

Zeeman Doppler mapping ⋆ deconstructed

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ABSTRACT

Aims. Magnetic and abundance maps of chemically peculiar (CP) stars, derived with the help of Zeeman Doppler mapping, have invariably been used as arguments against theories, in particular atomic diffusion theory. We intend to expose the fallacy of these claims.
Methods. We have identified in the literature those (5) CP stars for which multiple maps have been published, all based on the same Zeeman Doppler mapping strategy. For each of these stars we have then carried out inter comparisons between the recovered distributions of magnetic field and of abundances.
Results. Agreement between maps often turns out to be quite poor in regard to both abundances, field topology and absolute field strengths. Maps based on the same set of observations can differ considerably, even when they are coming from the same authors.
Conclusions. It becomes clear that Zeeman Doppler mapping cannot be guaranteed to yield unique results. When a number of physically impossible magnetic geometries all provide good fits to the observed Stokes $IQUV$ profiles, these solutions must necessarily be spurious and cannot be used as constraints to diffusion theory.

Key words. Stars: chemically peculiar – Stars: magnetic field – Stars: abundances – Methods: numerical

. Introduction

n the 1940s, a former student at Uppsala University, Hannes lfvén, developed the theory of magneto-hydrodynamics MHD), for which he received the Nobel Price in Physics in 970. This theory provided insight, among others, into the beaviour of strongly magnetic structures in the solar photosphere sunspots) and the loops and arcades in the highly ionised orona. It seemed however to bear little relevance to magnetic pper main sequence chemically peculiar (CP) stars, since the agnetic geometries of these stars were thought of and modlled in terms of low order potential fields. When Doppler maping, as introduced largely in its present form by Vogt et al. 1987), was extended to Zeeman Doppler mapping (ZDM) of P stars by Kochukhov & Piskunov (2002) the question of orce-free fields was completely overlooked, never to be disussed for a full 2 decades. Unnoticed by the ZDM commuity, Braithwaite & Spruit (2017) mentioned the fact that already t field strengths of 300 G, as in the weakest-field CP stars, agnetic pressure equals gas pressure at the photosphere, i.e. $= 8\,\pi\,P/B^2 = 1$. Substantially stronger fields in CP stars therefore must be force-free to be stable over years and decades. Some time later, Stift & Leone (2022) realised that the formulae underlying modern ZDM inversions were incompatible with the (very) strong fields observed in most target CP stars, as they violated the force-free condition.

The analysis of α^2 CVn by Kochukhov et al. (2002), based on Stokes IV only, employed a second-order multipolar expansion. Two years later, in the study of 53 Cam, Kochukhov et al. (2004a) abandoned this approach, assuming that unique recovery of a stellar magnetic map is possible with high quality observational data in $IQUV$, obviating any a priori assumptions about the magnetic geometry. The results were sobering, as the resulting field did not even prove divergence-free.

The decisive step away from force-free fields was made by Kochukhov & Wade (2010). Given the long-term (decades) stability of the magnetic fields of CP stars, potential fields (Jardine et al. 1999) should have been the choice on account of the absence of currents (see e.g. Winch et al. 2005). They however sidestepped potential fields, choosing the set of formulae given by Donati et al. (2006) instead, which include toroidal components and assume decoupling between radial and horizontal poloidal field components. As shown by Stift & Leone (2022), this added freedom (3 times the number of free parameters) violates the force-free condition.

From these considerations one far-reaching conclusion can be drawn: at variance with all previous claims concerning the alleged uniqueness of ZDM inversions, one is faced with the fact that there exist multiple solutions. Indeed, when magnetic maps yield good fits to all 4 Stokes parameters, although they are physically absolutely impossible, one cannot uphold the fiction of uniqueness. This logically leads to the next question, viz. whether ZDM offers even more spurious solutions for a given star. We will investigate this in detail in what follows.

2. The (Zeeman) Doppler mapping approach

It can be argued that the article by Vogt et al. (1987) (=VPH) started the field of Doppler mapping as carried out ever since. After an overview of first efforts to map equivalent widths of certain metal lines, the authors explain how to recover horizontal distributions of chemical abundances of various elements.

⋆ Instead of using the terms Zeeman Doppler imaging (ZDI) or magnetic Doppler imaging (MDI), we respect the original meaning given by Semel (1989) to ZDI, knowing that he always objected to its use for Doppler mapping.

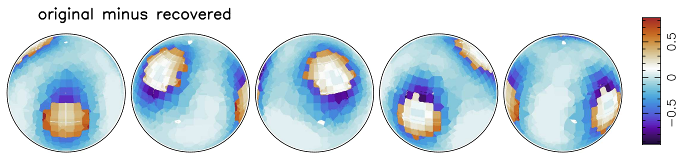

Fig. 1: Abundance differences between the original 2002 3-spot map and the recovered map.

The ill-posed inversion problem was constrained by the condition that the resulting map show maximum entropy. The inversion code was subjected to the famous "Vogtstar" test, based on "V O G T" written in black letters onto the white surface of a rotating sphere. These 4 letters were reasonably well retrieved by their algorithm, as were 7 black spots in a further test.

In the 1990s, this approach was developed further by, among others, Piskunov & Wehlau (1990) who had a look at the problem of mapping small and large rings. In contrast to VPH, they used Tikhonov regularisation (Tikhonov & Goncharsky 1987) instead of maximum entropy.

Please note that these tests deal with moderately complex patterns of horizontal inhomogeneities, but they do not include magnetic fields.

2.1. Constructing an artificial paradise

Zeeman Doppler mapping proper really took off with the series of 3 articles in 2002 by O. Kochukhov and N. Piskunov. In their second paper, Kochukhov & Piskunov (2002) (=KP) presented numerical experiments aimed at estimating the accuracy to which the geometry of the magnetic field could be established by inversion of Stokes $IQUV$ profiles, and the errors in the horizontal distribution of chemical abundances derived simultaneously by the same procedure.

The adopted magnetic topology is a combination of a centred dipole and a linear quadrupole. The Fe abundance distribution is characterised by

- either three largish Fe spots (two located at the equator, one at a northern latitude, all equally spaced in longitude);
- or a large single spot centred on the negative magnetic pole;
- an abundance contrast of 1.5 dex between the constant background and the monolithic spots.

These premises ensured reasonably plausible ZDM test results, but being in no way typical for magnetic CP stars, they cannot be used to confirm that the Doppler mapping code can actually recover "fairly complex magnetic and abundance distributions", as found for example in HR 3831 (Kochukhov et al. 2004b), HD 32633 (Silvester et al. 2015), or HD 119419 (Rusomarov et al. 2018), but also in 53 Cam (Kochukhov et al. 2004a).

The authors pointed out that Tikhonov regularisation does not impose any global constraints on the large scale field structure and that the resulting magnetic geometry therefore is not necessarily divergence-free. They professed however to "*believe* that in all real applications it is possible to find solutions within the uncertainty of the final magnetic map that would obey integral constraints imposed by Maxwell's equations" [verbatim, our emphasis]. There exists no proof for this supposition, nor was it followed up by further tests. Nevertheless, non-solenoidal magnetic maps were published more than a decade after 53 Cam by (Silvester et al. 2015).

2.2. Controversial claims

The authors claim that the *average* error in the recovered abundances is 0.04 dex and that the reconstruction of the field modulus is on *average* in agreement with the true map to within 9%.

Even the shortest glance at Fig. 5 of Kochukhov & Piskunov (2002) raises serious doubts as to these estimates, given the poor recovery of the spots and their surroundings. As can be seen in the difference map original vs. reconstructed spots (Fig.1), abundances are found to be locally in error by as much as ± 0.9 dex. It is important to realise that in these tests we are not dealing with errors in the usual statistical sense involving distributions, Gaussian or others. They are not caused by photon statistics nor by accumulated numerical errors in the inversion procedure. Even when exactly the same physics and the same numerical grids are employed in the forward synthesis of the Stokes $IQUV$ parameters used for input and in their inversion, and when the synthesised profiles are fitted to within 10^{-4} or better, we find *differences* (in a certain sense accurate ones), caused by the chosen regularisation functional in the correct execution of the ZDM algorithm (Stift & Leone 2017b), not *errors*. It follows that one cannot simply exclude "outliers" as done e.g. by Silvester et al. (2017) in the mapping of 49 Cam.

The authors maintain that the MDI/ZDM technique provides an *accurate*, self-consistent derivation of magnetic and abundance maps without positing a specific global magnetic geometry. Extrapolating straight from the simple magnetic maps discussed above and the somewhat modest results for the 3 monolithic spots, they declare that they "*believe* that the code can be successfully applied to the imaging of global stellar magnetic fields and abundance distributions of an arbitrary complexity" [verbatim, our emphasis].

One may legitimately disagree with the view that maps with local errors/differences of ± 0.9 dex can be called *accurate*. Yet even more debatable is the *belief* – not based on any published comprehensive test suites or for that matter on any tests at all involving truly complex magnetic topologies – that a method working reasonably well for field geometries defined by just 5 parameters would straightforwardly lead to unique results for magnetic topologies analysed as a function of about 400 free parameters, as e.g. in the cases of HD 32633 (Silvester et al. 2015) and HD 119419 (Rusomarov et al. 2018).

Once ZDM analyses had suggested chemical distributions of "stunning complexity" and magnetic geometries that could not be modelled by dipole plus quadrupole fields, extensive realistic tests should have followed the 2002 paper. The fact that the assessment of the credibility of almost the totality of published ZDM results and claims has been based exclusively on a *belief* enunciated back in 2002 is quite disconcerting.

2.3. More idealisations, more claims

There is just one other paper presenting numerical tests of Doppler mapping (Kochukhov 2017) (=K17) which however only deals with abundances, not with magnetic fields. This time the star features 4 spots, placed at −30°, 0°, +30° and +60° latitude, and equidistantly spaced in longitude (as back in 2002). The spots are no longer assumed monolithic but display a central part with a constant [Fe/H] = -2.5 and a smooth run from the inner radius of 15° to the outer radius of 30° where it merges into the constant background with [Fe/H] = -4.0. Regrettably, none of the spots in the test coincide with magnetic poles, a constellation found in a number of stars, e.g. in HR 3831 (Kochukhov et al. 2004b)(Ca, Ba), α^2 CVn

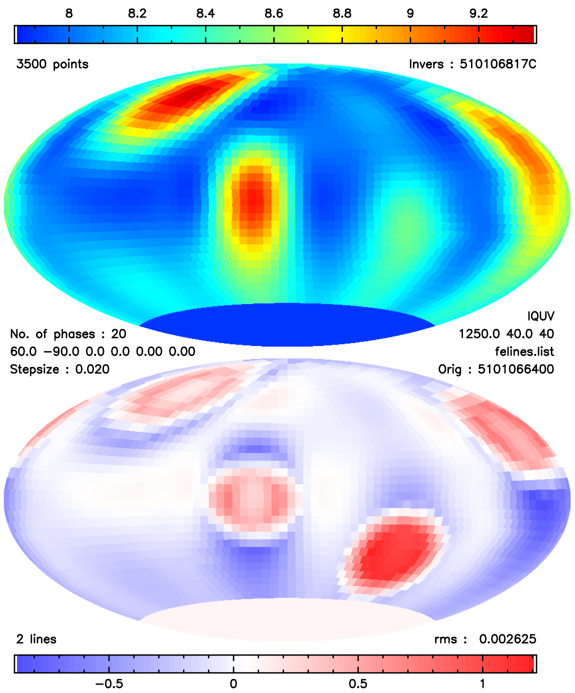

Fig. 2: Hammer projection of the recovered abundance map for the K17 4-spot test case, based on all 4 Stokes parameters at 20 phases, 20 mÅ spectral resolution and 2034 pixel spatial grid. **Top:** Unsmoothed ZDM output. **Bottom:** Difference map original minus inverted.

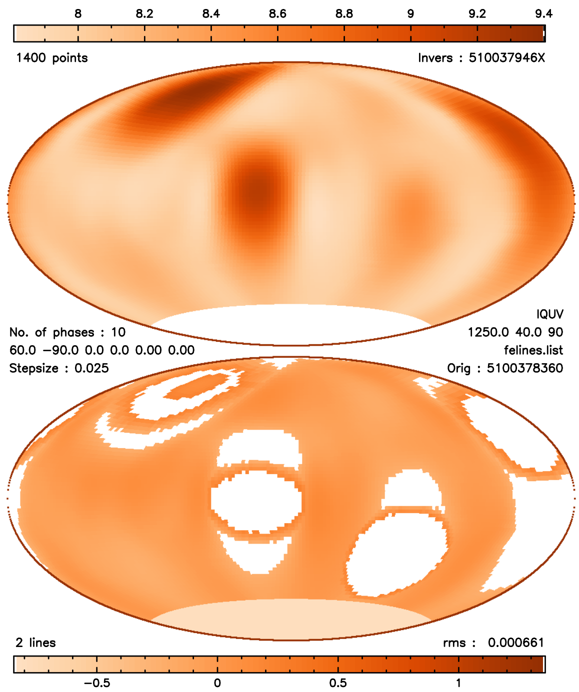

Fig. 3: Same as before, but now based on all 4 Stokes parameters at 10 phases, 25 mÅ spectral resolution and 10310 pixel spatial grid. **Top:** Unsmoothed ZDM output. **Bottom:** Difference map original minus inverted; the excluded pixels are displayed in white.

(Silvester et al. 2008)(Mg) or HD 125248 (Rusomarov et al. 2016)(Cr). The test is not applicable to the assessment of the pole-centred Li, O, Mn, Fe, and Nd spots in HD 3980, star credited with a field of 7 kG polar strength (Nesvacil et al. 2012).

There are conspicuous anomalies in the published test results. K17 clearly states that his test is based on a grid of 1876 approximately equal-area pixels in 38 latitude belts. This is in conflict with the spherical plots in his Fig. 3a which he labels as "true abundance maps" and with subsequent figures. The input maps for the synthesis of the $IQUV$ spectral line profiles to be inverted are unpixeled and in fact correspond exactly to the idealised spots at infinite resolution. The nature of the recovered maps is more difficult to ascertain: not only is there no trace whatsoever of the 1876 pixel inversion, but scans along the x- or y-coordinates reveal remarkably smooth RGB profiles with about 5 times the number of surface elements (and corresponding colours) compared to a 1876 pixel grid.

This immediately raises the question of how to interpret the inverted maps in Fig. 3b,c and later; they cannot be the product of heavy smoothing of a relatively coarse grid by an unspecified algorithm. Regardless of the method used to create these unpixeled maps from the 1876 pixel ZDM results, differences with respect to the original maps are inevitably ironed out to some degree. The published maps thus no longer exhibit the true differences/errors. In the present case the situation is further complicated by the ambiguous nature of the "original". Is it the 1876 pixel map or the idealised infinite-resolution map? We cannot choose between these scenarios without more information or detailed modelling.

There is an additional unknown to the appreciation of the difference maps: what has happened to abundance value outliers that according to Silvester et al. (2017) normally comprise 2–12% of all the surface elements. Have they been excluded prior to smoothing? And how in this context do we have to interpret the approach applied by K17? In view of the overabundance regions (the 4 spots) covering 27% of the stellar surface, he considered the 75th percentile of the unsigned difference map as a relevant measure of the maximum inversion errors. What happened to the 470 pixels exceeding the maximum error, have they been excluded, replaced or interpolated?

In order to elucidate these problems, we decided to revive the ZDM code presented in (Stift & Leone 2017b) and to simulate the K17 test for the 2 Fe lines. Adopting a 2034 pixel grid in 40 latitude belts, the same spot geometries and the same dipole magnetic field as in K17, we synthesised $IQUV$ profiles for various numbers of phases up to 20 and at spectral resolutions of 20 mÅ, 25 mÅ, and 50 mÅ. We did the same for a 10310 pixel grid in 90 latitude belts. Fig.2 shows the result of the inversion for the K17 fwo-line case. In sharp contrast to the K17 plots, the pixeled nature of the inverted map is clearly visible, and so is the pixeled difference map between the true input map and the true, unsmoothed recovered map. The $IQUV$ spectral line profiles at 3500 wavelength points (altogether 14000 values) have been fitted to better than $3\,10^{-3}$.

Our results are in serious disagreement with the "maximum error" estimates given by K17, viz. 0.13 to 0.15 dex, once the uppermost quartile of the "errors" is discarded. As pointed out above, in this kind of test we are dealing with *differences* that Fig.2 shows to be almost 8 times larger than claimed by K17. The abundance of large parts of the southern spot is underestimated by 1 dex and more, whereas the spot at +30° is stretched in the N-S direction beyond the equator, with the northern part quite underabundant, the southern part overabundant by up to 0.8 dex. Let us emphasise that these maximum differences change very little (±0.1 dex) with number of phases, spectral resolution and size of spatial grid.

Certainly, the poor performance of the K17 DM inversion regarding in particular the spot at $-30°$ latitude is not propitious to the credibility of the abundance maps published for several CP stars with inclination near 60°. In HD 3831, Na, Si, Ca, Mn, Fe, and Y spots are located well to the south of the equator (Kochukhov et al. 2004b). In HD 133880, the most conspicuous spots of Mg, Si, Ti, Cr, and Fe are likewise found in the partly hidden southern hemisphere (Kochukhov et al. 2017). Ca, Cr, and Eu in 49 Cam, fare similarly, but this time north of the equator (Silvester et al. 2017).

We also would like to add that the decision by K17 to throw out the pixels belonging to the uppermost "error" quartile is lacking a statistical foundation, making it impossible to assess the effects of this choice on the results. Taking our inversions, we would have to exclude those 470 pixels from the maps in Fig.2 that differ by more than 0.305 dex from the original. We illustrate the outcome with a 10310 pixel inversion (Fig.3) – 5600 wavelength points in $IQUV$ fitted to 6.6 10−4 – where the exclusion of 2577 pixels differing by more than 0.235 dex is seen to severely affect all spots (and their surroundings), rendering them virtually invisible.

2.4. Outdated and skewed numerical tests

As discussed above, the two numerical tests presented so far are unsatisfactory as both the adopted magnetic field geometries and the horizontal abundance inhomogeneities are overly simple. For the 2002 paper this can be considered acceptable since at that time people had only vague ideas on what kind of magnetic field geometry could be expected and horizontal chemical structures would look like. Not so for the K17 paper which in no way reflects the wealth and variety of maps published between 2002 and 2017. Please keep in mind that the entirety of ZDM data constitutes a fairly homogeneous set, having been analysed exclusively with the codes of O. Kochukhov; publications have without exception been written by himself or closely supervised.

Take a handful of examples. For 53 Cam Kochukhov et al. (2004a) found a magnetic field topology considerably more complex than any low order multipolar expansion. Kochukhov & Wade (2010) and later Silvester et al. (2014a) showed that the observed Stokes profiles of α^2 CVn could not be fitted with a dipole plus quadrupole geometry. This was repeated by (Silvester et al. 2015) for HD 32633 and by (Rusomarov et al. 2016) for HD 125248.

Kochukhov et al. (2004b) claimed a "stunning complexity" of the horizontal Fe abundance pattern in HR 3831. The surface distribution of Ca is fairly complex too, without large areas of overabundance or underabundance. In the case of HD 125248 (Rusomarov et al. 2016). the very strong surface abundance inhomogeneities also do not bear any resemblance to the 4-spot pattern adopted by K17.

For HR 3831, Kochukhov et al. (2004b) have assumed a dipole field of 2.5 kG polar strength. Small Li and Ba spots are concentrated in areas at or very close to the magnetic poles. In the case of HD 3980 (Nesvacil et al. 2012) with an estimated polar field strength of 7 kG, spots of Mn, Pr, O, Li, Nd have been found at the poles. Tests with large spots on the other hand are exclusively located away from the poles and are therefore misleading.

3. Assessing published, ZDM based maps

An interesting dichotomy emerges. On one side, two highly idealised numerical tests (Kochukhov & Piskunov 2002; Kochukhov 2017) seem to indicate "maximum errors" in the recovery of horizontal abundance distributions of just 0.13 to 0.15 dex. On the other side, what we prefer to call "maximum differences" can reach values in excess of 1 dex as demonstrated by Stift & Leone (2017b) and again in the present study, this time in a rerun of the 4-spot test of K17. Is it possible to obtain supplementary information that would allow us to choose between these conflicting estimates and provide a sound base for the realistic assessment of the validity and accuracy of abundance and magnetic field maps?

It is always challenging to compare results arrived at with different codes, but most fortunately there is a solution that entirely avoids these problems: stick solely to inversions effected with the INVERS family of codes. ZDM analyses carried out over the years with largely identical methods and codes by O. Kochukhov and a small group of researchers under his supervision will guarantee a high degree of internal consistency. Providentially for our endeavour, a number of magnetic CP stars have been analysed several times. Separate maps of the same star have been derived from data sets that can be identical, just slightly differing or completely distinct; published over the years or decades, they ensure meaningful comparisons that are not influenced by varying approaches to ZDM or by different codes.

Be aware that we do not look at the plausibility of the physical and mathematical assumptions underlying the ZDM procedure. As mentioned in section1, Stift & Leone (2022) have already convincingly shown that a number of ZDM analyses are violating the laws of magneto-hydrodynamics. Here we are exclusively interested in the interagreement between the ZDM results obtained and presented by O. Kochuknov and his group. Ideally, differences between abundance maps of the same star should not be markedly in excess of the values given by K17, i.e. 0.15 dex. For the magnetic field one would then expect errors of roughly 10-20%. For early critical reviews of ZDM analyses we refer to Stift et al. (2012) and to Stift & Leone (2017a,b).

4. Data extraction and its caveats

In contrast to the situation 10-20 years ago, high quality colour plots in online publications now make it possible to numerically recover detailed ZDM results on abundances and magnetic fields. Code of just a handful of lines written in a popular interpreted computing language helps to obtain data from which – after a few coordinate transformations – detailed maps can be established (see e.g. Stift & Leone 2022).

There are some stumbling blocks that render the process of data recovery challenging, if not sometimes impossible:

- In the worst case (Silvester et al. 2008), there is no colour wedge to convert colours to abundances or field strengths.
- Image compression artefacts in plots of poor quality – as encountered in Nesvacil et al. (2012)) – can always seriously complicate data recovery, but in the particular case of the first ZDM study of α^2 CVn (Kochukhov et al. 2002), all our efforts have failed.
- Maps are not always accompanied by vital data such as the inclination i of the rotational axis. Only after more than a decade, 49 Cam had a value of i published for the first time, disappointingly without any mention of the previously used, differing value(s).
- Isolines in plots of magnetic field components (radial, meridional, azimuthal) can display strange behaviour at the borders as in Fig. 6 of Silvester et al. (2015). Unfortunately we cannot simply ignore this and extract the underlying data

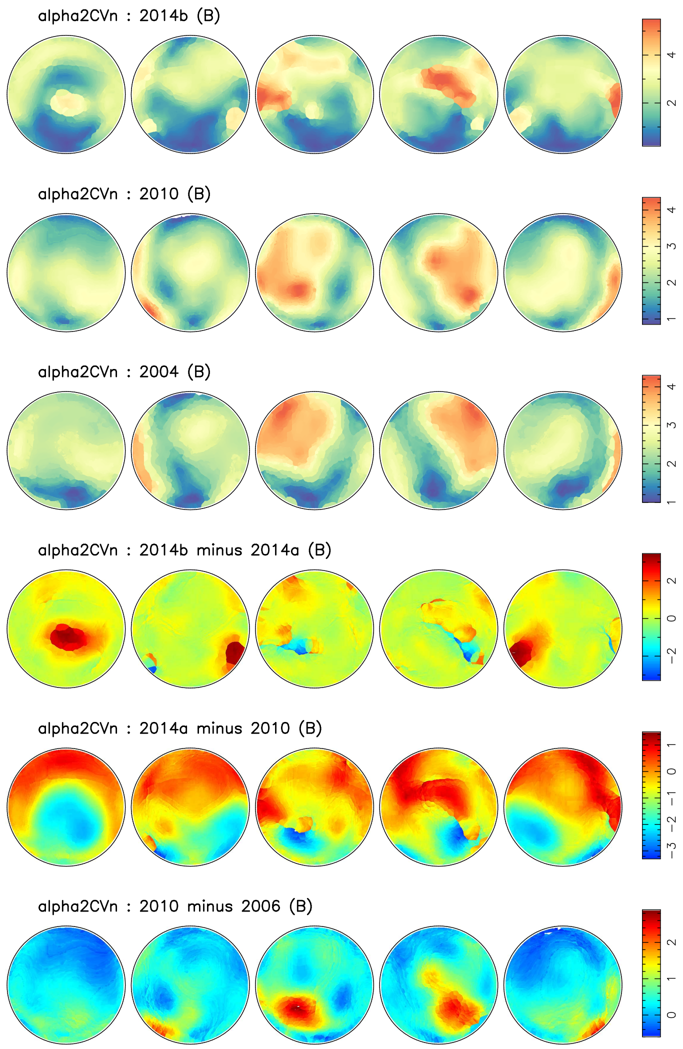

Fig. 4: (**top 3**) α^2 CVn : Absolute magnetic field strengths at 5 equidistant phases, recovered from 3 articles. (**bottom 3**) Differences in field strengths at 5 equidistant phases for combinations between 5 maps.

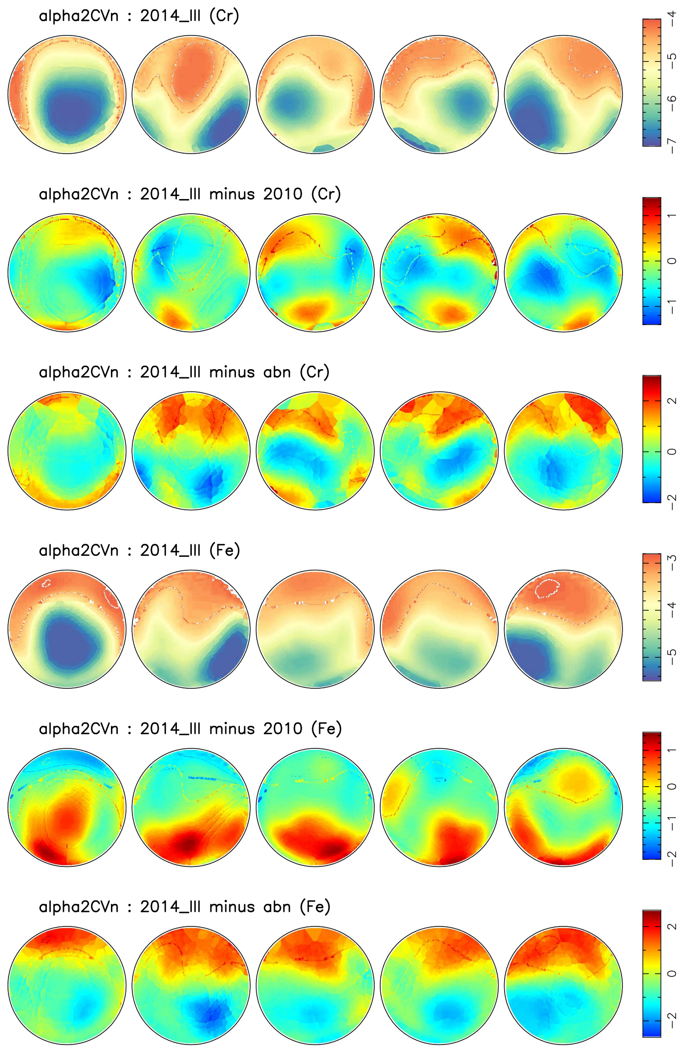

Fig. 5: (**top 3**) α^2 CVn : Cr abundance at 5 equidistant phases and abundance differences for combinations between 3 maps. (**bottom 3**) Same as before, but for Fe abundances.

(which are needed for example in the modelling of stratified abundances for the star), because discontinuities occur in all 3 components with small gaps of unspecified size around phase 0.0/1.0.

– Slide 19, entitled "Stability of Ap-star spots", in a lecture given by O. Kochukhov 2014 in Besançon, (lesia.obspm.fr/MagMas/cartography2014/kochukhov_lecture.pdf) displays Si spots on 56 Ari that seem to move in longitude. In the same lecture, on slide 39, we find the original 2002 test case exhibiting the same behaviour. Numerous other instances turn up in articles published between 2002 and 2014, among them Kochukhov et al. (2003), Piskunov (2008), and Lüftinger et al. (2010).

5. A detailed discussion of 5 stars

In the following we shall discuss 5 stars for which multiple abundance and/or magnetic maps could be found in journals, proceedings, and web pages. There are altogether 12 different maps of field strength and 20 different maps of Cr, Fe, and Nd available for our study. Very few of the contributions to proceedings have ever been cited in subsequent papers, refereed or not, so the following discussion will give, for the first time, an overview of these maps, their variety, their disparateness and their chronology.

5.1. α^2 CVn (HD 112413)

α^2 CVn is the star with the most published maps. The first magnetic map based on full Stokes *IQUV* spectral line profiles is due to Kochukhov (2004), followed shortly after by Kochukhov (2006). Different results were presented by Kochukhov & Wade (2010), superseded by 2 papers a couple of years later (Silvester et al. 2014a,b). Chromium and iron maps can be found in Kochukhov & Wade (2010), Silvester et al. (2014b). and in an undated plot (file name alpha2CVn_abn.jpg) that seems to correspond largely to the 2002 B/W plot, available until several years ago at an Uppsala University homepage. This plot is also found in a 2014 Uppsala lecture note on inverse problems.

The striking dissimilarities between the field maps stand out visually and do not require sophisticated tools to quantify (Fig.4). Still, only differential maps fully reveal the differences which in the case 2014b minus 2014a run from −3.3 kG to +3.4 kG. The 2014a maps are based on 5 iron and 3 chromium lines in Stokes $IQUV$, to which 2 oxygen and chlorine lines in Stokes IV were added for the 2014b maps. It is somewhat surprising that this modest increase in the number of lines would lead to such a substantial revision of the magnetic map, but similar discrepancies have already previously occurred when the addition of 2 Cr lines to the 2010 analysis (based on 5 Fe lines and 1 Cr line) led to the 2014a map that differs from 2010 by −3.3 kG to +1.5 kG.

It is true that the resolution (R=35.000) of the MuSiCoS spectra underlying the 2010 maps is no match to the resolution of ESPaDOnS and Narval (R=65.000). However, as Silvester et al. (2014a) have pointed out, there is good agreement between the two data sets, and the overall phase sampling is quite similar. It seemed to them therefore "reasonable" to compare field maps derived with the same atomic line list. The huge differences in field strengths were not pointed out at that time.

Abundances hardly fare any better than field strengths (Fig.5). The 2014b chromium map featuring a 3.0 dex contrast differs by −1.4 to +1.4 dex from the undated map and by −2.0 to +3.0 dex from the 2010 map. Fe displays a contrast of 2.7 dex, and differences are large again, reaching −2.0 to +1.5 dex in the first case, −2.7 to +2.7 dex in the second.

These results contradicting the conclusions by K17 that there is little difference between maps based on 5 and on 2 lines, we may therefore legitimately question the alleged quality of the recovered maps. Discussion of the other 4 stars will further nourish our doubts.

We note that none of the results published before 2010 has ever been cited in later articles, only the 2010 paper being mentioned in 2014. More than 10 years later, the mechanisms leading to conflicting field and abundance maps still remain unexplored. Additional evidence for multiple solutions will be presented below.

5.2. 49 Cam (HD 62140)

In all, 2 different Fe maps and 4 different magnetic field maps of 49 Cam can be found in three papers and one slide published between 2007 and 2017. Silvester et al. (2007) offered the very first iron and field maps, followed a few years later (Silvester et al. 2011) by a field map based on 14 spectra (compared to 8 spectra back in 2007). Finally, Silvester et al. (2017) came up with new Fe and field maps, derived from 19 spectra. With just 5 spectra added, one would expect little difference between the 2017 and the 2011 field maps, but see below. There exists one more field map, contained in a presentation by J. Silvester at the STARS2016 Conference (https://www.star.uclan.ac.uk/stars2016/slides/Silvester.pdf). Given the time stamp, this map ought to be based on the same 19 spectra as the 2017 map.

The magnetic map in the arXiv preprint and the largely identical corresponding final article (Silvester et al. 2008) differs substantially from the map presented in the contribution to the meeting "Astronomical Polarimetry 2008" (Silvester et al. 2011). where no mention is made of the 2007/2008 map.

The reasons for the discrepancies between the 2007/2008 and the 2011 results – both called "(very) preliminary" by the authors – have never been discussed. In 2017, no mention at all can be found of the 2007 and 2011 papers, a fact that also holds true for the 2016 slide. Whereas a mere 8 spectra might fall short of providing the necessary input for reliable abundance or field maps, 14 spectra should be sufficient, and 19 spectra can be expected to plainly confirm the 2011 findings.

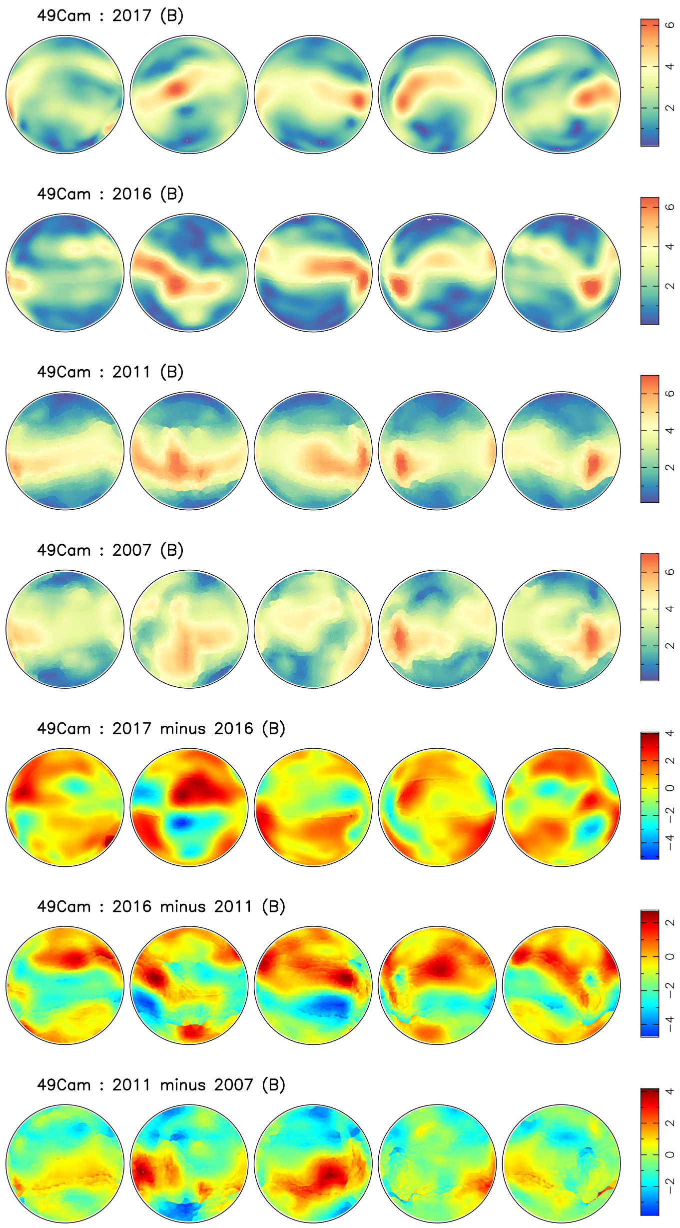

Fig. 6: (**top 4**) 49 Cam : Absolute magnetic field strengths at 5 equidistant phases. (**bottom 3**) Differences in field strengths at 5 equidistant phases for combinations between the 4 maps.

The abundance contrast in the 2 iron maps is almost identical at ≈ 4, 5 dex. but local differences 2017 minus 2007 are found to range between −3.6 dex and +4.2 dex (Fig.7). Similarly, all 4 field maps (Fig.6). agree on an approximate contrast of 6 to 7 kG. Yet, concerning the magnetic geometry, there is no agreement at all: differences in field strength 2017 minus 2011 cover a staggering −5.6 kG to +3.4 kG (remember the 19 vs. 14 spectra !). 2016 minus 2011 is only slightly less spectacular with −4.7 kG and +2.7 kG. For 2011 minus 2007 we find extrema of −3.8 kG and +4.2 kG.

This brings us to a curious feature in the remarkable evolution of the magnetic maps. Having adopted – as probably in all previous analyses – an inclination of 85° of the rotational axis for his 2016 slide, Silvester et al. (2017) suddenly changed this

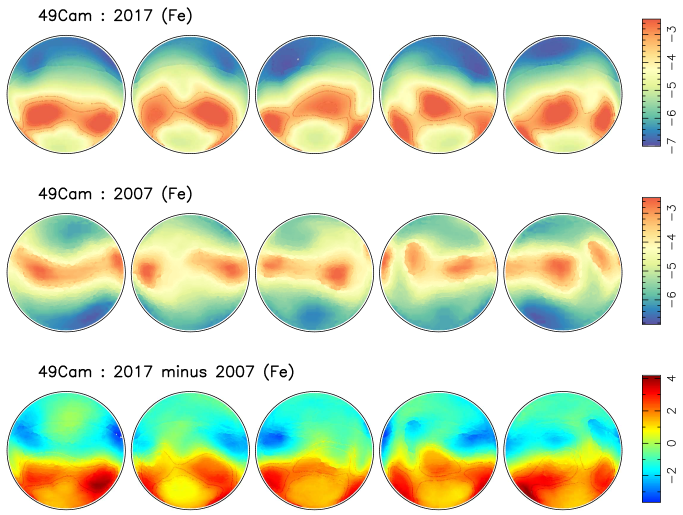

Fig. 7: (**top 2**) 49 Cam : Fe abundances at 5 equidistant phases. (**bottom**) Fe abundance differences between the 2 maps.

b

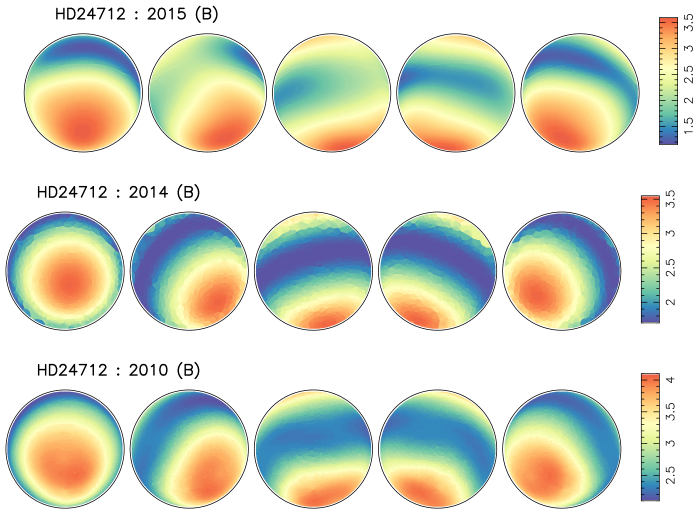

Fig. 9: (HD 24712 : Absolute magnetic field strengths at 5 equidistant phases.

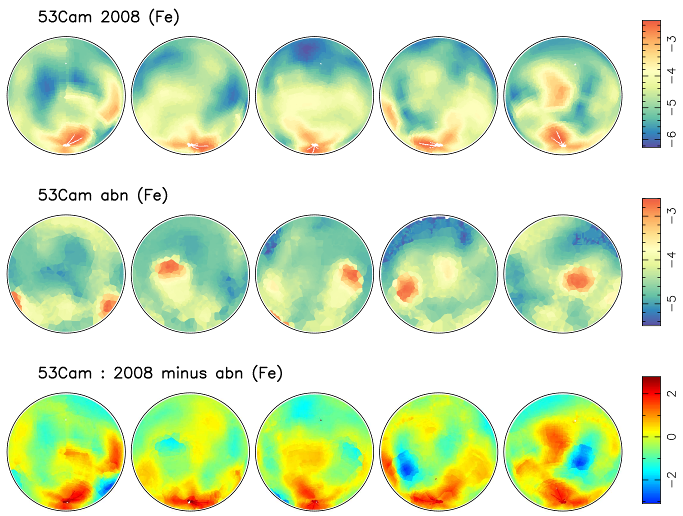

Fig. 8: (**top 2**) 53 Cam : Fe abundances at 5 equidistant phases. (**bottom**) Fe abundance differences between the 2 maps.

to 120°. Of course adjustments to the inclination are not unusual in ZDM: over the years, the inclination of α^2 CVn has changed by 10°, by 17° for HD 24712. One could also argue with the 2002 and 2017 numerical tests that showed magnetic inversions to be relatively insensitive to moderate uncertainties ($\approx 10 - 20°$) in the inclination angle. Here however the adjustment in the angle reaches perplexing 25 degrees, and it turns out that both 2017 field and Fe maps bear precious little resemblance to their predecessors from 2007 to 2016.

We think that the fascinating history of the 49 Cam ZDM maps should not slip into oblivion because of lack of citations in the definitive study. The discrepancies are sufficiently troubling to warrant an in-depth discussion. As stated in the previous subsection, a convincing case can be made again for the existence of multiple solutions to ZDM inversions.

5.3. 53 Cam (HD 65339)

53 Cam was the first CP star to be observed and analysed in Stokes $IQUV$ (Kochukhov et al. 2004a). The recovered magnetic topology exhibits field strengths of up to 27 kG. It is neither divergence-free, nor does it obey the force-free condition. It would thus seem that this star is of no use for our present study. However, there are 2 maps of iron abundances available, based on the same set of observations. In addition to the B/W plots in the 2004 paper as republished in colour by Piskunov (2008), there existed an undated plot (53Cam_abn.jpg), similarly to alpha2CVn_abn.jpg available on an Uppsala University homepage.

Here too, the maps prove discordant, both with respect to the location of the small spots and (to a lesser extent) to the contrast which amounts to 4.0 dex (2008) and 2.9 (undated/abn) respectively. The differences 2010 minus abn range from −3.1 dex to +2.8 dex. Fig.8 clearly reveals the size of the discrepancies. Once again, the optimistic error estimates given by K17 turn out unrealistic when it comes to real-life ZDM.

We note that Kochukhov & Wade (2010) mention an "updated" magnetic map of 53 Cam, produced in the same way as for α^2 CVn, that allegedly does not differ *appreciably* from the *average* field topology inferred back in 2004. The vagueness of this wording does not help in assessing the actual differences, and so is their Fig. 12 which mainly reveals a fairly strong (but unphysical) toroidal field. The relative strengths of the poloidal field components differ manifestly from those in Fig. 11 of the 2004 paper, and so do the toroidal components. We strongly encourage the authors to publish the "updated" magnetic and abundance maps which should clarify the true nature, structure and extent of the differences 2010 vs. 2004.

5.4. HD 24712

HD 24712 is a roAp star with a fairly low rotational velocity of $v \sin i = 5.6\,\mathrm{km\,s^{-1}}$ which is not conducive to good spatial resolution with respect to latitude. Relying on Stokes IV only and adopting an inclination angle of $i = 137°$, Lüftinger et al. (2010) determined a dipolar magnetic geometry and derived abundance

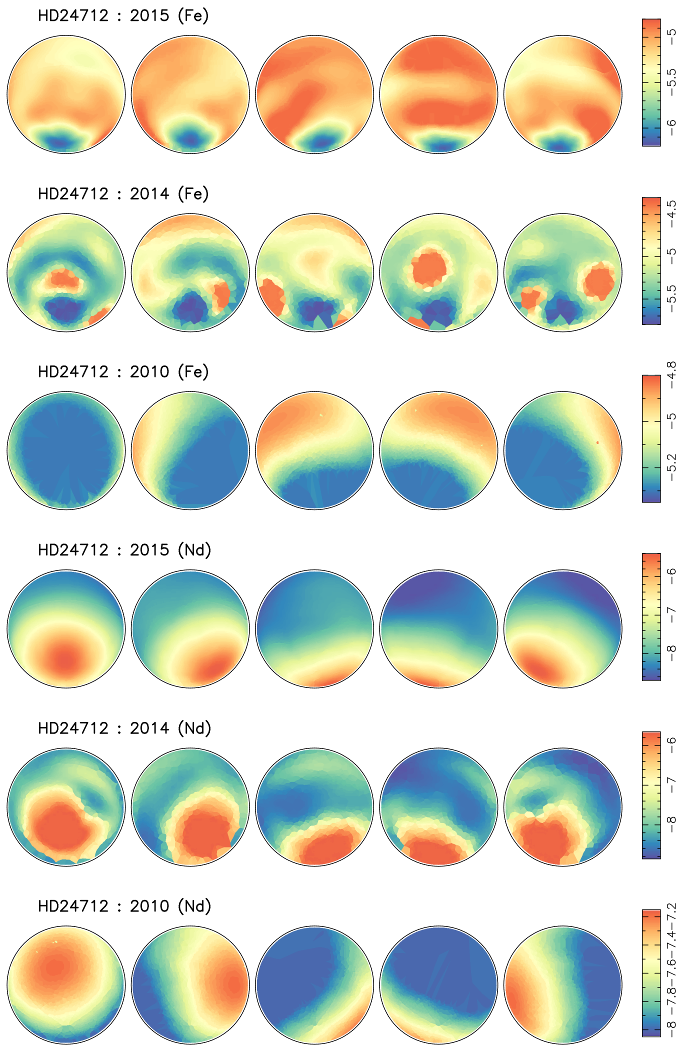

Fig. 10: (**top 3**) HD 24712 : Fe abundances at 5 equidistant phases.
(**bottom 3**) Nd abundances at 5 equidistant phases.

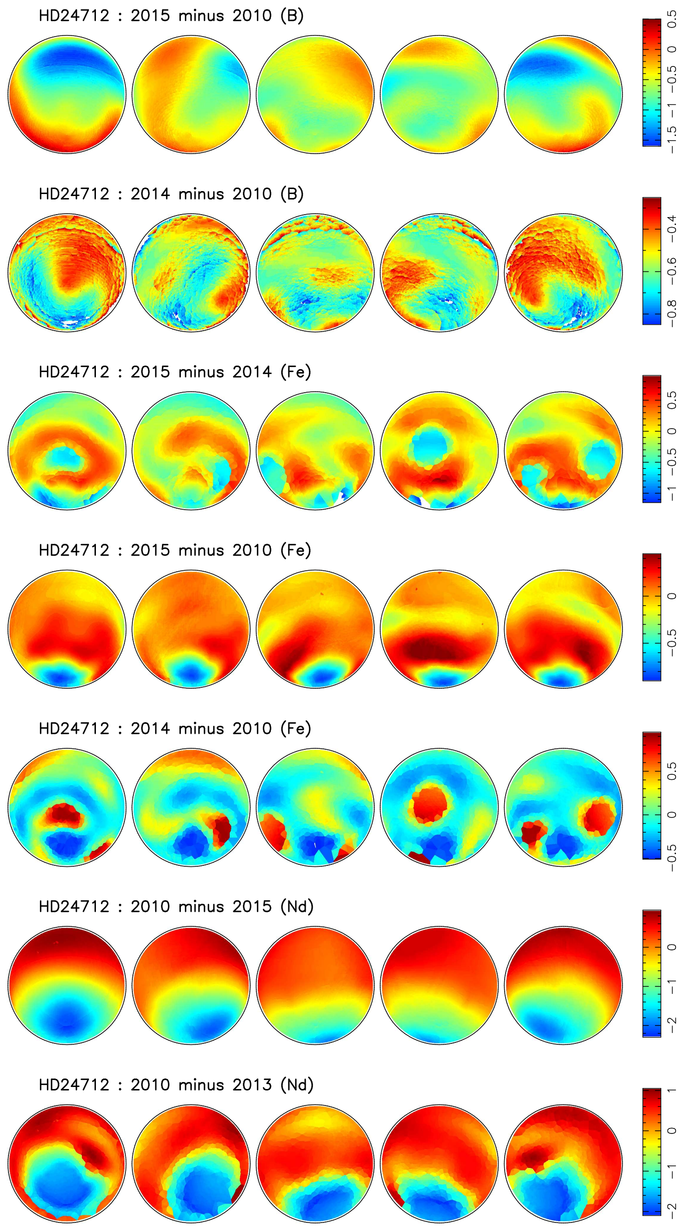

Fig. 11: (**top 2**) HD 24712 : Field strength differences between combinations of 3 maps at 5 equidistant phases. (**middle 3**) Same as above, but for Fe abundances. (**bottom 2**) Same as above, but for Cr abundances.

maps for a number of chemical elements, with emphasis on Fe and Nd. Iron was found to concentrate near the equator over a considerable interval in longitude, with neodymium almost at antiphase near −30° south. Inversion of high-quality (R=114.000) HARPSpol spectra in Stokes $IQUV$ however led Rusomarov et al. (2014) to results quite at variance with these 2010 maps. Fe now featured 2 high contrast spots of moderate size and an underabundant south polar region, whereas Nd could hardly be considered in antiphase. The position of the Nd spot had shifted by at least 30° in latitude as can be seen in Fig.10.

Using the same observations at 16 phases as in the 2014 paper, but with $i = 120°$, Rusomarov et al. (2015) arrived at yet another magnetic topology and corresponding set of Fe and Nd abundance maps. The magnetic field is no longer exclusively dipolar, but higher order modes contribute only weakly (their Fig. 5 and our Fig.9). As to the new Fe and Nd maps, the authors maintain that they *"confirm"* the 2010 findings and that they also show some *"details"* the previous study lacked. A look at Fig.10 however reveals *"details"* of more than 2 dex in the Nd maps, and of almost 1 dex for Fe. No mention is made of the huge differences with respect to the 2014 paper or of this paper at all. No explanation is given for the large shift in latitude of the Nd spot, the authors preferring a detailed discussion of a possible marginal shift in longitude instead.

Given the low rotational velocity, there is very little difference in the signal of a spot at −30° and a spot much further to the south. Looking at the quality (or the lack thereof) of the fits to the Stokes profiles in 2014 as compared to 2015, one must not be surprised to encounter multiple solutions to the inverse problem.

5.5. HD 3980

In a paper on Doppler mapping of HD 3980, Nesvacil et al. (2012) present abundance maps with stellar spots where sili-

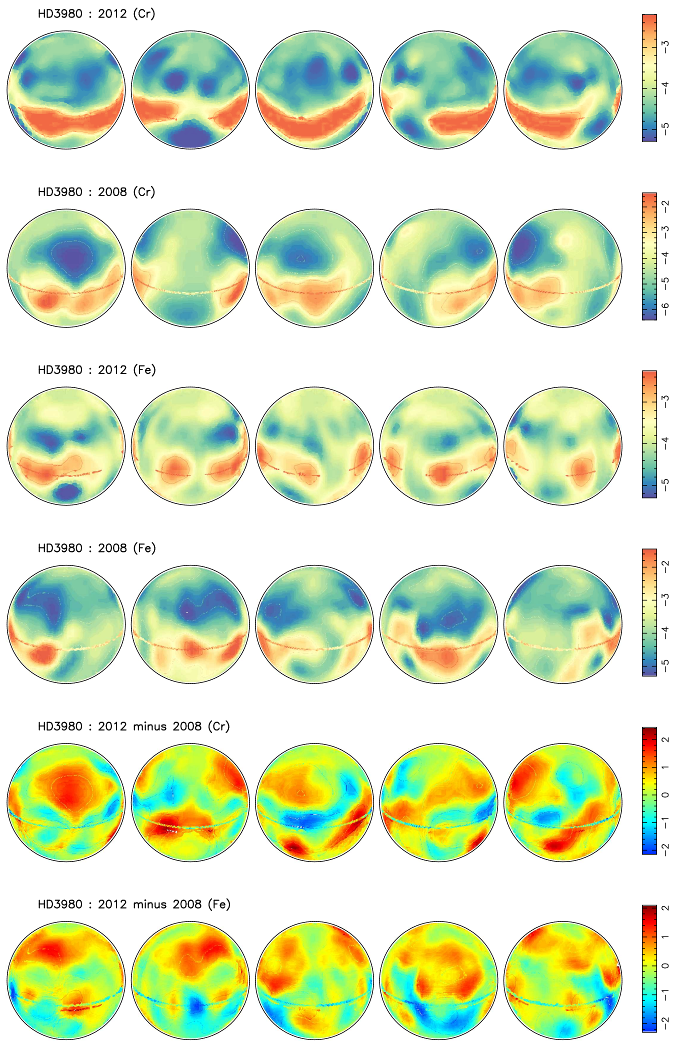

Fig. 12: (**top 2**) HD 3980 : maps of Cr, dated 2012 and 2008; (**middle 2**) the same as above, but for Fe; (**bottom 2**) abundance differences between the 2012 and 2008 maps.

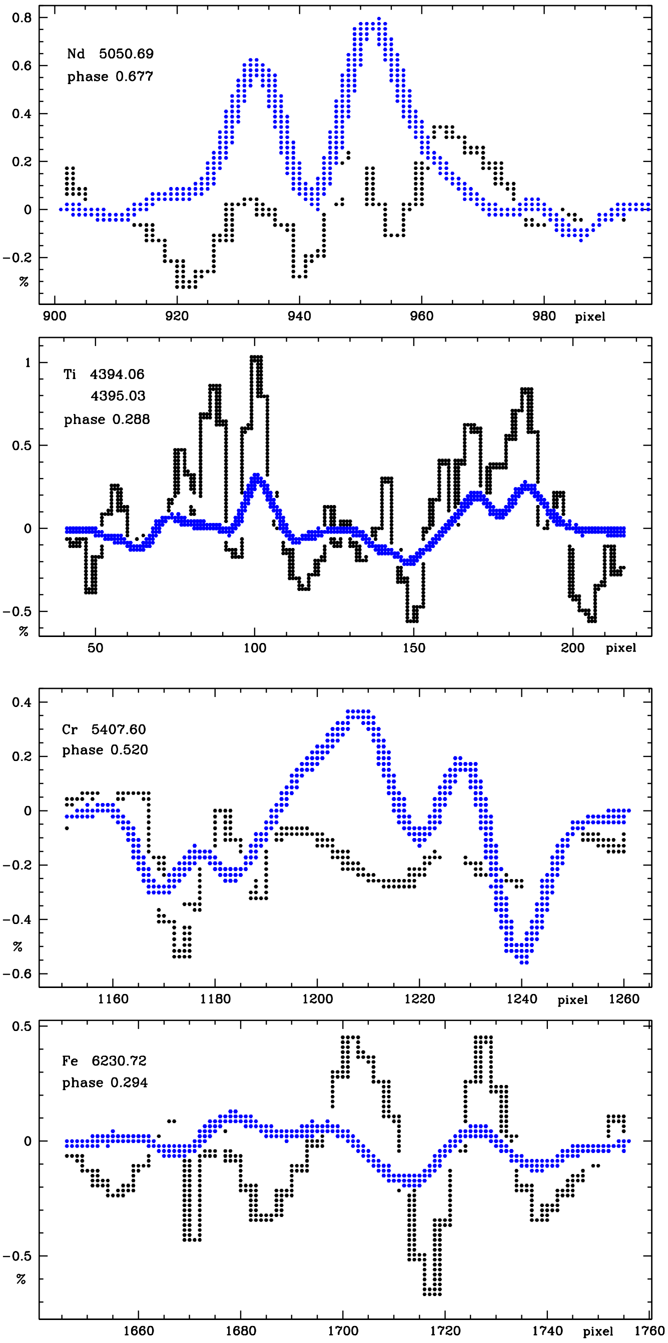

Fig. 13: 49 Cam : synthetic (blue) vs. observed (black) Stokes Q profiles of the elements Ti, Cr, Fe, and Nd according to Silvester et al. (2017).

con is as abundant as hydrogen throughout the atmosphere, oxygen and manganese abundant as helium. The intractable physical problems the existence of such spots would imply have been discussed in some detail by Stift & Leone (2017a,b). Note that a magnetic field of an estimated 7 kG polar strength has not been taken into account in the DM inversion.

HD 3980 would at first glance appear an unlikely object for our error analysis, but there exists another set of Doppler maps presented by Obbrugger et al. (2008) which allows us to carry out comparisons between the respective Cr and Fe maps. The fits to the observed profiles in the 2008 paper are of comparable quality to those in 2012, but as Fig.12 reveals, the abundance maps do not agree at all, differences reaching ±2 dex. In the light of the manifest physical inadequacies in the inversions and the conflicting Doppler maps, there is no foundation to the claim made by Nesvacil et al. (2012). that "a lack of up-to-date theoretical models" is responsible for the fact that no correlation is found between the abundance patterns in HD 3980 and predictions of diffusion theory.

6. Conclusions

In view of the evidence gathered in the course of this investigation, the gloomy outlook on ZDM as presented by Stift & Leone (2017b) is corroborated:

- It is an indisputable fact that ZDM inversions admit of multiple solutions, even when built on exactly the same set of high-quality observations.
- One has to accept that even magnetic topologies which are physically impossible – in particular strong fields that do not obey the force-free condition – can reproduce observed

Stokes $IQUV$ profiles quite well. Unphysical solutions also suffer non-uniqueness.
- The tests published by Kochukhov & Piskunov (2002) and Kochukhov (2017) do not guarantee the plausibility of published ZDM maps, including the stars discussed above.
- Not a single published test has explored inversions of truly complex magnetic geometries and horizontal abundance distributions. Tests never went beyond second-order magnetic topologies and idealised spots or rings.
- In the presence of multiple maps that yield good fits to the observations, the choice between (widely) different horizontal abundance structures and magnetic geometries is quite arbitrary, often depending on the choice of lines and phases where substantial misfits are allowed. 49 Cam is a telling example.

It follows that present-day ZDM results cannot provide constraints of any kind to the modelling of atomic diffusion. There remain also a few as yet unanswered questions:

- In ZDM solutions, can we really tolerate systematic discrepancies synthetic vs. observed profiles that attain levels by far exceeding the observational errors, as illustrated for 4 chemical elements in Fig.13?
- How does vertical stratification of chemical elements affect ZDM results? "Putting diffusion theory to the test" as done by Kochukhov & Ryabchikova (2018), assumes the most perfectly symmetric magnetic geometry, a centred dipole with its axis in the equatorial plane of the star. What is the relevance of this test to stars such as α^2 CVn, 49 Cam, 53 Cam, HD 32633, HD 75049, HD 119419, HD 125428, HD 133880?

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