Can Newtonian Gravity Produce Quantum Entanglement?

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In this note, we consider specific scenarios of matter-gravity interaction based on the source theory to clarify the issue regarding the Gravitationally Induced Entanglement (GIE) protocol, which posits that the detection of quantum entanglement induced by a pure gravitational effect serves as a probe of quantum gravity. Thus, the GIE protocol implies that classical gravity cannot produce or mediate quantum entanglement, to which a recent paper [1] shows the opposite. In all three scenarios of matter-gravity interactions, including mini-superspace, semiclassical gravity, and stochastic gravity approaches, our results agree with the GIE protocol and its implications. More precisely, we show that the mesoscopic quantum bodies with their parities of mass quadrupoles interacting via Newtonian gravity can get entangled only if the parity of the gravitational tidal field sourced by the quantum body is also quantized. We also point out a possible loophole that could seemingly violate the GIE protocol due to the truncation of higher-order terms in the perturbative expansion by orders of the Newton constant.

Contents

I. INTRODUCTION AND CONCLUSION

A recent paper in Nature [1] claims that classical gravity can produce quantum entanglement through the exchange of virtual quantum matter, even though the mediator, the gravity field, is classical. This raises the debate over whether classical gravity can produce or mediate quantum entanglement. Especially in [2, 3], the authors object to the claim based on a dynamics-independent quantum information scheme, called the Gravitationally Induced Entanglement (GIE) protocol [4, 5], which states that the detection of quantum entanglement involving only the pure gravity effect is a probe to quantum gravity. Therefore, the GIE protocol implies that the local classical mediator systems, such as classical gravity, cannot create quantum entanglement. The seemingly contradictory statements between [1] and [4, 5] deserve further clarification, which we address in this note. For some experimental proposals to realize the GIE protocol, see [6–8].

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Though the GIE argument is sound, it is very general without attribution to specific exemplar gravity-matter models. This can create loopholes in some technical, detailed calculations, such as the one in [1]. To address such issues, in this note, we consider a specific quantum-matter and gravity model from the perspective of source theory. In this way, we will see that all the subtle issues regarding GIE become transparent, thus justifying its validity in the concrete contexts. Firstly, we define the quantum bodies as the mesoscopic superposed states of a Z_2 parity of some physical observables, such as the sign of an electric dipole or a mass quadrupole. The GIE protocol is to examine whether gravity can entangle a pair of such quantum bodies that are spatially separated. These mesoscopic quantum bodies for the GIE protocol play a similar role to the microscopic spin-$1/2$ particles in the Stern-Gerlach experiment. Secondly, we will generalize the source theory for the classical Newtonian gravity to three possible scenarios of dealing with the gravity interactions between quantum bodies: (I) the mini-superspace approach to quantum gravity by just quantizing the parity of the gravitational tidal field sourced by the quantum mass quadrupole; (II) the semiclassical gravity approach by sourcing the Newtonian potential by the expectation value of the mass quadrupole; (III) the stochastic gravity approach by appending the semiclassical gravity with a Langevin-type noise due to fluctuation of the parity of quantum mass quadrupole.

By studying the unitarily evolved final state of the two quantum bodies dictated by the quadrupole-quadrupole interactions in the above three scenarios, we conclude that the quantum bodies can be entangled in scenario (I), but not in scenarios (II) and (III). Therefore, our result is consistent with the GIE protocol. Moreover, in scenarios (II) and (III), we point out a possible loophole based on the perturbative calculation, which will yield an artifact entanglement production due to the neglect of the subleading terms by truncating the expansion at a particular order of the Newton constant. This could be a cause of the disagreement between [1] and the GIE ones [4, 5].

II. MESOSCOPIC QUANTUM BODIES AND THEIR PREPARATION

All considerations regarding the detection of quantumness in gravity are based on a setup originally proposed by Feynman during the 1957 Chapel Hill conference on gravity [9] to prepare a massive object in a quantum superposition of two positions, which we will refer to as a quantum body. Physically, this mass superposition can be realized as distributed N00N states with spatial separation of two different modes located respectively at $\vec{x}_L$ and $\vec{x}_R$ [10, 11],

$$|\mathrm{N00N}\rangle = \frac{1}{\sqrt{2}}\big(|N;\vec{x}_L\rangle|0;\vec{x}_R\rangle + e^{iN\theta}|0;\vec{x}_L\rangle|N;\vec{x}_R\rangle\big) \ , \tag{1}$$

where N is either the photon or the atom numbers, depending on the experimental setup. This N00N state has recently been adopted to study its quantum decoherence by a black hole [12–15].

Moreover, if the N00N state also carries some electric charge, then by placing a charge of opposite sign at $\vec{x}_M = (\vec{x}_L - \vec{x}_R)/2$, the combination forms a quantum body with an electric dipole with its direction in a superposition state. As there is no negative mass, it is impossible to produce the quantum mass dipole analogue. However, it is possible to form the mass quadrupole with its sign in the superposition state as shown in Fig. 1. Therefore, we will consider only the quantum mass quadrupoles and their mutual gravitational interactions in this note.

Formally, we can associate with such a quadrupole observable a mini-superspace operator

$$\hat{Q}_{ij} = Q_{ij}\hat{Z} \ , \tag{2}$$

where $\hat{Z}$ is a Z_2 operator for measuring the sign of the quadrupole, satisfying

$$\hat{Z}|\pm;Q_{ij}\rangle = \pm|\pm;Q_{ij}\rangle \ , \qquad \text{with} \quad \langle\pm;Q_{ij}|\pm;Q_{ij}\rangle = 1 \ , \quad \text{and} \quad \langle\mp;Q_{ij}|\pm;Q_{ij}\rangle = 0 \ , \tag{3}$$

where $\hat{Q}_{ij}$'s eigenstates $|\pm;Q_{ij}\rangle$'s correspond to the classical quadrupole moments with definite signs. We can then define a general superposed state of quadrupole signs as follows,

$$|\theta,\phi;Q_{ij}\rangle := \cos\theta|+;Q_{ij}\rangle + \sin\theta e^{i\phi}|-;Q_{ij}\rangle \ . \tag{4}$$

The N00N state (1) is just a special case with $\theta = \pm\frac{\pi}{4}$, $\phi = 0$. Note that

$$\langle\theta,\phi;Q_{ij}|\hat{Q}_{ij}|\theta,\phi;Q_{ij}\rangle = Q_{ij}\langle\theta,\phi;Q_{ij}|\hat{Z}|\theta,\phi;Q_{ij}\rangle = Q_{ij}\cos 2\theta \ . \tag{5}$$

This mini-superspace approach to quantum bodies can be thought of as the mesoscopic version of microscopic quantum spin-$\frac{1}{2}$. Its simplicity helps elucidate the issue discussed in this note, much like the role of quantum spin-$\frac{1}{2}$ in Stern-Gerlach experiments in demonstrating quantum superposition.

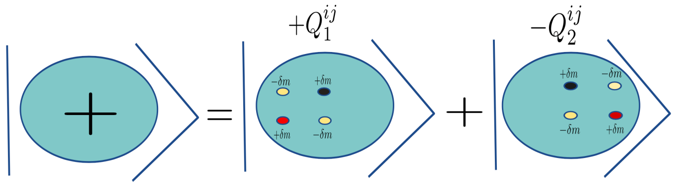

FIG. 1: A quantum body of mass quadrupole with its sign in the equal-weight superposed state.

III. NEWTONIAN GRAVITY OF QUANTUM BODIES

In Newtonian gravity, the Newton potential Φ in the multipole expansion takes the form

$$\Phi(\vec{x}) = \frac{G_N m}{r} + \frac{G_N p_i x_i}{r^3} + \frac{G_N Q_{ij} x_i x_j}{r^5} + \cdots \,, \tag{6}$$

where G_N is the Newton constant, m, p_i, and Q_{ij} are respectively the mass moments of monopole, dipole, and quadrupole of a compact body. Since there is no negative mass, the mass dipole can be made to vanish by placing the origin at the center of mass. Moreover, the quadrupole moments are symmetric and traceless, i.e., $Q_{ij} = Q_{ji}$ and $\sum_{i=1}^{3} Q_{ii} = 0$.

The interaction Hamiltonian of a compact body with the Newtonian gravity in the multipole expansion takes the form

$$H_{\rm N} = -m\Phi|_{\vec{x}=\vec{0}} - p_i \partial_i \Phi|_{\vec{x}=\vec{0}} - Q_{ij}\partial_i\partial_j\Phi|_{\vec{x}=\vec{0}} + \cdots \,. \tag{7}$$

Combining (6) and (7), we can obtain the interaction between two static quadrupole moments $Q_{ij}^{(1,2)}$ with the superscript labeling the quantum bodies,

$$H^c_{QQ} = -\frac{1}{2}\Big[Q_{ij}^{(1)}\partial_i\partial_j\Phi^{(2)}|_{\vec{x}=\vec{x}_2} + Q_{ij}^{(2)}\partial_i\partial_j\Phi^{(1)}|_{\vec{x}=\vec{x}_1}\Big] = -\frac{G_N}{r_{12}^5} Q_{ij}^{(1)} T_{ijkl} Q_{kl}^{(2)} \,, \tag{8}$$

where

$$T_{ijkl} := r^5 \partial_i\partial_j\big[\frac{x_k x_l}{r^5}\big]\Big|_{\vec{x}=r_{12}\hat{r}} = \big[\delta_{ik}\delta_{jl} + (i \to j)\big] - 5\big[\delta_{jk}\hat{r}_i\hat{r}_l + \delta_{il}\hat{r}_j\hat{r}_k + (i\to j)\big] - 5\delta_{ij}x_k x_l + 35 x_i x_j x_k x_l \,, \tag{9}$$

with $\hat{r} := \frac{\vec{x}_1 - \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|}$ and $r_{12} := |\vec{x}_1 - \vec{x}_2|$. This is the tidal interaction between two quadrupole moments, and can also be reproduced from the leading-order post-Newtonian approximation of Einstein gravity [15–18], with a possibly different T_{ijkl} due to the choice of gauge fixing. Moreover, the subleading PN terms will involve the motion of the compact bodies [19, 20], which we will not consider in this note.

We now aim to generalize the above source theory of Newtonian gravity for classical bodies to quantum bodies in the superposed states of the quadrupole sign. Due to the lack of a comprehensive, consistent approach to quantum gravity, there are three possible scenarios depending on how to treat the gravity field sourced by quantum bodies.

A. Mini-superspace approach

The first is to take the mini-superspace approach to quantum gravity, so that the induced Newtonian potential inherits the quantum nature of the sign associated with the quadrupole, i.e.,

$$\hat{\Phi}^{{\rm mini},(a)}_{\rm quad} = \frac{G_N Q_{ij}^{(a)} x_i x_j}{r^5}\hat{Z}^{(a)}, \qquad a = 1, 2 \,, \tag{10}$$

where $\hat{Z}^{(a)}$ is as defined in (2) and (3). This may not be a consistent treatment of quantum gravity. Still, it is analogous to the mini-superspace approach to quantum cosmology [21–23], which involves quantizing only the scale factor, not the full metric.

Generalizing (8) to this mini-superspace case, the Hamiltonian for the interaction between two such quantum bodies can be found to be

$$\hat{H}^{\rm mini}_{QQ} = -\hat{Q}_{ij}^{(1)}\partial_i\partial_j\hat{\Phi}^{{\rm mini},(2)}_{\rm quad}|_{\vec{x}=\vec{x}_2} = H^c_{QQ}\hat{Z}^{(1)}\otimes\hat{Z}^{(2)} \tag{11}$$

with H^c_{QQ} is given by (8).

B. Semi-classical gravity approach

Another popular approach is the so-called semiclassical gravity [24–26], in which matter, but not gravity, is quantized. Thus, the Newton potential is then sourced not by $\hat{Q}_{ij}$, but by its expectation value with respect to the N00N-like state. That is,

$$\Phi^{\mathrm{semi},(a)}_{\mathrm{quad}} = \frac{G_N x_i x_j}{r^5}\langle\theta^{(a)},\phi^{(a)};Q^{(a)}_{ij}|\hat{Q}^{(a)}_{ij}|\theta^{(a)},\phi^{(a)};Q^{(a)}_{ij}\rangle = \frac{G_N Q^{(a)}_{ij} x_i x_j}{r^5}\cos 2\theta^{(a)}\ , \qquad a=1,2\ . \tag{12}$$

Even though the Newton potential sourced by a quantum body is a classical quantity, this hybrid system exhibits bizarre behavior when a measurement collapses the quantum body, e.g., from $\theta=\frac{\pi}{4}$ to $\theta=0$. This will then result in a sudden jump of the Newton potential due to the wavefunction collapse [27]. In general relativity, this implies that the spacetime metric can be singular.

Due to the classical nature of $\Phi^{\mathrm{semi}}_{\mathrm{quad}}$, the generailization of (8) in the semiclassical gravity becomes

$$\hat{H}^{\mathrm{semi}}_{QQ} = \frac{1}{2}H^c_{QQ}\big[(I^{(1)}\otimes\hat{Z}^{(2)})\cos 2\theta^{(1)} + (\hat{Z}^{(1)}\otimes I^{(2)})\cos 2\theta^{(2)}\big]\ , \tag{13}$$

where $I^{(a)}$ is the identity operator for the a-th quantum body. Note that the quantum nature of $\hat{H}^{\mathrm{semi}}_{QQ}$ inherits from the acted quantum body, not from the source Newton gravity field. We also note that the N00N state (1) yields vanishing $\Phi^{\mathrm{semi},(a)}_{\mathrm{quad}}$ due to $\theta^{(q)}=\frac{\pi}{4}$.

C. Stochastic gravity approach

The third approach is stochastic gravity, an extension of semiclassical gravity that incorporates the stochastic noise $\xi_{\mu\nu}$ resulting from the quantum fluctuations of matter in the so-called Einstein-Langevin equation [28–30],

$$G_{\mu\nu} = 8\pi G_N\big(\langle T_{\mu\nu}\rangle + \xi_{\mu\nu}\big)\ . \tag{14}$$

The gravity field plays the role of a Brownian particle in the Langevin dynamics, kicked by the Gaussian random noise $\xi_{\mu\nu}$ due to the leading-order influence functional [28]. To the Newtonian order, this implies that the Newton potential of (12) should be appended by a random noise arising from the sign fluctuation of the quantum body's quadrupole. This noise can then contribute to the quadrupole-quadrupole interaction. This is equivalent to replacing $\theta^{(a)}$ in (12) and (13) by appending a white noise fluctuation

$$\theta^{(a)} \to \tilde{\theta}^{(a)} = \theta^{(a)} + \delta\theta \tag{15}$$

with $\delta\theta$ denote a Gaussian white noise $\mathcal{N}(0,1)$. Therefore, the quadrupole-quadrupole $\hat{H}^{\mathrm{stoc}}_{QQ}$ for the stochastic gravity still takes the form of (13) but replacing $\theta^{(a)}$ by $\tilde{\theta}^{(a)}$.

IV. ENTANGLING QUANTUM BODIES BY NEWTONIAN GRAVITY ?

With the descriptions of three different source theories for quantum bodies and the corresponding Newtonian quadrupole-quadrupole interaction Hamiltonians, we are now ready to examine whether these Newtonian interactions can entangle two spatially separated quantum bodies. This is basically to examine the evolved state

$$|\mathrm{final}\rangle = U_T|\mathrm{init}\rangle \tag{16}$$

with the unitary evolution operator

$$U_T := \mathcal{T}e^{-i\int_{-\infty}^{\infty} w_T(t)\hat{H}_{QQ}dt}\ , \tag{17}$$

where $\hat{H}_{QQ}$ can be either H^{mini}_{QQ}, H^{semi}_{QQ} or H^{stoc}_{QQ}, and w_T is some window function, which is chosen to be $w_T(t) = e^{-(t/T)^2}/(\pi/2)^{1/4}$ such that $\int_{-\infty}^{\infty}|w_T(t)|^2dt = T$. The initial state of the two quantum bodies is just

$$|\mathrm{init}\rangle = |\theta^{(1)},\phi^{(1)};Q^{(1)}_{ij}\rangle\otimes|\theta^{(2)},\phi^{(2)};Q^{(2)}_{ij}\rangle \tag{18}$$

with $\theta^{(a)}\neq n\pi/2$ for $n\in\mathbb{Z}$. Otherwise, the quantum bodies will be in their eigenstates, acting as classical bodies.

A. Entanglement production in mini-superspace approach

For the mini-superspace approach with $\hat{H}^{\rm mini}_{QQ}$ of (13), then

$$U_T = e^{-i\lambda \hat{Z}^{(1)}\otimes \hat{Z}^{(2)}} \tag{19}$$

with

$$\lambda := -H^c_{QQ}\int_{-\infty}^{\infty} dt\; w_T(t) = -(2\pi)^{1/4} H^c_{QQ} T \sim \mathcal{O}(1)\frac{G\delta M_1 \delta M_2}{r_{12}}\left(\frac{b}{r_{12}}\right)^4 T\ . \tag{20}$$

For simplicity, we consider the following initial state with $\theta^{(1)}=\theta^{(2)}=\frac{\pi}{4}$ and $\phi^{(1)}=\phi^{(2)}=0$, i.e.,

$$|{\rm init}\rangle = |++\rangle := |+;1\rangle \otimes |+;2\rangle\ , \qquad \text{with} \quad |+;a\rangle := |\theta^{(a)}=\pi/4, \phi^{(a)}=0; Q^{(a)}_{ij}\rangle\ . \tag{21}$$

Further define and note

$$|--\rangle := |-;1\rangle \otimes |-;2\rangle \qquad \text{with} \qquad |-;a\rangle := \hat{Z}^{(a)}|+;a\rangle \quad \text{and} \quad \langle \pm; a|\mp; a\rangle = 0\ . \tag{22}$$

Similarly, one can define $|+-\rangle$ and $|-+\rangle$. Then,

$$|{\rm final}\rangle = U_T|++\rangle = \cos\lambda\,|++\rangle - i\sin\lambda\,|--\rangle\ . \tag{23}$$

The Schmidt rank generally increases by one after evolution unless $\lambda = n\pi/2$ with $n\in\mathbb{Z}$, so that the mini-superspace quantum gravity entangles the quantum bodies. This is consistent with the GIE proposal, i.e., the production of quantum entanglement can be used as a probe to quantum gravity.

B. No entanglement production in semiclassical and stochastic gravity approaches

Now, for the semiclassical gravity and stochastic gravity, the $\hat{H}_{QQ}$ takes the form of (13). In such cases, U_T can be factorized into the product of two local unitaries, i.e.,

$$U_T = e^{-\lambda_2 \hat{Z}^{(1)}} \otimes e^{-\lambda_1 \hat{Z}^{(2)}}\ , \tag{24}$$

where

$$\lambda_a = -\frac{1}{2}(2\pi)^{1/4} H^c_{QQ} T\cos 2\Theta^{(a)}\ , \qquad a=1,2 \tag{25}$$

with $\Theta^{(a)}=\theta^{(a)}$ for semiclassical gravity, and $\tilde{\theta}^{(a)}$ for stochastic gravity. The product of local unitaries cannot produce entanglement as can be seen as follows,

$$|{\rm final}\rangle = e^{-\lambda_2 \hat{Z}^{(1)}}|\theta^{(1)},\phi^{(1)};Q^{(1)}_{ij}\rangle \otimes e^{-\lambda_1 \hat{Z}^{(2)}}|\theta^{(2)},\phi^{(2)};Q^{(2)}_{ij}\rangle\ , \tag{26}$$

which remains as a product state. This is consistent with the GIE proposal again, i.e., the classical gravity cannot produce quantum entanglement.

C. Confusing GIE-violating result due to perturbative calculations

The recent paper [1] claims that classical gravity can produce quantum entanglement, which is based on perturbative calculations in the context of the stochastic gravity approach. We now demonstrate a subtlety for reaching this GIE-violating result. For simplicity, we consider the initial state of (21) so that λ_a's of (25) vanish as $\cos 2\theta^{(a)}\big|_{\theta^{(a)}=\pi/4} = 0$ for semiclassical gravity approach, but not for stochastic gravity approach due to nontrivial $\delta\theta$. So, there is no evolution for the semiclassical gravity in this case. On the other hand, for the stochastic gravity, we have

$$|{\rm final}\rangle = e^{-\lambda_2 \hat{Z}^{(1)}}|+;1\rangle \otimes e^{-\lambda_1 \hat{Z}^{(2)}}|+;2\rangle\ , \tag{27}$$

which is unentangled.

Now, we can explain the possible origin of claiming the GIE-violating result as follows. In the perturbative approach as performed in [1], the resultant final state is obtained up to some order of G_N. This corresponds to expanding the right side of (27) in order of λ_a as $\lambda_a \sim \mathcal{O}(G_N)$. Suppose we are interested in the final state up to $\mathcal{O}(G_N)$, then we have

$$|\text{final}\rangle = \big(1 - i\lambda_2 \hat{Z}^{(1)} + \cdots\big)|+;1\rangle \otimes \big(1 - i\lambda_2 \hat{Z}^{(2)} + \cdots\big)|+;2\rangle \simeq |++\rangle - i\lambda_1|+-\rangle - i\lambda_2|-+\rangle + \mathcal{O}(G_N^2) \,. \tag{28}$$

We see that the above final state up to $\mathcal{O}(G_N)$ is now entangled due to the neglect of the $\lambda_1\lambda_2 \sim \mathcal{O}(G_N^2)$ term. The entanglement entropy of the final state is

$$S \sim \lambda_1^2 \lambda_2^2 \sim \mathcal{O}(G_N^4) \,. \tag{29}$$

This is tiny but still non-zero even after averaging over $\delta\theta$ as $\overline{\lambda_a^2} \sim \overline{\delta\theta\delta\theta} \sim \sigma_{\delta\theta}^2$ with overline denoting the average over Gaussian noise and $\sigma_{\delta\theta}$ its variance. However, the entanglement induced in this way is just an artifact due to the truncation of higher-order terms in the perturbative calculation.

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[1] J. Aziz and R. Howl, Nature **646**, 813 (2025), arXiv:2510.19714 [quant-ph] .
[2] C. Marletto and V. Vedral, (2025), arXiv:2510.19969 [quant-ph] .
[3] C. Marletto, J. Oppenheim, V. Vedral, and E. Wilson, (2025), arXiv:2511.07348 [quant-ph] .
[4] C. Marletto and V. Vedral, Phys. Rev. Lett. **119**, 240402 (2017), arXiv:1707.06036 [quant-ph] .
[5] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. Geraci, P. Barker, M. S. Kim, and G. Milburn, Phys. Rev. Lett. **119**, 240401 (2017), arXiv:1707.06050 [quant-ph] .
[6] R. J. Marshman, A. Mazumdar, and S. Bose, Phys. Rev. A **101**, 052110 (2020), arXiv:1907.01568 [quant-ph] .
[7] S. Bose, A. Mazumdar, M. Schut, and M. Toroš, Phys. Rev. D **105**, 106028 (2022), arXiv:2201.03583 [gr-qc] .
[8] U. K. Beckering Vinckers, Á. de la Cruz-Dombriz, and A. Mazumdar, Phys. Rev. D **107**, 124036 (2023), arXiv:2303.17640 [gr-qc] .
[9] C. M. Dewitt and D. Rickles (2011).
[10] J. P. Dowling, Contemp. Phys. **49**, 125 (2008), arXiv:0904.0163 [quant-ph] .
[11] D.-H. Kim, S. Hong, Y.-S. Kim, K. Oh, S.-Y. Lee, C. Lee, and H.-T. Lim, Phys. Rev. Lett. **135**, 050802 (2025), arXiv:2508.02070 [quant-ph] .
[12] D. L. Danielson, G. Satishchandran, and R. M. Wald, Phys. Rev. D **105**, 086001 (2022), arXiv:2112.10798 [quant-ph] .
[13] D. L. Danielson, G. Satishchandran, and R. M. Wald, Int. J. Mod. Phys. D **31**, 2241003 (2022), arXiv:2205.06279 [hep-th] .
[14] D. L. Danielson, G. Satishchandran, and R. M. Wald, Phys. Rev. D **108**, 025007 (2023), arXiv:2301.00026 [hep-th] .
[15] A. Biggs and J. Maldacena, (2024), arXiv:2405.02227 [hep-th] .
[16] T. Binnington and E. Poisson, Phys. Rev. D **80**, 084018 (2009), arXiv:0906.1366 [gr-qc] .
[17] B. Kol and M. Smolkin, JHEP **02**, 010 (2012), arXiv:1110.3764 [hep-th] .
[18] F.-L. Lin and S. Mondal, (2024), arXiv:2411.09409 [hep-th] .
[19] W. D. Goldberger, in *Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime* (2007) arXiv:hep-ph/0701129 .
[20] R. A. Porto, Phys. Rept. **633**, 1 (2016), arXiv:1601.04914 [hep-th] .
[21] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).
[22] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).
[23] A. Vilenkin, Phys. Rev. D **30**, 509 (1984).
[24] B. S. DeWitt, Physics Reports **19**, 295 (1975).
[25] L. Parker, Physical Review Letters **21**, 562 (1968).
[26] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975), [Erratum: Commun.Math.Phys. 46, 206 (1976)].
[27] R. Penrose, Gen. Rel. Grav. **28**, 581 (1996).
[28] B. L. Hu and S. Sinha, Phys. Rev. D **51**, 1587 (1995), arXiv:gr-qc/9403054 .
[29] B. L. Hu and E. Verdaguer, Living Rev. Rel. **11**, 3 (2008), arXiv:0802.0658 [gr-qc] .
[30] R. Martin and E. Verdaguer, Phys. Lett. B **465**, 113 (1999), arXiv:gr-qc/9811070 .