

Radiatively-Cooled Mass Transfer: Disk Properties and L2 outflows across Mass Transfer Rates

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ABSTRACT

High rates of stable mass transfer (MT) occur for some binary star systems, resulting in luminous transients and circumbinary outflows. We perform hydrodynamical simulations of a $10 M_{\odot}$ donor star and a $5 M_{\odot}$ point mass secondary, incorporating approximate effects of radiative cooling. By varying the orbital separation of the system, we probe MT rates between 10^{-5} and $10^{-1} M_{\odot}/\text{yr}$. Mass flows from the donor into an accretion disk, with significant equatorially-concentrated outflows through the outer Lagrange point L2 occurring for MT rates $\gtrsim 10^{-3} M_{\odot}/\text{yr}$, while the MT remaining mostly conservative for lower MT rates. In all cases, any outflowing gas approximately carries the specific angular momentum of L2. The gas cooling luminosity L and temperature increases with MT rate, with $L \sim 10^5 L_{\odot}$ and $T \sim 10^4 \text{ K}$ for simulations featuring the strongest outflows, with contributions from both the accretion disk and circumbinary outflow. The most luminous transients associated with mass outflows will be rare due to the high MT rate requirement, but generate significant optical emission from both the accretor’s disk and the circumbinary outflow.

1. INTRODUCTION

Mass transfer (MT) in binary star systems is common, especially for binaries containing massive stars (Sana et al. 2012). Episodes of MT precede the formation of gravitational wave sources (Postnov & Yungelson 2014; Korol et al. 2022; LIGO Scientific Collaboration et al. 2023) and a large fraction of core-collapse supernova (Schneider et al. 2021). A major uncertainty related to MT is the degree to which it is conservative (Huang 1963; Mink et al. 2007). Fully conservative MT results in the accreting star accepting all material from the donor, but when non-conservative MT occurs, material leaves the binary system, extracting angular momentum (AM). A consequent uncertainty is the amount of AM carried by the escaping mass.

If mass escapes by flowing through the outer Lagrange point, L2, which is a likely outcome in cases including an evolved massive star and stellar companion (Podsiadlowski et al. 1992), then its specific angular momentum may be comparable to that of the L2 point (e.g. Huang 1963; MacLeod et al. 2018a). In studies of stable MT that can precede the formation of binary black holes (BBHs), Marchant et al. (2021) and Picco et al. (2024) noted that L2 mass loss may occur and that the amount of specific AM lost is a significant uncertainty that will impact the observed BBH population.

L2 mass loss has been predicted for high rates of MT ($\gtrsim 10^{-4} M_{\odot}/\text{yr}$), where gas in the accretion disk is ex-

pected to cool inefficiently and be energetic enough to escape through L2, forming a circumbinary outflow (CBO) (Lu et al. 2023). The observational appearance of the MT is unclear, with Lu et al. (2023) predicting that photons from the accretion disk will be reprocessed into the infrared.

Recently, Scherbak et al. (2025) (hereafter Paper I) performed hydrodynamical simulations of rapid, stable MT that adopted an adiabatic assumption and therefore neglected cooling. Paper I found that material indeed flowed out through L2, carrying AM similar to that of L2, and with a relatively small average radial velocity (10-20% the orbital velocity). At the outer boundary of those simulations, the material remained marginally bound to the binary, with slightly negative specific energy. This meant it was unclear if L2 mass loss in a binary could be a mechanism behind pre-supernova mass loss and extended circumstellar material (CSM) that commonly interacts with supernova ejecta (e.g. Taddia et al. 2013, Clark et al. 2020).

A major assumption in Paper I was the lack of radiative cooling, which Paper I estimated to become important at moderate MT rates $\lesssim 10^{-2} M_{\odot}/\text{yr}$. While the MT in Paper I was highly non-conservative when the simulations reached a steady-state, Lu et al. (2023) has predicted that the degree of non-conservativeness should increase with the MT rate, further suggesting that behavior could differ at lower MT rates. In this paper, we extend the work of Paper I by incorporating radia-

tive cooling into hydrodynamical simulations, thereby expanding our simulations to cover lower and more common MT rates, where L2 mass loss may occur differently or not occur at all. In addition, the presence of cooling will result in more realistic gas temperatures, and allow for calculations of the observational appearance of the accretion disk and circumbinary outflows.

Observationally, L2 mass loss appears to be occurring in binaries SS433 (Blundell et al. 2001; Cherepashchuk et al. 2018) and W Serpentis (Shepard et al. 2024). In the case of SS433, radio/infrared images (Blundell et al. 2001; Bowler 2010, 2011a,b; Perez M. & Blundell 2010) suggest the presence of outflowing circumbinary material. Infrared images of W Serpentis (Davidge 2023) also find dust formation extending away from the central binary, suggesting the formation of a circumbinary disk that outflows through the outer Lagrange point on the side of the accretor (Shepard et al. 2024). The red transient V1309 Sco may also have undergone L2 mass loss, which would explain several peculiar features of its light curve (Pejcha 2014).

Numerous works have investigated mass loss through the outer Lagrange point (summarized in Paper I) but only a relatively small subset have incorporated cooling and investigated various MT rates. Pejcha et al. (2016) performed smoothed particle hydrodynamic (SPH) simulations of L2 mass-loss with radiative cooling and heating, but they initialized their material near L2 without including the donor star or accretor in their domain. Nazarenko et al. (2005) modeled MT rates between $\sim 10^{-7} - 10^{-5} M_{\odot}/\text{yr}$ in Algol-type binaries, noting that gas in the disk can be transferred to the vicinity of L2/L3 and form an outflow, but did not discuss the fraction of material lost from the accretion disk. Mohamed & Podsiadlowski (2012) studied MT in symbiotic binaries undergoing wind RLOF, finding that some of the wind is deflected away from the accretor and escapes through L2. Booth et al. (2016) extended this work to symbiotic binaries undergoing mass loss through a combination of winds and a Roche lobe overflow stream, incorporating a sophisticated radiative cooling model. They also found that part of the wind is channeled near L2 and escapes. Both Mohamed & Podsiadlowski (2012) and Booth et al. (2016) focused on MT rates $\sim 10^{-6} M_{\odot}/\text{yr}$, whereas we focus on higher MT rates. Although they did not include cooling, Bobrick et al. (2017) investigated realistic MT rates between a WD and a neutron star through SPH simulations, finding that mass is lost through a disk wind, forming a “common envelope” around both stars. The AM carried by these winds is greater than the specific orbital AM of the neutron star, but much less than the specific AM of L2. In contrast to these

works, we focus on the approximately steady-state behavior of stable MT where we track the outflow of gas, originating in the accretion disk, from the binary, and we vary MT rates to investigate its changing effects.

In Sec. 2 we discuss the numerical setup of our simulations, including the grid setup and our radiative cooling model. Sec. 3.1 characterizes the degree of conservativeness for different MT rates, Sec. 3.2 the temperature, luminosity and observational appearance of the MT episodes we model, and Sec. 3.3 the properties of any outflows through L2, with a comparison to previous work. We conclude in Sec. 4.

2. SIMULATION SETUP

We perform Newtonian hydrodynamic simulations using the PLUTO code (Mignone et al. 2007; Mignone et al. 2012), considering MT between a donor star with core mass M_1 and an accretor with mass M_2 , separated by a distance a in a fixed circular orbit. We solve the conservation equations of mass, momentum, and energy without any explicit viscosity, but including cooling due to radiative losses (Sec. 2.1). Our setup, including grid spacing, initial hydrostatic structure of the donor star, and injection of heat inside the donor star, is discussed in more detail in Paper I.

Our simulations are set in spherical coordinates r, θ, ϕ , centered at the donor star, in the co-rotating frame of the binary. The grid radius r extends from inside the donor’s envelope, not including the core, out to $5a$. θ extends from 0 to $\pi/2$, with a reflective boundary condition, so that we only simulate the top hemisphere of the binary, while ϕ wraps around from 0 to 2π . We refer to values in code units when we non-dimensionalize using $G = M_{\text{tot}} = M_1 + M_2 = a = 1$.

We use the Roche.tidal.equilibrium code of Lu (2025) to help construct our donor star setup. The density ρ and pressure P inside the donor star are initialized using the conditions: 1. $P = K\rho^\gamma$, for some entropy constant K and polytropic index γ ; 2. the star is initially in hydrostatic equilibrium in the Roche potential and underfills its Roche lobe. Outside the star, ρ and P are initialized to spatially constant small values, corresponding to a low sound speed c_s . P and the internal energy u_{int} are related by PLUTO’s “ideal” equation of state (EOS),

$$P = (\gamma - 1) \rho u_{\text{int}}, \quad (1)$$

where we use the same value of γ as the polytropic index above. We inject heat inside the envelope so that it expands on a Kelvin-Helmholtz timescale t_{KH} much greater than the orbital dynamical timescale.

The secondary is represented by a Plummer softening potential, with softening length ϵ such that $\epsilon/a = 0.05$.

The values M_1 and M_2 do not change over the course of the simulations, but the mass of the donor's envelope, which is much less than M_1 , is reduced as MT occurs.

The values controlling our setup are the same as in our fiducial simulation of Paper I, `q-0.5_mid_heat`: mass ratio $q \equiv \frac{M_2}{M_1} = 0.5$, $K = 0.578$ (code units), $\gamma = 1.4$ and an initial donor radius of about 75% the Roche radius. The total luminosity injected inside the donor is $4.0\text{e-}6$ (code units) and t_{KH} is about 80 orbital periods. Although Paper I found that varying q can impact outflow properties, we leave q fixed in this work to focus on the impact of cooling, but future work should incorporate cooling and vary MT rates with different q values.

Because our goal is to incorporate cooling and investigate different MT regimes, conversions are necessary to convert from code units and give physical values of temperatures, opacities, and cooling rates. This differs from the previous simulations of Paper I which assumed adiabaticity outside the donor star, did not incorporate cooling, and were therefore, in principle, scale-free. We choose $M_{\text{tot}} = 15 M_{\odot}$, which could correspond to a massive star plus stellar or black hole companion. We vary the value of a between our simulations, which effectively changes the physical MT rate between simulations via

$$\dot{M}_{\text{phys}} = \left(\frac{M_{\text{tot}}^{3/2} G^{1/2}}{a^{3/2}} \right) \dot{M}_{\text{code}}. \quad (2)$$

By raising a and keeping M_{tot} fixed and $\dot{M}_{\text{code}}$ approximately fixed, we lower $\dot{M}_{\text{phys}}$. Strictly speaking, this means that we do not sample all regimes of parameter space (e.g., low $\dot{M}_{\text{phys}}$ at short orbital period). However, it is reasonable to expect that $\dot{M}_{\text{phys}}$ will be the main factor in determining outflow properties, and that our results can be extended to various a . Similarly, setting the scale of a determines the physical values of density, pressure, temperature, opacity, etc.

Table 1 summarizes the simulations that we perform, demonstrating the difference in scales used to simulate different MT rates. Note that values such as K and L_{heat} , which are constant in code units across simulations, are different in physical units for different simulations. See also Sec. 3.1 for further discussion of where our simulations fall in the parameter space of MT rate and a .

2.1. Cooling Prescription

We include energy losses due to radiative cooling, assuming that the photons primarily escape perpendicular to the orbital plane in the $\hat{z}$ direction. We estimate that the cooling power per volume, $\dot{E}_{\text{cool}}$, is given by

$$\dot{E}_{\text{cool}} = -\frac{4\rho\sigma_{\text{SB}}T^4}{\frac{1}{\kappa} + \Sigma_z\tau_z} \quad (3)$$

where κ is the opacity, T the temperature of the gas, Σ_z is the gas surface density in the $\hat{z}$ direction, and τ_z is the optical depth in the $\hat{z}$ direction. This equation is inspired by several previous works that have incorporated radiative cooling (Stamatellos et al. 2007; Forgan et al. 2009; Wilkins & Clarke 2012; Lombardi et al. 2015; Pejcha et al. 2016), but that performed SPH simulations. Of these, Pejcha et al. (2016) applied this cooling term to their SPH simulations of L2 mass loss, noting that their particles are not self-gravitating and the original formalism of Stamatellos et al. (2007); Forgan et al. (2009); Lombardi et al. (2015) to develop this formula does not strictly apply.

Eq. 3 takes the correct forms in both the optically thick and optically thin limits. In the optically thin limit where $\frac{1}{\kappa} \gg \Sigma_z\tau_z$, the magnitude of the cooling reduces to

$$|\dot{E}_{\text{cool}}| \approx \frac{4\rho\sigma_{\text{SB}}T^4}{\frac{1}{\kappa}} \approx 4\rho\kappa\sigma_{\text{SB}}T^4 \quad (4)$$

where this form represents the emitted radiation per unit volume of an optically thin gas. In the optically thick limit, where $\frac{1}{\kappa} \ll \Sigma_z\tau_z$, Eq. 3 reduces to

$$|\dot{E}_{\text{cool}}| \approx \frac{4\rho\sigma_{\text{SB}}T^4}{\Sigma_z\tau_z} \approx \rho a T^4 / \left(\frac{H_z \rho \tau_z}{c} \right) = u_{\text{rad}} / \left(\frac{H_z \tau_z}{c} \right) \quad (5)$$

where we have estimated that $\Sigma_z = H_z \rho$ for some scale height H_z , the form of which we will discuss shortly. This represents the radiation energy density over the diffusion timescale in the $\hat{z}$ direction, $\frac{H_z \tau_z}{c}$.

We calculate T , by solving the quartic equation for the total pressure as a sum of gas and radiation pressure

$$P = \frac{\rho k_b T}{\mu m_p} + \frac{a_{\text{rad}} T^4}{3}, \quad (6)$$

which assumes gas and radiation are in equilibrium with the same T , where k_b is the Boltzmann constant, m_p is the proton mass, and μ is the mean molecular weight. In our simulations, $\mu \approx 0.62$, appropriate for ionized Solar-composition material, but does not take into account the formation of neutral hydrogen at low temperatures, which can occur in our low-MT rate simulations. This means the temperature calculation is not quite consistent at low T , but, compared to the impact of MT rate (Sec. 3.2), is likely a relatively small correction that will not severely impact our T estimates. Eq. 6 is solved at runtime at every cell, using a Newton-Raphson root solver.

The opacity κ is estimated by interpolating tables of opacity at solar metallicity ($Z_{\odot}$), using tables conveniently collected in the Modules for Experiments in

Simulation	$\dot{M}_{\text{code}}$	a (au)	P_{orb} (days)	$M_{\text{tot}}^{3/2} G^{1/2} a^{-3/2}$ ($M_{\odot} \text{ yr}^{-1}$)	$\dot{M}_{\text{phys}}$ ($M_{\odot} \text{ yr}^{-1}$)
highest_mdot	1.3e-5	0.1	3	1.1e4	1.5e-1
high_mdot	1.1e-5	0.3	15	2.2e3	2.3e-2
mid_mdot	0.7e-5	1	100	350	2.2e-3
low_mdot	0.4e-5	10	2970	5	2.0e-5

Table 1. Grid of 3D simulations, labeled “highest” “high” “mid” or “low” by their MT rate in physical units ($\dot{M}_{\text{phys}}$). The free parameter is the orbital separation a , while M_{tot} is fixed at $15 M_{\odot}$. The corresponding orbital period P_{orb} is calculated via Kepler’s Third Law. The factor $M_{\text{tot}}^{3/2} G^{1/2} a^{-3/2}$ is used to convert from $\dot{M}_{\text{code}}$ to $\dot{M}_{\text{phys}}$ (see Eq. 2), where both values are approximate steady-state values.

Stellar Astrophysics code (MESA Paxton et al. 2011, 2013, 2015, 2018, 2019; Jermyn et al. 2023). Specifically, we interpolate the table `gn93_z0.02_x0.7.data` in $\log T$ and $\log R \equiv \log \rho - 3 \log T + 18$ to estimate $\log \kappa$, using `lowT_fa05_gn93_z0.02_x0.7.data` for $\log T < 3.75$. For any values outside the tables, we use the value of κ at the outer edge of the table. The opacities we are using are Rosseland mean opacities, whereas they should technically be Planck mean opacities in the optically thin limit (see Stamatellos et al. 2007). We follow Pejcha et al. (2016) in neglecting this discrepancy, given the approximations already taken in using Eq. 3 for our purposes. The opacity table we use, and where the points in the simulations fall on it, is visualized further in Appendix C.

The final two ingredients needed in calculating Eq. 3 are H_z and τ_z . We estimate $\tau_z \approx \kappa \rho H_z$ at every point in the grid. This is a rough estimate of the local vertical optical depth, but doing a runtime integral over z -coordinate in spherical coordinates would be cumbersome and potentially introduce numerical artifacts. Our estimate of τ_z captures the fact that the greatest contribution to τ_z is where the density is largest in the midplane. We estimate H_z in a local fashion as well, using an estimate appropriate to an accretion disk,

$$H_z = \frac{c_s}{\Omega_{\text{eff}}} \quad (7)$$

where the sound speed c_s is defined as $c_s^2 = \frac{\gamma P}{\rho}$ and we estimate the effective orbital frequency as

$$\Omega_{\text{eff}}^2 = \frac{M_1}{|\vec{r}|^3} + \frac{M_2}{|\vec{r} - a\hat{x}|^3} \quad (8)$$

where $\vec{r}$ is the coordinate vector, as M_1 is located at the origin, and M_2 is located at $a\hat{x}$. This reduces to the correct limit in the accretion disk near M_2 when the second term dominates, and far out in the circumbinary disk surrounding both M_1 and M_2 when $|\vec{r} - a\hat{x}| \sim |\vec{r}|$.

We include cooling in PLUTO using the `radiat.c` cooling module, where we set the right hand side of the energy equation that is solved by PLUTO. We do not allow any cooling inside the Roche lobe of the donor

star, as that would be likely be a bad approximation given our cooling form is based off a disk structure.

Eq. 3 gives the nominal form for the cooling power per volume, which acts to reduce u_{int} . However, we also impose safeties and floors to avoid catastrophic cooling and unphysical values of T or c_s . We set that in all grid cells, u_{int} cannot be reduced by more than 50% in a given timestep. Test runs showed that our results are not sensitive to this value. Additionally, we set a pressure floor and a corresponding internal energy floor, which are related via the ideal EOS (Eq. 1). The cooling is capped such that u_{int} is not allowed to fall below the internal energy floor. In low density regions, where $\rho < 10^{-5}$ in code units, we set the pressure floor to be proportional to ρ , corresponding to a low sound speed floor. In higher density regions, we set a pressure floor corresponding to a temperature floor of 2000 K, with pressure and temperature related by Eq. 6.

We choose not to impose a temperature floor everywhere in the domain, because doing so in low density regions creates a high pressure and high sound speed. This leads to material seemingly having a large u_{int} and a spurious outflow of mass near the outer boundary. The downside to this approach is that T of some low-density material can become lower than covered by our opacity tables. However, these low-density regions are unimportant for the cooling luminosity and L2 outflow. If we do not impose a floor on sound speed in low density regions, the pressure and sound speed can become extremely small, leading to unphysically high Mach numbers.

Additionally, we enforce floors on ρ and P within the `UserDefBoundaries` section of PLUTO’s `init.c`. This is complementary to the previous discussion of floors, as those implied a limit on the cooling power, but did not enforce floors in general during runtime. The ρ floor is set to be 10^{-10} in code units, and the P floor is the same as above. More discussion of the floors is included in Appendix C.

In the `highest_mdot` simulation, numerical issues such as negative pressure occur near the pole, where the resolution is set to be low in order to avoid small timesteps.

Therefore, we do not include cooling at $\theta < 0.3$ for this simulation. Because the density sharply drops near the pole and the outflow originates in the midplane, this will not affect our main results.

In our main suite of simulations, we do not include any heating terms due to the luminosity of either the donor star or the companion. However, we performed a small suite of 2-dimensional simulations in the equatorial midplane, and added a heating term due to the donor star's luminosity. For a 2-dimensional simulation analogous to our `mid_mdot` simulation, the accretion disk around the companion begins to experience significant heating on the donor-facing side, that dominates over the cooling at luminosities $\gtrsim 5e38$ erg/s. Due to subsequent numerical difficulties we chose not to incorporate heating into our main results. We therefore caution that at high luminosities $\gtrsim 1e5 L_\odot$, our results could change, and this parameter space should be investigated further. There is also significant missing physics involving the accretor in our simulations, including the potential launching of a fast wind that then emits X-ray and UV photons (Lu et al. 2023).

3. ANALYSIS

3.1. Morphology and mass transfer conservativeness

Fig. 1 shows the density in the equatorial plane for our simulations, with the colormap scale shown in physical units. We show snapshots corresponding to about 100 orbital periods after initialization. By this time, the donor star has expanded due to heating and begun Roche lobe overflow, and the MT rates have reached a quasi-steady state.

In the `highest_mdot` simulation, most of the equatorial domain is flooded with material, although the densest outflow is centered near L2, the Lagrange point on the far side of M_2 . Mass easily escapes the puffy accretion disk and flows out through L2 in a broad stream. In addition, the stream passing through L1, the inner Lagrange point, is quite puffy, and the boundary between donor, accretion disk and the MT stream is blurred. A small amount of material also flows out in a stream originating near L3 (the outer Lagrange point on the side of the donor star), and much of it falls back onto the L2 stream, forming shocks. Note that the simulations are in the co-rotating frame, and the outflow lags behind the binary orbit in this frame.

In the `high_mdot` simulation, a thick stream still flows out near L2, but lower density voids are also visible between the spiral arms of the circumbinary outflow. The stream originating near L3 is also more diffuse.

The `mid_mdot` and `low_mdot` simulations are somewhat similar to one another, in that a relatively thin

stream of material flows out of the accretion disk near L2. The outflow through L3 is extremely tenuous in these cases. The boundary of the accretion disk, and the L1 stream, is more sharp in these simulations compared to `highest_mdot`. In the `low_mdot` simulation, the L2 outflow has a very low density, as does the outer part of the secondary's accretion disk.

We estimate the fraction β of transferred mass that is retained by the companion by calculating $\dot{M}_{\text{donor}}$, the MT rate from the donor star, and $\dot{M}_{\text{tot}}$, the MT rate through the outer boundary, and using

$$\beta = 1 - \frac{\dot{M}_{\text{tot}}}{\dot{M}_{\text{donor}}}. \quad (9)$$

Fig. 2 plots the steady-state average of β for our simulations. For the `highest_mdot` simulation, $\beta \approx 0$, meaning the MT is very non-conservative, which is the same conclusion of Paper I that did not include any cooling. β monotonically increases with decreasing MT rate, meaning that the MT becomes more conservative. For the `high_mdot` simulation, $\beta \approx 0.3 - 0.5$, meaning that most of the mass still forms a circumbinary outflow, but a large fraction stays in the accretion disk. The other two simulations feature mostly conservative MT rates, although a small amount of material still flows out through L2. Therefore, between MT rates of $\sim 10^{-3}$ and $\sim 10^{-2} M_\odot$, there is a significant jump in the degree of non-conservativeness (top panel of Fig. 2). This would constitute a threshold for highly non-conservative mass transfer in astrophysical binaries.

In our simulations, the outflowing mass $\dot{M}_{\text{tot}}$ is almost identical to the MT rate of material outward near L2, $\dot{M}_{\text{L2}}$, which we calculate by integrating the mass flux over a wedge about L2, similar to the procedure in Paper I. We also estimate the MT rate near L3, $\dot{M}_{\text{L3}}$, the Lagrange point on the side of the donor star. In all cases, the L2 outflow dominates, but to different degrees. For the `highest_mdot` and `high_mdot` simulations, $\dot{M}_{\text{L2}} \sim 10 - 20 \times \dot{M}_{\text{L3}}$, similar to our result in Paper I without any gas cooling. For the other simulations, $\dot{M}_{\text{L2}} \gtrsim 10^3 \times \dot{M}_{\text{L3}}$, meaning that it is extremely unfavorable for material to circle around the donor star on an equipotential that would allow it to escape near L3.

This behavior can be understood by comparing the MT rate to the Eddington-limited MT rate at the *outer* edge of the accretion disk (Lu et al. 2023). $\dot{M}_{\text{edd}}$ is defined by equating the outer disk accretion luminosity to the Eddington luminosity, such that

$$\frac{GM_2|\dot{M}_{\text{donor}}|}{r_d} = L_{\text{edd}} = \frac{4\pi cGM_2}{\kappa} \quad (10)$$

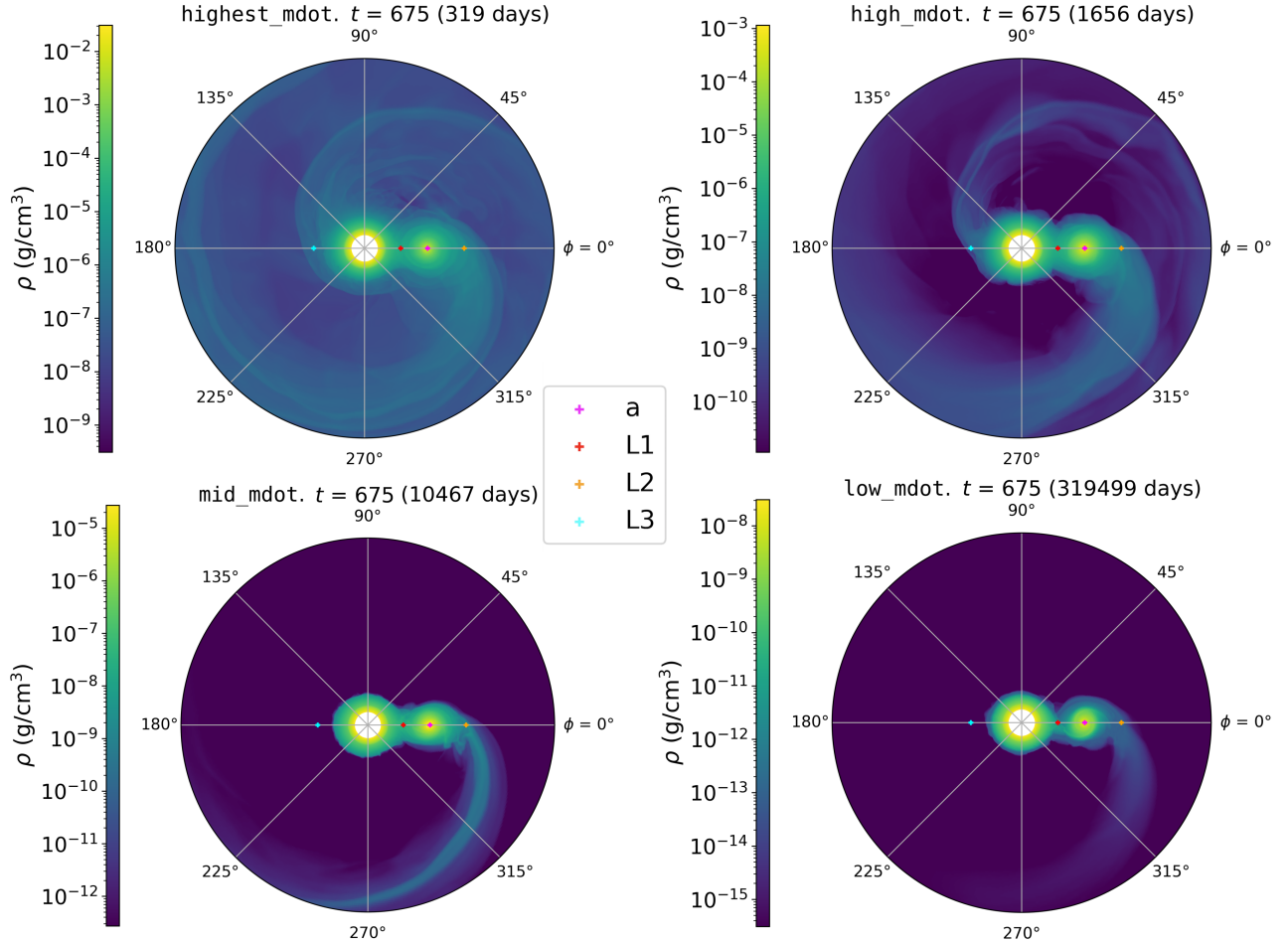


Figure 1. The density ρ in the equatorial plane, for simulations of varying MT rate. The orbital separation a and Lagrange points are labeled. The snapshots, representative of the simulations once they have reached a quasi-steady state, are labeled with t in both code units and physical units (days). The full domain extends to $r = 5a$, but these plots extend to $r = 3a$ to zoom in near the accretion disk. Note the different density scale in each case.

and solving for $|\dot{M}_{\text{donor}}|$, where r_d is the radius near the outer edge of the accretion disk. Therefore,

$$\dot{M}_{\text{edd}} = \frac{4\pi c r_d}{\kappa}. \quad (11)$$

For our simulations, we estimate $\dot{M}_{\text{edd}}$ by finding the density-weighted average of $\frac{4\pi c r_d}{\kappa}$ at r_d values (in units of a) of 0.2, 0.25, and 0.3 around M_2 and inside M_2 's Roche lobe. Using lower values r_d mildly changes $\dot{M}_{\text{edd}}$ but not enough to change the trend in Fig. 2. From the bottom panel of Fig. 2, the transition between conservative and significant non-conservative MT approximately occurs when the MT rate rises above $\dot{M}_{\text{edd}}$.

Fig. 3 shows a comparison of our simulations to the semianalytic predictions of Lu et al. (2023), by plotting f_{L2} , the fraction of transferred mass leaving the L2 point. Note that f_{L2} for our simulations is simply equal to $1 - \beta$. The colormap of Fig. 3 is generated us-

ing code¹ accompanying Lu et al. (2023) for the binary masses we simulate, and assuming a hydrogen rich gas of solar metallicity.

We find that f_{L2} is about 0.6 for the `high_mdod` simulation and about 0.1 for the `mid_mdod` simulation, but Lu et al. (2023) predicts values of > 0.9 and about 0.65, respectively. For our more extreme simulations, `low_mdod` and `highest_mdod`, f_{L2} approaches 0 and 1, in general agreement with Lu et al. (2023).

In other words, our simulations imply that the MT threshold where MT becomes strongly non-conservative is higher than predicted by Lu et al. (2023), by a factor of $\sim 2-5$. More densely-sampled simulations would help determine the differences throughout the entire parameter space (e.g. if the “wiggles” of Fig. 3, caused by

¹ <https://github.com/wenbinlu/L2massloss>

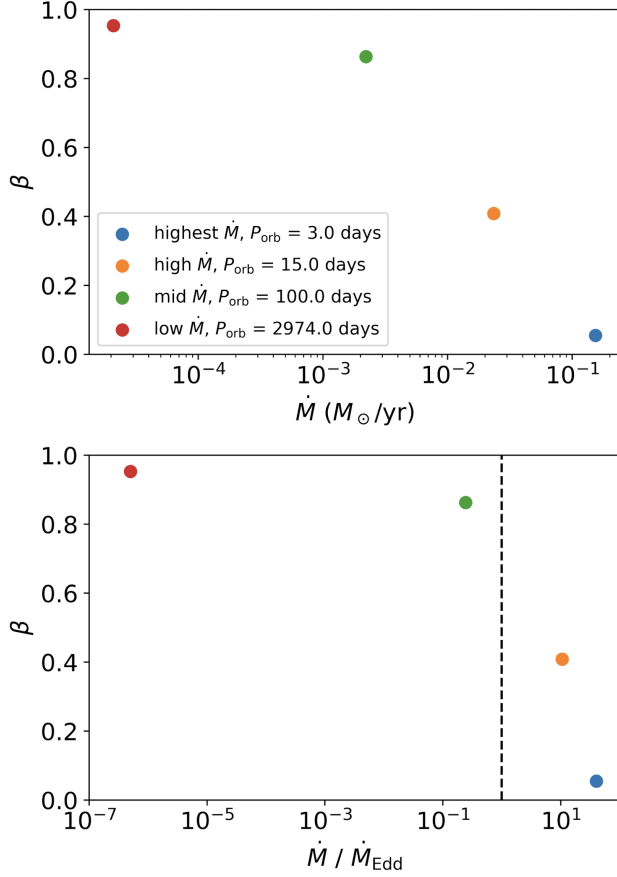


Figure 2. Top panel: The fraction of material retained in the accretion disk, β , versus the MT rate $\dot{M}_{\text{donor}}$ for our simulations. Bottom panel: the same quantity, plotted versus $\dot{M}_{\text{donor}}$ in units of the Eddington MT rate (Eq. 11). The vertical dashed line shows where $\dot{M}_{\text{donor}} = \dot{M}_{\text{edd}}$. The values shown are time-averaged once the simulations reach a quasi-steady state, at t greater than ~ 300 (code units).

opacity variations, are robust). The reason for the difference is unclear, but may be due to geometric factors of order unity, our approximate treatment of gas cooling, or the lack of viscous accretion power in our simulations.

3.2. Observational appearance: Temperatures and luminosities

We calculate the cooling luminosity of the accretion disk near M_2 by integrating the power per unit volume within a sphere of radius $0.3a$, with approximates the Roche lobe of M_2 , i.e.

$$L_{\text{cool,disk}} = \int_{|\vec{r}-a\hat{x}| \leq 0.3a} \dot{E}_{\text{cool}} dV \quad (12)$$

and the cooling luminosity of all gas in the domain by integrating over the entire spherical domain

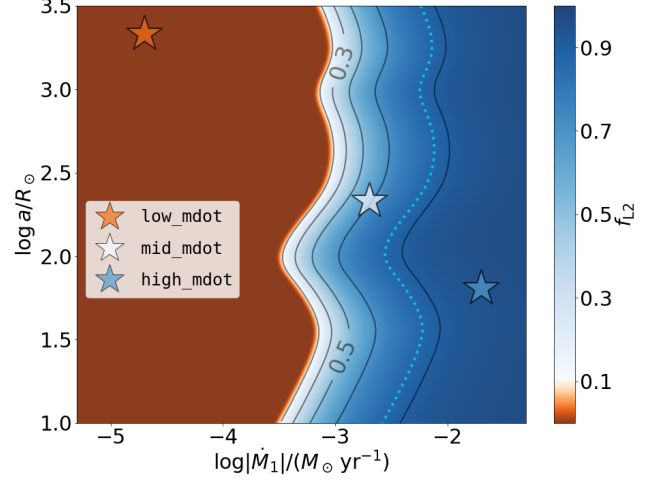


Figure 3. The predicted fraction of material escaping through L2, f_{L2} , for various a and MT rates. This follows the figures of Lu et al. (2023), but adapted for a $10 M_{\odot}$ primary and $5 M_{\odot}$ primary. Solid lines show contours of constant f_{L2} , and the cyan dotted line shows where $\dot{M}_{\text{donor}} = \dot{M}_{\text{edd}}$. Stars show where our simulations lie in this parameter space, with the internal color corresponding to their f_{L2} value. The **highest_mdots** simulation is not shown for convenience, but would lie outside the right border of this plot.

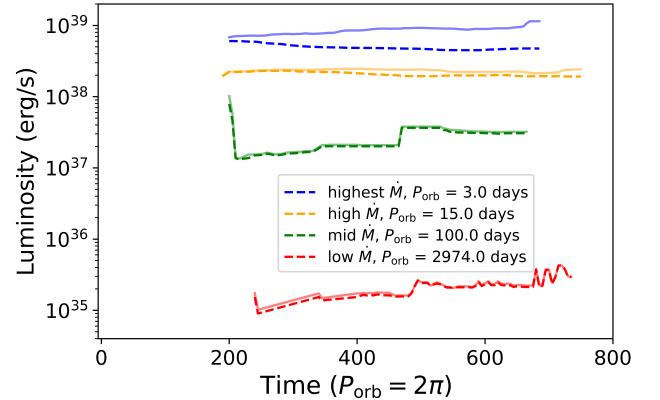


Figure 4. The cooling luminosity calculated within the accretion disk (dashed lines, Eq. 12, and the entire computational domain (solid lines, Eq. 13). Curves are smoothed using a rolling average, and values are not shown at $t < 200$ because the MT rate has not ramped up to its steady-state value.

$$L_{\text{cool,tot}} = \int_{\text{entire domain}} \dot{E}_{\text{cool}} dV. \quad (13)$$

Note that there is no cooling inside the Roche lobe of the donor M_1 .

Fig. 4 plots the values of luminosity, in erg/s, versus time for our different simulations. Because of the vastly different orbital timescales between our simulation, time is plotted in code units, such that 2π is one

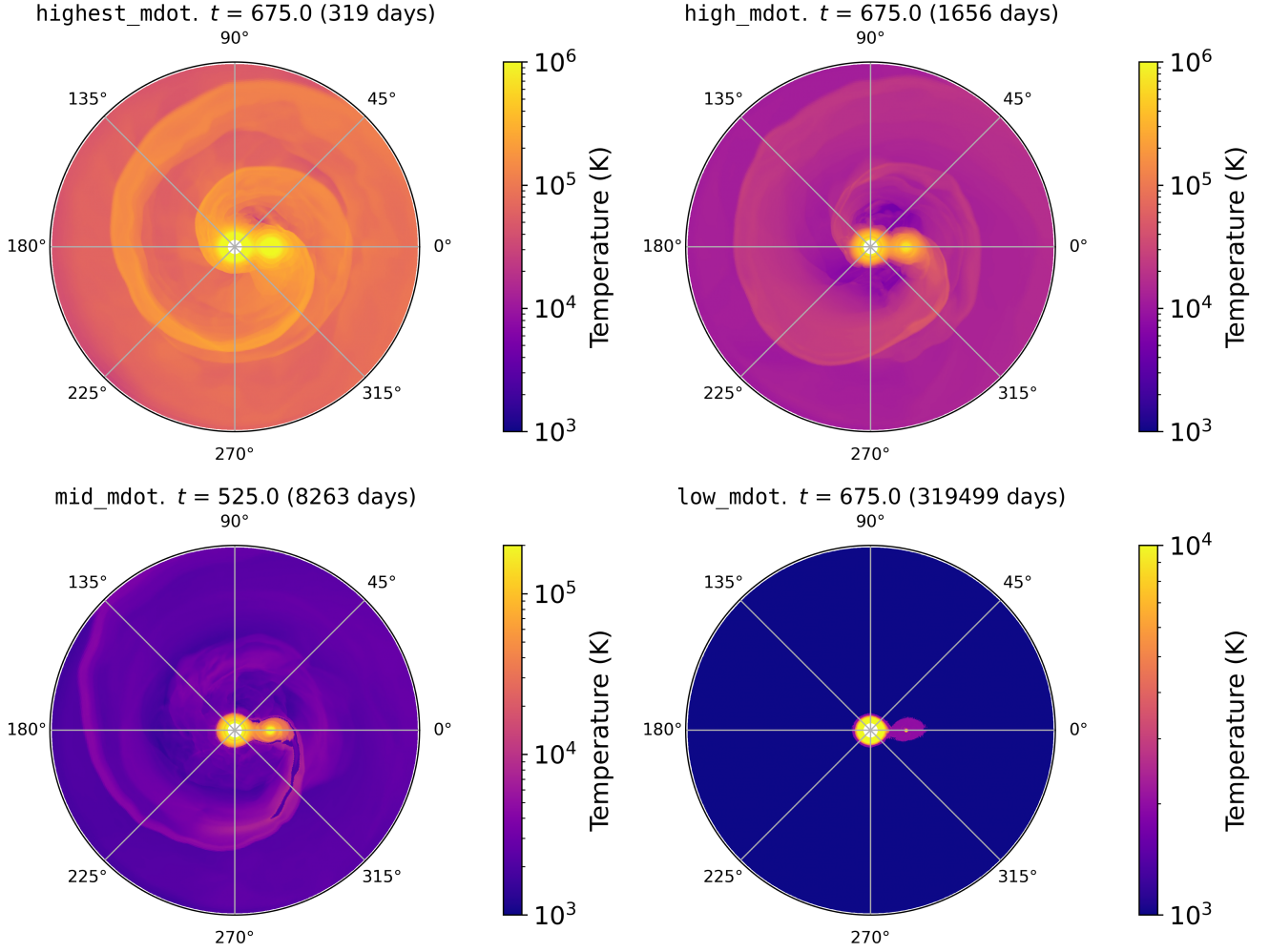


Figure 5. The gas temperature T in the equatorial plane for all simulations. The snapshots, representative of the simulations once they have reached a quasi-steady state, are labeled with t in both code units and physical units (days). The T floor is reached in the outer regions of the `mid_mdot` simulation and almost everywhere in the `low_mdot` simulation (see text for details).

orbit. Unsurprisingly, the luminosity increases with increasing MT rate, although note from Appendix B that for the `highest_mdot` simulation that the gas cooling timescale is long and it is more difficult for photons to escape. For the `mid_mdot` and `low_mdot` simulations, all the cooling power comes from the accretion disk, with almost nothing from the circumbinary outflow. In the `highest_mdot` case, including the full domain approximately doubles the overall luminosity, so the cooling of the outflow is quite important. This is because most of the equatorial domain is flooded with material (top panel of Fig. 1). The `high_mdot` simulation shows intermediate behavior, where the cooling power of the outflow makes a $\sim 20\%$ increase to the overall cooling power.

The increase of luminosity as a function of MT rate can be understood through an increase in accretion power from the disk surrounding M_2 . Note, however,

that we do not resolve the surface of the accretor, as it is represented by a point mass with a softening potential of length $\epsilon = 0.05a$ (Sec. 2). When we assume that $L_{\text{cool,disk}} \sim \frac{GM_2\dot{M}_2}{r_{\text{d,eff}}}$, we find that $r_{\text{d,eff}}/a \approx 0.05, 0.15, 0.2, 0.3$ for our simulations of increasing MT rate (`low_mdot`, `mid_mdot`, `high_mdot`, `highest_mdot`). For the lower MT rate simulations, $r_{\text{d,eff}}$ is closer to the softening radius, suggesting that the cooling power is mostly coming from the inner edge of the disk. For the higher MT rate simulations, $r_{\text{d,eff}}$ is closer to the Roche radius of M_2 . $\dot{M}_2$ is much less than $\dot{M}_{\text{donor}}$ in the case of the `highest_mdot` and `high_mdot` simulations, which show highly non-conservative MT.

The gas temperature T , defined by Eq. 6, also varies greatly between simulations. Fig. 5 plots T in the equatorial plane, at representative snapshots. For all simulations, T in the accretion disk is hotter than in the rest

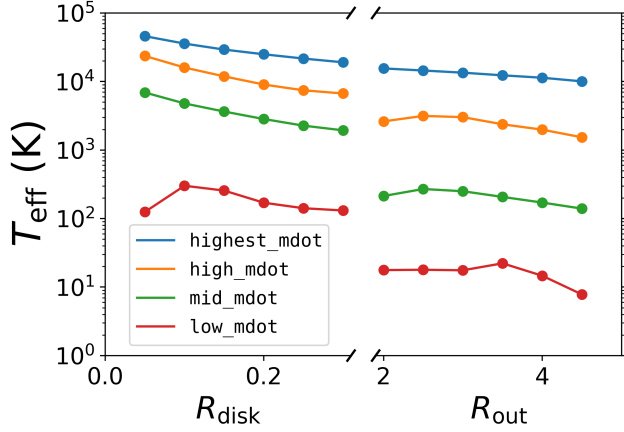


Figure 6. The average gas effective temperature T_{eff} in the accretion disk near M_2 and in the circumbinary disk. The curves shown are calculated over cylinders of constant cylindrical radius R_{disk} about M_2 and constant R_{out} about the origin (see text for details). T_{eff} values are averaged starting at $t = 300$ because the MT rate has not reached a steady-state value and the accretion disk has not yet formed at early times.

of the domain, due to shock heating and slower cooling. With decreasing MT rate, T is lower everywhere (note the difference in colorbar scales between subplots). For the `low_mdot` simulation, T outside the accretion disk reaches close to a temperature floor, which can be outside our opacity tables that extend down to 1,000 K. However, because there is only a small amount of gas outflowing in this case, its cooling is not important.

We also estimate the effective temperature T_{eff} of the emitted radiation by computing azimuthally averaged effective temperatures inside the disk around M_2 and also in the circumbinary gas, and estimating that

$$\sigma_{\text{SB}} T_{\text{eff}}^4 = \int \dot{E}_{\text{cool}} dz. \quad (14)$$

We perform this $\hat{z}$ integral for cylinders centered both around M_2 of approximately constant

$$R_{\text{disk}} \equiv ((x-1)^2 + y^2)^{1/2} \quad (15)$$

and around the origin, with constant

$$R_{\text{out}} \equiv (x^2 + y^2)^{1/2}. \quad (16)$$

To generate our quoted values, we then take the average of T_{eff} over the angle relative to the cylindrical axis (which is just ϕ in the origin-centered case). Note that Eq. 14 approximately reduces to

$$T_{\text{eff,est}}^4 = T^4 \left(\tau_z + \frac{1}{\tau_z} \right)^{-1} \quad (17)$$

given our cooling form.

Fig. 6 plots time-averaged values of T_{eff} both in the accretion disk and circumbinary disk, versus R_{disk} and R_{out} respectively. The typical values of T_{eff} decrease with decreasing MT rate - unsurprisingly given that higher MT rates correspond to a higher cooling luminosity (Fig. 4). The value of T_{eff} reaches very low values in the circumbinary gas for the `low_mdot` simulation, which should not be trusted. In the case of such low density gas, our assumptions related to radiative cooling adopted in the simulation setup (Sec. 2.1), e.g. that there is an equilibrium between gas and radiation, might break down. Since the circumbinary densities and optical depths are very low, any emission would likely occur via lines (e.g., molecular lines such as CO). Dust formation may also occur and change the opacity of the gas.

As a function of increasing cylindrical radius R_{disk} around M_2 , Fig 6 demonstrates that T_{eff} decreases monotonically, which is consistent with the equatorial plots of Fig. 5. The exception is the `low_mdot` simulation, for which T reaches the temperature floor. As a function of increasing cylindrical radius R_{out} around the origin, T_{eff} generally decreases, although the decrease is not quite monotonic for some of the simulations.

With the differences in T within the simulation domain, and between simulations, as well as the vastly different density scale between simulations (Fig. 1), we expect gas pressure and radiation pressure to respectively dominate in different regions and for different simulations (see Appendix A for further discussion).

3.2.1. Observable predictions

From the T_{eff} estimated in Sec. 3.2, we can estimate the observational appearance of the accretion disk and outflows we model, although making more precise predictions will require more realistic radiative transport modeling. For $T_{\text{eff}} \sim 5 \times 10^4$, 10^4 , and 5×10^3 K in the accretion disk of the `highest_mdot`, `high_mdot`, and `mid_mdot` simulations respectively, this roughly corresponds to peak emission wavelengths in the ultraviolet, near-ultraviolet/optical, and optical/near-infrared. For $T_{\text{eff}} \sim 10^4$, 2×10^3 , and 5×10^2 K in the circumbinary disk of the same simulations, this roughly corresponds to peak emission wavelengths in the near-ultraviolet/optical, near-infrared, and the mid-infrared, respectively.

The cooling luminosities associated with our simulations are $\sim 10^{39}$ erg/s for our `highest_mdot` simulation and $\sim 10^{38}$ erg/s for our `high_mdot` simulation. Even when just including the domain outside the accretion disk, luminosities for these two higher MT rate simulations are $\sim 5 \times 10^{38}$ erg/s and $\sim 2 \times 10^{37}$ erg/s. Our

lower MT rate simulations are associated with lower luminosities, dominated by the cooling power in the accretion disk. For instance, the `mid_mdot` simulation has luminosities of $\sim 10^{37}$ erg/s when including the entire domain, and $\sim 10^{36}$ erg/s outside the accretion disk.

Despite these predictions, there are multiple radiation processes that we have neglected. Although the cooling luminosity of the accretion disk tends to peak near its center (Sec. 3.2), we have neglected a detailed treatment of accretion power onto the surface of M_2 , especially if it is a compact object like a neutron star. In such a case, a fast wind will likely be launched due to viscous accretion, which will emit X-ray and UV photons. These photons will be reprocessed to longer wavelengths by the circumbinary outflow — in the infrared if dust is formed (Lu et al. 2023).

However, the production of dust in the outflow is unclear. Temperatures below around 1500 K are necessary, though not sufficient, to cause dust condensation. In the `mid_mdot` simulation, such low temperatures are reached in parts of the domain above the equatorial plane, and in the `low_mdot` simulation such temperatures are reached almost everywhere outside the accretion disk, which is kept hotter by an imposed temperature floor (Fig. 5). However, our low temperature estimates may not be accurate and are likely may be more associated with ambient gas rather than with the outflows we primarily seek to model. A more important question is whether dust forms somewhere in the equatorial plane of our simulations, where the outflow is strongest, especially for our higher MT simulations. Although T decreases moving radially outward (Fig. 6), our simulation domain does not extend far enough from the binary to capture its ultimate asymptotic temperature. Pejcha et al. (2016) shows that, for a MT rate out of L2 of $10^{-5} M_\odot/\text{yr}$, the gas temperature does not drop low enough to favor dust formation until $r = 8a$. This MT rate is intermediate to our `low_mdot` and `mid_mdot` simulations, and the gas temperatures in Pejcha et al. (2016) are approximately similar to ours where the simulation domains overlap. However our domain does not extend far enough to reach lower temperatures (especially in the equatorial plane), preventing us from making more detailed predictions of dust formation.

3.3. Energetics, velocities, and angular momentum losses

We compute the specific angular momentum (AM) carried away by the outflowing gas. The specific AM (along $\hat{z}$) of the L2 point is given by $h_{L2} = \Omega(r_{L2} - r_{\text{com}})^2$, where r_{L2} and r_{com} are the radial coordinates of

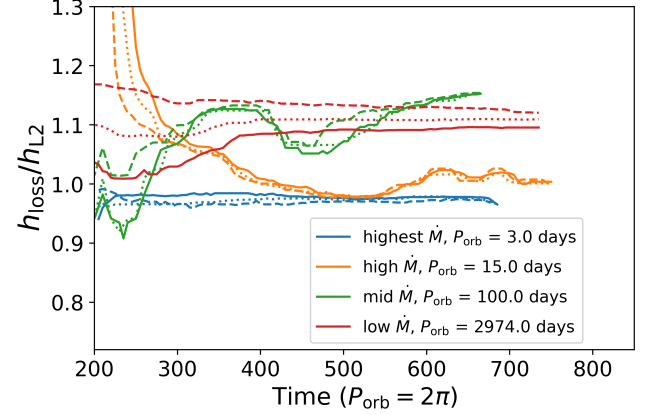


Figure 7. The specific AM of outflowing material h_{loss} , in units of h_{L2} , versus time in code units, where 1 orbit = 2π . h_{loss} is calculated over spheres centered at the COM, of radius 3, 4, and 4.65 (dashed, dotted, solid lines) and all curves are smoothed using a rolling average.

L2 and the COM. In units of $\sqrt{GM_{\text{tot}}/a}$, $h_{L2} = 1.56$ for the $q = 0.5$ binaries that we simulate.

The specific AM $\vec{h}$ of an arbitrary fluid element, with coordinate $\vec{r}$ and velocity $\vec{v}$ in the simulation frame, is given by

$$\vec{h} = (\vec{r} - \vec{r}_{\text{com}}) \times (\vec{v} + \vec{\Omega} \times (\vec{r} - \vec{r}_{\text{com}})) \quad (18)$$

where $\vec{r}_{\text{com}}$ is the coordinate of the binary's COM (in the co-rotating frame) and $\vec{\Omega}$ is the orbital frequency. We are interested predominantly in the $\hat{z}$ component of the AM, as the orbital axis of the binary is along $\hat{z}$. The rate of AM outflow $\dot{L}_z$ over a surface with area element $d\vec{A}$ is given by

$$\dot{L}_z = \int_A h_z \rho \vec{v} \cdot d\vec{A}. \quad (19)$$

Similarly, the rate of mass flow through the same surface can be written as

$$\dot{M} = \int_A \rho \vec{v} \cdot d\vec{A} \quad (20)$$

and the average specific AM h_{loss} of the outflowing material through this surface is the ratio of these two integrals, i.e.

$$h_{\text{loss}} = \frac{\dot{L}_z}{\dot{M}}. \quad (21)$$

As in Paper I, we find that the value of h_{loss} asymptotes with distance from the COM to an approximately constant value. We calculate h_{loss} over spheres of constant $|\vec{r} - \vec{r}_{\text{com}}| \approx 3, 4$, and ≈ 4.65 , where 4.65 is the largest COM-centered sphere that fits in the domain.

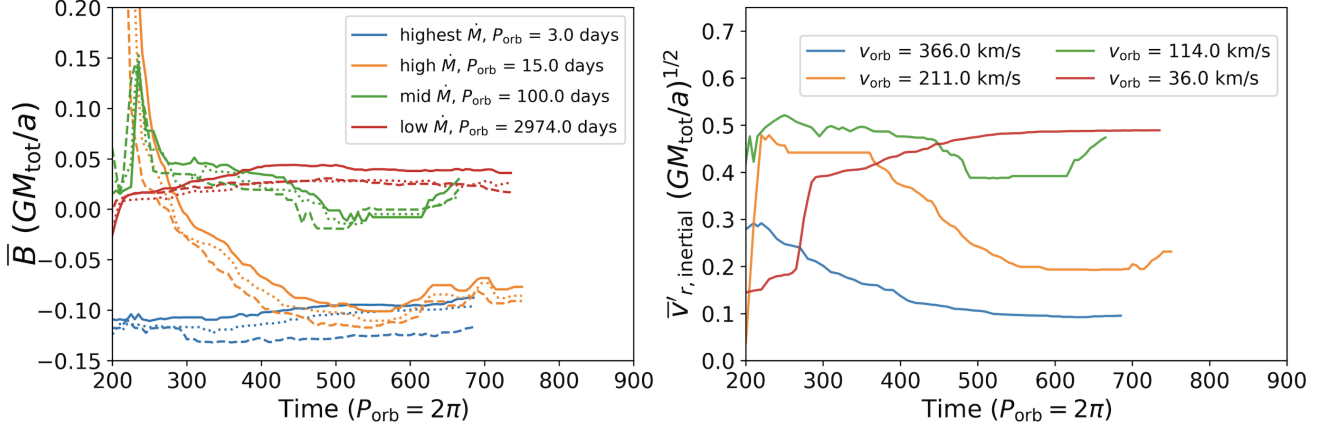


Figure 8. Properties related to the energy and velocity of outflowing gas. Curves are time-smoothed using a rolling average. Left: The Bernoulli parameter $\bar{B}$, in units of GM_{tot}/a , for COM-centered spheres of $|\vec{r} - \vec{r}_{\text{com}}| = 3, 4$ and $r_{\text{max}} \approx 4.65$, in units of $a = 1$ (dashed, dotted, solid lines). Right: The volume-averaged radial velocity of the outflow, $\bar{v}'_{r,\text{inertial}}$ (Eq. 25), in units of the orbital velocity for each simulation, $\sqrt{GM_{\text{tot}}/a}$.

Fig 7 plots the values of h_{loss} , in units of h_{L2} . In all cases, $h_{\text{loss}} \sim h_{\text{L2}}$, meaning that the outflow efficiently extracts AM from the binary - note that h_{L2} is much larger than the specific AM of the accretor, h_{M2} . For the **highest_mdot** and **high_mdot** simulations, $h_{\text{loss}}/h_{\text{L2}}$ is slightly less than unity, as in Paper I, whereas it is slightly larger than unity in the **low_mdot** and **mid_mdot** cases. However, even though AM may be mildly more efficiently extracted at these lower MT rates, far less material is actually flowing out through L2. These competing affects are discussed further in Section 3.4.

We also compute the velocity and energy of the outflowing gas. Velocities relative to the center of mass are given by $\vec{v}_{\text{inertial}} = \vec{v} + \vec{\Omega} \times (\vec{r} - \vec{r}_{\text{com}})$. The radial component $v_{r,\text{inertial}}$ is calculated as

$$v_{r,\text{inertial}} = \vec{v}_{\text{inertial}} \cdot \frac{\vec{r} - \vec{r}_{\text{com}}}{|\vec{r} - \vec{r}_{\text{com}}|}. \quad (22)$$

The Bernoulli parameter per unit mass, B , is calculated as

$$B = \frac{|\vec{v}_{\text{inertial}}|^2}{2} + u_{\text{int}} + \Phi_{\text{grav}} + \frac{P}{\rho}. \quad (23)$$

where $\Phi_{\text{grav}} = -\frac{M_1}{|\vec{r}|} - \frac{M_2}{|\vec{r} - a\hat{x}|}$. We define the mass-weighted angle-averaged Bernoulli parameter $\bar{B}$ as

$$\bar{B} = \frac{\int \rho B dA}{\int \rho dA}, \quad (24)$$

where the integral is performed over spheres at constant $|\vec{r} - \vec{r}_{\text{com}}|$.

The value of $\bar{B}$ asymptotes with radius, as in Paper I. Similar to the calculation above regarding h_{loss} , we calculate $\bar{B}$ over spheres of constant $|\vec{r} - \vec{r}_{\text{com}}|$. These values are plotted in the left hand panel of Fig. 8. The

low_mdot and **mid_mdot** simulations have near zero or slightly positive asymptotic $\bar{B}$, meaning a large fraction of the outflow is formally unbound, whereas the **highest_mdot** and **high_mdot** simulations are bound, similar to Paper I. However, although we do include radiative cooling, we do not include any momentum deposition due to irradiation of the circumbinary gas by the stars or the accretion disk that may help drive the gas outwards.

We calculate a characteristic radial velocity $\bar{v}'_{r,\text{inertial}}$ using the radial inertial velocity $v_{r,\text{inertial}}$ (Eq. 22),

$$\bar{v}'_{r,\text{inertial}} = \frac{\int_{r=3}^{r=r_{\text{out}}} \rho v_{r,\text{inertial}} dV}{\int_{r=3}^{r=r_{\text{out}}} \rho dV} \quad (25)$$

where we have performed a full volume integral for the average because, due to the spiral structure and shocks in the outflow, $v_{r,\text{inertial}}$ can show a wave pattern that oscillates about an average value. The values of $\bar{v}'_{r,\text{inertial}}$ are plotted in the right hand panel of Fig. 8. In units of the orbital velocity v_{orb} , there is a trend that the **highest_mdot** simulation has a low $\bar{v}'_{r,\text{inertial}}$ (10% v_{orb}), the **high_mdot** simulation a somewhat higher value (20% v_{orb}), and the lower MT simulations a substantially higher value (40–50% v_{orb}). This is consistent with decreasing MT rate corresponding to a higher specific energy and Bernoulli parameter (left panel). In cgs units, the trend is different due to lower MT rate simulations having a lower v_{orb} . The **low_mdot** simulation has the lowest outflow velocity at about 15 km/s, whereas the other 3 simulations have velocities clustered between 40–60 km/s.

Pejcha et al. (2016) performed smoothed particle hydrodynamic (SPH) simulations, where material was initialized near L2 in co-rotation with the binary's orbit,

and the outflow was tracked far from the binary to $r = 50a$. They found that the asymptotic velocity of the outflow is always proportional to v_{orb} . In contrast, we find that the average radial velocity in units of the escape speed, $\bar{v}'_{r,\text{inertial}}/v_{\text{orb}}$ tends to increase with decreasing MT rate. In addition, our outflows tend to be bound for `highest_mdot` and the `high_mdot` simulations, whereas the outflows in [Pejcha et al. \(2016\)](#) are unbound for the same mass ratio.

These discrepancies may be understood through [MacLeod et al. \(2018b\)](#) and our Paper I, which found that strong outflows through L2 tend to be initialized not at co-rotation, and are therefore more bound compared to the assumptions of [Pejcha et al. \(2016\)](#). Our higher MT rate simulations are in this regime. In contrast, the fact that $\bar{v}'_{r,\text{inertial}}/v_{\text{orb}}$ reaches a roughly constant ratio for our lower MT rate simulations seems to imply that they are in the regime consistent with [Pejcha et al. \(2016\)](#), although the value is a bit higher than calculated in [Pejcha et al. \(2016\)](#), where the asymptotic radial velocity is 0.2-0.35 v_{orb} .

Overall, our results detailing the properties of the outflow - AM, velocity, and energy - are similar to that of Paper I, which included no radiative cooling, in the case of our `highest_mdot` and `high_mdot` simulations. Indeed, we found in Paper I that our adiabatic assumptions were most valid at $\gtrsim 10^{-2} M_{\odot}/\text{yr}$. Therefore, the results of Paper I are only valid at the very high end of MT rates, and this work delineates more accurately when the results of Paper I break down.

3.4. Effect on binary orbit

Simulation	β	$h_{\text{loss}}/h_{\text{L2}}$	γ	ζ_{RL}
<code>highest_mdot</code>	0.05	0.95	6.7	115
<code>high_mdot</code>	0.4	1.0	7.0	13
<code>mid_mdot</code>	0.9	1.1	7.7	4
<code>low_mdot</code>	0.96	1.1	7.7	3

Table 2. For our grid of MT simulations, the fraction of mass retained in the accretion disk β (Eq. 9) and outgoing AM parameters h_{loss} , in units of h_{L2} , and γ (Eqs. 21 and 26). ζ_{RL} is the mass-radius exponent of the donor's Roche radius (Eq. 28), with higher values generally making unstable MT more likely. The values quoted are averaged over the steady-state behavior of the simulations.

The parameters β and h_{loss} , describing the fraction of the transferred mass that is accreted, and the AM carried by any escaping material, are of great consequence to the evolution of the binary's orbit. Because we are focused on snapshot of stable MT, we keep the orbital

separation fixed in our simulations, but here discuss how the orbit would change over long-term MT.

It is useful to express the AM losses in terms of the dimensionless quantity γ , expressing the specific AM lost, in units of the specific AM of the binary. Therefore, if J_{orb} is the orbital AM,

$$\gamma = \frac{h_{\text{loss}} M_{\text{tot}}}{J_{\text{orb}}} \quad (26)$$

Table 2 summarizes the approximate values of β and γ appropriate to the steady-state behavior of our simulations (see also Fig. 2). Assuming that a fraction β accretes onto M_2 , which will break down if M_2 is a compact object, the orbital evolution of the binary then can be expressed as²

$$\frac{\dot{a}}{a} = -2 \frac{\dot{M}_1}{M_1} \left(1 - \beta \frac{M_1}{M_2} - (1 - \beta) \left(\gamma + \frac{1}{2} \right) \frac{M_1}{M_1 + M_2} \right). \quad (27)$$

The last term on the right hand side of Eq. 27 represents orbital AM lost due to outflowing gas. Even though γ is high for the `mid_mdot` and `low_mdot` simulations, this term is close to 0 because β is close to 1. Therefore, the additional AM loss and the effect on $\dot{a}$ is minimal in these two cases. In contrast, the `high_mdot` and `highest_mdot` simulations are associated with high values of γ and low values of β , implying that this term will be large and $\dot{a}$ will be negative (as the term in parenthesis will be negative, but $\dot{M}_1$ is also negative). Although the orbit is expected to shrink in these cases, the stability of MT is determined also by the response of the donor star. This is beyond the scope of this work, as we drive MT through via heat injection and do not accurately model the donor's structural response.

However, we can estimate the value of ζ_{RL} , the mass-radius exponent for the Roche radius of the donor

$$\zeta_{\text{RL}} = \frac{d \ln R_L}{d \ln M_1} \quad (28)$$

where R_L is the Roche radius of the donor star. If ζ_{RL} is large compared to the exponent representing the response of the donor's radius R_* to mass loss, ζ_*

$$\zeta_* = \frac{d \ln R_*}{d \ln M_1} \quad (29)$$

the overfill factor of the donor will increase and MT is likely to be unstable. We calculate ζ_{RL} using the formula of [Soberman et al. \(1997\)](#) in the case of a disk

² For a derivation, see Chapter 7 of O. Pols' binary evolution notes, https://www.astro.ru.nl/~onnop/education/binaries_utrecht_notes/

ring formed from a fraction $1 - \beta$ of the transferred mass, with ζ_{RL} also depending on γ and q . Note that the definition of γ in [Soberman et al. \(1997\)](#) is different than ours, by a factor of $\frac{1/q}{(1+1/q)^2}$.

The values of ζ_{RL} calculated for our simulations are given in Table 2. We find that ζ_{RL} is quite large, over 100, for the `highest_mdot` simulation, such that MT is likely to be unstable unless ζ_* is similarly huge, which would correspond to a star that contracts very rapidly in response to MT. This would appear to violate our assumption of constant a and our focus on stable MT. However, for our `high_mdot` simulation, which is generally similar to the `highest_mdot` simulation but with the MT a bit more conservative, ζ_{RL} is only about 12. This means that a star with a radiative envelope, with $\zeta_* \gg 1$, may still undergo stable MT in this case. For the two lower MT rate simulations, ζ_{RL} drops even further. For convective-envelope stars, with $\zeta_* < 0$, MT is likely unstable for the q that we consider.

4. CONCLUSION

Episodes of rapid mass transfer (MT) ($\gtrsim 10^{-4} M_{\odot}/\text{yr}$) occur in many binaries containing a massive star, potentially leading to non-conservative MT and circumbinary outflows. We simulated stable MT between a $10 M_{\odot}$ donor star and an $5 M_{\odot}$ unresolved secondary, probing MT rates between $\sim 10^{-5}$ and $\sim 10^{-1} M_{\odot}/\text{yr}$ by varying the orbital separation of the binary (Table 1). Our 3D simulations are performed using the PLUTO hydrodynamic code. They resolve the flow of mass from the donor, into an accretion disk around the secondary, through the outer Lagrange point (L2), and into a circumbinary outflow. Radiative cooling is included at every grid cell, using tabulated opacities and an approximate treatment of photon diffusion perpendicular to the orbital plane, where the density is lowest. Our idealized simulations do not include detailed radiative transport or radiative forces.

The main results of our work are:

1. Mass from the accretion disk tends to outflow in a stream originating near L2, forming an equatorially-concentrated circumbinary disk outflow (Fig. 1). The fraction of gas staying in the accretion disk, versus outflowing near L2, is expressed with the parameter β , where $\beta = 1$ implies conservative MT. We find that β decreases strongly with increasing MT rate, meaning more material outflows, and in particular rises sharply between $\sim 10^{-3}$ and $\sim 10^{-2} M_{\odot}/\text{yr}$ (Fig. 2), approximately where the accretion luminosity of the outer edge of the disk becomes super-Eddington.

2. Across a wide range of MT rates, the outflowing gas has a specific angular momentum (AM) h_{loss} that asymptotes with radius to a value approximately equal to that of the L2 point, h_{L2} (Fig. 7) and much larger than the specific AM of the accretor. This means that, in the case of strongly non-conservative MT and significant outflows, AM will be efficiently extracted from the binary, which will have important consequences for orbital evolution.
3. The outflow reaches an asymptotic radial velocity that increases from 10% the orbital velocity to 50% the orbital velocity as the MT rate decreases from $\sim 10^{-1}$ to $\sim 10^{-3} M_{\odot}/\text{yr}$. In physical units, however, the radial velocity is about 30-50 km/s for our simulations, except for the `low_mdot` simulation where it is lower (Fig. 8).
4. The gas cooling luminosity is dominated by the accretion disk at low MT rates, but the outflow luminosity contributes strongly at $\sim 10^{-2} M_{\odot}/\text{yr}$ and dominates the luminosity at $\sim 10^{-1} M_{\odot}/\text{yr}$ (Fig. 4), leading to luminosities between 10^{38} and 10^{39} erg/s.
5. The gas temperature and T_{eff} decrease with decreasing MT rate (Figs. 5, 6), with T_{eff} of the circumbinary disk $\sim 10^4, 10^3, 10^2$ K for MT rates of $\sim 10^{-1}, 10^{-2}, 10^{-3} M_{\odot}/\text{yr}$. The T_{eff} of the accretion disk is an order of magnitude higher, peaking near the disk center, though we do not resolve the inner disk where we impose a softening potential. Dust formation may occur within the outflow, especially away from the equatorial plane and for our lower MT rate simulations.

Although there have been a few prior works that have performed hydrodynamic simulations involving outflows through L2 (and/or L3) that implemented radiative cooling losses, our work differs by systematically controlling the MT rate from the donor star and tracking its effect on the circumbinary outflow. Our measurement of the MT rates where MT transitions from conservative to non-conservative, as well as the AM carried by the outflowing gas, can be incorporated into population synthesis calculations investigating the stability of mass transfer, and orbital evolution during stable MT preceding mergers of binary compact objects (e.g. [Marchant et al. 2021](#); [Gallegos-Garcia et al. 2021](#)). Observationally, these simulations can be used to predict the formation of material following extreme mass loss from supernova progenitors ([Wu & Fuller 2022](#)), as well as the temperature and luminosity of disks associated with a

phase of rapid MT. However, more examination of the outflow farther away from the binary is needed to investigate the formation of dust, especially in the equatorial plane of the outflow, and the outflow's ultimate properties, which likely connect to luminous red novae (Pejcha et al. 2016). In addition, our simulations were performed for only a single combination of donor and accretor masses, and for solar metallicity and composition, which should be extended to other massive binaries.

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 7 Support (ACCESS) program, which is supported by
 8 U.S. National Science Foundation grants #2138259,
 9 #2138286, #2138307, #2137603, and #2138296.

Software: PLUTO, Python, NumPy (Harris et al. 2020), Matplotlib (Hunter 2007), SciPy (Virtanen et al. 2020), Roche lobe calculator (Leahy & Leahy 2015), Roche_tidal_equilibrium³ (Lu 2025), L2massloss⁴

APPENDIX

A. RATIO OF GAS TO RADIATION PRESSURE

Fig. 9 plots the ratio of gas pressure to radiation pressure in the equatorial plane, where, given ρ and P , the two terms are calculated via

$$P = \frac{\rho k_b T}{\mu m_p} + \frac{a_{\text{rad}} T^4}{3}. \quad (\text{A1})$$

See Sec. 2.1 for more details related to the calculation. In general, regions with higher ρ are more gas pressure dominated. This includes the accretion disk and the circumbinary outflow originating at L2 (and L3, for the `highest_mdot` and `high_mdot` simulations. Where the density is low, radiation pressure dominates (the purple voids of Fig. 9). As the circumbinary gas density decreases with decreasing MT rate, likewise the simulations with decreasing MT rate are less gas pressure dominated in the circumbinary regions. However, the opposite is true in the accretion disk near the accretor. For the `low_mdot` and `mid_mdot` simulations, the ratio of gas to radiation pressure reaches quite a high value in the center of the accretion disk, where the gas has collapsed to a cool, dense disk. Note that we are using an adiabatic index of 1.4 for our EOS (Eq. 1) due to limitations in the current EOS options in PLUTO. Future work should adopt a more dynamic equation of state that varies more self-consistently with T and ρ .

B. TIMESCALES

The ratio of the cooling timescale to mass advection timescale changes significantly between our simulations

of different MT rates. The cooling timescale t_{cool} is given by

$$t_{\text{cool}} = \frac{u_{\text{int}}}{|\dot{E}_{\text{cool}}|}. \quad (\text{B2})$$

The value of t_{cool} is estimated, with density-weighted averages, 1. within the accretion disk at several different radii surrounding M_2 and inside its Roche lobe; 2. at radii within the circumbinary outflow, at radii centered at the origin and larger than a .

The advective timescale t_{adv} is given by

$$t_{\text{adv}} = \frac{|\vec{r} - a\hat{x}|}{v_{r,M_2}} \quad (\text{B3})$$

near M_2 , where v_{r,M_2} is the component of velocity radially outward from M_2 . In the broader circumbinary disk surrounding both stars, we estimate t_{adv} as simply

$$t_{\text{adv}} = \frac{|\vec{r}|}{v_r} \quad (\text{B4})$$

where v_r is the radial component of velocity relative to the origin. We similarly perform density-weighted averages of t_{adv} at various radii in our domain, using the appropriate definition in the accretion disk or in the circumbinary disk.

For the `highest_mdot` and `high_mdot` simulations, we find that t_{cool} is larger than t_{adv} in both the accretion disk and circumbinary disk. Because the gas is optically thick everywhere, it is difficult for radiation to escape in this case. For the `mid_mdot` simulation, t_{adv} is smaller than t_{cool} in the accretion disk, but is larger in the circumbinary disk. This implies that the outflow cools quickly once it leaves the accretion disk. Finally, for the `low_mdot` simulation, t_{adv} is larger than t_{cool} in

³ https://github.com/wenbinlu/Roche_tidal_equilibrium

⁴ <https://github.com/wenbinlu/L2massloss>

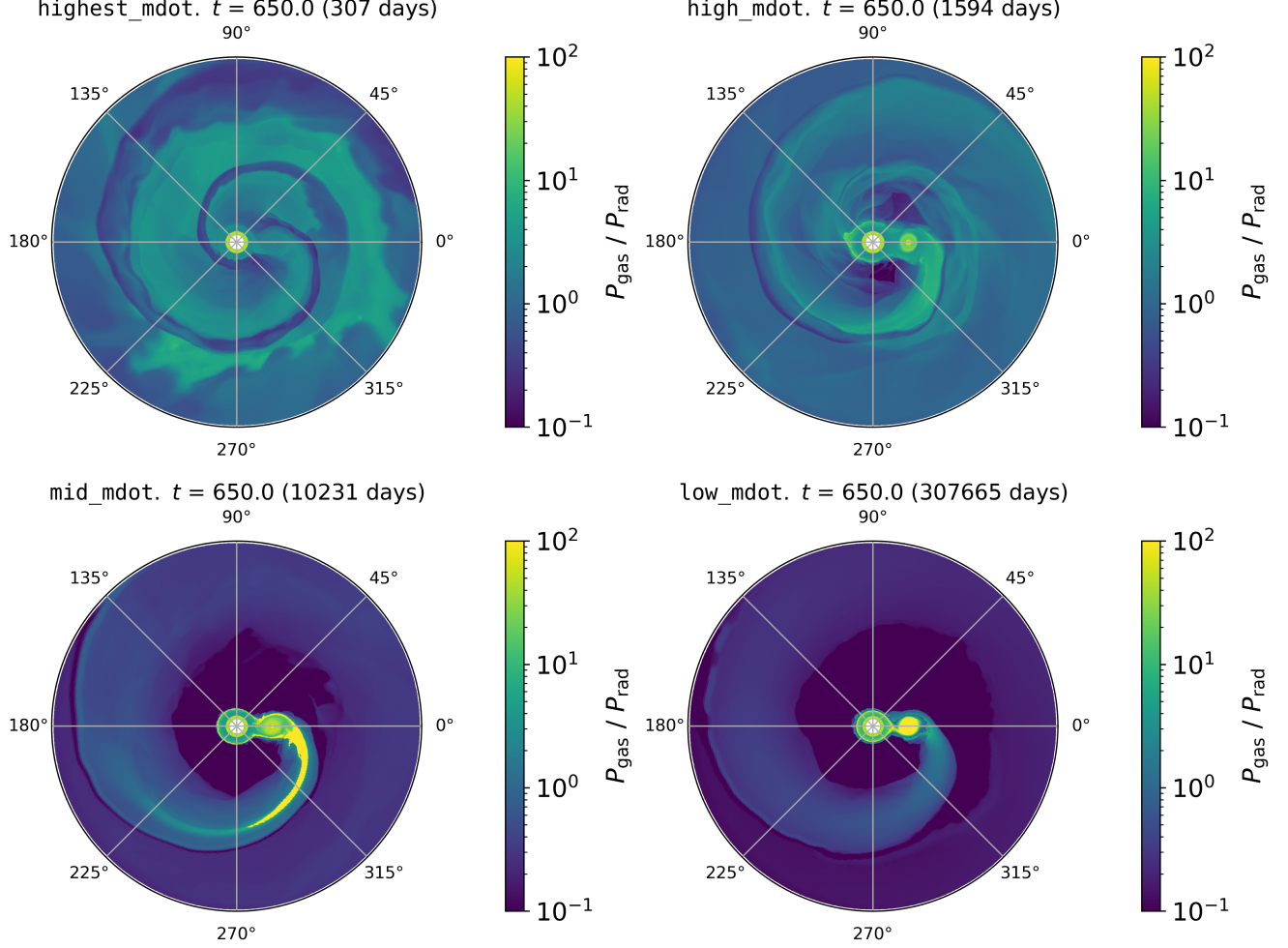


Figure 9. The ratio of gas pressure to radiation pressure (see Eq. A1) in the equatorial plane of the simulations, for snapshots representative of the quasi-steady state behavior.

both the accretion disk and in the circumbinary disk, meaning that material rapidly cools to near the internal energy floor (which is set via a temperature floor of 2000 K in the higher density regions such as the accretion disk).

C. ADOPTED OPACITIES

Fig. 10 plots the tabulated value of κ used in our work as a function of ρ and T , as well as some representative points showing where the cells in the simulation domain are located in this phase-space for a representative snapshot. The opacity rises at $\log T \gtrsim 3.7$ due to H^- opacity, and is dominated by dust for $\log T \lesssim 3.1$ (Pejcha et al. 2016).

We divide the grid domain into three representative regions: inside M_2 's Roche radius (cyan points); in the equatorial plane, where ρ is the highest, but outside M_2 's Roche radius (red points); everywhere else in the

domain, which are therefore points above the equatorial plane and not near M_2 (orange points).

The black dashed lines of Fig. 10 show the floors that we impose (see also Sec. 2.1 for the reasoning behind our floors). The vertical line at $T = 2000$ K represents our temperature floor imposed in relatively high ρ regions, whereas the curved line shows the floor on sound speed that is imposed in low density regions. The horizontal dashed line shows the density floor imposed everywhere. For the latter two lines, the floor values are the same in code units, but vary in physical units and so shift position between the two subplots.

For the *high_mdot* simulation shown in Fig. 10, points near M_2 are safely away from the imposed floors. In this region, the dominant opacity appears to from free-free and bound-free transitions, although this is a rough estimate based on a Kramer's opacity law. Some points in the equatorial plane and many points above the equatorial plane reach the sound speed floor, which is es-

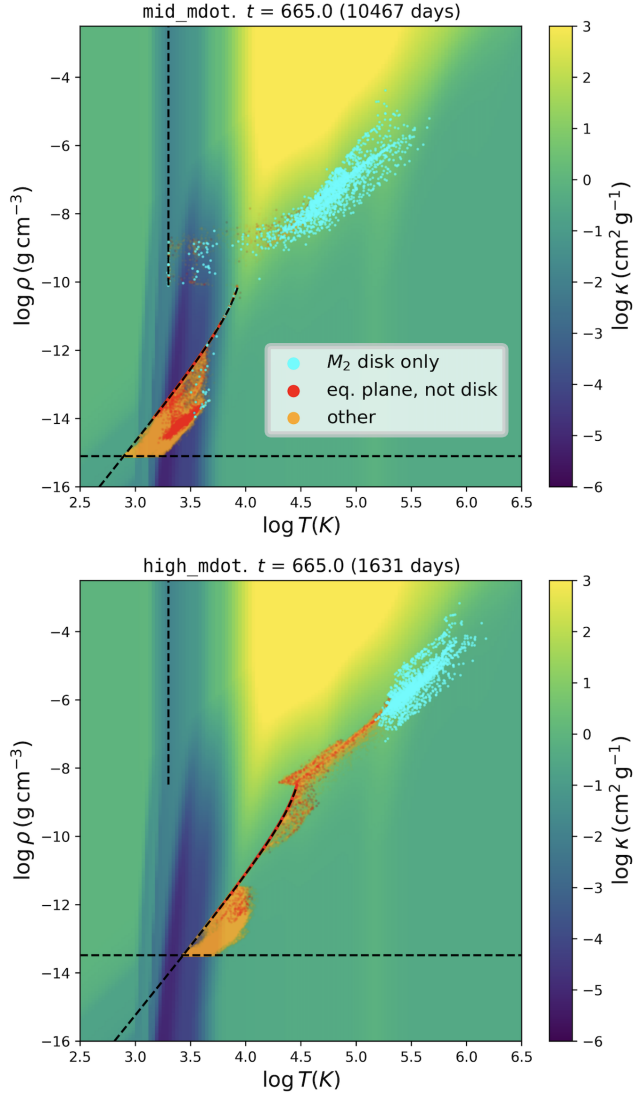


Figure 10. Plots of the opacity used in this work. Colored dots correspond to a sample of grid cells, corresponding to the labeled t , and where they lie in ρ, T space. Cyan points are located in the Roche lobe surrounding M_2 . Red points are in the equatorial plane, excluding any points falling into M_2 's Roche lobe. Orange points are sampled everywhere else in the domain, i.e. above the equatorial plane. The `highest_mdots` and `low_mdots` simulations are not shown because they are relatively similar to the subplots shown here. The black curves correspond to floors on ρ , T , and c_s (see text for details). The opacity tables used (see Sec. 2.1) extend down to 10^3 K, so points off the table use the value at the boundary.

essentially a density-dependent temperature floor. Points above the equatorial plane also tend to approach the density floor. For the `mid_mdots` simulation, most points near M_2 are above the temperature floor, although some approach it. Most points elsewhere in the domain approach the density and sound speed floor. The $T = 2000$

K floor is not relevant because of the low densities in these regions - were it imposed, it would lead to an artificially high internal energy and a spurious flux of mass in the outer domain. Overall, there is a progression where the high MT rate simulations never approach the temperature floor, the `mid_mdots` simulation reaches the floor only far away from the accretion disk, and the `low_mdots` simulation reaches the floor basically everywhere.

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