

Reciprocal swimming in granular media: the role of jamming and swimmer inertia

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We use particle simulations to reveal two distinct propulsion mechanisms for a scallop-like swimmer to locomote itself in granular media by reciprocally flapping its wings. Based on the discrete element method, we examine the kinematics and contact forces of particles near the swimmer to identify jamming effects induced by the swimmer in a frictional granular medium, which are less intense during the opening stroke than the closing. This broken symmetry is quantified by the difference in the number of strong particle contact forces formed during opening and closing, which shows a linear relation with the swimmer's net displacement across various swimmer and medium configurations, all favoring the opening stroke. We identify a secondary propulsion mechanism in a dynamic regime with significant swimmer inertia, as the flapping period approaches the coasting time for a moving swimmer to come to rest under the medium resistance. In this case, the swimmer's net displacement is correlated to the ratio between these two time scales, and the swimming direction favors the closing stroke due to the smaller medium resistance as the swimmer coasts with closed wings.

I. INTRODUCTION

Locomotion in granular media is challenging because granular particles can both flow like a complex fluid and jam into a solid [1, 2]. To navigate granular media, many swimmers adopt intricate gaits [3–5], such as undulation [6–11] and peristaltic body expansion [12–16]. On the other hand, gaits as simple as flapping an appendage also prove effective for granular locomotion of animals and bio-inspired robots [17–22]. By periodically oscillating rigid appendages such as rods and plates, swimmers can consistently propel themselves in granular media [21, 22]. This is likely not predictable by simplified models like certain resistive force theories [23], which assume time-reversal symmetry and often neglect the evolving state of the granular medium, e.g., due to the jamming transition [24]. This highlights the need to reveal and understand more locomotion mechanisms that utilize the complexity of the medium.

Swimming with reciprocally flapping wings recalls the scallop theorem [25], which states that reciprocal motions, consisting of time-reversible body deformation sequences, cannot produce net displacement in Newtonian fluids at low Reynolds numbers. In granular media, two preconditions for this theorem can be broken.

First, granular media have strongly non-Newtonian rheology [26–28]. While granular locomotion often occurs in a regime with rate-independent medium response [3], the jamming transition may result in elasto-plastic-like medium deformation, which manifests as formation of stagnant zones with jammed particles near a swimmer [29–37]. More complexities may arise due to overlapping length scales of the stagnant zone size and intruder size [34, 38], and due to the overlap between the required jamming transition time (or strain) and the swimming period.

Second, the swimmer's inertia can play a significant role in granular swimming, even when the medium exhibits rate-independent deformation. For a typical swimmer like a bacterium in a liquid at low Reynolds numbers, both the swimmer's and the fluid's inertia are negligible due to their similar densities. For granular locomotion, on the other hand, a swimmer denser than the surrounding particles can possibly be sustained without sinking, especially given that the medium tends to exert lift forces on laterally moving intruders [19, 39]. For a swimmer with non-negligible inertia, reciprocal strokes could potentially break the time-reversal symmetry and enable locomotion [40]. While this mechanism is under-explored in granular media, it has been demonstrated using a dense interfacial swimmer whose weight is supported by surface tension [41]. In this case, the swimmer's coasting time, the characteristic duration over which fluid drag halts its motion after a stroke, becomes comparable to the driving period. When the respective coasting times for the two reciprocal strokes differ, net locomotion occurs even if the surrounding fluid remains in the Stokes regime [40]. This type of inertial effect remains to be investigated in the context of granular locomotion.

In this study, we seek to reveal the respective roles of jamming and swimmer inertia in reciprocal granular swimming using discrete element method (DEM) simulations. Building on our recent experiments with a scallop-like robot that swims in granular media by reciprocally flapping its wings [22], we systematically vary jamming effects in DEM to explain how they enable reciprocal swimming, while identifying a new inertia-based swimming mechanism. In the quasi-static regime, we use microscopic force transmission analyses to show how the jamming transition breaks the

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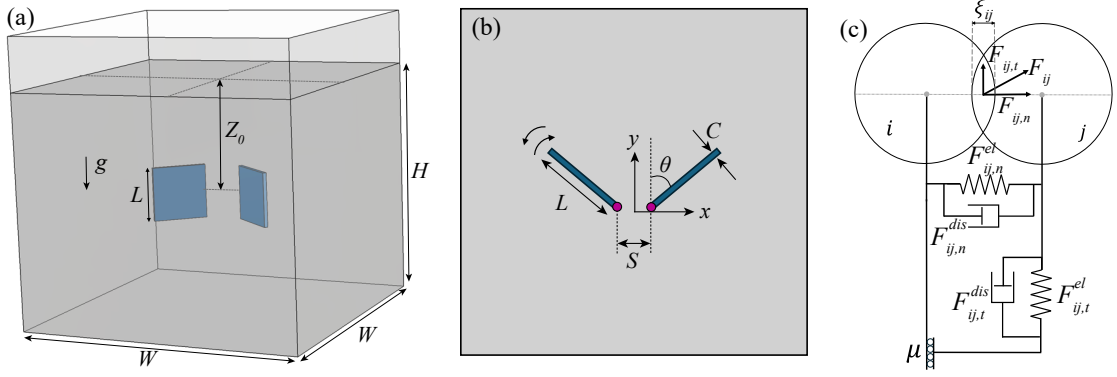


FIG. 1. Schematics of the simulation setup and particle contact models. (a) 3D view of the setup. (b) Top-down view of the setup. (c) Normal and tangential contact forces between particles i and j .

symmetry of the medium's states between the opening and closing strokes, which is reflected by the formation of asymmetric stagnant zones near the swimmer. This results in net propulsion that favors the opening stroke, agreeing with experiments [22]. The swimmer inertia becomes significant in a dynamic regime, where the coasting time approaches the wing flapping period. The swimmer experiences a longer coasting time with closed wings, resulting in net propulsion that favors the closing stroke. For both mechanisms, we establish physics-based order parameters that strongly correlate with the respective net locomotion displacements.

II. SIMULATION METHOD

We simulate a scallop-like swimmer submerged at depth Z_0 in a granular reservoir, see Fig. 1a. The swimmer has a pair of square-shaped wings with length L and thickness C . The wings rotate cyclically around their respective hubs (purple dots in Fig. 1b), which are set at a gap width S to each other. A coordinate system is established, with the origin set at the midpoint between the two hubs, the x direction pointing laterally, the y direction aligned with the swimmer translation, and the z direction against the gravitational acceleration g . We define the wing angle, θ , as the angle between the y -axis and a vector in the xy plane tangent to a wing's surface. In each simulation, the swimmer reciprocally opens and closes its wings within a period T , and the range of θ is bounded by the maximum and minimum angles, θ_o and θ_c , respectively. The wings rotate at a constant angular speed, $\omega = 2(\theta_o - \theta_c)/T$. In a subset of simulations, we also consider the swimmer to have inert components as discussed later.

We model the interaction between a pair of contacting particles, i and j , with a force, $\mathbf{F}_{ij} = \mathbf{F}_{ij,n} + \mathbf{F}_{ij,t}$, as depicted in Fig. 1c. The normal component is $\mathbf{F}_{ij,n} = (F_{ij,n}^{\text{el}} + F_{ij,n}^{\text{dis}})\mathbf{n}_{ij}$ and the tangential component is $\mathbf{F}_{ij,t} = (F_{ij,t}^{\text{el}} + F_{ij,t}^{\text{dis}})\mathbf{t}_{ij}$, where $\mathbf{n}_{ij}$ is the unit vector connecting particle centroids and $\mathbf{t}_{ij}$ is the tangential unit vector, set by the direction of the relative tangential velocity between i and j [42, 43]. For the scalar values of the normal force, the Hertz model [44] was used to calculate the elastic component, and a viscous model [45] was used for dissipation, which are

$$F_{ij,n}^{\text{el}} = \frac{2}{3} \frac{E}{1 - \nu^2} \sqrt{R_{\text{eff}}} \xi_{ij}^{3/2}, \quad (1)$$

$$F_{ij,n}^{\text{dis}} = \gamma_n \dot{\xi}_{ij} = \frac{E}{1 - \nu^2} \sqrt{R_{\text{eff}}} A \sqrt{\xi_{ij}} \dot{\xi}_{ij}, \quad (2)$$

where E is the Young's modulus, ν is the Poisson's ratio, $R_{\text{eff}} = \frac{R_i R_j}{R_i + R_j}$ is the effective particle radius, $\xi_{ij} = R_i + R_j - |\mathbf{r}_i - \mathbf{r}_j|$ is the overlap in the normal direction, $\dot{\xi}_{ij}$ is the relative normal velocity, R_i is the particle radius, $\mathbf{r}_i$ is the particle position, and A is a dissipation constant that can be calculated using the scheme in [45]. The tangential interaction combines a viscoelastic force with Coulomb friction [46–48], which is

$$F_{ij,t} = \begin{cases} \mu F_{ij,n} & \mu |F_{ij,n}| < F_{ij,t}^{\text{el}} \\ F_{ij,t}^{\text{el}} + F_{ij,t}^{\text{dis}} & \mu |F_{ij,n}| > F_{ij,t}^{\text{el}} \end{cases}, \quad (3)$$

$$F_{ij,t}^{\text{el}}(\delta) = \frac{4}{3} \frac{E}{1 - \nu^2} \sqrt{R_{\text{eff}}} \sqrt{\xi_{ij}} \delta_{ij}, \quad (4)$$

TABLE I. Parameters used in the DEM simulation. Bold values are used in the base case.

Parameter	Symbol (unit)	Value
mean particles diameter	d (mm)	1.0
wing width	L (mm)	15.0
wing thickness	C (mm)	1.0
wing gap width	S (mm)	{0.0, 2.75, 5.5, 11.0 , 22.0}
container width	W (mm)	80.0
free surface height	H (mm)	65.0
swimmer depth	Z_0 (mm)	32.5
close angle	θ_c (deg)	20
open angle	θ_o (deg)	80
swimming period	T (s)	{0.05, 0.1, 0.2, 0.5, 1.0, 2.0 }
Young's modulus	E (MPa)	1.0
Poisson's ratio	ν	0.34
friction coefficient	μ	{0.0, 0.4 }
dissipation coefficient	$\gamma_{n/t}$ (N.s/m)	1.63×10^{-5}
particle density	ρ (kg/m ³)	1050
swimmer mass	m_s (g)	{0.012, 0.60 , 1.19, 2.38, 9.53, 29.78, 1370.00}
time step size	Δt (s)	1×10^{-5}

$$F_{ij,t}^{\text{dis}} = \gamma_t \dot{\delta}_{ij}, \quad \gamma_t = \gamma_n, \quad (5)$$

where μ is the friction coefficient, δ_{ij} is the relative tangential displacement between particles i and j , and $\dot{\delta}$ is the relative tangential velocity. We adopted the DEM package *MercuryDPM* [43, 49] and simulated approximately 510,000 particles having a diameter of $d = 1.0 \pm 0.12$ mm and a density of $\rho = 1050$ kg/m³, which sets the lateral dimensions of the reservoir to be $W = 80d$ and a fill height of $H = 80d$. We used a reduced Young's modulus so that a large time step can be used [50]. We selected a common value of $\mu = 0.4$ for friction as a base value, but we will test the influence of μ later on. The bottom and side walls of the reservoir are frictional boundaries as in our previous experiment [22].

As for the swimmer, its wings interact with particles with the standard particle-boundary interaction, having identical properties as in the particle-particle interaction. We impose the rotation on the wings and integrate its translational motion using the Verlet scheme [51]. In our experimental setting [22], the swimmer's position was fixed, and the reservoir was lubricated so that it could move in the y direction due to the swimming force. Accordingly, the simulated swimmer has only one translational degree of freedom in y , while its mass (m_s) is set to be concentrated at the hubs. A parametric study was performed with systematically varied parameters as listed in Table I.

III. RESULTS

A. Swimmer and Particle Kinematics

We first compare the simulated swimmer locomotion and particle kinematics with results from experiments that have identical setups in the swimmer configuration and reservoir size, as described in our previous work [22]. As the experimental swimmer was mounted on two vertical rods, we included the rods in one DEM simulation for validation. Figure 2a shows the normalized displacement of the swimmer, y/L , over normalized time, t/T . The experimental measurement (dotted curve) and the corresponding simulation result (thick yellow curve) show qualitative agreement, with consistent net locomotion in the $+y$ direction and similar displacement amplitudes per stroke. The simulated net displacement per cycle is larger than that in the experiment. This is likely due to a higher initial packing fraction, $\phi_0 = 0.60$, in the simulation, compared to the experiment ($\phi_0 \approx 0.58$), as the experimental particle system was air-fluidized before each experiment [22], while particles settle under gravity in DEM. We also plot the result of a simulation without the rods (blue curve), which shows the same $+y$ locomotion direction, a bigger displacement amplitude per stroke, and less net locomotion in each cycle. For simplicity, we will later focus on this simulation in the following sections and refer to it as the *base case*.

To continue our validation, we visualize particle kinematics using DEM and a quasi-static experiment imaged by X-ray computed tomography (CT) [22], capturing one full 3D scan of the system after an incremental wing rotation. In Fig. 2b-e, we first highlight regions with significant particle displacement (panels b and c), and then plot the in-plane ($x - y$) displacement field in the mid-plane of the swimmer (panels d and e), both showing good agreement

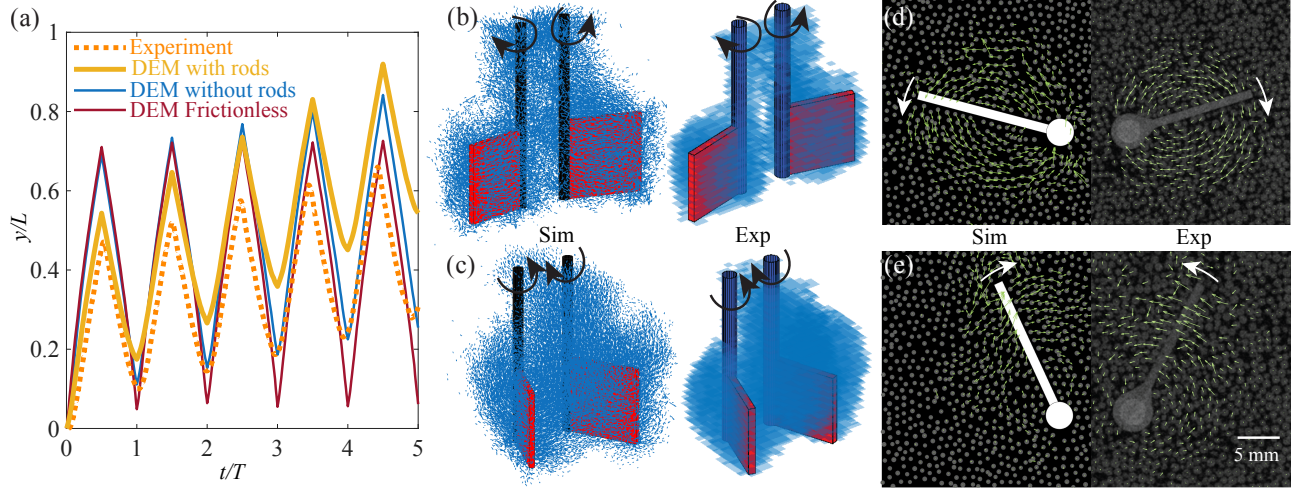


FIG. 2. Swimmer and particle kinematics from the experiment [22] and from DEM simulations, see configurations in Table I. (a) Swimmer displacement vs time in the experiment (dashed, orange), the DEM result with simulated mounting rods (yellow), the simplified simulation *base case* (blue), and a frictionless simulation (red). (b) and (c) Highlighting particles with a displacement larger than five times the average displacement in the DEM results (left panels) and the experimental X-ray CT results (right panels), at $\theta = 75^\circ$ during opening and $\theta = 25^\circ$ during closing, respectively. (d) and (e) Comparison of the particle displacement field at the mid-plane of the swimmer between DEM results (left panels) and X-ray CT results (right panels), at $\theta = 75^\circ$ during opening and $\theta = 25^\circ$ during closing, respectively.

between the simulation and experiment. During opening at $\theta = 75^\circ$, the wing rotation displaces a relatively small number of particles (Fig.2b), primarily in the x - y plane, forming a localized, vortex-like displacement field around the wing, see Fig.2d. In contrast, during closing at $\theta = 25^\circ$, the wing rotation affects a wider region, causing both in-plane and out-of-plane particle motion, see Fig.2c and e. The more extensive particle displacements indicate that localized particle rearrangements become more difficult, signaling intensified jamming effects[22]. Moving beyond kinematics, we next examine the role of jamming in greater detail using DEM simulations.

B. Jamming Induces Hysteresis in the Medium State During Swimming

In Fig. 3a, we visualize the inter-particle contact forces $|\mathbf{F}|$ and particle velocities $|\dot{\mathbf{r}}|$ in the base case (Table I) at five different θ during both the opening and closing strokes. Only the top 0.1% of the contact forces are shown, and only particles with velocities, $|\dot{\mathbf{r}}| > 0.33\omega L$, are shown. During each stroke, stagnant zones with percolated strong forces gradually form near the tip of the rotating wings on the leading edges that press into the medium, as in $\theta = 50^\circ$ for both strokes. These contact forces form chains that are rooted from the wings and propagate into the medium, which is a signature of jamming in granular media [24, 52]. As each stroke proceeds, a second set of force chains that resist the swimmer's translation appear near the wing hubs as seen in $\theta = 71^\circ$ for opening and $\theta = 29^\circ$ for closing. We refer to these zones as resistive stagnant zones (RSZ) and the other type as propulsive stagnant zones (PSZ), which are separated by the gyration point of a wing that has zero instantaneous velocity in the y direction. The distance from a gyration point to a hub is $r_g = |\dot{y}(\theta)| \sin(\theta)/\omega$, where $\dot{y}$ denotes the swimmer's translational velocity.

We then examine the structural signals of jamming by decomposing the immediate surroundings of the wings into zones I and II, separated by $r_g(\theta)$, as depicted in Fig. 3b. Both zones span the full height of the wings and extend $5d$ from the wings. In each zone, we average the coordination number Z , i.e., the number of contacts per particle, and plot the average as a function of θ in a swimming cycle in Fig. 3c and d for zones I and II, respectively.

In zone I, the coordination number Z increases during opening until $\theta \approx 29^\circ$, corresponding to the development of force chains in the PSZ in Fig. 3a. For $\theta > 29^\circ$, Z plateaus above $Z = 6$ as the PSZ becomes fully developed and the force chain intensity remains unchanged in $50^\circ \leq \theta \leq 80^\circ$. After the wing rotation direction changes, Z decreases towards a plateau around $Z = 5.7$, corresponding to the vanishing of force chains during an unjamming transition. Similar hysteresis in Z has been reported for jammed granular systems under cyclic shear [24, 53–55], and the plateaued value of $Z = 6$, i.e., two times the dimension, coincides with previously reported value for the jamming transition [24, 53, 56–60]. Thus, in reciprocal swimming, the hysteresis due to jamming and unjamming transitions breaks the symmetry in the medium state between opening and closing, creating the possibility for net locomotion.

In zone II, while a similar hysteresis in Z exists, a secondary increase up to $Z = 6.5$ occurs near the end of the closing stroke, which is due to the merging of two stagnant zones as in Fig. 3a from $\theta = 29^\circ$ to $\theta = 20^\circ$. During this period,

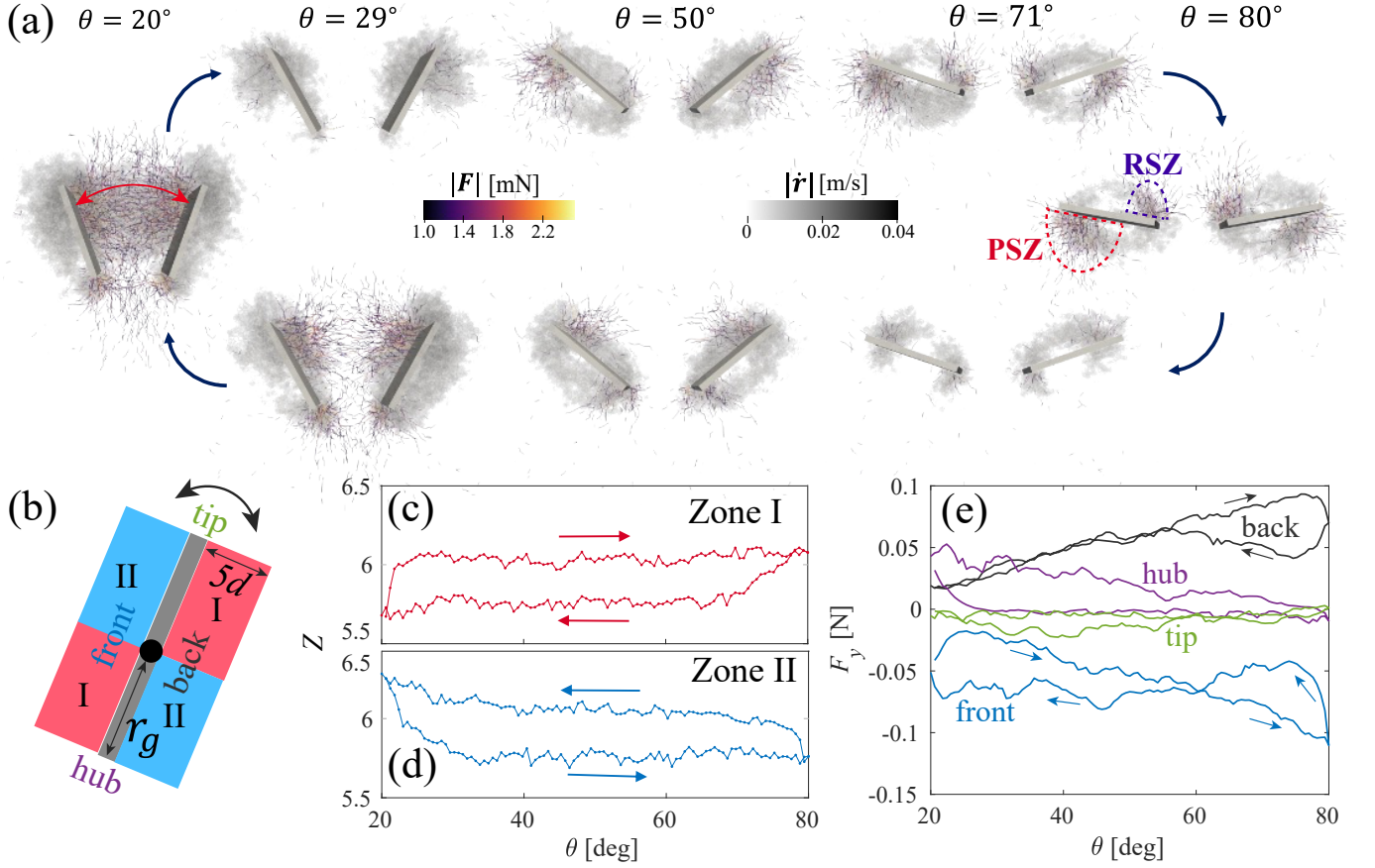


FIG. 3. Force transmission and structural analyses for the base simulation case. (a) Particle velocity and inter-particle contact forces visualized during one swimming cycle. (b) Sketch for spatial decomposition of the swimmer's surroundings. (c) and (d) Average coordination number Z over θ for zone I and II, respectively. (e) Total force exerted on different lateral surfaces of the wings.

force chains rooted from one wing land on the other, as sketched in $\theta = 20^\circ$, making them effectively “internal” forces of the swimmer with little contribution to propulsion. To demonstrate this, we separately examine the y component of the resistive forces, F_y , exerted on the four lateral wing surfaces as defined in Fig. 3b, which are plotted in Fig. 3e as a function of θ . While a monotonic increase can be observed in F_y on the back side during opening, the front side F_y plateaus near the end of closing. At this stage, a large F_y arises for the hub as its surrounding region serves as the RSZ. The increasing amount of jammed particles makes it difficult for the swimmer to move in the $-y$ direction, while triggering upward particle flows as in Fig. 2c.

To quantify the influence of jamming on locomotion, we performed simulations varying key jamming-related factors, including the swimmer geometry and the friction coefficient μ . For the geometry, we varied the wing gap width S that can influence the intensity of jamming between the wings. To approximate $S = \infty$, a case with only one wing was simulated. We also altered the swimmer geometry by inserting an inert mid-body to connect the wings to approximate a more realistically looking swimmer. In Fig. 4a, we show how the swimmer's normalized net displacement per cycle, $\Delta y/L$, depends on the varied factors. As the gap width increases, $\Delta y/L$ rises until $S/d \approx 6$, then decreases for $S/d > 6$ towards the value for $S/d = \infty$ (the black dashed line). This trend resembles the cooperative effects reported for two closely-spaced intruders in granular media, where both the drag force [61] and total work [34] peak at a separation of approximately $4d$. Similar to the result with inert rods in Fig. 2a, by filling this gap with an inert mid-body, a significant increase in $\Delta y/L$ occurs, accompanied by a stagnant zone (RSZ) with more intense force chains behind the mid-body.

By eliminating friction between particles, we can eliminate the jamming effects as friction is required for shear-related jamming [52]. In the simulation with $\mu = 0$, locomotion vanishes, along with the stagnant zones as seen in Fig. 2a and the inset snapshot in Fig. 4a, respectively. Different from the frictional cases in both Fig. 3a and Fig. 4a, a larger number of particles are mobilized uniformly around the swimmer in the absence of strong forces, which is a more fluid-like response. Note that we used a consistent threshold to select mobile particles and strong forces across all cases.

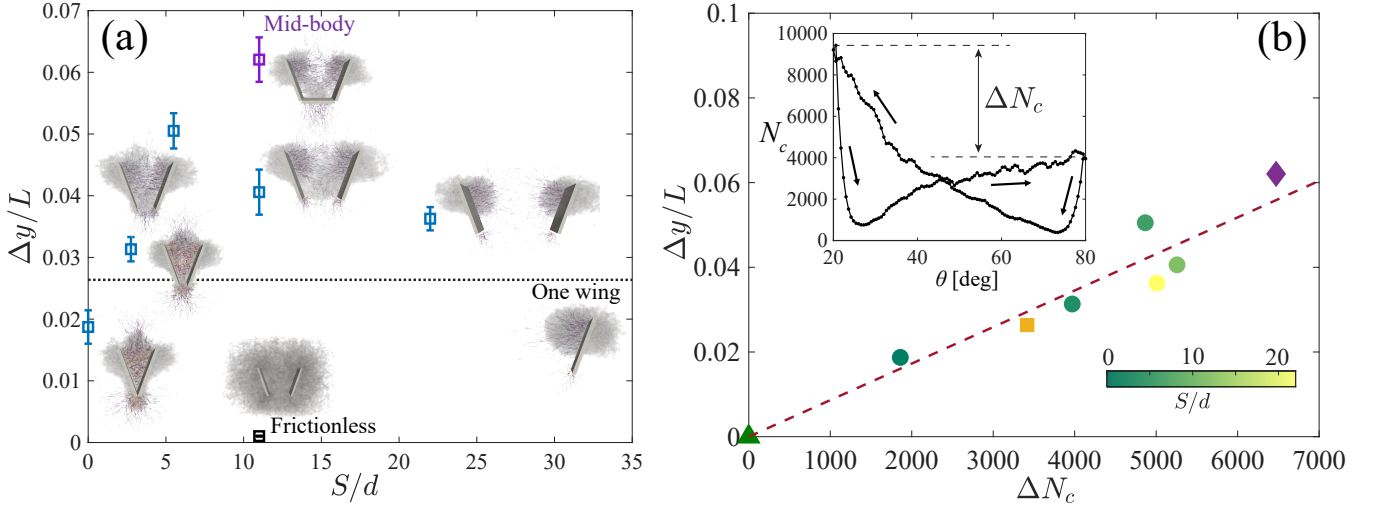


FIG. 4. The influence of jamming on net locomotion. (a) Cycle-averaged swimmer net displacement Δy for cases with different gap widths S , with error bars indicating cycle-cycle fluctuations, along with the case with inert mid-body (purple) and the case with $\mu = 0$ (black). The black dashed line marks the value of Δy for the one-wing case. (b) Cycle-averaged Δy vs. the contact number difference ΔN_c . Inset shows the N_c in the base case in a swimming cycle. The purple diamond represents the case with inert mid-body, the orange square represents the one-wing case, and the green triangle represents the case with $\mu = 0$. The red dashed line is a linear fit, $\Delta y/L = \alpha \Delta N_c$, where $\alpha = 8.63 \times 10^{-6}$.

To quantify the intensity of jamming induced by the swimmer, we define a (strong) contact number, N_c , as the number of contacts that have force magnitude, $|\mathbf{F}| > |\mathbf{F}_c|$, where $\mathbf{F}_c = 1.0 \text{ mN}$ is a threshold held consistent for all cases. The value of $\mathbf{F}_c$ corresponds to the value at the top 0.1% of the contact force distribution in the base case, which helps identify forces mostly in the stagnant zones, as in Fig. 3a and Fig. 4a. An example of N_c vs θ in a swimming cycle is shown in the inset of Figure 4b, showing a broken symmetry in the force transmission between the opening and closing strokes, agreeing with the structural signal based on Z in Fig. 3c and d. As argued earlier, the excess number of strong contacts towards the end of the closing stroke can limit the swimmer's translation. Therefore, we quantify the degree of jamming-induced symmetry breaking by calculating the difference of N_c between the fully closed and fully open states, yielding

$$\Delta N_c = N_c(\theta_c) - N_c(\theta_o). \quad (6)$$

Figure 4b shows the net displacement $\Delta y/L$ with respect to ΔN_c , revealing a linear relation between these two physical quantities across all cases with varied swimmer geometry and friction. This quantitatively demonstrates how jamming effects break the symmetry in the medium state between the two reciprocal strokes of the scallop swimmer and enable net locomotion in the $+y$ direction.

C. Role of Swimmer Inertia in Reciprocal Swimming

A second swimming mechanism, related to the swimmer's inertia, exists in a dynamic regime, which can be reached by increasing the swimmer mass m_s and decreasing the swimming cycle period T . To quantify the inertial effect, we consider a coasting time,

$$T_c = \frac{L/T}{P_s L^2/m_s} = \frac{m_s}{P_s L T}, \quad (7)$$

where L/T sets a characteristic swimmer translation speed that is comparable with the swimmer's speed near the end of a stroke, see real measurements, i.e., the slopes, in Fig. 2a. We then consider a characteristic acceleration, $P_s L^2/m_s$, where $P_s = \rho \phi_0 g Z_0$ is a hydrostatic-like pressure. Under this definition, T_c reflects a characteristic time for the swimmer to “coast” until its movement ceases due to the medium's drag [41].

Figure 5a shows the swimmer displacement with respect to time for a simulation in the dynamic regime with $T_c/T = 0.33$, along with the base case with $T_c/T = 5.16 \times 10^{-5}$. Contrary to the forward ($+y$) quasi-static locomotion [22],

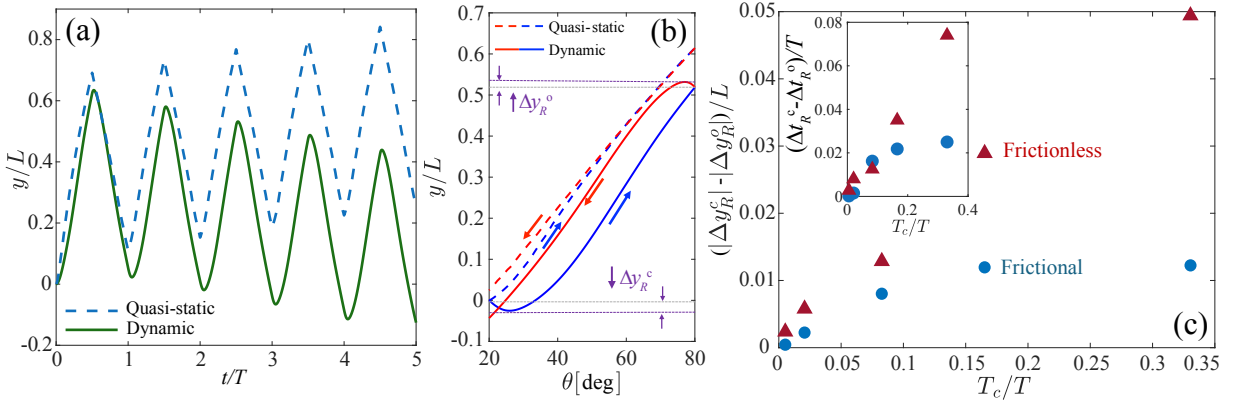


FIG. 5. Locomotion in the dynamic regime. (a) The normalized swimmer displacement Δy vs time in quasi-static (dashed blue curve) and dynamic (solid green curve) regimes. The quasi-static case is with $m_s = 0.60$ g, $T = 2.0$ s, and $T_c/T = 5.16 \times 10^{-5}$, and the dynamic case has $m_s = 2.38$ g, $T = 0.05$ s, and $T_c/T = 0.33$. (b) Normalized Δy vs θ in both quasi-static (dashed curve) and dynamic (solid curve) regimes. Blue represents opening, and red represents closing. (c) The difference in retardation distance vs the coasting time T_c in frictional (blue circle) and frictionless (red triangle) media. The inset shows the difference in the retardation time between the two strokes vs T_c .

a persistent locomotion occurs in the $-y$ direction, indicating a distinct mechanism in this regime. This direction agrees with the theoretical prediction of a scallop-like swimmer with non-negligible inertia in Newtonian fluids at low Reynolds numbers [40], despite the differences in the nature of forcing and problem setup.

We then scrutinize y/L within a swimming cycle in Fig. 5b, which shows the presence of a retardation time, Δt_R^i , measuring the delay between the instant when the wing rotation direction switches and the instant when the swimmer's translation direction switches, which can be seen after both the opening ($i = o$) and closing ($i = c$) strokes. This is an important indicator for reaching the dynamic regime where the swimmer's inertia is non-negligible. We also measured the corresponding retardation distance, Δy_R^i , as labeled in Fig. 5b.

To further investigate the role of the swimmer's inertia, a second parametric study was performed, varying T and m_s to determine the dependence of Δy_R^i and Δt_R^i on different T_c/T in both frictional and frictionless media. In particular, a frictionless medium can further promote the coasting of the swimmer as the medium deforms more viscously. We show $(|\Delta y_R^c| - |\Delta y_R^o|)/L$ in Fig. 5c, and $(\Delta t_R^c - \Delta t_R^o)/T$ in the inset, both demonstrating that the retardation is more prolonged after the end of the closing stroke. The differences in both retardation distance and time between closing and opening increase with T_c/T , showing a broken symmetry in the inertial coasting after the two reciprocal strokes in the dynamic regime. This is reminiscent of the result in [41] that a floating dumbbell swimmer can propel itself provided that the respective coasting times for its reciprocal strokes are different.

The symmetry breaking mechanism with our scallop swimmer is likely due to the difference in the resistive drag force that opposes the swimmer's translation during coasting after the two respective strokes. This force should be proportional to the swimmer's exposed area in the $x - z$ plane, $2L^2 \sin(\theta)$, which is larger after opening than closing, resulting in the differences in Δy_R^i and Δt_R^i . In Fig. 5c, the increase of these differences with T_c is linear in the frictionless cases, while it saturates in the frictional cases at large T_c , which is possibly due to the formation of the resistive stagnant zones as the swimmer's hubs press into the granular medium during backward translation.

In Fig. 6a, we relate the net swimmer locomotion to the coasting time T_c/T , which reflects both the significance of the swimmer's inertia and the difference between the coasting after the closing and opening strokes. In the frictional cases, a plateau exists at low T_c/T , showing a quasi-static regime as discussed in Sec. III B. At $T_c/T \approx 4 \times 10^{-3}$, a transition to a dynamic regime occurs, showing a decrease in forward locomotion or even the presence of backward locomotion. In the frictionless cases, the same plateau exists in the quasi-static regime, but without forward locomotion. A transition occurs at a similar T_c/T as in the frictional cases, after which the swimmer shows a significant increase in backward locomotion. The difference in $\Delta y/L$ between the frictional and the frictionless cases is related to the ΔN_c as shown in Fig. 4b. After subtracting $\alpha \Delta N_c$ from $\Delta y/L$, the frictional and frictionless data collapse in Fig. 6b, where a master power-law curve fits well to all data, as

$$\Delta y/L = \alpha \Delta N_c - \beta (T_c/T)^n, \quad (8)$$

with the fitted $\beta = 0.24$ and $n = 0.44$, highlighting the respective roles of jamming and swimmer inertia in locomotion. We note that Eq. 8 shall not be extrapolated to $T_c/T = \infty$. As the swimmer keeps increasing its flapping frequency, the granular medium can be strongly agitated and transform into a gaseous behavior, which can possibly lead to a

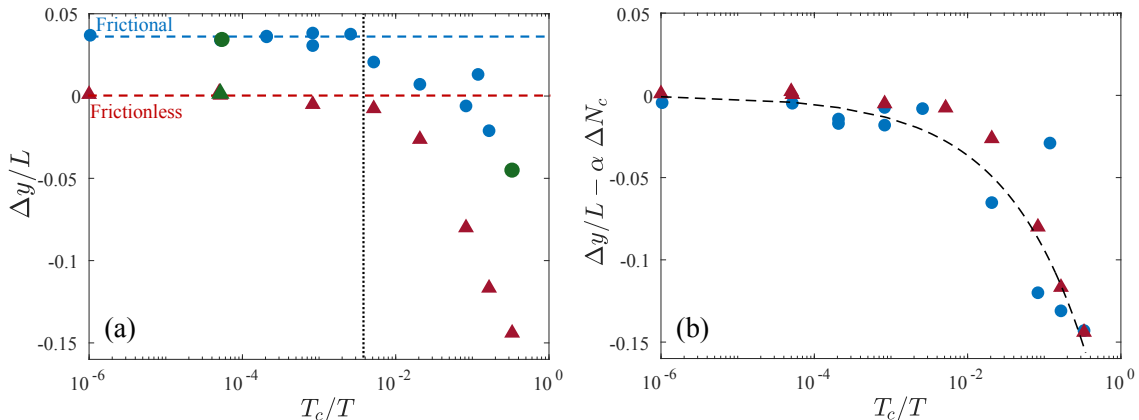


FIG. 6. Quantified relations between swimmer locomotion and inertial coasting. (a) Normalized swimmer displacement vs the coasting time in the frictional medium (blue circles) and the frictionless medium (red triangles). Green circle markers show the simulation cases demonstrated in Fig.5 and the green triangle marker shows the case in frictionless medium demonstrated in Fig.2a. (b) The same plot but with $\Delta y/L$ offset by the jamming contribution $\alpha \Delta N_c$. The dashed curve is the fitted Eq. 8.

decrease in $|\Delta y|$. Similarly, if the swimmer's mass m_s keeps increasing, the swimmer may not be able to gain enough momentum from a stroke, also resulting in decreased $|\Delta y|$.

IV. SUMMARY

For the scallop-inspired granular swimmer, reciprocal swimming is possible via two distinct mechanisms. The first is related to jamming in granular matter. The timescale (or wing rotation θ) over which jammed stagnant zones develop overlaps with the swimming cycle period (or $\theta_c - \theta_o$), inducing hysteresis in the medium response during swimming. The scallop-inspired geometry results in stronger jamming effects towards the end of the closing stroke, further breaking the symmetry in both the structure and the force transmission in the medium between opening and closing. The resulting forward locomotion has a net displacement that is linear to the difference in the number of strong particle contacts between closing and opening. In the dynamic regime where the swimmer's inertia becomes important, the second swimming mechanism relies on the difference between the swimmer's coasting times during closing and opening. The retardation time and distance at the end of closing are larger than those at the end of opening, resulting in an overall backward locomotion, with the net displacement related to the normalized coasting time, T_c/T .

Both mechanisms have essential implications in granular locomotion. The jamming-based mechanism in the quasi-static regime shows potential for designing the geometry of the swimmer to achieve optimal locomotion. Specifically, in Fig. 2a and Fig. 4a, we see that the swimmer can generate larger locomotion if its body has an inert component. As for the inertia-based mechanism, although we currently disabled vertical translation of the swimmer, it is possible that a truly free swimmer with a density higher than the medium can be sustained due to the solid-like nature of granular matter. Future research could explore the effects of varying geometry and mass of swimmers with more degrees of freedom unlocked.

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