|A\rangle$ and the final state $|B_f\rangle$ as

$$\mathcal{W}_{\mu\nu}^{(el)(cc)} = \sum_f \langle B_f, p_x | J_{\mu}^{(el)(cc)} | A \rangle \langle A | J_{\nu}^{(el)(cc)\dagger} | B_f, p_x \rangle \delta(\varepsilon_A + \omega - \varepsilon_x - \varepsilon_{B_f}), \quad (6)$$

where the sum is taken over undetected states.

The reduced cross section is given by

$$\sigma_{red} = \frac{d^5\sigma^{(el)(cc)}}{d\varepsilon_f d\Omega_f d\Omega_x} / K^{(el)(cc)} \sigma_{lN}, \quad (7)$$

where $K^{el} = Rp_x \varepsilon_x / (2\pi)^3$ and $K^{cc} = Rp_x \varepsilon_x / (2\pi)^5$ are phase-space factors for electron and neutrino scattering and

$$\sigma_{eN}^{(el)} = \frac{\varepsilon_f}{\varepsilon_i} \frac{\alpha^2}{Q^4} (L_{\mu\nu}^{(el)} W^{\mu\nu(el)})_{PWIA} \quad (8)$$

$$\sigma_{\nu N}^{(cc)} = \frac{k_f}{\varepsilon_i} \frac{(G_F \cos \theta_c)^2}{(2\pi)^2} (L_{\mu\nu}^{(cc)} W^{\mu\nu(cc)})_{PWIA} \quad (9)$$

are the corresponding elementary cross sections for the electron σ_{eN} and neutrino $\sigma_{\nu N}$ scattering on the moving free nucleon in the plane wave impulse approximation (PWIA) normalized to unit flux. The PWIA response tensor is computed for a free nucleon in the final state and is given by

$$W_{PWIA}^{\mu\nu} = \frac{1}{2} \text{Trace}(J^{\mu} J^{\nu+}), \quad (10)$$

where

$$J_{s_j, s_i}^{\mu} = \sqrt{\frac{m^2}{\varepsilon_i \varepsilon_j}} \bar{u}(\mathbf{p}_f, s_j) \Gamma^{\mu} u(\mathbf{p}_i, s_i) \quad (11)$$

is the single-nucleon current between free spinors normalized to unit flux. The initial momentum $\mathbf{p}_i = \mathbf{p}_f - \mathbf{q}_{eff}$ is obtained from the final ejectile momentum and the effective

momentum transfer $\mathbf{q}_{eff}$ in the laboratory frame, and the initial energy is placed on shell. The effective momentum transfer accounts for incoming electron acceleration in the nuclear Coulomb field and $\mathbf{q}_{eff} = \mathbf{q}$ for the incoming neutrino.

The single-nucleon charged current has the $V-A$ structure $J^{\mu(cc)} = J_V^\mu + J_A^\mu$. For calculation of vertex function $\Gamma^{\mu(cc)} = \Gamma_V^\mu + \Gamma_A^\mu$ of a moving but free nucleon we employ the CC1 de Forest prescription for off-shell vector current vertex function[16]

$$\Gamma_V^\mu = G_M(Q^2)\gamma^\mu - \frac{\bar{P}^\mu}{2m}F_M(Q^2) \quad (12)$$

and the axial current vertex function

$$\Gamma_A^\mu = F_A(Q^2)\gamma^\mu\gamma_5 + F_P(Q^2)q^\mu\gamma_5, \quad (13)$$

where $\bar{P} = (\epsilon_f + \bar{\epsilon}, 2\mathbf{p}_x - \mathbf{q})$ and $\bar{\epsilon} = \sqrt{m^2 + (\mathbf{p}_x - \mathbf{q})^2}$. The weak vector form factors F_V and F_M are related to corresponding electromagnetic ones for proton $F_{i,p}^{(el)}$ and neutron $F_{i,n}^{(el)}$ by the hypothesis of conserved vector current

$$F_i = F_{i,p}^{(el)} - F_{i,n}^{(el)} \quad (i = V, M). \quad (14)$$

The experimental momentum distribution is also obtained using Eq.(7) with off-shell electron-nucleon cross section σ_{cc1} developed by de Forest [16] that normally used for σ_{eN} . For the axial F_A and pseudoscalar F_P form factors we use the dipole approximation.

The reduced cross section can be regarded as the nucleon momentum distribution modified by FSI, i.e. as the distorted spectral function. Final-state interactions make the reduced cross sections $\sigma_{red}(\mathbf{p}_m, \mathbf{p}_x)$ depend upon ejectile momentum $\mathbf{p}_x$, the angle between the initial and final nucleon momentum and upon incident lepton energy. In the nonrelativistic PWIA limit, σ_{red} reduces to the bound-nucleon momentum distribution. These cross sections for (anti)neutrino scattering off nuclei are similar to the electron scattering apart from small differences at low beam energy due to effects of Coulomb distortion of the incoming electron wave function as shown in Refs. [9–11].

The factorization approximation to the knockout cross section stipulates that

$$\frac{d^5\sigma^{(el)(cc)}}{d\epsilon_f d\Omega_f d\Omega_x} = K^{(el)(cc)} \times \sigma_{lN} \times \sigma_{red}(\epsilon_m, \mathbf{p}_m, \mathbf{p}_x). \quad (15)$$

This factorization implies that the initial nuclear state and FSI effects are decoupled from the leptonic vertex with preserved correlations between the final lepton and nucleon. In the

nonrelativistic PWIA limit, the fivefold differential cross section Eqs.(1a) and (1b) may be expressed as a product of the phase-space factor K , elementary cross section σ_{eN} , and nucleon momentum distribution $S(\mathbf{p}_m, \boldsymbol{\varepsilon}_m)$. Such factorization is not strictly valid relativistically because the binding potential alters the relationship between lower and upper components of a Dirac wave function [17].

B. Neutrino event generator as a tool in reduced cross section study

To test nuclear models that are employed in the GENIE event generator version 3.4.0, we compare reduced cross sections data for oxygen nucleus with predictions made within GENIE simulation framework. This framework offers several models for nuclear ground state, several models for each of the eA or νA scattering mechanisms and several models for hadronic final interaction, i.e., intranuclear rescattering of the outgoing hadrons. Only CCQE neutrino interactions with oxygen nucleus were simulated. To select events and their kinematics in the stable final state which correspond to a $1\mu 1p$ signal, we choose events with one proton and any number of neutrons which have not been affected by inelastic FSI. Two methods are used for this. The first is based on the topology of the events in the stable final state, and second is based solely on the knowledge of the kinematics of events [15].

In the Saclay experiment Ref. [18] the $^{16}\text{O}(e, e'p)$ measurements have been performed in the perpendicular kinematics for $-100 \leq p_m \leq 300$ MeV/c and reduced cross section has been integrated in the intervals of missing energy $\boldsymbol{\varepsilon}_m = 10 - 15$ MeV and 15-20 MeV, where the $p_{1/2}$ and $p_{3/2}$ hole strengths are mostly centered in two peaks with separation energies of 12.2 MeV and 18.5 MeV, respectively. In perpendicular kinematics the incident energy and energy transferred are fixed, as are the electron scattering angle and the outgoing proton energy, whereas the missing momentum changes with the proton angle. It is necessary to choose the electron angle and outgoing proton energy such that $|\mathbf{p}_x| = |\mathbf{q}|$. The vectors $\mathbf{p}_m$ and $\mathbf{q}$ are almost perpendicular.

In the NIKHEF experiment Ref. [19–21] the ^{16}O reduced cross section has been measured in the range $0 \leq \boldsymbol{\varepsilon}_m \leq 40$ MeV and $-180 \leq p_m \leq 270$ MeV/c. The reduced cross sections for removal nucleons from $1p_{1/2}$ and $1p_{3/2}$ shells in $^{16}\text{O}(e, e'p)^{15}\text{N}$ were measured in quasielastic parallel kinematics at three different beam energies: $\boldsymbol{\varepsilon}_i = 304, 456$ and 521 MeV. In this kinematics $\mathbf{p}_x$ is parallel or antiparallel to $\mathbf{q}$. The distribution of missing momentum is

data set	ε_i (MeV)	ε_f (MeV)	θ_e (deg)	T_p (MeV)	θ_p (deg)	$\Delta\varepsilon_m$ (MeV)	notes
Saclay [18]	500	372	59	102	35 - 88	10	perpendicular; 1s is not available
NIKHEF [19, 20]	304-521	188-406	29-81	89-99	36 - 49	40	parallel; 1s is not available

TABLE I. Summary of data for $^{16}\text{O}(e, e'p)$. ε_i is the beam energy, ε_f is the central electron energy, θ_e is the central electron angle, T_p is the central proton kinetic energy, θ_p is the central proton angle, and $\Delta\varepsilon_m$ is the range of the missing energy.

obtained by keeping $|\mathbf{p}_x|$ constant and changing the transferred momentum, i.e., by varying the scattering angle of the two detected particles. The total kinetic energy in the center-of-mass system between the outgoing proton and the recoiling ^{15}N nucleus was kept constant at 90 MeV. In Ref. [20] in Table II list the relevant kinematics parameters of the experiment. The missing momentum is positive for $|\mathbf{q}| < |\mathbf{p}_x|$.

As in all electron scattering experiments the lepton and hadron spectrometers are set in plane, and the out-of-plane angles are fixed to $\phi_e = 0^\circ$ and $\phi_p = 180^\circ$. The sixfold exclusive cross section is given by

$$\frac{d^6\sigma}{d\varepsilon_\mu d\Omega_\mu dT_p d\Omega_p} = \frac{N_{1\mu 1p}}{N_{tot}} \frac{\sigma_{tot}(\varepsilon_\nu)}{\Delta\varepsilon_\mu \Delta T_p \Delta\Omega_\mu \Delta\Omega_p}, \quad (16)$$

where N_{tot} is the total number of generated CCQE neutrino events with a total CCQE cross section $\sigma_{tot}(\varepsilon_\nu)$ at energy ε_ν . $N_{1\mu 1p}$ is the number of selected $1\mu 1p$ events in the differential phase-space volume bin $\Delta V = \Delta\varepsilon_\mu \Delta T_p \Delta\Omega_\mu \Delta\Omega_p$ with the central values $\varepsilon_\mu, T_p, \Omega_\mu, \Omega_p$ and with size of differential bins $\Delta\varepsilon_\mu, \Delta T_p, \Delta\theta_\mu, \Delta\theta_p, \Delta\phi_\mu, \Delta\phi_p$. The central values and size of the differential bins of kinematic variables are given in Tables I and II, corresponding, for every dataset that is analyzed in this work.

To go from measured variables ε_μ and T_p to phase-space (ε_m, p_m)

$$\varepsilon_m = \omega - T_p - \varepsilon_B \quad (17)$$

and

$$p_m = [p_x^2 + k_\nu^2 + k_\mu^2 - 2k_\mu k_\nu \cos \theta_\mu - 2p_x k_\nu \cos \theta_p + 2k_\mu p_x \cos \theta_{\mu p}]^{1/2} \quad (18)$$

we use the formula Ref. [22]

$$\frac{d^6\sigma}{dp_m d\varepsilon_m d\Omega_\mu d\Omega_p} = \frac{d^6\sigma}{d\varepsilon_\mu dT_p d\Omega_\mu d\Omega_p} / J(\theta_p), \quad (19)$$

data set	$\Delta\varepsilon_f$	$\Delta\theta_e$	ΔT_p	$\Delta\theta_p$	$\Delta\phi_e$	$\Delta\phi_p$
	(%)	(deg)	(%)	(deg)	(deg)	(deg)
Saclay [18]	[-7, +25]	2	8	2	6	6
NIKHEF [19, 20]	[-4, +6]	4	[-5.4, +5]	6	6	6

TABLE II. Table of the experimental cuts that is used in simulation. $\Delta\varepsilon_f$ is the electron energy acceptance, $\Delta\theta_e$ is the electron angle acceptance, ΔT_p is the proton kinetic energy acceptance, $\Delta\phi_p$ is the proton angle acceptance, and $\Delta\phi_e$ is the electron plane angle acceptance.

where Jacobin $J = \partial(\boldsymbol{\varepsilon}_m, p_m)/\partial(\boldsymbol{\varepsilon}_\mu, T_p)$ is equal to [15]

$$J(\theta_p) = \frac{1}{p_m}(k_\mu - k_\nu \cos \theta_\mu + p_x \cos \theta_{\mu p}) - \frac{\boldsymbol{\varepsilon}_x}{2p_m} \frac{p_x^2 + p_m^2 - |\mathbf{q}|^2}{p_x^2}. \quad (20)$$

In Eq.(19), $\cos \theta_{\mu p} = \cos(\theta_\mu + \theta_p)$, and we assess positive (negative) values to p_m for the condition $\theta_p < \theta_q$ ($\theta_p > \theta_q$), where $\cos \theta_q = \mathbf{p}_x \cdot \mathbf{q}/(|\mathbf{p}_x||\mathbf{q}|)$. Then, we can determine the distorted spectral function as

$$S^D(p_m, \boldsymbol{\varepsilon}_m) = \frac{d^6\sigma}{dp_m d\boldsymbol{\varepsilon}_m d\Omega_\mu d\Omega_p} / K^{(el)(cc)} \sigma_{\nu N}, \quad (21)$$

where $K^{(el)(cc)}$ is phase-space factors. All nuclear models which are implemented in the GENIE version 3 generator use nucleon momentum distribution as a function of missing momentum at the fixed value of missing (binding) energy $\boldsymbol{\varepsilon}_m$. Therefore, we can evaluate only

$$\sigma_{red}(p_m) = \int S^D(p_m, \boldsymbol{\varepsilon}_m) d\boldsymbol{\varepsilon}_m \quad (22)$$

as a function of p_m , integrated over range $\Delta\boldsymbol{\varepsilon}_m$, that is given in Table I. Then, the reduced cross section can be written as

$$\sigma^{red}(p_m) = \Delta\boldsymbol{\varepsilon}_m S^D(p_m). \quad (23)$$

The calculation of $\sigma_{\nu N}$ is performed with the same nucleon form factors as the GENIE cross section calculation and using the CC1 de Forest prescription [16].

C. GENIE simulation framework

The GENIE version 3 simulation framework uses a few different models of nucleon momentum distribution in the nuclear ground state. It also offers several models of quasielastic lepton-nucleus interactions and several intranuclear cascade models for FSI.

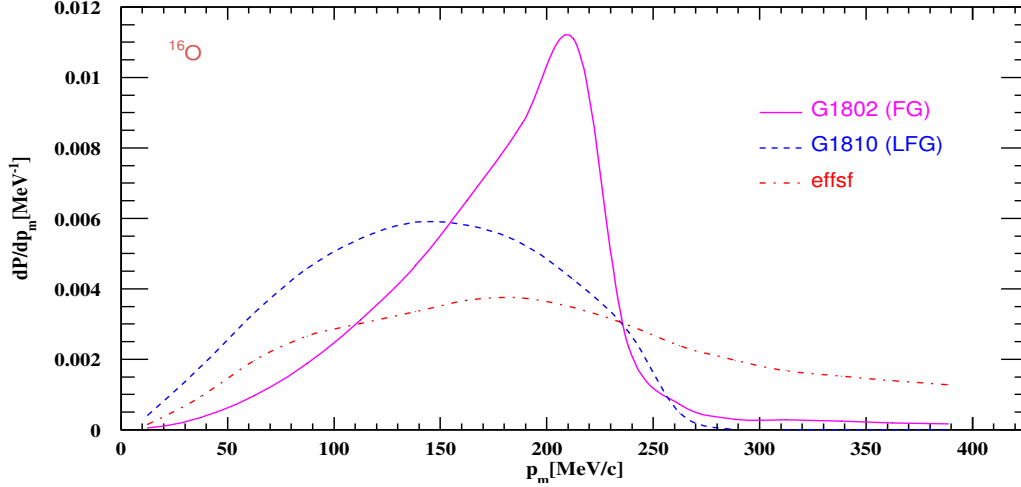


FIG. 1. Initial nucleon momentum distribution for ^{16}O according to the GENIE implementation of G18_02a (FG, solid line), G18_10a (LFG, dashed line), and effsf (dot-dashed) models.

We use the GENIE Version 3.4.0 and consider four distinct sets of GENIEs models for CCQE scattering on oxygen, namely, the G18-02a (RFG), G18-10a (LFG), G21-11a (SuSAv2-MEC), and effsf (effective spectral function) models [23–25]. These models are accompanied by the FSI models hA2018 and hN2018 available in GENIE [23]. We use the hA2018 model for each of our four considered CCQE scattering models.

The G18_02a configuration of GENIE used for CCQE interaction on nuclei relies on implementation of the relativistic Fermi gas model (RFG), which has been modified to incorporate short-range nucleon-nucleon (NN) correlations [27]. The CCQE scattering is simulated by the Llewellyn-Smith model [28]. The G18_10a model set includes the full Valencia model [29–31] for the local Fermi gas (LFG) nucleon momentum distribution. It also uses the random phase approximation (RPA) which is a description of long-range and short-range NN correlations.

The G21_11 (SuSAv2) model is based on a superscaling model that accurately describes QE inclusive scattering on a variety of targets ($A \geq 10$) for a wide range of electron energies. The version used in GENIE version 3 describes struck nucleon momenta with a mean field model (similar to LFG) at low momentum transfer and with a relativistic Fermi gas at high momentum transfer. For use in simulations of experiment, approximations were made describing the outgoing nucleon kinematics using the LFG nuclear model. The effective

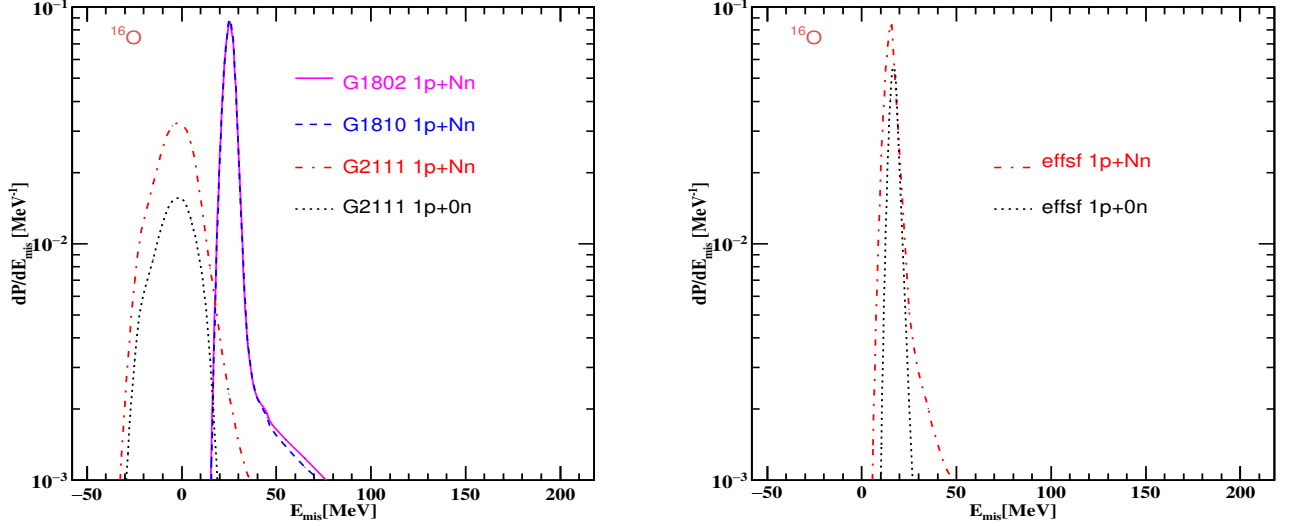


FIG. 2. Probability density distribution vs missing energy E_{mis} calculated for neutrino energy $\epsilon_\nu = 0.5$ GeV. Left panel: PDF for (1p+Nn) events calculated with the G18_02 (solid line), G18_10 (dashed line) and G21_11 (dot-dashed line) models and for (1p+0n) events calculated within G21_11 (dotted line) model. Right panel: PDF for (1p+0n) set calculated with effsf (dotted line) model and PDF for (1p+Nn) set calculated within the effsf (dot-dashed line) models.

spectral function (effsf) model with or without enhancement of transverse contribution [33, 34] is implemented in GENIE as an option EffectiveSF.

The bound nucleon momentum distributions in oxygen for the genuine CCQE events produced using the GENIE and the RFG, LFG, and effsf representations of the target nucleus are shown in Fig.1. It is clearly visible that in the range $150 \leq p_m \leq 220$ ($p_m \leq 50$) (MeV/c) the probability density function (PDF) dP/dp_m calculated with the RFG is much larger (smaller) than PDFs used in the other models, and correlation tails are observed in the RFG and effsf distributions. The missing energy $E_{mis} = \epsilon_\nu - \epsilon_\mu - T_p$ distribution for two sets of events in which neutrino knocks out a proton, calculated with neutrino energy $\epsilon_\nu = 0.5$ GeV using four GENIEs models are shown in Fig.2. The first set contains events with a muon and only one proton (1p + 0n) in the final state. Such events are selected in electron scattering experiments ($e, e'p$) to measure reduced cross sections. The second set contains events with a muon, one proton, and at least one neutron (1p + Nn) in the final

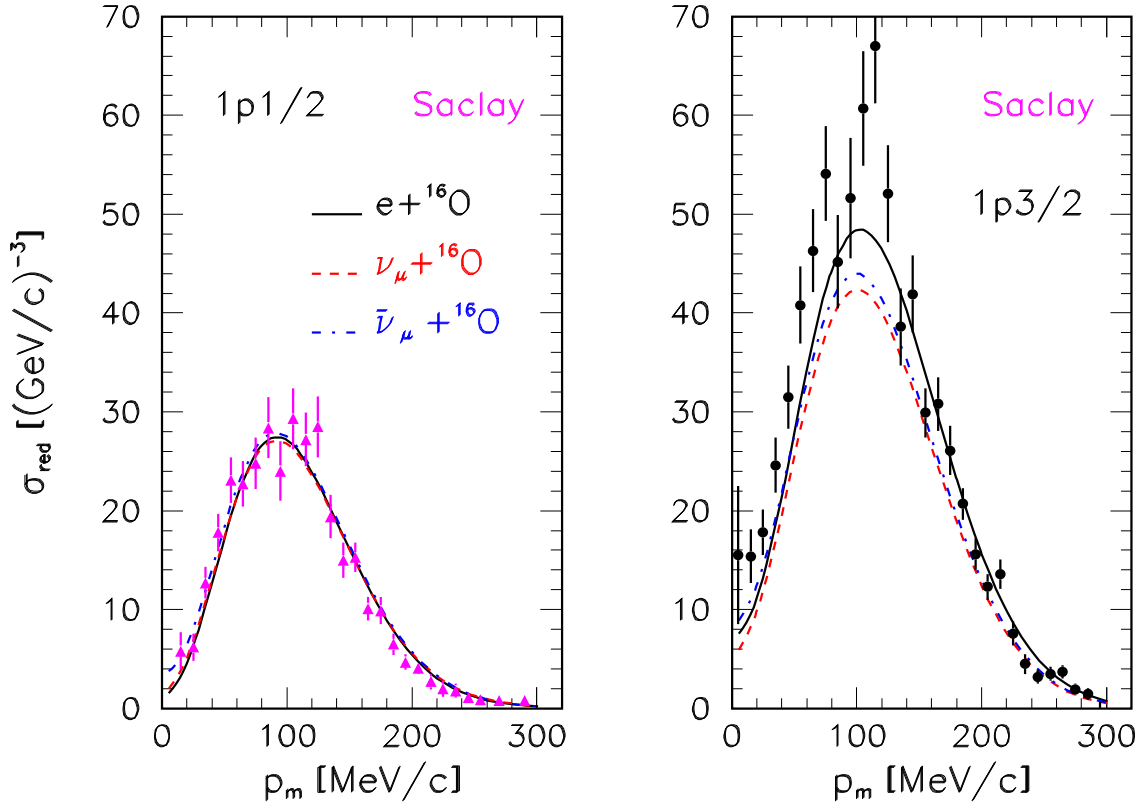


FIG. 3. Comparison of the RDWIA electron, neutrino, and antineutrino reduced cross sections [9] for the removal of nucleons from $1p_{1/2}$ and $1p_{3/2}$ shells of oxygen for Saclay [18] perpendicular kinematics. Saclay data for beam energy $E_{\text{beam}} = 500$ MeV, proton kinetic energy $T_p = 100$ MeV and $Q^2 = 0.3\text{GeV}^2$.

state. In neutrino experiments these two sets of events are not distinguished, because the incident neutrino energy is unknown. For all models (except G21.11 model), the maximum of distribution is located in the range $E_m \leq 40$ MeV. For the G18.02a and G18.10a models, the distributions of events in $(1p+0n)$ set have narrow peak in the range $20 \leq E_m \leq 30$ MeV. The inelastic scattering changes the energy of the knocked-out protons significantly, and therefore the distributions of events in the $(1p+Nn)$ set are continued into the higher missing region.

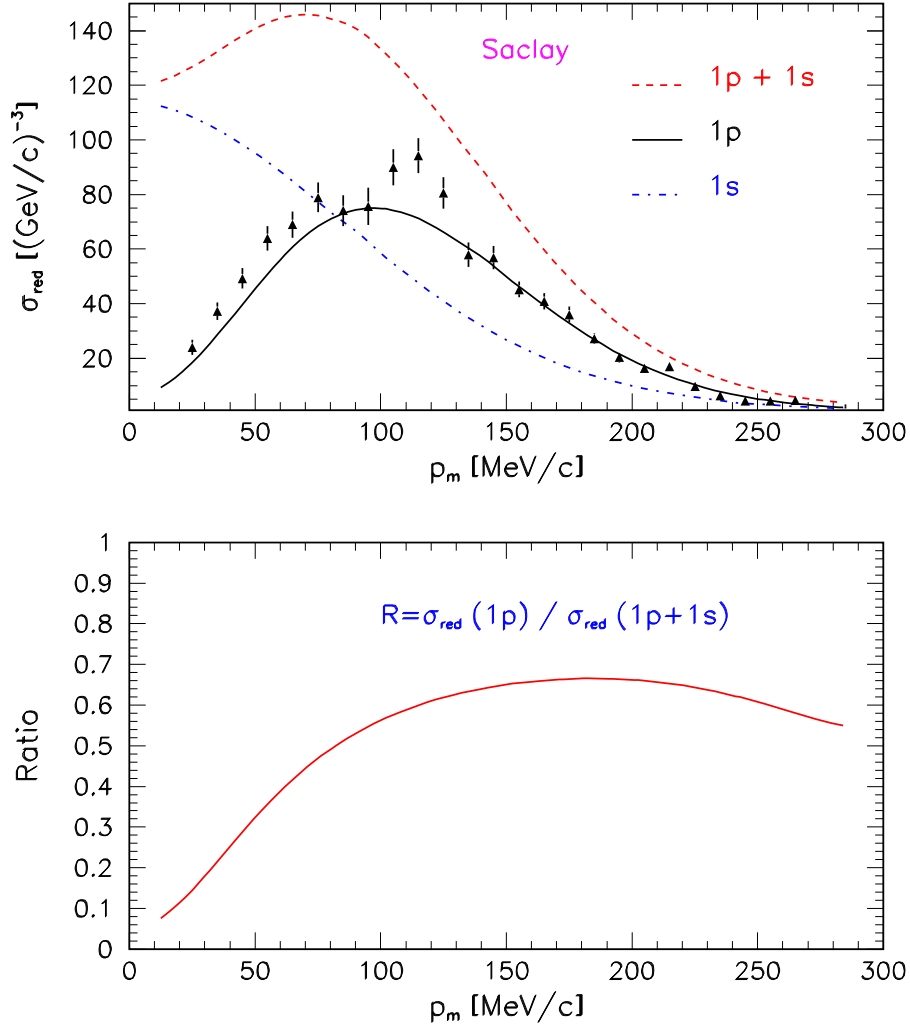


FIG. 4. The RDWIA electron reduced cross section [9] for the removal of protons from $1s$, $1p$, and $1s + 1p$ shells of ^{16}O for Saclay kinematics (upper panel) and ratio R_{1p} (lower panel) as functions of missing momentum. Also shown (upper panel) are Saclay data for the removal protons from $1p = 1p_{1/2} + 1p_{3/2}$ shell.

III. RESULTS AND ANALYSIS

In Saclay and NIKHEF experiments the reduced cross sections were measured as a function of missing momentum for the removal proton from $1p_{1/2}$ and $1p_{3/2}$ shells of ^{16}O that correspond to the intervals of missing energy $\epsilon_m = 10\text{--}15$ MeV and $15\text{--}20$ MeV, respectively. On the other hand all nuclear models which implemented in the GENIE generator allow calculate only the reduced cross sections for the knock out of nucleons from oxygen nucleus

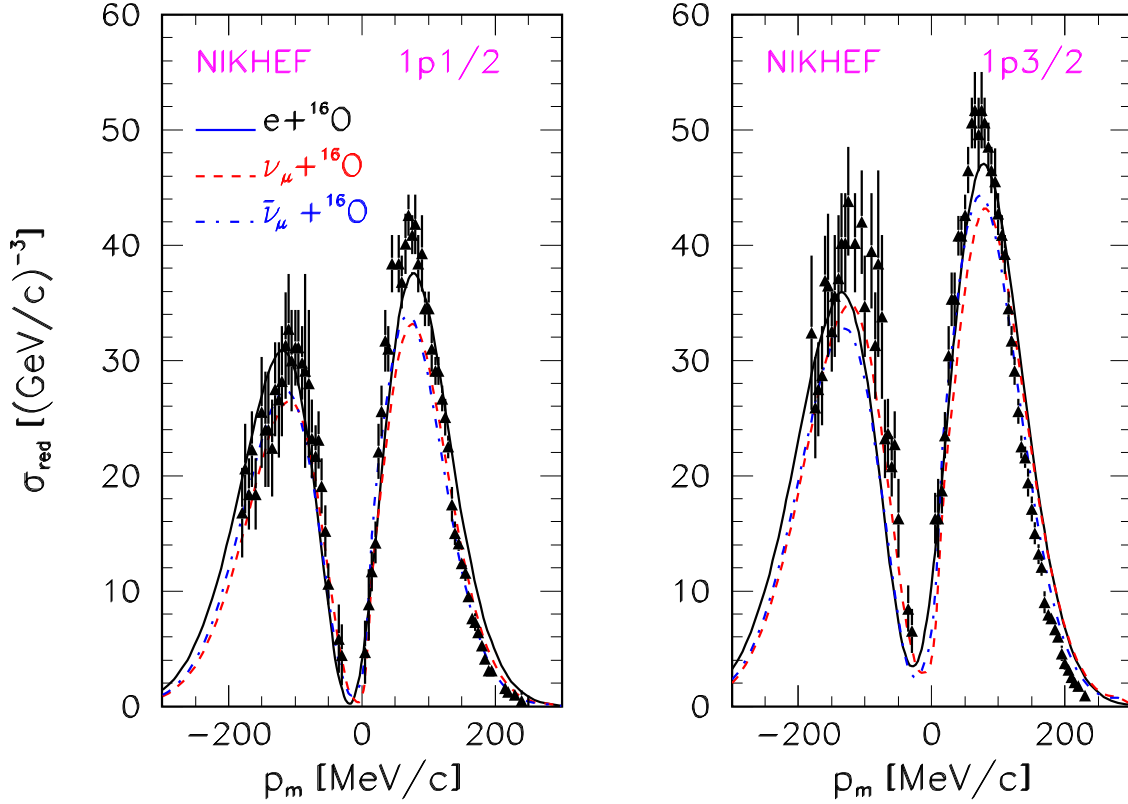


FIG. 5. The RDWIA calculation of electron, neutrino, and antineutrino reduced cross sections [9] for the removal of nucleons from $1p_{1/2}$ and $1p_{3/2}$ shells of ^{16}O for NIKHEF parallel kinematics [19, 20] as function of p_m . NIKHER data for beam energy $E_{beam} = 304 - 521$ MeV, proton kinetic energy $T_p = 89 - 99$ MeV and Q^2 is varied.

as a whole, i.e., from the $1p$ and $1s$ shells.

The reduced exclusive cross sections for the removal of nucleons from $1p_{1/2}$ and $1p_{3/2}$ shells in $^{16}\text{O}(e, e'p)^{15}\text{N}$, $^{16}\text{O}(\nu, \mu^-p)^{15}\text{O}$, and $^{16}\text{O}(\bar{\nu}, \mu^+n)^{15}\text{N}$ reactions calculated within the RDWIA [9] are shown in Fig. 3 together with Saclay data. The cross sections were calculated using the Saclay kinematic conditions with the normalization factors of data examined presented in Ref. [35].

The reduced cross sections for the removal of protons from $1p = 1p_{1/2} + 1p_{3/2}$ shell in $^{16}\text{O}(e, e'p)^{15}\text{N}$ reaction together with Saclay data are shown in Fig. 4. Also shown are the calculated in the perpendicular Saclay kinematics the reduced cross sections for the knock out of protons from $1s$ ($\sigma_{red}(1s)$) and $1s+1p$ ($\sigma_{red}(1p + 1s)$) shells in the $^{16}\text{O}(e, e'p)^{15}\text{N}$

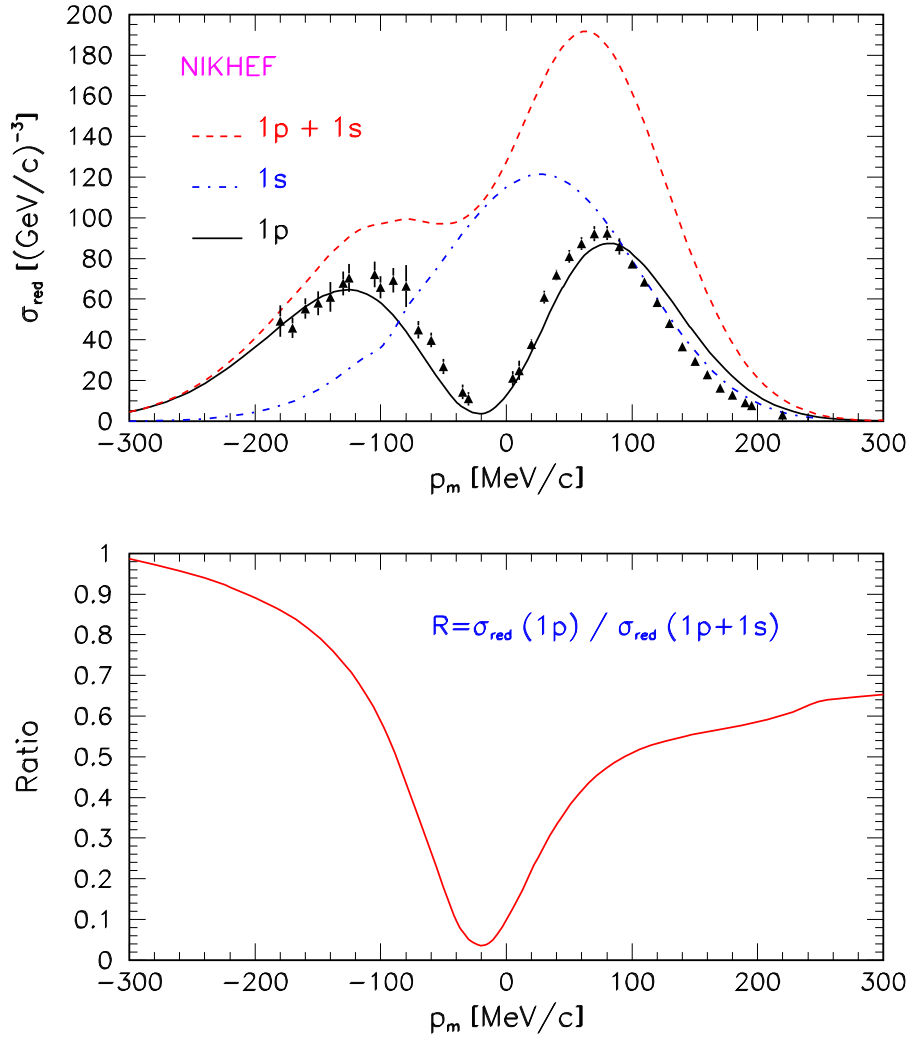


FIG. 6. The RDWIA electron reduced cross section [9] for the removal of protons from $1s$, $1p$, and $1s + 1p$ shells of ^{16}O for NIKHEF kinematics (upper panel) and ratio R_{1p} (lower panel) as functions of missing momentum. Also shown (upper panel) are Saclay data for the removal protons from $1p = 1p_{1/2} + 1p_{3/2}$ shell.

reaction, and a ratio $R_{1p} = \sigma_{red}(1p)/\sigma_{red}(1p + 1s)$ as functions of missing momentum p_m . The contribution of the $1p$ shell to the reduced cross section calculated with the GENIE models can be evaluated using this ratio as

$$\sigma_{red}^{MC}(1p) = R_{1p} \cdot \sigma_{red}^{MC}(1p + 1s), \quad (24)$$

where $\sigma_{red}^{MC}(1p + 1s)$ is determined by Eq.(23). In the following we compare $\sigma_{red}^{MC}(1p)$ with the measured reduced cross sections.

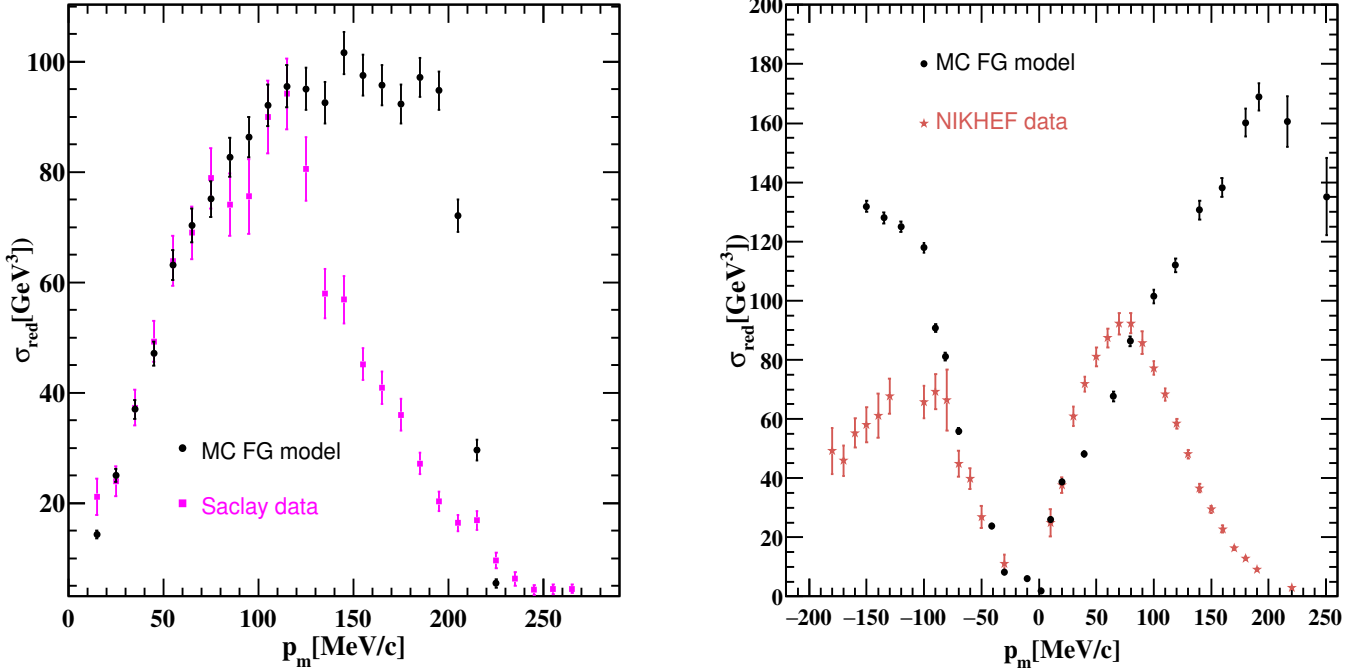


FIG. 7. Comparison of the GENIE F1802a (FG model) model calculation reduced cross sections with data as functions of missing momentum. As shown in key, cross sections were calculated for Saclay [18] and NIKHEF [19, 20] kinematics.

The reduced cross sections together with NIKHEF data are shown in Fig. 5. The cross sections were calculated using the normalization factors of data examined in Ref. [35]. The reduced cross sections for the knock out of protons from the $1p = 1p_{1/2} + 1p_{3/2}$ shell calculated in the parallel NIKHEF kinematics together with data are shown in Fig. 6. Also shown in this figure is the ratio R_{1p} as a function of missing momentum. This ratio is minimal at $p_m = 0$, where the contribution to the reduced cross section by removal nucleons from $1s$ shell is dominant and increases with missing momentum up to 0.6-0.7 at $p_m = 150$ MeV/c. This range corresponds to the knock out of nucleons from $1p$ shell where the reduced cross sections measured at Saclay and NIKHEF kinematics reach maximum value of about $80(\text{GeV}/c)^3$. There is an overall good agreement between calculated cross sections, but the value of electron cross sections at maximum is systematically higher (less than 10%) than (anti)neutrino ones with the exception of $1p_{1/2}$ state for Saclay kinematics. The small difference between neutrino and antineutrino reduced cross sections is due to the difference

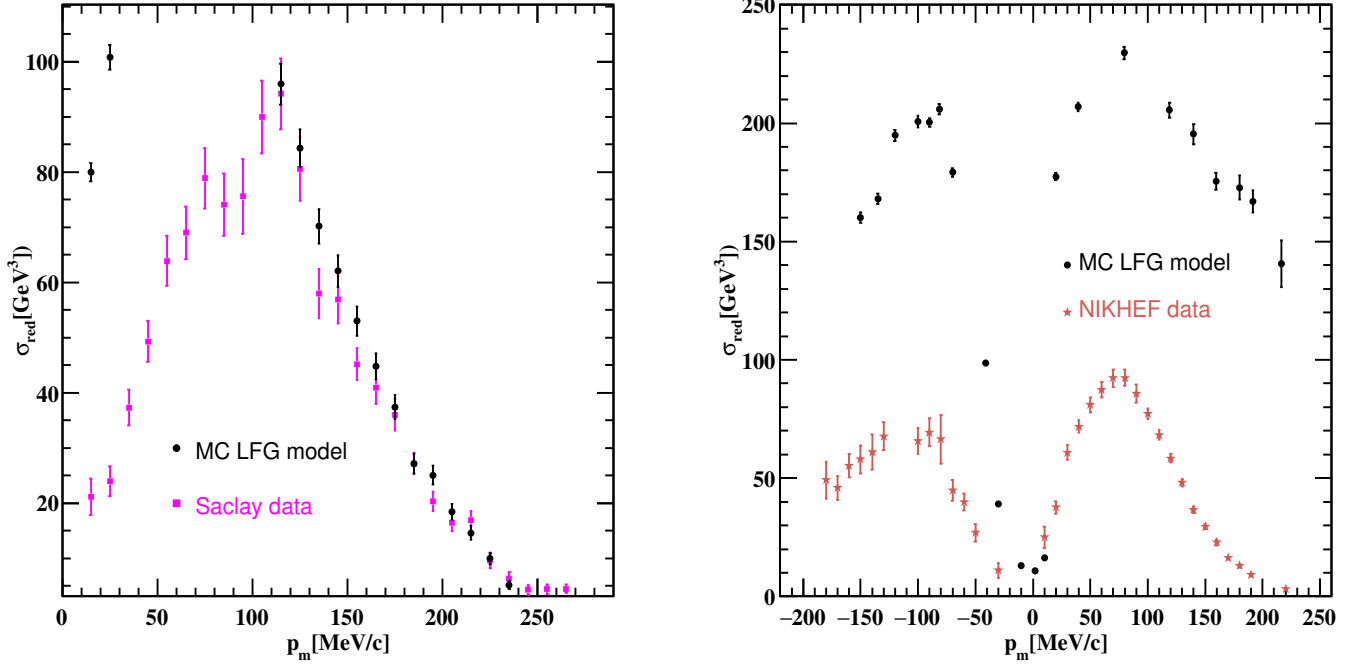


FIG. 8. Same as Fig. 7 but for the GENIE G18_10a (LFG model) model calculation.

in the FSI of proton and neutron with residual nucleus.

For each model considered we generated 10^8 CCQE neutrino events. The reduced cross sections calculated with the GENIE G18_02a model that uses the RFG nucleon momentum distribution are shown in Fig. 7 together with Saclay [18] and NUKHEF [19–21] data. The cross sections were calculated using the kinematic conditions of data examined that is presented in Tables I and II. At $|p_m| \leq 100$ MeV/c the values of calculated cross sections are in agreement with data and at higher missing momentum the calculated σ_{red} overestimate the measured ones significantly. This excess increases with missing momentum because of the uniform momentum distribution of the Fermi gas model.

The other features are observed in Fig. 8 where the comparison with data of the reduced cross sections calculated with the GENIE G18_10a model is presented. We observe agreement between GENIE predictions and Saclay data at $|p_m| \geq 120$ MeV/c. At low missing momentum the calculation overestimates data significantly. On the other hand the σ_{red} calculated within this GENIE model for NIKHEF parallel kinematics overestimate the measured electron scattering data in the range of $|p_m| \geq 50$ MeV/c. At $|p_m| \approx 100$ MeV/c

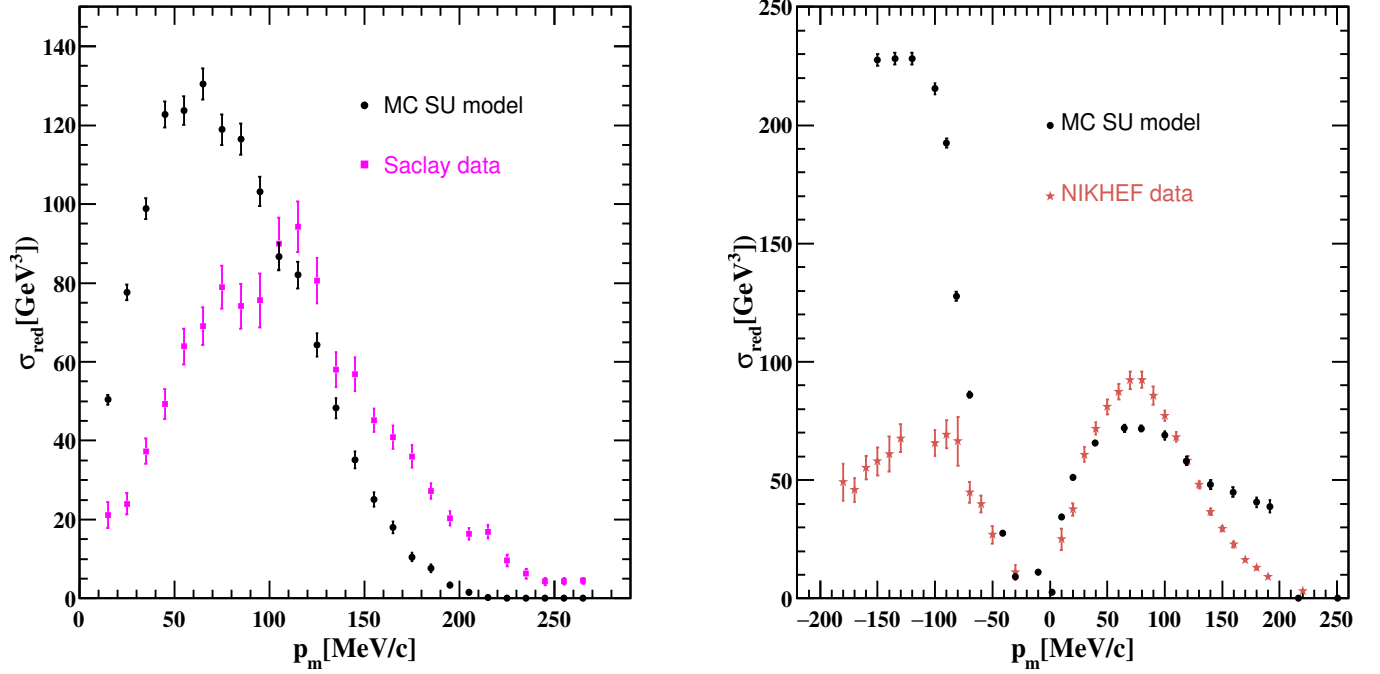


FIG. 9. Same as Fig. 7 but for the GENIE G21_11 (SuSAv2 model) model calculation.

where the maximum of the reduced cross section is observed the excess reaches up to 300%.

Figure 9 shows the same as Fig. 8 for the reduced cross sections calculated with the GENIE G21_11 model where the LFG nucleon momentum distribution is used as well. The σ_{red} calculated with this model for Saclay kinematics overestimates significantly measured cross sections at $p_m \leq 100$ MeV/c and underestimates the data in the range of $p_m \geq 150$ MeV/c. At NIKHEF kinematics the issue visible at $p_m \geq 150$ MeV/c resemble those for the G18_10a model seen in Fig. 8, but the excess is significant only in the range of $p_m \leq -100$ MeV/c. Approximately the same features are observed in Fig. 10, where the comparison with data of the reduced cross sections calculated with the GENIE effsf model is presented. The calculated cross sections overestimate the measured one at Saclay less than 60% at $p_m \leq 100$ MeV/c. The difference between the reduced cross sections calculated and measured at NIKHEF kinematics is significant at $|p_m| \geq 100$ MeV/c.

So, at Saclay perpendicular kinematics the agreement of the GENIE calculated reduced cross sections with data for neutrino scattering off oxygen and carbon [12] is approximately the same. For models considered (except the RFG), the issues are similar: the calculations

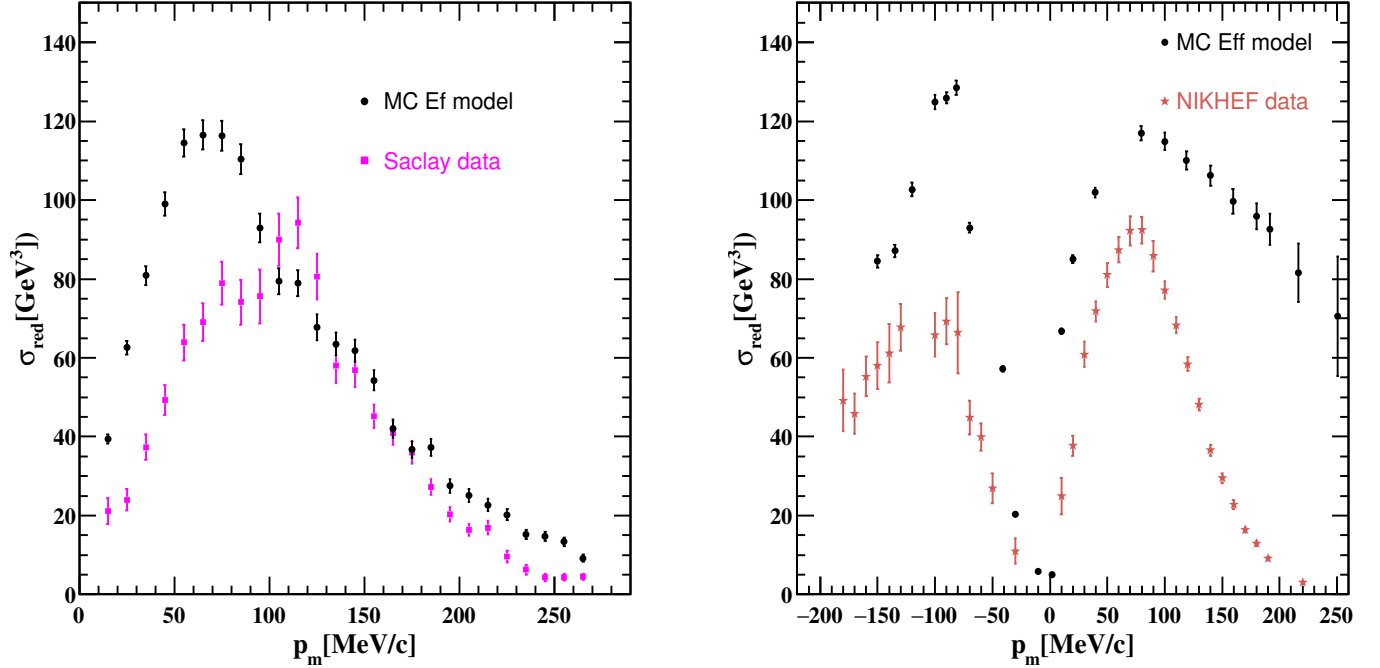


FIG. 10. Same as Fig. 7 but for the GENIE effsf model calculation.

overestimate data significantly at low missing momenta and underestimate them slightly (except SuSAv2 model) at $p_m \geq 100$ MeV/c. The σ_{red} calculated at NIKHEF parallel kinematics demonstrate absolutely different behavior (except the RFG model) than measured ones. We observe persistent disagreement between the GENIE predictions of the reduced cross sections and electron scattering data at low beam energy. On the other hand there is a good agreement between the RDWIA calculated and measured cross sections and this approach can be used to model both lepton-boson and boson-nucleus vertices [36]. It is obvious that this model describes better the semiexclusive $(l, l'p)$ lepton scattering process than the phenomenological models employed in the GENIE simulation framework.

IV. CONCLUSIONS

In this article, we carried out systematic comparison of the CCQE reduced cross sections calculated with models employed in the GENIE version 3 simulation framework with data measured in electrons scattering off the oxygen target. The reduced cross sections as func-

tions of missing momentum were measured only for the removal protons from $1p$ shells of ^{16}O as whole, i.e. from the $1p + 1s$ shells. The reduced cross sections for the removal proton from the $1p$ shells calculated with the GENIE models were evaluated using the relative contribution of the $1p$ shells R_{1p} predicted within the RDWIA model at Saclay and NIKHEF kinematics. We have observed persistent disagreement between the GENIE predictions and electron scattering data for reduced cross sections. The σ_{red} calculated at NIKHEF parallel kinematics demonstrate absolutely different behavior than measured ones. At Saclay perpendicular kinematics the agreement with data of reduced cross sections calculated within the GENIE v.3 simulation framework for neutrino scattering on oxygen and carbon targets is similar: the calculations overestimate data at low missing momenta and underestimate data at $p_m \geq 100$ MeV/c. Therefore the GENIE event generator can not simulate well two tracks CCQE events in all allowed kinematics region. In generators of neutrino event needs to use more sophisticated models to simulate CCQE semiexclusive processes. The direct comparison of spectral functions, implemented in neutrino event generators, with the precise electron reduced cross section data is original and promising method for testing the quality of nuclear physics models implemented in Monte Carlo generators for neutrino interaction.

V. DATA AVAILABILITY

The data that support the finding of this article are openly available [18–20]

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