

Magneto-Chiral Anisotropy in Josephson Diode Effect of All-Metallic Lateral Junctions with Interfacial Rashba Spin-Orbit Coupling

Maximilian Mangold,^{1,2} Lorenz Bauriedl,³ Johanna Berger,³ Chang Yu-Cheng,⁴ Thomas N.G. Meier,^{1,2} Matthias Kronseder,³ Pertti Hakonen,⁵ Christian H. Back,^{1,2} Christoph Strunk,^{3,*} and Dhavala Suri^{1,6,†}

¹*School of Natural Sciences, Technical University of Munich, 85748 Garching b. Munich, Germany*

²*Center for Quantum Engineering (ZQE), Technical University of Munich, 85748 Garching b. Munich, Germany*

³*Department of Physics, University of Regensburg, 93049 Regensburg, Germany*

⁴*Pico Group, QTF Centre of Excellence, Department of Applied Physics, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland*

⁵*Low Temperature Laboratory, Department of Applied Physics, Aalto University, P.O. Box 15100, FI-00076 Espoo, Finland*

⁶*Centre for Nanoscience and Engineering, Indian Institute of Science, Bengaluru 560012, India*

We explore the role of interfacial Rashba spin-orbit coupling (SOC) for the Josephson diode effect in all-metal diffusive Josephson junctions. Devices with Fe/Pt and Cu/Pt weak links between Nb leads reveal a Josephson diode effect in an in-plane magnetic field with magneto-chiral anisotropy according to the symmetry of Rashba SOC. The Rashba SOC originates from inversion symmetry breaking at the metal-metal interfaces. A control sample with a plain Cu-layer as weak link exhibits also a finite diode efficiency that, in contrast, is independent of the angle between current and field. The Fraunhofer patterns display an apparent inverted hysteresis which can be traced back to stray fields resulting from the conventional hysteretic vortex pinning in the Nb contacts.

Hybrid Josephson junctions (JJs) have enabled major advances in fundamental research of superconductivity and are central to contemporary developments in quantum technology. Milestone experiments have led to novel qubit architectures [1–3], memory devices based on magnetic JJs [4], and spin-polarized supercurrents [5, 6].

The recent discovery of the Josephson diode effect (JDE) [7–11] has added a new circuit element to future superconducting electronics [12–16]. The JDE can occur when time-reversal symmetry (TRS) and inversion symmetries are broken and is reflected in an asymmetry of positive and negative critical current. For equilibrium supercurrents, this asymmetry breaks Onsager reciprocity. If inversion symmetry and TRS are broken by the presence of an interface and an in-plane magnetic field $\vec{B}_{\text{ip}}$, respectively, the diode efficiency is proportional to the vector product $\vec{B}_{\text{ip}} \times \vec{j}$ with the supercurrent density $\vec{j}$. The dependence on the angle between $\vec{B}_{\text{ip}}$ and $\vec{j}$ is also called magneto-chiral anisotropy. So far, the diode effect in JJs was mostly observed in semiconductor weak links like InAs or InSb with strong Rashba spin-orbit coupling (SOC). In bi-layer metallic films, inversion symmetry is broken by the charge transfer between the metal films, leading to a short-ranged, but strong interface electric field and thus to a Rashba-type Hamiltonian $\mathcal{H}_{\text{R}} = \alpha(\vec{k} \times \hat{z}) \cdot \hat{s}$, where $\vec{k}$ and $\hat{s}$ are the electron's momentum and spin, respectively, and α is the Rashba constant [17–20]. In momentum space, a characteristic circular spin texture emerges at the Fermi level [21] [Fig. 1(a)].

Such interfacial SOC is reflected, *e.g.*, in spin-orbit torques [22, 23]. Moreover, spin injection techniques have demonstrated Rashba SOC at the Cu/Pt interface [24, 25]. Strong interfacial spin-orbit interactions have also been observed in Fe/Pt on GaAs in spin pumping measurements driven by ferromagnetic resonance [26, 27]. Hence, it is natural to ask whether the JDE can be induced by metallic bi-layer weak links in JJs.

So far, studies of SOC effects in metallic bi-layer weak links are sparse. Senapati *et al.* investigated Cu/Pt bilayers as weak link in Nb-based JJs [28]. They observed no diode effect, but reported an apparent inverse hysteresis. Similar inverse hysteresis was also found in Al-based JJs with Fe-doped InAs as weak links [29] and in epitaxial CoSi₂/TiSi₂ [30] normal metal/superconductor junctions. To date, no consensus on the origin of the inverse hysteresis has been reached.

In this Letter, we investigate effect of interfacial SOC on the supercurrent in Nb-based lateral JJs with metallic bi-layer weak links. The JDE is observed only for bi-layer Fe/Pt and Cu/Pt, where the breaking of inversion symmetry leads to interfacial Rashba SOC. All devices exhibit an apparent inverted hysteresis in out-of-plane fields that can be understood as a result of hysteretic vortex pinning in the Nb terminals.

We first examine an epitaxial heterostructure of GaAs(001)/Fe(2.9 nm)/Al(1 nm)/Pt(5 nm)/Nb(50 nm), (sample A). The specific stack is chosen for its strong interfacial SOC arising at both the GaAs/Fe and the Fe/Al/Pt interface [26, 27]. The thin layer of Al serves as a spacer, preventing magnetic proximity polarization of the Pt layer while remaining transparent for charge and spin transport [27]. A lateral Nb JJ is realized atop

* Corresponding Author: christoph.strunk@ur.de

† Corresponding Author: dsuri@iisc.ac.in

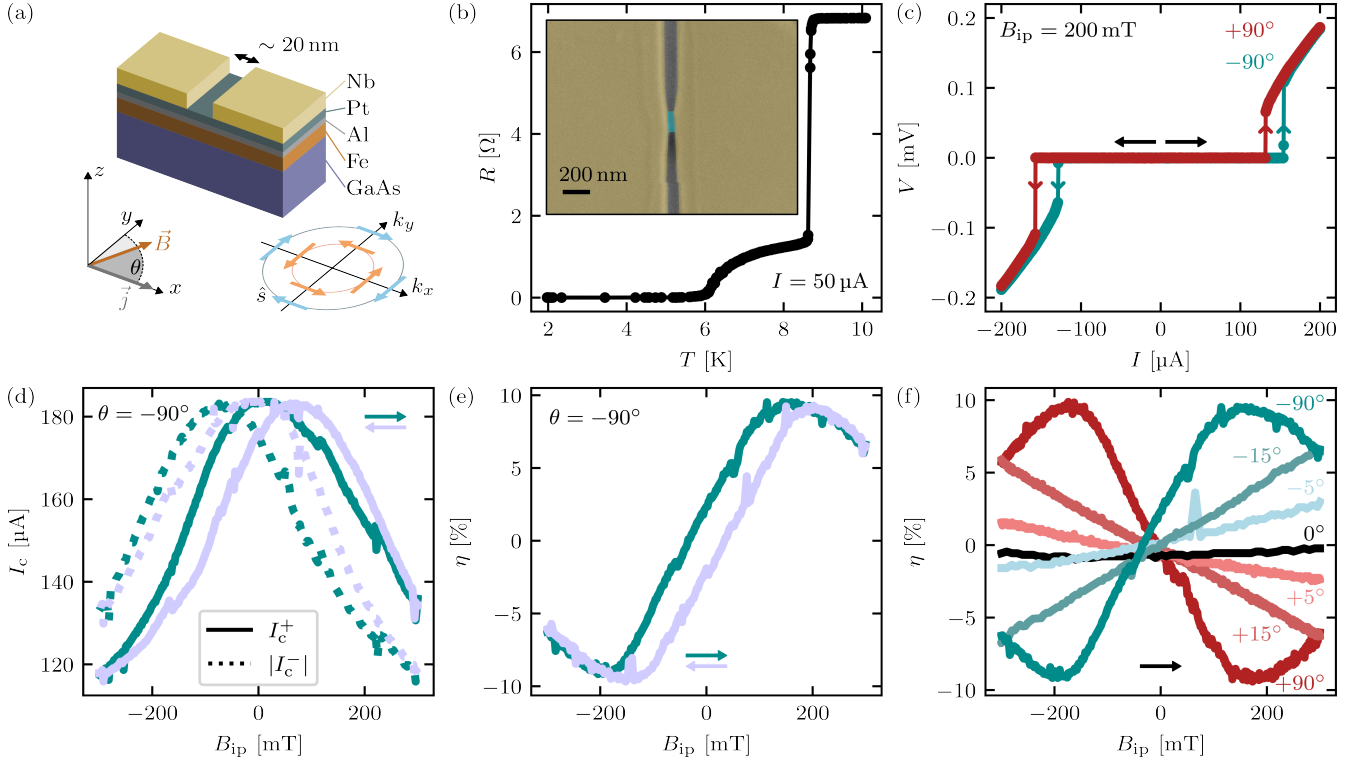


FIG. 1. Diode factor of sample A under B_{ip} . (a) Schematic stack and orientation of the junction. The direction of positive applied current defines the x -axis. The direction of B_{ip} applied in panels (c-f) is indicated in the coordinate system in the bottom left corner. θ measures the angle between B_{ip} and the positive current direction. The bottom right sketch symbolizes the chiral spin texture of the Rashba SOC at the Fermi level in momentum space. (b) Temperature-dependence of the device resistance measured at $I = 50 \mu\text{A}$ in zero magnetic field. Inset: SEM false-color image of the device. The Nb leads are highlighted in yellow, the blue area represents the junction, measuring around 20 nm in length and ~ 180 nm in width. (c) IV curves taken at $B_{ip} = 200 \text{ mT}$ applied at $\theta = +90^\circ$ (red) and -90° (blue) in forward field sweep direction. Arrows indicate the sweep direction of I for each trace, ramping from zero to $200 \mu\text{A}$ and from zero to $-200 \mu\text{A}$. (d) Field-dependence of I_c for fixed $\theta = -90^\circ$. Absolute values of the I_c under positive bias I_c^+ and under negative bias I_c^- are indicated by solid and dashed lines, respectively. The sweep direction of the applied field is indicated by the colored arrows. (e) Diode factor extracted from the data shown in panel (d) for both field sweep directions. (f) Diode factor η as a function of B_{ip} at various in-plane angles θ . The field is swept from negative to positive values. The I_c curves producing the depicted $\eta(B_{ip}, \theta)$ are shown in the Supplemental Material [31]. Measurements shown in panels (c-f) are taken at 30 mK.

the Pt film by etching a narrow gap into the Nb layer. Fig. 1(a) depicts the stack schematically, and an SEM image is shown in the inset of Fig. 1(b).

We study the device resistance R as a function of temperature and observe a two-step behavior characteristic of a hybrid JJ. At $T \approx 8.6 \text{ K}$, the Nb electrodes become superconducting first, followed by a broad transition to $R = 0$ near $T \approx 6.0 \text{ K}$ [see Fig. 1(b)]. To investigate the signatures of Rashba-type SOC in the present device, we study the critical current I_c under a magnetic field B_{ip} applied in the sample plane. IV curves are acquired by ramping the applied current I first from zero to $200 \mu\text{A}$ and then from zero to $-200 \mu\text{A}$ while measuring the voltage drop V across the junction in a 4-point geometry. At 30 mK, $I_c R_n \approx 150 \mu\text{V}$. Fig. 1(c) displays two curves taken at $B_{ip} = 200 \text{ mT}$ applied at an angle $\theta = +90^\circ$ (red) and -90° (blue) with respect to

I . The difference in I_c under positive (I_c^+) and negative bias (I_c^-) is quantified through the diode efficiency $\eta = (I_c^+ - |I_c^-|)/(I_c^+ + |I_c^-|) \times 100\%$. Here $\eta = -8.9\%$ and 9.0% at $\theta = \pm 90^\circ$, respectively. Fig. 1(d) depicts the magnetic field dependence of I_c^+ and $|I_c^-|$ measured at $\theta = -90^\circ$ in both forward (from $-B_{ip}$ to $+B_{ip}$, blue) and backward (from $+B_{ip}$ to $-B_{ip}$, lavender) field sweep direction. All curves show the expected bell-like shape of a JJ under B_{ip} [32–34]. Comparing the curves of I_c^+ measured with opposite field sweep directions (solid lines), a substantial hysteretic shift in magnetic field is observed. The dashed traces representing $|I_c^-|$ are shifted in the same fashion. Remarkably, this shift does not conform with conventional magnetic hysteresis; instead it is characterized by an inverted order. Inverted hysteresis of critical currents in JJs of type-II superconductors has been reported before [28, 29, 35] and shall be discussed

in more detail below.

In addition to the shift between the sweep directions, a considerable JDE is observed between I_c^+ and $|I_c^-|$ for a given sweep direction. The resulting diode efficiency $\eta(B_{ip})$ is depicted in Fig. 1(e). It features the characteristic shape of a diode effect [13, 34, 36]. A maximum diode efficiency of $\pm 10\%$ is reached around ± 175 mT. The inverted hysteresis described above is expressed by a finite and sweep-direction dependent $\eta(B_{ip}) \sim \pm 2\%$. Fig. 1(f) summarizes the field-dependent diode factors for various angles θ in forward field-sweep direction. At $\theta = -90^\circ$ (i.e. $\vec{B} \perp \vec{j}$), $\eta(B_{ip})$ exhibits positive polarity (positive cusp at positive field and vice versa). As the field direction is rotated towards $+90^\circ$, the diode efficiency is gradually reduced, vanishes and changes sign at $\theta = 0^\circ$ (i.e. $\vec{B} \parallel \vec{j}$) before returning to its initial strength with opposite polarity at $+90^\circ$, constituting the typical 2π -periodic magneto-chiral anisotropy. Since the reversal of polarity in η occurs at $\theta = 0^\circ$, the chirality matches that of an interfacial Rashba SOC inside the weak link [37].

To rule out the possibility that η is affected by the relative orientation between the applied field and the in-plane $\langle 110 \rangle$ anisotropy axis of GaAs/Fe, we repeat the same measurements with a weak link consisting only of non-magnetic materials. Cu(50 nm)/Pt(3.5 nm)/Nb(40 nm) films are sputtered on a Si/SiO₂ substrate (sample B). This combination induces Rashba SOC at the Cu/Pt interface [24, 25] while simultaneously allowing for long-range Cooper pair transport through the Cu layer [39, 40]. Lateral JJs [see inset of Fig. 2(a)] are fabricated following the same methods as for the first sample, and the same measurement routine is applied. The resistance of sample B is shown in Fig. 2(a) as a function of temperature and exhibits a sharp edge around $T = 8.0$ K, indicating the superconducting transition of the Nb leads, and a second, broader transition near 5.0 K that is attributed to the proximitized junction area. A Josephson current appears for $T < 5$ K and yield $I_c R_n \approx 26 \mu V$ (see Supplemental Material [31]). The low resistance $R \approx 30$ m Ω between 5.0 and 8.0 K reflects the thickness of the Cu layer. Above 8.0 K, $R = 1.3 \Omega$. Similarly to sample A, we acquire IV curves under B_{ip} at various angles θ . Before each field sweep the sample temperature is brought to 9 K – which is above the T_c of the Nb leads – and field-cooled at the respective field. We extract I_c and calculate the diode efficiency η shown in Fig. 2(b). Compared to Fig. 1(f), the maximum diode efficiency is reduced by more than half, and the noise level is increased. Nevertheless, the same qualitative chirality is observed, exhibiting a maximally negative polarity at $\theta = -75^\circ$, a vanishing diode factor around 0° , and an increasing effect of positive polarity as θ approaches $+45^\circ$. Larger angles were not experimentally accessible. Note that the individual η traces shown in Fig. 2(b) are offset to overlap at zero magnetic field. For raw data and discussion of the zero-field offset, see Supplemental Material [31].

The similarity of the results for both samples suggests that the effect is most likely related to Rashba-type SOC rather than the Fe-magnetism of sample A.

Next, we verify that interfacial SOC is absent in weak links based on 50 nm Cu films with no metal-metal interfaces and negligible bulk SOC (sample C) [41, 42]. Details on the fabrication process and device characterization are given in the Supplemental Material [31]. Also in this device a non-zero diode factor of about $\pm 10\%$ is observed. In contrast to the bi-layer devices with Rashba interface, the diode factor of sample C does not depend of the direction of B_{ip} (see Supplemental Material [31]). We conclude that only for those devices with bi-layer weak links (samples A and B), the observed diode factor exhibits a magneto-chiral anisotropy under rotation of B_{ip} that obeys the same symmetry as Rashba SOC.

For the GaAs/Fe/Al/Pt layer sequence of sample A, where strong Rashba SOC has been reported [26, 27], the diode effect is more pronounced than for Cu/Pt which is known to have weaker SOC [24, 25]. This strongly supports the conclusion that the JDE is tuned by purely changing the interfacial configuration, demonstrating that the bulk SOC does not play a significant role. It is also consistent with observations by Senapati *et al.* [28] on comparable structures.

To characterize the junction quality, we investigate the Fraunhofer patterns of sample B. Fig. 2(c) shows $I_c(B_z)$ which is swept from -20 mT to 20 mT (forward, dark gray) and from 20 mT to -20 mT (backward, light gray) after the respective field-cooling sequence. An inversely hysteretic shift of the interference patterns in forward and backward field sweep direction of around ± 10 mT is observed. Experimental artifacts from remnant fields stored in the coils are ruled out as the origin of the inverted hysteresis in the Supplemental Material [31]. Rather than regular Fraunhofer lobes, the result features several jumps of I_c while the field is swept. The switching pattern varies for multiple repetitions of the sweep (see Supplemental Material [31]). As explained below, we assign the jumps to flux instabilities in the junctions resulting from the stray magnetic field of vortices pinned in the Nb leads near the junction and their random depinning events as the applied field changes (see Supplemental Material [31] for more details).

To avoid the complication of flux jumps and access the intrinsic supercurrent diffraction pattern, we apply magnetic field essentially parallel to the stack, but with a small misalignment angle $\epsilon = 0.5^\circ$ from the sample plane. In this configuration, vortices thread the Nb film over the full width of the contacts. Hence, each vortex is expected to contribute the amount $b_z^s = B_{ip} \sin \epsilon \simeq 0.01 B_{ip}$ to the average field in the junction, leading to a smooth, but still inversely hysteretic Fraunhofer pattern with several side lobes in Fig. 2(d). We note a considerable asymmetry in magnitude and width between the left and right secondary lobes with respect to the field sweep direction

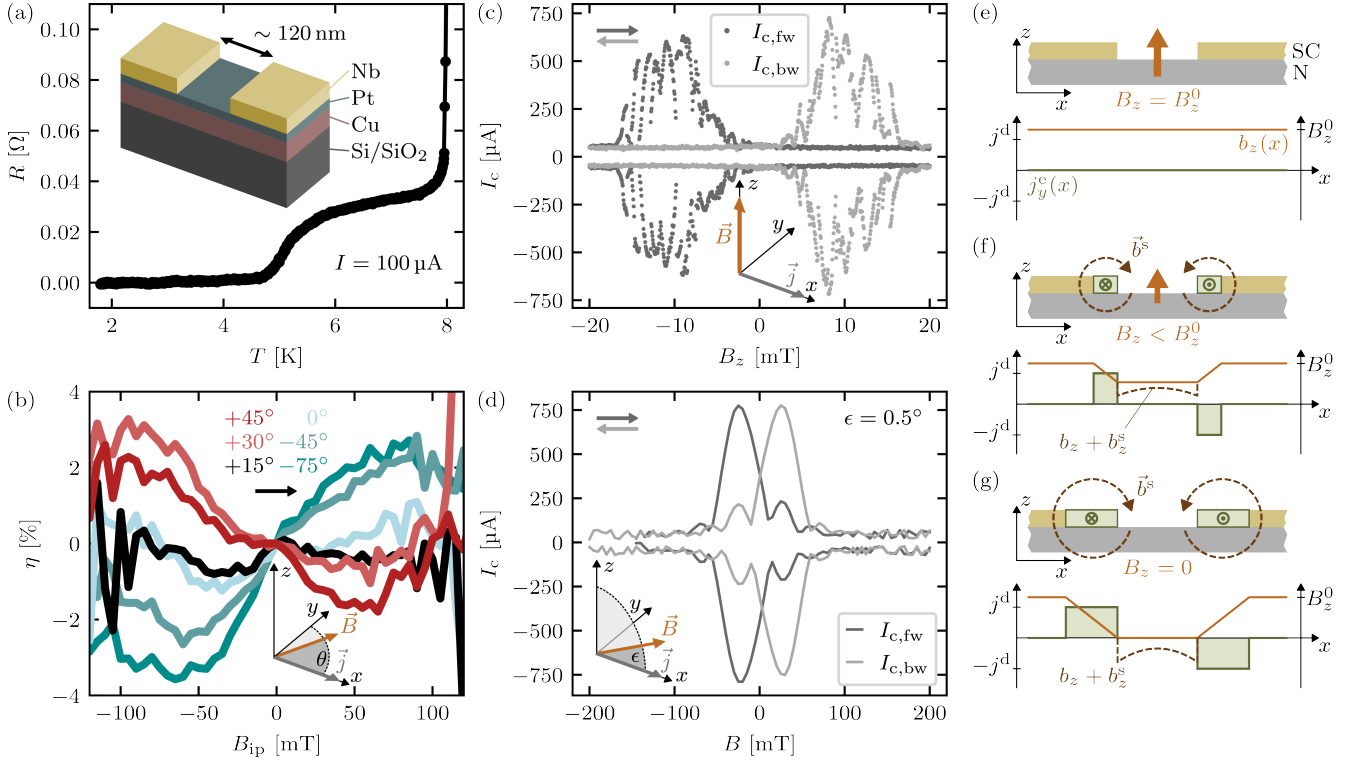


FIG. 2. Diode effect and inverted hysteresis in sample B. (a) Temperature-dependence of the device resistance measured at $I = 100 \mu\text{A}$ in zero magnetic field. A constant zero-bias voltage is subtracted. Inset: Schematic diagram of the Cu/Pt/Nb stack and the JJ on top of a Si/SiO₂ substrate. (b) Diode factor η as a function of B_{ip} at various angles θ . The field sweep direction is forward for all curves. $\eta(B_{\text{ip}} = 0)$ is subtracted to create an overlap at zero applied field. (c) $I_c(B_z)$ in forward (dark) and backward (light) field sweep direction. (d) $I_c(B)$ for B applied along the current direction with a small misalignment of $\epsilon = 0.5^\circ$ from the sample plane. All measurements are taken at 1.8 K. (e-g) Schematic cross-section of a generic JJ in a superconductor (SC) on top of a normal conductor (N) and corresponding profiles of the local external flux density $b_z(x)$ and edge current density $j_y^e(x)$ according to the critical state model [38] at different stages during a downward field sweep after field-cooling at B_z^0 . The dashed line qualitatively sketches the additional field contribution from b_z^s . For the discussion of the field profile, see Supplemental Material [31].

that is qualitatively comparable to the findings of Senapati *et al.* [28].

Inspecting Fig. 2(c) again reveals a smooth change of I_c in the junction over 1-3 mT until a flux jump occurs. In this way, short intervals of the intrinsically clean Fraunhofer pattern are probed many times, until the flux in the junction exceeds a few flux quanta and $I_c(B_x)$ is featureless (see also Supplemental Material [31]). The existence of a clean Fraunhofer pattern at a misalignment angle of only 0.5° demonstrates the importance of a precise field alignment routine for measurements in in-plane magnetic field. The alignment procedure applied to Cu/Pt and Cu samples is described in the Supplemental Material [31]. Following the same protocol, sample C is tested for hysteretic behavior under B_z , revealing qualitatively comparable inverted hysteresis, as shown in the Supplemental Material [31].

In recent reports, the phenomenon of inverted hysteresis in superconductor heterostructures has been linked to the presence of Rashba SOC inside the JJ [28] as well

as triplet-pair superconductivity [29, 30]. Since we observed inverted hysteresis in junctions with and without SOC, the effect appears to be not related to SOC. A much more conventional explanation takes into account the aforementioned pinned vortices near the edges of the Nb leads. The hysteretic penetration of vortices into the Nb strips is described by Bean's model [38]. This model considers strongly pinned vortices that gradually enter or exit a superconducting slab from the sides when the Meissner current induced by changes in applied field exceeds the local critical depinning current j^d . Confined in vortices, the flux penetrates much deeper than the London penetration depth. According to Maxwell's equation $\text{rot} \vec{B} = \mu_0 \vec{j}$, the gradual penetration of field introduces an edge current density $\vec{j}^e$ in the outer regions of the superconductor until $|\vec{j}^e|$ equilibrates at j^d .

Figures. 2(e-g) outline how the stray magnetic field $\vec{b}^s$ generated by the edge currents can oppose the externally applied field B_z in the junction area, causing an apparent inverted hysteresis in the flux applied to the

junction. The model considers a cross-section along the length of the junction in the xz -plane subject to an external field B_z ; the edge current $\vec{j}^e$ flows in y -direction. The local magnetic flux density $\vec{b}_z(x)$ is sketched together with the edge current $j_y^e(x)$. In this simplified geometry, Maxwell's equation reduces to

$$-\frac{db_z(x)}{dx} = \mu_0 j_y^e(x). \quad (1)$$

Initially, a field $B_z^0 < B_{c2}$ is applied in positive z -direction, see Fig. 2(e), where, B_{c2} is the upper critical field. By cooling below T_c in fixed B_z , the Nb leads are brought into a fully penetrated Abrikosov state, such that $b_z(x) = B_z^0$ both inside the junction as well as inside the leads. No edge currents are induced. Upon reducing the applied field below B_z^0 , the local external flux density $b_z(x)$ is lowered inside the junction, and vortices start to be pushed out of the leads near the edges such that a slope in b_z forms near the Nb edges. Thus we expect $j_y^e(x) \simeq \pm j^d$ in these regions as depicted in Fig. 2(f). The resulting stray field $\vec{b}^s$ adds a negative z -component in the area of the junction. When $B_z = 0$, the combined field $b_z + b_z^s$ is negative on the junction area [see Fig. 2(g)]. Consequently, the point of net zero flux enclosed in the junction must be reached while $B_z > 0$ during a backward field sweep and $B_z < 0$ during a forward sweep. Taking $j^d \simeq 50 \times 10^6 \text{ A/cm}^2$ (see Refs. [43–46] and Supplemental Material [31]), the magnitude of the excess field at zero applied field can be estimated to be $\sim 3\text{--}4 \text{ mT}$ for the case of sample B (see Supplemental Material [31]). This is compatible with the value $\pm 10 \text{ mT}$ seen in Fig. 2(c). Furthermore, the jumps in $I_c(B_z)$ visible in Fig. 2(c) as well as the minor loop behavior of sample C can be explained with this model, as discussed in the Supplemental Material [31]. As explained above, also for an in-plane applied field, the apparent inverted hysteresis is consistent with the Bean's model. Our interpretation is similar to the observations of [35].

Finally, we investigate the interference pattern of sample A. Figures. 3(a) and (b) illustrate the supercurrent interference pattern of the junction's IV characteristic as a function of B_z in forward and backward field sweep direction, respectively. Notably, $I_c(B_z)$ strongly differs from the standard Fraunhofer shape and exhibits a pronounced asymmetry about $B_z = 0$. The non-periodic behavior is attributed to the inherent inhomogeneity of the junction. Flux jumps are observed in I_c on the few mT-scale, in agreement with the critical state model, as demonstrated in the Supplemental Material [31]. Upon comparing the forward and backward sweeps, an apparent inverted hysteresis is seen again which is discussed in the context of the critical state model in the Supplemental Material [31].

We point out that, interestingly, the combined operations $I_c^+ \rightarrow I_c^-$ and $+B_z \rightarrow -B_z$ do not lead to the matching of the corresponding curves that is expected

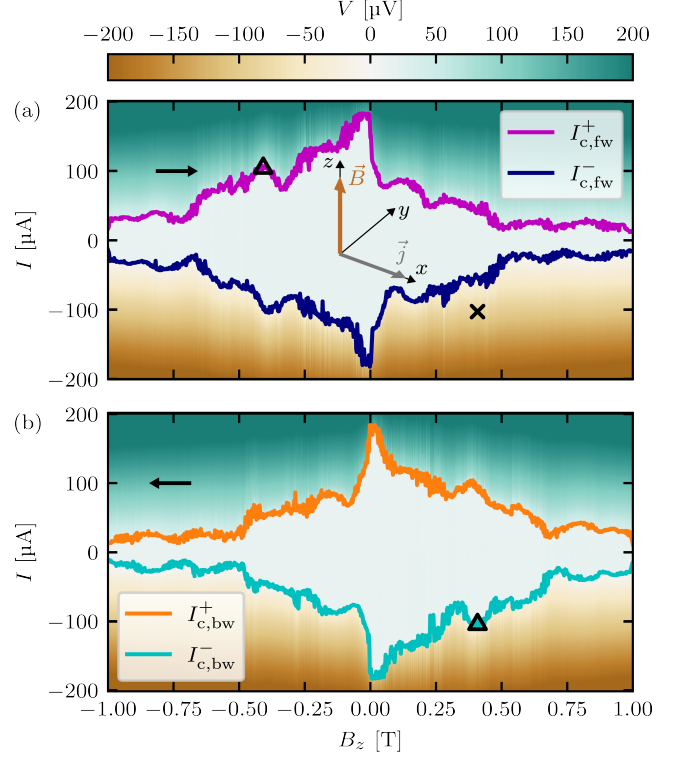


FIG. 3. Interference pattern of sample A under B_z in (a) forward and (b) backward field sweep direction, as indicated by the arrows. The extracted I_c curves are overlaid in different colors. The triangles indicate a feature that is reproduced only by the inversion of current, field and field sweep direction. The cross marks the position that is obtained by inverting only field and current direction. Both measurements are taken at 30 mK. The inset of panel (a) shows the direction of the applied field.

from TRS. The reason is that the hysteretic stray fields from the pinned vortices must be included to reverse *all* magnetic fields. Thus, a proper check for TRS has to reverse the sweep direction, not only I_c and the applied field B_z . As an example, the value of $I_{c, fw}^+$ near -0.4 T in Fig. 3(a) marked by the triangle is not reproduced in $-I_{c, fw}^-$ at $+0.4 \text{ T}$, as indicated by the cross. However, it is equal to $-I_{c, bw}^-$ at $+0.4 \text{ T}$ and the opposite sweep direction, as highlighted by the triangle in Fig. 3(b). In the Supplemental Material [31] it is demonstrated that the full I_c curves can be matched by comparing $I_{c, fw}^+$ and $I_{c, bw}^-$, and vice versa, whereas keeping the sweep direction fixed does not result in an adequate overlap.

To summarize, Nb-based Josephson junctions with three different thin-film stacks were investigated for the Josephson diode effect under rotating in-plane magnetic field. Rashba-type interfacial spin-orbit coupling is expected in stacks comprising Fe/Al/Pt and Cu/Pt. Our measurement reveals an anisotropic diode efficiency that signals the Rashba spin texture of the metal-metal interfaces. A control experiment with a Cu weak link,

i.e. without metal/metal interface, showed a finite, but nearly isotropic diode effect. Our results demonstrate the signatures of Rashba SOC in diffusive, all-metallic Josephson junctions. An apparently inverted hysteresis is attributed to the standard hysteresis of a hard superconductor, *i.e.* the Nb leads, which leads to stray fields that are opposite to the applied magnetic field. The consideration of this effect is crucial in superconducting electronics where the phase bias is a key parameter.

The authors thank Lin Chen, Nicola Paradiso and Jukka Pekola for insightful discussions. This work was funded by Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through Project-ID 314695032 - SFB 1277 (Subprojects A08 and B08), and by the European Union's Horizon 2020 Research and Innovation Programme, under Grant Agreement No. 824109 (EMP). DS acknowledges funding from Marie Skłodowska-Curie postdoctoral fellowship from EuroTechPostdoc2. The experimental work benefited from the Aalto University OtaNano/LTL infrastructure.

-
- [1] R. Aguado, Majorana quasiparticles in condensed matter, *La Rivista del Nuovo Cimento* **40**, 523 (2017).
 - [2] R. Aguado, A perspective on semiconductor-based superconducting qubits, *Applied Physics Letters* **117** (2020).
 - [3] S. Das Sarma, In search of Majorana, *Nature Physics* **19**, 165 (2023).
 - [4] N. O. Birge and N. Satchell, Ferromagnetic materials for Josephson π junctions, *APL Materials* **12**, 041105 (2024), <https://pubs.aip.org/aip/apm/article-pdf/doi/10.1063/5.0195229/19866826/041105.1.5.0195229.pdf>.
 - [5] M. Eschrig, Spin-polarized supercurrents for spintronics: a review of current progress, *Reports on Progress in Physics* **78**, 104501 (2015).
 - [6] J. Linder and J. W. Robinson, Superconducting spintronics, *Nature Physics* **11**, 307 (2015).
 - [7] C. Baumgartner, L. Fuchs, A. Costa, S. Reinhardt, S. Gronin, G. C. Gardner, T. Lindemann, M. J. Manfra, P. E. Faria Junior, D. Kochan, J. Fabian, N. Paradiso, and C. Strunk, Supercurrent rectification and magnetochiral effects in symmetric Josephson junctions, *Nature Nanotechnology* **17**, 39 (2022).
 - [8] B. Pal, A. Chakraborty, P. K. Sivakumar, M. Davydova, A. K. Gopi, A. K. Pandeya, J. A. Krieger, Y. Zhang, M. Date, S. Ju, *et al.*, Josephson diode effect from Cooper pair momentum in a topological semimetal, *Nature physics* **18**, 1228 (2022).
 - [9] H. Wu, Y. Wang, Y. Xu, P. K. Sivakumar, C. Pasco, U. Filippozzi, S. S. Parkin, Y.-J. Zeng, T. McQueen, and M. N. Ali, The field-free Josephson diode in a van der Waals heterostructure, *Nature* **604**, 653 (2022).
 - [10] A. Costa, J. Fabian, and D. Kochan, Microscopic study of the Josephson supercurrent diode effect in Josephson junctions based on two-dimensional electron gas, *Phys. Rev. B* **108**, 054522 (2023).
 - [11] M. Amundsen, J. Linder, J. W. Robinson, I. Žutić, and N. Banerjee, Colloquium: Spin-orbit effects in superconducting hybrid structures, *Reviews of Modern Physics* **96**, 021003 (2024).
 - [12] T. Golod and V. M. Krasnov, Demonstration of a superconducting diode-with-memory, operational at zero magnetic field with switchable nonreciprocity, *Nature Communications* **13**, 3658 (2022).
 - [13] D. Suri, A. Kamra, T. N. G. Meier, M. Kronseder, W. Belzig, C. H. Back, and C. Strunk, Non-reciprocity of vortex-limited critical current in conventional superconducting micro-bridges, *Applied Physics Letters* **121**, 102601 (2022).
 - [14] Y. Hou, F. Nichele, H. Chi, A. Lodesani, Y. Wu, M. F. Ritter, D. Z. Haxell, M. Davydova, S. Ilić, O. Glezakou-Elbert, *et al.*, Ubiquitous superconducting diode effect in superconductor thin films, *Physical Review Letters* **131**, 027001 (2023).
 - [15] J. Ingla-Aynés, Y. Hou, S. Wang, E.-D. Chu, O. A. Mukhanov, P. Wei, and J. S. Moodera, Efficient superconducting diodes and rectifiers for quantum circuitry, *Nature Electronics*, 1 (2025).
 - [16] M. Castellani, O. Medeiros, A. Buzzi, R. A. Foster, M. Colangelo, and K. K. Berggren, A superconducting full-wave bridge rectifier, *Nature Electronics*, 1 (2025).
 - [17] E. I. Rashba, Properties of semiconductors with an extremum loop. I. Cyclotron and combinational resonance in a magnetic field perpendicular to the plane of the loop, *Soviet Physics, Solid State* **2**, 1109 (1960).
 - [18] Y. A. Bychkov and E. I. Rashba, Properties of 2D electron gas with lifted spectral degeneracy, *JETP Lett.* **39**, 78 (1984).
 - [19] A. Manchon, H. C. Koo, J. Nitta, S. M. Frolov, and R. A. Duine, New perspectives for Rashba spin-orbit coupling, *Nature materials* **14**, 871 (2015).
 - [20] S. D. Ganichev and L. E. Golub, Interplay of Rashba/Dresselhaus spin splittings probed by photogalvanic spectroscopy - A review, *physica status solidi (b)* **251**, 1801 (2014).
 - [21] P. Gambardella and I. M. Miron, Current-induced spin-orbit torques, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **369**, 3175 (2011).
 - [22] A. Manchon and S. Zhang, Theory of nonequilibrium intrinsic spin torque in a single nanomagnet, *Physical Review B—Condensed Matter and Materials Physics* **78**, 212405 (2008).
 - [23] I. M. Miron, G. Gaudin, S. Auffret, B. Rodmacq, A. Schuhl, S. Pizzini, J. Vogel, and P. Gambardella, Current-driven spin torque induced by the Rashba effect in a ferromagnetic metal layer, *Nature Materials* **9**, 230 (2010).
 - [24] J.-C. Rojas-Sánchez, N. Reyren, P. Laczkowski, W. Savero, J.-P. Attané, C. Deranlot, M. Jamet, J.-M. George, L. Vila, and H. Jaffrès, Spin Pumping and Inverse Spin Hall Effect in Platinum: The Essential Role of Spin-Memory Loss at Metallic Interfaces, *Phys. Rev. Lett.* **112**, 106602 (2014).
 - [25] R. Yu, B. Miao, Q. Liu, K. He, W. Xue, L. Sun, M. Wu, Y. Wu, Z. Yuan, and H. Ding, Fingerprint of the inverse Rashba-Edelstein effect at heavy-metal/Cu interfaces, *Phys. Rev. B* **102**, 144415 (2020).
 - [26] L. Chen, M. Decker, M. Kronseder, R. Islinger, M. Gmitra, D. Schuh, D. Bougeard, J. Fabian, D. Weiss, and

- C. H. Back, Robust spin-orbit torque and spin-galvanic effect at the Fe/GaAs (001) interface at room temperature, *Nature Communications* **7**, 13802 (2016).
- [27] L. Chen, Y. Sun, S. Mankovsky, T. Meier, M. Kronseder, C. Sun, A. Orekhov, H. Ebert, D. Weiss, and C. Back, Signatures of magnetism control by flow of angular momentum, *Nature* **633**, 548 (2024).
- [28] T. Senapati, A. K. Karnad, and K. Senapati, Phase biasing of a Josephson junction using Rashba-Edelstein effect, *Nature Communications* **14**, 7415 (2023).
- [29] T. Nakamura, L. D. Anh, Y. Hashimoto, S. Ohya, M. Tanaka, and S. Katsumoto, Evidence for Spin-Triplet Electron Pairing in the Proximity-Induced Superconducting State of an Fe-Doped InAs Semiconductor, *Phys. Rev. Lett.* **122**, 107001 (2019).
- [30] S.-P. Chiu, C. C. Tsuei, S.-S. Yeh, F.-C. Zhang, S. Kirchner, and J.-J. Lin, Observation of triplet superconductivity in CoSi₂/TiSi₂ heterostructures, *Science Advances* **7**, eabg6569 (2021).
- [31] See Supplemental Material at links.aps.org for experimental details and supporting measurements, which includes Refs. [48–51].
- [32] M. Tinkham, *Introduction to superconductivity* (Courier Corporation, 2004).
- [33] H. J. Suominen, J. Danon, M. Kjaergaard, K. Flensberg, J. Shabani, C. J. Palmström, F. Nichele, and C. M. Marcus, Anomalous Fraunhofer interference in epitaxial superconductor-semiconductor Josephson junctions, *Phys. Rev. B* **95**, 035307 (2017).
- [34] B. Turini, S. Salimian, M. Carrega, A. Iorio, E. Strambini, F. Giazotto, V. Zannier, L. Sorba, and S. Heun, Josephson diode effect in high-mobility InSb nanoflags, *Nano Letters* **22**, 8502 (2022).
- [35] P. Mandal, N. Taufertshöfer, L. Lunczer, M. P. Stehno, C. Gould, and L. W. Molenkamp, Finite field transport response of a dilute magnetic topological insulator-based Josephson junction, *Nano Letters* **22**, 3557 (2022).
- [36] L. Bauriedl, C. Bäuml, L. Fuchs, C. Baumgartner, N. Paulik, J. M. Bauer, K.-Q. Lin, J. M. Lupton, T. Taniguchi, K. Watanabe, *et al.*, Supercurrent diode effect and magnetochiral anisotropy in few-layer NbSe₂, *Nature communications* **13**, 4266 (2022).
- [37] M. Nadeem, M. S. Fuhrer, and X. Wang, The superconducting diode effect, *Nature Reviews Physics* **5**, 558 (2023).
- [38] C. P. Bean, Magnetization of Hard Superconductors, *Phys. Rev. Lett.* **8**, 250 (1962).
- [39] T. Bergmann, K. Kuhl, B. Schröder, M. Jutzler, and F. Pobell, Proximity-effect-induced superconductivity at millikelvin temperatures, *Journal of low temperature physics* **66**, 209 (1987).
- [40] L. Schneider, C. von Bredow, H. Kim, K. T. Ton, T. Hänke, J. Wiebe, and R. Wiesendanger, High-resolution spectroscopy of proximity superconductivity in finite-size quantized surface states, *arXiv preprint arXiv:2402.08895* (2024).
- [41] C. Du, H. Wang, F. Yang, and P. C. Hammel, Systematic variation of spin-orbit coupling with d-orbital filling: Large inverse spin Hall effect in 3 d transition metals, *Physical Review B* **90**, 140407 (2014).
- [42] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Spin hall effects, *Reviews of modern physics* **87**, 1213 (2015).
- [43] R. Huebener, R. Kampwirth, R. Martin, T. Barbee Jr, and R. Zubeck, Critical current density in superconducting niobium films, *Journal of low temperature physics* **19**, 247 (1975).
- [44] V. Dedyu, A. Lykov, and S. Prischepa, Critical currents in niobium-based layer structures, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki* **97**, 872 (1990).
- [45] G. S. Park, C. E. Cunningham, B. Cabrera, and M. E. Huber, Vortex pinning force in a superconducting niobium strip, *Physical review letters* **68**, 1920 (1992).
- [46] A. Pautrat, J. Scola, C. Goupil, C. Simon, C. Villard, B. Domengès, Y. Simon, C. Guilpin, and L. Méchin, Quantitative analysis of the critical current due to vortex pinning by surface corrugation, *Physical Review B—Condensed Matter and Materials Physics* **69**, 224504 (2004).
- [47] M. Mangold, L. Bauriedl, J. Berger, C. Yu-Cheng, T. N. Meier, M. Kronseder, P. Hakonen, C. H. Back, C. Strunk, and D. Suri, Data pertaining to "Magnetochiral Anisotropy in Josephson Diode Effect of All-Metallic Lateral Junctions with Interfacial Rashba Spin-Orbit Coupling", *abcdefghijklmnopqrstuvwxyz* (2025).
- [48] G. Bayreuther, J. Prempfer, M. Sperl, and D. Sander, Uniaxial magnetic anisotropy in Fe/GaAs (001): Role of magnetoelastic interactions, *Physical Review B—Condensed Matter and Materials Physics* **86**, 054418 (2012).
- [49] T. I. Larkin, V. V. Bol'ginov, V. S. Stolyarov, V. V. Ryazanov, I. V. Vernik, S. K. Tolpygo, and O. A. Mukhanov, Ferromagnetic Josephson switching device with high characteristic voltage, *Applied Physics Letters* **100**, 222601 (2012).
- [50] V. V. Bol'ginov, V. Stolyarov, D. Sobanin, A. Karpovich, and V. V. Ryazanov, Magnetic switches based on Nb-PdFe-Nb Josephson junctions with a magnetically soft ferromagnetic interlayer, *JETP letters* **95**, 366 (2012).
- [51] B. Baek, W. H. Rippard, S. P. Benz, S. E. Russek, and P. D. Dresselhaus, Hybrid superconducting-magnetic memory device using competing order parameters, *Nature communications* **5**, 3888 (2014).

SUPPLEMENTAL MATERIAL

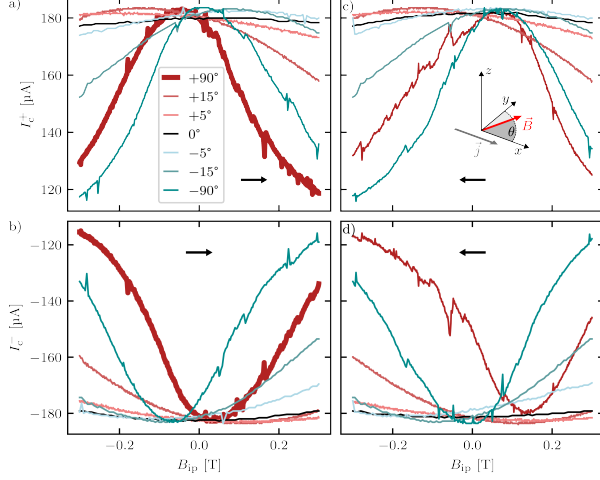
CRITICAL CURRENT CORRESPONDING TO
THE η VS B_{ip} 

FIG. S1. Critical current of sample A under in-plane magnetic field for various in-plane field angles θ under (a) positive bias and forward field sweep, (b) negative bias and forward field sweep, (c) positive bias and backward field sweep, and (d) negative bias and backward field sweep. Inset in panel (c) indicates field direction with respect to applied current and sample xy -plane.

This figure corresponds to the critical current curves which generate η in Fig. 1(f) in the main text.

The diode factor η discussed is calculated from measurements of I_c^+ and I_c^- in forward field sweep direction, plotted in Figs. SS1(a) and (b), respectively. The forward sweep at $\theta = 90^\circ$ is highlighted to visualize the diode effect, manifested in a horizontal shift between I_c^+ and I_c^- . Figs. SS1(c) and (d) show I_c^+ and I_c^- in backward sweep direction, also revealing a diode effect. Inverted hysteresis is observed at all angles θ .

DETAILS ON SAMPLE B

IV characteristics of sample B are measured by sweeping the applied current from zero to 1 mA and from zero to -1 mA. Fig. S2(a) shows a typical IV curve measured at $B_{ip} = 0.05$ T and $\theta = 15^\circ$. Fig. S2(b) depicts $I_c(B_z)$ as shown in the main text Fig. 2(c) in forward and backward magnetic field sweep direction, showing inverted hysteresis. In addition, the Al stripe resistance measured simultaneously (see Supplemental Note 5) is shown for both sweep directions in the corresponding colors. No hysteretic shift is observed in R_{Al} , demonstrating that the inverted hysteresis in $I_c(B_z)$ is not an experimental artifact.

The reproducibility of the $I_c(B_z)$ curves is investigated by comparing measurements of $I_c(B_z)$ from different runs, as shown in Fig. S2(c). Run A corresponds to the data plotted in the main text Fig. 2(c), run B is recorded separately on the same device. While inverted hysteresis of similar width is present in both experiments, the jumps in I_c occur at random fields. This indicates spontaneous re-ordering of the vortex distribution near the lead edges, leading to jumps in the flux through the junction, see also Supplemental Note 7.

Fig. S2(d) depicts the field-dependence of $I_c(B_{ip})$ of sample B for several in-plane field angles in forward field sweep direction, showing regular, ~ 200 mT wide single lobes. The diode factor η extracted from this data is shown in Fig. S2(e) before subtraction of a zero-field offset. While the θ -dependence is less clear than in the case of sample A, a polarity change between $\theta = 90^\circ$ and -90° is observed. To generate the main text Fig. 2(d), a zero-field offset is subtracted from the diode factor to align the curves at $B_{ip} = 0$. Fig. S2(f) shows this zero-field diode factor offset $\Delta\eta_{B=0}$. Since no clear angular dependence can be inferred, the offset is considered a random fluctuation, and subtracting it does not conflict with the interpretation of the diode factor as being related to Rashba SOC.

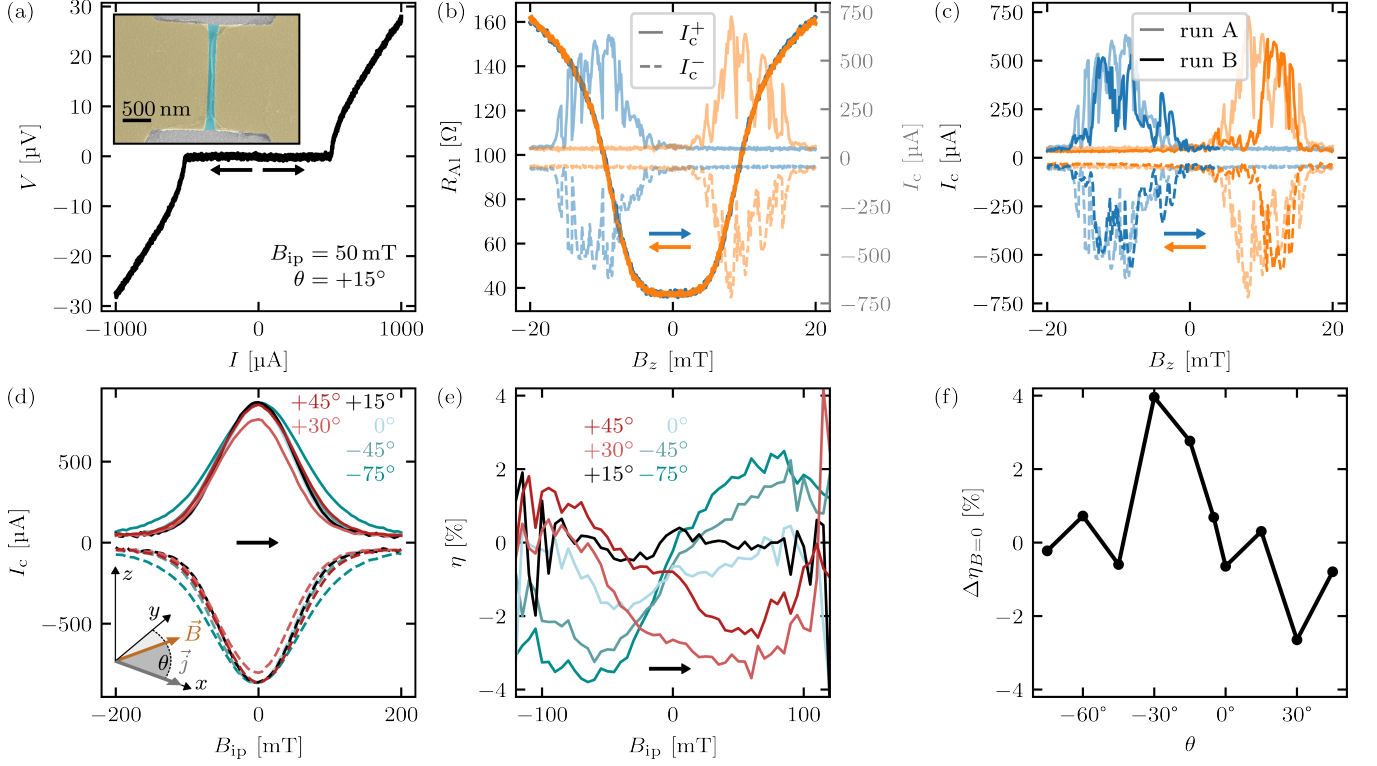


FIG. S2. Details and raw data on sample B. (a) Typical IV curve. The inset shows a false-color SEM image of the device where the Nb leads are highlighted in yellow and the junction area is colored in blue. (b) Right axis: $I_c(B_z)$, re-plotted from the main text Fig. 2(c). Left axis: Al stripe resistance, measured simultaneously with $I_c(B_z)$. The arrows indicate the field sweep direction. (c) $I_c(B_z)$ measured on the same device in two different runs at 1.8 K. (d) $I_c(B_{ip})$ at various in-plane angles θ . The field orientation is indicated by the inset. (e) Diode factor extracted from the data shown in panel (d) before subtracting the zero-field offset $\Delta\eta_{B=0}$. (f) Zero-field diode factor $\Delta\eta_{B=0}$ subtracted from the data shown in panel (e) to generate Fig. 2(b).

FABRICATION AND CHARACTERIZATION OF SAMPLE C

During the fabrication on the GaAs/Fe/Al/Pt/Nb and Cu/Pt/Nb stacks, electron beam lithography-defined junctions were etched into the Nb by SF_6 reactive ion etching, using the Pt layer as an etch stop. For the Cu/Nb sample, selective etching becomes more difficult due to the high etch rate of Cu in an SF_6 environment. To precisely control the etch depth, a Ga-based Focussed Ion Beam microscope (Zeiss Crossbeam 550) is employed at an acceleration voltage of 30 kV and a beam current of 1 pA. The etch rate is controlled to stop inside the Cu layer. SEM and Electron Dispersive X-ray spectroscopy (EDX) are applied in-situ immediately after the etch step. An SEM image of the central region of the Cu junction is depicted in Fig. S3(a). EDX point spectra are taken on the Nb leads and inside the junction and shown in Fig. S3(b) in blue and orange, respectively, as indicated by the colored circles in Fig. S3(a). Elementary spectra are fitted to the curves using the Oxford Instruments AZtec software tool. The characteristic peaks of

relevant elements are highlighted in Fig. S3 (b). Compared to the full stack on the leads, the Cu and Si peaks are enhanced inside the gap, while the Nb peak is largely suppressed. The small remaining Nb component inside the gap is attributed to unavoidable re-deposition during the etch process; even after etching deep into the Cu layer, the Nb peak does not fully disappear (not shown). In Fig. S3(c), the Cu and Nb parts of a line scan spectrum across the junction are compared. Inside the junction, the Nb part reduces to almost zero while the Cu counts increase. In Fig. S3(d), the temperature-dependent resistance of sample C shows a step decline when cooling below 8 K as the Nb leads turn superconducting. Around 5 K, a second step is observed, attributed to the junction entering the superconducting phase. A typical IV curve measured at $\theta = -90^\circ$ and zero field is shown in Fig. S3(e). The magneto-interference pattern under B_z in forward and backward sweep direction is depicted in Fig. S3(f), demonstrating inverted hysteresis of ~ 10 mT, comparable to sample B discussed in the main text. Spontaneous jumps in I_c are observed, as discussed further in Supplemental Note 7. For both sweep directions, the trailing side of the spectrum after reach-

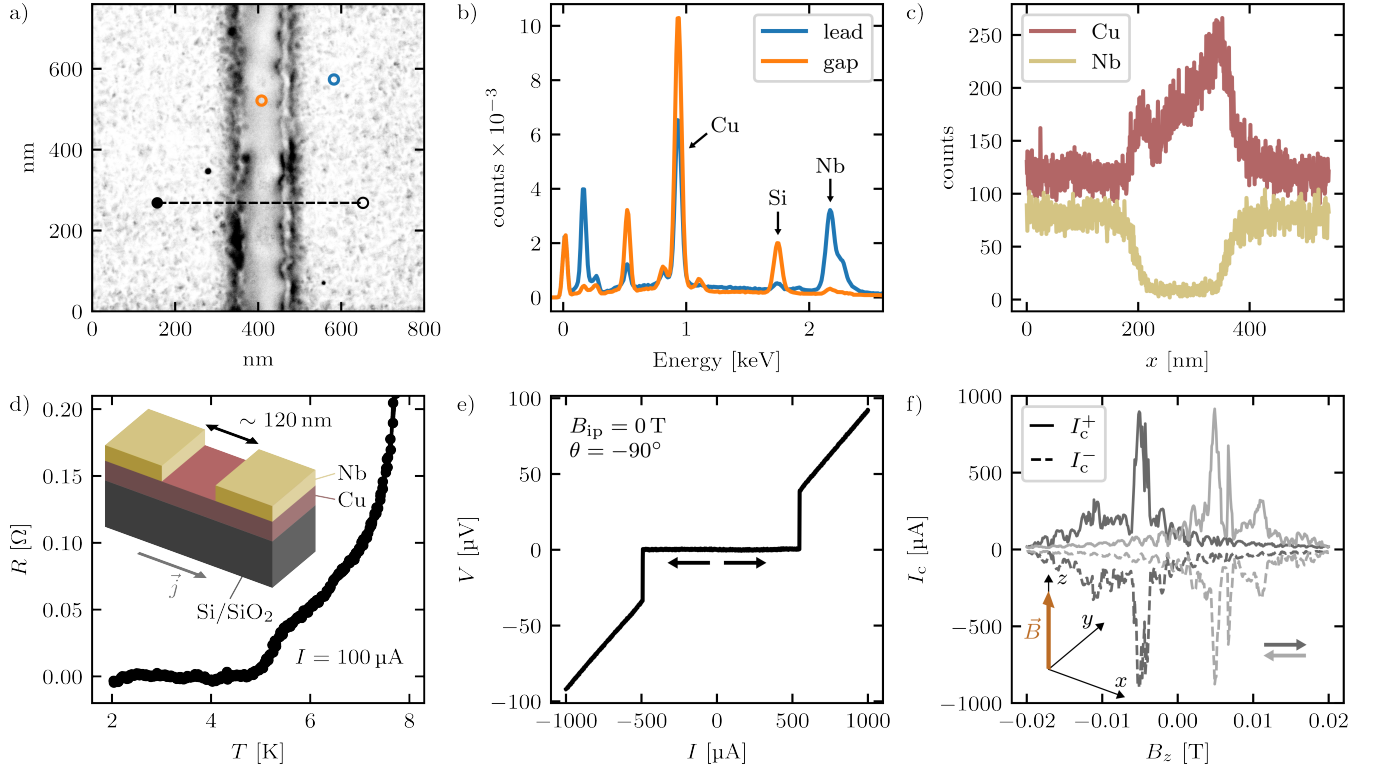


FIG. S3. Fabrication and characterization of sample C. (a) SEM image of central section of the junction. The colored circles mark the positions of the EDX spectra shown in panel (b). The dashed line indicates the path of the EDX line scan depicted in panel (c). (b) EDX spectra taken on the Nb leads and inside the Cu junction at an acceleration voltage of 5 kV. The spectral peaks of Cu, Si and Nb are indicated. (c) Line scan of element-specific counts across the length of the junction. (d) $R(T)$ curve, measured at $I = 100 \mu\text{A}$ and $B = 0 \text{ T}$. The inset shows the schematic device structure and the current direction. (e) IV characteristics measured at $B_{ip} = 0 \text{ T}$ and $\theta = -90^\circ$. The sweeping direction of the applied current is indicated by the arrows. (f) $I_c(B_z)$ in forward and backward field sweep direction, as indicated by the arrows.

ing maximum I_c exhibits a periodic Fraunhofer pattern which shows the Josephson junction character of the device.

COMPARISON OF IN-PLANE DIODE EFFECT BETWEEN GAAS/FE/AL/PT, CU/PT AND CU WEAK LINKS

The in-plane diode effect of the three measured samples, GaAs/Fe/Al/Pt (sample A), Cu/Pt (sample B) and Cu (sample C), is summarized in Fig. S4. To illustrate the fundamental difference between the devices A and B on one side and sample C on the other, η is plotted for various in-plane fields as a function of the in-plane angle θ . This corresponds to taking vertical line cuts in the $\eta(B_{ip})$ curves shown in the main text Fig. 1(f) and Fig. 2(b) for devices A and B, respectively. For sample A (Fig. S4 (a)), the sign change in η occurs at $\theta = 0^\circ$. Similarly, in sample B (Fig. S4(b)), a sign change is observed between zero and 20° . This offset angle is likely related to a misalignment between the nominal field di-

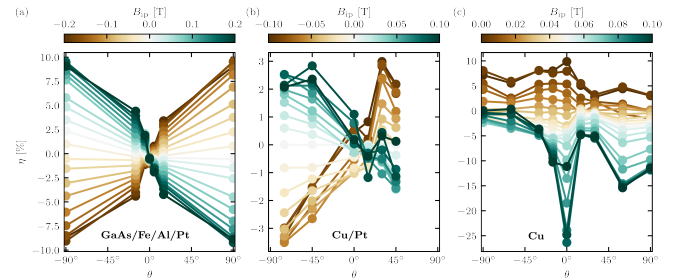


FIG. S4. Comparison of in-plane field-dependent diode effect of all samples presented in the main text. $\eta(\theta)$ is shown for (a) sample A, (b) sample B and (c) sample C at several magnetic field values. The data for devices A and B corresponds directly to the curves shown in main text Fig. 1(f) and Fig. 2(b), respectively.

rection and the current direction on the chip. For sample C, however, no sign change can be identified in Fig. S4(c). For an individual field, the diode factor is finite, but independent of the field angle. Thus, the diode effect in sample C does not comply with Rashba SOC.

ALIGNMENT METHOD FOR THE DEVICES

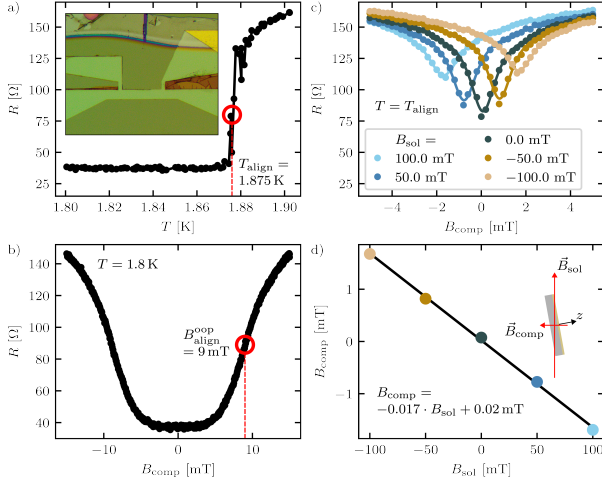


FIG. S5. Alignment procedure. (a) $R(T)$ of the Al stripe. The red circle highlights the temperature T_{align} at which the in-plane alignment is performed. The inset shows a micrograph of the Al stripe in a four-point geometry. (b) Al stripe resistance under variable compensation field at $T = 1.8$ K. The measurement is taken before the out-of-plane procedure with the sample aligned roughly perpendicular to B_{comp} . The red circle indicates the field at which out-of-plane alignment is conducted. (c) Typical in-plane alignment measurement conducted at $T = T_{\text{align}}$ for various solenoid fields. Solid lines indicate phenomenological Lorentzian fits to determine B_{comp} at minimum stripe resistance. (d) Minimum positions extracted from fits in panel (c). Linear fit yields parameters for alignment function $B_{\text{comp}} = a \cdot B_{\text{sol}} + b$. Inset shows orientation of sample in solenoid and compensation field.

As demonstrated by Fig. 2(c) in the main text, a small out-of-plane component caused by a sample misalignment of less than one degree from the field axis can lead to considerable narrowing of the central lobe in $I_c(B_{\text{ip}})$ and the formation of side lobes. These out-of-plane features mask the in-plane signature and may lead to a distortion in the diode factor. In order to optimize the in-plane alignment, the cryostat is equipped with a superconducting solenoid magnet used to apply the major component of the magnetic field $\vec{B}_{\text{sol}}$, as well as a pair of coils that generate an independent, perpendicular compensation field $\vec{B}_{\text{comp}}$. As a sensitive out-of-plane field probe, a 10 nm thick Al stripe is evaporated on the substrate after the junction fabrication and patterned into a $200 \mu\text{m} \times 50 \mu\text{m}$ stripe (see Fig. S5(a) inset), located less than 1 mm away from the junction device on the same chip. The longitudinal resistance of the stripe is measured using lock-in detection in a four-terminal geometry. Since the Al is a thin film type-I superconductor, its state is dominantly determined by the out-of-plane magnetic field and free of vortex-driven effects.

Initially, the temperature-dependence of the stripe resistance is determined at zero applied field, as shown in Fig. S5(a). The field compensation protocol operates at a temperature just below T_c where the Al stripe is most sensitive to any applied magnetic field. From Fig. S5(a), this alignment temperature is determined as $T_{\text{align}} = 1.875$ K. To align the sample plane normal to the compensation field, the stripe resistance is measured as a function of B_{comp} at a temperature well below T_c , see Fig. S5(b), with the sample being roughly aligned with the nominal field axis. The field is set to a constant value of around 9 mT determined by the point of the largest slope in resistance. The sample is then aligned perpendicular to $\vec{B}_{\text{comp}}$ by rotating the insert from outside the cryostat and maximizing the stripe resistance, corresponding to the largest out-of-plane component produced by $B_{\text{comp}} = 9$ mT. Since this is an iterative trial-and-error process, no data is available on this step. The angular accuracy of this method is estimated to be around 5° . These steps are performed once in the beginning of each cool-down.

To access various in-plane angles θ , the sample is mounted on a piezo rotator. Since the axis of the rotator is not guaranteed to be perpendicular to the sample plane, the following steps of the in-plane alignment have to be performed at each angle θ . An exemplary alignment measurement is shown in Fig. S5(c). At T_{comp} , $R(B_{\text{comp}})$ is measured for various constant values of the solenoid field B_{sol} . The minimum resistance of each curve represents the compensation field B_{comp} necessary to counteract the spurious out-of-plane component of the applied B_{sol} . A Lorentzian fit function is used to identify the minimum position. (It should be noted that there is no theoretical model supporting the use of this function; the Lorentz fit is merely used to interpolate the true minimum position between a limited number of data points.) The extracted compensation fields are plotted in Fig. S5(d) and fitted linearly, providing a function $B_{\text{comp}} = a \cdot B_{\text{sol}} + b$ to determine B_{comp} needed to compensate the out-of-plane component of a given B_{sol} during the field-dependent IV curves of the Josephson device under investigation.

The inset of Fig. S5(d) schematically depicts the relative orientation of solenoid field $\vec{B}_{\text{sol}}$, compensation field $\vec{B}_{\text{comp}}$ and sample normal z . Note that the notation B_{ip} and B_z is relative to the sample plane.

Instead of using the compensation function to align the applied field in the sample plane, one can also extract the misalignment angle ϵ between the sample and $\vec{B}_{\text{sol}}$ by $\epsilon = \arctan(a)$. The data shown in Fig. 2(d) of the main text is acquired by performing the alignment procedure to calculate the misalignment angle ϵ and applying only B_{sol} without compensation.

ESTIMATION OF THE MAGNITUDE OF THE STRAY FIELD INSIDE THE JUNCTION

The explanation of inverted hysteresis of the magnetic field interference patterns on all presented junctions based on the critical state model is discussed qualitatively in the main text. Using a set of crude assumptions, the magnitude of the magnetic field b^s produced by edge currents inside the lead edges acting on the junction area can be estimated.

First, we address the vortex depinning current density j^d which ultimately defines the penetration depth of the edge currents into the leads. At the smallest cross-section along the direction of the applied current, the leads have a width of $w = 2 \mu\text{m}$, as depicted in Fig. S6(a). Combined with the Nb thickness $d_{\text{Nb}} = 40 \text{ nm}$ and a critical current of $I_c^{\text{leads}} = 40 \text{ mA}$ determined on a different device, the critical depinning current density is $j^d = \frac{I_c^{\text{leads}}}{w d_{\text{Nb}}}$. Further

assuming j^d to be constant everywhere inside the leads, the edge current depth x^{max} of the vortices after ramping the applied field from B_z^0 to zero can be calculated, following Eq. (1) of the main text:

$$-\frac{db_z(x)}{dx} = -\frac{B_z^0}{x^{\text{max}}} = \mu_0 j_y^e(x) = \mu_0 j^d. \quad (2)$$

Using $B_z^0 = 20 \text{ mT}$, the applied out-of-plane field at which the sample is field-cooled for the measurements shown in main text Fig. 2(b), one obtains $x^{\text{max}} = 32 \text{ nm}$. This defines a volume $x^{\text{max}} \times w \times d_{\text{Nb}}$ in which a constant current density j^d flows in along the lead edges as depicted in Figs. S6(a) and (b). The z -component of the magnetic field generated by j_y^e of a single lead can be calculated (along the x -axis) based on Biot-Savart's law by

$$b_z^{s,i}(x) = \frac{\mu_0 j^d}{4\pi} \int_{-l/2-x^{\text{max}}}^{-l/2} \int_{-w/2}^{w/2} \int_0^{d_{\text{Nb}}} \frac{x-x'}{[(x-x')^2 + y'^2 + z'^2]^{3/2}} dz' dy' dx' \quad (3)$$

where the index i denotes the individual leads. Fig. S6(c) shows the result of a numeric integration applied for each lead, and the total stray field $b_z^s(x)$. The result implies that at zero applied field, the junction is subject to a field density of 3-4 mT which is comparable to the inverted hysteresis width of $\pm 10 \text{ mT}$ observed in Fig. 2(b).

Some remarks should be made on the obvious weaknesses of this simplistic model. First, it may be an oversimplification to assume a constant depinning current density. Especially near the Electron Beam Lithography- or Focussed Ion Beam-fabricated edges of the leads, local pinning centers should be expected. Second, no spontaneous redistribution of vortices is considered, as discussed in Supplemental Note 7. Finally, mutually and self-induced fields as well as edge effects are not taken into account. As a consequence, these results are only considered as an indication that the critical state model provides the correct order of magnitude to explain the observed inverted hysteresis.

EXPLANATION OF JUMPS IN $I_c(B_z)$ AND MINOR LOOPS IN B_z THROUGH CRITICAL STATE MODEL

The jumps in I_c observed for devices B and C under a perpendicular field B_z can be understood when considering the critical state model in combination with spon-

taneous reordering of the vortex distribution. Fig. S7(a) depicts $I_c(B_z)$ for sample B. The jumps in I_c occur at random fields, but simultaneously in I_c^+ and I_c^- . Furthermore, the data following a jump can be shifted backwards with respect to the field sweep direction to overlap with the data before the jump. The blue arrows in Fig. S7(a) indicate examples of such backward shifts. By applying this method to both forward and backward field sweeps, irregular, but mostly consistent Fraunhofer patterns can be restored, as shown in Fig. S7(b) and (c), respectively. It is important to highlight that the data is only shifted opposed to the sweep direction and never along it. This behavior can be explained based on vortices inside the Nb leads. Following the critical state model as outlined in the main text, an ordered vortex distribution builds up during the field sweep which creates an additional stray field component on the junction area. If changes in the applied field spontaneously bring this distribution to a less ordered state through the random depinning of vortices, the stray field is reduced. Consequently, the total field acting on the junction is increased again (*i.e.* more negative (positive) for the forward (backward) sweep before reaching zero) and a part of the junction's interference pattern is repeated at a lower applied field. In other words, the sections of the raw data in Fig. S7(a) constitute replica of a consistent Fraunhofer patterns. Therefore, shifting the curve backwards at each jump in I_c reconstructs the intrinsic diffraction pattern of the junction. Quantitative deviations between the individual sec-

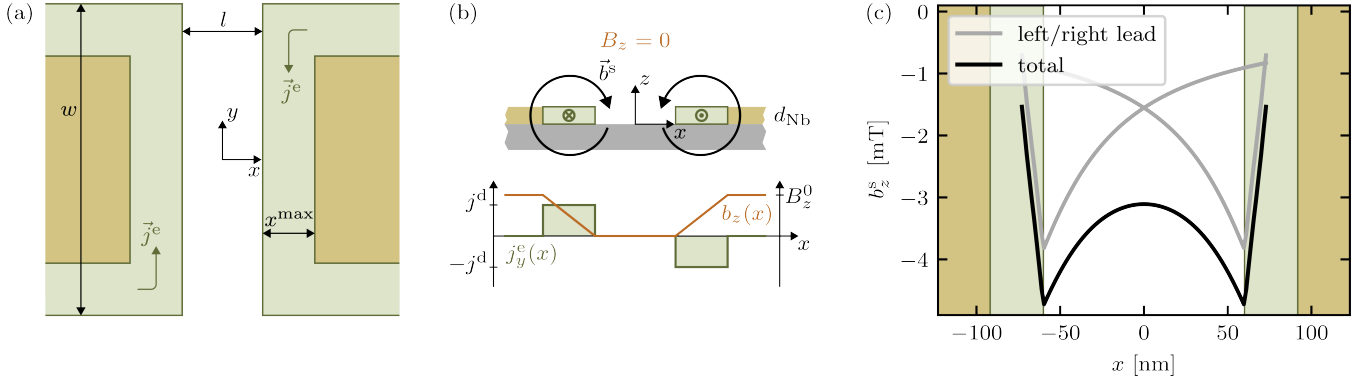


FIG. S6. Quantitative estimation of the inverted hysteresis from the critical state model. (a) Top-view sketch of a junction of length l with Nb leads of width w carrying the edge currents $\vec{j}^e$ down to a depth $x^{\max}$, following from the critical state model. Origin and orientation are indicated by the coordinate system. (b) Side-view sketch of external field density b_z , edge current density j_y^e and stray field $\vec{b}^s$ after decreasing the applied magnetic field B_z from B_z^0 to zero, analogue to the main text Fig. 2(g). (c) Result of the numerical integration following Eq. (S3) for the z -component of the stray field b_z^s for the situation depicted in panel (b).

tions may be caused by the local inhomogeneity of the vortex distribution.

Since the Nb leads of devices A and B are of comparable thickness and width, a similar effect is expected in $I_c(B_z)$ of sample A. Fig. S7(d) shows the same I_c curves as presented in the main text Fig. 3(a). A section of I_c^+ near $B_z = 0$ mT is shown in Fig. S7(e). At the small field scale, several jumps similar to those observed in Fig. S7(a) can be identified. We conclude that, due to the smaller junction size in sample A, the effect of a random reordering of the vortex distribution inside the leads is concealed by the ~ 1 T broad intrinsic interference pattern. Therefore, $I_c(B_z)$ is stable and reproducible, as opposed to the devices B and C where the flux change originating from vortex reordering is comparable to the width of the Fraunhofer pattern.

Further indication for the critical state model is found when gradually reducing the sweep range of the applied field. Minor loops in B_z are measured on sample C, starting from ± 20 mT ($b_{z,0}$ and $b_{z,1}$) and decreasing towards ± 0.5 mT ($b_{z,14}$ and $b_{z,15}$), see Fig. S8(a). No field-cooling is performed at the turn-around fields $b_{z,i}$. With reduced sweep range, the central lobes of the interference pattern shift closer together until the hysteretic effect vanishes and the central peak settles near zero field, as indicated by the gray dotted line. Upon increasing the field range from this state ($b_{z,16}$ onward), the lobe starts to shift away from zero and inverted hysteresis is recovered.

Fig. S8(b) describes a possible scenario for the local field distribution $b_z(x)$ under the premises of the critical state model. The following description is based on the lead to the left of the junction. The situation on the right-hand-side lead is antisymmetric, and the resulting stray fields of both sides add up on the area of the gap. After the initial field-cooling at B_z^0 , the superconductor

is in the fully penetrated vortex state and the field density $b_{z,0}$ is imprinted across the lead. As the field is swept towards the negative $b_{z,1}$, vortices leave the lead near the edges, creating a positive edge current distribution $j_{y,1}^e$, as sketched in the first graph of Fig. S8(b). As explained in the main text, this leads to inverted hysteresis effects. However, when returning to a positive field $b_{z,2} < b_{z,0}$ (second graph in Fig. S8(b)) without performing a field-cooling routine to delete the vortex trapping history, the edge current is inverted only in the region of the lead that is closer to the junction. As a result, areas with positive and negative j_y^e form. Since the region of negative j_y^e is closer to the junction, the stray field resulting from there is larger at the junction area and can, therefore, still create inverted hysteresis, albeit smaller than in the initial case. Gradually decreasing the field sweep range further effectively folds $b_z(x)$, which translates to a spatially alternating $j_y^e(x)$. The resulting stray fields of each sector average out, leaving no field component to counteract the external field and create an inverted hysteresis. As the field range is increased again, the innermost folds are straightened out by the increasing applied field and allow a re-building of the inverted hysteresis.

It is noted that for smaller field ranges, $I_c(B_z)$ changes from a distorted and irregular shape to a more Fraunhofer-like pattern as the spontaneous jumps in I_c disappear. The jumps return when the field range is increased again. This observation is consistent with the above explanation of the jumps in I_c since for small field sweep ranges, the applied field is not sufficient to disturb the vortex distribution by spontaneous depinning of vortices.

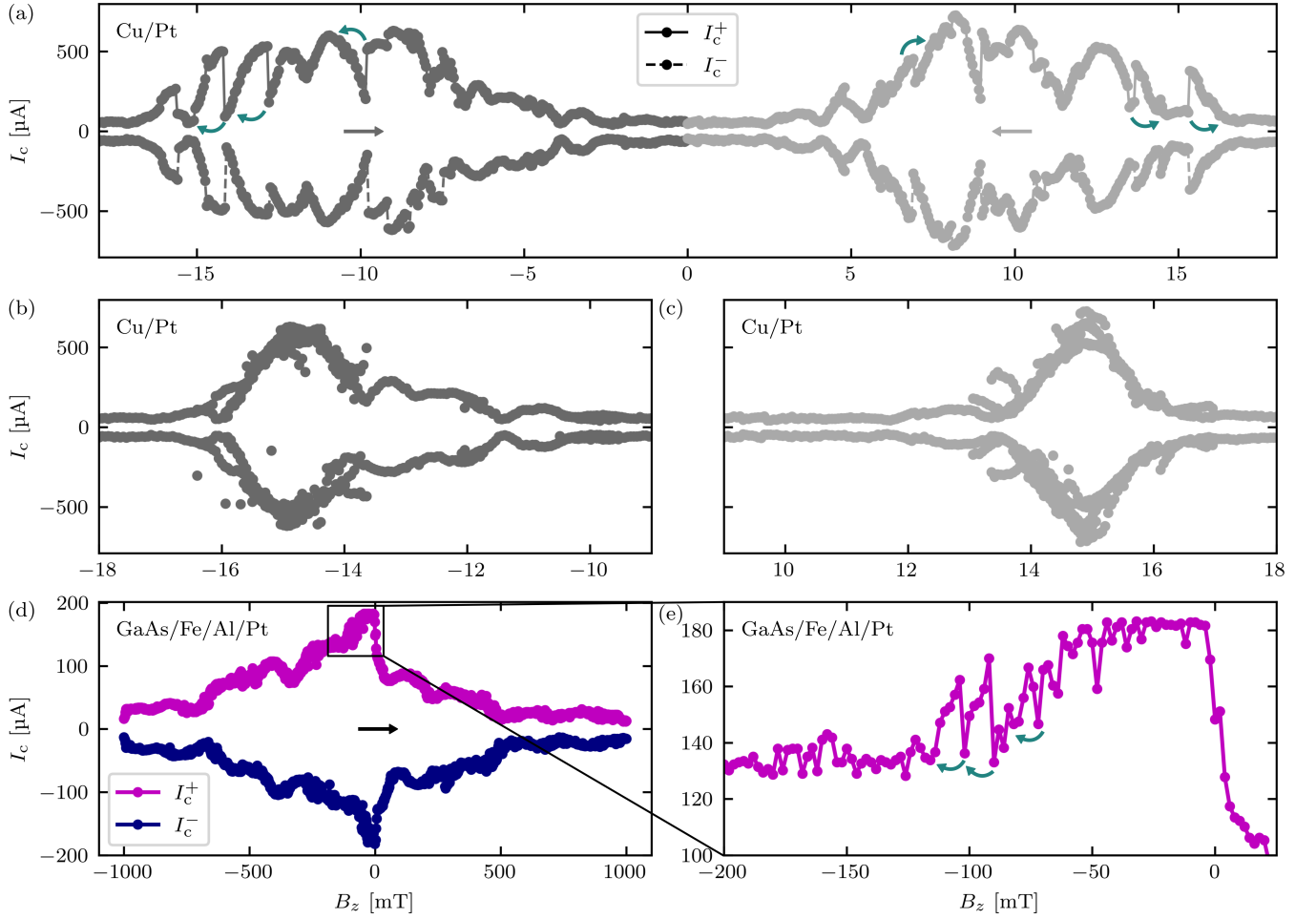


FIG. S7. Random jumps in $I_c(B_z)$ under consideration of critical state model. (a) $I_c(B_z)$ of sample B in both sweep directions. The data is the same as shown in the main text Fig. 2(c). The blue arrows indicate examples of backward shifts with respect to the field sweep direction. (b) and (c) show $I_c(B_z)$ of sample B in forward and backward field sweep direction, respectively, after performing the backward shifts. (d) $I_c(B_z)$ of sample A in forward field sweep direction, as shown in the main text Fig. 1(a). (e) Section of I_c^+ near zero field, as indicated in panel (d). The blue arrows indicate examples of backward shifts.

INVERTED HYSTERESIS IN SAMPLE A IN A PERPENDICULAR MAGNETIC FIELD

Figs. S9(a) and (b) compare I_c^+ and I_c^- of sample A between the sweep directions of B_z , respectively, showing an apparent inverted hysteresis. The horizontal separation of side features is on the order of 200 mT. For the estimation of the additional flux from the stray field b_z^s resulting from the critical state model, the approach outlined in Supplemental Note 6 is adapted to the case without field-cooling by adding a negative contribution from the current distribution built up during the previous sweep. Following eq. (2), the horizontal position of the sign change depends on the applied field B_z as $x^{\text{sg}} = |B_z^0 - B_z|/2\mu_0 j^d$. Using $B_z^0 = \pm 1$ T, $B_z \in [-1, +1]$ T and the same critical depinning current density j^d for the Nb leads as for sample B, one obtains $x^{\text{max}} = 1.6 \mu\text{m}$. Since this is much larger than

the junction length of ~ 20 nm, b_z^s is homogeneous across the length of the junction. Hence, it is sufficient to evaluate eq. (3) only in the center of the junction at $x = 0$. The result is shown in Fig. S9(c) as a function of B_z for both sweep directions. This suggests that, for the chosen parameters, the critical state model can account for an inverted hysteresis of the order of 50 mT. Taking into account the simplifications discussed in Supplemental Note 6, the quantitative deviation by a factor of ~ 4 is considered small enough to accept the qualitative explanation of the observed inverted hysteresis by the critical state model.

With respect to the Fe-layer of the stack [26, 27, 48], it should be noted that experiments on magnetic Josephson devices have demonstrated hysteretic Fraunhofer patterns caused by the stray field of the remnant vertical magnetization keeping the flux through the junction area finite even when the applied field has reached zero [49–

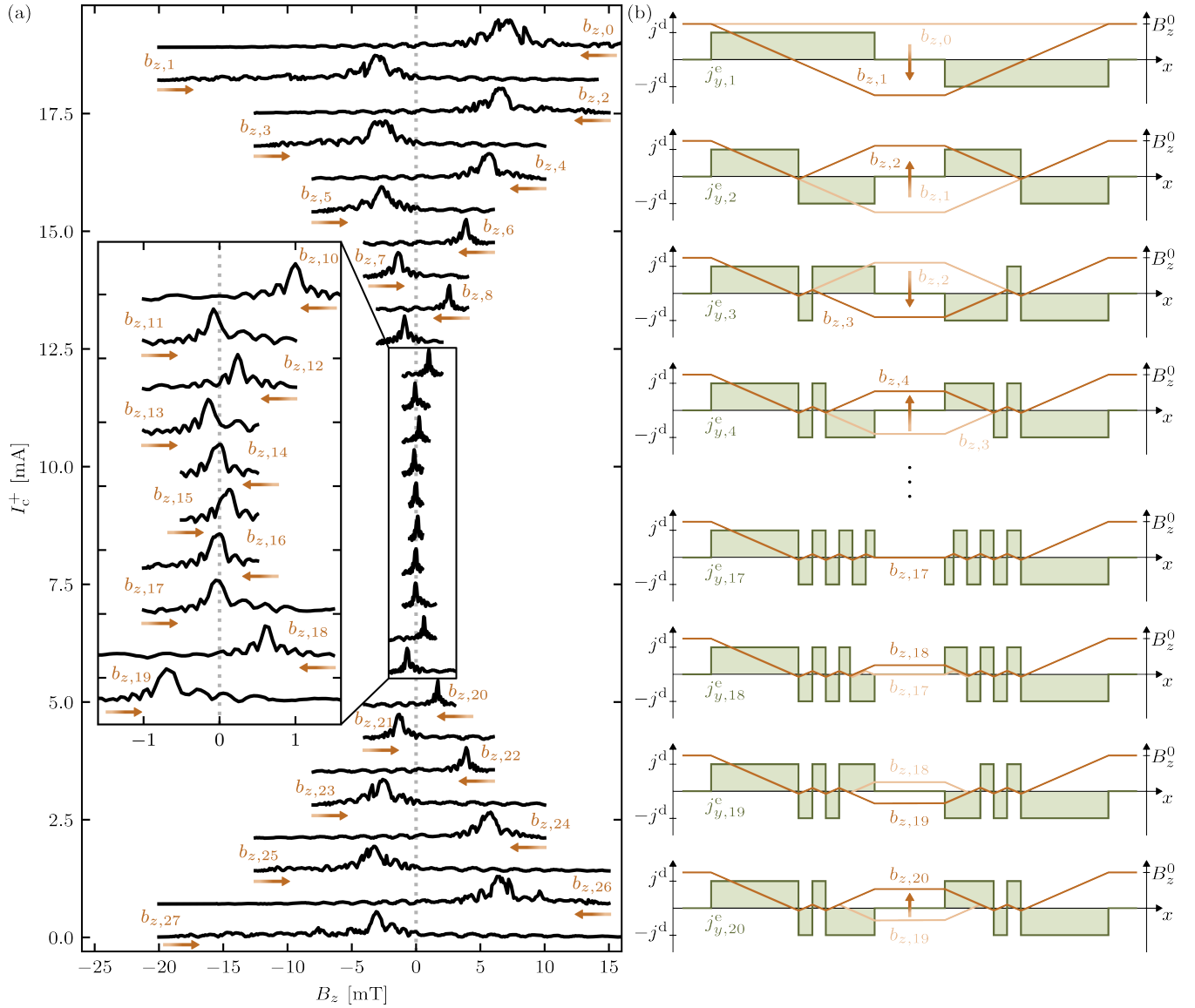


FIG. S8. Out-of-plane field minor loops in sample C. (a) $I_c(B_z)$ at different field ranges without field-cooling between sweeps (off-set for clarity). The sweep's starting point $b_{z,i}$ and direction are indicated for each trace. (b) Internal magnetic field distribution $b_z(x)$ and edge current density $j_y^e(x)$ across the leads and the junction according to the critical state model outlined in the main text. The field distributions before ($b_{z,i-1}$, beige) and after ($b_{z,i}$, brown) an individual sweep are sketched together with the edge current density $j_{y,i}^e$ after the sweep, corresponding to $b_{z,i}$.

51]. In the presented case, the hysteresis appears to be inverted and therefore is unlikely to be related to magnetization switching of the Fe layer.

TIME-REVERSAL SYMMETRY INCLUDING MAGNETIC FIELD SWEEP DIRECTION

Fig. S10(a) shows $I_{c,\text{fw}}^+(B_z)$ and $|I_{c,\text{bw}}^-|(-B_z)$ of sample A corresponding to Fig. 3. For the latter, the field axis is reversed to account for the backward sweep direction.

The curves overlap in all prominent features. Similarly, $|I_{c,\text{fw}}^-(B_z)|$ and $I_{c,\text{bw}}^+(-B_z)$ exhibit overlap, as shown in Fig. S10(b). In contrast, Figs. S10(c-f) demonstrate that other combinations lead to a significant mismatch. Therefore, the reversal of the field sweep direction must be included in the time reversal operations $I_c^+ \rightarrow I_c^-$ and $+B_z \rightarrow -B_z$.

The overlap of I_c curves between forward and backward field sweep direction is also investigated for the sample B. Due to the jumps in I_c discussed in Supplemental Note 7, the matching of I_c curves under time reversal operations is not possible in the case of a perpendicular

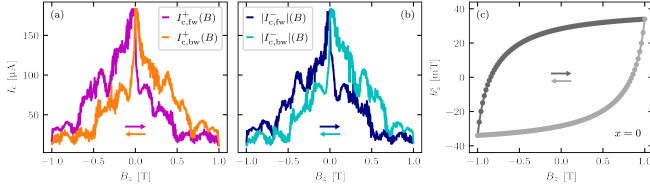


FIG. S9. Inverted hysteresis of I_c curves for sample A under B_z . (a) Comparison of I_c^+ between opposite sweep directions corresponding to the main text Fig. 3. The colored arrows indicate the respective sweep directions. (b) Comparison of I_c^- between opposite sweep directions, analogously to (a). (c) Additional field component b_z^S simulated in the center of the junction ($x = 0$) for both sweep directions without field-cooling.

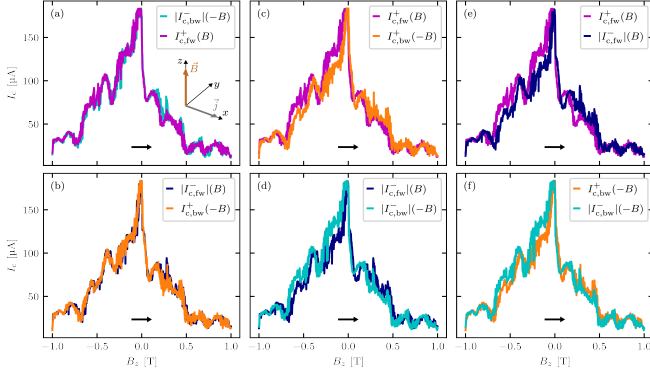


FIG. S10. Comparison of I_c curves for sample A under B_z in both field sweep directions, extracted from the data shown in Fig. 3 in the main text. The field axis of the backward sweep data is flipped to match the forward sweep direction. (a) and (b) compare I_c curves of opposite current, field and field sweep direction. (c-f) show alternative combinations where one of the TRS operations is neglected.

field. When applying the field close to the sample plane with only a small out-of-plane misalignment angle ϵ , vortices are pinned across the full width of the Nb leads, leading to a stable Fraunhofer pattern, as shown in the main text Fig. 2(d). Figs. SS11(a) and (b) depict the corresponding IV color maps in both field sweep directions. The comparison of I_c^+ and $|I_c^-|$ under the application of time reversal operations including the sweep directions is given in Figs. SS11(c) and (d) while the other combinations that neglect one of the symmetry operations are shown in Figs. SS11(e-h). No significant enhancement of overlap can be identified when including the sweep direction reversal.

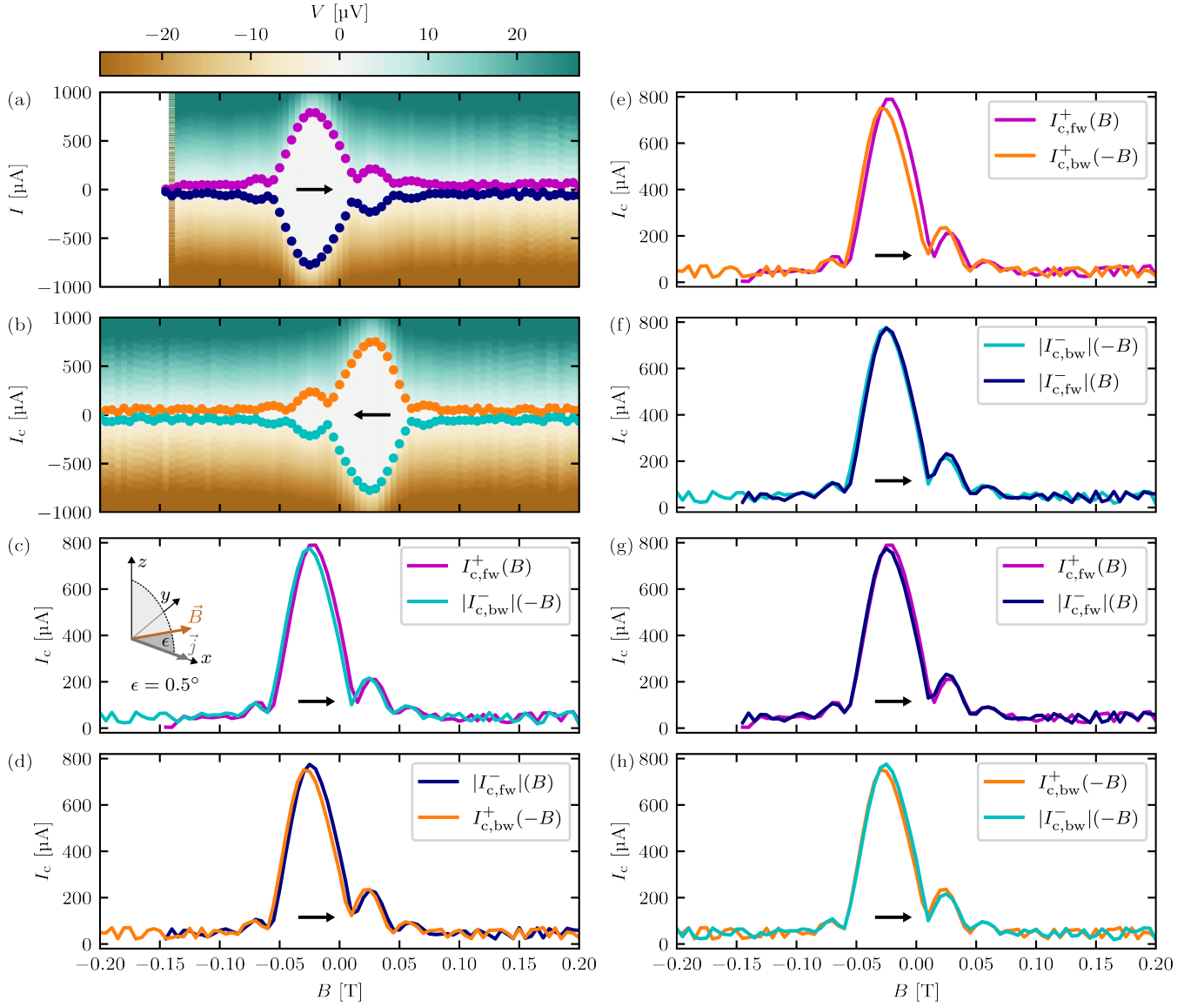


FIG. S11. TRS partners for small-tilt out-of-plane fields in sample B. (a) and (b) show the full field-dependent IV color map for $\theta = 0^\circ$ with a small misalignment angle $\epsilon = 0.5^\circ$ from the sample plane. The extracted critical current corresponds to the data shown in the main text Fig. 2(d). Panels (c) and (d) match the I_c curves with the respective TRS partner under consideration of the field sweep direction (analogue to the main text Fig. 3(c) and (d) in sample A). Panels (e-h) overlay the same data in other combinations.