Entanglement inequalities, black holes and the architecture of typical states

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Using holographic realizations of the Araki-Lieb (AL) inequality, we show that typical pure states in large N holographic CFTs possess two characteristic length scales determined solely by energy and conserved charges: a microscopic $L_{\rm UV}$ and an infrared $L_{\rm IR} > L_{\rm UV}$. Degrees of freedom between these scales effectively factorize — one purifying the ultraviolet (scales $< L_{\rm UV}$) and the other the infrared sector (scales $> L_{\rm IR}$). Remarkably, the pure state factor including the ultraviolet sector is determined only by the energy and conserved charges up to exponentially suppressed corrections. Our results imply that all black holes in anti-de Sitter space can be isolated from an asymptotic region, the corona, that is formed by the inclusion of entanglement wedges for which the AL inequality is saturated, and an effective factorization emerges in the buffer region between the corona and the outer horizon.

Introduction:- Thermalization of isolated quantum systems is best explained by quantum statistical properties of typical states, as particularly enunciated by the eigenstate thermalization hypothesis (ETH) [1, 2]. The holographic (AdS/CFT) duality, which maps strongly interacting large N conformal field theories (CFTs) to classical gravitational theories in one higher dimensional anti-de Sitter space (with generalizations extending to non-conformal field theories) [3], describes thermalization in terms of the ubiquitous process of black hole formation from typical initial conditions [4, 5]. It is therefore pertinent to ask whether one can extract the architecture of typical states in large N holographic theories from the universal geometry of black holes.

The holographic duality has recently led to a major progress [6–11] in resolving the black hole information loss paradox by demonstrating how the semi-classical black hole geometry can reproduce the Page curve [12, 13] (the time-dependence of the entanglement entropy of Hawking radiation which is consistent with unitarity) using holographic prescriptions for computing entanglement [14–16]. Microstate models [17, 18] have also been used to deepen our understanding of how black holes encode quantum information in consistency with the black hole complementarity principle [19, 20], which states that the validity of the semi-classical black hole geometry for an infalling observer requires that information of infalling matter is present both inside and outside an old black hole. These results implicitly assume an effective factorization of the Hilbert space which can isolate the black hole from an asymptotic observer. Although there are arguments [21] which rescue these current developments from criticisms [22–27] against such a factorization, the statistical nature of typical states can explain the emergence of the semi-classical factorization in the bulk.

In this letter, we use necessary and sufficient conditions for realizing saturation of entanglement inequalities in non-extremal black hole geometries to deduce quantum statistical properties of typical states in large N holographic CFTs with fixed energy and momentum. We also demonstrate the emergence of the split property which isolates the black hole from the asymptotic observer. Although we focus on two-dimensional CFTs, our basic claims generalize to higher dimensions and also generalize some statistical features of ETH.

Entanglement inequalities and their saturation:- We begin with a review of entanglement inequalities, and necessary and sufficient conditions for their saturation. Consider a state ρ^{AB} in a bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ with A and B denoting subsystems. $S(AB)$ denotes the von Neumann entropy of ρ^{AB} i.e. $S(AB) = -\mathrm{Tr}(\rho^{AB} \ln \rho^{AB})$. The reduced density matrix of A is defined as $\rho^A = \mathrm{Tr}_B(\rho^{AB})$ with partial trace in $\mathcal{H}_B$, and $S(A)$, the entanglement entropy of A is the von Neumann entropy of $\rho(A)$. Similarly, we can define ρ^B and $S(B)$. The Araki-Lieb inequality [28–30] states that

$$|S(A) - S(B)| \leq S(AB). \tag{1}$$

for any state ρ^{AB} of the bipartite system. The condition for saturation of the inequality is related to other inequalities involving $J(A|B)$ and the quantum discord $D(A|B)$, which are measures of classical and quantum correlations between the two subsystems in the pure/mixed state ρ^{AB}, respectively, that are invariant under local unitary operations [31–34]. Firstly,

$$J(A|B) = \max_{\{E_i^B\}} \left[S(A) - \sum_i p_i S(\rho_i^A) \right] \tag{2}$$

where the maximization is performed over all possible positive operator valued measures $\{E_i^B\}$ in $\mathcal{H}_B$ corresponding to measurements performed on B with $\rho_i^A = \mathrm{Tr}_B(E_i^B \rho^{AB})$ denoting the post measurement state corresponding to the ith outcome of the measurement which has the probability $p_i = \mathrm{Tr}(E_i^B \rho^{AB})$. The quantum mutual information of ρ^{AB} is defined as

$$I(A:B) = S(A) + S(B) - S(AB) \tag{3}$$

is symmetric in A and B, and equals to $2S(A) = 2S(B)$ iff ρ^{AB} is a pure state. The quantum discord $D(A|B)$ defined as

$$D(A|B) = I(A:B) - J(A|B) \tag{4}$$

is generically asymmetric but symmetric and $\frac{1}{2}I(A:B)$ for pure states. It satisfies the inequality

$$0 \le D(A|B) \le S(B) \tag{5}$$

for any bipartite state ρ^{AB} [35, 36]. The inequality

$$D(A|B) + J(C|B) \le S(B), \tag{6}$$

is valid for any tripartite state ρ^{ABC}, and is saturated iff ρ^{ABC} is pure, i.e. iff the subsystem C purifies ρ^{AB} [37, 38].

The upper bound on quantum discord is directly linked to the saturation of the Araki-Lieb inequality: the equality $D(A|B) = S(B)$ holds if and only if $S(A) - S(B) = S(AB)$. Obviously, this is realized only if B, the subsystem on which measurements are performed, has entropy not larger than A. Both maximal discord and saturation of the Araki-Lieb inequality occur iff $\mathcal{H}_A$ factorizes as $\mathcal{H}_A = \mathcal{H}_M \otimes \mathcal{H}_N$ such that

$$\rho^{AB} = \rho^M \otimes |\psi\rangle^{NB} \langle\psi| \tag{7}$$

where ρ^M is a (not necessarily pure) state in $\mathcal{H}_M$ and $|\psi\rangle^{NB} \langle\psi|$ is a *pure state* in $\mathcal{H}_N \otimes \mathcal{H}_B$, i.e. ρ^B is necessarily purified by a subsystem of A [34, 38].

We readily note that for the state (7), we obtain

$$\begin{aligned} D(B|A) &= D(A|B) = S(B) = S(A^N) \\ &= J(B|A) = J(A|B) = \frac{1}{2}I(A:B) \end{aligned} \tag{8}$$

where $S(A^N) = S(B)$ is the von Neumann entropy of $\mathrm{Tr}_B(|\psi\rangle^{NB} \langle\psi|)$ [38]. Note that for the state (7), both the quantum discord and classical correlations are symmetric in A and B.

For a state (7) saturating the Araki-Lieb inequality, let C be its purifier. Such a pure state admits a quantum Markov chain [39] structure,

$$\rho^{ABC} = |\tilde{\psi}\rangle^{CM} \langle\tilde{\psi}| \otimes |\psi\rangle^{NB} \langle\psi|, \tag{9}$$

with C purifying ρ^M. Using (8), we obtain from saturation of (6) that

$$\begin{aligned} &J(C|B) = D(C|B) = 0, \\ &J(C|A) = D(C|A) = S(AB) = S(C). \end{aligned} \tag{10}$$

The state (9) also saturates strong subadditivity as

$$S(CA) + S(AB) = S(A) + S(ABC) = S(A). \tag{11}$$

The BTZ black hole:- Consider a two-dimensional holographic CFT on a circle of radius L with equal central charge c for the left and right movers. $L_{n,+}$ ($L_{n,-}$) are the Virasoro generators for the left (right) movers, respectively. The thermal state with temperature β^{-1} and chemical potential $\beta\Omega$ for the momentum is given by

$$\rho = \frac{e^{-\beta(H-\Omega P)}}{\mathrm{Tr}\big(e^{-\beta(H-\Omega P)}\big)} = \frac{e^{-\frac{\beta_+}{L}\left(L_{0,+}-\frac{c}{24}\right)}e^{-\frac{\beta_-}{L}\left(L_{0,-}-\frac{c}{24}\right)}}{\mathrm{Tr}\left(e^{-\frac{\beta_+}{L}\left(L_{0,+}-\frac{c}{24}\right)}e^{-\frac{\beta_-}{L}\left(L_{0,-}-\frac{c}{24}\right)}\right)}, \tag{12}$$

where the Hamiltonian and momentum are

$$H = \frac{1}{L}\Big(L_{0,+} + L_{0,-} - \frac{c}{12}\Big), \qquad P = \frac{1}{L}\Big(L_{0,+} - L_{0,-}\Big),$$

and

$$\beta = \frac{\beta_+ + \beta_-}{2}, \qquad \Omega = \frac{\beta_- - \beta_+}{\beta_- + \beta_+} \tag{13}$$

give the temperatures β_+^{-1} (β_-^{-1}) of the left (right) movers. This state is holographically dual to the Bañados-Teitelboim-Zanelli (BTZ) black hole [40]

$$\mathrm{d}s^2 = -\frac{\ell^2}{r^2 r_+^2 r_-^2}(r_+^2 - r^2)(r_-^2 - r^2)\,\mathrm{d}t^2 + \frac{\ell^2 r_+^2 r_-^2}{r^2(r_+^2 - r^2)(r_-^2 - r^2)}\,\mathrm{d}r^2 + \frac{\ell^2}{r^2}\left(L\,\mathrm{d}\phi - \frac{r^2}{r_+ r_-}\,\mathrm{d}t\right)^2, \tag{14}$$

which is locally AdS_3 spacetime with radius ℓ and with the conformal boundary at $r = 0$. For $r_+ < r_-$, the outer and inner horizons are at r_+ and r_-, respectively. The CFT central charge is given by $c = 3\ell/2G_N$ (with large c implying large N), where G_N is the three-dimensional Newton constant. Let

$$\mu_\pm = \frac{1}{2}\left(\frac{1}{r_+} \pm \frac{1}{r_-}\right), \tag{15}$$

and we set $\ell = 1$ henceforth. With holographic renormalization [41, 42] we can extract the known non-vanishing components of the thermal stress tensor of the dual CFT with $\beta_\pm = \pi/\mu_\pm$. The thermal energy and momentum

$$E = \langle H\rangle = \frac{cL}{6}(\mu_+^2 + \mu_-^2),\ J = \langle P\rangle = \frac{cL}{6}(\mu_+^2 - \mu_-^2), \tag{16}$$

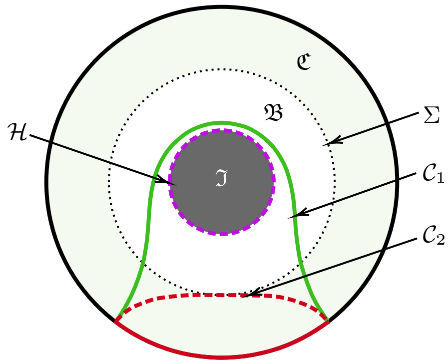

FIG. 1. The bulk geodesics ending on the boundary interval R (red) are $\mathcal{C}_1$ (green) and $\mathcal{C}_2$ (dashed red). $\mathcal{C}_1 \cup \mathcal{H}$ and $\mathcal{C}_2$ are homologous to R with $\mathcal{H}$ the outer horizon (dashed magenta). $\mathcal{C}_1$ and $\mathcal{C}_2 \cup \mathcal{H}$ are homologous to $\overline{R}$ (black), the complement of R. Here the BTZ black hole shown with compactified radial coordinate $r \to \cot\rho$ has $\mu_+ = \mu_- = 0.25$ (AdS radius $\ell = 1$).

correspond to the mass and momentum of the BTZ black hole whose horizon has Hawking temperature β and rotation Ω

$$\beta = \frac{\pi(\mu_+ + \mu_-)}{2\mu_+\mu_-}, \qquad \Omega = \frac{\mu_+ - \mu_-}{\mu_+ + \mu_-} \tag{17}$$

matching (13) also with $\beta_\pm = \pi/\mu_\pm$. The Bekenstein-Hawking entropy,

$$S_{BH} = S_L + S_R, \quad S_L = \frac{c}{3}\pi L\mu_+, \quad S_R = \frac{c}{3}\pi L\mu_-, \tag{18}$$

of the BTZ black hole reproduces the quantum statistical entropy in the CFT in the large c limit [43, 44]. For later convenience, we define

$$S_< = \min(S_L, S_R), \tag{19}$$

so that $S_< = S_{BH}/2$ for $\Omega = 0$ (no rotation). It is necessary that $T^{-1} = \beta < 2\pi L$ for the BTZ black hole (14) to represent the thermal ensemble (12) [45].

Results of numerical relativity [46–49] and analytic solutions [50–53] suggest that BTZ black holes are formed as a result of gravitational collapse for generic initial conditions, at least when the momentum is small compared to the energy. This suggests that the BTZ black hole can quantify the statistical features of typical states.

Holographic entanglement, the corona and the buffer:- In a large N holographic CFT, the entanglement entropy of a boundary subregion R is given by the Hubeny-Rangamani-Ryu-Takayanagi (HRRT) prescription, which equates $S(R)$ to $1/(4G_N)$ times the area of co-dimension two bulk extremal surface(s) of least area and homologous to R [14, 15]. This set of extremal surface(s) may have disconnected components but one must be anchored to ∂R. The entanglement wedge $E_W(R)$ is the bulk domain of dependence of a spatial region bounded by R and its HRRT surface(s). Bulk operators in $E_W(R)$ can be reconstructed from CFT operators in R, and this is called *entanglement wedge reconstruction* [11, 54–58].

For the BTZ geometry (14) whose boundary is a circle of length $2\pi L$, consider an interval R of length $l < 2\pi L$. The homology condition admits two competing configurations of extremal co-dimension two surfaces (curves): the geodesic $\mathcal{C}_1$, and a disconnected set consisting of the geodesic $\mathcal{C}_2$ together with the outer horizon $\mathcal{H}$ as shown in Fig. 1. The HRRT prescription yields

$$\begin{aligned} S(R) &= \frac{1}{4G_N}\min\left[\mathbf{L}(\mathcal{C}_2), \mathbf{L}(\mathcal{C}_1 \cup \mathcal{H})\right], \\ S(\overline{R}) &= \frac{1}{4G_N}\min\left[\mathbf{L}(\mathcal{C}_1), \mathbf{L}(\mathcal{C}_2 \cup \mathcal{H})\right], \end{aligned} \tag{20}$$

where $\mathbf{L}(C)$ denotes the length of a curve C and $\overline{R}$ is the complement of R. Let us define

$$\mathcal{P} = \frac{|S(R) - S(\overline{R})|}{S(R \cup \overline{R})} = \frac{|S(R) - S(\overline{R})|}{S_{BH}} \tag{21}$$

with $S(R \cup \overline{R}) = S_{BH} = \mathbf{L}(\mathcal{H})/(4G_N)$ given by (18).

Generically, for any BTZ geometry, there are three cases as shown in the Table I [45]. Note that $\mathcal{P}$ is symmetric under exchanging R and $\overline{R}$, and so $\mathcal{P} = 0$ for $l = \pi L$ ($\alpha^* \geq 0.5$ with equality only for global AdS where $\mu_\pm = 0$; note however that $\beta < 2\pi L$ and so we cannot take the global AdS limit). We readily note that

$l = \mathbf{L}(R)$	$S(R)$	$S(\overline{R})$	$\mathcal{P}$
$0 < l < 2\pi L(1-\alpha^*)$	$\frac{\mathbf{L}(\mathcal{C}_2)}{4G_N}$	$\frac{\mathbf{L}(\mathcal{C}_2)}{4G_N} + S_{BH}$	1
$2\pi L(1-\alpha^*) \leq l < 2\pi L\alpha^*$	$\frac{\mathbf{L}(\mathcal{C}_2)}{4G_N}$	$\frac{\mathbf{L}(\mathcal{C}_1)}{4G_N}$	< 1
$2\pi L\alpha^* \leq l \leq 2\pi L$	$\frac{\mathbf{L}(\mathcal{C}_1)}{4G_N} + S_{BH}$	$\frac{\mathbf{L}(\mathcal{C}_1)}{4G_N}$	1

TABLE I. Three cases of holographic entanglement in a generic BTZ geometry.

$$\alpha_{\rm UV} = 1 - \alpha^* = \max\left(\frac{\mathbf{L}(R)}{2\pi L}\,\middle|\, S(\overline{R}) = S(R) + S(R \cup \overline{R})\right). \tag{22}$$

Thus the microscopic scale $L_{\rm UV}$ defined as

$$L_{\rm UV} := 2\pi\alpha_{\rm UV}L \tag{23}$$

is the *largest* boundary interval whose entanglement wedge does *not* include the outer horizon, and for which the Araki-Lieb inequality is saturated ($\mathcal{P} = 1$). For any BTZ black hole, $\alpha_{\rm UV}$ (and thus $L_{\rm UV}$) can be obtained as a function of $\mu_+ L$ and $\mu_- L$ by solving

$$\begin{aligned} &\ln\Big[\sinh(2\pi\mu_+ L\alpha_{\rm UV})\sinh(2\pi\mu_- L\alpha_{\rm UV})\Big] + 2\pi L(\mu_+ + \mu_-) \\ &= \ln\Big[\sinh(2\pi\mu_+ L(1-\alpha_{\rm UV}))\sinh(2\pi\mu_- L(1-\alpha_{\rm UV}))\Big]. \end{aligned} \tag{24}$$

The above equation can be obtained readily from explicit computations of the renormalized lengths of the geodesics $\mathcal{C}_1$ and $\mathcal{C}_2$ via uniformization maps [15, 59].

We define the corona $\mathfrak{C}$ as the asymptotic region of the BTZ black hole that contains *all* entanglement wedges corresponding to boundary subregions R with length $l \leq L_{\rm UV}$. The buffer $\mathfrak{B}$ is the region between the outer horizon $\mathcal{H}$ and Σ, the inner boundary of the corona and the envelope of all entanglement wedges corresponding to subregions R with length $l = L_{\rm UV}$ (see Fig. 1).

Stringy and quantum gravity effects should modify $L_{\rm UV}$, and so the extent of the corona perturbatively in inverse powers of the (gauge) coupling constant of the dual field theory and N, respectively. However, the corona $\mathfrak{C}$ can still be defined in the perturbative expansion as the union of entanglement wedges of microscopic intervals for which the Araki-Lieb inequality is saturated.

The architecture of a typical state:- Consider typical states of the dual holographic CFT living on a circle of radius L in the Hilbert space $\mathcal{H}_{E,J}$ spanned by the eigenstates of the energy and momentum with eigenvalues E and J, respectively, coinciding with those of a BTZ black hole. The degrees of freedom of the CFT are naturally split into three sectors according to their wavelengths λ: (i) the ultraviolet (UV) sector $\lambda \leq L_{\rm UV}$ (ii) the intermediate (IM) sector $L_{\rm UV} < \lambda \leq L_{\rm IR}$ with $L_{\rm IR} = \beta$, the thermal wavelength and (iii) the infrared (IR) sector $L_{\rm IR} < \lambda \leq 2\pi L$. We denote the three factors by $\mathcal{H}_{\mathfrak{C}}$ (UV sector), $\mathcal{H}_{\mathfrak{B}}$ (IM sector), and $\mathcal{H}_{\mathfrak{I}}$ (IR sector) which correspond to the corona $\mathfrak{C}$, the buffer $\mathfrak{B}$ and the black hole interior $\mathfrak{I}$ regions of the BTZ black hole, respectively. This naturally reflects how the radial coordinate encapsulates the energy scale of the dual field theory [60–68]. The split $\mathcal{H}_{E,J} = \mathcal{H}_{\mathfrak{C}} \otimes \mathcal{H}_{\mathfrak{B}} \otimes \mathcal{H}_{\mathfrak{I}}$ holds by the virtue of locality of the semi-classical effective bulk theory. Clearly, $L_{\rm UV}$ and $L_{\rm IR}$ are determined solely by the conserved energy E and momentum J.

Consider a typical state $|\psi\rangle$ in $\mathcal{H}_{E,J}$. We expect that the union of the corona and the buffer comprising of the geometry exterior to the outer horizon of the BTZ black hole, which can be trusted in the semi-classical large N limit, should describe the quantum statistical properties of the marginal

$$\rho^{\mathfrak{B}\mathfrak{C}} = \mathrm{Tr}_{\mathfrak{I}}(|\psi\rangle\langle\psi|). \tag{25}$$

Based on our previous discussion, we will prove that the saturation of the Araki-Lieb inequality requires the existence of an effective factorization of $\mathcal{H}_{\mathfrak{B}}$, the buffer Hilbert space corresponding to intermediate wavelengths, in the form $\mathcal{H}_{\mathfrak{B}} = \mathcal{H}_{\mathfrak{L}} \otimes \mathcal{H}_{\mathfrak{K}}$ such that the factor $\mathcal{H}_{\mathfrak{K}}$ purifies the UV sector $\mathcal{H}_{\mathfrak{C}}$ represented by the corona $\mathfrak{C}$. Furthermore, a typical state $|\psi\rangle$ should be of the form

$$|\psi\rangle = \left(\sum_i \sqrt{p_i}\, |\tilde{\chi}_i^{\mathfrak{I}}\rangle \otimes |\chi_i^{\mathfrak{L}}\rangle\right) \otimes |\phi\rangle^{\mathfrak{K}\mathfrak{C}} + \mathcal{O}(e^{-S_<}), \tag{26}$$

where

- $|\tilde{\chi}_i^{\mathfrak{I}}\rangle \in \mathcal{H}_{\mathfrak{I}}$ and $|\chi_i^{\mathfrak{L}}\rangle \in \mathcal{H}_{\mathfrak{L}}$ form complete orthogonal bases of $\mathcal{H}_{\mathfrak{I}}$ and $\mathcal{H}_{\mathfrak{L}}$, repectively,
- $|\phi\rangle^{\mathfrak{K}\mathfrak{C}} \in \mathcal{H}_{\mathfrak{K}} \otimes \mathcal{H}_{\mathfrak{C}}$ is *independent* of $|\psi\rangle \in \mathcal{H}_{E,J}$,
- p_i is a probability distribution ($0 \leq p_i \leq 1$ and $\sum_i p_i = 1$), and also *independent* of $|\psi\rangle \in \mathcal{H}_{E,J}$, and
- $S_<$ denotes the entropy defined in (19).

We therefore claim that in the large N limit, the state dependence of the architecture of a typical state is solely in the Schmidt bases of the factor in $\mathcal{H}_{\mathfrak{I}} \otimes \mathcal{H}_{\mathfrak{L}}$ as the probabilities p_i and UV pure state factor $|\phi\rangle$ are determined *solely* by the conserved charges (energy and momentum). We validate these claims as follows.

Firstly, tracing over $\mathcal{H}_{\mathfrak{I}}$ yields

$$\rho^{\mathfrak{B}\mathfrak{C}} = \left(\sum_i p_i\, |\chi_i^{\mathfrak{L}}\rangle\langle\chi_i^{\mathfrak{L}}|\right) \otimes |\phi\rangle^{\mathfrak{K}\mathfrak{C}}\langle\phi| + \mathcal{O}(e^{-S_<}). \tag{27}$$

For any such state $\rho^{\mathfrak{B}\mathfrak{C}}$, $S(\mathfrak{B}) = S(\mathfrak{C}) + S(\mathfrak{B}\cup\mathfrak{C})$, with $S(\mathfrak{C}) = S(\mathrm{Tr}_{\mathfrak{K}}|\phi\rangle^{\mathfrak{K}\mathfrak{C}}\langle\phi|) = S(\mathrm{Tr}_{\mathfrak{C}}|\phi\rangle^{\mathfrak{K}\mathfrak{C}}\langle\phi|)$, and $S(\mathfrak{B}\cup\mathfrak{C}) = S_{BH}$, the BTZ black hole entropy (18) if $S_{BH} = -\sum_i p_i \ln p_i$. The black hole entropy emerges from tracing out the complex infrared degrees of freedom which cannot be probed by simple operators at the boundary. The expectation that the entanglement spectrum of $\rho^{\mathfrak{B}\mathfrak{C}}$ should be reproduced by the semi-classical black hole geometry (as in the case of Hawking radiation) suggests that the probabilities p_i are state-independent.

Secondly, (26) implies that the asymptotic corona region $\mathfrak{C}$ of the BTZ black hole should receive only $\mathcal{O}(e^{-S_<})$ corrections just like the pure state factor $|\phi\rangle$ that includes the UV sector when the black hole is non-extremal (i.e. when $S_< \neq 0$). As expectation values of operators such as the energy-momentum tensor are extracted from the asymptotic behavior of dual fields in the geometry [41, 42], it follows that these should coincide with the thermal expectation values up to $\mathcal{O}(e^{-S_<})$ reproducing the implication of ETH for vanishing J (as in this case $S_< = S/2$ with $S = S_{BH}$, the total entropy) and generalizes it for non-vanishing J.

Thirdly, to explain saturation of the Araki-Lieb inequality, consider a boundary interval R of length $l \leq L_{\rm UV}$. Its entanglement wedge $E_W(R)$ lies within the corona of the BTZ black hole. By virtue of bulk locality, we can factorize $\mathcal{H}_{\mathfrak{C}}$ as $\mathcal{H}_{\mathfrak{C}} = \mathcal{H}_{\mathcal{E}} \otimes \mathcal{H}_{\overline{\mathcal{E}}}$, with $\mathcal{H}_{\mathcal{E}}$ and $\mathcal{H}_{\overline{\mathcal{E}}}$ corresponding to $E_W(R)$ and its complement in the corona $\mathfrak{C}$ ($\overline{E}_W(R) \cap \mathfrak{C}$), respectively. The state (27) can be written as

$$\rho^{\mathfrak{B}\mathfrak{C}} = \left(\sum_i p_i\, |\chi_i^{\mathfrak{L}}\rangle\langle\chi_i^{\mathfrak{L}}|\right) \otimes |\phi\rangle^{\widetilde{\mathfrak{K}}\mathcal{E}}\langle\phi| + \mathcal{O}(e^{-S_<}), \tag{28}$$

where $\mathcal{H}_{\widetilde{\mathfrak{K}}} = \mathcal{H}_{\mathfrak{K}} \otimes \mathcal{H}_{\overline{\mathcal{E}}}$. (Note that $\mathcal{H}_{\mathfrak{C}} \otimes \mathcal{H}_{\mathfrak{K}} = \mathcal{H}_{\mathcal{E}} \otimes \mathcal{H}_{\widetilde{\mathfrak{K}}}$.) The necessary and sufficient condition for the saturation of the Araki-Lieb inequality is thus realized by $\rho^{\mathfrak{B}\mathfrak{C}}$ for an arbitrary boundary interval of length $l \leq L_{\mathrm{UV}}$ up to exponentially suppressed corrections.

Finally, the suppression factor $e^{-S_<}$ in (26) can be derived from the argument that the coherent state independent pure state factor $|\phi\rangle\langle\phi|$ of $\rho^{\mathfrak{B}\mathfrak{C}}$ including the UV degrees of freedom should determine quantum coherence in a typical $\rho^{\mathfrak{B}\mathfrak{C}}$. As shown in the End Matter, we can define the quantum coherence length L_{QC} from quantum mutual information of disjoint regions of arbitrary lengths, and argue that the ratio $(L_{\mathrm{UV}} - L_{\mathrm{QC}})/L_{\mathrm{UV}}$ should estimate the corrections to the leading term in (26). For $S_< \neq 0$ and large, we find that

$$\frac{L_{\mathrm{UV}} - L_{\mathrm{QC}}}{L_{\mathrm{UV}}} \sim k e^{-S_<} \tag{29}$$

with k a constant. When $S_< = 0$, the black hole is extremal and then $L_{\mathrm{UV}} - L_{\mathrm{QC}} = \tilde{k}\tilde{\mu}^{-1}$ with $\tilde{k} \approx 0.33$ and $\tilde{\mu}$ being the non-vanishing element of the set $\{\mu_+, \mu_-\}$. Therefore, the exponential suppression is realized only for non-extremal black holes.

The $\mathcal{O}(e^{-S_<})$ terms in (26) can be interpreted as contributions arising from *wormholes* representing entangled pairs of Hawking quanta (with one in the interior and another in the corona) [69] that are entropically enhanced by $e^{S_<}$ factor after Page time during Hawking radiation so that their suppression is removed [8, 10, 11]. Heuristically, the $e^{S_<}$ enhancement is due to the independence of the CFT left and right movers and appears the earliest post Page time.

Discussion:- Based on the necessary and sufficient conditions for the saturation of the Araki-Lieb inequality for boundary intervals of length $l \leq L_{\mathrm{UV}}$ realized in the BTZ black hole geometry for a two-dimensional large N holographic CFT, we have argued that a typical state with energy and momentum equal to that of a BTZ black hole should have the architecture given by (26). Both the microscopic length scale L_{UV} and the macroscopic length scale L_{IR} which define the ultraviolet, intermediate and infrared sectors depend only the conserved energy and momentum, and are represented by the corona, the buffer and the black hole interior regions of the BTZ black hole, respectively. Furthermore, an effective factorization of the buffer Hilbert space must exist with a factor purifying the corona in the same way for any state in the ensemble reproducing features of ETH.

The major implications of our result for any realization of non-extremal black hole microstates, as for instance in the fuzzball proposal [70], result from the identities (i) $J(\mathfrak{I}|\mathfrak{C}) = D(\mathfrak{I}|\mathfrak{C}) = \mathcal{O}(e^{-S_<})$ and (ii) $J(\mathfrak{I}|\mathfrak{B}) = D(\mathfrak{I}|\mathfrak{B}) = S_{BH} + \mathcal{O}(e^{-S_<})$ which follow from the replacements $C \mapsto \mathfrak{I}$, $B \mapsto \mathfrak{C}$, and $A \mapsto \mathfrak{B}$ in (10) as the state (9) has the same structure as (26). Particularly, $J(\mathfrak{I}|\mathfrak{C}) = D(\mathfrak{I}|\mathfrak{C}) = \mathcal{O}(e^{-S_<})$ (exponentially suppressed classical and quantum correlations between the black hole interior and the corona) suggests that the microstates of non-extremal BTZ black holes could be described by geometries where the interior of the BTZ black hole is replaced by singular sources, such as bound states of strings realized by multiway gravitational junctions [71], which do not affect the asymptotic geometry by usual corrections in powers of r/r_{H} (with r and r_{H} the radial coordinate and horizon radius, respectively). The equality $J(\mathfrak{I}|\mathfrak{B}) = D(\mathfrak{I}|\mathfrak{B}) = S_{BH} + \mathcal{O}(e^{-S_<})$ suggests that the non-extremal black hole microstates should support quantum hair in the near-horizon region, and should explain the equality of quantum and classical correlations between the black hole interior and the buffer quantitatively. These comments are consistent with the discussions in [72–77].

It should be interesting to investigate the dynamical architecture of typical states under quantum quenches generalizing (26). We can proceed by exploring how the corona behaves dynamically in BTZ-Vaidya geometries using the methodology of [59, 78, 79]. The dynamical evolution of the statistical suppression factor $\mathcal{O}(e^{-S_<})$ would be of particular interest, as it would be a novel result in quantum statistical mechanics. Similarly, given that our basic conclusions extend to rotating black holes in higher dimensions, it would be fascinating to investigate the statistical suppression factor in the presence of rotation in higher dimensions.

Furthermore, the identification of the asymptotic corona region (UV sector) that isolates the black hole can lead to rigorous application of holography to realistic strongly interacting systems [80, 81], especially via the semi-holographic approach [82–86] where the UV sector is replaced by perturbative degrees of freedom.

Finally, it would be interesting to understand how features of typical states implied by (26) can be reproduced by tensor networks, perhaps following the approaches in [87–89]. It is also necessary to understand better how quantum ergodicity that is necessary for the effective emergence of the canonical ensemble [90] is explicitly realized in holographic systems (see [91–93] for recent developments).

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END MATTER

Derivation of the suppression factor

To justify the $e^{-S_<}$ suppression in the ansatz for the typical state (26), we should compare two quantities that are expected to coincide only after ensemble averaging; their discrepancy indicates how deviations from typical behavior are suppressed. We propose that such a suitable comparison is between the ultraviolet length scale $L_{\rm UV}$ and the quantum correlation length $L_{\rm QC}$ defined below.

Consider two disjoint boundary intervals R_1 and R_2 separated by non-vanishing gaps of lengths $\mathbf{L}_{g1}$ and $\mathbf{L}_{g2}$. Denoting the boundary lengths of R_1 and R_2 as $\mathbf{L}(R_1)$ and $\mathbf{L}(R_2)$, the total boundary length satisfies $\mathbf{L}_{g1} + \mathbf{L}_{g2} + \mathbf{L}(R_1) + \mathbf{L}(R_2) = 2\pi L$ (see Fig. 2). We define the quantum correlation length $L_{\rm QC}$ as the largest possible value of $\min(\mathbf{L}_{g1}, \mathbf{L}_{g2})$ for which the mutual information $I(R_1 : R_2)$ remains non-vanishing:

$$L_{\rm QC} = \max(\min(\mathbf{L}_{g_1}, \mathbf{L}_{g_2})|\mathbf{L}_{g_1} + \mathbf{L}_{g_2} + \mathbf{L}(R_1) + \mathbf{L}(R_2) = 2\pi L \ \& \ I(R_1 : R_2) > 0). \tag{30}$$

In a typical state of the form (27), obtained after tracing out the long-wavelength modes represented by the black hole interior, the correlations between R_1 and R_2 arise through the coherent pure state factor $|\phi\rangle$ which includes the short-wavelength modes that are represented by the corona. We therefore expect that $L_{\rm QC}$ equals $L_{\rm UV}$ up to fluctuations around the ensemble average.

For a typical state, the value of $L_{\rm QC}$ can be computed using the BTZ geometry and the HRRT prescription [14, 15]. We find that $\min(\mathbf{L}_{g1}, \mathbf{L}_{g2})$ can take its maximum value when R_1 and R_2 have equal lengths and are placed symmetrically (so that $\mathbf{L}_{g1} = \mathbf{L}_{g2}$) for arbitrary $\mu_\pm$. Therefore, we obtain that $L_{\rm QC} = \pi\alpha_{\rm QC} L$ where $\alpha_{\rm QC}$ can be determined as a function of $\mu_+ L$ and $\mu_- L$ by solving the equation

$$\begin{aligned} &2\ln\Big[\sinh(\pi\mu_+ L\alpha_{\rm QC})\sinh(\pi\mu_- L\alpha_{\rm QC})\Big] \\ &+ 2\pi L(\mu_+ + \mu_-) \\ &= 2\ln\Big[\sinh(\pi\mu_+ L(1-\alpha_{\rm QC}))\sinh(\pi\mu_- L(1-\alpha_{\rm QC}))\Big] \end{aligned} \tag{31}$$

which can be derived via the uniformization maps for the appropriate geodesics (see Fig. 2) [15, 59]. This equation can be solved analytically for special values of the ratio of $\mu_</\mu_>$ (where $\mu_< = \min(\mu_+, \mu_-)$ and $\mu_> = \max(\mu_+, \mu_-)$). We find that $L_{\rm UV} > L_{\rm QC}$ and when $S_< \neq 0$ and large, we obtain (29) with k a function of $\mu_</\mu_>$. When $\mu_</\mu_> = 1$, then

$$k = \frac{1}{\ln 2} \approx 1.44$$

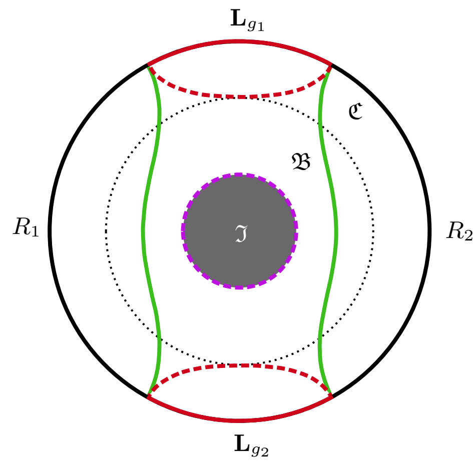

FIG. 2. Two non-overlapping intervals R_1 and R_2 at the boundary are shown above with two non-vanishing gaps $\mathbf{L}_{g1}$ and $\mathbf{L}_{g2}$ between them. The mutual information between R_1 and R_2 is non-vanishing if the bulk entanglement wedge is connected (the dotted region) bounded by the dotted red bulk geodesics and the outer horizon instead of the union of the two disconnected regions bounded by the green bulk geodesics. Here the BTZ black hole shown with compactified radial coordinate $r \to \cot\rho$ has $\mu_+ = \mu_- = 0.25$ (AdS radius $\ell = 1$).

and when $\mu_</\mu_> = 1/2$,

$$k = \frac{2}{5+\sqrt{5}}\frac{1}{\ln\left(\frac{1+\sqrt{5}}{2}\right)} \approx 0.57.$$

In the extremal case, where $S_< = 0$, we find that $L_{\rm UV} - L_{\rm QC} = \tilde{k}\tilde{\mu}^{-1}$, with $\tilde{k} \approx 0.33$ and $\tilde{\mu}$ is the non-vanishing member of the set $\{\mu_-, \mu_+\}$. We see that the exponential suppression from ensemble average is absent in the extremal case.

Explicit examples and the limit $L \to \infty$

For $\mu_+ = \mu_- = \mu$ (no rotation),

$$L_{\rm UV} = \frac{\operatorname{arccoth}[1 + 2\coth(2\pi\mu L)]}{\mu},$$
$$L_{\rm QC} = \frac{\operatorname{arccoth}[1 + 2\coth(\pi\mu L)]}{\mu}.$$

If we keep μ fixed and take the length of the boundary circle to infinity i.e. $L \to \infty$ (or equivalently $S_{BH} \to \infty$ at fixed β), then in this case

$$L_{\rm UV} = L_{\rm QC} = \frac{\ln 2}{2\mu} \sim \frac{0.35}{\mu}.$$

We recall that $L_{\rm IR} = \beta = \pi/\mu$ (see (17)), and therefore in the above limit $L_{\rm UV}/L_{\rm IR} = L_{\rm QC}/L_{\rm IR} = \ln 2/2\pi \sim 0.11$. Note that the entanglement wedge transition (when the

HRRT surface transits to the disconnected saddle) for a boundary interval of length l occurs when $l > 2\pi L - L_{\rm UV}$ which is infinite although $L_{\rm UV}$ and $L_{\rm QC}$ are both remarkably finite in this limit. Note that the corona is well defined in this limit as $L_{\rm UV}$ is finite. (Clearly in the $\mu \to 0$ limit at finite L (global AdS$_3$), we get $L_{\rm UV} = L\pi$ and $L_{\rm QC} = L\pi/2$.)

Generally, the corona is well defined even in the limit in which $L \to \infty$ at fixed μ_+ and μ_- ($S_{BH} \to \infty$ at fixed temperatures of the left/right movers) as in the non-rotating BTZ black hole case discussed above. As for instance, when $\mu_</\mu_> = 1/2$, we obtain that

$$L_{\rm UV} = L_{\rm QC} = \frac{\ln\left(\frac{1+\sqrt{5}}{2}\right)}{\mu_>} \sim \frac{0.48}{\mu_>}$$

while $L_{\rm IR} = \beta = 3\pi/2\mu_>$ (see (17)) and thus $L_{\rm UV}/L_{\rm IR} \sim 0.10$ in the limit $L \to \infty$ at fixed $\mu_>$.

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