

Cramér-Rao Bound Optimization for Fluid Antenna-Empowered Integrated Sensing and Uplink Communication System

Yuan Guo, Wen Chen, Qingqing Wu, Yang Liu, and Qiong Wu

Abstract—Integrated sensing and communication (ISAC) is a promising solution for the future sixth-generation (6G) system. However, classical fixed-position antenna (FPA) ISAC systems fail to fully utilize spatial degrees of freedom (DoFs), resulting in limited gains for both radar sensing and communication functionalities. This challenge can be addressed by the emerging novel fluid antenna (FA) technology, which can pursue better channel conditions and improve sensing and communication performances. In this paper, we aim to minimize the Cramér-Rao bound (CRB) for estimating the target's angle while guaranteeing communication performance. This involves jointly optimizing active beamforming, power allocation, receiving filters, and FA position configurations, which is a highly non-convex problem. To tackle this difficulty, we propose an efficient iterative solution that analytically optimizes all variables without relying on numerical solvers, i.e., CVX. Specifically, by leveraging cutting-edge majorization-minimization (MM) and penalty-dual-decomposition (PDD) methods, we develop a low-complexity algorithm to solve the beamformer configuration problem containing the fractional and quartic terms. Numerical simulation results demonstrate the effectiveness and efficiency of our proposed algorithm, highlighting significant performance improvements achieved by employing FA in the ISAC system.

Index Terms—Integrated sensing and communication (ISAC), fluid antenna (FA), Cramér-Rao bound (CRB), low-complexity algorithm.

I. INTRODUCTION

Recently, with the development of emerging sixth-generation (6G) applications, including smart homes, vehicle-to-everything (V2X) communications, and environmental monitoring, the demand for high-quality communication and precise sensing capabilities has been steadily increasing. In this context, integrated sensing and communication (ISAC) is widely regarded as a promising technique to meet these demands through the joint design of communication and sensing functions [1]. It has attracted significant interest from both academia and industry. By sharing spectrum resources and utilizing a unified hardware platform, ISAC aims to significantly enhance spectrum efficiency and reduce hardware costs and system complexity. Furthermore, many recent studies that extensively document dual-functional waveform designs

for joint radar sensing and communication can be found in [1]–[4], along with their references.

Nonetheless, traditional ISAC systems equipped with fixed-position antennas (FPAs) cannot fully exploit spatial diversity, which results in a loss of beamforming gains for both communication and sensing tasks. To address this limitation, an innovative concept is fluid antenna system (FAS) was proposed in [5]–[6]. This novel FAS architecture which utilizes flexible antenna technology such as liquid-based antenna [7]–[8], mechanically movable antenna [9]–[10], and etc., to adjust antenna's position flexibly within a predefined spatial area. By fully leveraging the additional degrees of freedom (DoFs) provided by the reconfiguration of wireless channels, the FA structure aims to enhance communication capabilities [9]–[10]. The significant potential of the FA technology to improve communication performance has been extensively demonstrated in recent studies [9]–[16], including uplink communication [10], [11], interference network [12], nonorthogonal multiple access (NOMA) [13], multicast communication [14], along with their references.

A. Related Works

Due to the significant benefits of FA technique, extensive research focused on its integration into ISAC systems to substantially enhance both communication capacity and sensing ability, e.g., [17]–[34]. The literature [17] designed a port selection strategy for the FA system to reduce transmit power. A FA-aided ISAC system was researched in [18]. The study aimed to maximize downlink (DL) communication rate while satisfying the sensing beampattern gain requirements. The authors of [19] first considered deploying the FA into unmanned aerial vehicle (UAV)-enabled ISAC system to support low-altitude economy (LAE) applications. Furthermore, an efficient algorithm, which jointly optimizes beamforming and the positions of the FA, has been proposed to promote both throughput capacity and beamforming gain. The work [20] focused on minimizing the Cramér-Rao bound (CRB) for estimating the direction of arrival (DoA) of a target in a DL communication ISAC system assisted by FA technology, and demonstrated that FA technology can significantly improve sensing performance. The paper [21] employed FA technology to enhance both communication and sensing performances in a reconfigurable intelligent surface (RIS)-assisted ISAC system. Besides, the numerical results demonstrated that FA efficiently reduces multiuser interference and mitigates the impact of

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multi-path effects. The authors of [22] adopted the deep reinforcement learning framework to jointly optimize the antenna port locations and precoding design in a multiuser multiple-input multiple-output (MIMO) downlink ISAC system. In [23], under the impact of the nature of the dynamic radar cross-section (RCS), the authors investigated the transmit power minimization problem while assuring the individual quality-of-service (QoS) requirement for communication and sensing in an FA-aided ISAC system. The authors of [24] adopted the emerging FA architecture in the ISAC system with low-altitude airborne vehicles and showed it could remarkably improve the communication capacity under the constraint of sensing signal-to-noise ratio (SNR). The work [25] firstly deployed the FA into a bi-static radar system with the objective of maximizing both the weighted communication rate and sensing mutual information (MI). Furthermore, the positions of the FA can be analytically updated by leveraging the Karush-Kuhn-Tucker (KKT) conditions. The authors of [26] aim to maximize the sensing signal-to-interference-plus-noise ratio (SINR) while assuring the minimum SINR of mobile users in an FA-assisted bi-static ISAC system. The paper [27] employed the FA to guard the communication security of RIS-aided ISAC network effectively, and proposed a two-layer penalty-based algorithm to solve the non-convex communication rate maximization problem. [28] utilized the FA to suppress the self-interference in a monostatic full-duplex (FD) ISAC system, and further promote the weighted sum of communication rate and sensing MI. In the FA-enabled FD ISAC system, the authors of [29] investigated the transmit power consumption minimization problem while considering discrete candidate positions for FA. Besides, in [30], the authors considered maximizing the weighted sum of sensing and communication rates (WSR) in an FA-aided near-field FD ISAC system and proposed an antenna position matching (APM) algorithm for reducing the antenna movement distance. The work [31] demonstrated that the FA could remarkably enhance the communication capacity for both DL and uplink (UL) users in the networked FD ISAC system. The paper [32] incorporated the FA into an MIMO ISAC system to maximize the signal-clutter-noise ratio (SCNR) while satisfying communication SINR requirements. The work [33] investigated the DL and UL sum-rate maximization problem in a FAS-assisted FD ISAC system. Lately, a novel optimization algorithm based on a two-timescale framework for FA-enabled ISAC systems, tackling key issues such as slow antenna movement speed, dynamic RCS variation, and imperfect channel state information (CSI), was developed in [34].

B. Motivations and Contributions

As seen above, although a substantial amount of research has explored joint beamforming and position optimization for FA-assisted ISAC systems, most of these works [17]–[27], [32]–[34] have focused on integrated sensing and *downlink* communication systems. Research on FA-enabled integrated sensing and *uplink* communication systems remains limited and is still emerging. Besides, recent investigations [28]–[31] have considered the FA-aided FD ISAC system containing DL

and UL communication. However, the sensing performance metrics used in [28]–[31], which include MI [28] and the sensing SINR [29]–[31], are challenging to quantify explicitly. In contrast, the well-known CRB is a widely accepted metric for estimating sensing performance, providing a lower bound on the variance of unbiased parameter estimators with closed-form expressions. Motivated by these gaps, our study focuses on an FA-aided uplink ISAC system aiming to enhance both its communication and estimation capabilities. The contributions of this paper are detailed as follows:

- This paper investigates the joint optimization of beamforming and the positions of FAs in a UL ISAC system enhanced by FA technology to boost communication and estimation performances. Our objective is to minimize the CRB for estimating the target's DoA while simultaneously assuring the sum-rate of all UL users via designing BS probing beamforming, UL users' power allocation, receiving filters and FAs' position coefficients. To the best of our knowledge, this problem has rarely been considered in the existing literature, e.g., [17]–[34].
- By introducing splitting variables and leveraging the penalty dual decomposition (PDD) [35] framework combined with the majorization-minimization (MM) [36] method, we develop a low-complexity algorithm to address the challenging beamformer configuration problem containing *fractional* and *quartic* terms. To the best of our knowledge, this has not been studied in the existing literature [17]–[34]. Besides, we also derive the closed-form solution for the FAs' position coefficients.
- Furthermore, we propose an iteration optimization algorithm that analytically optimizes all variables using convex optimization techniques to effectively tackle the non-convex CRB minimization problem, without relying on any numerical solvers such as CVX [37]. This approach is notably rare in the existing literature, e.g., [17]–[34].
- Extensive numerical results are presented to demonstrate the significant benefits of employing FAs to enhance estimation accuracy in the UL ISAC system. Additionally, our proposed PDD-based algorithm (i.e., Alg. 1) showcases the superior efficiency compared to the method presented in [38] (i.e., the Schur complement).

The rest of the paper is organized as follows. Section II will introduce the model of the UL ISAC system assisted by FA and formulate the joint beamforming and position coefficients design problem. Section III will propose a low-complexity iterative solution to tackle the proposed problem. Section IV and Section V will present numerical results and conclusions of the paper, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As shown in Fig. 1, we consider an FA-aided UL communication ISAC system comprising one base station (BS) equipped with uniform linear array (ULA) with N_t transmit FPAs and N_r receive FAs, K single-antenna uplink mobile

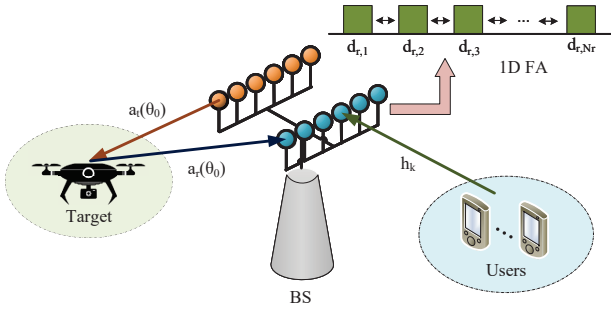


Fig. 1. An FA-aided uplink ISAC system.

users, and one point-like sensing target¹. In this system, BS sends probing waveform to estimate the target's angle parameter and receives the information from UL users and echo signal from the target simultaneously. During the whole procedure, all mobile users operate in uplink mode and transmit information symbols to the BS. For convenience, the sets of users, BS' transmit antennas and receive antennas are denoted as $\mathcal{K}$, $\mathcal{N}_t$ and $\mathcal{N}_r$, respectively.

Furthermore, the FAs, which are based on mechanically [9]–[10] or liquid-based [7]–[8] elements, can move within a local region in real time. The antenna positioning vectors (APVs) of BS transmit antennas and receive antennas are given by $\mathbf{d}_t \triangleq [d_{t,1}, d_{t,2}, \dots, d_{t,N_t}]^T$ and $\mathbf{d}_r \triangleq [d_{r,1}, d_{r,2}, \dots, d_{r,N_r}]^T$ within the given line segment of length d_{max} , respectively.

According to the far-field wireless channel model [9]–[10], changing the positions of FAs will influence the complex path coefficient, while does not affect the angle of departure (AoD), the angle of arrival (AoA), and the amplitude of the complex path coefficient. Moreover, we assume that the channel is quasi-static [9]–[10]. Let $L_{t,k}$ and $L_{r,k}$ denote the total number of transmit and receive channel paths at the BS from the k -th UL user, respectively. For the i -th transmit path between BS and user k , the AoD is given as $\theta_{k,i}^t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. For the i -th receive path between BS and user k , the elevation and azimuth AoAs are represented as $\theta_{k,i}^r \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\phi_{k,i}^r \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, respectively.

The transmit field response matrix of all N_r FAs from the BS to the k -th user is represented as

$$\mathbf{H}_{0,k}(\mathbf{d}_r) = [\mathbf{h}_{1,k,1}, \mathbf{h}_{1,k,2}, \dots, \mathbf{h}_{1,k,N_t}] \in \mathbb{C}^{L_{t,k} \times N_r}, \quad (1)$$

where the field-response vector is $\mathbf{h}_{1,k,n}(d_{r,n}) \triangleq [e^{j\frac{2\pi}{\lambda}d_{r,n}\sin(\theta_{k,1}^t)}, e^{j\frac{2\pi}{\lambda}d_{r,n}\sin(\theta_{k,2}^t)}, \dots, e^{j\frac{2\pi}{\lambda}d_{r,n}\sin(\theta_{k,L_{t,k}}^t)}]^T \in \mathbb{C}^{L_{t,k} \times 1}$, and λ is the wavelength. $\mathbf{h}_{0,k} \in \mathbb{C}^{L_{r,k} \times 1}$ is defined as the receive field-response vector from the BS to the k -th user. Therefore, the channel vector between the BS and the k -th UL user is given by

$$\mathbf{h}_k^H = \mathbf{h}_{0,k}^H \mathbf{\Sigma}_k \mathbf{H}_{0,k}(\mathbf{d}_r) \in \mathbb{C}^{1 \times N_r}, \quad (2)$$

where $\mathbf{\Sigma}_k \in \mathbb{C}^{L_{r,k} \times L_{t,k}}$ is denoted as the response of all transmit and receive paths from the BS to the k -th UL user.

¹Future work will consider an FA-aided ISAC system containing RCS fluctuations, multipath, and clutter, as well as accounting for propagation delays and Doppler effects.

Besides, the transmit and receive steering vectors of the BS towards the sensing target are respectively represented by

$$\mathbf{a}_{t,1} \triangleq [e^{j\frac{2\pi}{\lambda}d_{t,1}\sin(\theta_0)}, e^{j\frac{2\pi}{\lambda}d_{t,2}\sin(\theta_0)}, \dots, e^{j\frac{2\pi}{\lambda}d_{t,N_t}\sin(\theta_0)}]^T, \quad (3)$$

$$\mathbf{a}_{r,1} \triangleq [e^{j\frac{2\pi}{\lambda}d_{r,1}\sin(\theta_0)}, e^{j\frac{2\pi}{\lambda}d_{r,2}\sin(\theta_0)}, \dots, e^{j\frac{2\pi}{\lambda}d_{r,N_r}\sin(\theta_0)}]^T, \quad (4)$$

where θ_0 denotes the target's DoA. Furthermore, the complex path coefficients of the transmit and receive steering vectors are denoted as β_t and β_r , respectively. Then, the transmit and receive line-of-sight (LoS) channels are respectively given as

$$\mathbf{a}_t(\theta_0) = \beta_t \mathbf{a}_{t,1}(\theta_0), \mathbf{a}_r(\theta_0) = \beta_r \mathbf{a}_{r,1}(\theta_0), \quad (5)$$

The uplink signal transmitted at time slot l of the k -th UL user is represented as

$$x_k^u[l] = \sqrt{q_k} s_k^u[l], \forall k \in \mathcal{K}, \quad (6)$$

with $s_k^u[l]$ and q_k representing the information symbol and transmission power of the k -th UL user, respectively. Besides, $s_k^u[l]$ are assumed as mutually uncorrelated and each has zero mean and unit variance.

Let $\mathbf{W} \in \mathbb{C}^{N_t \times N_t}$ denote the beamforming matrix for radar sensing. The probing signal transmitted in the l -th time slot is given by

$$\mathbf{x}[l] = \mathbf{W} \mathbf{s}_r[l], \quad (7)$$

where $\mathbf{s}_r[l] \in \mathbb{C}^{N_t \times 1}$ denotes radar probing signal and has zero mean and covariance matrix $\mathbb{E}\{\mathbf{s}_r[l]\mathbf{s}_r^H[l]\} = \mathbf{I}_{N_t}$.

The received signal at the BS at time slot l is written as

$$\mathbf{y}[l] = \sum_{k=1}^K \mathbf{h}_k x_k^u[l] + \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{x}[l] + \mathbf{n}[l], \quad (8)$$

where $\alpha \in \mathbb{C}$ denotes the RCS coefficient and $\mathbf{n}[l] \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{N_r})$ denotes the additive white Gaussian noise (AWGN) at the BS receiver.

To decode UL users' information $\{s_k^u[l]\}$, the BS adopts the linear filter $\mathbf{u}_k \in \mathbb{C}^{N_r \times 1}$ to post-process the received signal. Therefore, the output of the k -th filter can be given as

$$y_k[l] = \mathbf{u}_k^H (\sum_{k=1}^K \mathbf{h}_k x_k^u[l] + \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{x}[l] + \mathbf{n}[l]), \forall k \in \mathcal{K}. \quad (9)$$

Then, the SINR of UL user k can be obtained as

$$\text{SINR}_k = \frac{q_k |\mathbf{u}_k^H \mathbf{h}_k|^2}{\sum_{i \neq k} q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \|\mathbf{u}_k^H \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2}, \quad (10)$$

and the achievable rate of k -th user is given as

$$R_k = \log(1 + \text{SINR}_k). \quad (11)$$

After BS recovers the communication information at the BS receiver, it will adopt the successive interference cancellation (SIC) technique [39]–[41] to eliminate² the uplink communication signals from the received signals (8). Therefore, the signal for estimating target's location information at the time slot l is written as

$$\mathbf{y}_r[l] = \alpha \mathbf{A}(\theta_0) \mathbf{x}[l] + \mathbf{n}[l]. \quad (12)$$

²This work adopts the perfect SIC assumption as an upper performance bound. The case of imperfect SIC will be studied in future work.

where $\mathbf{A}(\theta_0) \triangleq \mathbf{a}_r(\theta_0)\mathbf{a}_t^H(\theta_0)$. By stacking L coherent time slots, the received echo signals can be given as

$$\mathbf{Y}_r = \alpha \mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r + \mathbf{N}, \quad (13)$$

where $\mathbf{S}_r \triangleq [\mathbf{s}_r[1], \mathbf{s}_r[2], \dots, \mathbf{s}_r[L]]$ and $\mathbf{N} \triangleq [\mathbf{n}[1], \mathbf{n}[2], \dots, \mathbf{n}[L]]$. Moreover, when L is sufficiently large, the sample covariance matrix of $\mathbf{S}_r$ can be approximated by

$$\frac{1}{L} \mathbf{S}_r \mathbf{S}_r^H \approx \mathbf{I}. \quad (14)$$

Note that CRB is a lower bound of any unbiased estimator for describing the parameter estimation accuracy. In the following, we will derive the closed-form CRB expression for target's DoA θ_0 . Firstly, we define the target parameters as $\boldsymbol{\zeta} \triangleq [\theta_0, \alpha_R, \alpha_I]^T$, where $\alpha_R = \Re\{\alpha\}$, $\alpha_I = \Im\{\alpha\}$ and $\tilde{\boldsymbol{\alpha}} \triangleq [\alpha_R, \alpha_I]^T$.

According to [42], the Fisher information matrix (FIM) for the estimation of $\boldsymbol{\zeta}$ is given by

$$\mathbf{F} = \mathbb{E} \left[\frac{\partial \ln(f(\mathbf{Y}_r|\boldsymbol{\zeta}))}{\partial \boldsymbol{\zeta}} \left(\frac{\partial \ln(f(\mathbf{Y}_r|\boldsymbol{\zeta}))}{\partial \boldsymbol{\zeta}} \right)^H \right], \quad (15)$$

where $f(\mathbf{Y}_r|\boldsymbol{\zeta})$ denotes the conditional probability density function (PDF) of $\mathbf{Y}_r$ given $\boldsymbol{\zeta}$

Next, we will show the explicit expression of the FIM in (15). The joint conditional distribution of $\mathbf{Y}_r$ given $\boldsymbol{\zeta}$ can be written as

$$f(\mathbf{Y}_r|\boldsymbol{\zeta}) = \frac{1}{\pi^{N_r L} |\sigma^2 \mathbf{I}_{N_r L}|} \exp \left\{ - \frac{\|\mathbf{Y}_r - \alpha \mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r\|_F^2}{\sigma^2} \right\}. \quad (16)$$

Therefore, the log-likelihood function for estimating $\boldsymbol{\zeta}$ based on the observation $\mathbf{Y}_r$ is given by

$$\begin{aligned} \ln(f(\mathbf{Y}_r|\boldsymbol{\zeta})) &= -N_r L \ln(\pi \sigma^2) - \frac{\|\mathbf{Y}_r\|_F^2}{\sigma^2} \\ &\quad - \frac{|\alpha|^2 \|\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r\|_F^2}{\sigma^2} + \frac{2\Re\{\text{Tr}(\alpha^* \mathbf{S}_r^H \mathbf{W}^H \mathbf{A}(\theta_0)^H \mathbf{Y}_r)\}}{\sigma^2}, \end{aligned} \quad (17)$$

According to [43] and based on (15) and (17), $\mathbf{F}$ can be directly given as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\theta_0 \theta_0} & \mathbf{F}_{\theta_0 \alpha} \\ \mathbf{F}_{\theta_0 \alpha}^H & \mathbf{F}_{\alpha \alpha} \end{bmatrix} \in \mathbb{C}^{3 \times 3}. \quad (18)$$

Besides, the elements of $\mathbf{F}$ are given as

$$\mathbf{F}_{\theta_0 \theta_0} = \frac{2L|\alpha|^2}{\sigma^2} \text{tr}(\mathbf{A}_1 \mathbf{W} \mathbf{W}^H), \quad (19)$$

$$\mathbf{F}_{\theta_0 \alpha} = \frac{2L}{\sigma^2} \Re\{\alpha^* \text{tr}(\mathbf{A}_2 \mathbf{W} \mathbf{W}^H)[1, j]\} \quad (20)$$

$$\mathbf{F}_{\alpha \alpha} = \frac{2L}{\sigma^2} \text{tr}(\mathbf{A}_3 \mathbf{W} \mathbf{W}^H) \mathbf{I}_2, \quad (21)$$

respectively, where the newly introduced coefficients are defined in (22), respectively, with $\dot{\mathbf{a}}(\theta_0)$ denoting the derivative of $\mathbf{a}(\theta_0)$ with respect to (w.r.t.) θ_0 . The details of the derivation of the FIM $\mathbf{F}$ can be seen in Appendix A.

Next, the CRB for estimating the angle θ_0 , which corresponds to the first diagonal unit of $\mathbf{F}^{-1}$, can be given as

$$\text{CRB}_{\theta_0} = [\mathbf{F}^{-1}]_{1,1} \quad (23)$$

$$= \frac{\sigma^2}{2L|\alpha|^2 \left(\text{tr}(\mathbf{A}_1 \mathbf{W} \mathbf{W}^H) - \frac{|\text{tr}(\mathbf{A}_2 \mathbf{W} \mathbf{W}^H)|^2}{\text{tr}(\mathbf{A}_3 \mathbf{W} \mathbf{W}^H)} \right)}.$$

B. Problem Formulation

Our objective is to minimize the CRB of estimating the angle information θ_0 via jointly optimizing the transmit beamformer $\mathbf{W}$, the linear filters $\{\mathbf{u}_k\}$, the UL users' transmit power $\{q_k\}$ and the FA position coefficients $\{\mathbf{d}_r\}$. The optimization problem³ can be mathematically formulated as

$$(P0) : \min_{\mathbf{W}, \{\mathbf{u}_k\}, \{q_k\}, \mathbf{d}_r} \text{CRB}_{\theta_0} \quad (24a)$$

$$\text{s.t.} \sum_{k=1}^K R_k \geq R_t, \quad (24b)$$

$$\|\mathbf{W}\|_F^2 \leq P_{BS}, \quad (24c)$$

$$0 \leq q_k \leq P_{u,k}, \forall k \in \mathcal{K}, \quad (24d)$$

$$d_{r,1} \geq 0, d_{r,N_r} \leq d_{max}, \quad (24e)$$

$$d_{r,n} - d_{r,n-1} \geq d_{min}, n = 2, 3, \dots, N_r, \quad (24f)$$

where R_t is the predefined sum-rate threshold of all UL users, P_{BS} and $P_{u,k}$ denote the maximum transmission power of the BS and the k -th UL user, respectively, d_{max} represents the maximum moving distance of BS's FA and d_{min} is the minimum distance between adjacent antennas for avoiding antenna coupling effect.

III. ALGORITHM DESIGN

A. Problem Reformulation

Firstly, since minimizing CRB_{θ_0} is equivalent to maximizing $\text{tr}(\mathbf{A}_1 \mathbf{W} \mathbf{W}^H) - \frac{|\text{tr}(\mathbf{A}_2 \mathbf{W} \mathbf{W}^H)|^2}{\text{tr}(\mathbf{A}_3 \mathbf{W} \mathbf{W}^H)}$, we turn to solve the following problem:

$$(P1) : \max_{\substack{\mathbf{W}, \{\mathbf{u}_k\}, \\ \{q_k\}, \mathbf{d}_r}} \text{tr}(\mathbf{A}_1 \mathbf{W} \mathbf{W}^H) - \frac{|\text{tr}(\mathbf{A}_2 \mathbf{W} \mathbf{W}^H)|^2}{\text{tr}(\mathbf{A}_3 \mathbf{W} \mathbf{W}^H)} \quad (25a)$$

s.t. (24b) – (24f).

Besides, to make the problem (P1) more tractable, we will use the fractional programming (FP) method [44] to equivalently transform the sum-rate constraint (24b). Firstly, by leveraging the Lagrangian dual reformulation and introducing auxiliary variables $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_K]^T$, the original sum-rate constraint (24b) can be written in (26). And then, by exploiting the quadratic transform and introducing the auxiliary variables $\boldsymbol{\omega} = [\omega_1, \dots, \omega_K]^T$, equation (26) can be further converted to (27).

Based on the above transformation, the problem (P1) can be reexpressed as

$$(P2) : \max_{\substack{\mathbf{W}, \{\mathbf{u}_k\}, \\ \{q_k\}, \mathbf{d}_r}} \text{tr}(\mathbf{A}_1 \mathbf{W} \mathbf{W}^H) - \frac{|\text{tr}(\mathbf{A}_2 \mathbf{W} \mathbf{W}^H)|^2}{\text{tr}(\mathbf{A}_3 \mathbf{W} \mathbf{W}^H)} \quad (28a)$$

$$\text{s.t.} R_2 \geq R_t, \quad (28b)$$

$$\|\mathbf{W}\|_F^2 \leq P_{BS}, \quad (28c)$$

$$0 \leq q_k \leq P_{u,k}, \forall k \in \mathcal{K}, \quad (28d)$$

³The two-time scale optimization problem will be investigated in future work.

$$\begin{aligned}
\mathbf{A}_1 &\triangleq \sum_{n=1}^{N_r} \left(\frac{2\pi}{\lambda} d_{r,n} \cos(\theta_0) \right)^2 \mathbf{a}_t(\theta_0) \mathbf{a}_t^H(\theta_0) + \left(\sum_{n=1}^{N_r} -j \frac{2\pi}{\lambda} d_{r,n} \cos(\theta_0) \mathbf{a}_t(\theta_0) \dot{\mathbf{a}}_t^H(\theta_0) \right) \\
&+ \left(\sum_{n=1}^{N_r} j \frac{2\pi}{\lambda} d_{r,n} \cos(\theta_0) \dot{\mathbf{a}}_t(\theta_0) \mathbf{a}_t^H(\theta_0) \right) + N_r \dot{\mathbf{a}}_t(\theta_0) \dot{\mathbf{a}}_t^H(\theta_0), \\
\mathbf{A}_2 &\triangleq \left(\sum_{n=1}^{N_r} -j \frac{2\pi}{\lambda} d_{r,n} \cos(\theta_0) \right) \mathbf{a}_t(\theta_0) \mathbf{a}_t^H(\theta_0) + N_r \dot{\mathbf{a}}_t(\theta_0) \mathbf{a}_t^H(\theta_0), \mathbf{A}_3 \triangleq N_r \mathbf{a}_t(\theta_0) \mathbf{a}_t^H(\theta_0).
\end{aligned} \tag{22}$$

$$\mathbf{R}_1 = \sum_{k=1}^K \left(\log(1 + \gamma_k) - \gamma_k + \frac{(1 + \gamma_k) q_k |\mathbf{u}_k^H \mathbf{h}_k|^2}{\sum_{i=1}^K q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \|\mathbf{u}_k^H \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2} \right) \tag{26}$$

$$\mathbf{R}_2 = \sum_{k=1}^K \left(\log(1 + \gamma_k) - \gamma_k + 2\sqrt{(1 + \gamma_k)} \Re\{\omega_k^* \sqrt{q_k} \mathbf{u}_k^H \mathbf{h}_k\} - |\omega_k|^2 \left(\sum_{i=1}^K q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \|\mathbf{u}_k^H \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2 \right) \right) \tag{27}$$

$$d_{r,1} \geq 0, d_{r,N_r} \leq d_{max}, \tag{28e}$$

$$d_{r,n} - d_{r,n-1} \geq d_{min}, n=2, 3, \dots, N_r. \tag{28f}$$

In the next, we propose the block coordinate ascent (BCA)-based algorithm [45] to tackle the problem (P2).

B. Optimizing Auxiliary Variables

According to the derivation of FP method [44], the auxiliary variables γ and ω can be updated by analytical solutions that are respectively given as follows

$$\gamma_k^* = \frac{q_k |\mathbf{u}_k^H \mathbf{h}_k|^2}{\sum_{i \neq k} q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \|\mathbf{u}_k^H \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2}, \tag{29}$$

$$\omega_k^* = \frac{\sqrt{1 + \gamma_k} (\sqrt{q_k} \mathbf{u}_k^H \mathbf{h}_k)}{\sum_{i=1}^K q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \|\mathbf{u}_k^H \alpha \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2}. \tag{30}$$

C. Updating The BS Beamformer

In this subsection, when other variables are given, we investigate the update of the BS beamformer $\mathbf{W}$. By defining the new coefficients as follows

$$\mathbf{B}_2 \triangleq \sum_{k=1}^K (\mathbf{I}_{N_t \times N_t} \otimes (|\alpha|^2 \mathbf{a}_t \mathbf{a}_r^H \mathbf{u}_k \mathbf{u}_k^H \mathbf{a}_r \mathbf{a}_t^H)), \tag{31}$$

$$\begin{aligned}
c_1 &\triangleq \sum_{k=1}^K \left(\log(1 + \gamma_k) - \gamma_k + 2\sqrt{(1 + \gamma_k)} \Re\{\omega_k^* \sqrt{q_k} \mathbf{u}_k^H \mathbf{h}_k\} \right. \\
&\left. - |\omega_k|^2 \left(\sum_{i=1}^K q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + \sigma^2 \|\mathbf{u}_k^H\|_2^2 \right) \right) - R_t, \mathbf{w} \triangleq \text{vec}(\mathbf{W}), \\
\mathbf{B}_3 &\triangleq \mathbf{I}_{N_t \times N_t} \otimes \mathbf{A}_1, \mathbf{B}_4 \triangleq \mathbf{I}_{N_t \times N_t} \otimes \mathbf{A}_2, \mathbf{B}_5 \triangleq \mathbf{I}_{N_t \times N_t} \otimes \mathbf{A}_3.
\end{aligned}$$

Based on the above transformation, the optimization problem w.r.t. $\mathbf{w}$ can be rewritten as

$$(\text{P3}) : \min_{\mathbf{w}} \frac{(\mathbf{w}^H \mathbf{B}_4 \mathbf{w})(\mathbf{w}^H \mathbf{B}_4^H \mathbf{w})}{\mathbf{w}^H \mathbf{B}_5 \mathbf{w}} - \mathbf{w}^H \mathbf{B}_3 \mathbf{w} \tag{32a}$$

$$\text{s.t. } \mathbf{w}^H \mathbf{B}_2 \mathbf{w} \leq c_1, \tag{32b}$$

$$\mathbf{w}^H \mathbf{w} \leq P_{BS}. \tag{32c}$$

Obviously, the problem (P3) contains both the fractional and the quartic terms in the objective function (32a) is difficult to solve.

To address the challenge mentioned, we introduce auxiliary variables $\mathbf{f}$ and b to decouple the objective function (32a). Thus, the problem (P3) can be equivalently written as

$$(\text{P4}) : \min_{\mathbf{w}, \mathbf{f}, b} \frac{(\mathbf{w}^H \mathbf{B}_4 \mathbf{f})(\mathbf{f}^H \mathbf{B}_4^H \mathbf{w})}{b} - \mathbf{w}^H \mathbf{B}_3 \mathbf{w} \tag{33a}$$

$$\text{s.t. } \mathbf{w}^H \mathbf{B}_2 \mathbf{w} \leq c_1, \tag{33b}$$

$$\mathbf{f}^H \mathbf{f} \leq P_{BS}, \tag{33c}$$

$$\mathbf{w} = \mathbf{f}, \tag{33d}$$

$$\mathbf{w}^H \mathbf{B}_5 \mathbf{f} = b. \tag{33e}$$

Next, based on the PDD framework [35], [46]–[47], we propose a PDD-based efficient method to solve the problem (P4). Firstly, by penalizing the equality constraints (33d) and (33e) into the objective function, an augmented Lagrangian (AL) minimization problem can be given as

$$(\text{P5}) : \min_{\mathbf{w}, \mathbf{f}, b} \frac{(\mathbf{w}^H \mathbf{B}_4 \mathbf{f})(\mathbf{f}^H \mathbf{B}_4^H \mathbf{w})}{b} - \mathbf{w}^H \mathbf{B}_3 \mathbf{w} \tag{34a}$$

$$+ \frac{1}{2\rho} \|\mathbf{w} - \mathbf{f}\|_2^2 + \Re\{\lambda_1^H (\mathbf{w} - \mathbf{f})\}$$

$$+ \frac{1}{2\rho} |\mathbf{w}^H \mathbf{B}_5 \mathbf{f} - b|^2 + \Re\{\lambda_2^* (\mathbf{w}^H \mathbf{B}_5 \mathbf{f} - b)\}$$

$$\text{s.t. } \mathbf{w}^H \mathbf{B}_2 \mathbf{w} \leq c_1, \tag{34b}$$

$$\mathbf{f}^H \mathbf{f} \leq P_{BS}. \tag{34c}$$

Following the PDD method [35], [46]–[47], we conduct a two-layer iterative process, which its inner layer updates $\mathbf{w}$, $\mathbf{f}$ and b by using a block coordinate descent (BCD) method and its outer layer selectively updating the penalty coefficient ρ or the dual variables $\{\lambda_1, \lambda_2\}$. The PDD procedure will be detailed in the subsequent discussion.

Inner Layer Procedure

For the inner layer iteration, we will sequentially update $\mathbf{w}$, $\mathbf{f}$ and b . When $\mathbf{f}$ and b are fixed, the AL minimization problem w.r.t. $\mathbf{w}$ is reduced to as follows

$$(\text{P6}) : \min_{\mathbf{w}} \mathbf{w}^H \mathbf{B}_6 \mathbf{w} - 2\Re\{\mathbf{b}_1^H \mathbf{w}\} + c_3 - \mathbf{w}^H \mathbf{B}_3 \mathbf{w} \tag{35a}$$

$$\text{s.t. } \mathbf{w}^H \mathbf{B}_2 \mathbf{w} \leq c_1, \tag{35b}$$

where the above new coefficients are given as follows

$$\begin{aligned} \mathbf{B}_6 &\triangleq \mathbf{B}_4 \mathbf{f}^H \mathbf{B}_4^H / b + (\mathbf{I} + \mathbf{B}_5 \mathbf{f}^H \mathbf{B}_5^H) / (2\rho), \\ \mathbf{b}_1 &\triangleq (\mathbf{f} - \lambda_1 + b \mathbf{B}_5 \mathbf{f}) / (2\rho) - \lambda_2^* \mathbf{B}_5 \mathbf{f} / 2, \\ c_3 &\triangleq (\|\mathbf{f}\|_2^2 + |b|^2) / (2\rho) - \Re\{\lambda_1^H \mathbf{f} + \lambda_2^* b\}. \end{aligned} \quad (36)$$

Obviously, the problem (P6) is difficult to solve due to the non-convex objective function (35a). Inspired by the MM method [36], [48]–[49], we can establish a tight lower bound of the non-convex objective function (35a), which is given as

$$\begin{aligned} \mathbf{w}^H \mathbf{B}_3 \mathbf{w} &\geq \mathbf{w}_0^H \mathbf{B}_3 \mathbf{w}_0 + 2\Re\{\mathbf{w}_0^H \mathbf{B}_3 (\mathbf{w} - \mathbf{w}_0)\}, \\ &= 2\Re\{\mathbf{w}_0^H \mathbf{B}_3 \mathbf{w}\} - (\mathbf{w}_0^H \mathbf{B}_3 \mathbf{w}_0)^*. \end{aligned} \quad (37)$$

where $\mathbf{w}_0$ is obtained from the last iteration. Therefore, the term $\mathbf{w}^H \mathbf{B}_3 \mathbf{w}$ in the objective function (35a) can be replaced by (37). And then, we turn to optimize a tight convex upper bound of the objective function (35a), which is written as

$$(P7) : \min_{\mathbf{w}} \mathbf{w}^H \mathbf{B}_6 \mathbf{w} - 2\Re\{\mathbf{b}_2^H \mathbf{w}\} + c_4 \quad (38a)$$

$$\text{s.t. } \mathbf{w}^H \mathbf{B}_2 \mathbf{w} \leq c_1, \quad (38b)$$

where $\mathbf{b}_2 \triangleq \mathbf{b}_1 + \mathbf{B}_3^H \mathbf{w}_0$ and $c_4 \triangleq c_3 + (\mathbf{w}_0^H \mathbf{B}_3 \mathbf{w}_0)^*$. The problem (P7) is a typical second order cone program (SOCP) and can be solved by numerical solvers, i.e., CVX [37].

However, the above update method, relying on numerical solvers, e.g., CVX, has the following two main shortcomings: i) the complexity of solving SOCP problem via convex optimization solvers including CVX, which adopts the interior point (IP) method [38], will dramatically increase as the variables' dimension grows; ii) the application of third-party solvers unavoidably raises the cost and complicates the implementation of the algorithm due to licensing fees, the need for software installation and maintenance, and the platform requirements to support the solver.

Thus, we aim to develop an algorithm that ideally does not depend on CVX. Firstly, to solve the problem (P7) efficiently, we introduce the following lemma which has been proven in [50].

Lemma 1. *Consider the following convex trust region problem:*

$$(P_{Lm1}) : \min_{\mathbf{x}} \mathbf{x}^H \mathbf{Q} \mathbf{x} - 2\Re\{\mathbf{q}^H \mathbf{x}\} + q \quad (39a)$$

$$\text{s.t. } \|\mathbf{x}\|_2^2 \leq \bar{q}, \quad (39b)$$

where $\mathbf{Q} \succcurlyeq 0$ and Slater's condition holds. Besides, the eigenvalue decomposition of $\mathbf{Q}$ is given as $\mathbf{Q} = \mathbf{U} \Lambda \mathbf{U}^H$. Then the optimal solution to (P_{Lm1}) can be determined by

$$\mathbf{x}^* = \mathbf{U}(\Lambda + \nu^* \mathbf{I}) \mathbf{U}^H \mathbf{q}, \quad (40)$$

where the value of ν is non-negative and can be efficiently obtained by the Newton's method or bisection search.

To leverage Lemma 1, the constraint (38b) should be written into the standard bounded norm constraint (39b) of (P_{Lm1}) . Next, according to the singularity of the matrix $\mathbf{B}_2$, we need to consider the two different cases [51], which will be discussed later.

CASE-I: The matrix $\mathbf{B}_2$ is invertible. By introducing the following new notions

$$\tilde{\mathbf{w}} \triangleq \mathbf{B}_2^{\frac{1}{2}} \mathbf{w}, \tilde{\mathbf{B}}_6 \triangleq (\mathbf{B}_2^{-\frac{1}{2}})^H \mathbf{B}_6 \mathbf{B}_2^{-\frac{1}{2}}, \tilde{\mathbf{b}}_2 \triangleq (\mathbf{B}_2^{-\frac{1}{2}})^H \mathbf{b}_2, \quad (41)$$

and the problem (P7) can be equivalently rewritten as

$$(P8) : \min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H \tilde{\mathbf{B}}_6 \tilde{\mathbf{w}} - 2\Re\{\tilde{\mathbf{b}}_2^H \tilde{\mathbf{w}}\} + c_4 \quad (42a)$$

$$\text{s.t. } \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} \leq c_1, \quad (42b)$$

Obviously, by Lemma 1, the analytical solution $\tilde{\mathbf{w}}^*$ can be efficiently obtained. Therefore, via (41), the optimal solution $\mathbf{w}^*$ can be given as

$$\mathbf{w}^* = \mathbf{B}_2^{-\frac{1}{2}} \tilde{\mathbf{w}}^*. \quad (43)$$

CASE-II: The matrix $\mathbf{B}_2$ is singular. To invoke Lemma 1, by leveraging the MM methodology⁴, we conduct a tight upper bound of the constraint (38b), which is given as

$$\mathbf{w}^H \mathbf{B}_2 \mathbf{w} - c_2 = (\mathbf{w} - \mathbf{w}_0)^H \mathbf{B}_2 (\mathbf{w} - \mathbf{w}_0) \quad (44)$$

$$+ 2\Re\{(\mathbf{B}_2 \mathbf{w})^H (\mathbf{w} - \mathbf{w}_0)\} + \mathbf{w}_0^H \mathbf{B}_2 \mathbf{w}_0 - c_2$$

$$\leq (\mathbf{w} - \mathbf{w}_0)^H (\mathbf{B}_2 + \delta \mathbf{I}) (\mathbf{w} - \mathbf{w}_0)$$

$$+ 2\Re\{(\mathbf{B}_2 \mathbf{w})^H (\mathbf{w} - \mathbf{w}_0)\} + \mathbf{w}_0^H \mathbf{B}_2 \mathbf{w}_0 - c_2$$

$$= \mathbf{w}^H (\mathbf{B}_2 + \delta \mathbf{I}) \mathbf{w} - 2\Re\{\mathbf{w}_0^H \delta \mathbf{I} \mathbf{w}\} + \mathbf{w}_0^H (\delta \mathbf{I}) \mathbf{w}_0 - c_2$$

$$= \mathbf{w}^H \hat{\mathbf{B}}_2 \mathbf{w} - 2\Re\{\hat{\mathbf{b}}_2^H \mathbf{w}\} + \hat{c}_2,$$

where δ is a small positive constant, $\hat{\mathbf{B}}_2 \triangleq \mathbf{B}_2 + \delta \mathbf{I}$, $\hat{\mathbf{b}}_2 \triangleq \delta \mathbf{w}_0$ and $\hat{c}_2 \triangleq \mathbf{w}_0^H (\delta \mathbf{I}) \mathbf{w}_0 - c_2$.

Based on the above MM transformation, the constraint (38b) can be replaced by (44). And then, we turn to solve the following problem

$$(P9) : \min_{\mathbf{w}} \mathbf{w}^H \mathbf{B}_6 \mathbf{w} - 2\Re\{\mathbf{b}_2^H \mathbf{w}\} + c_4 \quad (45a)$$

$$\text{s.t. } \mathbf{w}^H \hat{\mathbf{B}}_2 \mathbf{w} - 2\Re\{\hat{\mathbf{b}}_2^H \mathbf{w}\} + \hat{c}_2 \leq 0, \quad (45b)$$

In order to adopt Lemma 1, we need to transfer the constraint (45b) into the standard form of Lemma 1. Firstly, we define the following new coefficients

$$\tilde{\mathbf{w}} \triangleq \hat{\mathbf{B}}_2^{\frac{1}{2}} \mathbf{w} - \hat{\mathbf{B}}_2^{-\frac{1}{2}} \hat{\mathbf{b}}_2, \tilde{\mathbf{B}}_2 \triangleq \hat{\mathbf{B}}_2^{-\frac{1}{2}} \mathbf{B}_6 \hat{\mathbf{B}}_2^{-\frac{1}{2}}, \quad (46)$$

$$\tilde{\mathbf{b}}_2 \triangleq \hat{\mathbf{B}}_2^{-\frac{1}{2}} \mathbf{b}_2 - \mathbf{B}_6^H \hat{\mathbf{B}}_2^{-\frac{3}{2}} \hat{\mathbf{b}}_2,$$

and then, the problem (P9) can be equivalently written as

$$(P10) : \min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H \tilde{\mathbf{B}}_2 \tilde{\mathbf{w}} - 2\Re\{\tilde{\mathbf{b}}_2^H \tilde{\mathbf{w}}\} \quad (47a)$$

$$\text{s.t. } \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} \leq \hat{\mathbf{b}}_2^H \hat{\mathbf{B}}_2^{-1} \hat{\mathbf{b}}_2 - \hat{c}_2. \quad (47b)$$

Up to here, we can obtain analytically the solution $\tilde{\mathbf{w}}^*$ via invoking Lemma 1. Furthermore, by (46), the optimal solution $\mathbf{w}^*$ is immediately given as

$$\mathbf{w}^* = \hat{\mathbf{B}}_2^{-\frac{1}{2}} \tilde{\mathbf{w}}^* + \hat{\mathbf{B}}_2^{-1} \hat{\mathbf{b}}_2. \quad (48)$$

When $\mathbf{w}$ and b are given, the update of $\mathbf{f}$ is reduced to solving the following problem

$$(P11) : \min_{\mathbf{f}} \mathbf{f}^H \mathbf{B}_7 \mathbf{f} - 2\Re\{\mathbf{b}_2^H \mathbf{f}\} + c_5 \quad (49a)$$

⁴This method guarantees that the subproblems relying on $\mathbf{B}_2$ satisfy the conditions of Lemma 1.

$$\text{s.t. } \mathbf{f}^H \mathbf{f} \leq P_{BS}, \quad (49b)$$

where the new coefficients are defined as

$$\begin{aligned} \mathbf{B}_7 &\triangleq (\mathbf{B}_4^H \mathbf{w} \mathbf{w}^H \mathbf{B}_4)/b + (\mathbf{I} + \mathbf{B}_5^H \mathbf{w} \mathbf{w}^H \mathbf{B}_5)/(2\rho), \quad (50) \\ \mathbf{b}_3 &\triangleq (\mathbf{w} + b \mathbf{B}_5^H \mathbf{w})/(2\rho) + (\lambda_1 - \lambda_2 \mathbf{B}_5^H \mathbf{w})/2, \\ c_5 &\triangleq (\mathbf{w}^H \mathbf{w} + |b|^2)/(2\rho) + \Re\{\lambda_1^H \mathbf{w} - \lambda_2^* b\} - \mathbf{w}^H \mathbf{B}_3 \mathbf{w}. \end{aligned}$$

The above problem is SOCP and can be solved by CVX. Furthermore, it is obvious that the problem (P11) can also be efficiently solved by using Lemma 1.

Next, we introduce the new definitions

$$\begin{aligned} c_6 &\triangleq (\mathbf{w}^H \mathbf{B}_5 \mathbf{f} \mathbf{f}^H \mathbf{B}_5^H \mathbf{w})/(2\rho) + \Re\{\lambda_2^* \mathbf{w}^H \mathbf{B}_5 \mathbf{f}\} - \mathbf{w}^H \mathbf{B}_3 \mathbf{w}, \quad (51) \\ a_1 &\triangleq 1/(2\rho), a_2 \triangleq -\mathbf{w}^H \mathbf{B}_5 \mathbf{f}/\rho - \Re\{\lambda_2^*\}, a_4 \triangleq \mathbf{w}^H \mathbf{B}_5 \mathbf{f} \mathbf{f}^H \mathbf{B}_5^H \mathbf{w}. \end{aligned}$$

The problem w.r.t. b is written as

$$(P12) : \min_b a_1 b^2 + a_2 b + c_6 + \frac{a_4}{b} \quad (52a)$$

The problem (P12) is an unconstrained convex problem and its optimal solution can be obtained by setting its derivation to zero.

The inner layer of the PDD method updates $\mathbf{w}$, $\mathbf{f}$ and b in a BCD manner until convergence.

Outer Layer Procedure

When the inner layer achieves convergence, the outer layer will selectively update the value of dual variables $\{\lambda_1, \lambda_2\}$ or the penalty coefficient ρ . Specifically,

- 1) when the equations $\mathbf{w} = \mathbf{f}$ and $\mathbf{w}^H \mathbf{B}_5 \mathbf{f} = b$ are nearly satisfied, the Lagrangian multipliers $\{\lambda_1, \lambda_2\}$ will be respectively updated in a gradient ascent manner as follows:

$$\lambda_1^{(k+1)} := \lambda_1^{(k)} + \rho^{-1}(\mathbf{w} - \mathbf{f}), \quad (53a)$$

$$\lambda_2^{(k+1)} := \lambda_2^{(k)} + \rho^{-1}(\mathbf{w}^H \mathbf{B}_5 \mathbf{f} - b). \quad (53b)$$

- 2) when the equality constraints $\mathbf{w} = \mathbf{f}$ and/or $\mathbf{w}^H \mathbf{B}_5 \mathbf{f} = b$ are not achieved, to force $\mathbf{w} = \mathbf{f}$ and/or $\mathbf{w}^H \mathbf{B}_5 \mathbf{f} = b$ to be approached in the subsequent iterations, the outer layer will update the penalty parameter ρ^{-1} as follows:

$$(\rho^{(k+1)})^{-1} := c^{-1} \cdot (\rho^{(k)})^{-1}, \quad (54)$$

where the positive constant c is usually smaller than 1 and is often selected within the range of [0.8, 0.9].

The previously developed algorithm based on the PDD framework to solve the problem (P3) is summarized in Alg.1.

D. Updating The Receiver Filter $\{\mathbf{u}_k\}$

In this subsection, we investigate the optimization of the user receive filter $\{\mathbf{u}_k\}$ while keeping other variables fixed. By introducing the new coefficients as follows

$$\mathbf{d}_{1,k} \triangleq \sqrt{(1+\gamma_k)} \omega_k^* \sqrt{q_k} \mathbf{h}_k, c_7 \triangleq \sum_{k=1}^K (\log(1+\gamma_k) - \gamma_k) - R_t, \quad (55)$$

$$\mathbf{D}_{1,k} \triangleq |\omega_k|^2 \left(\sum_{i=1}^K q_i |\mathbf{u}_k^H \mathbf{h}_i|^2 + |\alpha|^2 \mathbf{a}_r \mathbf{a}_t^H \mathbf{W} \mathbf{W}^H \mathbf{a}_t \mathbf{a}_r^H + \sigma^2 \|\mathbf{u}_k^H\|_2^2 \right),$$

the constraint (28b) is rewritten as

$$\sum_{k=1}^K (\mathbf{u}_k^H \mathbf{D}_{1,k} \mathbf{u}_k - 2\Re\{\mathbf{u}_k^H \mathbf{d}_{1,k}\}) - c_7 \leq 0 \quad (56)$$

Algorithm 1 PDD Method to Solve (P3)

```

1: initialize  $\mathbf{w}^{(0)}, \mathbf{f}^{(0)}, b^{(0)}, \lambda_1^{(0)}, \lambda_2^{(0)}, \rho^{(0)}$  and  $k = 1$ ;
2: repeat
3:   set  $\mathbf{w}^{(k-1,0)} := \mathbf{w}^{(k-1)}, \mathbf{f}^{(k-1,0)} := \mathbf{f}^{(k-1)},$ 
      $b^{(k-1,0)} := b^{(k-1)}, t = 0$ ;
4:   repeat
5:     update  $\mathbf{w}^{(k-1,t+1)}$  by solving (P7);
6:     update  $\mathbf{f}^{(k-1,t+1)}$  by solving (P11);
7:     update  $b^{(k-1,t+1)}$  by solving (P12);
8:      $t++$ ;
9:   until convergence
10:  set  $\mathbf{w}^{(k)} := \mathbf{w}^{(k-1,\infty)}, \mathbf{f}^{(k)} := \mathbf{f}^{(k-1,\infty)}, b^{(k)} :=$ 
      $b^{(k-1,\infty)}$ ;
11:  if  $\|\mathbf{w}^{(k)} - \mathbf{f}^{(k)}\|_\infty \leq \eta_k$  and  $|(\mathbf{w}^{(k)})^H \mathbf{B}_5 \mathbf{f}^{(k)} - b^{(k)}| \leq$ 
      $\eta_k$  then
12:     $\lambda_1^{(k+1)} := \lambda_1^{(k)} + \frac{1}{\rho^{(k)}}(\mathbf{w}^{(k)} - \mathbf{f}^{(k)}), \lambda_2^{(k+1)} := \lambda_2^{(k)} +$ 
      $\frac{1}{\rho^{(k)}}((\mathbf{w}^{(k)})^H \mathbf{B}_5 \mathbf{f}^{(k)} - b^{(k)}), \rho^{(k+1)} := \rho^{(k)}$ ;
13:  else
14:     $\lambda_1^{(k+1)} := \lambda_1^{(k)}, \lambda_2^{(k+1)} := \lambda_2^{(k)}, 1/\rho^{(k+1)} := 1/(c \cdot$ 
      $\rho^{(k)})$ ;
15:  end if
16:   $k++$ ;
17: until  $\|\mathbf{w}^{(k)} - \mathbf{f}^{(k)}\|_2$  and  $|(\mathbf{w}^{(k)})^H \mathbf{B}_5 \mathbf{f}^{(k)} - b^{(k)}|$  are
     sufficiently small simultaneously;

```

Therefore, the optimization of $\{\mathbf{u}_k\}$ is reduced to solve the following feasibility characterization problem as

$$(P13) : \text{Find } \{\mathbf{u}_k\} \quad (57a)$$

$$\text{s.t. } \sum_{k=1}^K (\mathbf{u}_k^H \mathbf{D}_{1,k} \mathbf{u}_k - 2\Re\{\mathbf{u}_k^H \mathbf{d}_{1,k}\}) - c_7 \leq 0. \quad (57b)$$

The problem (P13), also referred to as the Phase-I problem [38], is addressed by solving another closely related problem as follows:

$$(P14) : \min_{\{\mathbf{u}_k\}, \alpha_1} \alpha_1 \quad (58a)$$

$$\text{s.t. } \sum_{k=1}^K (\mathbf{u}_k^H \mathbf{D}_{1,k} \mathbf{u}_k - 2\Re\{\mathbf{u}_k^H \mathbf{d}_{1,k}\}) - c_7 \leq \alpha_1. \quad (58b)$$

As stated in [38], [52], minimizing (P14) is to identify more “feasible” $\{\mathbf{u}_k\}$, which offer a greater margin for satisfying the constraint (57b) and thereby facilitate the optimization of other variables. Clearly, optimality in problem (P14) is achieved only when equality is reached in constraint (58b). Consequently, solving (P14) equates to minimize the expression on the left side of (58b), which is given as

$$(P15) : \min_{\{\mathbf{u}_k\}} \sum_{k=1}^K (\mathbf{u}_k^H \mathbf{D}_{1,k} \mathbf{u}_k - 2\Re\{\mathbf{u}_k^H \mathbf{d}_{1,k}\}) - c_7 \quad (59a)$$

Furthermore, the problem (P15) can be decomposed into K independent sub-problems with each sub-problem defined as follows:

$$(P15_k) : \min_{\mathbf{u}_k} \mathbf{u}_k^H \mathbf{D}_{1,k} \mathbf{u}_k - 2\Re\{\mathbf{u}_k^H \mathbf{d}_{1,k}\} \quad (60a)$$

Since (P15_k) is an unconstrained convex quadratic problem,

the optimal solution $\mathbf{u}_k^*$ can be determined by setting its derivative to zero, which is formulated as

$$\mathbf{u}_k^* = \mathbf{D}_{1,k}^{-1} \mathbf{d}_{1,k}, \forall k \in \mathcal{K}. \quad (61)$$

E. Optimizing The User Transmission Power

After fixing other variables, the problem w.r.t. $\{q_k\}$ can be expressed as follows

$$(P16) : \text{Find } \{q_k\} \quad (62a)$$

$$\text{s.t. } \sum_{k=1}^K (a_{5,k} q_k - a_{6,k} \sqrt{q_k}) - c_8 \leq 0, \quad (62b)$$

$$0 \leq q_k \leq P_{u,k}, \forall k \in \mathcal{K}, \quad (62c)$$

with the newly introduced coefficients specified as follows

$$a_{5,i} \triangleq \sum_{k=1}^K |\omega_k|^2 |\mathbf{u}_k^H \mathbf{h}_i|^2, a_{6,k} \triangleq 2\sqrt{(1+\gamma_k)} \Re\{\omega_k^* \mathbf{u}_k^H \mathbf{h}_k\}, \quad (63)$$

$$c_8 \triangleq \sum_{k=1}^K (\log(1+\gamma_k) - \gamma_k - |\omega_k|^2 (\|\mathbf{u}_k^H \mathbf{a}_r \mathbf{a}_t^H \mathbf{W}\|_2^2 + \sigma^2) \|\mathbf{u}_k^H\|_2^2) - R_t.$$

Note that the problem (P16) is also a feasibility characterization problem. By introducing an auxiliary variable α_2 , (P16) can be rewritten as

$$(P17) : \min_{\{q_k\}, \alpha_2} \alpha_2 \quad (64a)$$

$$\text{s.t. } \sum_{k=1}^K (a_{5,k} q_k - a_{6,k} \sqrt{q_k}) - c_8 \leq \alpha_2, \quad (64b)$$

$$0 \leq q_k \leq P_{u,k}, \forall k \in \mathcal{K}, \quad (64c)$$

Furthermore, following the arguments (57a)-(59a) of the above subsection, we turn to solve the following problem

$$(P18) : \min_{\{q_k\}} \sum_{k=1}^K (a_{5,k} q_k - a_{6,k} \sqrt{q_k}) \quad (65a)$$

$$\text{s.t. } 0 \leq q_k \leq P_{u,k}, \forall k \in \mathcal{K}, \quad (65b)$$

which problem (P18) is convex and can be solved by CVX. Next, we will develop the closed-form solution to update $\{q_k\}$. Obviously, (P18) can also be decomposed into K independent sub-problems, which the sub-problem is formulated as

$$(P18_k) : \min_{q_k} a_{5,k} q_k - a_{6,k} \sqrt{q_k} \quad (66a)$$

$$\text{s.t. } 0 \leq q_k \leq P_{u,k}, \quad (66b)$$

Furthermore, we define $p_k \triangleq \sqrt{q_k}$ and $\bar{P}_k \triangleq \sqrt{P_{u,k}}$, and then the problem (P18_k) is rewritten as

$$(P19) : \min_{p_k} a_{5,k} p_k^2 - a_{6,k} p_k \quad (67a)$$

$$\text{s.t. } 0 \leq p_k \leq \bar{P}_k. \quad (67b)$$

Since $a_{5,k} > 0$, the closed solution can be determined by judge the position of the axis of symmetry, which is given as

$$p_k^* = \begin{cases} 0, & p_{k,x} < 0, \\ \bar{P}_k, & p_{k,x} > \bar{P}_k, \\ p_{k,x}, & \text{otherwise,} \end{cases} \quad (68)$$

where $p_{k,x} \triangleq \frac{a_{6,k}}{2a_{5,k}}$ and $q_k^* = (p_k^*)^2$.

F. Optimizing The Position of BS FA

With fixed other variables, the update for the position vector of the BS FA, i.e., $\mathbf{d}_r$, can be formulated as follows

$$(P20) : \min_{\mathbf{d}_r} \mathbf{d}_r^T \mathbf{A}_{4,3} \mathbf{d}_r + \mathbf{d}_r^T \mathbf{b}_4 - c_{16} \quad (69a)$$

$$\text{s.t. } \sum_{k=1}^K (\bar{\mathbf{h}}_{1,k}^H \mathbf{A}_{6,k} \bar{\mathbf{h}}_{1,k} + \Re\{\mathbf{b}_{7,k}^H \bar{\mathbf{h}}_{1,k}\}) + \mathbf{a}_r^H \mathbf{A}_9 \mathbf{a}_r - c_{20} \leq 0, \quad (69b)$$

$$d_{r,1} \geq 0, d_{r,N_r} \leq d_{max}, \quad (69c)$$

$$d_{r,n} - d_{r,n-1} \geq d_{min}, n=2, 3, \dots, N_r. \quad (69d)$$

where the above newly introduced notions are given in (70).

Note that the problem (P20) is non-convex. We proceed to update the positions of the FAs one by one. The sub-problem for optimizing n -th FA's position $d_{r,n}$ is given as

$$(P21) : \min_{d_{r,n}} a_{7,n} d_{r,n}^2 + b_{6,n} d_{r,n} - c_{19,n} \quad (71a)$$

$$\text{s.t. } \sum_{k=1}^K (\mathbf{h}_{1,k,n}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\}) + c_{21,k,n} + c_{22,k,n} + \Re\{\mathbf{b}_{15,n}^H \mathbf{a}_r[n]\} + c_{27,n} \leq 0, \quad (71b)$$

$$d_{r,n} - d_{r,n-1} \geq d_{min}, \quad (71c)$$

$$d_{r,n+1} - d_{r,n} \geq d_{min}, \quad (71d)$$

$$0 \leq d_{r,n} \leq d_{max}, \quad (71e)$$

where the new coefficients are defined in (72). Next, for simplicity, we define $d_{r,0} \triangleq -d_{min}$, $d_{r,N_r+1} \triangleq d_{max} + d_{min}$, and the constraints (71c)–(71e) can be rewritten as

$$\begin{cases} d_{r,n} - d_{r,n-1} \geq d_{min} \\ d_{r,n+1} - d_{r,n} \geq d_{min} \\ 0 \leq d_{r,n} \leq d_{max} \end{cases} \quad (73)$$

$$\iff d_{r,n-1} + d_{min} \leq d_{r,n} \leq d_{r,n+1} - d_{min}.$$

Therefore, the problem (P21) is rewritten by

$$(P22) : \min_{d_{r,n}} a_{7,n} d_{r,n}^2 + b_{6,n} d_{r,n} - c_{19,n} \quad (74a)$$

$$\text{s.t. } \sum_{k=1}^K (\mathbf{h}_{1,k,n}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\}) + c_{21,k,n} + c_{22,k,n} + \Re\{\mathbf{b}_{15,n}^H \mathbf{a}_r[n]\} + c_{27,n} \leq 0, \quad (74b)$$

$$d_{r,n-1} + d_{min} \leq d_{r,n} \leq d_{r,n+1} - d_{min}, \quad (74c)$$

Since the constraint (74b) is non-convex, we adopt the MM method to conduct a convex upper bound. Firstly, $d_{r,n,0}$ defines the value obtained from the previous iteration, $\lambda_{max}(\mathbf{A}_{7,k,n,n})$ is the largest eigenvalue of matrix $\mathbf{A}_{7,k,n,n}$ and $\mathbf{h}_{1,k,n,0} \triangleq \mathbf{h}_{1,k,n}(d_{r,n,0})$. The upper bound of the equation $\mathbf{h}_{1,k,n}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\}$ is presented in (75), where $\mathbf{b}_{11,k,n} \triangleq -2\lambda_{max}(\mathbf{A}_{7,k,n,n})\mathbf{h}_{1,k,n,0} + \mathbf{A}_{7,k,n,n}^H \mathbf{h}_{1,k,n,0}$, $c_{23,k,n} \triangleq 2\lambda_{max}(\mathbf{A}_{7,k,n,n})L_{r,k} - (\mathbf{h}_{1,k,n,0}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n,0})^*$ and $\mathbf{b}_{12,k,n} \triangleq \mathbf{b}_{10,k,n} + \mathbf{b}_{11,k,n}$, and then, by using the second-order Taylor expansion again, we can construct the convex surrogate functions of $\Re\{\mathbf{b}_{12,k,n}^H \mathbf{h}_{1,k,n}\}$ and $\Re\{\mathbf{b}_{15,n}^H \mathbf{a}_r[n]\}$ in (77) and (78), respectively, where

$$\nabla h_{3,k,n,0} \triangleq \frac{\partial h_{3,k,n,0}}{\partial d_{r,n,0}} = -\sum_{i=1}^{L_{r,k}} |\mathbf{b}_{12,k,n}^*[i]| \frac{2\pi}{\lambda} \sin(\theta_{k,i}^r) \quad (76)$$

$$\sin\left(\frac{2\pi}{\lambda} \sin(\theta_{k,i}^r) - \angle \mathbf{b}_{12,k,n}^*[i]\right), \tau_{2,k,n} \triangleq \frac{4\pi^2}{\lambda^2} \sum_{i=1}^{L_{r,k}} |\mathbf{b}_{12,k,n}^*[i]|,$$

$$\begin{aligned}
\bar{\mathbf{h}}_{1,k} &\triangleq \text{vec}(\mathbf{H}_{0,k}(\mathbf{d}_r)), \mathbf{D}_r \triangleq j \frac{2\pi}{\lambda} \text{diag}(\mathbf{d}_r) \cos(\theta_0), \mathbf{D}_t \triangleq j \frac{2\pi}{\lambda} \text{diag}(\mathbf{d}_t) \cos(\theta_0), \mathbf{A}_t \triangleq \mathbf{a}_t \mathbf{a}_t^H, \mathbf{W}_t \triangleq \mathbf{W} \mathbf{W}^H, \\
\mathbf{A}_{4,1} &\triangleq \left(\frac{2\pi}{\lambda}\right)^2 \text{tr}(\mathbf{A}_t \mathbf{W}_t), \mathbf{b}_1 \triangleq \text{tr}(\mathbf{A}_t \mathbf{D}_t^H \mathbf{W}_t) (-j \frac{2\pi}{\lambda} \cos \theta_0 \mathbf{1}), \mathbf{b}_2 \triangleq \text{tr}(\mathbf{D}_t \mathbf{A}_t \mathbf{W}_t) (j \frac{2\pi}{\lambda} \cos \theta_0 \mathbf{1}), c_{13} \triangleq \text{tr}(N_r \mathbf{D}_t \mathbf{A}_t \mathbf{D}_t^H \mathbf{W}_t), \\
\mathbf{b}_3 &\triangleq (-j \frac{2\pi}{\lambda} \cos \theta_0 \mathbf{1}) \text{tr}(\mathbf{A}_t \mathbf{W}_t), c_{14} \triangleq \text{tr}(N_r \mathbf{D}_t \mathbf{A}_t \mathbf{W}_t), c_{15} \triangleq \text{tr}(\mathbf{A}_3 \mathbf{W}_t), \mathbf{A}_{4,2} \triangleq \mathbf{b}_3 \mathbf{b}_3^T / c_{15}, \mathbf{b}_4 \triangleq 2c_{14} \mathbf{b}_3 / c_{15} - (\mathbf{b}_1 + \mathbf{b}_2), \\
c_{16} &\triangleq -|c_{14}|^2 / c_{15} + c_{13}, \mathbf{A}_{4,3} \triangleq \mathbf{A}_{4,2} - \mathbf{A}_{4,1}, \mathbf{A}_9 \triangleq \sum_{k=1}^K |\omega_k|^2 ((\mathbf{a}_t^H \mathbf{W}_t \mathbf{a}_t)^T \otimes \mathbf{u}_k \mathbf{u}_k^H), \mathbf{A}_{6,i} \triangleq \sum_{k=1}^K |\omega_k|^2 (q_i \mathbf{u}_k \mathbf{u}_k^H)^T \otimes (\Sigma_i^H \mathbf{h}_{0,i} \mathbf{h}_{0,i}^H \Sigma_i), \\
\mathbf{b}_{7,k} &\triangleq -2\sqrt{1+\gamma_k} (\mathbf{u}_k^T \otimes \omega_k \sqrt{q_k} \mathbf{h}_{0,k}^H \Sigma_k), c_{20} \triangleq \sum_{k=1}^K (\log(1+\gamma_k) - \gamma_k - |\omega_k|^2 (\sigma^2 \|\mathbf{u}_k^H\|_2^2)) - R_t.
\end{aligned} \tag{70}$$

$$\begin{aligned}
a_{7,n} &\triangleq \mathbf{A}_3[n, n], b_{6,n} \triangleq 2 \sum_{i \neq n}^{N_r} d_{r,i} \mathbf{A}_3[i, n] + \mathbf{b}_4[n], c_{19,n} \triangleq \sum_{i \neq n}^{N_r} \sum_{j \neq n}^{N_r} d_{r,i} \mathbf{A}_3[i, j] d_{r,j} - \sum_{i \neq n}^{N_r} d_{r,i} \mathbf{b}_4[i] + c_{16}, \\
\mathbf{A}_{7,k,n,m} &\triangleq \mathbf{A}_{6,k}[(n-1)L_{r,k} + 1 : nL_{r,k}, (m-1)L_{r,k} + 1 : mL_{r,k}], \mathbf{b}_{8,k,n} \triangleq \mathbf{b}_{7,k}[(n-1)L_{r,k} + 1 : nL_{r,k}], \\
\mathbf{b}_{9,k,n} &\triangleq 2 \sum_{m \neq n}^{N_r} \mathbf{A}_{7,k,m,n}^H \mathbf{h}_{1,k,m}, c_{21,k,n} \triangleq \sum_{i \neq n}^{N_r} \sum_{j \neq n}^{N_r} \mathbf{h}_{1,k,i}^H \mathbf{A}_{7,k,i,j} \mathbf{h}_{1,k,j}, c_{22,k,n} \triangleq \sum_{i \neq n}^{N_r} \Re\{\mathbf{b}_{8,k,i}^H \mathbf{h}_{1,k,i}\}, \\
\mathbf{b}_{10,k,n} &\triangleq \mathbf{b}_{8,k,n} + \mathbf{b}_{9,k,n}, b_{15,n} \triangleq 2 \sum_{i \neq n}^{N_r} a_r[i] \mathbf{A}_{9,k}[i, n]^*, c_{27,n} \triangleq \mathbf{A}_{9,k}[n, n] \sum_{i \neq n}^{N_r} \sum_{j \neq n}^{N_r} a_r[i]^* \mathbf{A}_{9,k}[i, j] a_r[j].
\end{aligned} \tag{72}$$

$$\begin{aligned}
&\mathbf{h}_{1,k,n}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\} \\
&= (\mathbf{h}_{1,k,n} - \mathbf{h}_{1,k,n,0})^H \mathbf{A}_{7,k,n,n} (\mathbf{h}_{1,k,n} - \mathbf{h}_{1,k,n,0}) + 2\Re\{\mathbf{h}_{1,k,n,0}^H \mathbf{A}_{7,k,n,n} (\mathbf{h}_{1,k,n} - \mathbf{h}_{1,k,n,0})\} + \mathbf{h}_{1,k,n,0}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n,0} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\} \\
&\leq \lambda_{\max}(\mathbf{A}_{7,k,n,n}) \|\mathbf{h}_{1,k,n} - \mathbf{h}_{1,k,n,0}\|_2^2 + 2\Re\{\mathbf{h}_{1,k,n,0}^H \mathbf{A}_{7,k,n,n} (\mathbf{h}_{1,k,n} - \mathbf{h}_{1,k,n,0})\} + \mathbf{h}_{1,k,n,0}^H \mathbf{A}_{7,k,n,n} \mathbf{h}_{1,k,n,0} + \Re\{\mathbf{b}_{10,k,n}^H \mathbf{h}_{1,k,n}\} \\
&= \Re\{\mathbf{b}_{12,k,n}^H \mathbf{h}_{1,k,n}\} + c_{23,k,n},
\end{aligned} \tag{75}$$

$$\begin{aligned}
\Re\{\mathbf{b}_{12,k,n}^H \mathbf{h}_{1,k,n}\} &= \sum_{i=1}^{L_{r,k}} |b_{12,k,n}^*[i]| \cos\left(\frac{2\pi}{\lambda} d_{r,n} \sin(\theta_0) - \angle b_{12,k,n}^*[i]\right) \triangleq h_{3,k,n} \\
&\leq h_{3,k,n,0}(d_{r,n,0}) + \nabla h_{3,k,n,0}(d_{r,n} - d_{r,n,0}) + \frac{1}{2} \tau_{2,k,n}(d_{r,n} - d_{r,n,0})(d_{r,n} - d_{r,n,0}) = \frac{1}{2} \tau_{2,k,n} d_{r,n}^2 + b_{13,k,n} d_{r,n} + c_{25,k,n}, \\
\Re\{b_{15,n}^* \mathbf{a}_r[n]\} &= |b_{15,n}^*| \cos\left(\frac{2\pi}{\lambda} d_{r,n} \sin(\theta_0) - \angle b_{15,n}^*\right) \triangleq \tilde{a}_r(d_{r,n}) \\
&\leq \tilde{a}_r(d_{r,n,0}) + \nabla \tilde{a}_r(d_{r,n,0})(d_{r,n} - d_{r,n,0}) + \frac{1}{2} \tau_{4,n}(d_{r,n} - d_{r,n,0})(d_{r,n} - d_{r,n,0}) = \frac{1}{2} \tau_{4,n} d_{r,n}^2 + b_{16,n} d_{r,n} + c_{28,n}
\end{aligned} \tag{77}$$

$$\begin{aligned}
&\leq \tilde{a}_r(d_{r,n,0}) + \nabla \tilde{a}_r(d_{r,n,0})(d_{r,n} - d_{r,n,0}) + \frac{1}{2} \tau_{4,n}(d_{r,n} - d_{r,n,0})(d_{r,n} - d_{r,n,0}) = \frac{1}{2} \tau_{4,n} d_{r,n}^2 + b_{16,n} d_{r,n} + c_{28,n}
\end{aligned} \tag{78}$$

$$\nabla \tilde{a}_r(d_{r,n,0}) \triangleq -|b_{15,n}^*| \frac{2\pi}{\lambda} \sin(\theta_0) \sin\left(\frac{2\pi}{\lambda} \sin(\theta_0) - \angle b_{15,n}^*\right),$$

$$\tau_{4,n} \triangleq \frac{4\pi^2}{\lambda^2} |b_{15,n}^*|, b_{13,k,n} \triangleq \nabla h_{3,k,n,0} - \tau_{2,k,n} d_{r,n,0},$$

$$c_{25,k,n} \triangleq \frac{1}{2} \tau_{2,k,n} d_{r,n,0}^2 - \nabla h_{3,k,n,0} d_{r,n,0} + h_{3,k,n,0},$$

$$b_{16,n} \triangleq \nabla \tilde{a}_r(d_{r,n,0}) - \tau_{4,n} d_{r,n,0}.$$

Based on the previous transformation, the problem w.r.t. $d_{r,n}$ can be written as

$$(P23) : \min_{d_{r,n}} a_{7,n} d_{r,n}^2 + b_{6,n} d_{r,n} - c_{19,n} \tag{79a}$$

$$\text{s.t. } \tau_3 d_{r,n}^2 + b_{14,n} d_{r,n} + c_{26,n} \leq 0, \tag{79b}$$

$$d_{r,n-1} + d_{\min} \leq d_{r,n} \leq d_{r,n+1} - d_{\min}, \tag{79c}$$

where $\tau_3 \triangleq \sum_{k=1}^K \frac{1}{2} \tau_{2,k,n} + \frac{1}{2} \tau_{4,n}$, $b_{14,n} \triangleq \sum_{k=1}^K b_{13,k,n} + b_{16,n}$ and $c_{26,n}$ is a constant. The problem (P23) is convex and can be solved by CVX. Furthermore, we proceed to present its analytic solution. Firstly, according to the structure of the constraints (79b)–(79c), the feasible area of (P23) can be rewritten as:

$$d_{r,n} \in [d_{r,n,\text{low}} \triangleq \max\{d_{r,n,r1}, d_{r,n-1} + d_{\min}\}, \tag{80}$$

$$d_{r,n,\text{up}} \triangleq \min\{d_{r,n,r2}, d_{r,n+1} - d_{\min}\}],$$

where $d_{r,n,r1}$ and $d_{r,n,r2}$ denote the roots of the equation $\tau_3 d_{r,n}^2 + b_{14,n} d_{r,n} + c_{26,n} = 0$ with $d_{r,n,r1} \leq d_{r,n,r2}$. The proof details of (80) can be seen in the Appendix B. Therefore, (P23) is rewritten as

$$(P24) : \min_{d_{r,n}} a_{7,n} d_{r,n}^2 + b_{6,n} d_{r,n} - c_{19,n} \tag{81a}$$

$$\text{s.t. } d_{r,n} \in [d_{r,n,\text{low}}, d_{r,n,\text{up}}]. \tag{81b}$$

And then, the closed solution of (P24) is presented in the following theorem.

Theorem 1. We define $d_{r,n,\text{sym}}$ as the axis of symmetry of the function $f(d_{r,n}) \triangleq a_{7,n} d_{r,n}^2 + b_{6,n} d_{r,n} - c_{19,n}$. The optimal values of $d_{r,n}$ are obtained according to one of the following three cases:

CASE-I: if $a_{7,n} = 0$, by investigating the value of $b_{6,n}$,

Algorithm 2 Overall Algorithm to Solve (P1)

- 1: initialize $i = 0$;
 - 2: randomly generate feasible $\mathbf{W}^0$, $\{\mathbf{u}_k^0\}$, $\{q_k^0\}$, and $\mathbf{d}_r^0$;
 - 3: **repeat**
 - 4: update $\{\gamma_k\}$, and $\{\omega_k\}$ by (29) and (30), respectively;
 - 5: update $\mathbf{W}^{i+1}$ by Alg. 1;
 - 6: update $\{\mathbf{u}_k^{i+1}\}$ by equation (61);
 - 7: update $\{q_k^{i+1}\}$ by equation (68);
 - 8: update $\{\mathbf{d}_{r,n}^{i+1}\}$ by Theorem 1;
 - 9: $i++$;
 - 10: **until** convergence
-

the optimal value of $d_{r,n}$ is given as

$$d_{r,n}^* = \begin{cases} d_{r,n,low}, & b_{6,n} > 0 \\ d_{r,n,up}, & b_{6,n} < 0 \\ \text{arbitrary value in } [d_{r,n,low}, d_{r,n,up}], & b_{6,n} = 0 \end{cases} \quad (82)$$

CASE-II: when $a_{7,n} > 0$, the solution $d_{r,n}^*$ can be directly given as

$$d_{r,n}^* = \begin{cases} d_{r,n,low}, & d_{r,n,sym} < d_{r,n,low} \\ d_{r,n,up}, & d_{r,n,sym} > d_{r,n,up} \\ d_{r,n,sym}, & d_{r,n,low} \leq d_{r,n,sym} \leq d_{r,n,up} \end{cases} \quad (83)$$

CASE-III: when $a_{7,n} < 0$, the solution $d_{r,n}^*$ is obtained as

$$d_{r,n}^* = \begin{cases} d_{r,n,up}, & d_{r,n,sym} < d_{r,n,low} \\ d_{r,n,low}, & d_{r,n,sym} > d_{r,n,up} \\ \arg \min\{f(d_{r,n,low}), f(d_{r,n,up})\} \end{cases} \quad (84)$$

Moreover, the overall algorithm to solve (P1) is demonstrated in Alg. 2.

IV. NUMERICAL RESULTS

In this section, we conduct numerical results to evaluate the effectiveness of our proposed algorithm. We assumed that the BS equipping with $N_t = N_r = 4$ transmit/receive antennas simultaneously serves $K = 2$ UL users and estimates the DoA of the target. The BS-target links are modeled as LoS channels. Moreover, we adopt a geometric channel model [9] for UL users, wherein the number of transmit and receive channel paths is consistently identical, i.e., $L_k^t = L_k^r \triangleq L_1 = 10$. Thus, the path-response matrix for each user is diagonal, i.e., the path-response matrix between the BS and k -th user is given by $\mathbf{\Sigma}_k = \text{diag}\{\sigma_{k,1}^0, \dots, \sigma_{k,L_1}^0\}$, where each $\sigma_{k,l}^0$ satisfying $\sigma_{k,l}^0 \sim \mathcal{CN}(0, C_0 d_k^{-\alpha_{loss}} / L_1)$, $l = 1, \dots, L_1$, where C_0 corresponds to the path loss at the reference distance of 1 m, d_k is the propagation distance between the BS and the k -th user, $\alpha_{loss} = 2.8$ is the path-loss exponent. The transmit power of BS is set as 20dBm. The noise power is set as $\sigma^2 = -80\text{dBm}$ [53].

Fig. 2 describes the converge behaviors of our proposed SOCP-based and analytic-based PDD methods updating the beamformer $\mathbf{W}$. To ensure a fair comparison, both the SOCP and analytic implementations start from the same initial point. As depicted in Fig. 2, the left and right subfigures

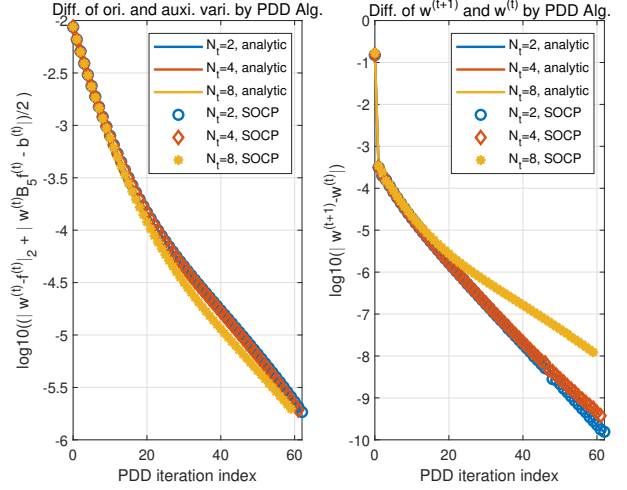


Fig. 2. Convergence of PDD method optimizing $\mathbf{W}$ (Alg. 1).

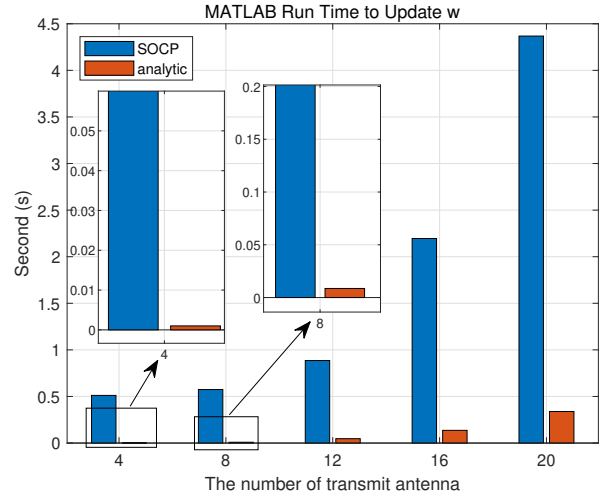


Fig. 3. Comparison of time complexity between SOCP and analytical methods for updating $\mathbf{w}$.

respectively display the differences $(\|\mathbf{w}^{(t)} - \mathbf{f}^{(t)}\| + |b^{(t)} - (\mathbf{w}^{(t)})^H \mathbf{B}_5 \mathbf{f}^{(t)}|)/2$ and $\|\mathbf{w}^{(t+1)} - \mathbf{w}^{(t)}\|$ in log domain for different settings of the number of BS transmit antenna N_t , alongside the progression of PDD iterations. As indicated by Fig. 2, both the SOCP and the analytic-based methods achieve nearly identical performance, typically converging well within 60 iterations.

Furthermore, we explore the complexity of our proposed analytic-based PDD algorithm under various N_t settings. The MATLAB runtime comparisons between the SOCP-based and analytic-based methods are detailed in Fig. 3 and Fig. 4. It is important to note that these methods achieve identical performance, as illustrated in Fig. 2. Moreover, Fig. 3 and Fig. 4 clearly show that our analytic solutions are highly efficient. The runtime of the analytic-based method is generally two or three orders of magnitude less than that of the SOCP-based method.

Firstly, we label the proposed PDD-based algorithm as

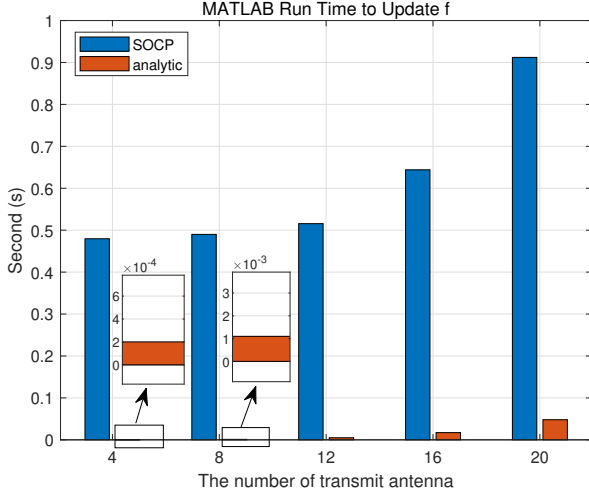


Fig. 4. Comparison of time complexity between SOCP and analytical methods for updating $\mathbf{f}$.

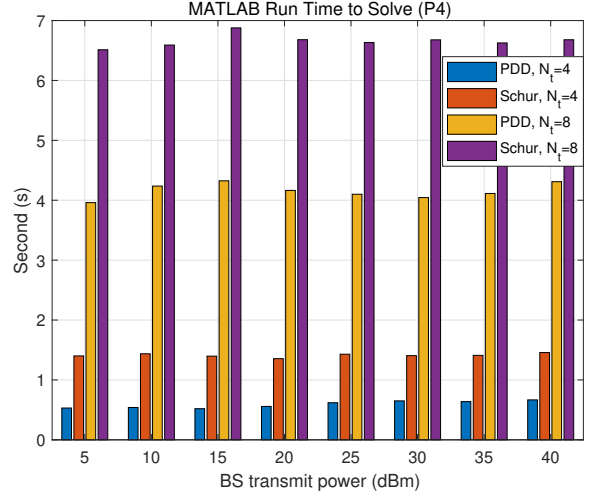


Fig. 6. Comparison of time complexity between PDD-based and Schur-based methods.

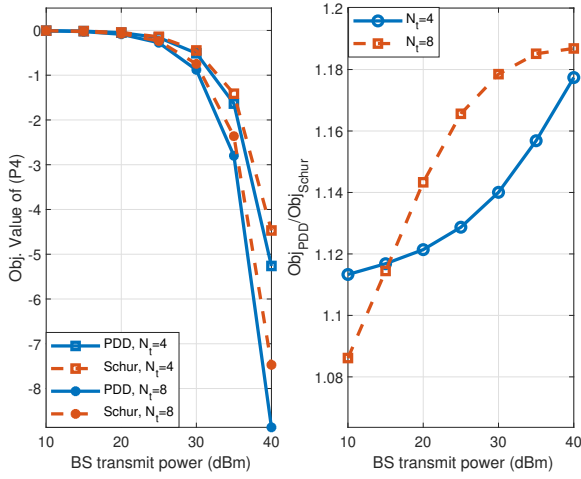


Fig. 5. Comparison of performance between PDD-based and Schur-based methods.

“PDD”. To verify the performance and time complexity of the proposed PDD-based method for solving (P3), we introduce the Schur complement proposed in [38] as a benchmark, labeling it as “Schur”. In Fig. 5, we evaluate the performance of the PDD-based and Schur-based algorithms across varying numbers of transmit antennas versus BS transmit power. The left and right sub-figures correspond respectively to the objective value for solving problem (P3) and the ratio of the objective values between the PDD-based and Schur-based algorithms, i.e., $\text{Obj}_{\text{PDD}}/\text{Obj}_{\text{Schur}}$. The left sub-figure illustrates that the curves for all cases decrease as the BS transmit power P_{BS} increases. Moreover, the performance achieved by the PDD-based method closely approximates that of the Schur-based method, thereby affirming the efficacy of the PDD-based approach.

Fig. 6 illustrates the computation time of the PDD-based and Schur-based algorithms across varying numbers of transmit

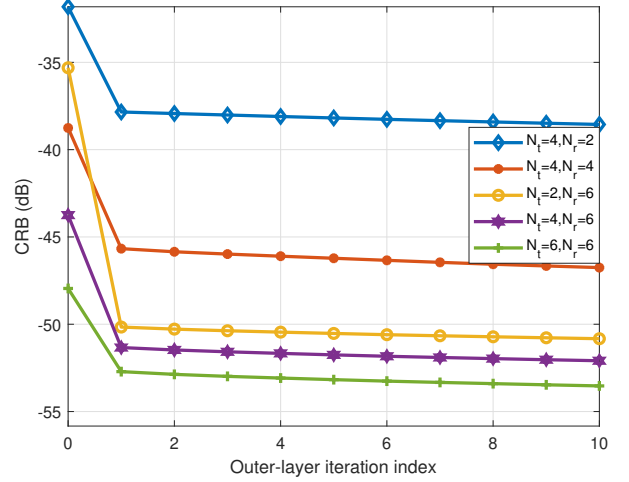


Fig. 7. Convergence of Alg. 2.

antennas. It is evident that the computation time required by the PDD-based approach is significantly lower than that of the Schur-based approach. In the case with “ $N_t = 4$ ” transmit antennas, the computation time of the PDD-based method is on average reduced by 59% compared to the Schur-based method. Similarly, in the “ $N_t = 6$ ” case, the computation time of the PDD-based algorithm is reduced by 38%. Moreover, combining the data from Fig. 5 and Fig. 6 allows us to verify the effectiveness and efficiency of the PDD-based method.

In Fig. 7, we present the overall convergence performance of the proposed algorithm to tackle the problem (P1) with different BS transmit and/or receive antenna number requirements. As seen from Fig. 7, the proposed algorithm can achieve monotonic decrease in CRB and generally achieves significant beamforming gain within around 10 outer iterations.

For comparison, we consider the following two cases: 1)“FA”: Our proposed algorithm (i.e., Alg. 2) in Sec. III. 2) “FPA”: both transmit and receive antennas of BS are equipped

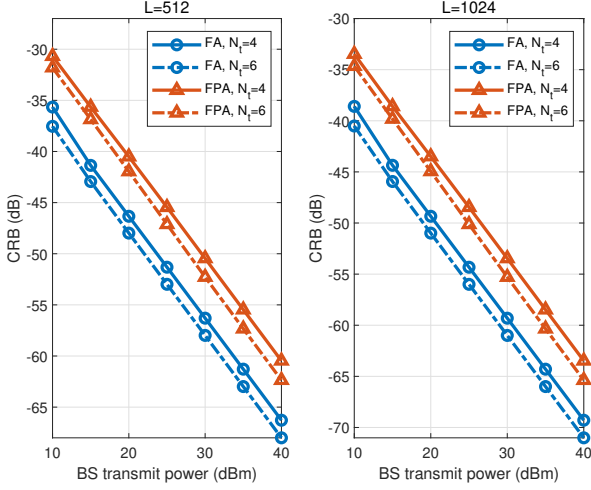


Fig. 8. The impact of BS transmit power.

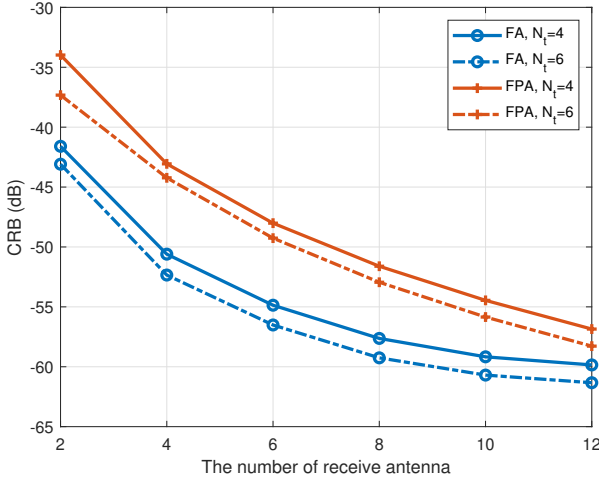


Fig. 9. The impact of BS receive antenna number N_r .

with FPA-based array. Fig. 8 describes the relationship between the BS transmit power P_{BS} and CRB. The left and right subplots correspond to different numbers of time slot L , respectively. In this test, the transmit power varies from 10dBm to 40dBm. The CRB of both the “FA” and “FPA” schemes decreases significantly as the transmit power increases, and the FA-based scheme improves substantially the estimated precision compared to the FPA-based case. Besides, compared to the cases with “ $L=512$ ” and “ $L=1024$ ”, a longer time slot will yield a greater estimated gain.

Fig. 9 depicts the impact of increasing the number of receive antenna at the BS. Initially, it is observed that as the number of receive antenna N_r increases from 2 to 12, there is a consistent decrease in the CRB across all schemes. This improvement can be attributed to the additional DoFs provided by the increased number of receive antenna. Then, the scheme employing FA significantly outperforms the FPA one. This superior performance is expected as FA are strategically positioned to modify the channel conditions between the BS and the target, thus

enhancing estimation accuracy. Additionally, the CRB for the “FA” scenario reaches saturation more quickly than that for the “FPA” scheme, indicating a quicker limit to performance gains with increased receive antenna numbers.

V. CONCLUSIONS

This paper explores the joint design of active beamforming and position coefficients in an FA-assisted uplink ISAC system, which simultaneously performs target angle estimation and uplink communication. We propose a toolbox-free and low-complexity solution to jointly design BS probing beamforming, users’ power allocation, receiving processors and FA position coefficients. This approach aims to enhance both target estimation and communication functionalities. Numerical results present the efficiency and effectiveness of our proposed algorithm and highlight the advantages of deploying FA in the uplink ISAC system.

APPENDIX

A. The Derivation of The FIM in (18)

Firstly, by vectorizing (13), we will have

$$\mathbf{a}_1 = \alpha \text{vec}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r) + \mathbf{n}_1, \quad (85)$$

where $\mathbf{n}_1 = \text{vec}(\mathbf{N}) \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{N_t L})$. Furthermore, the derivatives of $\mathbf{a}_1$ w.r.t. θ_0 and α can be formulated as

$$\frac{\partial \mathbf{a}_1}{\partial \theta_0} = \alpha \text{vec}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{S}_r), \quad (86)$$

$$\frac{\partial \mathbf{a}_1}{\partial \alpha} = [1, j] \otimes \text{vec}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r), \quad (87)$$

respectively, where $\dot{\mathbf{A}}(\theta_0)$ denotes the derivative of the cascade channel $\mathbf{A}(\theta_0)$ w.r.t. θ_0 , which is given as

$$\dot{\mathbf{A}}(\theta_0) \triangleq \frac{\partial \mathbf{A}(\theta_0)}{\partial \theta_0} = \dot{\mathbf{a}}_r(\theta_0) \mathbf{a}_t^H(\theta_0) + \mathbf{a}_r(\theta_0) \dot{\mathbf{a}}_t^H(\theta_0), \quad (88)$$

with $\dot{\mathbf{a}}_t(\theta_0)$ and $\dot{\mathbf{a}}_r(\theta_0)$ being defined as the derivatives of $\mathbf{a}_t(\theta_0)$ and $\mathbf{a}_r(\theta_0)$ with θ_0 as follows in (89)–(90), respectively.

The FIM extracted from the observed data (13) is given by [43]

$$[\mathbf{F}]_{i,j} = -\mathbb{E} \left[\frac{\partial^2 \ln(f(\mathbf{Y}_r|\zeta))}{\partial \zeta_i \partial \zeta_j} \right], i, j \in \{1, 2, 3\}. \quad (91)$$

Then, the elements of the FIM $\mathbf{F}$ can be expressed in (92)–(94).

B. Proof of (80)

Proof: Since the feasible set of the problem (P27) is existing, the equation $\tau_3 d_{r,n}^2 + b_{14,n} d_{r,n} + c_{26,n} = 0$ possesses at least one solution. Firstly, if the above equation only has one solution, i.e., $d_{r,n,r1} = d_{r,n,r2} \in [d_{r,n-1} + d_{min}, d_{r,n+1} - d_{min}]$, we can directly set $d_{r,n}^* = d_{r,n,r1}$.

Furthermore, if the equation $\tau_3 d_{r,n}^2 + b_{14,n} d_{r,n} + c_{26,n} = 0$ has two roots, i.e., $d_{r,n,r1} < d_{r,n,r2}$, the feasible area of (P27) can be determined by the following two cases:

CASE-1: When $d_{r,n-1} + d_{min} < d_{r,n,r1}$, we have two sub-cases:

$$\dot{\mathbf{a}}_t(\theta_0) \triangleq \frac{\partial \mathbf{a}_t}{\partial \theta_0} \quad (99)$$

$$= \beta_t \left[j \frac{2\pi}{\lambda} d_{t,1} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{t,1} \sin(\theta_0)}, j \frac{2\pi}{\lambda} d_{t,2} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{t,2} \sin(\theta_0)}, \dots, j \frac{2\pi}{\lambda} d_{t,N_t} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{t,N_t} \sin(\theta_0)} \right]^T,$$

$$\dot{\mathbf{a}}_r(\theta_0) \triangleq \frac{\partial \mathbf{a}_r}{\partial \theta_0} \quad (100)$$

$$= \beta_r \left[j \frac{2\pi}{\lambda} d_{r,1} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{r,1} \sin(\theta_0)}, j \frac{2\pi}{\lambda} d_{r,2} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{r,2} \sin(\theta_0)}, \dots, j \frac{2\pi}{\lambda} d_{r,N_r} \cos(\theta_0) e^{j \frac{2\pi}{\lambda} d_{r,N_r} \sin(\theta_0)} \right]^T.$$

$$\mathbf{F}_{\theta_0\theta_0} \quad (92)$$

$$= \frac{2}{\sigma^2} \Re\{(\text{avec}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{S}_r))^H \text{avec}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{S}_r)\} = \frac{2|\alpha|}{\sigma^2} \Re\{\text{tr}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{S}_r \mathbf{S}_r^H \mathbf{W}^H \dot{\mathbf{A}}^H(\theta_0))\}$$

$$= \frac{2L|\alpha|}{\sigma^2} \text{tr}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{W}^H \dot{\mathbf{A}}^H(\theta_0)) = \text{tr}(\dot{\mathbf{A}}^H(\theta_0) \dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{W}^H),$$

$$\mathbf{F}_{\theta_0\alpha} \quad (93)$$

$$= \frac{2}{\sigma^2} \Re\{(\text{avec}(\dot{\mathbf{A}}(\theta_0) \mathbf{W} \mathbf{S}_r))^H [1, j] \otimes \text{vec}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r)\} = \frac{2}{\sigma^2} \Re\{(\alpha^*[1, j] \text{tr}(\mathbf{S}_r^H \mathbf{W}^H \dot{\mathbf{A}}^H(\theta_0) \mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r))\}$$

$$= \frac{2L}{\sigma^2} \Re\{(\alpha^*[1, j] \text{tr}(\mathbf{W}^H \dot{\mathbf{A}}^H(\theta_0) \mathbf{A}(\theta_0) \mathbf{W}))\} = \frac{2L}{\sigma^2} \Re\{(\alpha^*[1, j] \text{tr}(\dot{\mathbf{A}}^H(\theta_0) \mathbf{A}(\theta_0) \mathbf{W} \mathbf{W}^H))\},$$

$$\mathbf{F}_{\alpha\alpha} \quad (94)$$

$$= \frac{2}{\sigma^2} \Re\{([1, j] \otimes \text{vec}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r))^H [1, j] \otimes \text{vec}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r)\} = \frac{2}{\sigma^2} \Re\{[1, j]^H [1, j] \text{tr}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{S}_r \mathbf{S}_r^H \mathbf{W}^H \mathbf{A}^H(\theta_0))\}$$

$$= \frac{2L}{\sigma^2} \text{tr}(\mathbf{A}(\theta_0) \mathbf{W} \mathbf{W}^H \mathbf{A}^H(\theta_0)) = \frac{2L}{\sigma^2} \text{tr}(\mathbf{A}^H(\theta_0) \mathbf{A}(\theta_0) \mathbf{W} \mathbf{W}^H) \mathbf{I}_2.$$

case-①: If $d_{r,n,r1} \leq d_{r,n+1} - d_{min} < d_{r,n,r2}$, the feasible set is $[d_{r,n,r1}, d_{r,n+1} - d_{min}]$.

case-②: If $d_{r,n,r2} \leq d_{r,n+1} - d_{min}$, the feasible set is expressed as $[d_{r,n,r1}, d_{r,n,r2}]$.

CASE-2: When $d_{r,n,r1} \leq d_{r,n-1} + d_{min}$, we have the following two sub-cases:

case-①: If $d_{r,n+1} - d_{min} \leq d_{r,n,r2}$, the set $[d_{r,n-1} + d_{min}, d_{r,n+1} - d_{min}]$ is feasible set.

case-②: If $d_{r,n,r2} \leq d_{r,n+1} - d_{min}$, $[d_{r,n-1} + d_{min}, d_{r,n,r2}]$ is a feasible area.

Based on **CASE-1** and **CASE-2**, we can obtain the feasible area of (P27) as follows:

$$d_{r,n} \in [\max\{d_{r,n,r1}, d_{r,n-1} + d_{min}\}, \min\{d_{r,n,r2}, d_{r,n+1} - d_{min}\}]. \quad (95)$$

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