

CMB observables and reheat temperature as a window to models of inflation and freeze-in dark matter production

Anish Ghoshal, Paweł Kozów, Marek Olechowski, Stefan Pokorski

Institute of Theoretical Physics, Faculty of Physics, University of Warsaw,
ul. Pasteura 5, 02-093 Warsaw, Poland

E-mail: anish.ghoshal@fuw.edu.pl, pawel.kozow@fuw.edu.pl,
marek.olechowski@fuw.edu.pl, stefan.pokorski@fuw.edu.pl

Abstract. A systematic approach is presented for using CMB observables and reheating temperature for discriminating between various models of inflation and certain freeze-in dark matter scenarios. It is applied to several classes of α -attractor models as an illustrative example. In the first step, all independent parameters of the inflationary potential are expressed in terms of the CMB observables (the three parameters - by the scalar spectral index n_s , scalar amplitude A_s and the tensor-to-scalar amplitude ratio r). For a standard reheating mechanism characterized by the inflaton equation of state parameter w and its effective dissipation rate Γ the reheating temperature is uniquely fixed in terms of the CMB observables measured for some pivot scale k_* . There are striking consequences of this fact. The model independent bounds on the reheating temperature, the BBN lower bound and the upper bound of the order of the GUT/Planck scale, translate themselves for each class of models into very narrow ranges of the allowed values of the spectral index $n_s(k_*)$, providing their strong tests by the present and future CMB data. The recent tension between Planck and DESI-ACT results has strong impact on our conclusions. Furthermore, given a class of inflation models satisfying those tests, the reheating temperature is an interesting portal to link the CMB observables to the particle physics scenarios that are sensitive to it. As an example, non-thermal dark matter (DM) production mechanisms are discussed. One obtains then a consistency check between theories of inflation and DM production. If the future precision of the CMB data will constrain the reheating temperature beyond the model independent bounds, further constraints on the DM production will follow.

Contents

1	Introduction	1
2	CMB, inflaton potentials and reheating temperature	3
3	$T_\times$ sensitive dark matter production	13
4	Conclusions	21
A	Formulae for α-attractor T- and P-models	22
B	Simplifying assumptions underlying this analysis	24
C	Concrete example: Higgs-portal	24

1 Introduction

The standard paradigm is that, in its early evolution, the universe underwent several distinct phases. Its accelerated expansion (inflation) was followed by a period when the energy stored in the inflaton field was converted to relativistic particles, broadly referred to as a reheating period, during the oscillations of the inflaton field around the minimum of its potential. After that, the standard radiation-dominated (and later, matter-dominated) evolution of the universe took place. It has been emphasized for a long time that the CMB data give us important insight into the inflationary and reheating periods, constraining theoretical models of both [1]. In this paper we propose a systematic approach for using CMB data for discriminating between different models of inflation and using the reheating temperature as a portal to constraining particle physics scenarios that are sensitive to it. As an example we discuss certain freeze-in DM scenarios.

As the first step, all independent parameters of an inflaton potential are expressed in terms of the CMB observables such as: the scalar metric perturbation amplitude $A_s(k)$, the scalar spectral index $n_s(k)$, its running $\alpha_s(k)$ and the ratio of the tensor to scalar perturbation amplitudes $r(k)$ etc. Here $k = k_*$ is a comoving wavenumber of the metric perturbation, measured by CMB experiments at some physical scale $\lambda_{\text{phys}} = a_0/k_*$ determined by the experimental angular resolution (the Planck pivot scale is $k_* = a_0 \times 0.05 \text{ Mpc}^{-1}$) where a_0 denotes the Friedman-Laimetre-Robertson-Walker (FLRW) scale factor today. The number of observables has of course to match the number of independent parameters of the potential and their best choice depends on the precision of their experimental values. As an illustrative example we apply our approach to several classes of α -attractor models, the so-called E, T and P-models, having three independent parameters, and choosing n_s, A_s, r as the three input CMB observables. Those potentials are well motivated in several cosmological scenarios. Some well known cases are the famous Starobinsky inflation [2], the Goncharov-Linde model [3, 4], the Higgs Inflation scenario [5], several superstring-inspired scenarios, [6–9]) and no-scale supergravity [10, 11].

In the next step we adopt a model for the reheating period. In the standard approach it is described by the inflaton equation of state parameter w and a single new parameter,

the inflaton effective dissipation rate Γ , or equivalently, by the reheating temperature at the beginning of the radiation era. It turns out that the value of Γ , or equivalently and more importantly the reheating temperature, are uniquely fixed in terms of the chosen (three for the α -attractor models) CMB observables. The logic of this procedure is nicely illustrated by figure 1, which shows a sketch of the evolution of the comoving Hubble scale $(H^{-1})_{com} = (aH)^{-1}$ as a function of the expansion of the universe described by the FLRW scale factor $a(t)$ (quantities on both axes are normalized by their present values). During inflation, the Hubble scale H^{-1} approximately corresponds to the size of causally connected universe (event horizon) and during the decelerated expansion it approximately gives the size of the particle horizon. The perturbation with the comoving wavenumber k_* leaves the causal region during inflation when its physical length $\lambda_{phys}^{inf} = a_{k_*}/k_* = H_{k_*}^{-1}$, hence for the value of the comoving Hubble scale $\frac{1}{H_{k_*} a_{k_*}} = \frac{1}{k_*}$. In units of $a_0 H_0$ and with $k_* = a_0 \times 0.05 \text{ Mpc}^{-1}$ one obtains the horizontal dashed line in figure 1. For a chosen inflaton potential, the value of H_{k_*} , in our approach, has already been calculated in terms of the CMB observables A_s and r . Knowing H_{k_*} , one can calculate the value of the scale factor when the perturbation left the horizon, $a_{k_*}/a_0 = 0.05 \text{ Mpc}^{-1}/H_{k_*}$, which determines the crossing of the line describing inflation in a given model with the dashed line corresponding to $k_* = a_0 \times 0.05 \text{ Mpc}^{-1}$. Similarly, the Hubble scale at the end of inflation is expressed for a given model of inflation in terms of the CMB parameters and gives us a_{end}/a_0 . On the right end of the plot, the running of Hubble scale as a function of a/a_0 in the matter domination and radiation domination eras is known. In between, we have a period of reheating which, for a given inflation model, is parameterized by the inflaton equation of state parameter w which determines the slope of a line describing reheating era in figure 1. The reheating temperature (temperature at the end of reheating and the beginning of the radiation domination era), T_{re} , is fixed by the consistency condition: the perturbation with the comoving wavenumber k_* , that left the horizon at the moment parameterized by the values of the CMB observables, re-entered it at some known time during the radiation domination era.¹ The reheating temperature is therefore determined in terms of the CMB parameters.²

There are striking consequences of this fact. The model independent bounds on the reheating temperature, the BBN lower bound, arising due to the precision measurements of primordial abundance of light nuclei, and the upper bound of the order of the GUT/Planck scale [15], translate themselves, for each class of models, into very narrow ranges of the spectral index $n_s(k)$ and with interesting dependence on the parameter r . That is a strong test of inflationary models by the present and future CMB data. Moreover, one can investigate the implications for models of inflation of imposing some constraints on the reheating temperature motivated by various physical arguments. For instance, successful standard thermal leptogenesis [16] requires $T_x \geq 10^9 \text{ GeV}$ [17].³ A different, contradictory, example are upper bounds on reheat temperature derived when solving the so-called gravitino problem in certain models of supersymmetry breaking in supergravity with $T_x \leq 10^7 \text{ GeV}$ [19–24], and axion and axion-like production as DM during reheating [25–28].

Furthermore, given a class of inflaton models, the reheating temperature is an interesting

¹Some consequences of this condition for α -attractor E- and T-models were investigated in [12–14].

²A model in a given class of α -attractor potentials has three independent parameters, which can be expressed in terms of the three CMB observables. Our approach can be generalized to models with more parameters, if more CMB observables are used, e.g. also the running of the spectral index $\alpha_s(k_*)$.

³It was shown in ref. [18] that in non-thermal leptogenesis during the reheating period, CMB observables could probe the scales of seesaw and leptogenesis.

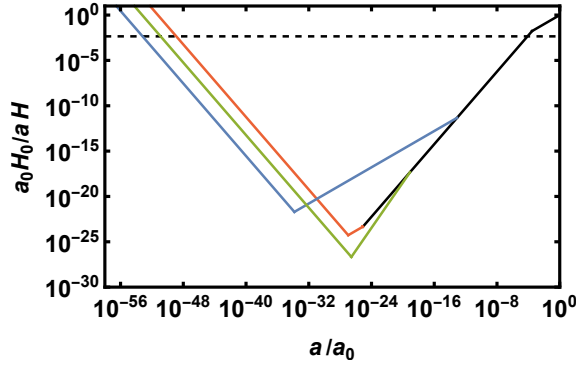


Figure 1. A schematic evolution of comoving Hubble radius aH as a function of the scale factor a . The blue line is for $r(k_*) = 0.04$ and $w = 0$; the green line is for $r(k_*) = 10^{-6}$ and $w = 1/2$; the red line is for $r(k_*) = 10^{-10}$ and $w = 0$; the obtained reheating temperatures are 10 GeV, 10^6 GeV and 10^{12} GeV, respectively. The horizontal dotted line corresponds to the comoving wavenumber value $k_* = a_0 \times 0.05 \text{ Mpc}^{-1}$.

portal to link the CMB observables with the physics scenarios sensitive to it. As an example we discuss some non-thermal DM production mechanisms. It has long been understood that freeze-in DM production is T_{re} -sensitive, provided the process is UV sensitive in the energy range accessible during reheating. This is the case when the DM production can be discussed in the effective field theory (EFT) approach, with its amplitude following from a dimension higher than four operator suppressed by appropriate powers of the decoupling scale $\Lambda_{\text{DM}} > T_{\text{re}}$. This means that a particular choice for the set of parameters associated with dark matter sector, such as the mass of the DM particle and the value of the scale Λ_{DM} , requires a unique value of reheating temperature of the Universe to yield the correct DM relic abundance. One obtains then a consistency check between theories of inflation and DM production. If the future precision of the CMB data will constrain the reheating temperature beyond the model independent bounds, further constraints on the DM production will follow. We present our results for a “prototype” model. They can be used for any concrete model under quantitative consideration, after being properly rescaled by its Wilson coefficients.⁴

The details of our approach and the power of the dependence of the reheating temperature on the CMB observables for discriminating between different models of inflation are discussed in section 2. The role of the reheating temperature T_{re} as a link between inflationary models and some DM production mechanisms is discussed in section 3.

2 CMB, inflaton potentials and reheating temperature

At present, the Planck CMB data give us the value of the spectral index $n_s(k_*)$ for the scalar perturbation mode with the (pivot) wavenumber $k_*/a_0 = 0.05 \text{ Mpc}^{-1}$, its amplitude $A_s(k_*)$ [30] and the upper bound on the ratio of the tensor to scalar perturbation amplitudes $r(k_*)$. Following the inflation paradigm, the values of those observables depend on the shape of the inflaton potential, its independent parameters and the value of the inflaton field ϕ_{k_*} when the mode k_* exits the horizon during inflation.

⁴While this project was ongoing, the link between CMB observables and freeze-in DM has been also addressed in ref. [29].

In the slow-roll approximation the observables $n_s(k)$, $r(k)$, and $A_s(k)$, where k is some comoving wavenumber of the metric perturbations, are related to the inflaton potential as follows:

$$\begin{aligned} n_s(k) &= 1 - 6\epsilon_k + 2\eta_k, \\ r(k) &= 16\epsilon_k, \\ A_s(k) &= \frac{V(\phi_k)}{24\pi^2\epsilon_k M_{\text{P}}^4}, \end{aligned} \tag{2.1}$$

where M_{P} is the reduced Planck mass. We also include in our discussion the running of the spectral index defined as

$$\alpha_s(k) = \frac{d n_s(k)}{d \log k} = (16\epsilon_k \eta_k - 24\epsilon_k^2 - 2\theta_k^2). \tag{2.2}$$

The parameters ϵ_k , η_k and θ_k read:

$$\begin{aligned} \epsilon_k &= \frac{1}{2} M_{\text{P}}^2 \left(\frac{\partial_\phi V(\phi)}{V(\phi)} \right)^2 \Big|_{\phi=\phi_k}, \\ \eta_k &= M_{\text{P}}^2 \frac{\partial_\phi^{(2)} V(\phi)}{V(\phi)} \Big|_{\phi=\phi_k}, \\ \theta_k^2 &= M_{\text{P}}^4 \frac{\partial_\phi V(\phi) \partial_\phi^{(3)} V(\phi)}{V^2(\phi)} \Big|_{\phi=\phi_k}. \end{aligned} \tag{2.3}$$

We focus now on three classes of the α -attractor inflaton models, the so-called E, T and P-models, with the following respective potentials [31–37]:

$$V(\phi, \alpha, n, \Lambda_{\text{inf}}) = \begin{cases} \Lambda_{\text{inf}}^4 \left(1 - \exp \left[-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{P}}} \right] \right)^{2n} & \text{E-model,} \\ \Lambda_{\text{inf}}^4 \left(\tanh \left[\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{P}}} \right] \right)^{2n} & \text{T-model,} \\ \Lambda_{\text{inf}}^4 \frac{\phi^{2n}}{\phi^{2n} + \left(\sqrt{\frac{3\alpha}{2}} M_{\text{P}} \right)^{2n}} & \text{P-model,} \end{cases} \tag{2.4}$$

where Λ_{inf} represents a mass scale that determines the energy scale of the inflation and $\sqrt{\alpha} M_{\text{P}}$ is an effective scale that can be higher than M_{P} .⁵ A special case of the E-model, with $\alpha = 1$ and $n = 1$, mimics the standard Higgs-Starobinsky inflaton potential [5].

In the following we shall present the necessary formulae for the E-model, relegating the analogous ones for the T and P-models to appendix A. For the E-model, all the perturbation parameters $n_s(k)$ etc., expressed in terms of the potential parameters, read (see eqs. (2.1)-

⁵We use the same normalization of α in all three models. Observe that it is different from the normalization for T-model used e.g. in [33].

(2.4)):

$$n_s(k) = n_s(\alpha, n, \phi_k) = 1 - \frac{8n \left(\exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) + n \right)}{3\alpha \left(\exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) - 1 \right)^2}, \quad (2.5)$$

$$r(k) = r(\alpha, n, \phi_k) = \frac{64n^2}{3\alpha \left(\exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) - 1 \right)^2}, \quad (2.6)$$

$$A_s(k) = A_s(\alpha, n, \phi_k, \Lambda_{\text{inf}}) = \frac{\alpha}{32\pi^2 n^2} \frac{\Lambda_{\text{inf}}^4}{M_P^4} \exp \left(2\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) \left(1 - \exp \left(-\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) \right)^{2(n+1)}, \quad (2.7)$$

$$\alpha_s(k) = \alpha_s(\alpha, n, \phi_k) = -\frac{32n^2 \exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) \left(\exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) + 2n + 1 \right)}{9\alpha^2 \left(\exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P} \right) - 1 \right)^4}. \quad (2.8)$$

The exponent $2n$ in the potential (2.4) is an even integer and we consider it as a number which defines the model.⁶ The other parameters Λ_{inf} , α and the inflaton field value ϕ_k when the mode with the co-moving wavenumber k left the horizon can be expressed in terms of the three observables by the inverse relations. It is convenient to define the following combination of the CMB observables, n_s and r , and the integer potential parameter n :

$$\xi = n \left(8 \frac{1 - n_s}{r} - 1 \right). \quad (2.9)$$

We get then:

$$\alpha = \frac{64n^2}{3r} \frac{1}{(\xi - 1)^2}, \quad (2.10)$$

$$\phi_k = M_P \sqrt{\frac{32}{r}} \frac{n}{(\xi - 1)} \ln(\xi), \quad (2.11)$$

$$\Lambda_{\text{inf}}^4 = \frac{3\pi^2}{2} M_P^4 r A_s \left(\frac{\xi}{\xi - 1} \right)^{2n}, \quad (2.12)$$

Thus, the values of the three observables for some comoving wavenumber k fully determine the parameters of the α -attractor potential for a given choice of the exponent $2n$. In this case the running $\alpha_s(k)$ is the prediction of the model:

$$\alpha_s = -\frac{r^2 \xi (\xi + 2n + 1)}{128n^2}. \quad (2.13)$$

In models with four independent parameters, it would be needed to fix the model parameters.

The observables are measured by the Planck experiment for the comoving wavenumber $k = k_* = a_0 \times 0.05 \text{ Mpc}^{-1}$. The results are usually presented as the experimentally allowed regions (at some confidence level) in the plane $(n_s(k_*), r(k_*))$ or $(n_s(k_*), \alpha_s(k_*))$. It is customary to compare them with models of inflation by checking if those allowed regions are

⁶We thank Renata Kallosh and Andrei Linde for drawing our attention to the fact that for polynomial potentials like P-model fractional values of n are also of theoretical interest.

consistent with the **assumed** number of e-folds, usually $50 \div 60$, during the rolling down of the inflaton field from its value ϕ_{k_*} to the value ϕ_{end} at the end of inflation. The weakness of this method of comparing the CMB data with the models of inflation is that the a priori acceptable range of N_k can actually be very large and is correlated with the expansion during the reheating period (see e.g. textbooks [15, 38], it is also evident from figure 1) and the often used range $50 \div 60$ has no model-independent justification. Instead of assuming a value (or range of values) of N_k one may and should calculate it, given some assumptions about the reheating period. Let us illustrate this with the example of α -attractor E-model (expressions for T- and P-models may be found in appendix A).

The value of ϕ_{end} at the end of (slow-roll) inflation can be estimated by the conditions for the slow-roll parameters $\epsilon = 1$ or $|\eta| = 1$, whichever is reached earlier. In the case of E-models, for the end of inflation defined by $\epsilon = 1$, one gets:

$$\phi_{\text{end}} = M_{\text{P}} \frac{4\sqrt{2}n}{(\xi-1)\sqrt{r}} \ln \left(1 + \frac{(\xi-1)\sqrt{r}}{4} \right). \quad (2.14)$$

The number of e-folds, N_k , is expressible via ϕ_k and ϕ_{end} :

$$N_k = -\frac{1}{M_{\text{P}}^2} \int_{\phi_k}^{\phi_{\text{end}}} \frac{V(\phi)}{\partial_{\phi} V(\phi)} d\phi. \quad (2.15)$$

Thus, using eqs. (2.11) and (2.14), one finds that N_k is determined by the observables n_s and r as follows:

$$N_k = \frac{4n(4-\sqrt{r})}{r(\xi-1)} - \frac{16n}{r(\xi-1)^2} \ln \left(\frac{4\xi}{4+\sqrt{r}(\xi-1)} \right), \quad (2.16)$$

with ξ given by eq. (2.9). Some curves of constant N_{k_*} in the $(n_s(k_*), r(k_*))$ plane are shown for several models in figure 2 (black dashed and dotted curves).

For each considered model the number of e-folds N_{k_*} may be calculated for each pair of values of the CMB observables $n_s(k_*)$ and $r(k_*)$. However, not every pair of such values is consistent with the model of inflation used for calculating N_{k_*} and the assumed description of the reheating process. One of the reasons is related to the model independent bounds on the reheating temperature which characterizes the transition from the reheating era to the radiation domination era. In our approach this temperature is a function of the CBM observables and the bounds on it constrain their acceptable range.

The observables are measured by the Planck and other experiments for the comoving wavenumber $k = k_* = a_{k_*} H_{k_*}$, with a_{k_*} and H_{k_*} denoting the scale factor and the Hubble scale at the moment of exit beyond the horizon of the mode with comoving wavenumber k_* . One has an identity

$$0 = \ln \left(\frac{k_*}{a_{k_*} H_{k_*}} \right) = \ln \left(\frac{a_{\text{end}}}{a_{k_*}} \frac{a_{\text{re}}}{a_{\text{end}}} \frac{a_0}{a_{\text{re}}} \frac{k_*}{a_0 H_{k_*}} \right), \quad (2.17)$$

where a_{end} and a_{re} are scale factors at the end of inflation and at the completion of reheating. The first three factors under the logarithm on the r.h.s. of the above formula correspond to three regions in figure 1: inflation, reheating and standard evolution after reheating, respectively. It is a consistency relation reflecting the fact that the observed perturbation left the (event) horizon at the scale factor a_{k_*} and reentered the (particle) horizon when the scale factor had some known value (depending of the choice of the pivot scale k_*). The number of e-folds $N_{k_*} = \ln \left(\frac{a_{\text{end}}}{a_{k_*}} \right)$ from the horizon exit of the mode k_* to the end of inflation

has already been given in eq. (2.15) in terms of the inflaton potential parameters and the value of the inflaton ϕ_{end} at the end of inflation, expressed by the CMB observables $n_s(k_*)$, $r(k_*)$ and $A_s(k_*)$. Regarding the reheating period, we adopt the standard description by the Boltzmann equation of the energy density transfer to radiation, with the inflaton dissipation rate Γ as a free parameter. We trade Γ for the reheating temperature T_{re} . It is then clear that for a given model of inflation the identity eq. (2.17) gives us the reheating temperature (or Γ) fixed in terms of the observables $A_s(k_*)$, $n_s(k_*)$ and $r(k_*)$. Conversely, some well motivated bounds on the reheating temperature, as we see later, can be translated into very severe tests of inflaton models by the CMB data.

We describe now the details of the above outlined procedure. We define the reheating temperature $T_{\text{re}} \equiv T_{\times}$ as the temperature of radiation at the moment when $\rho_{\phi} = \rho_R$, where ρ_{ϕ} and ρ_R are the inflaton and radiation energy densities, respectively, that is, the time or the temperature at which the energy densities cross each other. The value of $T_{\times}$ enters into eq. (2.17) in the ratios $\left(\frac{a_{\times}}{a_{\text{end}}}\right)$ and $\left(\frac{a_0}{a_{\times}}\right)$ (now $a_{\text{re}} \equiv a_{\times}$). The latter has the standard form:

$$\frac{a_{\times}}{a_0} = \left(\frac{43}{11g_{s*}}\right)^{1/3} \frac{T_0}{T_{\times}}, \quad (2.18)$$

where T_0 is the present temperature of the universe. The ratio $\left(\frac{a_{\times}}{a_{\text{end}}}\right)$ requires more attention. Classical reheating is described by the following set of Boltzmann equations⁷:

$$\dot{\rho}_{\phi} = -3(1+w)H\rho_{\phi} - \Gamma\rho_{\phi}, \quad (2.19)$$

$$\dot{\rho}_R = -4H\rho_R + \Gamma\rho_{\phi}, \quad (2.20)$$

$$H^2 = \frac{\rho_{\phi} + \rho_R}{3M_{\text{P}}^2}, \quad (2.21)$$

where dots denote derivatives with respect to the cosmic time and w is the equation of state parameter for the *inflaton* following from the inflaton potential. In the leading order of the expansion in the inflaton field⁸ it is given by the following function of the potential exponent n :

$$w \approx \frac{n-1}{n+1}. \quad (2.22)$$

It occurs that it is more convenient to analyze the above set of equations using the cosmic scale factor, a , instead of the cosmic time as the independent variable. We will apply also the usual approximation consisting in neglecting the radiation contribution to the total energy density during the reheating process, i.e. $\rho_R \ll \rho_{\phi}$, resulting in the following approximate expression for the Hubble parameter

$$H^2 \approx \frac{\rho_{\phi}}{3M_{\text{P}}^2}. \quad (2.23)$$

This way equations (2.19) and (2.20) with H given by (2.23) may be rewritten in the following form:

$$a\rho'_{\phi} = -3(1+w)\rho_{\phi} - \sqrt{3}M_{\text{P}}\Gamma\sqrt{\rho_{\phi}}, \quad (2.24)$$

$$a\rho'_R = -4\rho_R + \sqrt{3}M_{\text{P}}\Gamma\sqrt{\rho_{\phi}}, \quad (2.25)$$

⁷We assume Γ to remain constant during the entire reheating process.

⁸Here, the contributions of the inflaton fluctuations to w have been neglected. However, these can give corrections to the equation of state w , see [39].

where primes denote derivatives with respect to the cosmic scale factor a . The above set of equations may be solved analytically. The solution reads ⁹

$$\rho_\phi = \rho_{\text{end}} \left[(1 + \gamma) \left(\frac{a}{a_{\text{end}}} \right)^{-\frac{3}{2}(1+w)} - \gamma \right]^2, \quad (2.26)$$

$$\rho_R = \rho_{\text{end}} \frac{\Gamma}{H_{\text{end}}} \left[\frac{2(1 + \gamma)}{5 - 3w} \left(\left(\frac{a}{a_{\text{end}}} \right)^{-\frac{3}{2}(1+w)} - \left(\frac{a}{a_{\text{end}}} \right)^{-4} \right) - \frac{\gamma}{4} \left(1 - \left(\frac{a}{a_{\text{end}}} \right)^{-4} \right) \right], \quad (2.27)$$

where

$$\gamma = \frac{\Gamma}{3(1 + w)H_{\text{end}}}, \quad (2.28)$$

and we used obvious initial conditions at the beginning of reheating (identified with the end of inflation): $\rho_R(a_{\text{end}}) = 0$, $\rho_\phi(a_{\text{end}}) = \rho_{\text{end}}$. For a given model energy density at the end of inflation may be calculated in terms of the CBM observables. In the case of E-models it reads

$$\rho_{\text{end}} = 2\pi^2 M_{\text{P}}^4 r A_s \left(\frac{\xi \sqrt{r}}{(\xi - 1)\sqrt{r} + 4} \right)^{2n}. \quad (2.29)$$

We define the end of reheating as the moment when the cosmic scale factor is equal $a_\times$ for which $\rho_\phi(a_\times) = \rho_R(a_\times)$. It is possible to calculate $a_\times$ after neglecting terms proportional to $(a/a_{\text{end}})^{-4}$ in (2.27). We obtain ¹⁰

$$\frac{a_\times}{a_{\text{end}}} \approx \left[\frac{\gamma (16 + 3(1 + w)\sqrt{9 - 3w})}{2(1 + \gamma)(5 - 3w)} \right]^{-\frac{2}{3(1+w)}}. \quad (2.30)$$

Energy density at $a_\times$ equals $\rho_\times = \rho_\phi(a_\times) + \rho_R(a_\times) = 2\rho_\phi(a_\times)$. Substituting $a_\times$ given by (2.30) into (2.26) we obtain the following simple result

$$\rho_\times = \frac{\rho_{\text{end}}}{2} \left[3(1 + w)\gamma \frac{2 + \sqrt{9 - 3w}}{5 - 3w} \right]^2. \quad (2.31)$$

From the equality $\rho_\phi(a_\times) = \rho_R(a_\times)$ one may calculate the reheating temperature as a function of γ (defined in eq. (2.28))

$$T_\times^4 = \frac{30}{g_*\pi^2} \frac{\rho_\times}{2} = \frac{30}{g_*\pi^2} \rho_{\text{end}} \left[\frac{3}{2}(1 + w)\gamma \frac{2 + \sqrt{9 - 3w}}{5 - 3w} \right]^2, \quad (2.32)$$

which may be easily inverted:

$$\gamma = \frac{T_\times^2}{\sqrt{\rho_{\text{end}}}} \sqrt{\frac{2g_*}{15}} \frac{\pi(5 - 3w)}{3(1 + w)(2 + \sqrt{9 - 3w})}. \quad (2.33)$$

⁹These results reduce to those from standard textbooks (see e.g. [38]) if Γ is set to zero in (2.28), but explicit Γ is kept in (2.27). This corresponds to neglecting Γ term in (2.24) but keeping it in (2.25).

¹⁰This approximation is good when two conditions are met. First: $w < \frac{5}{3}$ which is always the case in the α -attractor models considered in this paper. Second: γ is not too large. We checked numerically that accuracy of analytical results is good if Γ is not bigger than about $0.1H_{\text{end}}$.

In calculations with fixed reheating temperature $T_\times$ one should replace γ with the above function of $T_\times$. Notice that our procedure avoids using the effective equation of state parameter w_{eff} , often used in the literature ¹¹.

The result (2.30) is important for us because the ratio $a_\times/a_{\text{end}}$, after substituting γ given by (2.33), may be directly used in the relation (2.17) (with $a_{\text{re}} \equiv a_\times$). Finally, using the Friedman equation and definitions of the parameters r and A_s , one obtains the following expression for H_k

$$H_k = \frac{\pi}{\sqrt{2}} M_{\text{P}} \sqrt{r A_s}. \quad (2.34)$$

This completes our task of relating the values of the observables n_s, r, A_s to a value of $T_\times$ in an α -attractor inflation potential with a given exponent n . This is an important prediction due to the following: Firstly, there are model independent bounds on the reheat temperature of the universe

$$10 \text{ MeV} \lesssim T_\times \lesssim 2 \cdot 10^{15} \text{ GeV}. \quad (2.35)$$

The lower bound is the BBN energy scale while the upper one follows from the fact that the consistency condition (2.17) with higher reheat temperatures has no physically meaningful solutions (it can be formally solved but only with negative number of e-folds during reheating). They provide strong tests via the CMB data on the models of inflation. Secondly, that explicit link of the CMB data to the reheat temperature may have important particle physics implications. Several theoretical consistency conditions and new physics ball-park values which are often conveyed through reheat temperatures will readily find observable quantities (in a given model of inflation) like spectral index, tensor-to-scalar ratio, etc. which either will be constrained or will be tested further with upcoming precision CMB measurements.

Since at present only the upper bound on r is known experimentally, it is interesting to illustrate the above results in the plane (n_s, r) as a function $r(n_s) = f(n, A_s, n_s, T_\times)$, for several fixed values of the exponent n . First, we compare our approach to the selection of the acceptable inflationary models based on the reference to the reheating temperature $T_\times$ with the one using an ansatz for the e-fold number N_k to be in the range $50 \div 60$. In figure 2 two sets of curves in the (n_s, r) plane are compared: colored curves with fixed $T_\times$, $r(n_s) = f(n, A_s, n_s, T_\times)$, and black ones with fixed N_k , $r(n_s) = f(n, A_s, n_s, N_k)$. For $r \gtrsim 10^{-3}$ the two sets of curves have similar r -dependence, with the fixed temperature curves consistent with some fixed N_k , but not always in the range $50 \div 60$. The characters of those two sets of curves as functions of r are quite different for smaller values of r . A given model may be self-consistent only if it predicts a point on the (n_s, r) plane between the outermost colored curves because only then the consistency condition (2.17) and the model independent bound on the reheat temperature (2.35) are fulfilled.¹² In many models the allowed region

¹¹Results (2.30) and (2.31) may be used to calculate w_{eff} which is defined by $\rho(a_1)/\rho(a_2) = (a_1/a_2)^{-3(1+w_{\text{eff}})}$. In the case of reheating we have $a_1 = a_\times$ and $a_2 = a_{\text{end}}$. The expression for effective w reads

$$w_{\text{eff}} = \frac{\ln \left(\frac{3(1+w)(2+\sqrt{9-3w})}{\sqrt{2}(5-3w)} \gamma \right)}{\ln \left(\frac{(16+3(1+w)\sqrt{9-3w})}{2(5-3w)} \frac{\gamma}{1+\gamma} \right)} - 1.$$

This formula gives good approximation of numerical results for Γ not (much) bigger than about $0.1 H_{\text{end}}$

¹²The highest $T_\times$ presented in figures in this section equals $1.6 \cdot 10^{15} \text{ GeV}$ and is somewhat smaller than the model-independent upper bound (2.35). The corresponding curves in figure 2 are quite short because the consistency condition (2.17) with so high $T_\times$ have solutions only for relatively big values of r . For $T_\times$

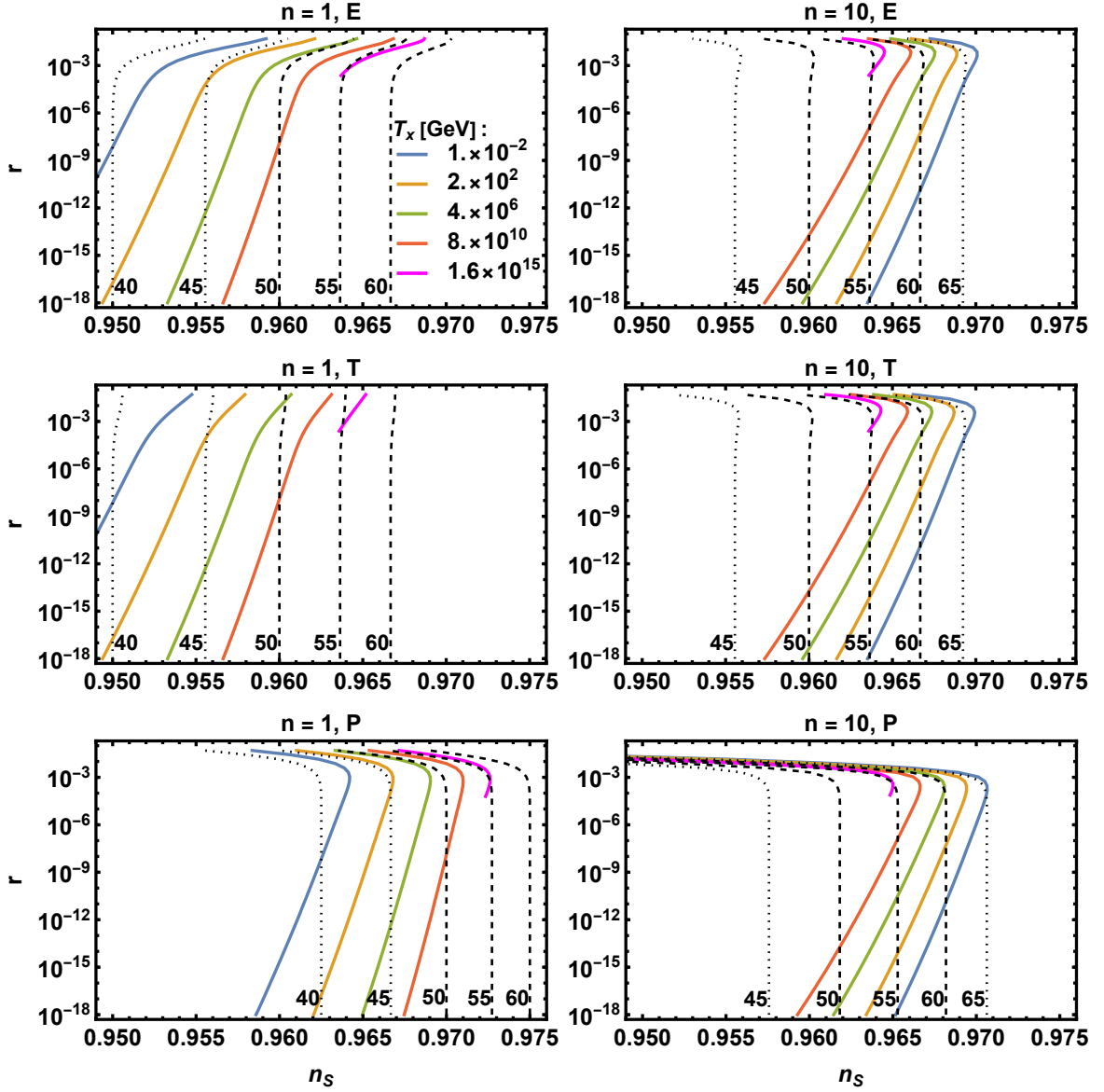


Figure 2. Correlation between n_s and r for three α -attractor models (E, T, and P) and two values of the exponent n (1 and 10). Colorful curves correspond to fixed values of the reheating temperature $T_\times$, while black curves represent fixed values of the number of e-folds N_k .

in the (n_s, r) plane has only some partial overlap with the region corresponding to the often used condition $50 \leq N_k \leq 60$. Clearly, the approach based on well motivated bounds on $T_\times$ may give significantly different results compared to an (essentially arbitrary) ansatz for N_k . Thus, in the following the dependence of the results on the reheating temperature (and not on N_k) will be investigated.

There are several interesting observations following from figure 2. Very narrow ranges of n_s are predicted, that depend on the model and on r . In most of the considered cases there is quite strong qualitative change in the behaviour of the curves of constant $T_\times$ for some,

approaching the upper bound the length of the corresponding curve shrinks to a point.

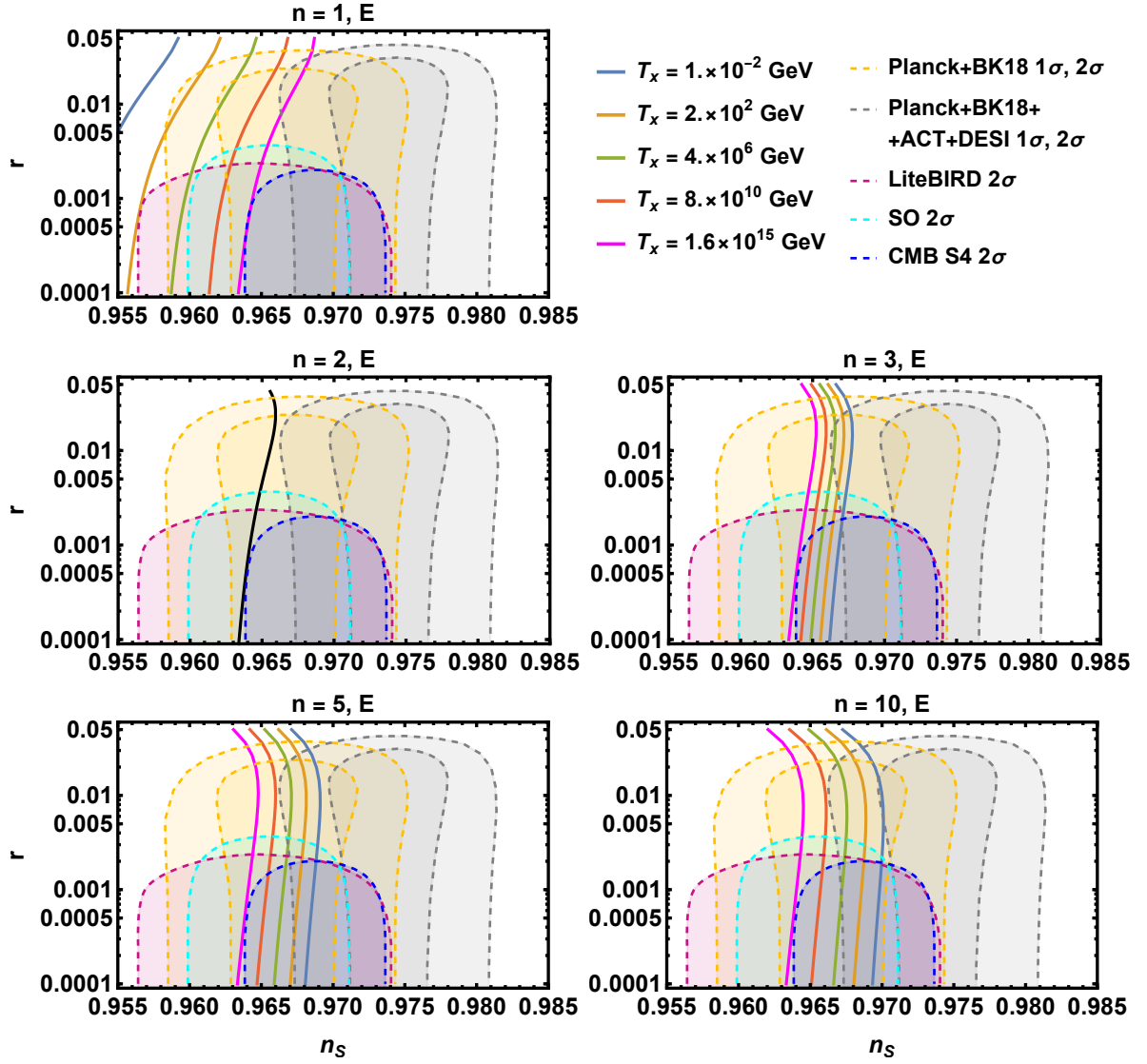


Figure 3. Correlation between n_s and r as a function of the reheating temperature T_x in the case of E-model for several values of the exponent n . Results for $n = 2$ do not depend on T_x and are represented by the black curve. The experimental 1σ and 2σ allowed regions from *Planck*, *BK15* and *BAO* [30, 40] are shown. Future sensitivity reaches of *LiteBIRD*, *CMB-S4*, and *SO* [41–43] experiments are also indicated.

depending on the model, value of r in the range $10^{-4} \div 10^{-2}$. For smaller r the spectral index n_s increases with increasing r for all models. The correlations between n_s and r are quite similar for E- and T-models. Such correlations are different in the case of P-models, especially for $n = 1$ or for relatively big values of r .

Results for $r \geq 10^{-4}$ are presented in figures 3 and 4 where curves of constant T_x are shown together with present experimental bounds and sensitivity reaches of future experiments. The present 2σ contours of the Planck+BK18 data differ quite strongly from the global fit including also ACT and DESI data (for recent discussion see e.g. [44]). The latter fit has more discriminative power for the considered models. It shows some preference for

P-model with $n = 1$ and for high reheating temperature $T_\times$. It can also be reconciled with E and T-models with large values of n and low reheating temperature. Actually, we see that in the considered models there is the upper bound on n_s around 0.97, to be compared with the 2σ lower bound of the global fit at ~ 0.965 . One can also ask how constraining are the present CMB data for the reheating temperature. Given the large difference between the Planck data and the global fit one concludes that the whole range of temperatures is acceptable. Taken Planck and the global fit separately, some constraints on the reheating temperatures can be seen in the figures. Furthermore, it is worth noting the change of ordering of the $T_\times = \text{const}$ curves from small to large values, as a function of n_s . This is related to the fact that $w < \frac{1}{3}$ for $n = 1$ while $w > \frac{1}{3}$ for $n > 2$. The case of $n = 2$ is a special one, where $w = \frac{1}{3}$ and the function $r(n_s)$ does not depend on $T_\times$ (the black curve in figure 3). Improving the precision of n_s measurements, as expected for the CMB S4 experiment, would be a stronger test of the models. The expected 2σ range (0.964, 0.974) would be particularly constraining if, in addition, the upper bound on r was pushed down, say to $r < 10^{-5}$ (see figure 2). It also follows from figure 2 that, if the present 2σ lower bound is valid for any value of r , then E- and T-models with $n = 1$ predict a lower bound on r of the order of 10^{-10} . The corresponding bounds for P-models and all considered models with big values of n are much weaker (e.g. of the order of 10^{-26} for $n = 10$). Some differences in the predictions for $r(n_s)$ for a fixed reheating temperature $T_\times$, particularly between P-model compared to E- and T-models, can be traced back to the shape of the inflaton potential in those models.

Finally, in figure 5 we show the curves of constant $T_\times$ in the plane (n_s, α_s) for E- and P-models. The bounds on the reheating temperature give the lower bound on the absolute value of α_s around 5×10^{-4} , independently of the considered model. Comparing figure 5 with figure 2 we see that the maximum absolute values of α_s allowed by the $T_\times$ bounds are increasing with decreasing values of r . This may be understood from the expressions for α_s in terms of the CMB observables. Substituting ξ given by eq. (2.9) into eqs. (2.13) and (A.19) we get the following simple expressions

$$\alpha_s = \begin{cases} -\frac{(8(1-n_s)-r)(n(8(1-n_s)-r)-r)}{128n} & \text{for E-model,} \\ -\frac{64(1-n_s)^2 + n(8(1-n_s)-r)^2 + r^2}{64(2n+1)} & \text{for P-model.} \end{cases} \quad (2.36)$$

The dependence on r is very weak for $r \ll (1-n_s)$ with the following simple limits:

$$\lim_{r \rightarrow 0} \alpha_s = \begin{cases} -\frac{1}{2}(1-n_s)^2 & \text{for E-model,} \\ -\frac{n+1}{2n+1}(1-n_s)^2 & \text{for P-model.} \end{cases} \quad (2.37)$$

In the case of E-models α_s for small r depends only on n_s and does not depend on the exponent n . This is visible on upper panels of figure 5 where the behavior of α_s for small r (corresponding to small n_s) is almost the same for $n = 1$ and $n = 10$. Situation in P-models is somewhat different. The running of the spectral index does depend on n . The first factor in the lower line of the r.h.s. of (2.37) approaches $-\frac{1}{2}$ present in upper line of (2.37) in the limit of large n . One can see in figure 5 that the behavior of α_s for small r in P-models with $n = 10$ is similar to that in E-models. The biggest difference is between P-model with $n = 1$ and E-models (with arbitrary n). Comparing two lines in eq. (2.37) one can see that $|\alpha_s|$ in such P-model is bigger by the factor of $\frac{4}{3}$. The present 2σ lower bound on n_s may

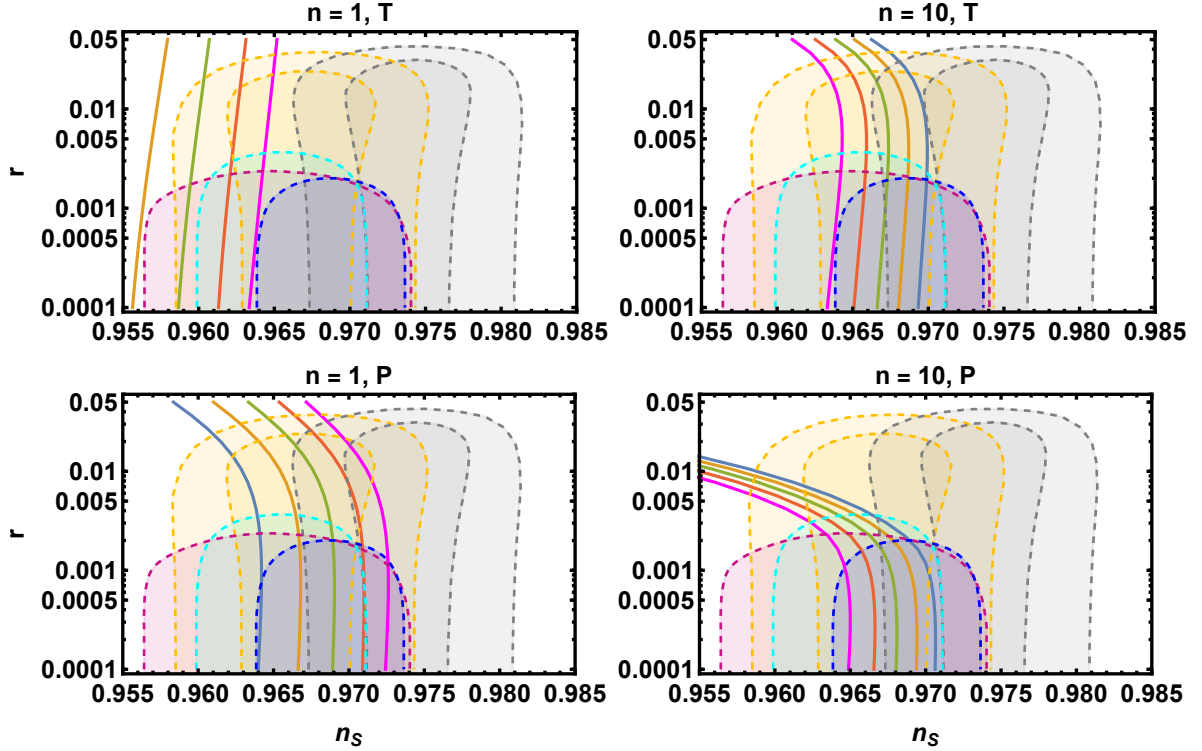


Figure 4. Correlation between n_s and r as a function of the reheating temperature $T_\times$ in the case of E-model for several values of the exponent n . Results for $n = 2$ do not depend on $T_\times$ and are represented by the black curve. The experimental 1σ and 2σ allowed regions from *Planck*, *BK15* and *BAO* [30, 40] are shown. Future sensitivity reaches of *LiteBIRD*, *CMB-S4*, and *SO* [41–43] experiments are also indicated.

be translated into upper bound on $|\alpha_s|$. Depending on the model this bound is between $8 \cdot 10^{-5}$ and $1.1 \cdot 10^{-4}$ with the highest value for P-model with $n = 1$. These numbers give us some intuition about the required sensitivity of the α_s measurements for being useful as a supplement to n_s in discriminating between different models of inflation, if r turns out to be very small.

3 $T_\times$ sensitive dark matter production

In this section we discuss the reheating temperature as a portal to link some DM production mechanisms to the CMB observables. One obtains then an interesting consistency check between theories of inflation and DM production. That happens when DM production is sensitive to the reheating temperature $T_\times$, as is the case in some freeze-in models. We consider the DM production in a process of annihilation of some particles (e.g. SM particles) in thermal bath during the reheating process and after reaching the temperature $T_\times$. To consider both periods on equal footing, it is useful to recall that the reheating period is also characterized by some temperature usually denoted by $T_{\max}$. That temperature is calculable in terms of the $T_\times$ so that overall, given a DM production model, requiring $\Omega_{\text{DM}} h^2 = 0.12$ indeed fixes the value of $T_\times$. The temperature $T_{\max}$ may be easily obtained from maximization of (2.27) – of course this time NOT neglecting any terms. Value of the cosmic scale factor

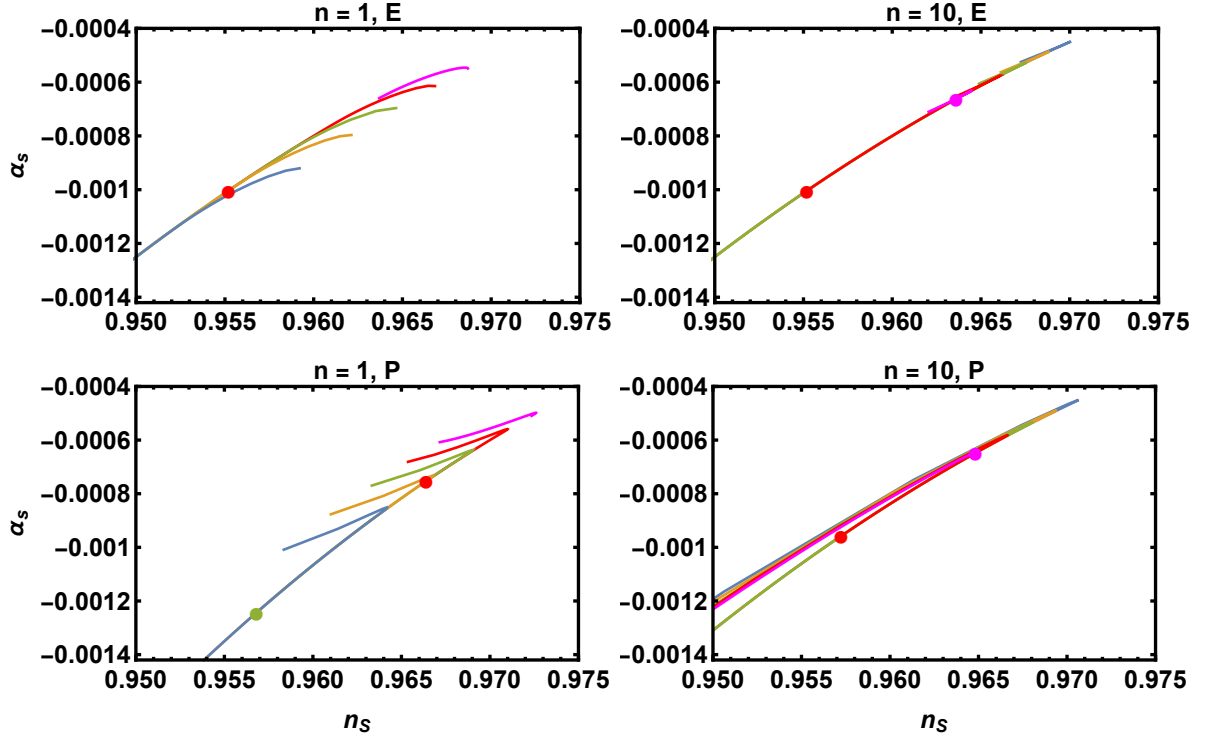


Figure 5. Correlation between scalar spectral index n_s and its running α_s , following from eq. (2.36), in the case of E- and P-model for two values of the exponent $n = 1, 10$ for fixed reheate temperature $T_\times$ (color coding as in figures 2-4). As the curves in different colours partially overlap, the end of some of them is denoted by the dot of the same colour. The absence of a dot in a given color means that the corresponding curve ends outside the plot.

for which ρ_R is maximal equals:

$$a_{\max} = a_{\text{end}} \left(\frac{8 + 3(1+w)\gamma}{3(1+w)(1+\gamma)} \right)^{\frac{2}{5-3w}} \quad (3.1)$$

This should be used in expression for ρ_R given by (2.27) with a replaced with the above $a_{\max}$. The results are shown in figure 6, for a large range of values of Γ and several values of the inflaton w . The dependence on those parameters is very weak, it corresponds to the thickness of the blue line.

If the DM particles are weakly enough coupled to the thermal bath they never reach thermal equilibrium with it but nevertheless they may reach the observed DM relic abundance in that freeze-in process. The necessary condition for the $T_\times$ dependence of that production process is that in the accessible range of energies (determined by the value of $T_\times$) its amplitude can effectively be derived from a higher dimension operator, $\mathcal{L} \sim \mathcal{O}_{4+d}/\Lambda_{\text{DM}}^d$, (we take d to be integer)¹³ where Λ_{DM} is some physical cut-off scale to the effective theory. It is clear on dimensional grounds that, for production of relativistic particles considered here, the thermal average cross section, relevant for the Boltzmann equation describing the production yield,

¹³Parameter d can also be fractional motivated from conformal field theory operators where the DM sector resides in CFT, investigated recently in refs. [45–47] with intriguing consequences in DM experiments.

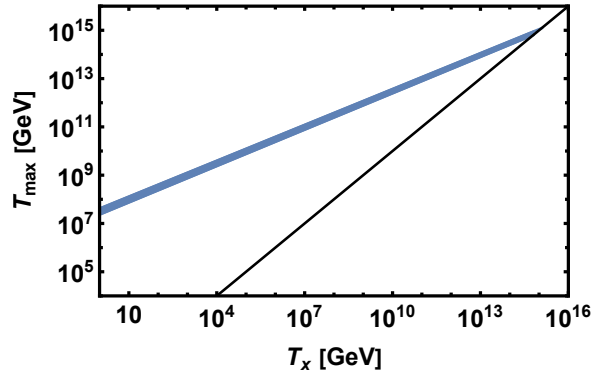


Figure 6. The blue line shows $T_{\max}$ vs $T_{\times}$. Its thickness reflects some (very weak) dependence on the parameters Γ and w . The black line denotes $T_{\max} = T_{\times}$.

reads

$$\langle\sigma v\rangle\sim\frac{T^{2(d-1)}}{\Lambda_{DM}^{2d}}. \quad (3.2)$$

There are several comments in order here. Firstly, the period of the universe evolution when the dominant contribution to the production of DM comes from depends on the dimension of the operator. If it dominantly comes from the reheating period it is sensitive to the maximal temperature $T_{\max}$ reached then, before the radiation dominance era (see figure 6). Secondly, such “theory” of production makes sense only for $\Lambda_{DM} \gg T_{\max}, T_{\times}$. And finally, and importantly for the presentation of our results, the numerical coefficient in the above equation, not explicitly written in it, may strongly depend on the considered process of the DM production, such as some powers of the coupling constants, phase space factors etc.. We present our results for its value taken to be 1, keeping in mind that for our results to be used in concrete models they have to be properly rescaled.

With regard to the dependence of the DM abundance on $T_{\max}$ during the reheating period, the question has been investigated in detail in refs. [48–50]. We show the results for the production amplitudes of $\dim 4 + d = 5, 6$ and $4 + d = 10$. In the first case, the production during the reheating period can approximately be neglected, in the second one it is the dominant one [50, 51]. Correspondingly, in the first case the DM abundance depends on $T_{\times}$. In the second case it depends on $T_{\max}$ but can still be parametrized by $T_{\times}$. The number density of DM particles produced via scatterings of particles in the thermal bath, with $\langle\sigma v\rangle$ of the processes given by (3.2), is determined by the Boltzmann equation

$$\frac{dn_{DM}}{dt} + 3H n_{DM} = \langle\sigma v\rangle ((n_{DM}^{eq})^2 - n_{DM}^2). \quad (3.3)$$

Here, n_{DM}^{eq} denotes the equilibrium DM number density. In this paper we assume that this is the only process for production of the DM and that initially $n_{DM} = 0$. In particular, we assume that the inflaton direct decays to DM can be neglected. In a more general analyses those assumptions can be relaxed.¹⁴

For the comoving DM particle density $Y \equiv n_{DM}/s$ one has

$$\frac{dY}{dT} = \frac{\langle\sigma v\rangle s}{HT} (Y^2 - Y_{eq}^2), \quad (3.4)$$

¹⁴Thermalization and number-changing processes within the dark sector may modify the DM abundance [52–59], we do not consider such cases as well.

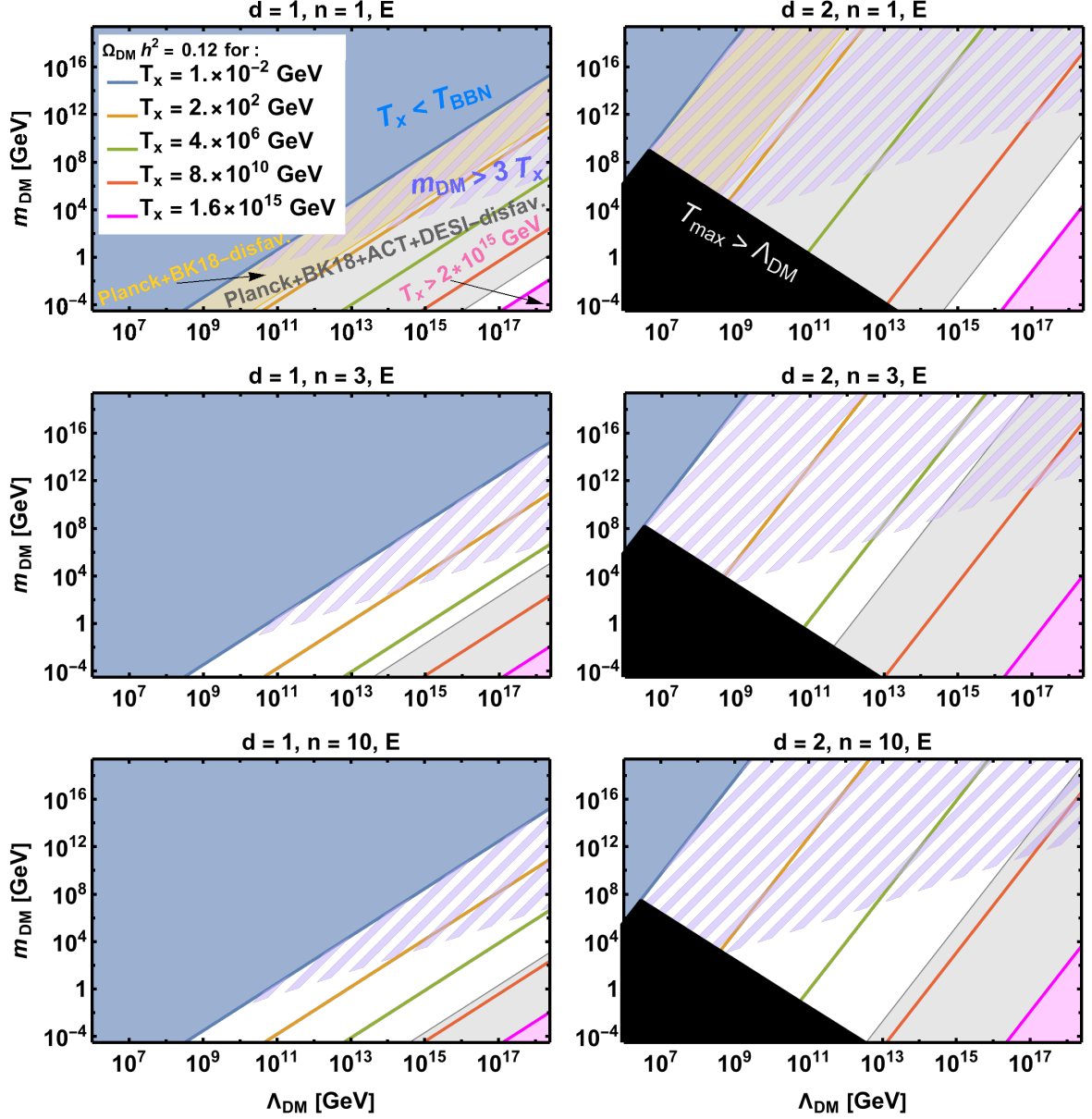


Figure 7. Various constraints in the parameter space $(\Lambda_{\text{DM}}, m_{\text{DM}})$ for the prototype model, eq. (3.2), with $d = 1, 2$ and E-model. Solid color lines correspond to the values of the reheating temperature necessary for the correct relic abundance, eqs. (3.6), (3.7); blue shaded region is excluded by $T_x < 10^{-2}$ GeV; dashed purple shaded region is excluded by $m_{\text{DM}} > 3T_x$; black shaded region is excluded by $T_{\text{max}} > \Lambda_{\text{DM}}$; yellow (gray) band is disfavored by current Planck+BK18 (Planck+BK18+ACT) data; pink-shaded region is excluded by $T_x > 2 \cdot 10^{15}$ GeV.

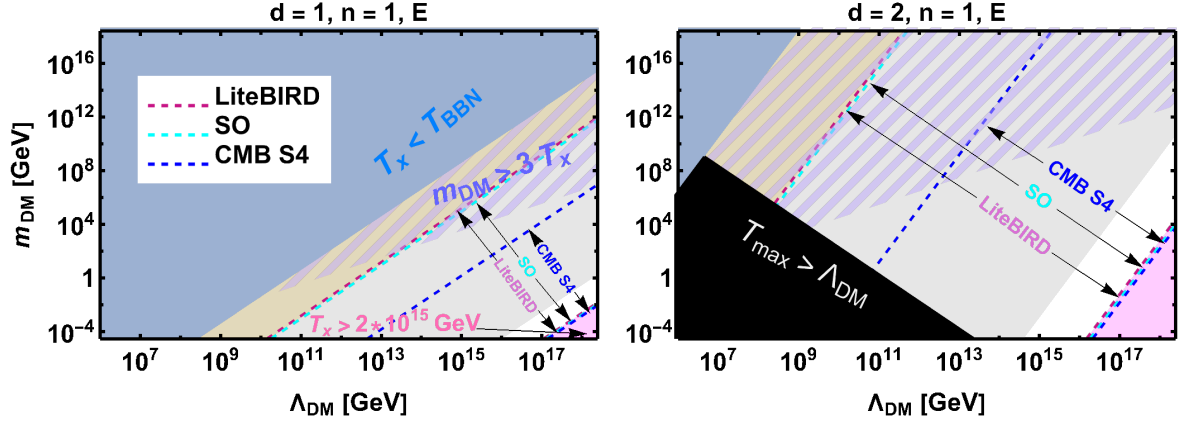


Figure 8. Same models as in two upper panels of figure 7. Regions between pairs of dashed same-color lines indicate DM parameter range allowed by the corresponding experiment. Notice that the dashed lines of different planned experiments often overlap at the high temperature end at $2 \cdot 10^{15}$ GeV.

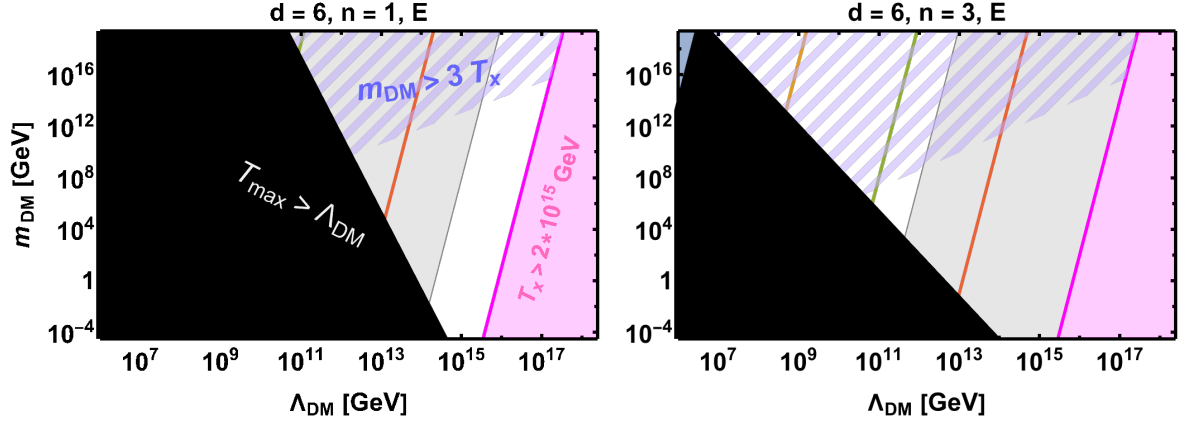


Figure 9. The same as figure 7, for E-model, $d = 6$.

where s is the entropy density

$$s(T) \equiv \frac{2\pi^2}{45} g_{*s}(T) T^3 \quad (3.5)$$

with $g_{*s}(T)$ being the effective number of relativistic degrees of freedom in the SM plasma [60]. The DM relic abundance has been calculated by numerical integration. However, for a qualitative understanding of the results it is useful to present its approximate analytical solutions. At a very qualitative level, since DM never reaches the equilibrium distribution in freeze-in production, i.e. $Y \ll Y_{\text{eq}}$, we have [25]

$$Y \sim \int_0^{T_\times} \frac{M_P T^{2(d-1)}}{\Lambda_{\text{DM}}^{2d}} \sim \frac{M_P T_\times^{2d-1}}{\Lambda_{\text{DM}}^{2d}}. \quad (3.6)$$

The upper limit in the integral is in principle given by the maximal temperature T_{max} reached during the reheating process. However, as we already above discussed, the relative importance of the contribution to the DM abundance coming from the reheating period respectively to the post-heating time, depends on the dimension of the operator describing the production process. For low dimension operators, for instance $d = 1, 2$, the reheating contribution is

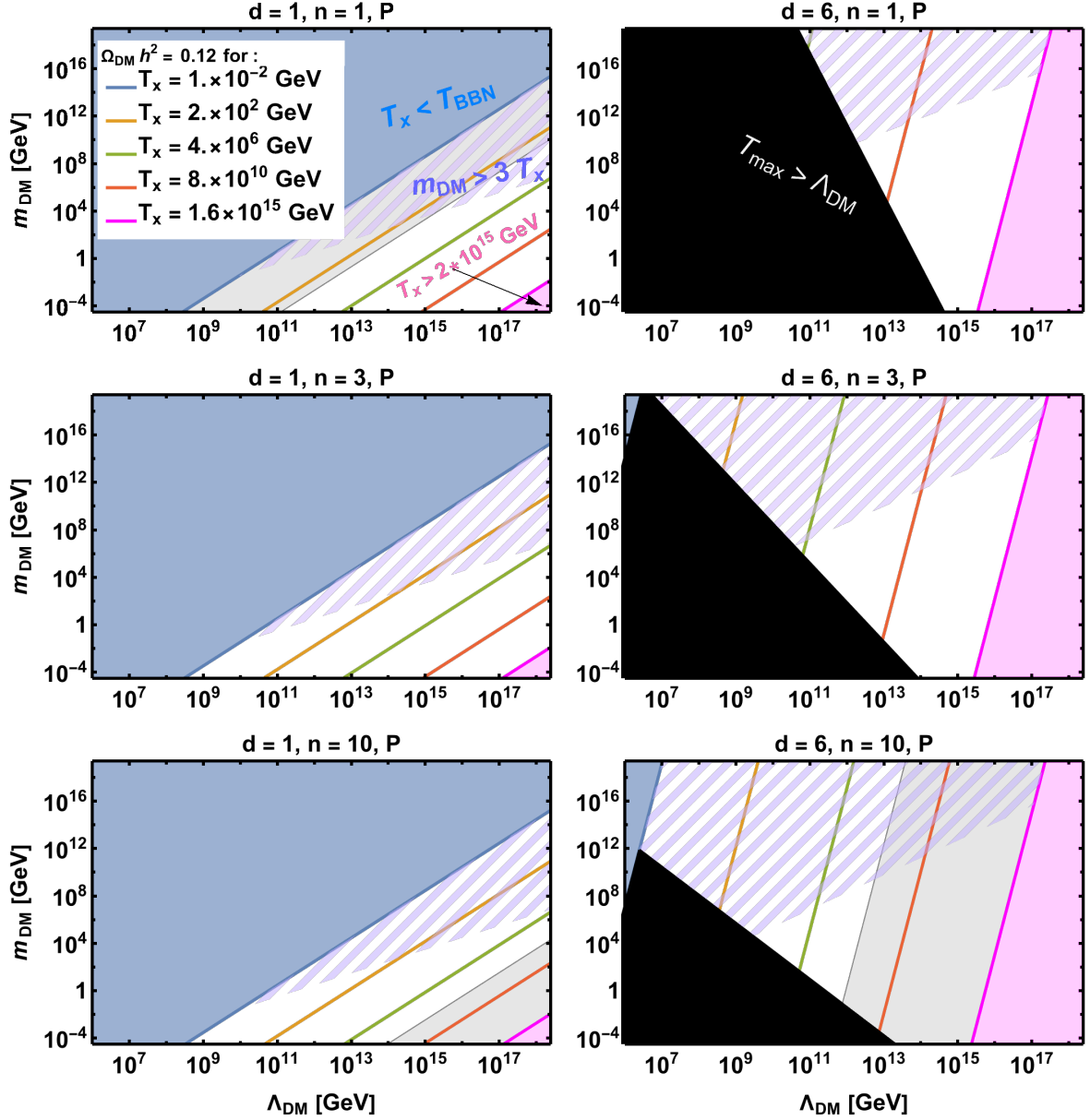


Figure 10. The same as figure 7, P-model, $d = 1$ (left) and $d = 6$ (right).

negligible and to a very good approximation the upper limit of integration can be taken as T_x .

The DM yield Y is related to the relic density via

$$\Omega_{\text{DM}} = \frac{m_{\text{DM}} Y s_0}{\rho_c} \simeq 0.2 \times \left(\frac{m_{\text{DM}}}{1 \text{ TeV}} \right) \left(\frac{Y}{10^{-13}} \right), \quad (3.7)$$

where ρ_c represents the critical density of the Universe, s_0 denotes entropy density at present epoch at $T_0 = 2.75 \text{ K} \sim 10^{-4} \text{ eV}$. We have also approximated $\sqrt{g_*} g_{*s} \sim 10^3$. The Planck experiment gives $\Omega_{\text{DM}} h^2 \sim 0.12$.

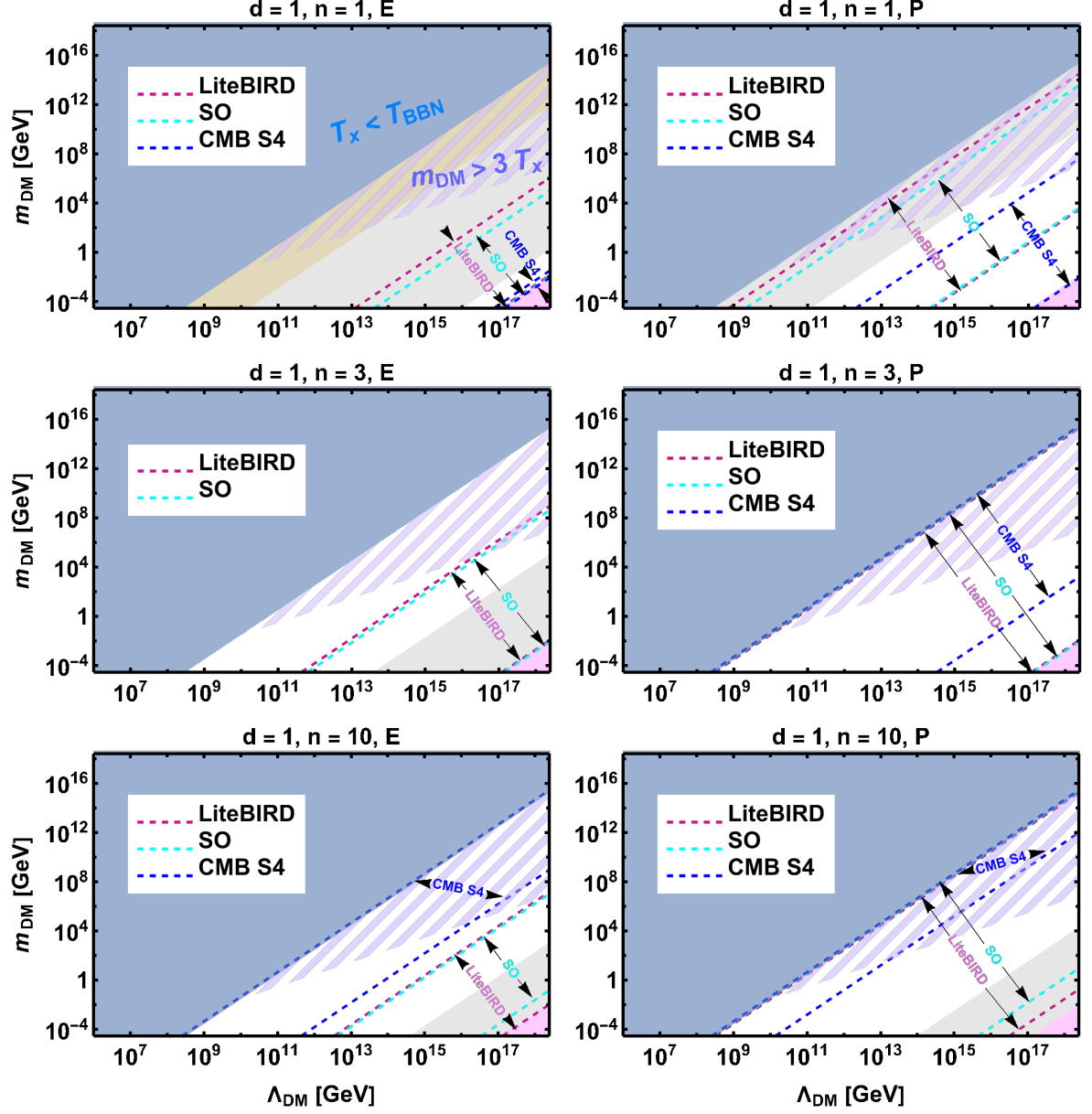


Figure 11. The same as figure 8, E-model (left), P-model (right), $d = 1$. We assume that the precision in n_s would be 10 times better (around the central values, for each planned experiment); if a pair of dashed lines for an experiment is absent (reflected also in the legend), this means the experiment would constraint n_s to values incompatible with the inflation model in question.

The DM abundance is dependent on the reheating temperature as evident from eq. (3.6). This feature gives us the scope of correlating the DM phenomenology with the cosmological quantities (n_s, r) as earlier stated, for a chosen model of inflation. Those results are illustrated in figures 7-11 for the “prototype” models given by eq. (3.2).

We show the results as the plots of m_{DM} vs Λ_{DM} . They illustrate two kinds of constraints on our prototype models. Firstly, those are the constraints following from the DM production mechanisms, and secondly, those following from the requirement of consistency with the chosen inflationary model and the CMB observables.

To the first category belong colored thin solid lines representing the $T_\times$ values required to satisfy the observed relic density of DM. Along with this, there are denoted regions excluded by theoretical consistency conditions, namely for $m_{\text{DM}} > 3T_\times$ needed for the assumption of the produced DM particles to be relativistic and $T_{\text{max}} < \Lambda_{\text{DM}}$ for the validity of EFT of UV-freeze-in DM. Moreover, there are also shown the model independent bounds on the reheat temperature of the universe $T_\times$, as in eq. (2.35), translated into the $(\Lambda_{\text{DM}}, m_{\text{DM}})$ parameter space. To the second category belong the constraints on the reheating temperature obtained in the considered inflationary models confronted in section 2 with the CMB observables from Planck and Planck-ACT observations as well as future sensitivity reaches of CMB experiments like LiteBIRD, SO and CMB-S4.

In all cases shown in figures 7-11 a large region of the parameter space is ruled out from the requirement of theoretical consistencies and by the model independent bounds on the reheating temperature. The main effect visible here is the dependence of the constraints on the dimension d of the operators responsible for DM production process: the higher the dimension d the stronger the dependence on the reheating temperature (eq. (3.2)). In consequence, value of m_{DM} necessary to get the correct relic abundance for a given T_{max} (so also given reheat temperature $T_\times$ – see figure 6) grows faster with Λ_{DM} when d is bigger. In addition, stronger dependence on T_{max} (and $T_\times$) results in bigger distances between lines of two fixed temperatures. As a result, the regions excluded by the condition $T_{\text{max}} < \Lambda_{\text{DM}}$ (black regions in figures 7-10) are bigger for bigger d . For big values of d there is additional dependence on the value of the exponent n . It follows from the fact that the inflaton energy density red-shifts faster when n is bigger (at early stages of reheating $\rho_\phi \propto a^{-3(1+w)}$ with w given by eq. (2.22)). Faster decrease of ρ_ϕ gives faster decrease of H which results in bigger coefficient in eq. (3.4). Thus, for bigger n we obtain bigger DM yield so smaller m_{DM} is needed to obtain the observed relic abundance of DM.

In the remaining regions the second category of constraints becomes relevant and the picture is more diversified. The constraints on the reheating temperature from the CMB observables, from either Planck+BK18 data or from Planck+BK18+ACT+DESI data give some dependence on the inflationary models and particularly on the value of n . The details of the plots can be understood with help of figures 3 and 4. We stress again that the results presented here are for the prototype model, eq. (3.2), and for a concrete complete model the numbers following from figures 7-11 have to be properly rescaled by its Wilson coefficients. The expected improvement of the constraints in the $(\Lambda_{\text{DM}}, m_{\text{DM}})$ parameter space due to the future CMB experiments is indicated figures 8 and 11.

Many freeze-in models with the DM production described by dim 5 and dim 6 effective operators have been discussed in the literature. They include, for instance, two scalar thermal bath particles Φ annihilating into a pair of DM fermions χ , $\mathcal{L} = |\Phi|^2 \bar{\chi}\chi/\Lambda_{\text{DM}}$, or two thermal bath fermions Ψ annihilating either into a pair of scalar DM particles S , $\mathcal{L} = \bar{\Psi}\Psi SS/\Lambda_{\text{DM}}$ or into a pair of DM fermions χ , $\mathcal{L} = \bar{\Psi}\Psi \bar{\chi}\chi/\Lambda_{\text{DM}}^2$.

This has been studied in the context of Higgs-portal, Spin 2 -portal, gravitino-portal, sterile neutrino-portal, scotogenic DM where DM loop exchange generates tiny SM neutrino mass, with correlation with neutrino oscillation data, axion and axino-portal, see ref. [50, 61] for a review. One important scenario in our context is the gravitational production scenarios, with dim 5 or dim 6 effective operators [62, 63], where simply $\Lambda_{\text{DM}} = M_{\text{P}}$ in figures 7-11. These models have their characteristic dependencies on the reheat temperature $T_{\times}$ and our prescription presented in the paper will readily probe those scenarios. We discuss one concrete example involving Higgs-portal in appendix C.

4 Conclusions

The present and future experimental programmes will explore the properties of the Cosmic Microwave Background in a great detail. In this paper we have proposed a systematic approach to use the CMB observables and the model-independent bounds on the reheating temperature a) for discriminating between different models of inflation, b) to constrain a reheating temperature dependent particle physics scenario, freeze-in DM production models. In the first step of this approach, all independent parameters of an inflationary model are expressed in terms of the CMB observables. Next, adopting a one parameter effective description (by an effective dissipation rate Γ), a consistency relation fixes the reheating temperature in terms of the measured values of the CMB observables. For a given model of inflation the bounds on those observables then follow from model-independent bounds on the reheating temperature (see section 2). The striking consequence of those bounds is that a given model of inflation gives very narrow ranges of the power spectrum index n_s , different for different inflationary models and with very interesting dependence on the value of r . In this paper, the above described strategy has been applied to α -attractor inflaton potentials, with three free parameters. We have found that the ACT data have more discriminative power than the Planck data for the considered models. They show preference for the polynomial α -attractor P-model with $n = 1$ and high reheat temperature, corresponding to $N_k \approx 55$.¹⁵ We stress that those results are obtained for a very simple reheating model, which strongly correlates the effective equation of state parameter w during the reheating period with the inflaton potential (see eq. (2.22)).

Next, expressing the reheating temperature $T_{\times}$ in terms of the CMB observables gives an interesting probe of the $T_{\times}$ dependent post-inflationary processes by the CMB observables. In this context UV-freeze-in DM production models have been discussed, with the DM production process described by higher dimensional effective operators. The impact of the measured CMB observables may give interesting constraints on the DM candidate mass versus the scale of the effective operators and on the consistency of the DM production models with models of inflation. However, we also observe that important role in constraining the $(\Lambda_{\text{DM}}, m_{\text{DM}})$ parameter space is played by such theoretical constraints like the validity of EFT and the assumed relativistic nature of DM. We observe that for concrete values of the scale Λ_{DM} there are upper bounds on m_{DM} , strongly dependent on the dimension of the EFT operator and on the parameter n of the inflaton potential (for instance in our “prototype” models for the dimension five operators, they never exceed $\mathcal{O}(10^7)$ GeV, and are much lower for low values of Λ_{DM} (see figures 10 and 11). Particularly, we have shown that for the simple Higgs-portal UV-freeze-in scenario in the E- and P-model inflationary framework (see

¹⁵We thank Renata Kallosh and Andrei Linde for the discussion on this point.

figure 12), the mass of DM must be smaller than about 1 GeV for $n = 1$ and 10^7 GeV for $n = 10$. The lower bound on m_{DM} of order 10 keV follows from the Lyman- α observations.

Our approach can be extended in several interesting directions, such as for example a) inclusion of a hidden sector, with its own reheating temperature, b) more complex reheating mechanisms c) more parameters in the inflaton potential d) other $T_{\times}$ sensitive processes, for instance freeze-out during the reheating period.

Our final remark is that with the advent of Gravitational Wave (GW) astronomy, beyond the CMB experiments, to pulsar timing array and interferometer based GW detectors, we may aspire to achieve complementary probes of inflationary phenomenology and the reheating era. The first step towards this direction in the context of DM and reheating has been taken in refs. [27, 28].

Acknowledgments

The authors would like to thank Ranata Kallosh and Andrei Linde for many discussions. A.G. thanks S K Jeessun for comments. P.K. thanks Adeela Afzal and Shiladitya Porey for sharing their Mathematica codes used to produce the experimental contours in section 2.

A Formulae for α -attractor T- and P-models

Formulae related inflaton potential parameters with the CMB-related quantities for α -attractor E-model were presented in section 2. Here we present analogous formulae for other α -attractor models.

From the potential of the T-model

$$V_T(\phi) = \Lambda_{\text{inf}}^4 \left(\tanh \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{P}}} \right) \right)^{2n} \quad (\text{A.1})$$

the following expressions for the CMB parameters follow

$$A_s = \frac{\alpha}{128\pi^2 n^2} \frac{\Lambda_{\text{inf}}^4}{M_{\text{P}}^4} \left(\sinh \left(2\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{\text{P}}} \right) \right)^2 \left(\tanh \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{\text{P}}} \right) \right)^{2n}, \quad (\text{A.2})$$

$$n_s = 1 - \frac{32n}{3\alpha} \left(n + \cosh \left(2\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{\text{P}}} \right) \right) \left(\sinh \left(2\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{\text{P}}} \right) \right)^{-2}, \quad (\text{A.3})$$

$$r = \frac{256n^2}{3\alpha} \left(\sinh \left(2\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{\text{P}}} \right) \right)^{-2} \quad (\text{A.4})$$

They may be inverted giving the results

$$\alpha = \frac{256n^2}{3r} \frac{1}{\xi^2 - 1}, \quad (\text{A.5})$$

$$\phi_k = M_{\text{P}} \sqrt{\frac{32}{r(\xi^2 - 1)}} n \ln \left(\xi + \sqrt{\xi^2 - 1} \right), \quad (\text{A.6})$$

$$\Lambda_{\text{inf}}^4 = \frac{3\pi^2}{2} M_{\text{P}}^4 r A_s \left(\frac{\xi + \sqrt{\xi^2 - 1} + 1}{\xi + \sqrt{\xi^2 - 1} - 1} \right)^{2n}, \quad (\text{A.7})$$

$$\alpha_s = - \frac{r^2 \left(2\xi^2 + 2\xi\sqrt{\xi^2 - 1} - 1 \right) (2n\xi + \xi^2 + 1) \left(\xi + \sqrt{\xi^2 - 1} \right)^4}{8n^2 \left(\xi\sqrt{\xi^2 - 1} (16\xi^4 - 16\xi^2 + 3) + (2\xi^2 + 1) (16\xi^4 - 16\xi^2 + 1) \right)}. \quad (\text{A.8})$$

The condition $\epsilon = 1$ (“usual” end of inflation) leads to the following values of the inflaton field ϕ_{end} , the energy density ρ_{end} and the number of e-folds during inflation after the pivot scale crossed the horizon N_k :

$$\phi_{\text{end}} = M_{\text{P}} \frac{4\sqrt{2}n}{\sqrt{r(\xi^2 - 1)}} \ln \left(\frac{1}{4} \left(\sqrt{r(\xi^2 - 1)} + \sqrt{16 + r(\xi^2 - 1)} \right) \right), \quad (\text{A.9})$$

$$\rho_{\text{end}} = 2\pi^2 M_{\text{P}}^4 r A_s \left(\frac{\left(\xi + \sqrt{\xi^2 - 1} + 1 \right) \left(\sqrt{r(\xi^2 - 1)} + \sqrt{16 + r(\xi^2 - 1)} - 4 \right)}{\left(\xi + \sqrt{\xi^2 - 1} - 1 \right) \left(\sqrt{r(\xi^2 - 1)} + \sqrt{16 + r(\xi^2 - 1)} + 4 \right)} \right)^{2n}, \quad (\text{A.10})$$

$$N_k = \frac{8n}{r(\xi^2 - 1)} \left(2\xi - \frac{\sqrt{16 + r(\xi^2 - 1)} + \sqrt{r(\xi^2 - 1)}}{4} - \frac{4}{\sqrt{16 + r(\xi^2 - 1)} + \sqrt{r(\xi^2 - 1)}} \right). \quad (\text{A.11})$$

For P-models with the potential

$$V_P(\phi) = \Lambda_{\text{inf}}^4 \frac{\left(\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{P}}} \right)^{2n}}{1 + \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{P}}} \right)^{2n}} \quad (\text{A.12})$$

one obtains

$$A_s = \frac{1}{48\pi^2 n^2} \frac{\Lambda_{\text{inf}}^4 \phi_k^2}{M_{\text{P}}^6} \left(\frac{2\phi_k^2}{3\alpha M_{\text{P}}^2} \right)^n \left(1 + \left(\frac{2\phi_k^2}{3\alpha M_{\text{P}}^2} \right)^n \right), \quad (\text{A.13})$$

$$n_s = 1 - 4n \frac{M_{\text{P}}^2}{\phi_k^2} \left(n + 1 + (2n + 1) \left(\frac{2\phi_k^2}{3\alpha M_{\text{P}}^2} \right)^n \right) \left(1 + \left(\frac{2\phi_k^2}{3\alpha M_{\text{P}}^2} \right)^n \right)^{-2}, \quad (\text{A.14})$$

$$r = 32n^2 \frac{M_{\text{P}}^2}{\phi_k^2} \left(1 + \left(\frac{2\phi_k^2}{3\alpha M_{\text{P}}^2} \right)^n \right)^{-2} \quad (\text{A.15})$$

and

$$\alpha = \frac{64n^2}{3r} \left(\frac{2n+1}{2n+\xi} \right)^2 \sqrt[n]{\frac{2n+1}{\xi-1}}, \quad (\text{A.16})$$

$$\phi_k = M_{\text{P}} \sqrt{\frac{32}{r}} n \frac{2n+1}{2n+\xi}, \quad (\text{A.17})$$

$$\Lambda_{\text{inf}}^4 = \frac{3\pi^2}{2} M_{\text{P}}^4 r A_s \left(\frac{\xi-1}{2n+\xi} \right) \quad (\text{A.18})$$

$$\alpha_s = -\frac{r^2 (\xi^2(n+1) + 2n(\xi+n))}{64n^2(2n+1)}. \quad (\text{A.19})$$

It is not possible to get closed expression for $\phi_{\text{end}}^{(\epsilon)}$ in P-models because it is related to the solution of equation $x(1+x^{2n}) = \text{const}$ (explicit solutions in the case of $n=1$ can be found but they are rather lengthy and complicated).

B Simplifying assumptions underlying this analysis

In this paper we have made several simplifying assumptions:

- We have assumed perturbative reheating. There can be that, due to the inflaton couplings and DM couplings this simple picture breaks down and non-perturbative particle production may become important. Some discussion of non-perturbative effects can be found in [64, 65, 65–71]. However, generally this can be ignored roughly when inflaton couplings are of order 10^{-6} or below.
- We have assumed that the tree level and loop level decays of the inflaton to DM particles are negligible.
- We have assumed that the Coleman-Weinberg loop corrections to the inflaton potential generated by the inflaton couplings do not destabilize it and also do not significantly contribute to changing the shape of the potential, to an extent to impact the inflationary CMB observables.
- We have ignored the effects of possible changes of the dissipation rate Γ during reheating and during inflation, their contributions to CMB observables, for instance on primordial non-gaussianities.
- We have assumed the gravitational production of DM during inflation to be negligible, see ref. [27] for a discussion when it is justified.
- In the calculations regarding DM we have assumed instant and local thermal equilibrium to have been reached during the reheating period. We have not considered detailed thermalisation processes like those in refs. [72, 73].

C Concrete example: Higgs-portal

Let us take a very simple Higgs-portal model defined by the following interaction and mass terms, H denoting the SM Higgs doublet and χ is a Dirac fermion DM, [74–76]

$$\mathcal{L} \supset \frac{1}{\Lambda_{\text{DM}}} H^\dagger H \bar{\chi} \chi + m_{\chi 0} \bar{\chi} \chi, \quad (\text{C.1})$$

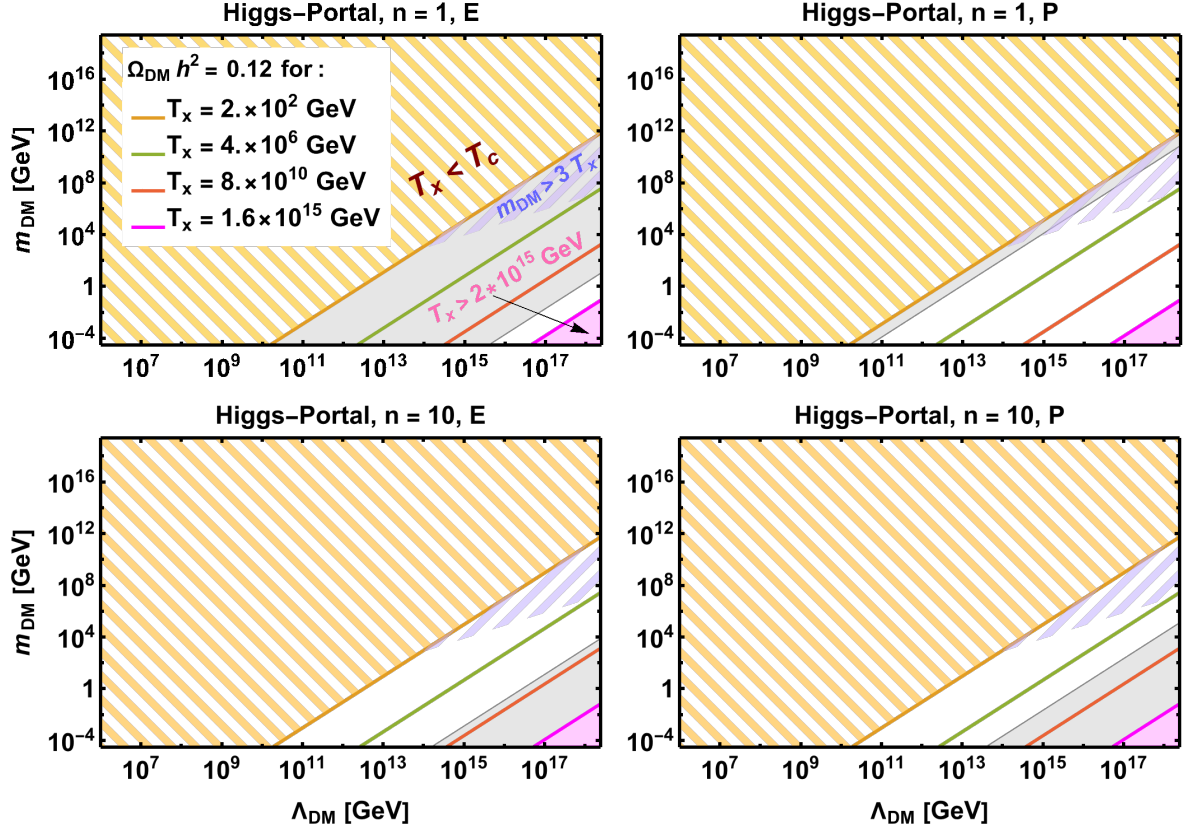


Figure 12. Constraints in the Λ_{DM} vs m_{DM} plane for the Higgs-portal, for exemplary E- and P-models of inflation. We take the SM critical temperature, T_c , below which the electroweak symmetry is broken, to be 160 GeV.

where $m_{\chi 0}$ is the bare mass term. For large temperatures, in particular larger than the critical temperature for the SM EW symmetry breaking, $T_c \simeq 160$ GeV, the thermally averaged cross section can be approximated by the relativistic cross-section for annihilation $H^\dagger H \rightarrow \bar{\chi}\chi$,

$$\langle \sigma_H v \rangle \simeq \frac{1}{8\pi\Lambda_{\text{DM}}^2}, \quad (\text{C.2})$$

which is, up to a factor, exactly of the form described in eq. (3.2). In what follows, we will focus on this regime.

We start with a qualitative understanding of the results discussing approximate analytical solutions. The total production rate of χ DM fermions from the radiation bath is described by the rate equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = 2\langle \sigma_H v \rangle (n_H^{\text{eq}})^2 \simeq \frac{(n_H^{\text{eq}})^2}{4\pi\Lambda_{\text{DM}}^2}, \quad (\text{C.3})$$

where $n_H^{\text{eq}}(T) = \zeta(3)T^3/\pi^2$ is the equilibrium number density of the SM Higgs. The above Boltzmann equation has an approximate analytic solution of the form (reflecting the fact that the dominant DM production occurs at temperatures close to $T_\times$)

$$\frac{n_\chi}{T^3} \simeq \frac{1}{4\pi\Lambda_{\text{DM}}^2} \left(\frac{1.2}{\pi^2} \right)^2 \frac{M_{\text{P}} T_\times}{k T_\times}, \quad (\text{C.4})$$

where $k_{T_\times} = (\pi^2 g_*(T_\times)/90)^{1/2}$ and $g_*(T_\times)$ is the number of relativistic degrees of freedom in the thermal bath. The total DM number density Ω_χ is due to χ plus $\bar{\chi}$ at the present temperature of the universe T_γ ,

$$\Omega_\chi = \frac{2m_\chi n_\chi(T_\gamma)}{\rho_c}, \quad (\text{C.5})$$

where ρ_c is the critical density and $n_\chi(T_\gamma)$ is given by (see (C.4))

$$\frac{n_\chi(T_\gamma)}{T_\gamma^3} = \frac{g(T_\gamma)}{g(T_\times)} \frac{n_\chi(T)}{T^3} = \frac{g(T_\gamma)}{g(T_\times)} \frac{1}{k_{T_\times}} \left(\frac{1.2}{\pi^2} \right)^2 \frac{M_P T_\times}{4\pi \Lambda_{\text{DM}}^2}, \quad (\text{C.6})$$

and DM mass, m_χ , at late times reads

$$m_\chi = m_{\chi 0} + \frac{v^2}{2\Lambda_{\text{DM}}} \approx m_{\chi^0}. \quad (\text{C.7})$$

where v is the EW scale. Thus, using (C.5) and (C.6),

$$T_\times \simeq \left(\frac{10 \text{ MeV}}{m_{\text{DM}}} \right) \left(\frac{\Lambda_{\text{DM}}}{10^{12} \text{ GeV}} \right)^2 \quad (\text{C.8})$$

Numerical solutions for DM abundance are shown in figure 12, for the E-model with $n = 1$ and $n = 10$. We see that the mass of the DM is quite restricted to be smaller than about 1 GeV for $n = 1$ and 10^7 GeV for $n = 10$ and heavier than $\mathcal{O}(\infty)$ keV (from the Lyman- α data). For the P-model and $n = 1$ the CMB constraints are quite loosened and higher masses of m_{DM} remain viable and testable within the future sensitivities of CMB experiments.

As it was mentioned in appendix B, the inflaton tree level and loop level induced decays are neglected in this scenario.

References

- [1] V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, Oxford (2005), [10.1017/CBO9780511790553](#).
- [2] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett. B* **91** (1980) 99.
- [3] A.S. Goncharov and A.D. Linde, *CHAOTIC INFLATION OF THE UNIVERSE IN SUPERGRAVITY*, *Sov. Phys. JETP* **59** (1984) 930.
- [4] A. Linde, *Does the first chaotic inflation model in supergravity provide the best fit to the Planck data?*, *JCAP* **02** (2015) 030 [[1412.7111](#)].
- [5] F.L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, *Phys. Lett. B* **659** (2008) 703 [[0710.3755](#)].
- [6] M. Cicoli, C.P. Burgess and F. Quevedo, *Fibre Inflation: Observable Gravity Waves from IIB String Compactifications*, *JCAP* **03** (2009) 013 [[0808.0691](#)].
- [7] S. Ferrara and R. Kallosh, *Seven-disk manifold, α -attractors, and B modes*, *Phys. Rev. D* **94** (2016) 126015 [[1610.04163](#)].
- [8] R. Kallosh, A. Linde, T. Wrase and Y. Yamada, *Maximal Supersymmetry and B-Mode Targets*, *JHEP* **04** (2017) 144 [[1704.04829](#)].

- [9] R. Kallosh, A. Linde, D. Roest, A. Westphal and Y. Yamada, *Fibre Inflation and α -attractors*, *JHEP* **02** (2018) 117 [[1707.05830](#)].
- [10] J. Ellis, D.V. Nanopoulos, K.A. Olive and S. Verner, *Unified No-Scale Attractors*, *JCAP* **09** (2019) 040 [[1906.10176](#)].
- [11] J. Ellis, B. Nagara, D.V. Nanopoulos, K.A. Olive and S. Verner, *From Minkowski to de Sitter in Multifield No-Scale Models*, *JHEP* **10** (2019) 161 [[1907.09123](#)].
- [12] S.S. Mishra, V. Sahni and A.A. Starobinsky, *Curing inflationary degeneracies using reheating predictions and relic gravitational waves*, *JCAP* **05** (2021) 075 [[2101.00271](#)].
- [13] G. Germán, R.G. Quaglia and A.M.M. Colorado, *Model independent bounds for the number of e -folds during the evolution of the universe*, *JCAP* **03** (2023) 004 [[2212.03730](#)].
- [14] M.A.G. Garcia, G. Germán, R. Gonzalez Quaglia and A.M.M. Colorado, *Reheating constraints and consistency relations of the Starobinsky model and some of its generalizations*, *JCAP* **12** (2023) 015 [[2306.15831](#)].
- [15] S. Weinberg, *Cosmology*, Oxford University Press (2008).
- [16] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, *Phys. Lett. B* **174** (1986) 45.
- [17] W. Buchmuller, P. Di Bari and M. Plumacher, *Leptogenesis for pedestrians*, *Annals Phys.* **315** (2005) 305 [[hep-ph/0401240](#)].
- [18] A. Ghoshal, D. Nanda and A.K. Saha, *CMB imprints of high scale non-thermal leptogenesis*, *Phys. Lett. B* **849** (2024) 138484 [[2210.14176](#)].
- [19] M.Y. Khlopov and A.D. Linde, *Is It Easy to Save the Gravitino?*, *Phys. Lett. B* **138** (1984) 265.
- [20] T. Moroi, H. Murayama and M. Yamaguchi, *Cosmological constraints on the light stable gravitino*, *Phys. Lett. B* **303** (1993) 289.
- [21] T. Asaka, K. Hamaguchi and K. Suzuki, *Cosmological gravitino problem in gauge mediated supersymmetry breaking models*, *Phys. Lett. B* **490** (2000) 136 [[hep-ph/0005136](#)].
- [22] L. Roszkowski, R. Ruiz de Austri and K.-Y. Choi, *Gravitino dark matter in the CMSSM and implications for leptogenesis and the LHC*, *JHEP* **08** (2005) 080 [[hep-ph/0408227](#)].
- [23] D.G. Cerdeno, K.-Y. Choi, K. Jedamzik, L. Roszkowski and R. Ruiz de Austri, *Gravitino dark matter in the CMSSM with improved constraints from BBN*, *JCAP* **06** (2006) 005 [[hep-ph/0509275](#)].
- [24] F.D. Steffen, *Gravitino dark matter and cosmological constraints*, *JCAP* **09** (2006) 001 [[hep-ph/0605306](#)].
- [25] F. Elahi, C. Kolda and J. Unwin, *UltraViolet Freeze-in*, *JHEP* **03** (2015) 048 [[1410.6157](#)].
- [26] P. Arias, N. Bernal, J.K. Osiński and L. Roszkowski, *Dark matter axions in the early universe with a period of increasing temperature*, *JCAP* **05** (2023) 028 [[2207.07677](#)].
- [27] A. Ghoshal, D. Paul and S. Pal, *Primordial gravitational waves as probe of dark matter in interferometer missions: Fisher forecast and MCMC*, *JHEP* **12** (2024) 150 [[2405.06741](#)].
- [28] A. Cheek, A. Ghoshal and D. Paul, *Axion Dark Matter Archaeology with Primordial Gravitational Waves*, [2505.04614](#).
- [29] A. Biswas, S. Ganguly, D. Nanda and S.K. Sahoo, *Viability of post-inflationary freeze-in with precision cosmology*, [2505.13624](#).
- [30] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [[1807.06209](#)].

- [31] R. Kallosh and A. Linde, *Superconformal generalizations of the Starobinsky model*, *JCAP* **06** (2013) 028 [[1306.3214](#)].
- [32] R. Kallosh and A. Linde, *Universality Class in Conformal Inflation*, *JCAP* **07** (2013) 002 [[1306.5220](#)].
- [33] R. Kallosh, A. Linde and D. Roest, *Superconformal Inflationary α -Attractors*, *JHEP* **11** (2013) 198 [[1311.0472](#)].
- [34] R. Kallosh and A. Linde, *Superconformal generalization of the chaotic inflation model $\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$* , *JCAP* **06** (2013) 027 [[1306.3211](#)].
- [35] R. Kallosh and A. Linde, *Non-minimal Inflationary Attractors*, *JCAP* **10** (2013) 033 [[1307.7938](#)].
- [36] M. Galante, R. Kallosh, A. Linde and D. Roest, *Unity of Cosmological Inflation Attractors*, *Phys. Rev. Lett.* **114** (2015) 141302 [[1412.3797](#)].
- [37] R. Kallosh and A. Linde, *Polynomial α -attractors*, *JCAP* **04** (2022) 017 [[2202.06492](#)].
- [38] E.W. Kolb and M.S. Turner, *The Early Universe*, vol. 69, Taylor and Francis (5, 2019), [10.1201/9780429492860](#).
- [39] J.A.R. Cembranos, A.L. Maroto and S.J. Núñez Jareño, *Cosmological perturbations in coherent oscillating scalar field models*, *JHEP* **03** (2016) 013 [[1509.08819](#)].
- [40] BICEP2, KECK ARRAY collaboration, *BICEP2 / Keck Array x: Constraints on Primordial Gravitational Waves using Planck, WMAP, and New BICEP2/Keck Observations through the 2015 Season*, *Phys. Rev. Lett.* **121** (2018) 221301 [[1810.05216](#)].
- [41] LITEBIRD collaboration, *Probing Cosmic Inflation with the LiteBIRD Cosmic Microwave Background Polarization Survey*, *PTEP* **2023** (2023) 042F01 [[2202.02773](#)].
- [42] CMB-S4 collaboration, *CMB-S4 Science Book, First Edition*, [1610.02743](#).
- [43] SIMONS OBSERVATORY collaboration, *The Simons Observatory: Science goals and forecasts*, *JCAP* **02** (2019) 056 [[1808.07445](#)].
- [44] E.G.M. Ferreira, E. McDonough, L. Balkenhol, R. Kallosh, L. Knox and A. Linde, *The BAO-CMB Tension and Implications for Inflation*, [2507.12459](#).
- [45] S. Hong, G. Kurup and M. Perelstein, *Conformal Freeze-In of Dark Matter*, *Phys. Rev. D* **101** (2020) 095037 [[1910.10160](#)].
- [46] S. Hong, G. Kurup and M. Perelstein, *Dark matter from a conformal Dark Sector*, *JHEP* **02** (2023) 221 [[2207.10093](#)].
- [47] C. Csáki, S. Hong, G. Kurup, S.J. Lee, M. Perelstein and W. Xue, *Continuum dark matter*, *Phys. Rev. D* **105** (2022) 035025 [[2105.07035](#)].
- [48] D.J.H. Chung, E.W. Kolb and A. Riotto, *Production of massive particles during reheating*, *Phys. Rev. D* **60** (1999) 063504 [[hep-ph/9809453](#)].
- [49] G.F. Giudice, E.W. Kolb and A. Riotto, *Largest temperature of the radiation era and its cosmological implications*, *Phys. Rev. D* **64** (2001) 023508 [[hep-ph/0005123](#)].
- [50] N. Bernal, F. Elahi, C. Maldonado and J. Unwin, *Ultraviolet Freeze-in and Non-Standard Cosmologies*, *JCAP* **11** (2019) 026 [[1909.07992](#)].
- [51] N. Bernal and F. Hajkarim, *Primordial Gravitational Waves in Nonstandard Cosmologies*, *Phys. Rev. D* **100** (2019) 063502 [[1905.10410](#)].
- [52] X. Chu, Y. Mambrini, J. Quevillon and B. Zaldivar, *Thermal and non-thermal production of dark matter via Z' -portal(s)*, *JCAP* **01** (2014) 034 [[1306.4677](#)].

- [53] N. Bernal, X. Chu, C. Garcia-Cely, T. Hambye and B. Zaldivar, *Production Regimes for Self-Interacting Dark Matter*, *JCAP* **03** (2016) 018 [[1510.08063](#)].
- [54] N. Bernal and X. Chu, $\mathbb{Z}_2$ *SIMP Dark Matter*, *JCAP* **01** (2016) 006 [[1510.08527](#)].
- [55] N. Bernal, X. Chu and J. Pradler, *Simply split strongly interacting massive particles*, *Phys. Rev. D* **95** (2017) 115023 [[1702.04906](#)].
- [56] A. Falkowski, E. Kuflik, N. Levi and T. Volansky, *Light Dark Matter from Leptogenesis*, *Phys. Rev. D* **99** (2019) 015022 [[1712.07652](#)].
- [57] S. Heeba, F. Kahlhoefer and P. Stöcker, *Freeze-in production of decaying dark matter in five steps*, *JCAP* **11** (2018) 048 [[1809.04849](#)].
- [58] C. Mondino, M. Pospelov, J.T. Ruderman and O. Slone, *Dark Higgs Dark Matter*, *Phys. Rev. D* **103** (2021) 035027 [[2005.02397](#)].
- [59] N. Bernal, *Boosting Freeze-in through Thermalization*, *JCAP* **10** (2020) 006 [[2005.08988](#)].
- [60] M. Drees, F. Hajkarim and E.R. Schmitz, *The Effects of QCD Equation of State on the Relic Density of WIMP Dark Matter*, *JCAP* **06** (2015) 025 [[1503.03513](#)].
- [61] N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen, *The Dawn of FIMP Dark Matter: A Review of Models and Constraints*, *Int. J. Mod. Phys. A* **32** (2017) 1730023 [[1706.07442](#)].
- [62] F. Koutroulis, O. Lebedev and S. Pokorski, *Gravitational production of sterile neutrinos*, *JHEP* **04** (2024) 027 [[2310.15906](#)].
- [63] O. Lebedev, *Scalar overproduction in standard cosmology and predictivity of non-thermal dark matter*, *JCAP* **02** (2023) 032 [[2210.02293](#)].
- [64] K.D. Lozanov and M.A. Amin, *Equation of State and Duration to Radiation Domination after Inflation*, *Phys. Rev. Lett.* **119** (2017) 061301 [[1608.01213](#)].
- [65] D. Maity and P. Saha, *(P)reheating after minimal Plateau Inflation and constraints from CMB*, *JCAP* **07** (2019) 018 [[1811.11173](#)].
- [66] P. Saha, S. Anand and L. Sriramkumar, *Accounting for the time evolution of the equation of state parameter during reheating*, *Phys. Rev. D* **102** (2020) 103511 [[2005.01874](#)].
- [67] S. Antusch, D.G. Figueroa, K. Marschall and F. Torrenti, *Energy distribution and equation of state of the early Universe: matching the end of inflation and the onset of radiation domination*, *Phys. Lett. B* **811** (2020) 135888 [[2005.07563](#)].
- [68] R. Easther, R. Flauger and J.B. Gilmore, *Delayed Reheating and the Breakdown of Coherent Oscillations*, *JCAP* **04** (2011) 027 [[1003.3011](#)].
- [69] K. Freese, E.I. Sfakianakis, P. Stengel and L. Visinelli, *The Higgs Boson can delay Reheating after Inflation*, *JCAP* **05** (2018) 067 [[1712.03791](#)].
- [70] M. Drewes, J.U. Kang and U.R. Mun, *CMB constraints on the inflaton couplings and reheating temperature in α -attractor inflation*, *JHEP* **11** (2017) 072 [[1708.01197](#)].
- [71] M. Drewes, *Measuring the inflaton coupling in the CMB*, *JCAP* **09** (2022) 069 [[1903.09599](#)].
- [72] K. Harigaya, M. Kawasaki, K. Mukaida and M. Yamada, *Dark Matter Production in Late Time Reheating*, *Phys. Rev. D* **89** (2014) 083532 [[1402.2846](#)].
- [73] K. Harigaya, K. Mukaida and M. Yamada, *Dark Matter Production during the Thermalization Era*, *JHEP* **07** (2019) 059 [[1901.11027](#)].
- [74] E.W. Kolb and A.J. Long, *Superheavy dark matter through Higgs portal operators*, *Phys. Rev. D* **96** (2017) 103540 [[1708.04293](#)].

- [75] O. Lebedev, H.M. Lee and Y. Mambrini, *Vector Higgs-portal dark matter and the invisible Higgs*, *Phys. Lett. B* **707** (2012) 570 [[1111.4482](#)].
- [76] J. McDonald, *Warm Dark Matter via Ultra-Violet Freeze-In: Reheating Temperature and Non-Thermal Distribution for Fermionic Higgs Portal Dark Matter*, *JCAP* **08** (2016) 035 [[1512.06422](#)].