Generation and read-out of many-body Bell correlations with a probe qubit

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As demand for quantum technologies increases, so does the need to generate and classify non-classical correlations in complex many-body systems. We introduce a simple and versatile method for creating and certifying entanglement and many-body Bell correlations. This method relies on a single qubit interacting with an N-qubit system. We demonstrate that: (i) such pairwise interaction is sufficient to induce many-body quantum correlations, and (ii) the qubit can serve as a probe to extract all information about these correlations. Thus, single-qubit measurements reveal multipartite entanglement and N-body Bell correlations, enabling the rapid and efficient certification of non-classicality in complex systems.

Introduction.—Significant technological advances quickly followed the development of quantum theory. The invention of semiconducting transistors paved the way for the creation of personal computers and smartphones. Nowadays, these devices are connected by laser pulses propagating through waveguides, creating the Internet. Magnetic spin resonance scanners took the medical diagnoses to the next level. All of this and more was developed within the framework of *the First Quantum Revolution* which primarily utilizes single-body or collective non-classical effects [1, 2]. The *Second Quantum Revolution* [3] builds on engineered many-body quantum systems whose distinctive resources—quantum coherence [4], quantum correlations/entanglement [5, 6] and Bell-type nonlocality [7]—are absent in classical systems [8–12]. Coherence supports controlled dynamics and interference. Meanwhile, quantum correlations offer advantages in nonclassical information processing, facilitating tasks like unconditionally secure communication [13–18], and sub–shot noise metrology [19–23]. For these purposes, quantum correlations must be reliably generated and certified. The former task can be accomplished using the one-axis twisting (OAT) interaction of N qubits. This method is useful for generating spin-squeezed states, which find application in quantum metrology [24–31] and many-body Bell correlations [32–38].

The latter requires more complex techniques, such as resource certification [39–45]. Complementary methods include steering tests [46], device-independent Bell tests and self-testing [47, 48], measurement-device-independent witnesses [49, 50], as well as scalable reconstructions via compressed sensing and classical shadows [51, 52]. Additionally, there are resource-specific tools, such as coherence witnesses [53–55]. As an alternative, the fidelity of a quantum state to the desired target can be estimated in order to certify quantum resources [56–59]. Experiments have shown that quench protocol-based methods are a scalable way to certify quantum correlations [60, 61]. Quench processes have also made it possible to directly measure Rényi entropies [62, 63] and reconstruct entanglement growth over time using randomized measurements [64]. These processes also reveal the light-cone spreading of correlations, exposing how entanglement is generated and transported in many-body systems [65].

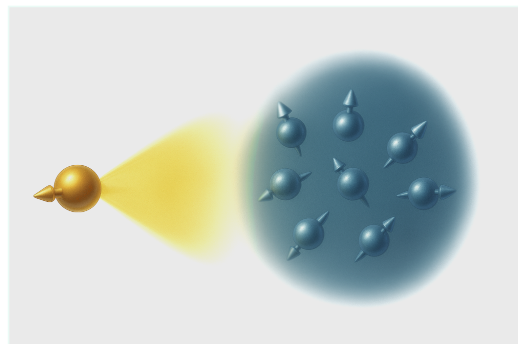

FIG. 1. A visualization of a single-qubit probe "observing" a system of N qubits.

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In this work, we show that a single qubit interacting with an N-qubit ensemble can generate and certify quantum correlations. First, it can effectively initiate the OAT dynamics within the system. Second, we show that a single qubit can act as a probe to extract information from the system. Simple measurements on the probe enable us to verify the entanglement depth and many-body Bell correlations of a wide range of systems governed by Lipkin-Meshkov-Glick (LMG) Hamiltonians [66–68]. These LMG models include OAT systems and describe interacting Bose-Einstein condensates (BECs) and long-range spin-1/2 chains in the lowest momentum mode. Additionally, they describe phonon- and photon-mediated interactions in spin-1/2 chains where

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the Dicke manifold is protected by an energy gap.

The probe qubit has been used to locate quantum critical points in Ising-type chains and to diagnose phase transitions by monitoring its coherence over time [69, 70]. It can detect nonergodicity and many-body localization in disordered spin chains [71, 72] as well as determine ground-state phases and phase boundaries in the XXZ chain [73]. Furthermore, it provides access to the generation of entanglement and its spread across spatial cuts [74–76]. Meanwhile, qubit-only protocols determine whether a qubit becomes entangled with the environment during pure dephasing [77–80]. Little is known about using a single-qubit probe to generate and detect many-body quantum correlations. This work aims to bridge this gap by providing an experimentally friendly recipe for accomplishing these important steps for emerging quantum technologies.

We consider a collection of N qubits in a state $\hat{\varrho}_N$, a natural platform for many quantum-enhanced tasks. In the first scenario the qubits are non-interacting and form a separable state. An external probe qubit mediates interactions and *generates* many-body Bell correlations in the composite (N-qubits)+probe system. In the second scheme, the probe extracts information about the *existing* correlations in $\hat{\varrho}_N$.

Generation of correlations.—To begin, we take the so called *central-spin* Hamiltonian describing probe-qubit collectively interacting with the N-qubit system [81]

$$\hat{H} = \frac{\Omega}{2}\,\hat{\sigma}_z^{(pr)} + \omega\,\hat{J}_z + g\big(\hat{J}_+\hat{\sigma}_-^{(pr)} + \hat{J}_-\hat{\sigma}_+^{(pr)}\big), \qquad (1)$$

where $\hat{\sigma}_z^{(pr)}$ and $\hat{\sigma}_\pm^{(pr)} = \frac{1}{2}(\hat{\sigma}_x^{(pr)} \pm i\hat{\sigma}_y^{(pr)})$ are the probe-qubit operators. The collective N-spin operator is $\hat{J}_z = \frac{1}{2}\sum_{i=1}^{N}\hat{\sigma}_z^{(i)}$ and analogically for $\hat{J}_\pm$. In the dispersive regime, when $\Delta = \omega - \Omega$, $|\Delta| \gg g$ and for as long as the evolution time t satisfies $t \ll \chi^{-1}$, where $\chi = g^2/\Delta$, the second–order Schrieffer–Wolff (SW) transformation allows to express Eq. (1) as [81, 82]

$$\hat{H}_{\rm OAT} \simeq \chi\hat{S}_z^2. \qquad (2)$$

Here $\hat{S}_z = \hat{J}_z + \frac{1}{2}\hat{\sigma}_z^{(pr)}$ is the composite spin operator. This is the OAT Hamiltonian that can correlate the $(N+1)$-qubit system. To characterize its potency and compare the exact evolution generated by the Hamiltonian (1) with the effective OAT dynamics (2) we take a product state of N qubits and the probe in the form $|+1\rangle_x^{\otimes(N+1)}$. Here, $|+1\rangle_x$ is a single-qubit eigenstate of $\hat{\sigma}_x$. Next, we evolve this state with either of the Hamiltonians and calculate the following correlator

$$\mathcal{E}_\mu^{(q)} = |\langle\bigotimes_{k=1}^{\mu}\hat{\sigma}_+^{(k)}\rangle|^2, \qquad (3)$$

where $\mu = N+1$ is the total number of qubits. Note that $\mathcal{E}_\mu^{(q)} \leqslant 2^{-\mu}$ is an μ-body Bell inequality, hence if

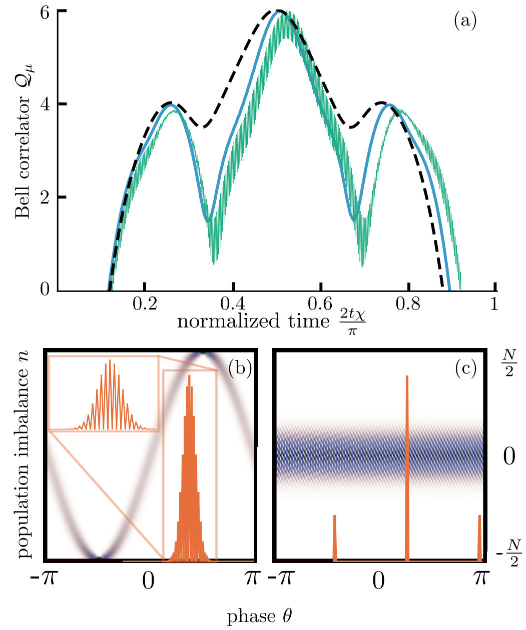

FIG. 2. (a): Generation of many-body quantum correlations via single-qubit in central-spin model, Eq. (1). Lines present the Bell correlator $\mathcal{Q}_\mu$ as a function of the normalized time for $\mu = 8$ qubits (including the probe qubit). The dashed black line denotes the ideal OAT scheme, while the solid blue is for $g = 0.05$ and the quickly oscillating green one uses $g = 0.1$. For both cases, $\Omega = 11$ and $\omega = 1$. (b) and (c): probabilities $p_n(\theta)$ for the separable state (b) and the GHZ state (c), shown for $N = 64$. The insets show the Fourier transform of $p_0(\theta)$. The emergence of two peaks in the latter case is a signature of Bell correlations.

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$\mathcal{Q}_\mu = \log_2(\mathcal{E}_\mu^{(q)}2^\mu) > 0$, the system is Bell correlated, see Refs. [38, 83–95] and [82]. In Fig. 2(a) we plot $\mathcal{Q}_\mu$ as a function of time for both cases of Eq. (1) and Eq. (2) for a total of $\mu = 8$ particles. The correlator in both cases is a very similar function of time. As expected from the validity of the SW transformation, the agreement is best for short times. Most importantly, $\mathcal{Q}_\mu$ crosses the Bell threshold and reaches the maximal value $Q_\mu = \mu - 2$ that is attainable only with the μ-qubit Greenberger-Horne-Zeilinger (GHZ) state [82]. Therefore, we confirm that a single-qubit probe connected to an N-qubit ensemble via the Hamiltonian (1) can closely mimic the OAT dynamics and generate many-body Bell correlations.

Certification of correlations.—Next, we demonstrate that a single-qubit probe can certify the presence of non-classical correlations embedded in $\hat{\varrho}_N$. To this end, the probe qubit governed by $\hat{\varrho}_{pr}$ is "connected" to the system so that, initially, a product state is formed

$$\hat{\varrho} = \hat{\varrho}_N \otimes \hat{\varrho}_{pr}\,. \qquad (4)$$

Next, the two parts interact in order to transfer the information about $\hat{\varrho}_N$ to the probe via the Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} J_i \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(pr)}, \tag{5}$$

where J_i's are the coupling strengths. After time t the density matrix of the probe becomes [82]

$$\hat{\varrho}_{pr}(\theta, t) = \begin{pmatrix} \mathcal{P} & a \\ a^* & 1-\mathcal{P} \end{pmatrix}. \tag{6}$$

The probability $\mathcal{P}$ does not depend t, while

$$a = \sum_{\vec{s}=\pm 1} p_{\vec{s}}\, e^{-2it \sum\limits_{i=1}^{N} J_i s_i}. \tag{7}$$

Here, the summation runs through 2^N elements of the eigen-basis of the $\hat{\sigma}_z^{(i)}$ operators and $\vec{s} = (s_1, \ldots, s_N)$ are their ± 1 eigen-values. The diagonal element of the system's density matrix, $p_{\vec{s}} \equiv \varrho_N^{(\vec{s},\vec{s})}$ is the probability of the system occupying a state $|\vec{s}\rangle = |s_1\rangle \otimes \ldots |s_N\rangle$. If the system undergoes specific local operations prior to interacting with the probe, the element a becomes sensitive to the off-diagonal elements of $\hat{\varrho}_N$ and provides information about its non-classicality.

For illustration, we pick a phase-θ imprint on each of N qubits and the subsequent mixing of the signal using a $\pi/2$-pulse, namely

$$\hat{\varrho}_N(\theta) = e^{-i\frac{\pi}{2}\hat{J}_x} e^{-i\theta\hat{J}_z} \hat{\varrho}_N e^{i\theta\hat{J}_z} e^{i\frac{\pi}{2}\hat{J}_x}. \tag{8}$$

We now make a crucial observation: if the state $\hat{\varrho}_N$ is permutationally invariant and all the couplings are equal, $J_i = J$, then the matrix element from Eq. (7) becomes

$$a = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} p_n(\theta)\, e^{-i2\pi\frac{\tau}{N+1}n}, \tag{9}$$

where $n = \frac{1}{2}(n_{+1} - n_{-1})$ is the difference between the number of qubits occupying the $s = +1$ and the $s = -1$ state and $\tau = \frac{J}{\pi}(N+1)t$. Equation (9) is a discrete Fourier transform of the probability $p_n(\theta)$ and its inversion yields $p_n(\theta)$—a many-body probability from a measurement on a single-qubit probe. We will now demonstrate how to characterize non-classical correlations using the off-diagonal element a.

The first quantity derived from $p_n(\theta)$ that gives access to information on the multi-partite entanglement is the spin-squeezing parameter [19, 24, 25]

$$\xi^2 = N \frac{\Delta^2 \hat{J}_z(\theta)}{\left|\frac{\partial}{\partial\theta}\langle\hat{J}_z(\theta)\rangle\right|^2}, \tag{10}$$

where both the numerator and the denominator can be obtained by calculating the two lowest moments of the probability, $\langle \hat{J}_z^\alpha(\theta)\rangle = \sum_n p_n(\theta) n^\alpha$ with $\alpha = 1, 2$. When $\xi < 1$, the system is entangled and useful for quantum metrology [19]. Furthermore, from the value of ξ one can deduce the entanglement depth of the system [27, 43, 96].

For non-Gaussian states that cannot be characterized only by the mean and the variance of p_n, higher moments can be calculated as in [97]. The complete probability yields the Fisher information [98]

$$\mathcal{I} = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} \frac{1}{p_n(\theta)} \left(\frac{\partial p_n(\theta)}{\partial\theta}\right)^2. \tag{11}$$

Finally, according to [99], the off-diagonal element a provides a lower bound for $\mathcal{I}_q[\varrho_N]$—the quantum Fisher information (QFI) associated with the system's state [100], namely

$$\mathcal{I}_q[\hat{\varrho}_N(\theta)] \geqslant 4\left(\frac{\mathtt{Re}\left[\dot{a}e^{-i\phi}\right]^2}{1-|a|^2} + \mathtt{Im}\left[\dot{a}e^{-i\varphi}\right]^2\right), \tag{12}$$

where $\varphi = \arg(a)$. By setting the susceptibility of $\hat{\varrho}_N(\theta)$ to changing θ, the QFI sets the ultimate precision $\Delta\theta$ of phase estimation that can be achieved with it. According to [99], the above bound is tight for a large family of relevant multi-qubit states.

The above set of quantities, which all can be obtained from a, form a hierarchy

$$\frac{N}{\xi^2} \leqslant \mathcal{I} \leqslant \mathcal{I}_q[\hat{\varrho}_N(\theta)] \leqslant (\Delta\theta)^{-2}. \tag{13}$$

Depending on the setting, any of these parameters can be used to determine the lower bound of the system's entanglement depth and metrological usability.

Next, we will demonstrate how to use the probe to detect the system's many-body Bell correlations, which are quantified by Eq. (3). (Note that now $\mu = N$, rather than $N+1$ as before, since we characterize the Bell correlations of the N-qubit system using the probe, instead of correlations generated in an N-qubit system connected to a probe.) The Bell correlator is characterized by a single element of the density matrix [82], $|\varrho_{\frac{N}{2},-\frac{N}{2}}|^2$, that governs the GHZ coherence of the term $|-1\rangle\langle+1|^{\otimes N}$. To access this element using $p_n(\theta)$, we note that this probability, according to Eq. (8), is

$$p_n(\theta) = \sum_{m,m'=-\frac{N}{2}}^{\frac{N}{2}} d_{nm'}^* d_{mn} \varrho_{mm'} e^{-i(m'-m)\theta}, \tag{14}$$

where d_{mn} is a matrix element of the mixing operator $e^{-i\frac{\pi}{2}\hat{J}_x}$. The Fourier transform $\mathcal{F}[p_n(\theta)]$ with respect to θ reveals terms oscillating at different frequencies. In particular, the fastest-varying term $\mathcal{F}_{\max}[p_n(\theta)]$ is related to one element of the above sum: when $m' = -m = \frac{N}{2}$, and is proportional to the corresponding matrix element

$\varrho_{\frac{N}{2},-\frac{N}{2}}$. Using the symmetry of the rotation matrix $d_{n,\frac{N}{2}} = d_{-\frac{N}{2},n}$ and the expression for the Bell correlator from Eq. (EM.14), we obtain that

$$\mathcal{E}_N^{(q)}[\hat{\varrho}_N] = |\varrho_{\frac{N}{2},-\frac{N}{2}}|^2 = \frac{|\mathcal{F}_{\max}\left[p_n(\theta)\right]|^2}{n_\theta |d_{n,\frac{N}{2}}|^4}, \tag{15}$$

where n_θ is the number of grid points on which θ is sampled. This way, the Bell correlations, but also the k-locality and k-separability [93] can be deduced from the measurements of a single-qubit probe. In Fig. 2 (b) and (c), we plot the probability $p_n(\theta)$ as a function of θ and n for two different pure states $\hat{\varrho}_N = |\psi_N\rangle\langle\psi_N|$. One is a separable state

$$|\psi_N\rangle = \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)^{\otimes N} \tag{16}$$

while the other is the GHZ state

$$|\psi_N\rangle = \frac{|\uparrow\rangle^{\otimes N} + |\downarrow\rangle^{\otimes N}}{\sqrt{2}} \tag{17}$$

Clearly, the $p_n(\theta)$ for the former is a slowly-varying function of θ, while the latter, reveals fine structures, characteristic for strongly entangled states [101]. The Fourier transforms of $p_0(\theta)$ with respect to θ are shown for both cases as insets. The emergence of two side-band peaks is a direct and experimentally accessible confirmation of the GHZ-like coherence that drives the Bell correlations.

Applications.—This method is ideal for characterizing multi-qubit states described by LMG models [66–68]. Their ground, thermal, and dynamically generated states all support many-body quantum correlations [89, 102–116]. LMG models have been implemented on different experimental platforms, including superconducting qubits and BECs [116–123]. Furthermore, LMG systems are well-suited for studying quantum phase transitions. The related quantities, like the geometric phase, can be immediately deduced from the probability $p_n(\theta)$ [124–127].

Moreover, the proposed protocol is useful for more than just initializing the OAT dynamics; it can also be used to detect entanglement and Bell correlations in such systems. The OAT can be generated using a two-component Bose-Hubbard model [128, 129], as well as in spinful Fermi-Hubbard model with external lasers [130]. In both scenarios, the probe should interact with the system's zero mode, resulting in collective coupling. The proposed protocol can be also utilized in short-range spin chains, in particular the staggered XXX model that realizes the OAT dynamics [130, 131]. In Rydberg tweezer arrays, an XXX chain can be engineered by tuning microwave-programmable XXZ interactions [132] or by realizing Heisenberg-type exchange via off-resonant dressing in 1D chains [133]. Other applications are the OAT systems based on the resonant dipole–exchange evolution accompanied by the Floquet/control protocols to balance the couplings [134–137] or on dual-species architectures and long-lived circular Rydberg implementations [138, 139]. In trapped-ion systems, global Mølmer–Sørensen and geometric-phase gates were used to realize collective, effectively all-to-all OAT dynamics [140–147].

Discussion.—The single-qubit probe gives access to the distribution of N parties, $p_n(\theta)$, among their two modes. Naturally, $p_n(\theta)$ could be accessed by directly performing measurements on each of the qubits. While such on-site measurements are feasible in chain/lattice configurations [148], they become much harder when the system is a Bose gas [97, 98]. Hence the method presented here, requiring a measurement on only one qubit, can significantly facilitate the detection of many-body correlations in complex systems.

In principle, it can also serve for remote tomography of N-body permutation-symmetric states. The computational complexity of the tomography of such states scales like N^2, since it uses collective measurements [149–151]. Similar scaling is expected for the single-probe method, as both the inversion of the Fourier transform in Eq. (9) and the resolution of θ's in Eq. (14) require $\sim N$ points each. Nevertheless, direct tomography requires accurate collective detection prone to measurement errors that scale with N. By contrast, single-qubit tomography that yields access to the off-diagonal element a, is elementary [152–154].

Conclusions.—In this work we have shown that many-body quantum correlations, such as entanglement or Bell correlations, can be generated and certified using a single-qubit probe that interacts with the system. The effective OAT dynamics that stems from a coupling of the probe to each of the qubits yields a complex Bell-correlated system. Furthermore, quantities like the spin-squeezing or the (quantum) Fisher information can be derived from a one-body measurement and provide information about the entanglement depth and the presence of correlations useful for quantum metrology. Moreover, a many-body Bell correlator can also be deduced within this scheme, giving access to the most non-classical quantum correlations. The single-qubit probe can also detect the quantum phase transition in the ground state of a broad family of LMG systems. This protocol competes with other methods of generating and detecting quantum many-body effects by eliminating the need for cumbersome many-body measurements.

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END MATTER

Central-spin model to OAT

In the following we outline the second–order Schrieffer–Wolff (SW) transformation that maps the isotropic central–spin Hamiltonian to an effective one–axis twisting (OAT) form. We consider $\hat{H} = \hat{H}_0 + \hat{V}$, where

$$\hat{H}_0 = \frac{\Omega}{2}\,\hat{\sigma}_z^{(pr)} + \omega\,\hat{J}_z \tag{EM.1}$$

$$\hat{V} = g\big(\hat{J}_+\hat{\sigma}_-^{(pr)} + \hat{J}_-\hat{\sigma}_+^{(pr)}\big), \tag{EM.2}$$

which uses the notation from the main text. We focus on the dispersive regime $\Delta = \Omega - \omega$, $|\Delta| \gg g$, so that direct spin exchange is off–resonant. Projectors onto the probe-spin manifolds are

$$\hat{P} = \frac{1-\hat{\sigma}_z^{(pr)}}{2}, \qquad \hat{Q} = \frac{1+\hat{\sigma}_z^{(pr)}}{2}, \qquad \hat{P}+\hat{Q} = \mathbb{I}. \tag{EM.3}$$

To get rid of the off–diagonal couplings between low- and high-energy subspaces at first order, we perform a Schrieffer–Wolff transformation $\hat{H}_{\rm eff} = e^{\hat{S}}\hat{H}e^{-\hat{S}}$ with an anti-Hermitian generator $\hat{S}$.

At the leading order we choose $\hat{S}$ to eliminate the $\mathcal{O}(g)$ $\hat{P}$–$\hat{Q}$ couplings, i.e. we fix the first–order generator $\hat{S}^{(1)}$ to satisfy $[\hat{H}_0, \hat{S}^{(1)}] = -\hat{V}$, which yields

$$\hat{S}^{(1)} = \frac{g}{\Delta}\big(\hat{J}_+\hat{\sigma}_-^{(pr)} - \hat{J}_-\hat{\sigma}_+^{(pr)}\big). \tag{EM.4}$$

Expanding $e^{\hat{S}}\hat{H}e^{-\hat{S}}$ to second order gives

$$\hat{H}_{\rm eff}^{(2)} = \hat{H}_0 + \frac{1}{2}\,[\hat{S}^{(1)}, \hat{V}]. \tag{EM.5}$$

With $\hat{X} = \hat{J}_+\hat{\sigma}_-^{(pr)}$ and $\hat{V} = g(\hat{X} + \hat{X}^\dagger)$, one finds

$$\frac{1}{2}\,[\hat{S}^{(1)}, \hat{V}] = \frac{g^2}{\Delta}\,[\hat{X}, \hat{X}^\dagger]. \tag{EM.6}$$

Using $\hat{J}_\pm\hat{J}_\mp = \hat{J}^2 - \hat{J}_z^2 \pm \hat{J}_z$, $\hat{\sigma}_-^{(pr)}\hat{\sigma}_+^{(pr)} = \frac{1}{2}(1-\hat{\sigma}_z^{(pr)})$, and $\hat{\sigma}_+^{(pr)}\hat{\sigma}_-^{(pr)} = \frac{1}{2}(1+\hat{\sigma}_z^{(pr)})$, we obtain

$$[\hat{X}, \hat{X}^\dagger] = \hat{J}_z - \hat{\sigma}_z^{(pr)}\big(\hat{J}^2 - \hat{J}_z^2\big). \tag{EM.7}$$

The effective Hamiltonian becomes

$$\hat{H}_{\rm eff}^{(2)} = \frac{\Omega}{2}\,\hat{\sigma}_z^{(pr)} + \omega\,\hat{J}_z + \frac{g^2}{\Delta}\,\hat{J}_z - \frac{g^2}{\Delta}\,\hat{\sigma}_z^{(pr)}\big(\hat{J}^2 - \hat{J}_z^2\big). \tag{EM.8}$$

Projecting onto the $\hat{P}$ sector, $\hat{H}_\downarrow = \hat{P}\hat{H}_{\rm eff}^{(2)}\hat{P}$, yields

$$\hat{H}_\downarrow = -\frac{\Omega}{2} + \Big(\omega + \frac{g^2}{\Delta}\Big)\hat{S}_z + \frac{g^2}{\Delta}\hat{S}^2 - \frac{g^2}{\Delta}\hat{S}_z^2, \tag{EM.9}$$

where $\hat{S}_i$ is the collective N-spin/probe operator. Within the symmetric manifold ($S = (N+1)/2$), $\hat{S}^2$ is a constant and can be dropped. Defining $\tilde{\omega} \equiv \omega + \chi$ and $\chi \equiv \frac{g^2}{\Delta}$, we

$$\hat{H}_\downarrow = \tilde{\omega}\,\hat{S}_z - \chi\,\hat{S}_z^2. \tag{EM.10}$$

Finally, by moving to the rotating frame via $\hat{U}_z(t) = e^{+i\tilde{\omega}t\hat{S}_z}$, we obtain an effective OAT model

$$\hat{H}_{\rm coll} = \hat{U}_z\hat{H}_\downarrow\hat{U}_z^\dagger - i\hat{U}_z\partial_t\hat{U}_z^\dagger = -\chi\,\hat{S}_z^2. \tag{EM.11}$$

Accuracy requires $g/|\Delta| \ll 1$ and evolution times $t \ll |\Delta|/g^2$ so that $\mathcal{O}(g^3/\Delta^2)$ terms remain negligible.

Bell correlator

A many-body Bell inequality is constructed under the assumption of binary outcomes of measurements of two quantities $\sigma_{x/y}^{(k)} = \pm 1$, where $k \in \{1\dots N\}$ labels the particles [38, 83–93, 95]. If the N-body correlation function

$\mathcal{E}_N \equiv |\langle\prod_{k=1}^N \sigma_+^{(k)}\rangle|^2$ is consitent with the postulates of local realism, it satisfies

$$\begin{aligned}\mathcal{E}_N &= |\int d\lambda\, p(\lambda) \prod_{k=1}^{N} \sigma_+^{(k)}(\lambda)|^2 \\ &\leqslant \int d\lambda\, p(\lambda) \prod_{k=1}^{N} |\sigma_+^{(k)}(\lambda)|^2 = 2^{-N},\end{aligned} \tag{EM.12}$$

where $p(\lambda)$ is the probability density of a hidden variable, $\sigma_+^{(k)} = \frac{1}{2}(\sigma_x^{(k)} + i\sigma_y^{(k)})$ and in the last step we used the Cauchy-Schwarz inequality and the fact that $|\sigma_+^{(k)}(\lambda)|^2 = 1/2$. This is the many-body Bell inequality as its violation defies the postulate of local realism. For qubits $\sigma_+^{(k)}$'s are replaced with the rising Pauli operators and the Bell inequality becomes

$$\mathcal{E}_N^{(q)}[\hat{\varrho}_N] = |\mathrm{Tr}\left[\hat{\varrho}_N \bigotimes_{k=1}^{N} \hat{\sigma}_+^{(k)}\right]|^2 \leqslant 2^{-N}. \tag{EM.13}$$

The above product of N rising operators is non-zero only if it acts on $|-1\rangle^{\otimes N}$, giving $|+1\rangle^{\otimes N}$. Therefore $\mathcal{E}_N^{(q)}$ is a measure of coherence between these two, which is maximal for the GHZ state, $|\psi\rangle = \frac{1}{\sqrt{2}}(|-1\rangle^{\otimes N} + |+1\rangle^{\otimes N})$, and can be expressed as follows

$$\mathcal{E}_N^{(q)}[\hat{\varrho}_N] = |\varrho_{\frac{N}{2},-\frac{N}{2}}|^2. \tag{EM.14}$$

Here, $|\varrho_{\frac{N}{2},-\frac{N}{2}}|^2$ denotes the matrix element of $\hat{\varrho}_N$ multiplying $|-1\rangle\langle+1|^{\otimes N}$.

Expression for the antenna's density matrix

The general expression for the N-qubit state is

$$\hat{\varrho}_N = \sum_{\vec{s},\vec{s}'=\pm 1} \varrho_{\vec{s},\vec{s}'}(\theta) |\vec{s}\rangle\langle\vec{s}'|. \tag{EM.15}$$

Its coupling to the single-qubit probe as in Eq. (4) gives

$$\hat{\varrho} = \sum_{\vec{s},\vec{s}'=\pm 1} \sum_{s_p,s_p'=\pm 1} \varrho_{\vec{s},\vec{s}'}\varrho_{s_p,s_p'} |\vec{s}\rangle\langle\vec{s}'| \otimes |s_p\rangle\langle s_p'|. \tag{EM.16}$$

The subsequent time-evolution generated by the Hamiltonian (5) yields

$$\begin{aligned}\hat{\varrho}(t) = &\sum_{\vec{s},\vec{s}'=\pm 1} \sum_{s_p,s_p'=\pm 1} \varrho_{\vec{s},\vec{s}'}\varrho_{s_p,s_p'} |\vec{s}\rangle\langle\vec{s}'| \otimes |s_p\rangle\langle s_p'| \times \\ &\times e^{-it\sum\limits_{i=1}^{N} J_i(s_i s_p - s_i' s_p')}.\end{aligned} \tag{EM.17}$$

Once we trace-out the N-qubits' degrees of freedom we are left with

$$\hat{\varrho}_{pr}(t) = \sum_{s_p,s_p'=\pm 1} \varrho_{\vec{s},\vec{s}}\varrho_{s_p,s_p'} |s_p\rangle\langle s_p'| e^{-it\sum\limits_{i=1}^{N} J_i s_i (s_p - s_p')} \tag{EM.18}$$

The off-diagonal term is obtained when setting $s_p = 1 = -s_p'$ (or vice-versa), which yields Eq. (7).

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