

LETTER TO THE EDITOR

Eppur si eclissa: Eccentric low-mass companions and time-in-dust selection explain long secondary periods

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ABSTRACT

Context. Long Secondary Periods (LSPs) are observed in $\sim 1/3$ of pulsating red giants yet remain unexplained. Four key observational constraints anchor the discussion: (i) a $\sim 30\%$ occurrence rate in semi-regular variable AGB stars (SRVs) with a much lower rate (or absence) in regularly pulsating Mira-type AGB stars (Miras), (ii) $\sim 50\%$ of LSP stars show a secondary mid-IR minimum; (iii) Keplerian fits to radial-velocity (RV) curves favour the argument of periastron $\omega > 180^\circ$; and (iv) the RV–light curve phase lag clusters around $-\pi/2$.

Aims. We test whether a close-in, eccentric low-mass companion that only spends part of its orbit within the giant’s dust-formation (wind-launching) zone can match all four empirical facts.

Methods. Guided by observed RV amplitudes and periods of ~ 500 – 1500 days, we adopt a companion’s mass $M_2 \in [0.08, 0.25] M_\odot$, orbital separation $a \in [1.5, 3]$ au, eccentricity $e \leq 0.6$, and take the dust condensation radius $R_{\text{cond}} \sim 2.5$ – 3 au for SRVs (larger for Miras via scaling with luminosity). We compute the time-in-dust fraction f_{dust} (time with $r \geq R_{\text{cond}}$) and apply line-of-sight criteria: an LSP requires orbital inclination $i \geq i_{\text{LSP}}$ and $f_{\text{dust}} \geq f_{\text{min}}$; a secondary mid-IR minimum interpreted as secondary eclipse further needs $i \geq i_{\text{ecl}} > i_{\text{LSP}}$ and superior conjunction. We test the first three empirical facts analytically, then model the RV–light phase offset with 3D hydrodynamical simulations.

Results. Our proposed scenario explains the observed excess of $\omega > 180^\circ$. For SRV-like parameters we obtain an LSP detectability of $\sim 31.6 \pm 0.1\%$, while Mira-type conditions yield $\sim 3.0 \pm 0.1\%$; for both scenarios the conditional secondary mid-IR eclipse fraction is $\sim 44\%$. Our hydrodynamical models place the optical-depth peak just downstream of the companion near apastron, then shift it to ~ 90 – 225° phase offsets later in the orbit – consistent with the RV–light offsets.

Conclusions. A time-in-dust geometric selection for low-mass companions in close eccentric orbits explains the four key empirical facts constraining the LSP mechanism.

Key words. stars: AGB and post-AGB – stars: variables: general – binaries: general – circumstellar matter

1. Introduction

The long-secondary-period (LSP) phenomenon in red giants was first noted by O’Connell (1933). LSPs occur in $\sim 1/3$ of pulsating red giants, with periods ~ 5 – 10 times the primary pulsation period. In time-domain surveys, LSP stars delineate sequence D in the period–luminosity (P – L) plane (Wood et al. 1999). The physical origin remains debated, with two leading hypotheses: i) non-radial oscillatory convective modes in the outer atmosphere (Saio et al. 2015; Takayama & Ita 2020), and (ii) a binary with a close-in, low-mass companion and a *co-orbiting* dusty cloud that periodically obscures the giant (Wood et al. 2004; Soszyński & Udalski 2014; Soszyński et al. 2021).

Four empirical facts strongly constrain viable models. First, LSPs are common among semi-regular AGB variables (SRVs, typically double-mode pulsators, with periods between ~ 60 – 250 days) but rarer among the often more luminous, regularly pulsating Mira-type AGB stars (Miras, single-mode pulsators, periods between ~ 250 – 1000 days) (Soszyński 2022). Miras typically have larger pulsation-enhanced dust-driven wind velocities and mass-loss rates than SRVs owing to their higher luminosities. Second, about half of LSP stars exhibit secondary

mid-IR minima without systematic phase lag between the optical and primary mid-IR minima (Soszyński 2022). Third, Keplerian fits to LSP radial-velocity (RV) curves yield a markedly non-uniform distribution of the argument of periastron, ω , clustered at values $> 180^\circ$ (median $\sim 227^\circ$), i.e., at periastron the red giant is closest to the observer and the lower-mass companion is farther away (Wood et al. 2004; Nicholls et al. 2009). Fourth, observed phase lags between the RV and I band photometric variations cluster near $\Delta\phi \approx -\pi/2$, implying that a potential companion must be $\sim 180^\circ$ out of phase with the dust and gas that induce the brightness minimum of the LSP (Goldberg et al. 2024, Soszyński et al., in prep.).

Current oscillatory convective-mode calculations struggle to reproduce sequence D periods under standard convection parameters and do not naturally account for the combined RV, colour-magnitude, and mid-IR eclipse phenomenology. By contrast, the first two points emerge naturally from binary–dust geometries, whereas the third and fourth points have been cited as a challenge against the binary picture (Nicholls et al. 2009; Goldberg et al. 2024). More recently, the binary interpretation has been strengthened by the finding that $\sim 50\%$ of LSP stars exhibit mid-IR secondary minima (Soszyński et al. 2021), a natural signature of a dusty structure co-orbiting with a companion and producing

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a secondary eclipse. Meanwhile, RV surveys report full amplitudes of only a few km s^{-1} (e.g., Hinkle et al. 2002; Wood et al. 2004; Nicholls et al. 2009). Interpreted as orbital motion, this implies companions near the brown-dwarf/very-low-mass main-sequence boundary ($M_2 \sim 0.08\text{--}0.25 M_\odot$) on AU-scale orbits ($a \sim 1.5\text{--}3$ au), with typical eccentricities around $e \sim 0.3$ when fitted with Keplerian models (Nicholls et al. 2009); see App. A. This sits in the “brown-dwarf desert”, i.e. a relative paucity of companions in this mass range around solar-type stars (e.g., McCarthy & Zuckerman 2004; Grether & Lineweaver 2006), apparently at odds with the high ($\sim 30\%$) LSP incidence among low-mass stars as they ascend the AGB.

One reconciliation proposed by Soszyński et al. (2021) is that many LSP companions did not form at their present mass: planets orbiting AGB stars can accrete from the stellar wind and/or via direct primary-to-companion mass transfer, growing into brown dwarfs or even very low-mass stars (Retter 2005). This view meshes with demographics showing that planets around evolved hosts tend to be more massive than those around main-sequence stars (Jones et al. 2014; Niedzielski et al. 2015). Updated occurrence studies also point in this direction: giant planets at a few au are relatively common in field FGK samples (Fulton et al. 2021), and a sample of low-luminosity giants (median mass of $1.21 \pm 0.16 M_\odot$) shows $33.3^{+9.0}_{-7.1}\%$ occurrence rate for Jovian planets within 5 au (Jones et al. 2021). For higher-mass primaries (initial mass $\geq 1.5 M_\odot$) the close-binary fraction around giants will be higher due to the primordial multiplicity-mass relation. This suggests that the fraction of AGB progenitors hosting a low-mass companion capable of producing a dusty wake at $a \sim 1\text{--}few$ au is substantial, plausibly $f_{\text{comp}} \sim 40\text{--}60\%$ (or higher). Writing the SRV LSP incidence as $f_{\text{LSP}} = f_{\text{comp}} \times \langle P(\text{detect} | \text{comp}) \rangle$, the observed $f_{\text{LSP}} \approx 0.30$ then implies an order-of-magnitude requirement $\langle P(\text{detect} | \text{comp}) \rangle \sim 0.7\text{--}0.9$, which we will use as an empirical benchmark to constrain the geometry in the sections that follow.

The unresolved challenge is that Keplerian binary fits reproduce the observed RV amplitudes but, for random orientations, predict a uniform distribution of ω (Nicholls et al. 2009); the observed excess at $\omega > 180^\circ$ is therefore puzzling. Moreover, the binary-eclipse picture predicts an RV- I -band phase lag of $\Delta\phi \approx +\pi/2$ at inferior conjunction (companion in front; see Fig. E.1), contrary to the observed phase lag which mainly lies between $-\frac{1}{5}\pi$ and $-\frac{4}{5}\pi$ (see Fig. 3 of Goldberg et al. 2024). Here we argue that this is a selection effect set by where dust forms around the AGB star and by the companion spending only part of its orbit within that dust-formation region. We propose a simple geometric-selection model in which an eccentric, close-in low-mass companion captures most dust near apastron, producing a trailing spiral wake whose optical depth then peaks slightly downstream of the companion. As the orbit advances, the dominant optical depth column shifts to the opposite spiral arm, nearly anti-phased with the companion. The resulting time-in-dust gate, coupled with line-of-sight projection, can explain all four key LSP observational constraints.

2. Model and geometric analysis

2.1. Proposed model and the ω bias

In AGB systems, dust condenses at radii of a few stellar radii ($R_\star \sim 1\text{--}1.5$ au), where radiation pressure on grains helps drive the wind (Höfner & Olofsson 2018). Close-in companions lie in the dust-free cavity for $r < R_{\text{cond}}$; conversely, only $r \geq R_{\text{cond}}$ samples the dust-formation zone. In short-period *eccentric* binaries, the periastron may lie inside the dust-free cavity and

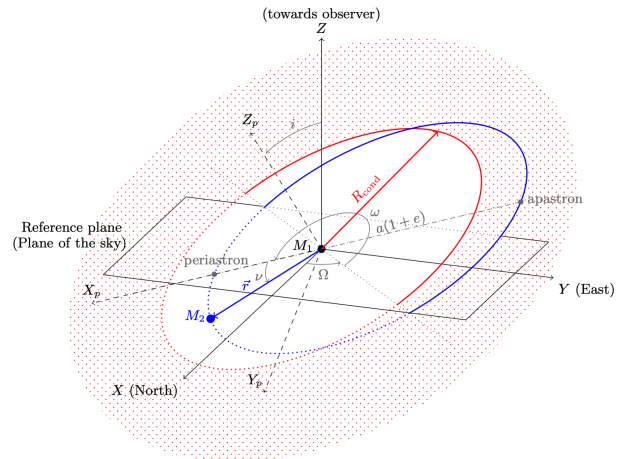


Fig. 1: **Elliptic orbit of a low-mass companion (blue) through the AGB dust-forming region (red).** The orbital-plane frame (X_p, Y_p, Z_p) (dashed) and focal frame (X, Y, Z) (solid) are shown. The position vector $\mathbf{r}$ from M_1 to M_2 is set by Ω (longitude of the ascending node), i (inclination), ω (argument of periastron; here 225°)¹, a (semi-major axis), e (eccentricity), and T_0 (time of periastron passage), alongside the true anomaly ν . The companion and its dusty wake cross the spherical dust zone (red dotted) over an apastron-centred phase interval.

the companion intersects the dust-formation region only over a limited portion of the orbit, with residence time biased towards apastron (Fig. 1). For $e = 0.3$, the ratio of orbital speeds is $v_p/v_a = (1+e)/(1-e) \approx 1.86$. Therefore, once $r \geq R_{\text{cond}}$ is reached, the longer residence time near apastron – together with the denser dusty wake – maximises the line-of-sight optical depth near the apastron sector. Light-curve modulations caused by dust captured in the companion’s gravitational wake will then be stronger. For configurations with $\omega > 180^\circ$ (as in Fig. 1), the companion’s passage through the dusty zone occurs at *inferior conjunction*, producing a deeper light-curve minimum than for $\omega < 180^\circ$, where inferior conjunction typically finds the companion in the dust-free region with a much weaker spiral wake (see also Sect. 2.6). *This scenario naturally explains the observed bias in ω as an observational selection effect, not an intrinsic property of binary orbital configurations.*

Our scenario assumes eccentric orbits, supported by Nicholls et al. (2009) and the $(e - \log P)$ diagram for post-AGB binaries with main-sequence companions, which shows large eccentricities for $P \gtrsim 100$ d (Van Winckel 2025). For companions down to $\sim 0.1\text{--}0.2 M_\odot$, the $(e - \log P)$ distribution at $P \gtrsim 100$ d remains broad – with a large fraction at $e \gtrsim 0.1$ (Moe & Di Stefano 2017). Because tidal circularization is far less efficient for such low-mass companions (Zahn 1977), eccentric orbits are expected.

In what follows, we assess the proposed scenario in the light of the other three empirical facts mentioned in Sect. 1. Because circumstellar dust around an AGB star can exist only at or beyond the condensation radius, the detectability of a companion’s dusty wake is set by (i) the fraction of the orbit spent at $r \geq R_{\text{cond}}$ (Sect. 2.2) and (ii) a favourable line-of-sight geometry (Sect. 2.3). Our goal here is not to map the full parameter space, but to test the viability of the proposed scenario under representative AGB-like conditions. In Sect. 2.4–2.5 we quantify the resulting detection fractions, constrained by the empirical $\langle P(\text{detect} | \text{comp}) \rangle \sim 0.7\text{--}0.9$. The phase offset between RV and light curves is discussed in Sect. 2.6.

¹By convention, RV solutions quote the argument of periastron of the observed star, ω ; the companion’s is $\omega_c = \omega + 180^\circ$. For values of $\omega > 180^\circ$, the red giant is closest to the observer at periastron.

2.2. Dust condensation radii and the crossing condition

For AGB stars, interferometry data place the dust-condensation radius at a few stellar radii (e.g. Wittkowski et al. 2007; Sacuto et al. 2013; Karovicova et al. 2013); a simple scaling being

$$R_{\text{cond}} \simeq \left(\frac{L_{\star}}{16\pi\sigma T_{\text{cond}}^4} \right)^{1/2} \sqrt{\frac{Q_{\text{abs}}(T_{\star})}{Q_{\text{abs}}(T_{\text{sub}})}}, \quad (1)$$

with T_{cond} the dust-condensation temperature ($T_{\text{cond}} \sim 1200$ – 1500 K) and Q_{abs} the dust-grain absorption efficiency (Lamers & Cassinelli 1999). Using representative luminosities $L_{\text{SRV}} \sim (3\text{--}5) \times 10^3 L_{\odot}$ and $L_{\text{Mira}} \sim 10^4 L_{\odot}$ gives representative values $R_{\text{cond}} \sim 2.5\text{--}3$ au for SRVs and $3.5\text{--}4.5$ au for Miras.

In our model, *gravitational capture* of circumstellar dust by the companion peaks near apastron (Fig. 1, Sect. 2.6). At the same time, *new-grain production* is maximal in the shock wake driven by the companion near the primary (Fig. H1 in Danilovich et al. 2025). At other phases the companion resides inside the dust-free cavity, while the previously formed dusty wake is advected outward; the spiral arm broadens, its density contrast fades, and detectability drops. The orbital time fraction spent in the dusty zone is (see App. B)

$$f_{\text{dust}} = \begin{cases} 1, & R_{\text{cond}} \leq r_p = a(1 - e), \\ 0, & R_{\text{cond}} \geq r_a = a(1 + e), \\ \frac{(2\pi - 2\theta) + 2e \sin \theta}{2\pi}, & \text{otherwise,} \end{cases} \quad (2)$$

where $\theta = \arccos Y \in [0, \pi]$, and $Y = (1 - R_{\text{cond}}/a)/e$. For SRV-like numbers ($a = 2.3$ au, $e = 0.3$, $R_{\text{cond}} = 2.7$ au), Eq. (2) gives $f_{\text{dust}} \approx 0.38$. For Mira-like $R_{\text{cond}} \in 3.5\text{--}4.5$ au, one obtains $f_{\text{dust}} = 0$; only larger e (and/or larger a) yields non-zero dusty phases. *This naturally explains why the LSP phenomenon has a higher occurrence rate in SRVs than in more luminous Miras.*

For Miras, higher mass-loss rates and hence larger circumstellar optical depths reduce the contrast of the companion's wake against the optically thick wind, making wake-induced minima harder to isolate. This offers an additional, observation-driven reason for the lower LSP detection rate in Miras.

2.3. Geometric selection model

For an LSP to be observable the system must be sufficiently inclined and the companion must spend a minimum fraction of the orbit in the dusty zone:

$$i \geq i_{\text{LSP}} \quad \text{and} \quad f_{\text{dust}} \geq f_{\text{min}}. \quad (3)$$

For an average detection fraction $\langle P(\text{detect} \mid \text{comp}) \rangle \approx 0.7\text{--}0.9$, this implies that (see App. C)

$$i_{\text{LSP}} \approx \arccos(0.7 - 0.9) = 26^\circ\text{--}46^\circ. \quad (4)$$

A secondary mid-IR eclipse further requires $i \geq i_{\text{ecl}}$ and that the dusty wake then lies at superior conjunction. Conditioning on systems that already satisfy $i \geq i_{\text{LSP}}$, the fraction that also exceed the mid-IR secondary eclipse threshold is (see App. C)

$$f_{\text{ecl}|\text{LSP}} \approx \frac{1}{2} \frac{\cos i_{\text{ecl}}}{\cos i_{\text{LSP}}}. \quad (5)$$

2.4. Analytic estimate of the LSP geometric properties

Constraining i_{LSP} : At conjunction, the sky-projected separation (impact parameter) between the stellar centre and the companion (or its dusty wake) is

$$b \equiv r_{\perp} = r \sqrt{\cos^2(\omega + \nu) + \sin^2(\omega + \nu) \cos^2 i}. \quad (6)$$

When conjunction happens near apastron $b \simeq a(1 + e) \cos i$. An occultation requires the wake to cross the stellar disk,

$$b \leq R_{\star} + w_{\text{eff}}, \quad (7)$$

where $R_{\star}$ is the stellar radius and w_{eff} the effective half-width of the dusty wake. This yields the inclination threshold

$$i_{\text{LSP}} \simeq \arccos \left(\frac{R_{\star} + w_{\text{eff}}}{a(1 + e)} \right). \quad (8)$$

The density scale height of the dusty wake can be approximated by the Bondi–Hoyle–Lyttleton (BHL) capture radius. When the companion's motion is mostly transverse to the flow (as near apastron/periastron for a radial wind)

$$R_{\text{BHL}} \approx \frac{2GM_2}{v_{\text{orb}}^2 + v_w^2 + c_s^2}.$$

At apastron, for a chosen representative SRV example ($M_1 = 1.5 M_{\odot}$, $M_2 = 0.125 M_{\odot}$, $a = 2.3$ au, $e = 0.3$, $v_w = 10 \text{ km s}^{-1}$) and an adiabatic sound speed $c_s = 2 \text{ km s}^{-1}$, one obtains $v_{\text{orb}} = v_a \approx 18.4 \text{ km s}^{-1}$ and $R_{\text{BHL}} \approx 0.5$ au. However, the BHL approximation is not valid in the case of low wind speeds when the system is in the wind-Roche lobe overflow regime. In that case, the accretion radius can be larger than the Roche Lobe radius of the companion. A convenient upper scale is the companion's Hill radius (Decin et al. 2020)

$$w_{\text{eff}} \approx R_H \sim r \left(\frac{M_2}{3M_1} \right)^{1/3},$$

which, for our SRV example, gives $R_H \approx 0.49$ au at periastron and $R_H \approx 0.91$ au at apastron. In general we adopt a BHL-based scale for the wake and parameterise the effective half-width as $w_{\text{eff}} = k R_{\text{BHL}}$, with $k > 1$ to reflect that the occulting structure may be broader than the strict accretion cylinder. For the chosen representative SRV example, and taking $R_{\star} \approx 1.5$ au and $k \in [0.25, 2]$ (see Sect. 2.6), yields $i_{\text{LSP}} \approx 33\text{--}57^\circ$, *in good agreement overall with the geometric estimate* (Eq. 4).

Secondary mid-IR eclipse and i_{ecl} : A mid-IR secondary eclipse occurs when, at (near) superior conjunction, the warm IR-emitting core of the dusty wake is partly occulted by the stellar disk. This requires a slightly smaller impact parameter than for detectable optical obscuration because the mid-IR dip is an occultation of the more compact, warm dust emission. We parametrise the effective half-width of the IR-emitting core as $w_{\text{IR}} = \zeta w_{\text{eff}}$. Because the mid-IR emission is centrally concentrated around the warmest part of the wake, $\zeta \lesssim 1$ is expected. The corresponding inclination threshold is

$$i_{\text{ecl}} \simeq \arccos \left(\frac{R_{\star} + w_{\text{IR}}}{a(1 + e)} \right) = \arccos \left(\frac{R_{\star} + \zeta w_{\text{eff}}}{r_a} \right). \quad (9)$$

Using the same SRV values as before for $k = 1$ (hence $i_{\text{LSP}} \sim 48^\circ$) and $\zeta \approx 0.6\text{--}0.8$ yields $i_{\text{ecl}} \approx 51\text{--}53^\circ$, hence i_{ecl} is typically only a few degrees larger than i_{LSP} . Eq. (5) then gives $f_{\text{ecl}|\text{LSP}} \approx 45\text{--}47\%$, *matching the observed $\sim 50\%$ of mid-IR eclipse events.*

2.5. Monte-Carlo analysis

We perform a Monte-Carlo experiment drawing 10^5 systems per run and repeating the experiment over 125 independent random seeds. For each draw we sample orbital/stellar/wind parameters from the ranges listed below, assign random viewing geometry, and apply the detectability criteria of Sects. 2.2–2.3.

As will be discussed in Sect. 2.6, k can range between 0.1 and 2, with larger values further downstream the spiral flow. Sampling $a \in [1.5, 3]$ au, $e \lesssim 0.6$ (median ~ 0.3 , Nicholls et al. 2009),

$M_1 \in [1.0, 1.5] M_\odot$, $M_2 \in [0.08, 0.25] M_\odot$, $v_w \in [8, 12] \text{ km s}^{-1}$, $R_\star \in [1.0, 1.5] \text{ au}$, $R_{\text{cond}} \in [2.5, 3] \text{ au}$, $k \in [0.25, 2]$, $\zeta \in [0.5, 0.9]$, and $f_{\text{dust}} \geq 0.1$, we obtain an SRV LSP detectability fraction $f_{\text{LSP}} \approx 31.6 \pm 0.1\%$ and a conditional secondary mid-IR eclipse fraction $f_{\text{ec||LSP}} \approx 44.8 \pm 0.3\%$. Increasing $a \in [3.5, 4.5] \text{ au}$ and $v_w \in [10, 20] \text{ km s}^{-1}$ to mimic Mira-type conditions yields $f_{\text{LSP}} \approx 3.0 \pm 0.1\%$ and a conditional secondary mid-IR eclipse fraction $f_{\text{ec||LSP}} \approx 44.0 \pm 0.9\%$. The eclipse fraction near one-half emerges naturally from the geometry, while the overall LSP incidence decreases as R_{cond}/a increases (Mira-like conditions), consistent with the observed SRV–Mira contrast.

2.6. Phase offset between RV and light curves

To study the origin of the observed phase offset between RV curves and I -band photometry, we run three-dimensional smoothed-particle hydrodynamics simulations with PHANTOM (Price et al. 2018), with numerical binary setup as described in Malfait et al. (2024). The opacity is calculated using the analytical expression proposed by Bowen (1988) with the dust equilibrium temperature being calculated under the Lucy approximation, as implemented by Esseldeurs et al. (2023). Four representative snapshots are shown in Fig. 2. In this simulation, the effective half-width of the inner spiral wake is $w_{\text{eff}} \approx 0.25\text{--}2.0 R_{\text{BHL}}$ depending on the cut angle (see App. D)

As the companion approaches apastron within the dust-formation region, the wake densifies and the optical depth τ increases; near periastron the wake is more rarefied and the overall τ is lower (see Fig. 2, panel (c)). This supports our interpretation of the observed $\omega > 180^\circ$ bias. The locus of maximum optical depth varies with orbital phase. Near apastron, the brightest ridge usually lies in the wake, slightly downstream of the companion. For much of the orbit, however, the dominant peak occurs $\sim 90^\circ\text{--}225^\circ$ out of phase with the companion, i.e. the phase offset between the RV and photometric variations occurs at $[-\pi, -\frac{1}{4}\pi]$. Because the observed flux integrates extinction along the entire line of sight, this geometry naturally produces *photometric minima that are approximately anti-phased with the companion, consistent with the observed RV– I phase lag reported by Goldberg et al. (2024).*

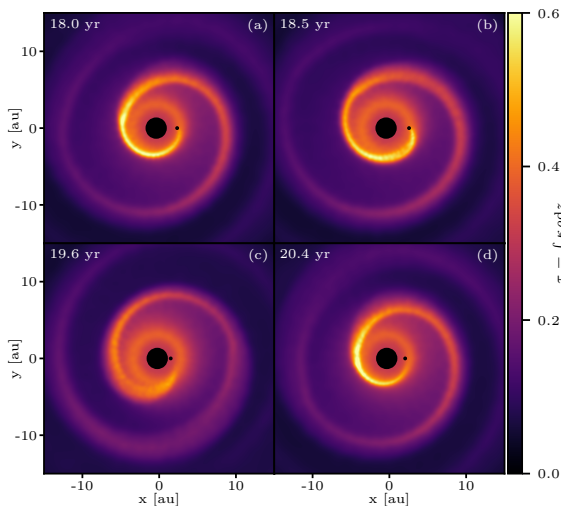


Fig. 2: Optical-depth maps along the z axis for a slice through the orbital plane for a binary setup with $M_1 = 1.5 M_\odot$, $M_2 = 0.125 M_\odot$, $a = 2.3 \text{ au}$, $e = 0.3$, $R_{\text{cond}} = 3 \text{ au}$ and mass-loss rate of $5 \times 10^{-7} M_\odot \text{ yr}^{-1}$. Plots are in the co-moving frame, with the center of mass at $(0, 0, 0)$ and the low-mass companion to the right. Video available at Video F.1.

3. Conclusions

A geometric and time-in-dust criterion – set by the orbit’s fraction of an eccentric, close-in low-mass companion with $r \geq R_{\text{cond}}$ and by line-of-sight geometry from a gravitationally focused dusty wake – simultaneously accounts for (i) the $\sim 30\%$ LSP occurrence rate in SRVs versus the much lower rate in Miras, (ii) the $\sim 50\%$ fraction of LSP stars that show a secondary mid-IR eclipse, (iii) the observed excess of fitted $\omega > 180^\circ$, and (iv) the phase lag between RV and I band light curves clustering around $-\pi/2$. The framework yields testable trends with wind speed, companion mass, and wake breadth.

Our selection model is intentionally minimal and does not account for (i) time variability of R_{cond} , which introduces scatter in the eclipse fraction but does not change the qualitative SRV–Mira contrast or the ω bias. Moreover, (ii) the effective occulting width, w_{eff} , encapsulates bow-shock compression and the dusty sheath, and is uncertain. The parameters k and ζ depend on the specific orbital configuration as well as on the abundance and size distribution of the newly formed dust grains. Lowering k reduces the SRV LSP detectability fraction, whereas decreasing ζ reduces the conditional secondary mid-IR eclipse fraction. In addition, (iii) non-radial/convective pulsation modes (Saio et al. 2015) may contribute to colour/phase relations in some objects. Future work should couple 3D hydrodynamics and radiative transfer to predict multi-band light curves and RV line-shape diagnostics in function of the system parameters.

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Appendix A: Orbital parameters

Published LSP RV curves typically have semi-amplitudes $K_1 \sim 1\text{--}3 \text{ km s}^{-1}$ (full amplitudes $\sim 2\text{--}6 \text{ km s}^{-1}$) (Hinkle et al. 2002; Wood et al. 2004; Nicholls et al. 2009). For a representative full amplitude of 3.5 km s^{-1} (i.e. $K_1 = 1.75 \text{ km s}^{-1}$) and periods $P \sim 500\text{--}1500 \text{ d}$, the spectroscopic mass function

$$f(M) \equiv \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = 1.036 \times 10^{-7} K_1^3 P_d (1 - e^2)^{3/2} M_\odot, \quad (\text{A.1})$$

(with K_1 in km s^{-1} and P_d in days) yields $f(M) \sim (2.4\text{--}7.2) \times 10^{-4} M_\odot$ for $e \sim 0.3$ (the median eccentricity reported by Nicholls et al. 2009). Assuming $M_1 \sim 1\text{--}1.5 M_\odot$ and $\sin i \sim 1$ gives $M_2 \approx 0.08\text{--}0.14 M_\odot$ (edge-on minima), rising to $\sim 0.12\text{--}0.25 M_\odot$ for a more typical inclination $i \sim 60^\circ$. Kepler's third law then implies $a \sim 1.5\text{--}3 \text{ au}$. In our analysis, we adopt $e \lesssim 0.6$ as a prior, as derived from the LSP RV fits (Nicholls et al. 2009).

Appendix B: Time-in-dust gate

For an ellipse with semi-major axis a and eccentricity e , the orbital radius as a function of eccentric anomaly E is

$$r(E) = a[1 - e \cos E], \quad M = E - e \sin E, \quad (\text{B.1})$$

with M the mean anomaly. Because M advances uniformly in time, we define the time fraction spent in the dusty zone as

$$f_{\text{dust}}(a, e; R_{\text{cond}}) \equiv \frac{\Delta t(r \geq R_{\text{cond}})}{P} = \frac{\Delta M}{2\pi}, \quad (\text{B.2})$$

where the boundary $r = R_{\text{cond}}$ is reached when $1 - e \cos E = R_{\text{cond}}/a$, i.e.

$$\cos E \equiv Y = \frac{1 - R_{\text{cond}}/a}{e}. \quad (\text{B.3})$$

Hence,

$$f_{\text{dust}} = \begin{cases} 1, & R_{\text{cond}} \leq r_p = a(1 - e), \\ 0, & R_{\text{cond}} \geq r_a = a(1 + e), \\ \frac{(2\pi - 2\theta) + 2e \sin \theta}{2\pi}, & \text{otherwise,} \end{cases} \quad (\text{B.4})$$

where $\theta = \arccos Y \in [0, \pi]$.

Appendix C: Geometric selection model

Photometric and radial-velocity observables (e.g., light-curve depth, mid-IR eclipse visibility, and RV semi-amplitude K) are invariant under $(i, \Omega) \mapsto (\pi - i, \Omega + \pi)$. We may therefore fold the inclination to $i \in [0, \pi/2]$ without loss of generality. For an isotropic distribution of orbital planes, $\cos i \sim \mathcal{U}(0, 1)$ on $[0, 1]$, which implies a probability density function $p(i) = \sin i$, and thus the probability of having $i \geq i_0$

$$\Pr(i \geq i_{\text{LSP}}) = \int_{i_{\text{LSP}}}^{\pi/2} \sin i \, di = \cos i_{\text{LSP}}$$

If we require an average detection fraction $\langle P(\text{detect} | \text{comp}) \rangle \approx 0.7\text{--}0.9$, then

$$i_{\text{LSP}} \approx \arccos(0.7\text{--}0.9) = 26^\circ\text{--}46^\circ. \quad (\text{C.1})$$

Conditioning on systems that already satisfy $i \geq i_{\text{LSP}}$, the fraction that also exceed the mid-IR eclipse threshold i_{ecl} (with

$i_{\text{ecl}} \geq i_{\text{LSP}}$; see Sect. 2.3) is

$$\Pr(i \geq i_{\text{ecl}} | i \geq i_{\text{LSP}}) = \frac{\cos i_{\text{ecl}}}{\cos i_{\text{LSP}}}.$$

Because a mid-IR secondary dip additionally requires the far-side (superior-conjunction) configuration and, for random nodes, that occurs half the time, we include a factor $\approx \frac{1}{2}$:

$$f_{\text{ecl|LSP}} \approx \frac{1}{2} \frac{\cos i_{\text{ecl}}}{\cos i_{\text{LSP}}}. \quad (\text{C.2})$$

Appendix D: Effective half-width of the inner spiral arm

We measure the effective half-width of the inner spiral wake, w_{eff} , from the hydrodynamical simulation (Sect. 2.6). We define w_{eff} as the density HWHM: the distance from the ridge (peak density) to where $\rho = \frac{1}{2}\rho_{\text{max}}$, using cuts perpendicular to the local arm tangent. As in Sect. 2.4 we write

$$w_{\text{eff}} \simeq k R_{\text{BHL}}.$$

In general,

$$R_{\text{BHL}} = \frac{2GM_2}{v_{\text{rel}}^2 + c_s^2}, \quad \mathbf{v}_{\text{rel}} \equiv \mathbf{v}_{\text{wind}} - \mathbf{v}_{\text{comp}},$$

evaluated in the instantaneous rest frame of the companion. For a radial wind and Keplerian motion,

$$v_{\text{rel}}^2 = (v_w - v_r)^2 + v_t^2, \quad v_r = \frac{\mu_\star}{h} e \sin \nu, \quad v_t = \frac{\mu_\star}{h} (1 + e \cos \nu),$$

with

$$h = \sqrt{\mu_\star a(1 - e^2)}, \quad \mu_\star \equiv G(M_1 + M_2),$$

and G the gravitational constant. At the apsides ($\nu = 0, \pi$), $v_r = 0$ so $v_{\text{rel}}^2 = v_w^2 + v_{\text{orb}}^2$.

The inner-arm compression ridge peaks only a few degrees downstream of the radial. We therefore set the downstream sampling angle by the flow geometry and define

$$\mu \equiv \arcsin\left(\frac{c_s}{v_{\text{rel}}}\right),$$

so that, with angles measured in the orbital plane from the primary to the companion (increasing prograde), the downstream limb for a prograde orbit is at

$$\theta = -\mu.$$

We measure w_{eff} at $\theta = -\mu$ on a cut perpendicular to the arm.

For comparison, we also measure w_{eff} along the arm–normal set by the local pitch angle ψ , being the angle between the arm tangent and the azimuthal (see Fig. 6 of El Mellah et al. 2020), with

$$\tan \psi = \frac{1}{r} \frac{dr}{d\phi} \simeq \frac{v_w}{v_{\text{orb}}}.$$

With angles measured from the radial and increasing prograde, the arm–normal lies at $\theta = -\psi$. Sampling at $\theta = -\psi$ gives the intrinsic cross-arm width and, in our setup, falls a few–tens of degrees downstream. Larger ψ corresponds to a more open, radially expanding segment (material launched when the companion's angular speed was lower, near apastron), while smaller ψ indicates a tighter, more wound segment (launched near periastron). We also sample the lateral trailing direction at $\theta = -90^\circ$ to

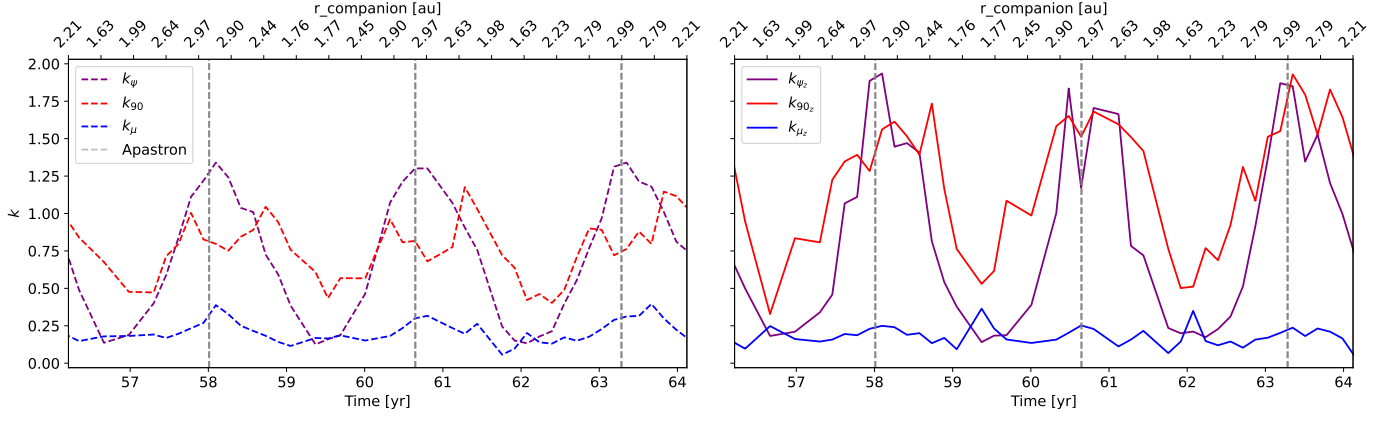


Fig. C.1: Evolution of $k \equiv w_{\text{eff}}/R_{\text{BHL}}$ over ~ 3 orbits for cut angles: $\theta = -\mu$ (red), $\theta = -\psi$ (purple), and $\theta = -90^\circ$ (blue). Bottom x-axis: time; top x-axis: orbital radius r . Left panel: rays in the orbital plane. Right panel: rays perpendicular to the plane (z -direction). In each panel, the apastron passage is indicated with a grey dashed vertical line. Curves are computed from discrete hydro snapshots (only every few time steps retained), so the small-scale jaggedness reflects the sampling cadence rather than intrinsic variability.

gauge how the width changes away from the downstream focusing.

Fig. C.1 shows that k is largest near apastron and decreases towards periastron. Over one orbit we find k to vary between ~ 0.25 and ~ 2.0 for $\theta = -\psi$ and $\theta = -90^\circ$, being only between ~ 0.15 and ~ 0.25 for $\theta = -\mu$.

We note that R_{BHL} is a capture/focusing scale for rectilinear upstream flow, not a lower bound on the morphological thickness of the post-shock ridge. Along the strongly compressed downstream limb ($\theta = -\mu$) the inner arm behaves as a thin, shock-bounded sheet whose thickness is set by compression and cooling in supersonic flow rather than by R_{BHL} . Orbital curvature and shear further concentrate material, while the instantaneous v_{rel} – largest near periastron – both reduces R_{BHL} and enhances compression. Consequently, $k < 1$ at $\theta = -\mu$ is physically expected. For the Monte-Carlo simulations presented in Sect. 2.5, we use the k -range associated with rays perpendicular to the plane (z -direction) at $\theta = -\psi$ since this sampling direction best represents the intrinsic structural width of the spiral arm, rather than the compressed post-shock layer. It captures the effective spread of bound and marginally bound material around the arm’s density ridge, and thus provides a physically meaningful measure of the morphological arm thickness relevant for radiative transfer and dust formation modelling.

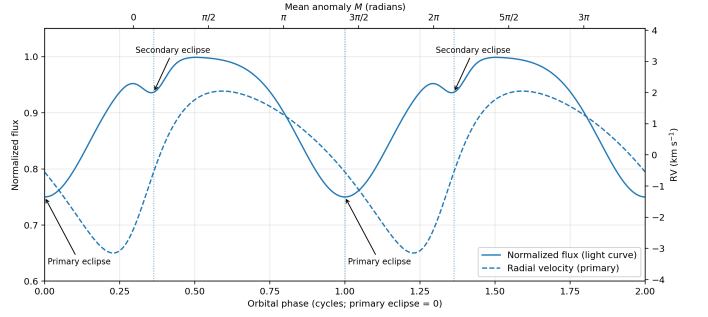
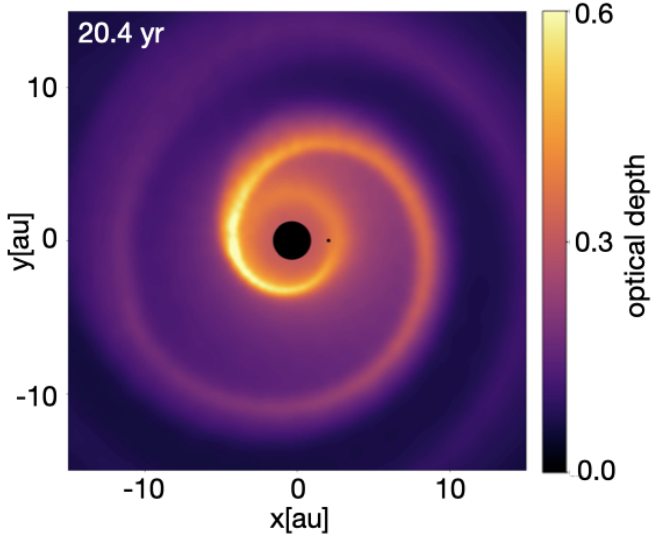


Fig. E.1: Relation between the orbital light curve (solid) and the radial-velocity curve (dashed) for an eclipsing binary with $e = 0.3$, $\omega = 225^\circ$, and $i = 60^\circ$. The bottom axis shows the normalized orbital phase, while the top axis indicates the corresponding mean anomaly M . Dotted vertical lines mark the epochs of primary and secondary eclipses. For a circular, edge-on orbit, the primary’s RV minimum is a quarter of an orbital cycle ($\pi/2$) later in phase than the light curve minimum (primary eclipse); for eccentric orbits, this offset depends on both e and ω .

Appendix E: Phase lag between light curve and radial velocity for an eclipsing binary system

Appendix F: Online material



Video F.1: 3D hydrodynamical simulation of an eccentric binary with a red-giant primary ($M_1 = 1.5 M_\odot$) and a low-mass companion ($M_2 = 0.125 M_\odot$). The orbit has semi-major axis $a = 2.3$ au and eccentricity $e = 0.3$; the dust-condensation radius is $R_{\text{cond}} = 3$ au, the orbital period is 1000 days. The video shows the optical-depth maps in a slice through the orbital plane. The red-giant primary is at the origin $(0, 0)$; the low-mass companion lies to the right in each panel. *If the embedded video does not render, a copy is available on request from leen.decin@kuleuven.be.*