Can the diffeomorphism and Gauss constraints be holonomy corrected in the deformed algebra approach to modified gravity?

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Deforming the algebra of constraint is a well known approach to effective loop quantum cosmology. More generally, it is a consistent way to modify gravity from the Hamiltonian perspective. In this framework, the Hamiltonian (scalar) constraint is usually the only one to be holonomy corrected. As a heuristic hypothesis, we consider the possibility to also correct the diffeomorphism and Gauss constraints. It is shown that it is impossible to correct the diffeomorphism constraint without correcting the Gauss one, while maintaining a first-class algebra. However, if all constraints are corrected, the algebra can be closed. The resulting differential equations to be fulfilled by the corrections (of the background and of the perturbations) are derived.

I. INTRODUCTION

This article focuses on the deformation of the algebra of general relativity. This approach has been recently intensively considered as a way to effectively describe loop quantum cosmology [1–17]. In fact, it is much more general and can be viewed as a rigorous way to study modified gravity in the Hamiltonian setting. We intentionally keep this work as brief and concise as possible. As such, we do not go through the motivations and the framework behind. We refer any interested reader to [18] and references therein both for the full picture and for the details about the notations.

In the loop view, quantum geometry corrections are effectively taken into account by replacing the Sen-Ashtekar-Barbero connection by a generalized function which encodes the holonomy along a graph, the so-called holonomy correction. Following recent works – in particular remarks made in [19–22] – we implement this correction into the classical algebra of general relativity, without specifying its explicit form *a priori*. Then, we ensure the consistency of the model by requiring the closure of the algebra of constraints – that is in making sure that each Poisson bracket between constraints is proportional to a constraint, or a combination of constraints: the evolution of a physical solution yields another physical solution. This approach actually extends far beyond its initial purpose and offers a powerful framework for studying modified gravity.

To ensure the closure of the algebra of constraints, we introduce counterterms to cancel the anomalies induced in the algebra by the holonomy correction. These counterterms can also be regarded as additional terms introduced to construct the most general theory — under some natural hypotheses — that would incorporate the relevant corrections and would lead to general relativity in the classical limit. In this view, our results are not limited to the loop framework.

At the end of the day, one should obtain a first class Poisson algebra and the subtle cancellation game between anomalies and counterterms could lead to restrictions on the allowed forms of the correction functions. This might have observable consequences as the Mukhanov-Sasaki equation for gauge-invariant perturbations is expected to be modified accordingly. Not to mention a possible disappearance of time [23, 24] around the quantum bounce [25].

Usually, the Hamiltonian constraint is the only one to receive corrections. Although there are reasons for this [26], we investigate here, at the heuristic level, the possibility that the diffeomorphism and Gauss constraints may themselves receive corrections. We begin by focusing on the correction of the diffeomorphism constraint alone. We then proceed by considering the full set of corrections affecting all the constraints that define the theory. The differential equations to be fulfilled by the corrections are investigated in details. We finally conclude with some remarks on possible future studies.

II. CORRECTING THE HAMILTONIAN AND DIFFEOMORPHISM CONSTRAINTS ONLY

A. Expression of the constraints

As in [18], the Hamiltonian constraint $\mathbb{H}$ is corrected by two distinct functions $g(\mathfrak{c},\mathfrak{p})$ and $\tilde{g}(\mathfrak{c},\mathfrak{p})$, which respectively replace the curvature $\mathfrak{c}$ at the background and perturbative levels. The variables $\mathfrak{p}$ and $\mathfrak{c}$, defined right after Eq. (2), respectively describe the scale factor and the expansion rate.

In this work, in addition to the Hamiltonian constraint, the diffeomorphism constraint is also corrected by a function $h(\mathfrak{c},\mathfrak{p})$. Following the usual procedure, three new counter-terms $\xi_i(\mathfrak{c},\mathfrak{p})$, $i = 1, 2, 3$, are then introduced. The geometrical constraint reads :

$$\mathbb{D}_{\mathfrak{g}}\left[N^a\right] = \kappa^{-1}\int d\mathbf{x}\delta N^a \mathcal{D}_a^{\mathfrak{g}(1)}, \qquad (1)$$

where

$$\mathcal{D}_a^{\mathfrak{g}(1)} = \mathfrak{p}(1+\xi_1)\partial_a \delta K_b^b - \mathfrak{p}(1+\xi_2)\partial_i \delta K_a^i - (h+\xi_3)\delta_a^j \partial_b \delta E_j^b, \tag{2}$$

with $N^a = \mathbf{N}^a + \delta N^a$ the shift function, $E_i^a = \mathfrak{p}\delta_i^a + \delta E_i^a$ the densitized triads and $K_a^i = \mathfrak{c}\delta_a^i + \delta K_a^i$ their canonically conjugate variables. Each of those quantities are perturbed around a homogeneous and isotropic background.

As usual, the matter sector of the diffeomorphism constraint is uncorrected – as is the Gauss constraint for now.

B. Cancellation of the anomalies

Correcting the diffeomorphism constraint leads to a modification of the algebra of constraint and, in particular, to additional anomalies when compared to [18]. Those new anomalies are listed in Appendix A. In order to keep this article as short as possible, we do not make explicit the heavy calculations and we simply focus on the solutions. The approach consists in expressing all the counterterms as functions of the corrections, and then constraining the corrections if any anomalies remain.

The conditions $\mathcal{A}_2^{\{\mathbb{H},\mathbb{D}\}} = 0$ and $\mathcal{A}_7^{\{\mathbb{H},\mathbb{D}\}} = 0$ (A.7 and A.12) imply $\xi_1 = \xi_2$ as α_9 and α_3 cannot be constant so as to ensure a correct classical limit. The same limit considered for α_3 and α_1, together with $\mathcal{A}_5^{\{\mathbb{H},\mathbb{H}\}} = 0$ (A.27), implies

$$\xi_1 = 0 \tag{3}$$

thus

$$\xi_2 = 0. \tag{4}$$

These counterterms are therefore “not physical” or, if one sees them as mere corrections, not necessary. Then, $\mathcal{A}_1^{\{\mathbb{D},\mathbb{G}\}} = 0$ (A.1), leads, with Eq. (4), to:

$$\xi_3 = \mathfrak{c} - h. \tag{5}$$

Since h and ξ_3 appear only as $h+\xi_3 = \mathfrak{c}$ in the anomalies, the system reduces to the one obtained without correcting the diffeomorphism constraint (see [18] for the end of the resolution). There is no other condition on h and ξ_3 to ensure that $\mathcal{A}_1^{\{\mathbb{D},\mathbb{G}\}}$ vanishes. As long as the counterterm ξ_3 compensates h exactly, the algebra is first class and the theory remains consistent. This kind of trivial compensation – which does not happen in the usual case where only the Hamiltonian constraint is corrected – is equivalent to not correcting the diffeomorphism constraint. Interestingly, had ξ_3 not been introduced initially, Eq. (5) would impose $h = \mathfrak{c}$, but with ξ_3 included, it becomes possible for h to differ from $\mathfrak{c}$.

It is important to underline that there are no additional degrees of freedom when compared to [18]. All the counterterms except α_9 are fixed by the corrections.

With a holonomy-corrected Hamiltonian constraint, it is therefore not possible to correct the diffeomorphism constraint *alone*[1] while maintaining a first-class algebra.

III. CORRECTING ALL CONSTRAINTS

A. Expression of the constraints

We now investigate what happens if all the constraints are holonomy corrected. In particular, Eq. (5) links the diffeomorphism correction to the Gauss constraint, which makes the situation quite different if the latter is also corrected. This anomaly changes and the algebra of constraints can be closed differently.

Is is now assumed that the Gauss constraint is corrected with a new function $l(\mathfrak{c},\mathfrak{p})$. Associated counterterms $\chi_1(\mathfrak{c},\mathfrak{p})$ and $\chi_2(\mathfrak{c},\mathfrak{p})$ are also introduced, so that the constraint reads,

$$\mathbb{G}[\Lambda^i] = \kappa^{-1} \int d\mathbf{x}\, \delta\Lambda^i \mathcal{G}_i, \tag{6}$$

with

$$\mathcal{G}_i = (\mathfrak{p}+\chi_1)\varepsilon_{ij}^a \delta K_a^j + (l+\chi_2)\varepsilon_{ia}^j \delta E_j^a, \tag{7}$$

$\Lambda^i = \mathbf{\Lambda}^i + \delta\Lambda^i$ being the perturbed Lagrange multipliers.

B. Cancellation of the anomalies

The new list of anomalies is given in Appendix B. This section focuses on the resolution of the associated system.

First, $\mathcal{A}_5^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.28) implies $\xi_1 = 0$. Combined with $\mathcal{A}_2^{\{\mathbb{H},\mathbb{D}\}} = 0$ and $\mathcal{A}_7^{\{\mathbb{H},\mathbb{D}\}} = 0$ (B.8 and B.13), the classical limits of α_9 and α_3 imply

$$\xi_1 = \xi_2 = 0. \tag{8}$$

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[1] By ”alone” we mean without correcting the Gauss constraint.

This causes the anomalies $\mathcal{A}_8^{\{\mathbb{H},\mathbb{D}\}}$ and $\mathcal{A}_9^{\{\mathbb{H},\mathbb{D}\}}$ (B.14 and B.15) to vanish immediately.

Considering the classical limits, the cancellation of $\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{D}\}}$ and $\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{D}\}}$ (B.22 and B.23) gives

$$\beta_8 = \beta_4 = 0, \tag{9}$$

leading to $\mathcal{A}_9^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.32). The cancellations of $\mathcal{A}_{10}^{\{\mathbb{H},\mathbb{D}\}}$, $\mathcal{A}_{11}^{\{\mathbb{H},\mathbb{D}\}}$ and $\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{D}\}}$ (B.16, B.17 and B.21) imply, respectively,

$$\beta_2 = 2\beta_1, \tag{10}$$
$$\beta_5 = \beta_6, \tag{11}$$
$$\beta_{11} = \beta_{12}. \tag{12}$$

Adding $\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{D}\}} = 0$ and $\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{D}\}} = 0$ (B.18 and B.19) gives $\beta_{10} = \beta_{12} = \beta_{11}$. Re-injecting this result in $\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{D}\}} = 0$ and subtracting the latter to $\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{D}\}} = 0$ leads to

$$\beta_{10} = \beta_{11} = \beta_{12} = \partial_{\mathfrak{c}}(h + \xi_3) - 1. \tag{13}$$

Then, in order to be consistent with the classical limit of α_3, $\mathcal{A}_4^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.27) implies

$$\alpha_4 = \alpha_5. \tag{14}$$

Cancelling $\mathcal{A}_6^{\{\mathbb{H},\mathbb{D}\}}$ (B.12) then gives

$$\alpha_7 = \alpha_8. \tag{15}$$

Since $\beta_4 = 0$ (9), $\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.38) implies

$$\partial_{\mathfrak{c}}(\tilde{g} + \alpha_1) = \frac{1}{2}(2 - \alpha_4 + 3\alpha_5). \tag{16}$$

Re-injecting this result in $\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.37), and making use of $\alpha_4 = \alpha_5$, leads to $2\beta_2(1 + \alpha_4) = 0$. Thus, recovering the correct classical limit for α_4 imposes $\beta_2 = 0$. Since $2\beta_1 = \beta_2$, this gives

$$\beta_1 = \beta_2 = 0. \tag{17}$$

Considering $\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.36) leads to

$$\beta_5 = \beta_6 = -\frac{\beta_3}{1 + \beta_3}, \tag{18}$$

since $\beta_5 = \beta_6$, from Eq. (11). The cancellation of $\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{H}\}}$ (B.40) and $\beta_1 = 0$ imply

$$\beta_3 = \frac{\tilde{g} + \alpha_1}{g\partial_{\mathfrak{c}}g} - 1. \tag{19}$$

Inserting this expression with $\mathcal{A}_7^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.30), which implies $\partial_{\mathfrak{c}}\beta_3 = 0$, in $\mathcal{A}_{20}^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.43) leads to

$$\partial_{\mathfrak{p}}\beta_3 = 0. \tag{20}$$

This also relies on the fact that g and $\partial_{\mathfrak{c}}g$ cannot vanish, in order to get the correct classical limit ($g \to \mathfrak{c}$). The term β_3 is then a constant since it depends neither on $\mathfrak{p}$ nor on $\mathfrak{c}$ ($\mathcal{A}_7^{\{\mathbb{H},\mathbb{H}\}} = 0$). As a consequence, the only way to get the correct classical limit is to have

$$\beta_3 = 0. \tag{21}$$

Considering Eq. (18), this gives

$$\beta_5 = \beta_6 = 0. \tag{22}$$

This cancellation applied to $\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{D}\}}$ (B.20) implies $\beta_{10} = -\beta_9$, thus

$$\beta_{10} = -\beta_9 = \beta_{12} = \beta_{11} = \partial_{\mathfrak{c}}(h + \xi_3) - 1. \tag{23}$$

Equation (19) must hold with $\beta_3 = 0$ and implies

$$\alpha_1 = g\partial_{\mathfrak{c}}g - \tilde{g}. \tag{24}$$

The cancellation of $\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{H}\}}$ (B.35) leads to

$$\beta_7 = 0. \tag{25}$$

The only non-vanishing matter counterterms are thus β_{13}, which is still undetermined, and β_{10}, β_9, β_{12} and β_{11} which are related, by Eq. (23), to the diffeomorphism correction. Those latter counterterms would vanish if the diffeomorphism constraint was not corrected.

At this level, $\mathcal{A}_1^{\{\mathbb{H},\mathbb{D}\}} = 0$ (B.7) implies

$$\alpha_2 = 3g^2 - \tilde{g}^2 - 2g(h + \xi_3)\partial_{\mathfrak{c}}g. \tag{26}$$

Taking into account that $\alpha_4 = \alpha_5$ in Eqs. (16) and (24) gives

$$\alpha_4 = \alpha_5 = (\partial_{\mathfrak{c}}g)^2 + g\partial_{\mathfrak{c}}^2 g - 1. \tag{27}$$

Cancelling $\mathcal{A}_3^{\{\mathbb{H},\mathbb{D}\}}$ (B.9) then implies:

$$\alpha_6 = 2g\partial_{\mathfrak{c}}g - \tilde{g} - (h + \xi_3)\left[(\partial_{\mathfrak{c}}g)^2 + g\partial_{\mathfrak{c}}^2 g\right]. \tag{28}$$

It can be noticed that because of the two previous relations, $\mathcal{A}_4^{\{\mathbb{H},\mathbb{D}\}}$ (B.10) vanishes without any new condition. Then, injecting the previous expression in $\mathcal{A}_5^{\{\mathbb{H},\mathbb{D}\}} = 0$ (B.11) leads to an expression for α_7, and thus also for α_8 :

$$\begin{aligned}\alpha_7 = \alpha_8 = &\, 4g\partial_{\mathfrak{c}} g \left[(h+\xi_3) + \mathfrak{p}\partial_{\mathfrak{p}}(h+\xi_3)\right] - \tilde{g}^2 \\ &- 2(h+\xi_3)^2 \left[(\partial_{\mathfrak{c}} g)^2 + g\partial_{\mathfrak{c}}^2 g\right] \\ &- \left[g^2 + 4\mathfrak{p} g \partial_{\mathfrak{p}} g\right] \partial_{\mathfrak{c}}(h+\xi_3). \end{aligned} \tag{29}$$

We next combine the expressions of α_1, α_5 and α_6 in $\mathcal{A}_3^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.26), then account that cancelling the anomaly $\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.39) implies that $\partial_{\mathfrak{c}}\alpha_3 = 0$. This yields to a relation between α_3 and α_9,

$$2g(\partial_{\mathfrak{c}} g)\left[\alpha_3 + 2\mathfrak{p}\partial_{\mathfrak{p}}\alpha_3 - \alpha_9\right] = 0. \tag{30}$$

Since $g \neq 0$ and $\partial_{\mathfrak{c}} g \neq 0$, this implies

$$\alpha_3 + 2\mathfrak{p}\partial_{\mathfrak{p}}\alpha_3 = \alpha_9. \tag{31}$$

It is worth noticing that, since α_3 does not depend on $\mathfrak{c}$, α_9 does not either. Solving this equation leads to :

$$\alpha_3(\mathfrak{p}) = \frac{1}{\sqrt{\mathfrak{p}}}\left[K + \int_1^p \frac{\alpha_9(u)}{2\sqrt{u}} du\right], \tag{32}$$

where K is an integration constant.

At this stage, β_{13} is still undetermined. To constrain this counterterm, we require the $\{\mathbb{H},\mathbb{H}\}$ bracket to be proportional to the full diffeomorphism constraint (see [18]), which implies that the structure coefficients in front of $\mathbb{D}_{\mathfrak{g}}$ and $\mathbb{D}_{\mathfrak{m}}$ must be the same,

$$(1+\alpha_3)(1+\alpha_5) = (1+\beta_1)(1+\beta_{13}). \tag{33}$$

Using the previous results, this leads to

$$\beta_{13} = \left[1+\alpha_3(\mathfrak{p})\right]\partial_{\mathfrak{c}}(g\partial_{\mathfrak{c}} g) - 1. \tag{34}$$

As $\alpha_9(\mathfrak{p})$ does not appear in any other anomaly and is not constrained in any way, it remains a free function of the theory, which determines, together with the explicit expression of the corrections, α_3 and thus β_{13}. All the other counterterms of the Hamiltonian constraint are fully determined by the correction functions and the counterterms from the other constraints. In order to constrain ξ_3, χ_1, χ_2, and the corrections, it is important to check the remaining anomalies.

$\mathcal{A}_1^{\{\mathbb{D},\mathbb{G}\}} = 0$ (B.1) implies

$$l + \chi_2 = \frac{1}{\mathfrak{p}}(h+\xi_3)(\mathfrak{p}+\chi_1). \tag{35}$$

As β_{10} and β_{12} are related to $(h+\xi_3)$, if $(l+\chi_2)$ fulfils Eq. 35, then $\mathcal{A}_4^{\{\mathbb{H},\mathbb{G}\}} = 0$ and $\mathcal{A}_5^{\{\mathbb{H},\mathbb{G}\}} = 0$ (B.5 and B.6) are automatically verified since $\mathcal{A}_3^{\{\mathbb{H},\mathbb{G}\}} = 0$ (B.4) implies $\partial_{\mathfrak{c}}\chi_1 = 0$.

Using this later condition, Eq. (35) and the expressions of α_4 and α_6 in $\mathcal{A}_1^{\{\mathbb{H},\mathbb{G}\}} = 0$ (B.2) lead to

$$4g(\partial_{\mathfrak{c}} g)\left[\chi_1 - \mathfrak{p}\partial_{\mathfrak{p}}\chi_1\right] = 0. \tag{36}$$

Since, once again, $g \neq 0$ and $\partial_{\mathfrak{c}} g \neq 0$, this implies

$$\chi_1 - \mathfrak{p}\partial_{\mathfrak{p}}\chi_1 = 0. \tag{37}$$

Recalling that χ_1 is a function of $\mathfrak{p}$ only, this leads to

$$\chi_1(\mathfrak{p}) = (L-1)\mathfrak{p}, \tag{38}$$

with L another integration constant. This allows to re-write Eq. (35) as

$$\chi_2 = L(h+\xi_3) - l. \tag{39}$$

This is an important result of this section. The relevant *effective* corrections that appear at the level of the constraints are the terms $(l+\chi_2)$ and $(h+\xi_3)$, and Eq. (39) shows that they must be proportional one to the other. More details on this proportionality are given in section III C 3.

Using this in $\mathcal{A}_2^{\{\mathbb{H},\mathbb{G}\}} = 0$ (B.3) directly cancels the anomaly without any new condition on the counter-terms or on the correction function. For $\mathcal{A}_{18}^{\{\mathbb{H},\mathbb{H}\}}$ (B.41), this implies

$$\left[\tilde{g} - g\partial_{\mathfrak{c}} g\right]\left[1 - \partial_{\mathfrak{c}}(h+\xi_3)\right] = 0. \tag{40}$$

To discuss this new condition, two cases must be distinguished:

- First, we consider $\tilde{g} = g\partial_{\mathfrak{c}} g$. If we take into account the fact that the physical situation corresponds to $\tilde{g} = g$, this is actually equivalent to having no correction. Equation (40) is then directly satisfied, and $\xi_3(\mathfrak{c},\mathfrak{p})$ is a new degree of freedom, as there is no additional anomaly to constrain it.

- The second case is $\tilde{g} \neq g\partial_{\mathfrak{c}} g$. Then, Eq. (40) leads to

$$\xi_3(\mathfrak{c},\mathfrak{p}) = \mathfrak{c} - h(\mathfrak{c},\mathfrak{p}) + f(\mathfrak{p}), \tag{41}$$

with $f(\mathfrak{p})$ an arbitrary function of $\mathfrak{p}$. This function introduces a new degree of freedom into the theory, but it must be chosen so that the correct classical limit of ξ_3 is preserved.

Finally, $\mathcal{A}_1^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.24) leads to a restriction on g,

$$g\partial_{\mathfrak{c}}^2 g + 2\mathfrak{p}\left[(\partial_{\mathfrak{c}} g)(\partial_{\mathfrak{c}}\partial_{\mathfrak{p}} g) - (\partial_{\mathfrak{p}} g)(\partial_{\mathfrak{c}}^2 g)\right] = 0, \tag{42}$$

which is the same condition as the one obtained with $\mathcal{A}_2^{\{\mathbb{H},\mathbb{H}\}} = 0$ (B.25). Those two anomalies are redundant. This condition is very different from the one obtained in [11, 18]. In particular, Eq. (42) is more general than the condition derived in previous works. Solutions presented in those articles were indeed sufficient to cancel the anomalies. However, if one was looking for the most general solution, Eq. (42) – which is mandatory in our case – would have been found.

It is also worth noting that the standard $\bar{\mu}$ scheme, where $g = \sin(\bar{\mu}\mathfrak{c})/\bar{\mu}$, with $\bar{\mu} = \sqrt{\Delta l_{Pl}^2/\mathfrak{p}}$, does satisfy Eq. (42).

This concludes the closure of the algebra for arbitrary correction functions g, $\tilde{g}$, h and l, as the other anomalies are automatically vanishing with the previous conditions. In summary, the theory requires only one condition on the correction functions given by Eq. (42). Specifically, l and h are unconstrained by the algebra as long as the counterterms are included. Furthermore, there are two new degrees of freedom with respect to the case where only the Hamiltonian constraint is corrected [18].

The first new degree of freedom is the constant L which appears in Eqs. (38) and (39). The latter gives the physical interpretation of L: it links the *effective* Gauss and diffeomorphism corrections at the level of constraints. In particular, it imposes that $(h+\xi_3)$ must be proportional to $(l+\chi_2)$ – in other words that the Gauss and diffeomorphism corrections must be the same, up to a proportionality factor.

The second new degree of freedom is linked to ξ_3. As described earlier, in the case where $\tilde{g} = g\partial_{\mathfrak{c}} g$, then $\xi_3(\mathfrak{c},\mathfrak{p})$ is a free function of the theory. If this is not the case, then ξ_3 is determined by Eq. (41), with $f(\mathfrak{p})$ the new degree of freedom, depending only on $\mathfrak{p}$.

One should also notice the $\alpha_9(\mathfrak{p})$ term which is still a free function of the theory together with the constant K, as in [18].

As a summary, the algebra for arbitrary correction functions g, $\tilde{g}$, h and l – if the counterterms fulfill the previous relations (see Appendix C) reads:

$$\{\mathbb{G},\mathbb{G}\} = 0, \tag{43}$$
$$\{\mathbb{D},\mathbb{D}\} = 0, \tag{44}$$
$$\{\mathbb{D},\mathbb{G}\} = 0, \tag{45}$$
$$\{\mathbb{H},\mathbb{G}\} = 0, \tag{46}$$
$$\{\mathbb{H}[N],\mathbb{D}[N^a]\} = \mathbb{H}[\delta N\partial_a\delta N^a], \tag{47}$$
$$\{\mathbb{H}[N_1],\mathbb{H}[N_2]\} = \frac{1}{2}\partial_{\mathfrak{c}}^2 g^2\left[1+\alpha_3\left[K,\alpha_9\right](\mathfrak{p})\right] \tag{48}$$
$$\times\, \mathbb{D}\left[\frac{\mathbf{N}}{\mathfrak{p}}\partial^a(\delta N_2 - \delta N_1)\right], \tag{49}$$

with $N = \mathbf{N} + \delta N$ the lapse function perturbed around a homogeneous background.

It is clear that the symplectic structure is mostly unchanged, even with these new degrees of freedom, with respect to [18].

C. Special cases

1. $\tilde{g} = g$

In the case of identical corrections for the background and for the perturbations in the Hamiltonian constraint, Eq. (40) becomes

$$g[1-\partial_{\mathfrak{c}} g][1-\partial_{\mathfrak{c}}(h+\xi_3)] = 0. \tag{50}$$

To fulfill this condition, either $g = \mathfrak{c} + m_1(\mathfrak{p})$, with m_1 an arbitrary function of $\mathfrak{p}$ vanishing at the classical limit, or ξ_3 has to fulfill Eq. (41). It should be underlined that the usual correction does not match this requirement, which makes the situation rather "unnatural". Of course, $g = \tilde{g}$ must satisfy Eq. (42). The other corrections (h and l) remain free.

The same conclusions can be obtained in the case $\tilde{g} = g = h = l$.

2. $\tilde{g} = g = \mathfrak{c}$

In this case, Eq. (40) is trivially satisfied, as is Eq. (42). There are no restrictions on h and l as long as the counterterms are included. This interestingly shows that correcting the Gauss and diffeomorphism constraint is possible without correcting the Hamiltonian one.

3. *Case* $\xi_3 = \chi_2 = 0$

This subsection focuses on the case where the two counterterms ξ_3 and χ_2 are both vanishing. This allows to underline the connection between the different corrections.

In particular, this modifies Eq. (39), leading to

$$l = L \times h. \tag{51}$$

The corrections appearing in the Gauss and diffeomorphism constraints must be proportional one to the other. In particular, in the case where the Gauss constraint is not corrected, Eq. (51) leads to $h = \mathfrak{c}/L$. In order to get the classical limit of h, L must be equal to one, leading to $h = \mathfrak{c}$. Thus, there cannot be a diffeomorphism correction without correcting the Gauss constraint (the reciprocal is also true). Furthermore, in this case, $L = 1$ implies that $\chi_1 = 0$ through Eq. 38.

Equation (40) is the only other equation of the previous section for which the physical implications are modified, such that

$$[\tilde{g} - g\partial_{\mathfrak{c}} g]\,[1 - \partial_{\mathfrak{c}} h] = 0. \tag{52}$$

If $\tilde{g} \neq g\partial_{\mathfrak{c}} g$, this implies $h = \mathfrak{c} + m_2(\mathfrak{p})$, where m_2 is an arbitrary function of $\mathfrak{p}$ which vanishes at the classical limit ($\mathfrak{p} \to +\infty$). This leads to Eq. (51) implying that $l = \mathfrak{c} + m_2(\mathfrak{p})$, as $L = 1$ for the same reason as earlier (leading to $\chi_1 = 0$).

However, if $\tilde{g} = g\partial_{\mathfrak{c}} g$, then the diffeomorphism and Gauss constraints are free, as long as they are proportional to each other, see Eq. (51). This case is particularly noteworthy since, if $\tilde{g} = g$, then Eq. (40) yields $g = \mathfrak{c} + m_3(\mathfrak{p})$, with m_3 a function of $\mathfrak{p}$ which vanishes at the classical limit.

Obviously, in all cases, Eq. (42) must be fulfilled.

IV. SUMMARY AND CONCLUSION

If only the diffeomorphism and Hamiltonian constraints are corrected, the cancellation of anomalies implies that:

- the diffeomorphism correction is absorbed by its associated counterterm, Eq. (5), and leads to no *effective* correction at the level of the constraint[2]. Without the counterterm, it is impossible to consistently correct the diffeomorphism constraint.
- the solution of the system is the same as the one obtained without diffeomorphism correction (see [18]).
- the condition

$$g - 2\mathfrak{p}\partial_{\mathfrak{p}} g - \mathfrak{c}\partial_{\mathfrak{c}} g = 0, \tag{53}$$

derived in previous works [11, 18], is sufficient to ensure the cancellation of $\mathcal{A}_1^{\{\mathbb{H},\mathbb{H}\}}$ and $\mathcal{A}_2^{\{\mathbb{H},\mathbb{H}\}}$.

If all the constraints are corrected, then the cancellation of the anomalies implies that :

- the Gauss and diffeomorphism corrections, added with their associated counterterms (see the previous footnote), must be proportional to each other – Eq. (39). In particular, without counterterms, the Gauss and diffeomorphism corrections are exactly proportional – Eq. (51). The associated proportionality constant is a new degree of freedom with respect to the previous case which can be fixed in some specific cases.
- $\xi_3(\mathfrak{c}, \mathfrak{p})$ is a new degree of freedom of the theory. If $\tilde{g} = g\partial_{\mathfrak{c}} g$, then ξ_3 is a fully undetermined function of $\mathfrak{c}$ and $\mathfrak{p}$. However, if $\tilde{g} \neq g\partial_{\mathfrak{c}} g$, then ξ_3 is partially fixed by the corrections up to a function of $\mathfrak{p}$ (see Eq. (41)).
- equation (53) is not enough to get a first-class algebra. It is the condition (42) that needs to be fufiled by g.

In this article, we have shown that it is impossible to correct only the diffeomorphism constraint (assuming, of course, that the Hamiltonian constraint is also corrected, as it should be). The general relativity expression must be maintained.

However, if the Gauss constraint is also modified, a first class Poisson algebra can be obtained. Interestingly, the differential equations for the correction functions are not the usual ones.

This study shows the richness of the deformed algebra approach. In the future, two related investigations should be considered. First, it would be important to understand better the meaning – at the fundamental level – of the correction incorporated here at the effective level. Second, it would be interesting to study phenomenological consequences by deriving the gauge-invariant equations for cosmological perturbations and the associated power spectrum.

[2] In previous studies, the philosophy was, in a way, to focus on the correction applied to the constraint and to consider the counterterms as a side effect ensuring consistency. Taking this viewpoint, we could claim here that the diffeomorphism constraint *can* be consistently corrected. However, in this specific case, the associated counterterm strictly removes the correction – this does not happen in usual case where only the Hamiltonian constraint is corrected. This is why we prefer to emphasize in this work that the correction *effectively* appearing in the diffeomorphism constraint actually vanishes. Considering that it is "present" but entirely cancelled by the counterterm would be very artificial.

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Appendix A: List of anomalies without correcting the Gauss constraint

There are no anomalies for the $\{\mathbb{G},\mathbb{G}\}$ and $\{\mathbb{D},\mathbb{D}\}$ brackets. For the $\{\mathbb{D},\mathbb{G}\}$ bracket one anomaly appears :

$$\mathcal{A}_1^{\{\mathbb{D},\mathbb{G}\}} = (1+\xi_2)\mathfrak{c} - h - \xi_3. \tag{A.1}$$

For the $\{\mathbb{H},\mathbb{G}\}$ bracket :

$$\mathcal{A}_1^{\{\mathbb{H},\mathbb{G}\}} = g^2 - 2\mathfrak{c}(\tilde{g}+\alpha_6) + (\tilde{g}^2+\alpha_7) + 4\mathfrak{p}g\partial_{\mathfrak{p}}g \tag{A.2}$$
$$\mathcal{A}_2^{\{\mathbb{H},\mathbb{G}\}} = \mathfrak{c}(1+\alpha_4) + (\tilde{g}+\alpha_6) - 2g\partial_{\mathfrak{c}}g \tag{A.3}$$
$$\mathcal{A}_3^{\{\mathbb{H},\mathbb{G}\}} = -\beta_{10} \tag{A.4}$$
$$\mathcal{A}_4^{\{\mathbb{H},\mathbb{G}\}} = \beta_{12}. \tag{A.5}$$

Then for the $\{\mathbb{H},\mathbb{D}\}$ bracket,

$$\mathcal{A}_1^{\{\mathbb{H},\mathbb{D}\}} = (\tilde{g}^2+\alpha_2)(\xi_2-2-3\xi_1) - 4(\tilde{g}+\alpha_1)(h+\xi_3) + 6g^2 \tag{A.6}$$
$$\mathcal{A}_2^{\{\mathbb{H},\mathbb{D}\}} = (1+\alpha_3)(\xi_1-\xi_2) \tag{A.7}$$
$$\mathcal{A}_3^{\{\mathbb{H},\mathbb{D}\}} = 2(1+\alpha_4)(h+\xi_3) + 2(\tilde{g}+\alpha_6)(1+\xi_2) - 4g(1+\xi_2+\mathfrak{p}\partial_{\mathfrak{p}}\xi_2)\partial_{\mathfrak{c}}g + (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\xi_2 \tag{A.8}$$
$$\mathcal{A}_4^{\{\mathbb{H},\mathbb{D}\}} = 4g(1+\xi_1+\mathfrak{p}\partial_{\mathfrak{p}}\xi_1)\partial_{\mathfrak{c}}g - (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\xi_1 - 2(1+\alpha_5)(h+\xi_3) - 2(\tilde{g}+\alpha_6)(1+\xi_1) \tag{A.9}$$
$$\mathcal{A}_5^{\{\mathbb{H},\mathbb{D}\}} = (\tilde{g}^2+\alpha_7)(1+\xi_2) - 2(\tilde{g}+\alpha_6)(h+\xi_3) - 4\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(h+\xi_3) + (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(h+\xi_3) \tag{A.10}$$
$$\mathcal{A}_6^{\{\mathbb{H},\mathbb{D}\}} = (\tilde{g}^2+\alpha_8)(2+3\xi_1-\xi_2) - 2(\tilde{g}^2+\alpha_7)(1+\xi_1) \tag{A.11}$$
$$\mathcal{A}_7^{\{\mathbb{H},\mathbb{D}\}} = (1+\alpha_9)(\xi_1-\xi_2). \tag{A.12}$$
$$\mathcal{A}_8^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}\xi_1 \tag{A.13}$$
$$\mathcal{A}_9^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}\xi_2 \tag{A.14}$$
$$\mathcal{A}_{10}^{\{\mathbb{H},\mathbb{D}\}} = 1+2\beta_1-(1+\xi_1)(1+\beta_2) \tag{A.15}$$
$$\mathcal{A}_{11}^{\{\mathbb{H},\mathbb{D}\}} = 1+\beta_5-(1+\xi_1)(1+\beta_6) \tag{A.16}$$
$$\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}(h+\xi_3)-(1+\xi_2)(1+\beta_{10}) \tag{A.17}$$
$$\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_2)(1+\beta_{12})-\partial_{\mathfrak{c}}(h+\xi_3) \tag{A.18}$$
$$\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(2+\beta_{10}+\beta_9)-2-2\beta_6 \tag{A.19}$$
$$\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(\beta_{11}-\beta_{12}) \tag{A.20}$$
$$\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(1+\beta_8)-1 \tag{A.21}$$
$$\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(1+\beta_4)-1. \tag{A.22}$$

Finally the $\{\mathbb{H},\mathbb{H}\}$ bracket :

$$\begin{aligned}\mathcal{A}_1^{\{\mathbb{H},\mathbb{H}\}} =& 4(\tilde{g}+\alpha_1)\left(g\partial_{\mathfrak{c}}g-\alpha_6-\tilde{g}\right) - 2(g^2+4g\mathfrak{p}\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) + 8\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(\tilde{g}+\alpha_1)\\ &+ (\tilde{g}^2+\alpha_2)(2+3\alpha_5-\alpha_4)\end{aligned} \tag{A.23}$$
$$\begin{aligned}\mathcal{A}_2^{\{\mathbb{H},\mathbb{H}\}} =& 2(\tilde{g}^2+\alpha_2)(\alpha_6-g\partial_{\mathfrak{c}}g+\tilde{g}) - (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(\tilde{g}^2+\alpha_2) + 4\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(\tilde{g}^2+\alpha_2)\\ &+ 2(\tilde{g}+\alpha_1)(\tilde{g}^2-2\alpha_7+3\alpha_8)\end{aligned} \tag{A.24}$$
$$\begin{aligned}\mathcal{A}_3^{\{\mathbb{H},\mathbb{H}\}} =& 2(1+\alpha_3)\left[\tilde{g}+\alpha_6-g\partial_{\mathfrak{c}}g+(h+\xi_3)(1+\alpha_5)\right] - 2(\tilde{g}+\alpha_1)(1+\alpha_9) + 4\mathfrak{p}g(\partial_{\mathfrak{p}}\alpha_3)(\partial_{\mathfrak{c}}g)\\ &- (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\alpha_3\end{aligned} \tag{A.25}$$
$$\mathcal{A}_4^{\{\mathbb{H},\mathbb{H}\}} = (1+\alpha_3)\left[(1+\xi_2)(1+\alpha_5)-1-\alpha_4\right] \tag{A.26}$$
$$\mathcal{A}_5^{\{\mathbb{H},\mathbb{H}\}} = -\xi_1(1+\alpha_3)(1+\alpha_5) \tag{A.27}$$
$$\mathcal{A}_6^{\{\mathbb{H},\mathbb{H}\}} = \partial_{\mathfrak{c}}\beta_1 \tag{A.28}$$

$$
\begin{aligned}
\mathcal{A}_{7}^{\{\mathbb{H},\mathbb{H}\}} &= \partial_{\mathfrak{c}}\beta_3 && \text{(A.29)}\\
\mathcal{A}_{8}^{\{\mathbb{H},\mathbb{H}\}} &= -\partial_{\mathfrak{c}}\beta_2 && \text{(A.30)}\\
\mathcal{A}_{9}^{\{\mathbb{H},\mathbb{H}\}} &= -\partial_{\mathfrak{c}}\beta_4 && \text{(A.31)}\\
\mathcal{A}_{10}^{\{\mathbb{H},\mathbb{H}\}} &= \partial_{\mathfrak{c}}(\beta_2+\beta_4) && \text{(A.32)}\\
\mathcal{A}_{11}^{\{\mathbb{H},\mathbb{H}\}} &= \beta_1-\beta_3-\beta_5-\beta_3\beta_5 && \text{(A.33)}\\
\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{H}\}} &= \beta_1+\beta_7+\beta_1\beta_7-\beta_3 && \text{(A.34)}\\
\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{H}\}} &= \beta_1+\beta_3+\beta_6+\beta_3\beta_6+\beta_8+\beta_1\beta_8-\beta_2-\beta_4 && \text{(A.35)}\\
\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{H}\}} &= [1+\beta_2]\,(2-\alpha_4+3\alpha_5)-2\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) && \text{(A.36)}\\
\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{H}\}} &= [1+\beta_4]\,(-2+\alpha_4-3\alpha_5)+2\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) && \text{(A.37)}\\
\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{H}\}} &= \partial_{\mathfrak{c}}\alpha_3 && \text{(A.38)}\\
\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{H}\}} &= -6\,[\tilde{g}+\alpha_1]\,[1+\beta_6]+2g(\partial_{\mathfrak{c}}g)(3+3\beta_1-2\mathfrak{p}\partial_{\mathfrak{p}}\beta_1)+g^2\partial_{\mathfrak{c}}\beta_1+4g\mathfrak{p}(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_1) && \text{(A.39)}\\
\mathcal{A}_{18}^{\{\mathbb{H},\mathbb{H}\}} &= 12\tilde{g}+10\alpha_1+2\alpha_6+4(\tilde{g}+\alpha_1)\beta_{10}+2(\tilde{g}+\alpha_6)\beta_2+6\tilde{g}\beta_9-2\tilde{g}\partial_{\mathfrak{c}}\tilde{g}-2g(\partial_{\mathfrak{c}}g)(5+5\beta_2-2\mathfrak{p}\partial_{\mathfrak{p}}\beta_2)\\
&\quad -\partial_{\mathfrak{c}}\alpha_2-g^2\partial_{\mathfrak{c}}\beta_2-4g\mathfrak{p}(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_2) && \text{(A.40)}\\
\mathcal{A}_{19}^{\{\mathbb{H},\mathbb{H}\}} &= 2\alpha_1-2\alpha_6+6(\tilde{g}+\alpha_1)\beta_{11}-4(\tilde{g}+\alpha_1)\beta_{12}-2(\tilde{g}+\alpha_6)\beta_4+2\tilde{g}\partial_{\mathfrak{c}}\tilde{g}-2g(\partial_{\mathfrak{c}}g)(1+\beta_4+2\mathfrak{p}\partial_{\mathfrak{p}}\beta_4)\\
&\quad +\partial_{\mathfrak{c}}\alpha_2+g^2\partial_{\mathfrak{c}}\beta_4+4g\mathfrak{p}(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_4) && \text{(A.41)}\\
\mathcal{A}_{20}^{\{\mathbb{H},\mathbb{H}\}} &= 3\mathfrak{p}(\tilde{g}+\alpha_1)+3\mathfrak{p}(\tilde{g}+\alpha_1)\beta_8-g(\partial_{\mathfrak{c}}g)\left[3\mathfrak{p}(1+\beta_3)+2\mathfrak{p}^2\partial_{\mathfrak{p}}\beta_3\right]+\frac{\mathfrak{p}}{2}g^2\partial_{\mathfrak{c}}\beta_3+2\mathfrak{p}^2g(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_3). && \text{(A.42)}
\end{aligned}
$$

Appendix B: List of anomalies when correcting every constraints

In this case, there are no anomalies for the $\{\mathbb{G},\mathbb{G}\}$ and $\{\mathbb{D},\mathbb{D}\}$ brackets. For the $\{\mathbb{D},\mathbb{G}\}$ bracket, there is one anomaly :

$$\mathcal{A}_{1}^{\{\mathbb{D},\mathbb{G}\}} = \mathfrak{p}(1+\xi_2)(l+\chi_2)-(h+\xi_3)(\mathfrak{p}+\chi_1). \tag{B.1}$$

For the $\{\mathbb{H},\mathbb{G}\}$ bracket, the anomalies read :

$$
\begin{aligned}
\mathcal{A}_{1}^{\{\mathbb{H},\mathbb{G}\}} &= g^2\partial_{\mathfrak{c}}\chi_1+4\mathfrak{p}g\left[(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\chi_1)-(\partial_{\mathfrak{c}}g)(1+\partial_{\mathfrak{p}}\chi_1)\right]+2\mathfrak{p}(1+\alpha_4)(l+\chi_2)+2(\tilde{g}+\alpha_6)(\mathfrak{p}+\chi_1) && \text{(B.2)}\\
\mathcal{A}_{2}^{\{\mathbb{H},\mathbb{G}\}} &= (\tilde{g}^2+\alpha_7)(\mathfrak{p}+\chi_1)-2\mathfrak{p}(\tilde{g}+\alpha_6)(l+\chi_2)+\mathfrak{p}g^2\partial_{\mathfrak{c}}(l+\chi_2)+4\mathfrak{p}^2g\left[(\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(l+\chi_2)-(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(l+\chi_2)\right] && \text{(B.3)}\\
\mathcal{A}_{3}^{\{\mathbb{H},\mathbb{G}\}} &= \partial_{\mathfrak{c}}\chi_1 && \text{(B.4)}\\
\mathcal{A}_{4}^{\{\mathbb{H},\mathbb{G}\}} &= \mathfrak{p}\partial_{\mathfrak{c}}(l+\chi_2)-(\mathfrak{p}+\chi_1)(1+\beta_{10}) && \text{(B.5)}\\
\mathcal{A}_{5}^{\{\mathbb{H},\mathbb{G}\}} &= (\mathfrak{p}+\chi_1)(1+\beta_{12})-\mathfrak{p}\partial_{\mathfrak{c}}(l+\chi_2). && \text{(B.6)}
\end{aligned}
$$

For the $\{\mathbb{H},\mathbb{D}\}$ bracket,

$$
\begin{aligned}
\mathcal{A}_{1}^{\{\mathbb{H},\mathbb{D}\}} &= (\tilde{g}^2+\alpha_2)(\xi_2-2-3\xi_1)-4(\tilde{g}+\alpha_1)(h+\xi_3)+6g^2 && \text{(B.7)}\\
\mathcal{A}_{2}^{\{\mathbb{H},\mathbb{D}\}} &= (1+\alpha_3)(\xi_1-\xi_2) && \text{(B.8)}\\
\mathcal{A}_{3}^{\{\mathbb{H},\mathbb{D}\}} &= 2(1+\alpha_4)(h+\xi_3)+2(\tilde{g}+\alpha_6)(1+\xi_2)-4g(1+\xi_2+\mathfrak{p}\partial_{\mathfrak{p}}\xi_2)\partial_{\mathfrak{c}}g+(g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\xi_2 && \text{(B.9)}\\
\mathcal{A}_{4}^{\{\mathbb{H},\mathbb{D}\}} &= 4g(1+\xi_1+\mathfrak{p}\partial_{\mathfrak{p}}\xi_1)\partial_{\mathfrak{c}}g-(g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\xi_1-2(1+\alpha_5)(h+\xi_3)-2(\tilde{g}+\alpha_6)(1+\xi_1) && \text{(B.10)}\\
\mathcal{A}_{5}^{\{\mathbb{H},\mathbb{D}\}} &= (\tilde{g}^2+\alpha_7)(1+\xi_2)-2(\tilde{g}+\alpha_6)(h+\xi_3)-4\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(h+\xi_3)+(g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(h+\xi_3) && \text{(B.11)}
\end{aligned}
$$

$$\mathcal{A}_6^{\{\mathbb{H},\mathbb{D}\}} = (\tilde{g}^2+\alpha_8)(2+3\xi_1-\xi_2) - 2(\tilde{g}^2+\alpha_7)(1+\xi_1) \tag{B.12}$$
$$\mathcal{A}_7^{\{\mathbb{H},\mathbb{D}\}} = (1+\alpha_9)(\xi_1-\xi_2). \tag{B.13}$$
$$\mathcal{A}_8^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}\xi_1 \tag{B.14}$$
$$\mathcal{A}_9^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}\xi_2 \tag{B.15}$$
$$\mathcal{A}_{10}^{\{\mathbb{H},\mathbb{D}\}} = 1+2\beta_1-(1+\xi_1)(1+\beta_2) \tag{B.16}$$
$$\mathcal{A}_{11}^{\{\mathbb{H},\mathbb{D}\}} = 1+\beta_5-(1+\xi_1)(1+\beta_6) \tag{B.17}$$
$$\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{D}\}} = \partial_{\mathfrak{c}}(h+\xi_3)-(1+\xi_2)(1+\beta_{10}) \tag{B.18}$$
$$\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_2)(1+\beta_{12})-\partial_{\mathfrak{c}}(h+\xi_3) \tag{B.19}$$
$$\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(2+\beta_{10}+\beta_9)-2-2\beta_6 \tag{B.20}$$
$$\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(\beta_{11}-\beta_{12}) \tag{B.21}$$
$$\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(1+\beta_8)-1 \tag{B.22}$$
$$\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{D}\}} = (1+\xi_1)(1+\beta_4)-1. \tag{B.23}$$

Finally for the $\{\mathbb{H},\mathbb{H}\}$ bracket,

$$\begin{aligned}\mathcal{A}_1^{\{\mathbb{H},\mathbb{H}\}} =& 4(\tilde{g}+\alpha_1)\left(g\partial_{\mathfrak{c}}g-\alpha_6-\tilde{g}\right) - 2(g^2+4g\mathfrak{p}\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) + 8\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(\tilde{g}+\alpha_1)\\ &+(\tilde{g}^2+\alpha_2)(2+3\alpha_5-\alpha_4)\end{aligned} \tag{B.24}$$
$$\begin{aligned}\mathcal{A}_2^{\{\mathbb{H},\mathbb{H}\}} =& 2(\tilde{g}^2+\alpha_2)(\alpha_6-g\partial_{\mathfrak{c}}g+\tilde{g}) - (g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}(\tilde{g}^2+\alpha_2) + 4\mathfrak{p}g(\partial_{\mathfrak{c}}g)\partial_{\mathfrak{p}}(\tilde{g}^2+\alpha_2)\\ &+2(\tilde{g}+\alpha_1)(\tilde{g}^2-2\alpha_7+3\alpha_8)\end{aligned} \tag{B.25}$$
$$\begin{aligned}\mathcal{A}_3^{\{\mathbb{H},\mathbb{H}\}} =& 2(1+\alpha_3)\left[\tilde{g}+\alpha_6-g\partial_{\mathfrak{c}}g+(h+\xi_3)(1+\alpha_5)\right] - 2(\tilde{g}+\alpha_1)(1+\alpha_9) + 4\mathfrak{p}g(\partial_{\mathfrak{p}}\alpha_3)(\partial_{\mathfrak{c}}g)\\ &-(g^2+4\mathfrak{p}g\partial_{\mathfrak{p}}g)\partial_{\mathfrak{c}}\alpha_3\end{aligned} \tag{B.26}$$
$$\mathcal{A}_4^{\{\mathbb{H},\mathbb{H}\}} = (1+\alpha_3)\left[(1+\xi_2)(1+\alpha_5)-1-\alpha_4\right] \tag{B.27}$$
$$\mathcal{A}_5^{\{\mathbb{H},\mathbb{H}\}} = -\xi_1(1+\alpha_3)(1+\alpha_5) \tag{B.28}$$
$$\mathcal{A}_6^{\{\mathbb{H},\mathbb{H}\}} = \partial_{\mathfrak{c}}\beta_1 \tag{B.29}$$
$$\mathcal{A}_7^{\{\mathbb{H},\mathbb{H}\}} = \partial_{\mathfrak{c}}\beta_3 \tag{B.30}$$
$$\mathcal{A}_8^{\{\mathbb{H},\mathbb{H}\}} = -\partial_{\mathfrak{c}}\beta_2 \tag{B.31}$$
$$\mathcal{A}_9^{\{\mathbb{H},\mathbb{H}\}} = -\partial_{\mathfrak{c}}\beta_4 \tag{B.32}$$
$$\mathcal{A}_{10}^{\{\mathbb{H},\mathbb{H}\}} = \partial_{\mathfrak{c}}(\beta_2+\beta_4) \tag{B.33}$$
$$\mathcal{A}_{11}^{\{\mathbb{H},\mathbb{H}\}} = \beta_1-\beta_3-\beta_5-\beta_3\beta_5 \tag{B.34}$$
$$\mathcal{A}_{12}^{\{\mathbb{H},\mathbb{H}\}} = \beta_1+\beta_7+\beta_1\beta_7-\beta_3 \tag{B.35}$$
$$\mathcal{A}_{13}^{\{\mathbb{H},\mathbb{H}\}} = \beta_1+\beta_3+\beta_6+\beta_3\beta_6+\beta_8+\beta_1\beta_8-\beta_2-\beta_4 \tag{B.36}$$
$$\mathcal{A}_{14}^{\{\mathbb{H},\mathbb{H}\}} = \left[1+\beta_2\right](2-\alpha_4+3\alpha_5)-2\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) \tag{B.37}$$
$$\mathcal{A}_{15}^{\{\mathbb{H},\mathbb{H}\}} = \left[1+\beta_4\right](-2+\alpha_4-3\alpha_5)+2\partial_{\mathfrak{c}}(\tilde{g}+\alpha_1) \tag{B.38}$$
$$\mathcal{A}_{16}^{\{\mathbb{H},\mathbb{H}\}} = \partial_{\mathfrak{c}}\alpha_3 \tag{B.39}$$
$$\mathcal{A}_{17}^{\{\mathbb{H},\mathbb{H}\}} = -6\left[\tilde{g}+\alpha_1\right]\left[1+\beta_6\right] + 2g(\partial_{\mathfrak{c}}g)(3+3\beta_1-2\mathfrak{p}\partial_{\mathfrak{p}}\beta_1) + g^2\partial_{\mathfrak{c}}\beta_1 + 4g\mathfrak{p}(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_1) \tag{B.40}$$
$$\begin{aligned}\mathcal{A}_{18}^{\{\mathbb{H},\mathbb{H}\}} =& 12\tilde{g}+10\alpha_1+2\alpha_6+4(\tilde{g}+\alpha_1)\beta_{10}+2(\tilde{g}+\alpha_6)\beta_2+6\tilde{g}\beta_9-2\tilde{g}\partial_{\mathfrak{c}}\tilde{g}-2g(\partial_{\mathfrak{c}}g)(5+5\beta_2-2\mathfrak{p}\partial_{\mathfrak{p}}\beta_2)\\ &-\partial_{\mathfrak{c}}\alpha_2-g^2\partial_{\mathfrak{c}}\beta_2-4g\mathfrak{p}(\partial_{\mathfrak{p}}g)(\partial_{\mathfrak{c}}\beta_2)\end{aligned} \tag{B.41}$$

$$\begin{aligned}
\mathcal{A}_{19}^{\{\mathbb{H},\mathbb{H}\}} =&2\alpha_1 - 2\alpha_6 + 6(\tilde{g}+\alpha_1)\beta_{11} - 4(\tilde{g}+\alpha_1)\beta_{12} - 2(\tilde{g}+\alpha_6)\beta_4 + 2\tilde{g}\partial_\mathfrak{c}\tilde{g} - 2g(\partial_\mathfrak{c} g)(1+\beta_4+2\mathfrak{p}\partial_\mathfrak{p}\beta_4) \\
&+ \partial_\mathfrak{c}\alpha_2 + g^2\partial_\mathfrak{c}\beta_4 + 4g\mathfrak{p}(\partial_\mathfrak{p} g)(\partial_\mathfrak{c}\beta_4) \qquad \text{(B.42)}\\
\mathcal{A}_{20}^{\{\mathbb{H},\mathbb{H}\}} =&3\mathfrak{p}(\tilde{g}+\alpha_1) + 3\mathfrak{p}(\tilde{g}+\alpha_1)\beta_8 - g(\partial_\mathfrak{c} g)\left[3\mathfrak{p}(1+\beta_3) + 2\mathfrak{p}^2\partial_\mathfrak{p}\beta_3\right] + \frac{\mathfrak{p}}{2}g^2\partial_\mathfrak{c}\beta_3 + 2\mathfrak{p}^2 g(\partial_\mathfrak{p} g)(\partial_\mathfrak{c}\beta_3). \qquad \text{(B.43)}
\end{aligned}$$

Appendix C: Summary of the counter-terms in the case of all the constraints being corrected

Here $\xi_3(\mathfrak{c},\mathfrak{p})$ is left arbitrary for generality purposes. First, the matter counter-terms :

$$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0 \qquad \text{(C.1)}$$
$$\beta_9 = 1 - \partial_\mathfrak{c}(h+\xi_3) \qquad \text{(C.2)}$$
$$\beta_{10} = \beta_{11} = \beta_{12} = \partial_\mathfrak{c}(h+\xi_3) - 1 \qquad \text{(C.3)}$$
$$\beta_{13} = \alpha_3[K,\alpha_9(\mathfrak{p})]\partial_\mathfrak{c}(g\partial_\mathfrak{c} g) + \partial_\mathfrak{c}(g\partial_\mathfrak{c} g) - 1. \qquad \text{(C.4)}$$

For the Hamiltonian counter-terms :

$$\alpha_1 = g\partial_\mathfrak{c} g - \tilde{g} \qquad \text{(C.5)}$$
$$\alpha_2 = 3g^2 - \tilde{g}^2 - 2g(h+\xi_3)\partial_\mathfrak{c} g \qquad \text{(C.6)}$$
$$\alpha_3 = \frac{1}{\sqrt{\mathfrak{p}}}\left[K + \int_1^p \frac{\alpha_9(u)}{2\sqrt{u}}du\right] \qquad \text{(C.7)}$$
$$\alpha_4 = \alpha_5 = (\partial_\mathfrak{c} g)^2 + g\partial_\mathfrak{c}^2 g - 1 \qquad \text{(C.8)}$$
$$\alpha_6 = 2g\partial_\mathfrak{c} g - \tilde{g} - (h+\xi_3)\left[(\partial_\mathfrak{c} g)^2 + g\partial_\mathfrak{c}^2 g\right] \qquad \text{(C.9)}$$
$$\alpha_7 = \alpha_8 = 4g\partial_\mathfrak{c} g\left[(h+\xi_3) + \mathfrak{p}\partial_\mathfrak{p}(h+\xi_3)\right] - \tilde{g}^2 - 2(h+\xi_3)^2\left[(\partial_\mathfrak{c} g)^2 + g\partial_\mathfrak{c}^2 g\right] - \left[g^2 + 4\mathfrak{p}g\partial_\mathfrak{p} g\right]\partial_\mathfrak{c}(h+\xi_3). \qquad \text{(C.10)}$$
$$\alpha_9 = \alpha_9(\mathfrak{p}). \qquad \text{(C.11)}$$

For the gauss counter-terms :

$$\chi_1 = (L-1)\mathfrak{p} \qquad \text{(C.12)}$$
$$\chi_2 = L(h+\xi_3) - l. \qquad \text{(C.13)}$$

Finally, the diffeomorphism counter-terms :

$$\xi_1 = \xi_2 = 0, \qquad \text{(C.14)}$$

and

$$\xi_3 = \begin{cases} \xi_3(\mathfrak{c},\mathfrak{p}) \text{ if } g\partial_\mathfrak{g} = \tilde{g} \\ \mathfrak{c} - h + f(\mathfrak{p}) \text{ if } g\partial_\mathfrak{g} \neq \tilde{g} \end{cases} \qquad \text{(C.15)}$$

with L, K, $f(\mathfrak{p})$ (or $\xi_3(\mathfrak{c},\mathfrak{p})$) and $\alpha_9(\mathfrak{p})$ being degrees of freedom. The only restriction on the correction functions affects g and is given by :

$$g\partial_\mathfrak{c}^2 g + 2\mathfrak{p}\left[(\partial_\mathfrak{c} g)(\partial_\mathfrak{c}\partial_\mathfrak{p} g) - (\partial_\mathfrak{p} g)(\partial_\mathfrak{c}^2 g)\right] = 0. \qquad \text{(C.16)}$$