

Near-extremal membranes in M-theory

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Abstract

We consider near-extremal membranes embedded in M-theory, consistently truncated to gauged $\mathcal{N} = 2$ supergravity in four dimensions on the coset space $M^{1,1,0}$. These are holographically dual to $2 + 1$ dimensional superconformal gauge theory with $U(1)_R \times U(1)_B$ global symmetry. Turning on the chemical potential to either the R -symmetry or the baryonic symmetry gives access to the quantum critical regime of the boundary gauge theory. We study perturbative stability of the extremal limit, and demonstrate that membranes with topological (baryonic) charge are free from all known instabilities. R -charged membranes are free from the superconducting instabilities, but have unstable charge transport and instabilities associated with the condensation of the axions.

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1 Introduction and summary

Holographic correspondence [1, 2] provides an interesting realization of quantum criticality: seemingly violating the third law of thermodynamics, a strongly coupled phase of matter in the limit of vanishing temperature $T \rightarrow 0$ has finite entropy density, while lacking the supersymmetry. In the gravitational dual, such phases are described by charged black branes in string theory/M-theory with non-supersymmetric extended extremal horizons. The best explored holographic example is that of the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) plasma in four spacetime dimensions. Here, the equilibrium states of the gauge theory plasma, with the same chemical potential μ for all $U(1)$ factors of the maximal Abelian subgroup $U(1)^3 \subset SU(4)$ R -symmetry, reach the quantum critical regime as $\frac{T}{\mu} \rightarrow 0$. In the gravitational dual, such states are represented by a Reissner–Nordström (RN) black brane in asymptotically AdS_5 spacetime.

A possible resolution of the extremal entropy paradox is the modification of the $T \rightarrow 0$ limit by the quantum gravity corrections [3]. As such corrections only become important at exponentially suppressed temperatures, it is natural to ask if a more mundane resolution exists. Indeed, one can only sensibly talk about quantum gravity

corrections in a consistent theory of quantum gravity — the string theory. Embedding extremal horizons in consistent string theory backgrounds (in the classical supergravity approximation) typically involves plethora of additional charged and neutral fields. It is then possible that the extremal limit is never reached due to various perturbative and non-perturbative instabilities, triggered by these spectator fields. Generic examples are the holographic superconducting instabilities [4], associated with the condensation of the operators (in the gravitational dual bulk scalars) charged under the global $U(1)$ symmetry that supports the extremal limit.¹ In the case of $\mathcal{N} = 4$ SYM such an operator is a chiral primary gaugino bilinear [5, 6].

More subtle examples are extremal horizons supported by gauge fields realizing “topological” global symmetries of the boundary gauge theory. Such holographic models arise from compactifications of string theory/M-theory on $AdS_{p+2} \times Y$ manifolds with nonzero p th Betti number b_p , leading to $U(1)^{b_p}$ “baryonic” global symmetry. Non-supersymmetric extremal quantum states supported by the baryonic $U(1)^{b_p}$ chemical potentials do not have superconducting instabilities. As an example, consider strongly coupled $\mathcal{N} = 1$ $SU(N) \times SU(N)$ gauge theory in four spacetime dimensions, the Klebanov–Witten (KW) model [7]. The theory has $U(1)_R \times U(1)_B$ global symmetry, which supports quantum critical states charged under either of the $U(1)$ s. The R -symmetry charged quantum critical states are unstable due to the condensation of the chiral primary $\mathcal{O}_F \equiv \text{Tr}(W_1^2 + W_2^2)$, where W_i are the gauge superfields corresponding to the two gauge group factors of $SU(N) \times SU(N)$ quiver [8]. The gauge-invariant operators of the KW theory charged under $U(1)_B$ have conformal dimensions of order² N , with the charge-to-mass ratio too small to trigger the superconducting instability [9]. Nonetheless, quantum critical states with a baryonic charge of the KW theory are unstable [10]: even though such states have zero R -symmetry charge density, at low temperatures R -charge starts “clumping”, breaking the homogeneity of $U(1)_B$ charged thermal equilibrium state.³

In this paper we study extremal horizons in a close analog of the KW model — a

¹We will use the term “superconducting” instability exclusively when *charged* bulk fields condense. We will also encounter the condensation of the (neutral) axions at, what we refer to as, “threshold” instabilities.

²The smallest such operators involve determinants of the bifundamental matter fields of the KW quiver gauge theory. This justifies the nomenclature “baryonic symmetry”.

³This is a direct consequence of the thermodynamic instabilities of the underlying thermal states [11]. For charged plasma this was originally explained in [12, 13].

membrane theory of Klebanov, Pufu and Tesileanu (KPT) [14]. The KPT model is a holographic example of a three dimensional superconformal gauge theory arising from compactification of M-theory on regular seven-dimensional Sasaki–Einstein manifold with fluxes [15]. The full list of such manifolds is given in [16], and we focus on a particular example. The starting point is $\frac{SU(2)^3}{U(1)^2}$ coset, known as $Q^{1,1,1}$, which is a $U(1)$ fibration over $\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1$. This manifold has second Betti number $b_2 = 2$, and we further consistently truncate one Betti vector of $Q^{1,1,1}$ to arrive⁴ at $\frac{SU(3) \times SU(2)}{SU(2) \times U(1)}$ coset, known as $M^{1,1,0}$, with a single topological $U(1)$. Much like the KW theory, the holographic membrane model of M-theory on $M^{1,1,0}$ has $U(1)_R \times U(1)_B$ global symmetry. We show that there are 3 distinct near-extremal regimes: one supported by the $U(1)_R$ charge density, and the other two supported by the $U(1)_B$ charge density. The reason for the distinct baryonic near-criticality comes from the fact that the dual gravitational backgrounds have nontrivial support from the bulk scalar with $m^2 L^2 = -2$, corresponding to an operator of conformal dimension $\Delta = (2, 1)$. Depending on whether one uses normal or alternative quantization [17], one obtains either of two field theory duals, each with a near-extremal regime.

We now report the results of our extensive analysis of the perturbative stability of the model:

- $U(1)_R$ quantum criticality:
 - While there is a single R -charged operator in the theory of conformal dimension $\Delta = 5$, its R -charge is too small to cause its condensation at the extremality.
 - We study baryonic charge transport and demonstrate that the $U(1)_B$ diffusion coefficient D_B becomes negative below some critical temperature T_{crit} , relative to the R -charge chemical potential μ_R of the near-critical thermal equilibrium states,

$$\begin{cases} D_B > 0, & \frac{T}{\mu_R} > \frac{T}{\mu_R} \Big|_{crit} \\ D_B < 0, & \frac{T}{\mu_R} < \frac{T}{\mu_R} \Big|_{crit} \end{cases}, \quad (1.1)$$

This triggers an instability of a diffusive mode in the hydrodynamic sound channel with a dispersion relation⁵

$$\mathfrak{w} = -iD_B \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^2), \quad (1.2)$$

⁴Baryonic black membrane theory of KPT is precisely such compactification.

⁵We use notations $\mathfrak{w} \equiv \frac{w}{2\pi T}$ and $\mathfrak{q} \equiv \frac{|\vec{k}|}{2\pi T}$ where $e^{-i\omega t + i\vec{k} \cdot \vec{x}}$ is the profile of the hydrodynamic perturbation.

resulting in the spatial baryonic charge clumping. The precise value of the critical temperature depends on what quantization condition is used for gravitational dual bulk pseudoscalar coupling the electric components of the $U(1)_B$ gauge field with the magnetic components of the $U(1)_R$ gauge field. The critical temperature is larger if the pseudoscalar is quantized so that the dual operator has a conformal dimension $\Delta = 1$.

- The pseudoscalars mentioned above are neutral under the $U(1)_R$ symmetry. However, we show that there is a critical temperature at which their homogeneous and isotropic fluctuations become normalizable — this signals an onset of the instability, potentially leading to new low-temperature phases of the model⁶. Once again, the critical temperature here depends on the pseudoscalar quantization. For both the normal and the alternative quantizations it is lower than the corresponding critical temperature for the $U(1)_B$ charge clumping instability (1.1), which sets in first when the temperature is lowered.

- $U(1)_B$ quantum criticality (the KPT model):

- Since there are no fields of dimension $\Delta \sim \mathcal{O}(1)$ charged under $U(1)_B$ symmetry, there can not be perturbative superconducting instabilities of the model.
- We study $U(1)_R$ charge transport of the model — in all cases, and for all values of $\frac{T}{\mu_B}$, the diffusion coefficient $D_R > 0$. In the extremal $T \rightarrow 0$ limit it either remains constant, $2\pi T D_R \propto +\frac{T}{\mu_B}$, or vanishes, $2\pi T D_R \propto +\left(\frac{T}{\mu_B}\right)^2$, depending on what quantization is used for scalars supporting the baryonic black membrane background, as well as what quantization is used for pseudoscalars (see section 3.3 for further details).
- There are no threshold instabilities associated with the pseudoscalar condensation in the model (see section 4.3 for further details).

Our main conclusion is that the KPT model [14] is free from perturbative instabilities in the (exotic — *i.e.*, with finite entropy density) quantum critical regime. Since it was already checked in [14] that the model is free⁷ from the non-perturbative “Fermi seasickness” instability [18], it appears to be the first example of the classically stable non-supersymmetric extremal horizon in string theory/M-theory.

The rest of the paper is organized as follows. In section 2 we follow [15] (CKV) and

⁶The detailed exploration of these phases will be reported elsewhere.

⁷We would like thank Igor Klebanov for emphasizing this point.

review the consistent truncation of M-theory on the $Q^{1,1,1}$ coset. We find it convenient to follow [19] and further dualize the massive 2-form B (in the expansion of the 11-dimensional 3-form gauge potential A_3 (2.4)) to a massive vector A^H . Section 3 deals with the baryonic black membranes. In section 3.1 we start with the CKV effective action, see (3.8), and reproduce the baryonic black membrane solution of [14]. Note that the solution is supported by the non-trivial profiles of the two bulk scalars v_1 and v_2 . The boundary gauge theory operator dual to $\ln \frac{v_1}{v_2}$ can be identified either with an operator $\mathcal{O}_2$ (of the conformal dimension $\Delta = 2$) or $\mathcal{O}_1$ (of the conformal dimension $\Delta = 1$). Only the former quantization was used in [14]. We construct black membrane solutions in both quantizations in section 3.2. As expected, the quantum criticality (precise $T = 0$ geometry) is identical for either of the quantizations, but the near-extremal limits, $\frac{T}{\mu_B} > 0$, differ. In section 3.3 we identify a decoupled set of fluctuations associated with the R -charge transport: besides the excitation of the electric components of the A^0 gauge field, dual to a conserved R -symmetry current of the boundary gauge theory [12], one must include the fluctuations of the massive gauge field A^H , the magnetic components of $A^1 - A^2$, dual to a conserved baryonic symmetry current, and the pseudoscalars (the axions) b_1 and b_2 arising from the expansion of the 11D supergravity 3-form A_3 , see (2.4). The latter naturally combine with the scalars v_i into complex scalars $t_i = v_i + ib_i$, see (2.13). Like for the v_i , one can impose different quantizations for the axions, leading to the identification of the linearized fluctuation $(b_1 - b_2)$ with either $\delta\mathcal{O}_2^b$ or $\delta\mathcal{O}_1^b$ operators. We compute the R -charge dimensionless diffusion coefficient $\mathcal{D}_R \equiv 2\pi T D_R$ for all four possible quantizations $\{\ln \frac{v_1}{v_2}, b_1 - b_2\} \iff \{\mathcal{O}_2, \delta\mathcal{O}_2^b\}$, or $\{\mathcal{O}_2, \delta\mathcal{O}_1^b\}$, $\{\mathcal{O}_1, \delta\mathcal{O}_2^b\}$, $\{\mathcal{O}_1, \delta\mathcal{O}_1^b\}$, see fig. 3. In section 3.4 we study the decoupled set involving the homogeneous fluctuations of $(b_1 - b_2)$ and demonstrate the absence of the threshold instabilities. We move to discussing R -charge supported quantum criticality of our model in section 4. In section 4.1 we derive Reissner–Nordström black membrane solution from the CKV effective action: a peculiar feature is the necessity to turn on both the electric component of A^0 gauge field and the magnetic component of the $A^1 = A^2$ gauge fields [20] (see (4.1)) in order to decouple the massive vector A^H and the axions b_i . Section 4.2 is a detailed analysis of the baryonic charge transport in this R -charged background. The structure of the decoupled set of linearized fluctuations closely resembles the discussion of that in section 3.3 with the roles of the two massless bulk gauge fields reversed $A^0 \leftrightarrow (A^1 - A^2)$. Once again, consistency of the truncation requires the excitation of the massive gauge

field A^H and the axions b_i . Since the scalars v_i are trivial in the RN background, $v_i \equiv 1$ (4.2), there are only two different cases for the diffusion coefficient of the baryonic charge transport, depending on what quantization we choose for the fluctuations $(b_1 - b_2)$: $\delta\mathcal{O}_2^b$ or $\delta\mathcal{O}_1^b$. As we indicated earlier, the baryonic charge transport in the RN membrane background is unstable at low temperatures, see (1.1) and fig. 5. Additional instabilities in the model are associated with the homogeneous and isotropic condensation of the neutral pseudoscalar $(b_1 - b_2)$, although these instabilities occur at lower temperatures than the corresponding critical temperatures for the baryonic charge clumping, see section 4.3. Finally, in section 4.4 we consider potential holographic superconductor [4] instabilities in the model: CKV effective action includes a complex bulk scalar $\chi \equiv \frac{1}{\sqrt{3}}(\xi^0 + i\tilde{\xi}^0)$ of conformal dimension $\Delta_\chi = 5$ with⁸ $R(\chi) = 4$. Since this is the only R -charged field, its linearized fluctuations decouple from the other fields of the model. We explicitly verified that χ does not condense. This agrees with the comprehensive probe analysis in [21] once we appropriately match the conventions.

In this paper we analyzed perturbative stability of the extremal horizons of M-theory compactified on $M^{1,1,0}$. The stability analysis is specific to the model, and thus it is interesting to extend the quest for classically stable extremal horizons to other examples of Sasaki-Einstein manifolds [16]. Of course, one must keep an open mind for additional instabilities that eluded the current analysis.

2 Effective action

We follow notations of [15] and review the consistent truncations of 11D supergravity on seven-dimensional Sasaki-Einstein coset $M^{1,1,0}$ with fluxes. The resulting $\mathcal{N} = 2$ gauged supergravity in four dimensions embeds the holographic duality of the KPT membrane model [14] with $U(1)_R \times U(1)_B$ global symmetry.

The starting point is 11D supergravity

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \left(R \star 1 - \frac{1}{2} G_4 \wedge \star G_4 - \frac{1}{6} A_3 \wedge G_4 \wedge G_4 \right). \quad (2.1)$$

We consider consistent truncation of (2.1) on the coset $Q^{1,1,1}$:

- The 11D metric is

$$ds^2 = e^{2V} \mathcal{K}^{-1} ds_4^2 + e^{-V} \sum_{i=1}^3 \frac{v_i}{8} \left((d\theta_i)^2 + \sin^2 \theta_i (d\phi_i)^2 \right) + e^{2V} \left(\theta + A^0 \right)^2, \quad (2.2)$$

⁸Our conversion for the R -charge normalization differs by a factor of 2 from the one used in [15].

where ds_4^2 is the 4D metric, A^0 is a 1-form on $\mathcal{M}_4$, and where

$$\mathcal{K} \equiv v_1 v_2 v_3, \quad \theta \equiv d\psi + \frac{1}{4} \sum_i \cos \theta_i d\phi_i. \quad (2.3)$$

We will furthermore define the scalar field $\phi = \frac{3}{2}V - \frac{1}{2} \sum_i \ln v_i$ as well as the 2-forms $\omega_i = \frac{1}{8} \sin \theta_i d\phi_i \wedge d\theta_i$ so that $d\theta = \sum_i m_i \omega_i$ with $m_i = \{2, 2, 2\}$.

- The fluxes are

$$A_3 = C_3 + B \wedge (\theta + A^0) - A^i \wedge \omega_i + b_i \omega_i \wedge (\theta + A^0), \quad (2.4)$$

with C_3 , B , A^i and b_i being correspondingly the 3-, 2-, 1- and the 0- forms on $\mathcal{M}_4$.

Under this ansatz, the action (2.1) reduces into the following components:

- The 11D Einstein–Hilbert term becomes

$$S_{EH} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} R \star 1 \equiv \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} \left[\frac{1}{2} R_4 \star 1 + \mathcal{L}_{kin,geo} - V_{geo} \star 1 \right], \quad (2.5)$$

where $\kappa_4^{-2} = \kappa_{11}^{-2} \int_{Q^{1,1,1}} d\psi \wedge \omega_1 \wedge \omega_2 \wedge \omega_3$ and

$$\mathcal{L}_{kin,geo} = -(\partial\phi)^2 \star 1 + \frac{1}{4} \sum_i (\partial \ln v_i)^2 \star 1 - \frac{1}{4} \mathcal{K} \mathcal{F}^0 \wedge \star \mathcal{F}^0, \quad (2.6)$$

$$V_{geo} = e^{4\phi} v_1 v_2 v_3 \cdot \sum_i v_i^{-2} - 8e^{2\phi} \cdot \sum_i v_i^{-1}, \quad (2.7)$$

where $\mathcal{F}^0 \equiv dA^0$.

- The kinetic flux term can be written as

$$-\frac{1}{4\kappa_{11}^2} \int_{\mathcal{M}_{11}} G_4 \wedge \star G_4 = \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} (\mathcal{L}_{kin,flux} - V_{flux} \star 1). \quad (2.8)$$

In the following, we denote the field strengths as $\mathcal{F}^I \equiv dA^I$ and the generalized field strengths as $F^I = \mathcal{F}^I - m^I B$, with $I = \{0, 1, 2, 3\}$ and $m^I = \{0, 2, 2, 2\}$. We then get

$$\begin{aligned} \mathcal{L}_{kin,flux} = & -\frac{1}{4} \sum_i v_i^{-2} (\partial b_i)^2 \star 1 - \frac{1}{4} e^{-4\phi} dB \wedge \star dB \\ & + \frac{1}{4} (\text{Im} \mathcal{N}_{IJ} + \mathcal{K} \delta_I^0 \delta_J^0) F^I \wedge \star F^J + \frac{1}{4} \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J, \end{aligned} \quad (2.9)$$

$$\text{Re} \mathcal{N}_{00} = -\frac{1}{3} \mathcal{K}_{ijk} b_i b_j b_k, \quad \text{Re} \mathcal{N}_{0i} = \frac{1}{2} \mathcal{K}_{ijk} b_j b_k, \quad \text{Re} \mathcal{N}_{ij} = -\mathcal{K}_{ijk} b_k,$$

$$\text{Im} \mathcal{N}_{00} = -\mathcal{K}(1 + 4g_{ij} b_i b_j), \quad \text{Im} \mathcal{N}_{0i} = 4\mathcal{K} g_{ij} b_j, \quad \text{Im} \mathcal{N}_{ij} = -4\mathcal{K} g_{ij},$$

where $\mathcal{K}_{ijk} = 1$ for $i \neq j \neq k$ and 0 otherwise, and $g_{ij} = \frac{1}{4}v_i^{-2} \delta_{ij}$. The potential V_{flux} is

$$V_{flux} = \frac{e^{4\phi}}{4\mathcal{K}} \cdot \sum_k \left[\sum_{ij} \mathcal{K}_{ijk} b_i m_j v_k \right]^2 + \frac{e^{4\phi}}{4} \mathcal{K}^{-1} \cdot \left[e_0 + \frac{1}{2} \sum_{i,j,k} \mathcal{K}_{ijk} b_i b_j m_k \right]^2, \quad (2.10)$$

where we dualized dC_3 as in [19],

$$\frac{e^{-4\phi}\mathcal{K}}{2} \star \left(dC_3 + B \wedge F^0 \right) = -\frac{1}{2} \left(e_0 + \frac{1}{2} \sum_{i,j,k} b_i b_j m_k \mathcal{K}_{ijk} \right). \quad (2.11)$$

The constant e_0 will set the radius of the asymptotic AdS_4 spacetime. Below, we will choose $e_0 = 6 \Rightarrow L = 1/2$.

- The topological term is

$$-\frac{1}{12\kappa_{11}^2} \int_{\mathcal{M}_{11}} A_3 \wedge G_4 \wedge G_4 = \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} \mathcal{L}_{top} = -\frac{e_0}{2\kappa_4^2} \int_{\mathcal{M}_4} dB \wedge A^0. \quad (2.12)$$

Combining the gravitational and flux contributions, we reproduce exactly the effective action of [15]:

$$\begin{aligned} S_{CKV} &= \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} \left[\frac{1}{2} R_4 \star 1 - \left\{ (\partial\phi)^2 + g_{ij} \partial t^i \partial \bar{t}^j \right\} \star 1 - \frac{1}{4} e^{-4\phi} dB \wedge \star dB \right. \\ &\quad \left. + \frac{1}{4} \text{Im} \mathcal{N}_{IJ} F^I \wedge \star F^J + \frac{1}{4} \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J - \frac{1}{2} e_0 dB \wedge A^0 - V_{CKV} \star 1 \right], \\ V_{CKV} &= e^{4\phi} \mathcal{K} \cdot \sum_i v_i^{-2} - 8e^{2\phi} \cdot \sum_i v_i^{-1} + \frac{e^{4\phi}}{4} \mathcal{K}^{-1} \cdot \sum_k \left[\sum_{ij} \mathcal{K}_{ijk} b_i m_j v_k \right]^2 \\ &\quad + \frac{e^{4\phi}}{4} \mathcal{K}^{-1} \cdot \left[e_0 + \frac{1}{2} \sum_{i,j,k} \mathcal{K}_{ijk} b_i b_j m_k \right]^2, \end{aligned} \quad (2.13)$$

with $t^i \equiv v_i + ib_i$.

Note that the effective action (2.13) is invariant under the 1-form gauge transformations (with the 0-form gauge parameters α^0, α^i):

$$A^0 \rightarrow A^0 + d\alpha^0, \quad A^i \rightarrow A^i + d\alpha^i. \quad (2.14)$$

For the next step we would like to follow [19] and dualize the massive 2-form B to a massive vector A_H .⁹ They treat an action of the general form

$$\mathcal{L}_B = - \left[h dB \wedge \star dB + M^2 B \wedge \star B + M_T^2 B \wedge B + B \wedge J_2 \right], \quad (2.15)$$

⁹Here we deviate from the logic of [15, 20], which dualized the 2-form while preserving the standard matter-coupled $\mathcal{N} = 2$ Lagrangian form by first performing an electric-magnetic symplectic transformation on the vector fields. Instead we prefer to keep the gauging magnetic.

where in our case:

$$\begin{aligned}
h &= \frac{1}{4}e^{-4\phi}, \quad M^2 = -\frac{1}{4}\text{Im}\mathcal{N}_{IJ} m^I m^J, \quad M_T^2 = -\frac{1}{4}\text{Re}\mathcal{N}_{IJ} m^I m^J, \\
J_2 &= J_a + \star J_b, \\
J_a &= -\frac{1}{2}e_0 \mathcal{F}^0 + \frac{1}{4}\text{Re}\mathcal{N}_{IJ} (m^I \mathcal{F}^J + m^J \mathcal{F}^I), \quad J_b = \frac{1}{4}\text{Im}\mathcal{N}_{IJ} (m^I \mathcal{F}^J + m^J \mathcal{F}^I).
\end{aligned} \tag{2.16}$$

Introducing a massive vector $A^H = h \star dB$ we rewrite (2.15) as [19]

$$\begin{aligned}
-\mathcal{L}_B = -\mathcal{L}_{AH} &\equiv \frac{1}{h} A^H \wedge \star A^H + \frac{M^2}{M^4 + M_T^4} \left(dA^H - \frac{1}{2} J_2 \right) \wedge \star \left(dA^H - \frac{1}{2} J_2 \right) \\
&\quad - \frac{M_T^2}{M^4 + M_T^4} \left(dA^H - \frac{1}{2} J_2 \right) \wedge \left(dA^H - \frac{1}{2} J_2 \right).
\end{aligned} \tag{2.17}$$

Finally, from here on we consider the trivial consistent sub-truncation $Q^{1,1,1} \rightarrow M^{1,1,0}$ which means identifying

$$A^3 \equiv A^1, \quad v_3 \equiv v_1, \quad b_3 \equiv b_1. \tag{2.18}$$

3 Baryonic black membranes

3.1 Truncation to KPT

In this section we describe the truncation of the effective action of section 2 to the one used in [14]. We emphasize that this is a truncation of M-theory membranes with topological charge for the equilibrium thermal homogeneous solutions only, and is inconsistent at the level of fluctuations. The solutions considered in [14] are homogeneous and isotropic black membranes of 11D supergravity on $AdS_4 \times M^{1,1,0}$ with a *baryonic* chemical potential:

- The 11D metric is a warped product of $\mathcal{M}_4$ and a squashed $M^{1,1,0}$

$$\begin{aligned}
ds^2 = e^{-7\chi/2} ds_4^2 + 4L^2 e^\chi &\left[\frac{e^{\eta_1}}{8} \sum_{i=1,3} \left((d\theta_i)^2 + \sin^2 \theta_i (d\phi_i)^2 \right) \right. \\
&\quad \left. + \frac{e^{\eta_2}}{8} \left((d\theta_2)^2 + \sin^2 \theta_2 (d\phi_2)^2 \right) + e^{-4\eta_1 - 2\eta_2} \theta^2 \right],
\end{aligned} \tag{3.1}$$

$$ds_4^2 = -\mathcal{G} e^{-w} dt^2 + \frac{r^2}{L^2} [d(x_1)^2 + d(x_2)^2] + \frac{dr^2}{\mathcal{G}}; \tag{3.2}$$

- The 4-form flux G_4 is given by¹⁰

$$G_4 = \frac{3}{L} e^{-\frac{21}{2}\chi} \star_4 1 - 8QL^3 \frac{e^{-\frac{w}{2} - \frac{3}{2}\chi}}{r^2} dt \wedge dr \wedge \left(e^{2\eta_1}(w_1 + w_3) - 2e^{2\eta_2}w_2 \right), \quad (3.3)$$

where the $\mathcal{M}_4$ metric warp factors $\mathcal{G}, w$ and the bulk scalars χ, η_i are functions of the radial coordinate r only. Furthermore, L is the radius of the asymptotic AdS_4 and Q is the baryonic charge of the black membranes.

To compare with the notation of the previous section, we begin by comparing the metric (3.1) to (2.2), which implies $A^0 = 0$ as well as

$$e_0 = 6 \quad \Longleftrightarrow \quad L = \frac{1}{2}, \quad (3.4)$$

and

$$\begin{aligned} \chi &= -\frac{8}{21}\phi + \frac{4}{21}\ln v_1 + \frac{2}{21}\ln v_2, \\ \eta_1 &= -\frac{2}{7}\phi + \frac{1}{7}\ln v_1 - \frac{3}{7}\ln v_2, \\ \eta_2 &= -\frac{2}{7}\phi - \frac{6}{7}\ln v_1 + \frac{4}{7}\ln v_2. \end{aligned} \quad (3.5)$$

Matching dA_3 in (2.4) with (3.3) furthermore requires

$$b_i \equiv 0, \quad B \equiv 0, \quad (3.6)$$

(i.e. $A^H = 0$) as well as $\mathcal{F}^1 \wedge \mathcal{F}^2 = 0$; however, this is consistent only if J_2 in (2.15) vanishes as well. This, in turn, implies

$$J_2 \equiv 0 \quad \Longrightarrow \quad \frac{1}{4} \text{Im} \mathcal{N}_{ij} m_i \mathcal{F}^j \equiv 0 \quad \Longleftrightarrow \quad \sum_i \frac{\mathcal{F}^i}{v_i^2} = 0. \quad (3.7)$$

As we show shortly, the last equality in (3.7) is indeed satisfied on solutions (3.1)-(3.3) of [14], but alas, it can not be imposed at the level of fluctuations.

Under these identifications, the effective action (2.13) takes the form

$$\begin{aligned} \mathcal{L}_{CKV} \rightarrow \mathcal{L}_{KPT} &= \frac{1}{2}R_4 - (\partial\phi)^2 - \frac{1}{2}(\partial\ln v_1)^2 - \frac{1}{4}(\partial\ln v_2)^2 - \frac{v_2}{4} \mathcal{F}_{\mu\nu}^1 \mathcal{F}^{1\mu\nu} \\ &\quad - \frac{v_1^2}{8v_2} \mathcal{F}_{\mu\nu}^2 \mathcal{F}^{2\mu\nu} - V_{KPT}, \end{aligned} \quad (3.8)$$

¹⁰We changed the overall sign of G_4 for consistency with the action of section 2.

with

$$V_{KPT} = e^{4\phi} \left[\frac{e_0^2}{4v_1^2 v_2} + v_1^2 v_2 \left(\frac{2}{v_1^2} + \frac{1}{v_2^2} \right) \right] - 8e^{2\phi} \left(\frac{2}{v_1} + \frac{1}{v_2} \right), \quad (3.9)$$

subject to additional constraint (3.7)

$$\frac{2\mathcal{F}^1}{v_1^2} + \frac{\mathcal{F}^2}{v_2^2} = 0. \quad (3.10)$$

This constraint is consistent with the 2-form equations of motion derived from (3.8), which read

$$d \star (v_2 \mathcal{F}^1) = 0, \quad d \star \left(\frac{v_1^2}{v_2} \mathcal{F}^2 \right) = 0, \quad (3.11)$$

resulting in

$$d \star \left[\mathcal{K} \cdot \left(\frac{2\mathcal{F}^1}{v_1^2} + \frac{\mathcal{F}^2}{v_2^2} \right) \right] = 0. \quad (3.12)$$

However, it is generically violated by Bianchi identities $d\mathcal{F}^i = 0$. Indeed,

$$d\mathcal{F}^2 = d \left(-\frac{2v_2^2}{v_1^2} \mathcal{F}^1 \right) = -2 d \left(\frac{v_2^2}{v_1^2} \right) \wedge \mathcal{F}^1 - 2 \frac{v_2^2}{v_1^2} d\mathcal{F}^1 \neq 0, \quad (3.13)$$

unless v_i are functions of $\{t, r\}$ exclusively (for purely electric $\mathcal{F}^1$). For this reason, the action (3.8) does not adequately describe the fluctuations around the background.

Going back to (3.3), we read off the ansatz

$$\mathcal{F}^1 \equiv \frac{Q}{r^2} e^{2\eta_1 - \frac{w}{2} - \frac{3}{2}\chi} dt \wedge dr, \quad \mathcal{F}^2 = -2 \frac{v_2^2}{v_1^2} \mathcal{F}^1 = -2e^{2(\eta_2 - \eta_1)} \mathcal{F}^1, \quad (3.14)$$

from which we recover from (3.8) the effective one-dimensional Lagrangian of [14],

$$\begin{aligned} \mathcal{L} = & \frac{r^2}{L^2} e^{-\frac{w}{2}} \left[\frac{63\mathcal{G}}{8} \chi'^2 + \frac{\mathcal{G}}{2} (2\eta_1'^2 + \eta_2'^2) + \mathcal{G}(2\eta_1' + \eta_2')^2 + \frac{2\mathcal{G}}{r} w' - \frac{2}{r} \mathcal{G}' \right. \\ & \left. - \frac{2\mathcal{G}}{r^2} + V_Q + V_s \right], \end{aligned} \quad (3.15)$$

where

$$\begin{aligned} V_Q &= \frac{4L^2}{r^4} e^{-\frac{3}{2}\chi} (e^{2\eta_1} + 2e^{2\eta_2}) Q^2, \\ V_s &= \frac{9}{2L^2} e^{-\frac{21}{2}\chi} - \frac{4}{L^2} e^{-\frac{9}{2}\chi} (2e^{-\eta_1} + e^{-\eta_2}) + \frac{1}{2L^2} e^{-2(2\eta_1 + \eta_2) - \frac{9}{2}\chi} [2e^{-2\eta_1} + e^{-2\eta_2}]. \end{aligned} \quad (3.16)$$

The Lagrangian $\mathcal{L}$ needs to be supplemented with the zero-energy constraint, coming from the rr -component of the Einstein equations,

$$\frac{2}{r} \mathcal{G}' - \mathcal{G} \left[\frac{63}{8} \chi'^2 + \frac{1}{2} (2\eta_1'^2 + \eta_2'^2) + (2\eta_1' + \eta_2')^2 + \frac{2}{r} w' - \frac{2}{r^2} \right] + V_Q + V_s = 0. \quad (3.17)$$

As an independent check, we reproduce from (3.15)-(3.17) the analytic extremal $AdS_2 \times \mathbb{R}^2$ solution of [14],

$$\begin{aligned} \mathcal{G} &= \mathbf{4} \cdot \frac{2^{\frac{5}{4}}(r^4 - 1)^2}{3^{\frac{7}{4}}r^{12}}, & w &= w_0 - 14 \ln r, & \eta_1 &= \frac{1}{7} \ln 3 \\ \eta_2 &= \frac{1}{7} \ln 3 - \ln 2, & \chi &= \frac{5}{14} \ln 3 - \frac{1}{2} \ln 2 + \frac{4}{3} \ln r, & Q &= \mathbf{4} \cdot \frac{2^{\frac{7}{4}}}{3^{\frac{5}{4}}}. \end{aligned} \quad (3.18)$$

The coefficients marked in boldface in (3.18) differ from [14], owing to the fact that their background had AdS length $L = 1$.

3.2 Background

Assuming an equivalent form of the 4D metric and field strengths as in the previous section,

$$ds_4^2 = -\frac{4\alpha^2 f}{r^2} dt^2 + \frac{4\alpha^2}{r^2} d\mathbf{x}^2 + \frac{s^2}{4r^2 f} dr^2, \quad \mathcal{F}^1 = \frac{q\alpha s}{v_2} dr \wedge dt, \quad \mathcal{F}^2 = -\frac{2v_2^2}{v_1^2} \mathcal{F}^1, \quad (3.19)$$

where α, q are coefficients (related to the temperature and the baryonic charge), and $f, s, v_i, g \equiv e^\phi$ are all functions of r , we derive the following equations of motion:

$$\begin{aligned} 0 &= f' + f \left(\frac{rv_2'^2}{4v_2^2} + \frac{rv_1'^2}{2v_1^2} + \frac{rg'^2}{g^2} - \frac{3}{r} \right) - \frac{s^2 r^3 (2v_2^2 + v_1^2) q^2}{8v_2 v_1^2} - \frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4v_2 v_1^2 r} \\ &\quad + \frac{2g^2 s^2 (2v_2 + v_1)}{v_2 v_1 r}, \end{aligned} \quad (3.20)$$

$$0 = s' + \frac{sr}{4} \left(\frac{v_2'^2}{v_2^2} + \frac{2v_1'^2}{v_1^2} + \frac{4g'^2}{g^2} \right), \quad (3.21)$$

$$\begin{aligned} 0 &= v_1'' - \frac{v_1'^2}{v_1} + v_1' \left(\frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2s^2 g^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\ &\quad - \frac{s^2 g^4 (v_1^4 - 9)}{2v_1 f v_2 r^2} + \frac{s^2 r^2 v_2 q^2}{2v_1 f} - \frac{4s^2 g^2}{f r^2}, \end{aligned} \quad (3.22)$$

$$\begin{aligned} 0 &= v_2'' - \frac{v_2'^2}{v_2} + v_2' \left(\frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2s^2 g^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\ &\quad - \frac{s^2 (2v_2^2 v_1^2 - v_1^4 - 9) g^4}{2f v_1^2 r^2} - \frac{4s^2 g^2}{f r^2} - \frac{s^2 q^2 r^2 (2v_2^2 - v_1^2)}{4f v_1^2}, \end{aligned} \quad (3.23)$$

$$\begin{aligned}
0 = g'' - \frac{g'^2}{g} + g' & \left(\frac{g^4 s^2 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2g^2 s^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\
& - \frac{s^2 g^5 (2v_2^2 v_1^2 + v_1^4 + 9)}{2f v_2 v_1^2 r^2} + \frac{2g^3 s^2 (2v_2 + v_1)}{v_1 f v_2 r^2}.
\end{aligned} \tag{3.24}$$

Following [15], the holographic spectroscopy relates the scalars $\{v_1, v_2, g\}$ to the boundary gauge theory operators $\mathcal{O}_\Delta$ of conformal dimension Δ as in table 1. Notice that

Table 1: Holographic spectroscopy of the background scalars

mass eigenstate	$m^2 L^2$	Δ	$U(1)$ R-charge
$\ln[v_1 v_2^{-1}]$	-2	(2, 1)	0
$\ln[v_1^2 v_2 g^3]$	4	4	0
$\ln[v_1^2 v_2 g^{-4}]$	18	6	0

the bulk scalar $\ln[v_1 v_2^{-1}]$ can be identified [17] either with the operator $\mathcal{O}_2$, the *normal quantization*, or with the operator $\mathcal{O}_1$, the *alternative quantization*. In [14] the authors consider the normal quantization only; here, we discuss both cases.

Eqs. (3.20)-(3.24) should be solved subject to the following asymptotic expansion

- In the UV, *i.e.*, as $r \rightarrow 0$, and with the identification $\ln[v_1 v_2^{-1}] \iff \mathcal{O}_2$, we have

$$f = 1 + f_3 r^3 + \frac{3}{8} q^2 r^4 - \frac{1}{6} v_{1,2} q^2 r^6 + \mathcal{O}(r^7), \quad s = 1 - \frac{3}{2} v_{1,2}^2 r^4 + \frac{1}{6} v_{1,2} q^2 r^6 + \mathcal{O}(r^7), \tag{3.25}$$

$$\begin{aligned}
v_1 = 1 + v_{1,2} r^2 + & \left(v_{1,4} + \left(\frac{24}{35} v_{1,2}^2 - \frac{1}{35} q^2 \right) \ln r \right) r^4 - \frac{1}{3} f_3 v_{1,2} r^5 + \left(v_{1,6} \right. \\
& \left. + \left(-\frac{13}{350} v_{1,2} q^2 + \frac{156}{175} v_{1,2}^3 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r),
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
v_2 = 1 - 2v_{1,2} r^2 + & \left(\frac{3}{2} v_{1,2}^2 + v_{1,4} + \frac{1}{8} q^2 + \left(\frac{24}{35} v_{1,2}^2 - \frac{1}{35} q^2 \right) \ln r \right) r^4 + \frac{2}{3} f_3 v_{1,2} r^5 + \left(v_{1,6} \right. \\
& \left. - \frac{39}{10} v_{1,2} v_{1,4} + \frac{4647}{3500} v_{1,2}^3 - \frac{653}{3500} v_{1,2} q^2 + \left(\frac{13}{175} v_{1,2} q^2 - \frac{312}{175} v_{1,2}^3 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r),
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
g = 1 + & \left(-\frac{3}{56} v_{1,2}^2 + \frac{3}{4} v_{1,4} + \frac{1}{56} q^2 + \left(\frac{18}{35} v_{1,2}^2 - \frac{3}{140} q^2 \right) \ln r \right) r^4 + \left(-v_{1,6} + \frac{13}{10} v_{1,2} v_{1,4} \right. \\
& \left. - \frac{1549}{3500} v_{1,2}^3 - \frac{37}{1750} v_{1,2} q^2 \right) r^6 + \mathcal{O}(r^7 \ln r),
\end{aligned} \tag{3.28}$$

i.e. the UV part of the solution is characterized (given q) by

$$\left\{ f_3, v_{1,2}, v_{1,4}, v_{1,6} \right\}; \quad (3.29)$$

■ in the UV, *i.e.*, as $r \rightarrow 0$, and instead with the identification $\ln[v_1 v_2^{-1}] \iff \mathcal{O}_1$, we have

$$f = 1 + f_3 r^3 + \frac{3}{8} q^2 r^4 + \left(-\frac{9}{20} v_{1,1}^2 f_3 - \frac{3}{10} v_{1,1} q^2 \right) r^5 + \frac{37}{120} v_{1,1}^2 q^2 r^6 + \mathcal{O}(r^7), \quad (3.30)$$

$$\begin{aligned} s = 1 - \frac{3}{4} v_{1,1}^2 r^2 + \frac{489}{800} v_{1,1}^4 r^4 + \left(v_{1,1}^5 + \frac{2}{5} v_{1,1}^2 f_3 + \frac{1}{10} v_{1,1} q^2 \right) r^5 + \left(\frac{5661}{22400} v_{1,1}^6 \right. \\ \left. + \frac{1}{8} v_{1,1}^3 f_3 - \frac{269}{1680} v_{1,1}^2 q^2 + \frac{3}{4} v_{1,1}^2 v_{1,4} + \left(-\frac{51}{70} v_{1,1}^6 - \frac{3}{140} v_{1,1}^2 q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \end{aligned} \quad (3.31)$$

$$\begin{aligned} v_1 = 1 + v_{1,1} r - \frac{1}{5} v_{1,1}^2 r^2 - \frac{31}{20} v_{1,1}^3 r^3 + \left(v_{1,4} + \left(-\frac{34}{35} v_{1,1}^4 - \frac{1}{35} q^2 \right) \ln r \right) r^4 + \left(-\frac{103}{800} v_{1,1}^5 \right. \\ \left. + \frac{19}{60} v_{1,1}^2 f_3 + \frac{11}{120} v_{1,1} q^2 + \frac{3}{2} v_{1,1} v_{1,4} + \left(-\frac{3}{70} v_{1,1} q^2 - \frac{51}{35} v_{1,1}^5 \right) \ln r \right) r^5 + \left(v_{1,6} \right. \\ \left. + \left(-\frac{51}{70} v_{1,1}^6 - \frac{3}{140} v_{1,1}^2 q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \end{aligned} \quad (3.32)$$

$$\begin{aligned} v_2 = 1 - 2v_{1,1} r + \frac{13}{10} v_{1,1}^2 r^2 + \frac{1}{10} v_{1,1}^3 r^3 + \left(\frac{131}{40} v_{1,1}^4 + \frac{1}{2} v_{1,1} f_3 + v_{1,4} + \frac{1}{8} q^2 + \left(-\frac{34}{35} v_{1,1}^4 \right. \right. \\ \left. \left. - \frac{1}{35} q^2 \right) \ln r \right) r^4 + \left(-\frac{4597}{400} v_{1,1}^5 - \frac{14}{15} v_{1,1}^2 f_3 - \frac{13}{30} v_{1,1} q^2 - 3v_{1,1} v_{1,4} + \left(\frac{3}{35} v_{1,1} q^2 \right. \right. \\ \left. \left. + \frac{102}{35} v_{1,1}^5 \right) \ln r \right) r^5 + \left(\frac{166743}{14000} v_{1,1}^6 - \frac{29}{40} v_{1,1}^3 f_3 + \frac{8061}{14000} v_{1,1}^2 q^2 + \frac{39}{20} v_{1,1}^2 v_{1,4} + v_{1,6} \right. \\ \left. + \left(-\frac{459}{175} v_{1,1}^6 - \frac{27}{350} v_{1,1}^2 q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \end{aligned} \quad (3.33)$$

$$\begin{aligned} g = 1 - \frac{3}{10} v_{1,1}^2 r^2 - \frac{1}{2} v_{1,1}^3 r^3 + \left(\frac{2047}{1400} v_{1,1}^4 + \frac{1}{8} v_{1,1} f_3 + \frac{3}{4} v_{1,4} + \frac{1}{56} q^2 + \left(-\frac{51}{70} v_{1,1}^4 \right. \right. \\ \left. \left. - \frac{3}{140} q^2 \right) \ln r \right) r^4 + \left(-\frac{73}{40} v_{1,1}^5 + \frac{1}{10} v_{1,1}^2 f_3 \right) r^5 + \left(-\frac{6761}{14000} v_{1,1}^6 + \frac{283}{240} v_{1,1}^3 f_3 \right. \\ \left. + \frac{2837}{21000} v_{1,1}^2 q^2 + \frac{19}{40} v_{1,1}^2 v_{1,4} - v_{1,6} + \left(\frac{187}{700} v_{1,1}^6 + \frac{11}{1400} v_{1,1}^2 q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \end{aligned} \quad (3.34)$$

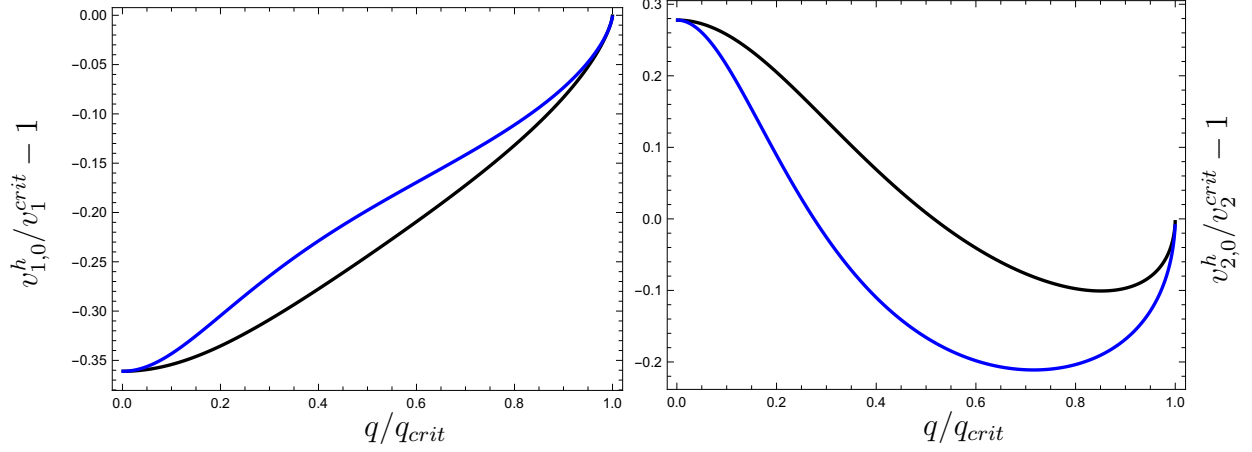


Figure 1: The values of the bulk scalars v_1 and v_2 at the horizon for different quantizations of the mode $\ln[v_1 v_2^{-1}]$ (see table 1): $\mathcal{O}_2$ (black) and $\mathcal{O}_1$ (blue). The limit $q/q_{crit} \rightarrow 1$ is a quantum critical regime corresponding to the zero-temperature limit $T \rightarrow 0$ of the baryonic black membranes (3.39) and (3.40); the regime $q/q_{crit} \rightarrow 0$ is the black membrane solution with vanishing baryonic charge density — the AdS_4 -Schwarzschild background with the trivial profile for the scalars $v_1 = v_2 \equiv 1$ (3.38).

characterized (given q) by

$$\left\{ v_{1,1}, f_3, v_{1,4}, v_{1,6} \right\}; \quad (3.35)$$

■ in the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0$, we have

$$\begin{aligned} f &= -\frac{(s_0^h)^2}{8v_{2,0}^h(v_{1,0}^h)^2} \left(2(g_0^h)^4 \left((v_{1,0}^h)^4 + 2(v_{1,0}^h)^2(v_{2,0}^h)^2 + 9 \right) - 16(g_0^h)^2 v_{1,0}^h \left(v_{1,0}^h + 2v_{2,0}^h \right) \right. \\ &\quad \left. + q^2 \left((v_{1,0}^h)^2 + 2(v_{2,0}^h)^2 \right) \right) y + \mathcal{O}(y^2), \\ s &= s_0^h + \mathcal{O}(y), \quad v_i = v_{i,0}^h + \mathcal{O}(y), \quad g = g_0^h + \mathcal{O}(y), \end{aligned} \quad (3.36)$$

characterized (given q) by

$$\left\{ s_0^h, v_{1,0}^h, v_{2,0}^h, g_0^h \right\}. \quad (3.37)$$

There are two exact analytic solutions of (3.20)-(3.24):

- an AdS_4 -Schwarzschild solution,

$$q = 0 : \quad f = 1 - r^3, \quad s = v_i = g = 1; \quad (3.38)$$

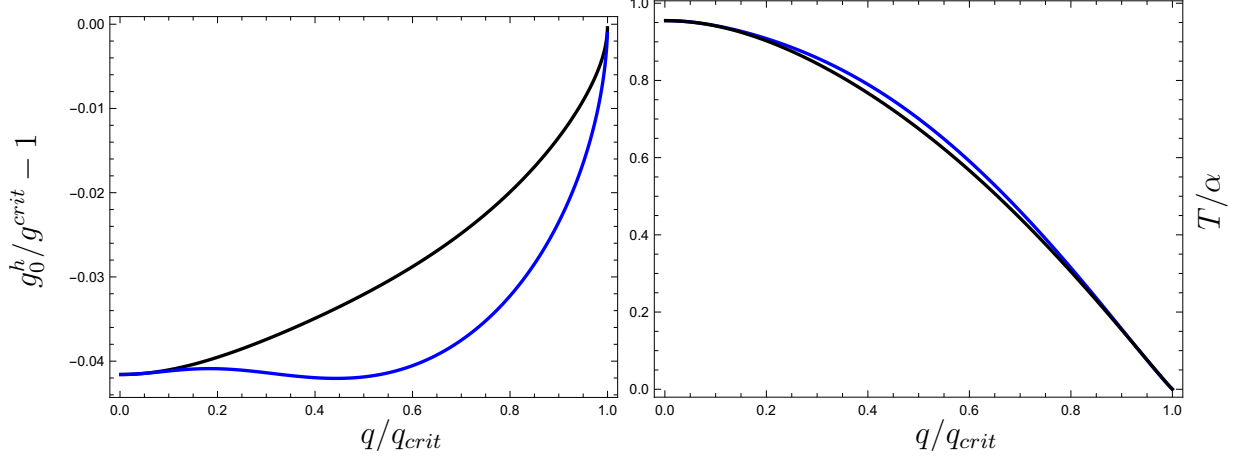


Figure 2: The values of the bulk scalar g at the horizon and the reduced temperature T/α (see (3.41)) for different quantizations of the mode $\ln[v_1 v_2^{-1}]$ (see table 1): $\mathcal{O}_2$ (black) and $\mathcal{O}_1$ (blue). The limit $q/q_{crit} \rightarrow 1$ is a quantum critical regime corresponding to $T \rightarrow 0$ of the baryonic black membranes (3.39) and (3.40); the regime $q/q_{crit} \rightarrow 0$ is the black membrane solution with vanishing baryonic charge density — the AdS_4 -Schwarzschild background with the trivial profile for the scalar $g \equiv 1$ (3.38).

- the extremal $AdS_2 \times \mathbb{R}^2$ solution of [14] given in (3.18)

$$q = q_{crit} = \frac{2^{\frac{15}{4}}}{3^{\frac{5}{4}}} : \quad f = \frac{32(s_0^h)^2 (1 - r^4)^2}{27q_{crit} r^8}, \quad s = \frac{s_0^h}{r^7}, \quad v_i = \frac{v_i^{crit}}{r^2}, \quad g = g^{crit} r^2, \quad (3.39)$$

where

$$v_1^{crit} = \frac{16}{3q_{crit}}, \quad v_2^{crit} = \frac{8}{3q_{crit}}, \quad g^{crit} = \frac{32}{9q_{crit}}. \quad (3.40)$$

A baryonic black membrane solution is an interpolating solution for $q \in [0, q_{crit}]$. We use numerical technique developed in [22] to construct such solutions. Some results of such computations, which validate (3.39), are presented in figs. 1-2.

Given q , a numerical solution is characterized by (3.29) (or (3.35)) and (3.37), which determine the black membrane Hawking temperature T ,

$$\begin{aligned} \frac{T}{\alpha} = & \frac{s_0^h}{8\pi v_{2,0}^h (v_{1,0}^h)^2} \left(2(g_0^h)^2 \left(8v_{1,0}^h (v_{1,0}^h + 2v_{2,0}^h) - (g_0^h)^2 ((v_{1,0}^h)^4 + 2(v_{1,0}^h)^2 (v_{2,0}^h)^2 + 9) \right) \right. \\ & \left. - \left((v_{1,0}^h)^2 + 2(v_{2,0}^h)^2 \right) q^2 \right). \end{aligned} \quad (3.41)$$

From (3.39) and (3.40), as $q \rightarrow q_{crit}$, we have from (3.41) (see also the right panel of fig. 2)

$$\lim_{q \rightarrow q_{crit}} \frac{T}{\alpha} \propto \lim \left(1 - \frac{q}{q_{crit}} \right) = 0. \quad (3.42)$$

3.3 Fluctuations

Given these backgrounds, we now discuss fluctuations of the R -charge density. To this end, we need to supplement the Lagrangian $\mathcal{L}_{KPT}$ of (3.8) to quadratic order in $\{ \mathcal{F}^0, B, b_i \}$ obtained from the full effective action $\mathcal{L}_{CKV}$

$$\begin{aligned} \mathcal{L}_{KPT} &\rightarrow \mathcal{L}_{KPT} + \delta \mathcal{L}_2[\mathcal{F}^0, B, b_i] \equiv \mathcal{L}_{KPT} + \delta \mathcal{L}_{2,kin} - \delta V_2 \star 1, \\ \delta \mathcal{L}_{2,kin} &= -\frac{1}{4} v_i^{-2} (\partial b_i)^2 \star 1 - \frac{1}{4} K_{ijk} b_k \mathcal{F}^i \wedge \mathcal{F}^j - \frac{\mathcal{K}}{4} \mathcal{F}^0 \wedge \star \mathcal{F}^0 + \frac{\mathcal{K}}{2} \mathcal{F}^0 \wedge \star \sum_i \frac{b_i}{v_i^2} \mathcal{F}^i \\ &\quad - \frac{1}{4} e^{-4\phi} dB \wedge \star dB - \mathcal{K} \sum_i v_i^{-2} B \wedge \star B + B \wedge \left\{ \star \left(\sum_i \mathcal{K} v_i^{-2} \mathcal{F}^i \right) + \frac{1}{2} \mathcal{K}_{ijk} b_k m_i \mathcal{F}^j \right. \\ &\quad \left. + \frac{e_0}{2} \mathcal{F}^0 \right\}, \\ \delta V_2 &= e^{4\phi} \mathcal{K}^{-1} \left(2v_1^2 (b_1 + b_2)^2 + 4b_1^2 v_2^2 \right) + e_0 e^{4\phi} \mathcal{K}^{-1} \left(2b_1 b_2 + b_1^2 \right). \end{aligned} \quad (3.43)$$

Dualizing B as per (2.15) and (2.17) we get

$$\begin{aligned} \delta \mathcal{L}_{2,kin} &= -\frac{1}{4} v_i^{-2} (\partial b_i)^2 \star 1 - \frac{1}{4} K_{ijk} b_k \mathcal{F}^i \wedge \mathcal{F}^j - \frac{\mathcal{K}}{4} \mathcal{F}^0 \wedge \star \mathcal{F}^0 + \frac{\mathcal{K}}{2} \mathcal{F}^0 \wedge \star \sum_i \frac{b_i}{v_i^2} \mathcal{F}^i \\ &\quad - \frac{1}{M^2} \left(dA_H - \frac{1}{2} J_2 \right) \wedge \star \left(dA_H - \frac{1}{2} J_2 \right) - 4e^{4\phi} A_H \wedge \star A_H, \end{aligned} \quad (3.44)$$

with

$$M^2 = 2v_2 + \frac{v_1^2}{v_2}, \quad J_a = -2(b_1 + b_2) \mathcal{F}^1 - 2b_1 \mathcal{F}^2 - \frac{e_0}{2} \mathcal{F}^0, \quad J_b = -2v_2 \mathcal{F}^1 - \frac{v_1^2}{v_2} \mathcal{F}^2. \quad (3.45)$$

Within the effective action (3.43) we consider linearized fluctuations

$$\begin{aligned} A^0 &= \delta A_t^0 dt + \delta A_{x_2}^0 dx_2 + \delta A_r^0 dr, & A_H &= \delta A_t^H dt + \delta A_{x_2}^H dx_2 + \delta A_r^H dr, \\ \delta A^i &= \delta A_{x_1}^i dx_1 + \delta A_r^i dr, & b_i &= \delta b_i, \end{aligned} \quad (3.46)$$

about the black membrane background (3.19), which we take to be functions of t , r , and x_2 as follows:

$$A_{t,x_2,r}^{0,H} = e^{-iwt+ikx_2} \cdot \mathcal{A}_{t,2,r}^{0,H}(r) \quad \delta A_{x_1,r}^i = e^{-iwt+ikx_2} \cdot \mathcal{A}_{1,r}^i(r), \quad \delta b_i = e^{-iwt+ikx_2} \cdot \mathcal{B}_i(r). \quad (3.47)$$

Note that δA^i is turned on “magnetically”, in the sense that it is the only form with support in the x_1 direction, while we are considering fluctuations with a spatial profile in the x_2 direction. It is straightforward to verify that the set (3.46) will decouple from the remaining fluctuations in the helicity-0 (the sound channel) sector. We use the bulk gauge transformations (2.14) to set

$$\delta \mathcal{A}_r^i = \delta \mathcal{A}_r^0 \equiv 0. \quad (3.48)$$

which lead to the constraints

$$0 = (\mathcal{A}_2^H)' + \frac{c_2^2 w}{c_1^2 k} (\mathcal{A}_t^H)' + \frac{3}{2} (\mathcal{A}_2^0)' + \frac{3c_2^2 w}{2c_1^2 k} (\mathcal{A}_t^0)' - \frac{c_2^2 w (\mathcal{B}_1(2v_2^2 - v_1^2) - \mathcal{B}_2 v_1^2) F}{c_1^2 v_1^2 k} \\ - \frac{i}{c_1^2 v_2 k} \left(4c_1^2 (2v_2^2 + v_1^2) c_2^2 g^4 - v_2 (c_2^2 w^2 - c_1^2 k^2) \right) \mathcal{A}_r^H, \quad (3.49)$$

$$0 = (2v_2^2 v_1^2 + v_1^4 + 9) \left((\mathcal{A}_t^0)' + \frac{c_1^2 k}{c_2^2 w} (\mathcal{A}_2^0)' \right) + 6(\mathcal{A}_t^H)' + \frac{6c_1^2 k}{c_2^2 w} (\mathcal{A}_2^H)' + F \left(-\frac{2}{v_1^2} (2v_2^2 v_1^2 \right. \\ \left. + v_1^4 + 6v_2^2 - 3v_1^2) \mathcal{B}_1 + 2\mathcal{B}_2 (2v_2^2 + v_1^2 + 3) \right) - \frac{6i (c_1^2 k^2 - c_2^2 w^2)}{w c_2^2} \mathcal{A}_r^H, \quad (3.50)$$

as well as 2 more equations which can be solved for the metric components. The equations of motion for the remaining fluctuations take the form

$$0 = (\mathcal{A}_t^0)'' + \left(\frac{2v_1'}{v_1} - \frac{c_3'}{c_3} + \frac{2c_2'}{c_2} - \frac{c_1'}{c_1} + \frac{v_2'}{v_2} \right) (\mathcal{A}_t^0)' - \frac{c_3^2 k}{c_2^2} (\mathcal{A}_t^0 k + \mathcal{A}_2^0 w) + \frac{24g^4 c_3^2}{v_2 v_1^2} \mathcal{A}_t^H \\ - \frac{2F'}{v_1^2} (\mathcal{B}_1 - \mathcal{B}_2) + F \left(-\frac{2}{v_1^2} (\mathcal{B}_1' - \mathcal{B}_2') + \left(-\frac{2v_2'}{v_2 v_1^2} + \frac{2c_3'}{v_1^2 c_3} - \frac{4c_2'}{c_2 v_1^2} + \frac{2c_1'}{v_1^2 c_1} \right) (\mathcal{B}_1 - \mathcal{B}_2) \right), \quad (3.51)$$

$$0 = (\mathcal{A}_2^0)'' + \left(\frac{2v_1'}{v_1} + \frac{v_2'}{v_2} - \frac{c_3'}{c_3} + \frac{c_1'}{c_1} \right) (\mathcal{A}_2^0)' + \frac{c_3^2 w}{c_1^2} (\mathcal{A}_t^0 k + \mathcal{A}_2^0 w) + \frac{24g^4 c_3^2}{v_2 v_1^2} \mathcal{A}_2^H, \quad (3.52)$$

$$\begin{aligned}
0 = & (\mathcal{A}_t^H)'' + \left(\frac{2c_2'}{c_2} - \frac{c_1'}{c_1} - \frac{c_3'}{c_3} + \frac{v_1^2 - 2v_2^2}{(2v_2^2 + v_1^2)v_2} v_2' - \frac{2v_1}{2v_2^2 + v_1^2} v_1' \right) (\mathcal{A}_t^H)' - \left(4g^4 c_2^2 (2v_2^2 v_1^2 \right. \\
& + v_1^4 + 9) + v_2 v_1^2 k^2 \left. \right) \frac{c_3^2}{v_1^2 c_2^2 v_2} \mathcal{A}_t^H + iw(\mathcal{A}_r^H)' + iw\mathcal{A}_r^H \left(\frac{2c_2'}{c_2} - \frac{c_1'}{c_1} - \frac{c_3'}{c_3} - \frac{2v_1}{2v_2^2 + v_1^2} v_1' \right. \\
& + \frac{v_1^2 - 2v_2^2}{(2v_2^2 + v_1^2)v_2} v_2' \left. \right) - \frac{6(v_1 v_2 v_2' + v_1'(v_1^2 + v_2^2))}{v_1(2v_2^2 + v_1^2)} (\mathcal{A}_t^0)' + \frac{(v_1^2 - 2v_2^2 + 3)F}{v_1^2} \mathcal{B}_1' \\
& + \frac{(v_1^2 - 3)F}{v_1^2} \mathcal{B}_2' + \frac{2ic_1 c_3 v_1 k (v_2' v_1 - v_2 v_1')}{(2v_2^2 + v_1^2) c_2^2} (\mathcal{A}_1^1 - \mathcal{A}_1^2) + \left(\frac{(v_1^2 - 2v_2^2 + 3)F}{v_1^2} \left(\frac{2c_2'}{c_2} - \frac{c_1'}{c_1} \right. \right. \\
& - \frac{c_3'}{c_3} + \frac{F'}{F} \left. \right) - \frac{2F(v_1^4 - 4v_2^2 v_1^2 - 4v_2^4)}{(2v_2^2 + v_1^2) v_1^3} v_1' + v_2' \frac{F(v_1^4 - 8v_2^2 v_1^2 - 4v_2^4 + 3v_1^2 + 6v_2^2)}{v_1^2 (2v_2^2 + v_1^2) v_2} \left. \right) \mathcal{B}_1 \\
& - \frac{c_3^2 k w}{c_2^2} \mathcal{A}_2^H + \left(\frac{(v_1^2 - 3)F}{v_1^2} \left(\frac{2c_2'}{c_2} - \frac{c_1'}{c_1} - \frac{c_3'}{c_3} + \frac{F'}{F} \right) - \frac{2Fv_1}{2v_2^2 + v_1^2} v_1' \right. \\
& + \left. \frac{F(v_1^4 - 2v_2^2 v_1^2 - 3v_1^2 - 6v_2^2)}{v_1^2 (2v_2^2 + v_1^2) v_2} v_2' \right) \mathcal{B}_2,
\end{aligned} \tag{3.53}$$

$$\begin{aligned}
0 = & (\mathcal{A}_2^H)'' + \left(\frac{c_1'}{c_1} - \frac{c_3'}{c_3} - \frac{2v_1}{2v_2^2 + v_1^2} v_1' - \frac{2v_2^2 - v_1^2}{v_2(2v_2^2 + v_1^2)} v_2' \right) (\mathcal{A}_2^H)' - \left(4c_1^2 g^4 (2v_2^2 v_1^2 \right. \\
& + v_1^4 + 9) - v_2 v_1^2 w^2 \left. \right) \frac{c_3^2}{v_2 v_1^2 c_1^2} \mathcal{A}_2^H + \frac{c_3^2 k w}{c_1^2} \mathcal{A}_t^H - ik(\mathcal{A}_r^H)' \\
& + ik \left(\frac{c_3'}{c_3} - \frac{c_1'}{c_1} + \frac{2v_1}{2v_2^2 + v_1^2} v_1' + \frac{2v_2^2 - v_1^2}{v_2(2v_2^2 + v_1^2)} v_2' \right) \mathcal{A}_r^H \\
& - \frac{6(v_1 v_2 v_2' + v_1'(v_1^2 + v_2^2))}{v_1(2v_2^2 + v_1^2)} (\mathcal{A}_2^0)' - \frac{2ic_3 v_1 w (v_2 v_1' - v_1 v_2')}{c_1(2v_2^2 + v_1^2)} (\mathcal{A}_1^2 - \mathcal{A}_1^1),
\end{aligned} \tag{3.54}$$

$$\begin{aligned}
0 = & (\mathcal{A}_1^1 - \mathcal{A}_1^2)'' + \left(\frac{c_1'}{c_1} - \frac{c_3'}{c_3} + \frac{4v_2^2}{v_1(2v_2^2 + v_1^2)} v_1' - \frac{2v_2^2 - v_1^2}{v_2(2v_2^2 + v_1^2)} v_2' \right) (\mathcal{A}_1^1 - \mathcal{A}_1^2)' \\
& - \frac{2i(v_2 v_1' - v_1 v_2') c_3}{c_1 v_1 (2v_2^2 + v_1^2)} \left(2k\mathcal{A}_t^H + 2w\mathcal{A}_2^H + 3k\mathcal{A}_t^0 + 3w\mathcal{A}_2^0 \right) + \frac{iFkc_3}{c_1 v_2} \left(\mathcal{B}_2 - \frac{4v_2^2}{v_1^2} \mathcal{B}_1 \right) \\
& - \frac{(c_2^2 w^2 - c_1^2 k^2) c_3^2}{c_1^2 c_2^2} (\mathcal{A}_1^2 - \mathcal{A}_1^1),
\end{aligned} \tag{3.55}$$

$$\begin{aligned}
0 = (\mathcal{B}_1)'' + \left(\frac{2c_2'}{c_2} + \frac{c_1'}{c_1} - \frac{c_3'}{c_3} - \frac{2v_1'}{v_1} \right) (\mathcal{B}_1)' + \left(-4c_2^2 c_1^2 c_3^2 v_1^2 (2v_2^2 + v_1^2 + 3)(2v_2^2 + v_1^2) g^4 \right. \\
+ c_3^2 v_2 v_1^2 (2v_2^2 + v_1^2) (c_2^2 w^2 - c_1^2 k^2) + 2c_2^2 F^2 v_2^2 (2v_2^2 - v_1^2)^2 \left. \right) \left(c_1^2 v_1^2 v_2 (2v_2^2 + v_1^2) c_2^2 \right)^{-1} \mathcal{B}_1 \\
- \frac{F v_2 (2v_2^2 v_1^2 + v_1^4 + 6v_2^2 - 3v_1^2)}{c_1^2 (2v_2^2 + v_1^2)} (\mathcal{A}_t^0)' - \frac{2F v_2 (2v_2^2 - v_1^2)}{c_1^2 (2v_2^2 + v_1^2)} (\mathcal{A}_t^H)' - 2 \left(2c_1^2 c_3^2 (v_1^2 + 3)(2v_2^2 \right. \\
+ v_1^2) g^4 + F^2 v_2^2 (2v_2^2 - v_1^2) \left. \right) \left(c_1^2 v_2 (2v_2^2 + v_1^2) \right) \mathcal{B}_2 - \frac{2iF v_2 w (2v_2^2 - v_1^2)}{c_1^2 (2v_2^2 + v_1^2)} \mathcal{A}_r^H \\
- \frac{4iv_1^2 c_3 F v_2^2 k}{c_1 (2v_2^2 + v_1^2) c_2^2} \left(\mathcal{A}_1^2 - \mathcal{A}_1^1 \right), \tag{3.56}
\end{aligned}$$

$$\begin{aligned}
0 = (\mathcal{B}_2)'' + \left(\frac{2c_2'}{c_2} + \frac{c_1'}{c_1} - \frac{c_3'}{c_3} - \frac{2v_2'}{v_2} \right) (\mathcal{B}_2)' + \left(-8c_2^2 c_1^2 c_3^2 v_2 (2v_2^2 + v_1^2) g^4 + c_3^2 (2v_2^2 \right. \\
+ v_1^2) (c_2^2 w^2 - c_1^2 k^2) + 4c_2^2 F^2 v_2^3 \left. \right) \left(c_1^2 (2v_2^2 + v_1^2) c_2^2 \right)^{-1} \mathcal{B}_2 + \frac{2F v_2^3 (2v_2^2 + v_1^2 + 3)}{c_1^2 (2v_2^2 + v_1^2)} (\mathcal{A}_t^0)' \\
+ \frac{4F v_2^3}{c_1^2 (2v_2^2 + v_1^2)} (\mathcal{A}_t^H)' - 4 \left(2c_1^2 c_3^2 (v_1^2 + 3)(2v_2^2 + v_1^2) g^4 + F^2 v_2^2 (2v_2^2 - v_1^2) \right) v_2 \left(c_1^2 v_1^2 (2v_2^2 \right. \\
+ v_1^2) \left. \right)^{-1} \mathcal{B}_1 + \frac{4iv_2^3 F w}{c_1^2 (2v_2^2 + v_1^2)} \mathcal{A}_r^H + \frac{2iv_2^2 c_3 F v_1^2 k}{c_1 (2v_2^2 + v_1^2) c_2^2} \left(\mathcal{A}_1^2 - \mathcal{A}_1^1 \right), \tag{3.57}
\end{aligned}$$

where, compare with (3.19),

$$c_1 = \frac{2\alpha\sqrt{f}}{r}, \quad c_2 = \frac{2\alpha}{r}, \quad c_3 = \frac{s}{2r\sqrt{f}}, \quad F = \frac{q\alpha s}{v_2}. \tag{3.58}$$

We explicitly verified that (3.49) and (3.50) are consistent with (3.51)-(3.57). Fluctuations of the $U(1)$ R -charge bulk potential A^0 excite the axions b_1 and b_2 (see (3.56) and (3.57)). Following [15], the holographic spectroscopy relates the pseudoscalars $\{b_1, b_2\}$ to the boundary gauge theory operators $\delta\mathcal{O}_\Delta$ of conformal dimension Δ as in table 2. Here again, we have the choice to quantize one of the fluctuations so that

Table 2: Holographic spectroscopy of the pseudoscalars

mass eigenstate	$m^2 L^2$	Δ	$U(1)$ R-charge
$b_1 - b_2$	-2	(2, 1)	0
$2b_1 + b_2$	10	5	0

it corresponds either to a CFT operator of dimension 2 (*normal quantization*) or of

dimension 1 (*alternative quantization*). This choice is independent from the choice of quantization for the background solution.

To proceed we introduce

$$Z \equiv \mathfrak{q} \mathcal{A}_t^0 + \mathfrak{w} \mathcal{A}_2^0, \quad \mathcal{A} \equiv \mathcal{A}_1^1 - \mathcal{A}_1^2, \quad (3.59)$$

where

$$\mathfrak{w} = \frac{w}{2\pi T}, \quad \mathfrak{q} = \frac{k}{2\pi T}, \quad (3.60)$$

and T is the Hawking temperature of the black membrane. We use the constraints (3.49) and (3.50) to eliminate $\mathcal{A}_r^H$ and obtain from (3.51)-(3.57) a decoupled set of the second-order equations for

$$\{ Z, \mathcal{A}_t^H, \mathcal{A}_2^H, \mathcal{A}, \mathcal{B}_1, \mathcal{B}_2 \}. \quad (3.61)$$

Solutions of the resulting equations with appropriate boundary conditions determine the spectrum of baryonic black membranes quasinormal modes — equivalently the physical spectrum of linearized fluctuations in membrane gauge theory plasma with a baryonic chemical potential. Following [23,24] we impose the incoming-wave boundary conditions at the black membrane horizon, and 'normalizability' at asymptotic AdS_4 boundary. Focusing on the $\text{Re}[\mathfrak{w}] = 0$ diffusive branch, and introducing

$$\begin{aligned} Z &= (1-r)^{-i\mathfrak{w}/2} z, & \mathcal{A}_t^H &= (1-r)^{-i\mathfrak{w}/2} a_t^H, & \mathcal{A}_2^H &= i(1-r)^{-i\mathfrak{w}/2} a_2^H, \\ \mathcal{A} &= i(1-r)^{-i\mathfrak{w}/2} a, & \mathcal{B}_i &= (1-r)^{-i\mathfrak{w}/2} B_i, & \mathfrak{w} &= -iv \mathfrak{q}, \end{aligned} \quad (3.62)$$

we solve the fluctuation equations subject to the asymptotics:

■ in the UV, *i.e.*, as $r \rightarrow 0_+$, and with the identifications¹¹ $\ln[v_1 v_2^{-1}] \iff \mathcal{O}_2$ and $(b_1 - b_2) \iff \delta\mathcal{O}_2^b$,

$$\begin{aligned} z &= \mathfrak{q}r - \frac{1}{2}\mathfrak{q}^2 v r^2 + \mathcal{O}(r^3), & a_t^H &= 2qb_{1,2}r^3 + \left(a_{t,4}^h - \frac{2}{7}a_1\mathfrak{q}T\pi v_{1,2} \ln r \right) r^4 + \mathcal{O}(r^5 \ln r), \\ a_2^H &= \left(a_{2,4}^h - \frac{2}{7}T\pi v_{1,2}\mathfrak{q}v a_1 \ln r \right) r^4 + \mathcal{O}(r^5 \ln r), & a &= a_1 r - \frac{1}{2}a_1\mathfrak{q}v r^2 + \mathcal{O}(r^3), \\ B_1 &= b_{1,2}r^2 - \frac{1}{2}b_{1,2}\mathfrak{q}v r^3 + \frac{1}{24(v^2+1)} \left(\pi^2 T^2 b_{1,2} \mathfrak{q}^2 (v^2+1)^2 + 3b_{1,2}(v^2+1)(\mathfrak{q}^2 v^2 - 2\mathfrak{q}v \right. \\ &\quad \left. - 8v_{1,2}) + 2q \right) r^4 + \left(b_{1,5} + \frac{1}{84}a_1\mathfrak{q}T\pi q \ln r \right) r^5 + \mathcal{O}(r^6 \ln r), \\ B_2 &= -2b_{1,2}r^2 + b_{1,2}\mathfrak{q}r^3 v + \mathcal{O}(r^4), \end{aligned} \quad (3.63)$$

¹¹Likewise, we develop the UV expansions for the alternative quantization of either the background, $\ln[v_1 v_2^{-1}]$, or the fluctuation, $(b_1 - b_2)$, (pseudo)scalars: $\{\mathcal{O}_2, \delta\mathcal{O}_1^b\}$, $\{\mathcal{O}_1, \delta\mathcal{O}_2^b\}$, and $\{\mathcal{O}_1, \delta\mathcal{O}_1^b\}$.

specified, for a fixed background and a momentum $\mathbf{q}$, by

$$\left\{ v, a_{t,4}^h, a_{2,4}^h, a_1, b_{1,2}, b_{1,5} \right\}; \quad (3.64)$$

■ in the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0_+$,

$$z = z_0^h + \mathcal{O}(y), \quad a_{t,2}^H = a_{t,2;0}^{H,h} + \mathcal{O}(y), \quad a = a_0^h + \mathcal{O}(y), \quad B_i = b_{i;0}^h + \mathcal{O}(y), \quad (3.65)$$

specified by

$$\left\{ z_0^h, a_{t;0}^{H,h}, a_{2;0}^{H,h}, a_0^h, b_{1;0}^h, b_{2;0}^h \right\}. \quad (3.66)$$

Note that in total we have $6 + 6 = 12$ parameters, see (3.64) and (3.66), which is precisely what is necessary to identify a solution of a coupled system of 6 second-order ODEs for $\{z, a_t^H, a_2^H, a, B_1, B_2\}$. Furthermore, since the equations are linear in the fluctuations, we can, without loss of generality, normalize the solutions so that

$$\lim_{r \rightarrow 0} \frac{dz}{dr} = \mathbf{q}. \quad (3.67)$$

Once we fix the background, and solve the fluctuation equations of motion, we obtain $v = v(\mathbf{q})$. Given v we extract the R -charge diffusion coefficient $\mathcal{D}$, as

$$\mathfrak{w} = -i \cdot \underbrace{2\pi T D}_{\equiv \mathcal{D}} \cdot \mathbf{q}^2 + \mathcal{O}(\mathbf{q}^3), \quad \mathcal{D} \equiv \left. \frac{dv}{d\mathbf{q}} \right|_{\mathbf{q}=0}. \quad (3.68)$$

For general values of q we have to solve the fluctuation equations numerically. At $q = 0$, an analytic solution is possible in the limit $\mathbf{q} \rightarrow 0$ — which is precisely what is needed to extract $\mathcal{D}$, see (3.68). Specifically, at $q = 0$, we have¹² $a_t^H = a_2^H = a = B_1 = B_2 = 0$ and

$$\begin{aligned} 0 = z'' + \frac{(\mathbf{q}(r^2 + r + 1)(r^3 - v^2 - 1) + 3r^2 v)v}{(r^3 - v^2 - 1)(1 - r^3)} z' - \frac{\mathbf{q}}{4(1 - r^3)(r^3 - v^2 - 1)(r^2 + r + 1)} \\ \times \left((r^3 v^2 + 3r^2 v^2 + 6rv^2 + 9r^2 + 8v^2 + 9r + 9)(r^3 - v^2 - 1)\mathbf{q} + 2v(r^2 + r + 1)(r^4 \right. \\ \left. + 2r^3 + 2rv^2 + 3r^2 + v^2 + 2r + 1) \right) z. \end{aligned} \quad (3.69)$$

¹²The background geometry is given by (3.38).

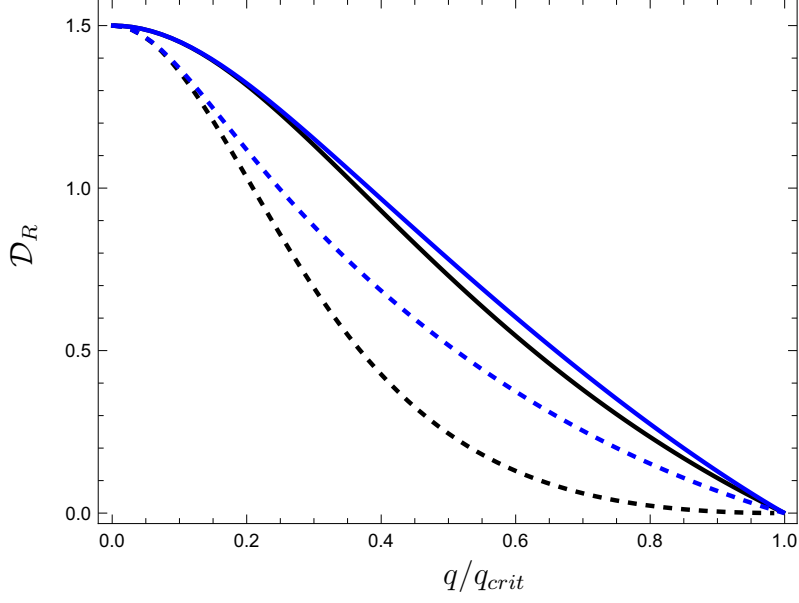


Figure 3: R -charge dimensionless diffusion coefficient $\mathcal{D}_R = 2\pi T D$ of the baryonic membrane theory plasma for different quantizations of the gravitational dual (pseudo)scalars $\{\ln[v_1 v_2^{-1}], b_1 - b_2\}$: $\{\mathcal{O}_2, \delta\mathcal{O}_2^b\}$ (black,solid), $\{\mathcal{O}_2, \delta\mathcal{O}_1^b\}$ (black,dashed), $\{\mathcal{O}_1, \delta\mathcal{O}_2^b\}$ (blue,solid), $\{\mathcal{O}_1, \delta\mathcal{O}_1^b\}$ (blue,dashed). At $q = 0$, $\mathcal{D}_R = \frac{3}{2}$ (3.72), while it vanishes in the quantum critical regime $q \rightarrow q_{crit}$, correspondingly $T \rightarrow 0$.

Furthermore, in the hydrodynamics limit,

$$z = \mathbf{q} \, r + \mathbf{q}^3 \, z_2(r) + \mathcal{O}(\mathbf{q}^5), \quad v = \mathcal{D} \, \mathbf{q} + \mathcal{O}(\mathbf{q}^3), \quad (3.70)$$

we find (imposing the UV boundary condition (3.63))

$$\begin{aligned} z_2 = & -\frac{1}{12}(2\mathcal{D} - 3)(2\mathcal{D} - 3r + 3) \ln(1 - r) + \frac{3}{8}(r + 2) \ln(r^2 + r + 1) \\ & + \frac{\sqrt{3}}{12}(4\mathcal{D}^2 - 9r) \arctan \frac{2r + 1}{\sqrt{3}} + \frac{1}{8}r(\sqrt{3}\pi - 8\mathcal{D}^2) - \frac{\sqrt{3}}{18}\pi\mathcal{D}^2 + \frac{1}{6}\mathcal{D}^2 \ln(r^2 + r + 1). \end{aligned} \quad (3.71)$$

Finally, the regularity of z_2 at the horizon, *i.e.*, as $r \rightarrow 1$, determines

$$\mathcal{D} \Big|_{q=0} = \frac{3}{2}. \quad (3.72)$$

For $q \in (0, q_{crit})$ (3.39) the R -charge diffusion coefficient of the baryonic membrane theory plasma is computed numerically, see fig. 3. We use the same color scheme as

for the background in figs. 1 and 2: the black curves correspond to quantization of the background mode $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_2$ and the blue curves to the identification $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_1$. Furthermore, the (solid,dashed) curves represent the pseudoscalar quantization $(b_1 - b_2) \Leftrightarrow (\delta\mathcal{O}_2^b, \delta\mathcal{O}_1^b)$ correspondingly. In all cases we find the diffusion coefficient $\mathcal{D} > 0$ for $q \in (0, q_{crit})$: the R -charge transport is stable. In the quantum critical regime $(q_{crit} - q) \ll q_{crit}$, equivalently $T/\alpha \rightarrow 0$ (3.41), $\mathcal{D} \propto (q_{crit} - q) \propto T/\alpha$ (the solid curves and the blue dashed curve), except for the quantization $\{\ln[v_1 v_2^{-2}], (b_1 - b_2)\} \Leftrightarrow \{\mathcal{O}_2, \delta\mathcal{O}_1^b\}$ (the dashed black curve) when $\mathcal{D} \propto (q_{crit} - q)^2 \propto (T/\alpha)^2$.

3.4 Threshold instabilities from condensation of $(b_1 - b_2)$

Consider spatially homogeneous and isotropic fluctuations of the bulk pseudoscalars b_1 and b_2 about baryonic black membrane of section 3.2. The corresponding equations of motion can be obtained from (3.49)-(3.57) in the limit

$$\{w, k\} \rightarrow 0, \quad (3.73)$$

provided¹³ we replace (3.50) with $\mathcal{A}_r^H = 0$. The decoupled set of linearized equations containing $\mathcal{B}_1$ and $\mathcal{B}_2$ is:

$$\begin{aligned} 0 = & \mathcal{B}_1'' + \left(\frac{v_2 g^4 s^2}{2rf} + \frac{r^3 v_2 s^2 q^2}{4f v_1^2} + \frac{v_1^2 g^4 s^2}{4rf v_2} + \frac{r^3 s^2 q^2}{8f v_2} - \frac{4g^2 s^2}{rf v_1} - \frac{2g^2 s^2}{rf v_2} + \frac{9g^4 s^2}{4rf v_2 v_1^2} - \frac{2v_1'}{v_1} \right. \\ & + \left. \frac{1}{r} \right) \mathcal{B}_1' - \frac{sq r^2 (2v_2^2 - v_1^2)}{2f(2v_2^2 + v_1^2)} (\mathcal{A}_t^H)' - \frac{sr^2 q (2v_2^2 v_1^2 + v_1^4 + 6v_2^2 - 3v_1^2)}{4f(2v_2^2 + v_1^2)} a \\ & + \left(-\frac{(2v_2^2 + v_1^2 + 3)s^2 g^4}{f v_2 r^2} + \frac{s^2 r^2 q^2 (2v_2^2 - v_1^2)^2}{2(v_1^2 f(2v_2^2 + v_1^2) v_2)} \right) \mathcal{B}_1 + \left(-\frac{(v_1^2 + 3)s^2 g^4}{f v_2 r^2} \right. \\ & \left. - \frac{r^2 q^2 s^2 (2v_2^2 - v_1^2)}{2v_2 f(2v_2^2 + v_1^2)} \right) \mathcal{B}_2, \end{aligned} \quad (3.74)$$

$$\begin{aligned} 0 = & \mathcal{B}_2'' + \left(\frac{v_2 g^4 s^2}{2rf} + \frac{r^3 v_2 s^2 q^2}{4f v_1^2} + \frac{v_1^2 g^4 s^2}{4rf v_2} + \frac{r^3 s^2 q^2}{8f v_2} - \frac{4g^2 s^2}{rf v_1} - \frac{2g^2 s^2}{rf v_2} + \frac{9g^4 s^2}{4rf v_2 v_1^2} - \frac{2v_2'}{v_2} \right. \\ & + \left. \frac{1}{r} \right) \mathcal{B}_2' + \frac{qsr^2 v_2^2}{f(2v_2^2 + v_1^2)} (\mathcal{A}_t^H)' + \frac{qsr^2 v_2^2 (2v_2^2 + v_1^2 + 3)}{2f(2v_2^2 + v_1^2)} a + \left(-\frac{2(v_1^2 + 3)s^2 v_2 g^4}{f v_1^2 r^2} \right. \\ & \left. - \frac{v_2 s^2 r^2 q^2 (2v_2^2 - v_1^2)}{v_1^2 f(2v_2^2 + v_1^2)} \right) \mathcal{B}_1 - \frac{v_2 s^2 (4v_2^2 g^4 + 2g^4 v_1^2 - r^4 q^2)}{f r^2 (2v_2^2 + v_1^2)} \mathcal{B}_2, \end{aligned} \quad (3.75)$$

¹³We explicitly verified this.

$$0 = a' + \left(\frac{r(v_2')^2}{4v_2^2} + \frac{r(v_1')^2}{2v_1^2} + \frac{r(g')^2}{g^2} + \frac{v_2'}{v_2} + \frac{2v_1'}{v_1} \right) a + \frac{6s^2g^4}{v_2v_1^2r^2f} \mathcal{A}_t^H - \frac{2sq}{v_2v_1^2} (\mathcal{B}_1' - \mathcal{B}_2'), \quad (3.76)$$

$$\begin{aligned} 0 = & (\mathcal{A}_t^H)'' + \left(\frac{r(g')^2}{g^2} + \frac{r(v_1')^2}{2v_1^2} + \frac{r(v_2')^2}{4v_2^2} - \frac{(2v_2^2 - v_1^2)v_2'}{v_2(2v_2^2 + v_1^2)} - \frac{2v_1'v_1}{2v_2^2 + v_1^2} \right) (\mathcal{A}_t^H)' \\ & - \frac{(2v_2^2v_1^2 + v_1^4 + 9)g^4s^2}{fv_2v_1^2r^2} \mathcal{A}_t^H - \frac{(2v_2^2 - v_1^2 - 3)sq}{v_1^2v_2} \mathcal{B}_1' + \frac{(v_1^2 - 3)sq}{v_1^2v_2} \mathcal{B}_2' \\ & - \frac{6(v_2'v_2v_1 + v_2^2v_1' + v_1'v_1^2)}{(2v_2^2 + v_1^2)v_1} a + \left(\frac{2sq(4v_2^4 + 4v_2^2v_1^2 - v_1^4)v_1'}{v_2(2v_2^2 + v_1^2)v_1^3} - \frac{8v_2'sq}{2v_2^2 + v_1^2} \right) \mathcal{B}_1 \\ & - \frac{2sq(2v_2'v_2 + v_1'v_1)}{v_2(2v_2^2 + v_1^2)} \mathcal{B}_2, \end{aligned} \quad (3.77)$$

where we introduced $a \equiv (\mathcal{A}_t^0)'$.

- In the UV, *i.e.*, as $r \rightarrow 0_+$, and with the identification¹⁴ $\ln[v_1v_2^{-1}] \iff \mathcal{O}_2$,

$$\begin{aligned} \mathcal{B}_1 = & b_{1,1}r + b_{1,2}r^2 - \frac{1}{5}v_{1,2}b_{1,1}r^3 + \left(-\frac{1}{6}f_3b_{1,1} - v_{1,2}b_{1,2} \right) r^4 + \left(b_{1,5} + \left(-\frac{1}{70}q^2b_{1,1} \right. \right. \\ & \left. \left. + \frac{12}{35}v_{1,2}^2b_{1,1} \right) \ln r \right) r^5 + \mathcal{O}(r^6 \ln r), \end{aligned} \quad (3.78)$$

$$\begin{aligned} \mathcal{B}_2 = & -2b_{1,1}r - 2b_{1,2}r^2 + \frac{14}{5}v_{1,2}b_{1,1}r^3 + \left(\frac{1}{3}f_3b_{1,1} + 2v_{1,2}b_{1,2} \right) r^4 + \left(-\frac{39}{20}v_{1,2}^2b_{1,1} \right. \\ & \left. + f_3b_{1,2} - \frac{1}{5}q^2b_{1,1} + b_{1,5} - \frac{3}{2}b_{1,1}v_{1,4} + \left(\frac{1}{35}q^2b_{1,1} - \frac{24}{35}v_{1,2}^2b_{1,1} \right) \ln r \right) r^5 + \mathcal{O}(r^6 \ln r), \end{aligned} \quad (3.79)$$

$$\begin{aligned} a = & \frac{12}{5}qb_{1,1}r - (6b_{1,1}qv_{1,2} + 2a_{t,4}^h)r^3 + \left(\frac{1}{2}f_3qb_{1,1} - 6qv_{1,2}b_{1,2} \right) r^4 + \mathcal{O}(r^5 \ln r), \\ \mathcal{A}_t^H = & \frac{3}{5}qb_{1,1}r^2 + 2qb_{1,2}r^3 + a_{t,4}^hr^4 + \mathcal{O}(r^5). \end{aligned} \quad (3.80)$$

Notice that $\lim_{r \rightarrow 0} a = 0$ — this ensures that the fluctuations $\{\mathcal{B}_i, \mathcal{A}_t^0, \mathcal{A}_t^H\}$ have the vanishing R -charge. In the quantization where $(b_1 - b_2)$ is identified with the boundary gauge theory operator $\delta\mathcal{O}_2^b$ the coefficient $b_{1,1}$ is the source, while in the identification $(b_1 - b_2) \iff \delta\mathcal{O}_1^b$ the source term is $b_{1,2}$.

- In the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0$,

$$\mathcal{B}_1 = b_{1,0}^h + \mathcal{O}(y), \quad \mathcal{B}_2 = b_{2,0}^h + \mathcal{O}(y), \quad \mathcal{A}_t^H = a_{t,1}^{h,h} y + \mathcal{O}(y^2), \quad a = a_0^h + \mathcal{O}(y). \quad (3.81)$$

¹⁴Likewise, we develop the UV expansions for the alternative quantization of the background scalars $\ln[v_1v_2^{-1}] \iff \mathcal{O}_1$.

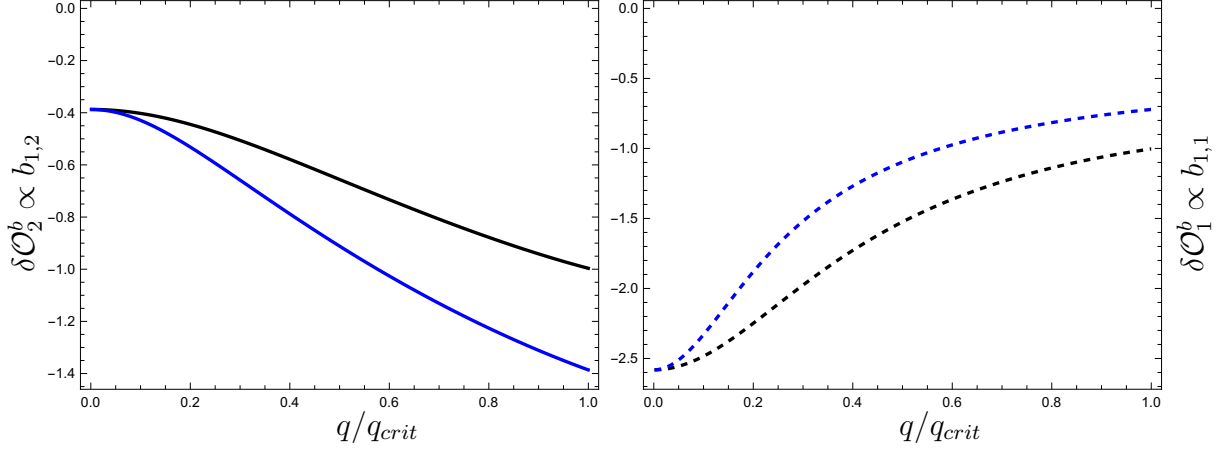


Figure 4: We test the instability of the baryonic black membranes due to the condensation of the neutral ($b_1 - b_2$) mode: the instability would be signaled by the divergence of the response of the corresponding operator for its fixed source, as we vary q/q_{crit} . The color coding is as in fig. 3.

Following [25], to identify the onset of the instability associated with the condensation of $\delta\mathcal{O}_2^b$ (or $\delta\mathcal{O}_1^b$) we keep fixed the source term of the operator, $b_{1,1} = 1$ (or $b_{1,2} = 1$), and scan q (correspondingly T , see (3.41)) looking for the divergence of the expectation value of the corresponding operator $\langle\delta\mathcal{O}_2^b\rangle \propto b_{1,2}$ (or $\langle\delta\mathcal{O}_1^b\rangle \propto b_{1,1}$). A divergence signals the presence of a homogeneous and isotropic normalizable mode of the fluctuations of $(b_1 - b_2)$ — the threshold for the instability. Results of such scans are presented in fig. 4: there are no divergences of the expectation values of $\delta\mathcal{O}^b$ operators.

4 Reissner–Nordström black membrane

Besides the baryonic black membranes described in the previous section, the truncation described in section 2 also features black membranes of the Reissner–Nordström type charged under the “non-topological” R-charge $U(1)$. In this section we describe these backgrounds and analyze whether they are stable with respect to *baryonic* charge density fluctuations, the threshold instability, and the superconducting instability.

4.1 Background

Minimal $\mathcal{N} = 4$ $D = 4$ gauged supergravity can be obtained from (2.13) truncating the complex scalars $t^1 = t^2 = 1 + i0$, and the massive 2-form B (equivalently the massive vector A^H in (2.17)). This is achieved with the gauge fields $\mathcal{F}^0, \mathcal{F}^1, \mathcal{F}^2$ satisfying (see also [20])

$$\mathcal{F}^1 = \mathcal{F}^2 = \star \mathcal{F}^0. \quad (4.1)$$

We will be interested in $U(1)_R$ -charged black membrane solutions of the resulting effective action. Using the metric ansatz as in (3.19), the background geometry is given by

$$\begin{aligned} ds_4^2 &= -\frac{4\alpha^2 f}{r^2} dt^2 + \frac{4\alpha^2}{r^2} (dx_1^2 + dx_2^2) + \frac{1}{4r^2 f} dr^2, \quad f = 1 - r^3(1 + q^2) + q^2 r^4, \\ \mathcal{F}^0 &= a'_0 dr \wedge dt, \quad \mathcal{F}^1 = \mathcal{F}^2 = \mathcal{B} dx_1 \wedge dx_2, \quad a_0 = 2q\alpha(1 - r), \\ \mathcal{B} &= -8\alpha^2 q, \quad b_1 = b_2 = 0, \quad v_1 = v_2 = 1. \end{aligned} \quad (4.2)$$

From (4.2) we extract an R -charge chemical potential of $\mu_R = 2q\alpha$ and a Hawking temperature of the black membrane $T = \alpha(3 - q^2)/\pi$, leading to

$$\frac{T}{\mu_R} = \frac{3 - q^2}{2\pi q}. \quad (4.3)$$

Here the extremal $AdS_2 \times \mathbb{R}^2$ limit is reached as $q \rightarrow q_{crit} = \sqrt{3}$.

4.2 Fluctuations

In this section we discuss fluctuations of the baryonic charge density fluctuations about the RN black membrane of section 4.1. Within the effective action (2.13) and (2.17) we consider linearized fluctuations

$$\begin{aligned} A^0 &= \delta A_{x_1}^0 dx_1 + \delta A_r^0 dr, \quad A_H = \delta A_{x_1}^H dx_1 \\ \delta A^i &= \delta A_t^i dt + \delta A_{x_2}^i dx_2 + \delta A_r^i dr, \quad b_i = \delta b_i, \end{aligned} \quad (4.4)$$

where

$$\delta A_{t,x_2,r}^i = e^{-i\omega t + ikx_2} \mathcal{A}_{t,2,r}^i(r), \quad \delta A_{x_1,r}^{0,H} = e^{-i\omega t + ikx_2} \mathcal{A}_{1,r}^{0,H}(r), \quad \delta b_i = e^{-i\omega t + ikx_2} \mathcal{B}_i(r), \quad (4.5)$$

about the black membrane background (4.2). It is straightforward to verify that the set (4.4) will decouple from the remaining fluctuations in the helicity-0 (the sound channel) sector. We use the bulk gauge transformations (2.14) to set

$$\delta\mathcal{A}_r^i = \delta\mathcal{A}_r^0 \equiv 0. \quad (4.6)$$

The equations of motion for the remaining fluctuations take the form

$$0 = \left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)'' + \left(-\frac{c'_3}{c_3} - \frac{c'_1}{c_1} + \frac{2c'_2}{c_2}\right)\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)' - \frac{c_3^2 k^2}{c_2^2}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right) - \frac{c_3^2 k w}{c_2^2}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right) - \frac{1}{2}a'_0(4\mathcal{B}'_1 - \mathcal{B}'_2), \quad (4.7)$$

$$0 = \left(\mathcal{A}_2^1 - \mathcal{A}_2^2\right)'' + \left(-\frac{c'_3}{c_3} + \frac{c'_1}{c_1}\right)\left(\mathcal{A}_2^1 - \mathcal{A}_2^2\right)' + \frac{c_3^2 w^2}{c_1^2}\left(\mathcal{A}_2^1 - \mathcal{A}_2^2\right) + \frac{c_3^2 k w}{c_1^2}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right), \quad (4.8)$$

$$0 = \left(\mathcal{A}_1^H + 2\mathcal{A}_1^0\right)'' + \left(-\frac{c'_3}{c_3} + \frac{c'_1}{c_1}\right)\left(\mathcal{A}_1^H + 2\mathcal{A}_1^0\right)' + \frac{c_3^2(c_2^2 w^2 - c_1^2 k^2)}{c_1^2 c_2^2}\left(\mathcal{A}_1^H + 2\mathcal{A}_1^0\right), \quad (4.9)$$

$$0 = \left(\mathcal{A}_1^H + \frac{3}{2}\mathcal{A}_1^0\right)'' + \left(-\frac{c'_3}{c_3} + \frac{c'_1}{c_1}\right)\left(\mathcal{A}_1^H + \frac{3}{2}\mathcal{A}_1^0\right)' + \left(\frac{c_3^2 w^2}{c_1^2} - \frac{c_3^2 k^2}{c_2^2}\right)\left(\mathcal{A}_1^H + \frac{3}{2}\mathcal{A}_1^0\right) - 12c_3^2 \mathcal{A}_1^H - \frac{ic_3 a'_0 k}{2c_1}(2\mathcal{B}_1 + \mathcal{B}_2), \quad (4.10)$$

$$0 = \mathcal{B}_1'' + \left(-\frac{c'_3}{c_3} + \frac{c'_1}{c_1} + \frac{2c'_2}{c_2}\right)\mathcal{B}_1' + \left(-24c_3^2 + \frac{c_3^2 w^2}{c_1^2} - \frac{c_3^2 k^2}{c_2^2} - \frac{2(a'_0)^2}{3c_1^2}\right)\mathcal{B}_1 - \frac{2a'_0}{3c_1^2}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)' + \left(-16c_3^2 - \frac{4(a'_0)^2}{3c_1^2}\right)\mathcal{B}_2 + \frac{2ic_3 a'_0 k}{3c_1 c_2^2}\mathcal{A}_1^H, \quad (4.11)$$

$$0 = \mathcal{B}_2'' + \left(-\frac{c'_3}{c_3} + \frac{c'_1}{c_1} + \frac{2c'_2}{c_2}\right)\mathcal{B}_2' + \left(-8c_3^2 + \frac{c_3^2 w^2}{c_1^2} - \frac{c_3^2 k^2}{c_2^2} + \frac{2(a'_0)^2}{3c_1^2}\right)\mathcal{B}_2 + \frac{a'_0}{3c_1^2}\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)' + \left(-32c_3^2 - \frac{8(a'_0)^2}{3c_1^2}\right)\mathcal{B}_1 + \frac{2ic_3 a'_0 k}{3c_1 c_2^2}\mathcal{A}_1^H, \quad (4.12)$$

along with the constraint

$$0 = \left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)' + \frac{c_1^2 k}{c_2^2 w}\left(\mathcal{A}_2^1 - \mathcal{A}_2^2\right)' - \frac{1}{2}a'_0(4\mathcal{B}_1 - \mathcal{B}_2), \quad (4.13)$$

where, compare with (4.2),

$$c_1 = \frac{2\alpha\sqrt{f}}{r}, \quad c_2 = \frac{2\alpha}{r}, \quad c_3 = \frac{1}{2r\sqrt{f}}. \quad (4.14)$$

We explicitly verified that (4.13) is consistent with (4.7)-(4.12). Fluctuations of the $U(1)$ baryonic current dual gauge potential $A^1 - A^2$, see [15], excite the axions b_1 and b_2 (see (4.11) and (4.12)). The holographic spectroscopy relates these scalars to the boundary gauge theory operators $\delta\mathcal{O}_\Delta^b$ of conformal dimension Δ as in table 2.

Notice that (4.9) can be solved trivially with

$$\mathcal{A}_1^H = -2\mathcal{A}_1^0. \quad (4.15)$$

Similar to section 3.3, we introduce

$$Z \equiv \mathfrak{q} \left(\mathcal{A}_t^1 - \mathcal{A}_t^2 \right) + \mathfrak{w} \left(\mathcal{A}_2^1 - \mathcal{A}_2^2 \right). \quad (4.16)$$

We use the constraint (4.13) to obtain from (4.7)-(4.12) a decoupled set of the second-order equations for

$$\{ Z, \mathcal{A}_1^0, \mathcal{B}_1, \mathcal{B}_2 \}. \quad (4.17)$$

Solutions of the resulting equations with appropriate boundary conditions determine the spectrum of R -charged black membranes quasinormal modes — equivalently the physical spectrum of linearized fluctuations in membrane gauge theory plasma with a baryonic chemical potential. Following [23,24] we impose the incoming-wave boundary conditions at the black membrane horizon, and 'normalizability' at asymptotic AdS_4 boundary. Focusing on the $\text{Re}[\mathfrak{w}] = 0$ diffusive branch, and introducing

$$Z = (1-r)^{-i\mathfrak{w}/2} z, \quad \mathcal{A}_1^0 = i(1-r)^{-i\mathfrak{w}/2} a, \quad \mathcal{B}_i = (1-r)^{-i\mathfrak{w}/2} B_i, \quad \mathfrak{w} = -iv \mathfrak{q}, \quad (4.18)$$

we solve the fluctuation equations subject to the asymptotics:

■ in the UV, *i.e.*, as $r \rightarrow 0_+$, and with the identification¹⁵ $(b_1 - b_2) \Longleftrightarrow \delta\mathcal{O}_2^b$,

$$\begin{aligned} z &= \mathfrak{q}r - \frac{1}{2}\mathfrak{q}^2 v r^2 + \mathcal{O}(r^3), \quad a = a_4 r^4 + \mathcal{O}(r^5), \quad B_2 = -2b_{1,2}r^2 + b_{1,2}\mathfrak{q}r^3 v + \mathcal{O}(r^4), \\ B_1 &= b_{1,2}r^2 - \frac{1}{2}b_{1,2}\mathfrak{q}v r^3 + \frac{b_{1,2}\mathfrak{q}}{24} \left(\mathfrak{q}(v^2 + 1)(q^2 + 1)^2 - 8\mathfrak{q}(v^2 + 1)(q^2 + 1) + 19\mathfrak{q}v^2 + 16\mathfrak{q} \right. \\ &\quad \left. - 6v \right) r^4 + b_{1,5}r^5 + \mathcal{O}(r^6), \end{aligned} \quad (4.19)$$

¹⁵Likewise, we develop the UV expansions for the alternative quantization of the fluctuation $(b_1 - b_2)$: $\{\delta\mathcal{O}_1^b\}$.

specified, for a fixed background and a momentum $\mathbf{q}$, by

$$\left\{ v, a_4, b_{1,2}, b_{1,5} \right\}; \quad (4.20)$$

■ in the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0_+$,

$$z = z_0^h + \mathcal{O}(y), \quad a = a_0^h + \mathcal{O}(y), \quad B_i = b_{i;0}^h + \mathcal{O}(y), \quad (4.21)$$

specified by

$$\left\{ z_0^h, a_0^h, b_{1;0}^h, b_{2;0}^h \right\}. \quad (4.22)$$

Note that in total we have $4+4 = 8$ parameters, see (4.20) and (4.22), which is precisely what is necessary to identify a solution of a coupled system of 4 second-order ODEs for $\{z, a, B_1, B_2\}$. Furthermore, without the loss of generality we normalized the solutions as in (3.67).

Once we fix the background, and solve the fluctuation equations of motion, we obtain $v = v(\mathbf{q})$. Given v we extract the baryonic charge diffusion coefficient $\mathcal{D}$ as in (3.68). For general values of q we have to solve the fluctuation equations numerically. In the limit $q = 0$ the diffusion coefficient can be computed analytically, see (3.72).

Results for the baryonic charge diffusion coefficient of the R -charged membrane theory plasma are presented in fig. 5. The black curve corresponds to the axion $(b_1 - b_2)$ identification with $\delta\mathcal{O}_2^b$ boundary operator, and the blue curve corresponds to the quantization $(b_1 - b_2) \Leftrightarrow \delta\mathcal{O}_1^b$. Note that in both cases $\mathcal{D}$ vanishes at certain value of q/q_{crit} (represented by vertical red lines), correspondingly the temperature, see (4.3),

$$\left. \frac{T}{\mu_R} \right|_{(b_1-b_2) \Leftrightarrow \delta\mathcal{O}_2^b}^{black} = 0.13(7), \quad \left. \frac{T}{\mu_R} \right|_{(b_1-b_2) \Leftrightarrow \delta\mathcal{O}_1^b}^{blue} = 0.46(0), \quad (4.23)$$

and becomes negative at yet lower temperatures. The negativity of the diffusion coefficient indicates unstable transport, physically realized as a baryonic charge clumping.

4.3 Threshold instabilities from condensation of $(b_1 - b_2)$

Consider spatially homogeneous and isotropic fluctuations of the bulk pseudoscalars b_1 and b_2 about R -charged black membrane (4.2). The corresponding equations of motion can be obtained from (4.7)-(4.12) in the limit

$$\{w, k\} \rightarrow 0, \quad (4.24)$$

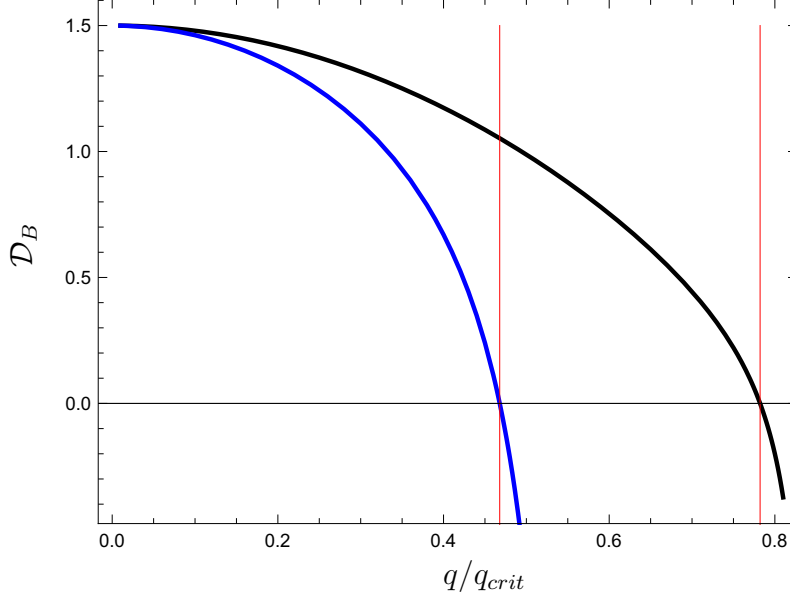


Figure 5: Dimensionless baryonic charge diffusion coefficient $\mathcal{D}_B = 2\pi T D$ of the R -charged membrane theory plasma for different quantizations of the gravitational dual pseudoscalar $(b_1 - b_2)$: $\{\delta\mathcal{O}_2^b\}$ (black), $\{\delta\mathcal{O}_1^b\}$ (blue). The vertical red lines indicate the onset of the baryonic charge clumping instability, see (4.23).

provided we drop¹⁶ the constraint (4.13). Solving (4.7) in the limit (4.24) we find

$$\left(\mathcal{A}_t^1 - \mathcal{A}_t^2\right)' = \text{const} + q(\mathcal{B}_2 - 4\mathcal{B}_1). \quad (4.25)$$

No matter what quantization is used for $(b_1 - b_2)$ bulk pseudoscalar, the constant in (4.25) is related to the baryonic charge of the black membrane (in addition to the R -charge determined by q). Thus, we must set $\text{const} = 0$ in (4.25). Using (4.25), we identify the decoupled set of linearized equations for $\mathcal{B}_1$ and $\mathcal{B}_2$ from (4.11) and (4.12) in the limit (4.24):

$$\begin{aligned} 0 &= \mathcal{B}_1'' + \left(\frac{f'}{f} - \frac{2}{r}\right)\mathcal{B}_1' - \frac{2q^2r^4 + 6}{r^2f}\mathcal{B}_1 - \frac{q^2r^4 + 4}{r^2f}\mathcal{B}_2, \\ 0 &= \mathcal{B}_2'' + \left(\frac{f'}{f} - \frac{2}{r}\right)\mathcal{B}_2' - \frac{4 - q^2r^4}{2r^2f}\mathcal{B}_2 - \frac{2(q^2r^4 + 4)}{r^2f}\mathcal{B}_1, \end{aligned} \quad (4.26)$$

¹⁶Much like in the related analysis in [10], this constraint equation is multiplied by w , and is trivially satisfied for spatially homogeneous and isotropic fluctuations.

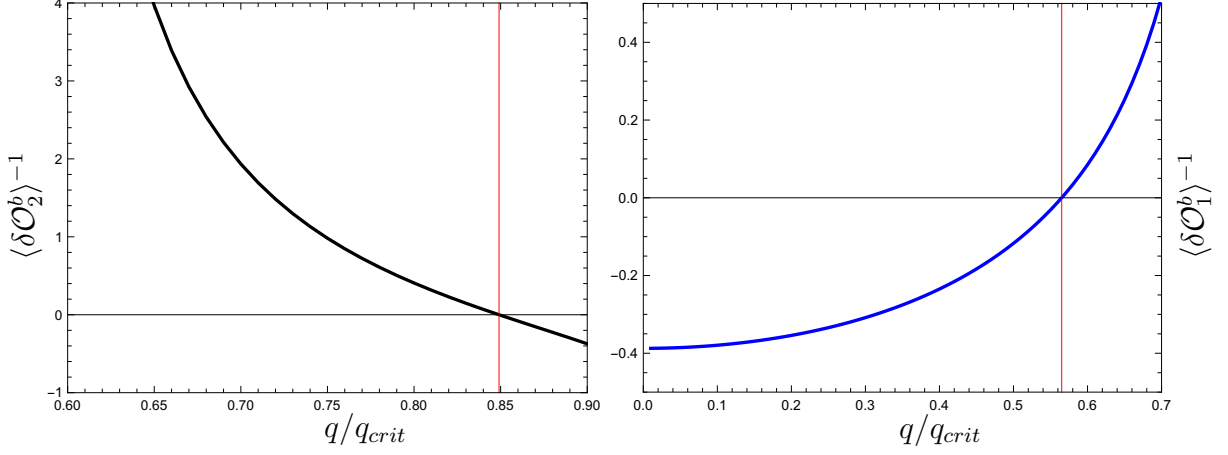


Figure 6: Divergence of the expectation value of the operator dual to spatially homogeneous and isotropic fluctuations of the bulk pseudoscalar $(b_1 - b_2)$ for different quantizations: $\{\delta\mathcal{O}_2^b\}$ (black, the left panel), and $\{\delta\mathcal{O}_1^b\}$ (blue, the right panel). The vertical red lines indicate the onset of the instability, see (4.30).

where from (4.2) $f = 1 - r^3(1 + q^2) + q^2r^4$.

■ In the UV, *i.e.*, as $r \rightarrow 0_+$, the general solution of (4.26) takes the form

$$\mathcal{B}_1 = b_{1,1} r + b_{1,2} r^2 + \frac{1}{6}(q^2 + 1)b_{1,1} r^4 + \left(b_{1,5} + \frac{1}{7}q^2 b_{1,1} \ln r\right) r^5 + \mathcal{O}(r^6), \quad (4.27)$$

$$\begin{aligned} \mathcal{B}_2 = & -2b_{1,1} r - 2b_{1,2} r^2 - \frac{1}{3}(q^2 + 1)b_{1,1} r^4 + \left(\frac{3}{4}q^2 b_{1,1} - (q^2 + 1)b_{1,2} + b_{1,5} \right. \\ & \left. + \frac{1}{7}q^2 b_{1,1} \ln r\right) r^5 + \mathcal{O}(r^6). \end{aligned} \quad (4.28)$$

In the quantization where $(b_1 - b_2)$ is identified with the boundary gauge theory operator $\delta\mathcal{O}_2^b$ the coefficient $b_{1,1}$ is the source, while in the identification $(b_1 - b_2) \Longleftrightarrow \delta\mathcal{O}_1^b$ the source term is $b_{1,2}$.

■ In the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0$,

$$\mathcal{B}_1 = b_{1,0}^h + \mathcal{O}(y), \quad \mathcal{B}_2 = b_{2,0}^h + \mathcal{O}(y). \quad (4.29)$$

Following [25], to identify the onset of instability associated with the condensation of $\delta\mathcal{O}_2^b$ (or $\delta\mathcal{O}_1^b$) we keep fixed the source term of the operator, $b_{1,1} = 1$ (or $b_{1,2} = 1$), and scan q (correspondingly T/μ_R , see (4.3)) looking for the divergence of the expectation value of the corresponding operator $\langle\delta\mathcal{O}_2^b\rangle \propto b_{1,2}$ (or $\langle\delta\mathcal{O}_1^b\rangle \propto b_{1,1}$). A

divergence signals the presence of a homogeneous and isotropic normalizable mode of the fluctuations of $(b_1 - b_2)$ — the threshold for the instability. Results of such scans are presented in fig. 6. We observe the onset of the instabilities at temperatures

$$\left. \frac{T}{\mu_R} \right|_{(b_1-b_2) \Leftrightarrow \delta \mathcal{O}_2^b}^{black} = 0.09(1), \quad \left. \frac{T}{\mu_R} \right|_{(b_1-b_2) \Leftrightarrow \delta \mathcal{O}_1^b}^{blue} = 0.33(2), \quad (4.30)$$

represented by the vertical red lines for the corresponding values of q/q_{crit} . The temperatures (4.30) are lower for the corresponding quantizations of $(b_1 - b_2)$ gravitational pseudoscalar than (4.23) — thus, the baryonic charge clumping occurs prior to the condensation of $\delta \mathcal{O}^b$ in the RN black membrane background. New phases of the R -charged black membranes in our model with $\langle \delta \mathcal{O}^b \rangle \neq 0$ will be discussed elsewhere.

4.4 Superconducting instability

In this section we complete discussion of the potential instabilities of the R -charged black membranes. The effective action (2.13) reviewed in section 2 does not contain any $U(1)_R$ charged matter. The most general $\mathcal{N} = 2$ gauged supergravity obtained from the consistent truncation of M-theory on $M^{1,1,0}$ coset includes a pair of R -charged real scalars ξ^0 and $\tilde{\xi}^0$ [15]. It is technically more transparent to discuss this charged sector using the effective action of [20].

From [20], the effective action is

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left[R - 24(\nabla U)^2 - \frac{3}{2}(\nabla V)^2 - 6\nabla U \cdot \nabla V \right. \\ & - \frac{3}{2}e^{-4U-2V}(\nabla h)^2 - \frac{3}{2}e^{-6U}|D\chi|^2 - \frac{1}{4}e^{6U+3V}F_{\mu\nu}F^{\mu\nu} \\ & - \frac{1}{12}e^{12U}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{3}{4}e^{2U+V}H_{\mu\nu}H^{\mu\nu} + 48e^{-8U-V} - 6e^{-10U+V} \\ & \left. - 24h^2e^{-14U-V} - 18(1+h^2+|\chi|^2)^2e^{-18U-3V} - 24e^{-12U-3V}|\chi|^2 \right] \\ & + \int \left[-3hH_2 \wedge H_2 + 3h^2H_2 \wedge F_2 - h^3F_2 \wedge F_2 + 6A_1 \wedge H_3 \right. \\ & \left. - \frac{3i}{4}H_3 \wedge (\chi^* D\chi - \chi(D\chi)^*) \right], \end{aligned} \quad (4.31)$$

where

$$H_3 = dB_2, \quad H_2 = dB_1 + 2B_2 + hF_2, \quad F_2 = dA_1, \quad D\chi \equiv d\chi - 4iA_1\chi. \quad (4.32)$$

Effective action (4.31) can be matched to the CKV effective action of [15] as follows:

$$\begin{aligned}
U &= -\frac{1}{3}\phi, & V &= \frac{2}{3}\phi + \ln v, & \text{where} & & v_1 = v_2 = v_3 \equiv v, \\
B_2 &= B, & A_1 &= A^0, & B_1 &= -A, & \text{where} & & A^1 = A^2 = A^3 \equiv A, \\
\chi &= \frac{1}{\sqrt{3}} \left(\xi^0 + i\tilde{\xi}^0 \right), & h &= b, & \text{where} & & b_1 = b_2 = b_3 \equiv b.
\end{aligned} \tag{4.33}$$

From (4.31), quadratic effective action S_χ for the complex scalar $\chi \equiv \eta e^{i\Theta}$, dual to the boundary membrane operator $\mathcal{O}_\chi$ of conformal dimension $\Delta = 5$ and R -charge $R(\chi) = 4$, about R -charged black membrane (4.2) takes the form

$$\begin{aligned}
S_\chi &= \int dx^4 \sqrt{-g} \left[-\frac{3}{2} |D\chi|^2 - 60 |\chi|^2 \right] \\
&= \int dx^4 \sqrt{-g} \left[-\frac{3}{2} (\nabla\eta)^2 - \frac{3}{2} \eta^2 (\nabla\Theta - 4A^0)^2 - 60\eta^2 \right].
\end{aligned} \tag{4.34}$$

The phase Θ of the complex scalar χ can be gauged away, and we arrive at the linearized equation for η in the RN black membrane background (4.2):

$$0 = \eta'' + \left(\frac{f'}{f} - \frac{2}{r} \right) \eta' - \frac{2(q(r^3 - r^2) + 5f)}{f^2 r^2} \eta, \tag{4.35}$$

where f is specified in (4.2).

■ In the UV, *i.e.*, as $r \rightarrow 0_+$, the general solution of (4.35) takes form

$$\begin{aligned}
\eta &= \eta_{-2} r^{-2} + \frac{1}{5} \eta_{-2} q - \frac{1}{6} \eta_{-2} (2q^2 + q + 2) r + \frac{1}{5} q^2 \eta_{-2} r^2 \\
&\quad + \frac{1}{150} \eta_{-2} q (20q^2 - 11q + 20) r^3 + \frac{1}{180} \eta_{-2} (10q^4 - 83q^3 + 30q^2 - 35q + 10) r^4 \\
&\quad + \left(\eta_5 + \frac{4}{175} q^2 \eta_{-2} (5q^2 - 7q + 5) \ln r \right) r^5 + \mathcal{O}(r^6).
\end{aligned} \tag{4.36}$$

In (4.36) the coefficients η_{-2} is the source for the dual operator $\mathcal{O}_\chi$, while its expectation value $\langle \delta \mathcal{O}_\chi \rangle \propto \eta_5$.

■ In the IR, *i.e.*, as $y \equiv 1 - r \rightarrow 0$,

$$\eta = \eta_0^h + \mathcal{O}(y). \tag{4.37}$$

Once again, to identify the onset of instability [25] associated with the condensation of $\mathcal{O}_\eta$ we keep fixed its source term, $\eta_{-2} = 1$, and scan q (correspondingly T/μ_R , see (4.3)) looking for the divergence of the expectation value coefficient η_5 . The result of such scan is presented in fig. 7. The lack of the instability is consistent with the analysis of [21] (once we match the conventions).

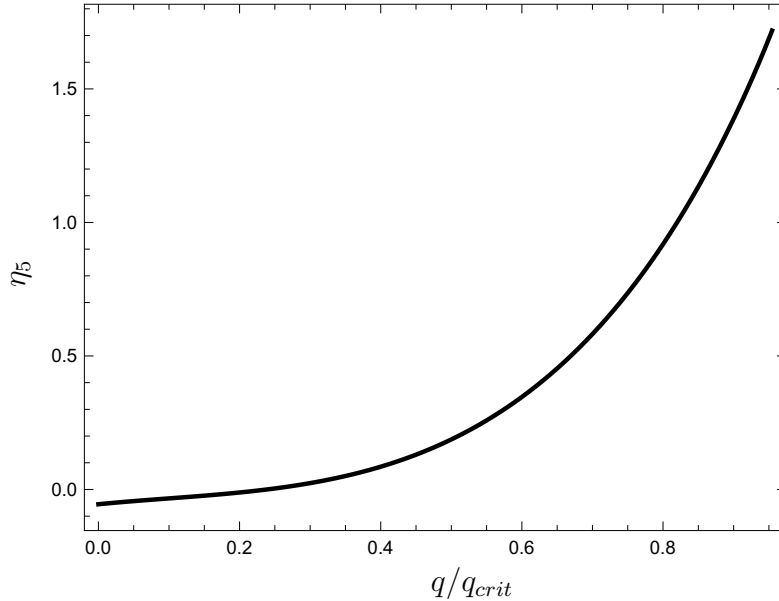


Figure 7: The normalizable coefficient η_5 does not diverge at finite values of q , correspondingly $\frac{T}{\mu_R} \neq 0$ — the dual operator $\mathcal{O}_\chi$ does not condense at nonzero temperature.

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