

# PCP-GAN: Property-Constrained Pore-scale image reconstruction via conditional Generative Adversarial Networks

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## Abstract

Obtaining truly representative pore-scale images that match bulk formation properties remains a fundamental challenge in subsurface characterization, as natural spatial heterogeneity causes extracted sub-images to deviate significantly from core-measured values. This challenge is compounded by data scarcity, where physical samples are only available at sparse well locations. This study presents a multi-conditional Generative Adversarial Network (cGAN) framework that generates representative pore-scale images with precisely controlled properties, addressing both the representativeness challenge and data availability constraints. The framework was trained on thin section samples from four depths (1879.50 m to 1943.50 m) of a carbonate formation, simultaneously conditioning on porosity values and depth parameters within a single unified model. This approach captures both universal pore network principles and depth-specific geological characteristics, from grainstone fabrics with interparticle-intercrystalline porosity to crystalline textures with anhydrite inclusions. The model achieved exceptional porosity control ( $R^2 = 0.95$ ) across all formations with mean absolute errors of 0.0099–0.0197. Morphological validation confirmed preservation of critical pore network characteristics including average pore radius, specific surface area, and tortuosity, with statistical differences remaining within acceptable geological tolerances, validating geological authenticity. Most significantly, when validated against core sample properties, generated images demonstrated superior representativeness with dual-constraint errors (combined porosity-permeability deviations) of 1.9–11.3% compared to 36.4–578% for randomly extracted real sub-images, showing tighter clustering around target porosity-permeability values. This capability to generate geologically authentic images that better represent bulk formation properties than traditional sampling provides transformative tools for subsurface characterization, particularly valuable for modeling processes in carbon storage, geothermal energy, and groundwater management, where knowing the representative morphology of the pore space is critical for implementing digital rock physics.

**Keywords:** Conditional Generative Adversarial Networks, Digital rock, Porous media, Porosity control, Deep learning, Subsurface imaging, Thin sections

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## 1. Introduction

Subsurface fluid flow analysis constitutes a fundamental component of geological and engineering applications, spanning carbon storage, groundwater management, geothermal energy development, and contaminant remediation. Central to these applications is digital rock analysis, which provides critical insights into complex pore-scale structures governing fluid transport phenomena [1], but faces persistent challenges due to visual pore-scale data scarcity despite advanced imaging technologies (micro-CT, FIB-SEM, laser scanning confocal microscopy) being limited by practical constraints [2, 3].

These data are typically only available at specific depths and wells due to high core extraction costs, technical difficulties, and physical sampling constraints. This scarcity creates significant gaps in our understanding of subsurface formations, particularly when attempting to characterize formations between sampled points. The challenge is further compounded by the heterogeneous nature of porous media, where physical properties can vary signifi-

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cantly within the material, requiring numerous experiments with limited reproducibility [4] and making digital rock analysis and fluid flow simulations particularly difficult.

Traditional approaches to address this data scarcity often rely on interpolation or statistical methods, such as the Joshi-Quiblier-Adler (MJQA) method [5, 6] and simulated annealing techniques [7]. However, these methods may not adequately capture the complex heterogeneity of subsurface formations. For instance, the MJQA method tends to underestimate pore-space connectivity and struggles to replicate the inherent anisotropy of real porous structures [8], while simulated annealing, despite its versatility, is limited by significant computational demands [9].

More sophisticated methods are needed to generate representative pore-scale images for depths lacking physical samples. Deep learning methods demonstrate remarkable capabilities in extracting complex data features and establishing intricate mappings between input and output patterns [10, 11], enabling diverse pattern capture within porous images that makes them particularly valuable for geological microstructure reconstruction [12, 13, 14, 15, 16].

Among these deep learning approaches, Generative Adversarial Networks (GANs) represent a particularly powerful framework, employing an adversarial process between generator and discriminator networks to capture and replicate complex patterns in training data [17]. While GANs have shown promise in geological modeling tasks, including porous media reconstruction and rock parameter prediction [13], most existing studies focus on unconditional generation and do not incorporate specific constraints on attributes that are crucial for accurate subsurface characterization.

Conditional Generative Adversarial Networks (cGANs) extend traditional GAN frameworks by incorporating additional input information, enabling targeted and controllable image synthesis [18]. This conditional approach enables porous media image generation with specific desired properties, offering practical tools for geoscientists and engineers. However, existing applications in subsurface characterization remain limited, particularly regarding comprehensive validation of generated images' geological authenticity and quantitative property preservation.

The development of cGANs has seen significant advancement in material science applications, particularly in controlling specific properties during generation. Early implementations focused on basic property control, with researchers demonstrating the ability to condition generation on fundamental characteristics such as phase distribution and volume fractions. Kishimoto et al. [19] developed a conditional framework for generating three-dimensional porous structures of solid oxide fuel cell (SOFC) anodes with controllable volume fractions, demonstrating the capability to generate realistic microstructures with precise phase distributions, even for compositions outside the training data range.

Building on single-property control, researchers have integrated physical constraints and process parameters into conditional generation frameworks. Yang et al. [20] developed a multi-scale reconstruction method using low-resolution images as conditional input for a cGAN framework, preserving both micro and macro features while maintaining physical accuracy. Tang et al. [21] implemented regression-based conditioning using numerical processing parameters as conditions for predicting microstructures during laser sintering, allowing interpolation between trained conditions and enabling microstructure prediction for unexplored processing parameters.

A distinct approach has emerged focusing on statistical property control and hybrid methodologies. Shams et al. [22] introduced ST-CGAN, a hybrid approach that combines statistical methods with cGANs for 3D porous media reconstruction from 2D images, where the statistical component provides conditional input to cGAN, allowing more realistic reconstructions of homogeneous and heterogeneous samples while improving network stability and efficiency compared to conventional methods.

The most sophisticated implementations achieve simultaneous control of multiple material characteristics. Zhou and Wu [23] introduced a multi-conditional generative adversarial network (MCGAN) for 3D reconstruction of digital rocks guided by multiple petrophysical parameters, incorporating porosity, specific surface area, fractal dimension, and tortuosity through a switch structure that allows selective parameter enabling. Similarly, Zheng and Zhang [24] demonstrated simultaneous control over rock type classification, porosity, and correlation length (both isotropic and anisotropic) while leveraging progressive growing architecture for improved stability.

Despite these advances in conditional generation, existing implementations have not addressed the simultaneous control of porosity with depth-specific geological characteristics within a unified framework, nor have they utilized RGB thin section images that preserve critical petrographic information essential for accurate pore network characterization. Previous studies employing grayscale or binary representations fundamentally lose the ability to

distinguish key geological features such as the differentiation between non-porous anhydrite (white) and porous dolomite (gray) matrices, the identification of grain boundaries crucial for fabric classification (grainstone vs. crystalline), and most importantly, the distinction between interparticle and intraparticle porosity systems that exhibit different flow behaviors [25]. Color-coded thin sections with blue-dyed epoxy enable simultaneous visualization of mineralogical composition, diagenetic alterations, and pore architecture information that cannot be recovered from simplified representations. This integration is particularly challenging as it requires the model to learn both universal pore network principles and formation-specific features that vary with burial depth and diagenetic history, while preserving these complex color-encoded geological relationships.

This study develops a multi-conditional GAN framework that generates representative pore-scale images from sparse petrography data. By utilizing multiple thin section images from various depths as training data and simultaneously conditioning on both porosity values and sample depths, our unified network captures depth-specific geological characteristics alongside porosity-dependent features. This dual conditioning enables the extraction of precise spatial relationships from the multi-dimensional conditional information while maintaining geological authenticity across different formations.

This work extends beyond methodological advancement to practical applications in subsurface characterization. By generating representative images for specific porosity values at trained depths, the framework provides valuable insight into complex subsurface formation heterogeneity. Most significantly, our approach demonstrates the ability to generate images that better represent bulk formation properties than randomly extracted real sub-images, achieving errors of 1.9-11.3% compared to 36.4-578% for real samples. This capability to produce images more representative of core-measured properties than naturally heterogeneous sub-samples addresses a fundamental challenge in digital rock analysis. The ability to produce geologically authentic images with precisely controlled properties directly enhances geological formation analysis, fluid flow simulation accuracy, and decision-making processes across various geological applications.

## 2. Methodology

### 2.1. Data Description and Preparation

This section describes the dataset and data preparation process for training the cGAN model. Our goal is to train the cGAN to generate realistic, colour-coded porous medium images with specific characteristics, effectively capturing the complex structure of rock porosity as revealed through specialized imaging techniques.

In the current study, the dataset for training the developed cGAN model was prepared from petrography images (Figure 1) comprising four distinct sets of thin-section samples from a carbonate formation [26]. These samples are designated as Sample 1 (1879.50 m), Sample 2 (1881.90 m), Sample 3 (1918.50 m), and Sample 4 (1943.50 m), where the values in parentheses indicate the sampling depths within the carbonate formation. The thin section images consist of dolomite, limestone, and anhydrite lithology. Carbonate rocks exhibit complex pore networks resulting from various geological processes [27], providing geologically challenging training data. In these images, the blue areas represent porous regions that result from the blue-dyed epoxy resin injected during the preparation of the sample to differentiate the pore space from rock minerals.

Table 1 presents the comprehensive geological and petrophysical characteristics of the four carbonate samples used in this study. The samples exhibit diverse geological features ranging from grainstone to crystalline fabrics, with core porosity values spanning from 10.58% to 24.77% and permeability ranging from 12.09 to 181.44 mD. This diversity in pore structure characteristics and lithological properties provides an ideal dataset for training the multi-conditional GAN model across different geological formations within the carbonate sequence.

Throughout this manuscript, samples are referenced by their numerical designation (1-4) with geological and depth details available in Table 1.

Table 1: Geological and petrophysical characteristics of carbonate samples used for cGAN training.

| Sample Number                                                              | Depth (m) | Core Porosity (%) | Core Permeability (mD) | Fabric Type | Main Pore Type | Lithology          |
|----------------------------------------------------------------------------|-----------|-------------------|------------------------|-------------|----------------|--------------------|
| 1                                                                          | 1879.50   | 15.73             | 33.64                  | Grainstone  | IP-IT          | Dolomite           |
| 2                                                                          | 1881.90   | 24.77             | 181.44                 | Grainstone  | IP-IT          | Dolomite           |
| 3                                                                          | 1918.50   | 10.58             | 13.39                  | Crystalline | IT             | Dolomite-Anhydrite |
| 4                                                                          | 1943.50   | 13.32             | 12.09                  | Crystalline | IT             | Dolomite           |
| <b>Note:</b> IP-IT = Interparticle-Intercrystalline; IT = Intercrystalline |           |                   |                        |             |                |                    |

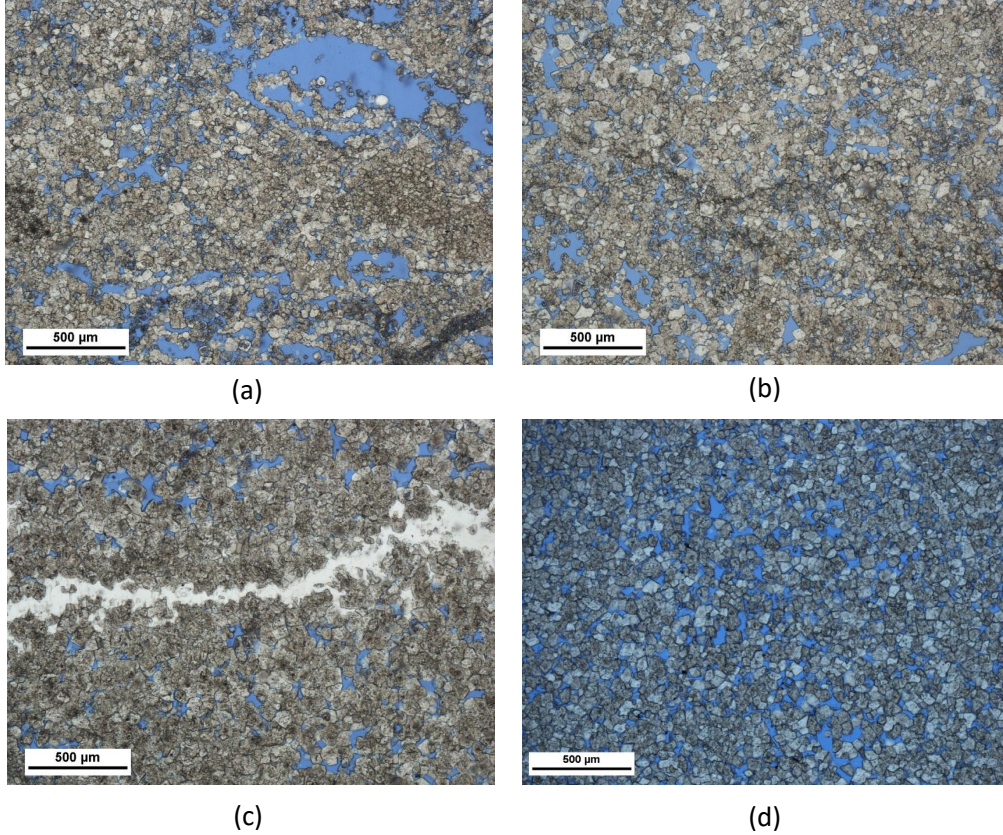


Figure 1: Examples of thin-section images (768×516 pixels) from the carbonate formation from different core samples: (a) Sample 1 (1879.50 m), (b) Sample 2 (1881.90 m), (c) Sample 3 (1918.50 m), and (d) Sample 4 (1943.50 m). Blue areas represent porous regions visualized by blue-dyed epoxy resin, with each image demonstrating the characteristic pore structure variability at different depths.

Prior to model training, a Representative Elementary Volume (REV) analysis was conducted to determine the optimal sub-image size balancing statistical representativeness with porosity variability preservation for cGAN training.

Sub-image sizes ranging from 64×64 to 516×516 (maximum extractable size) pixels were systematically evaluated using approximately 10 original images (768×516 pixels) extracted from the core sample at each depth, with porosity standard deviation analyzed across all samples following established REV determination methodologies [28].

The analysis incorporated these multiple images from each core sample to capture intra-sample spatial heterogeneity within individual formations. This approach accounts for the natural heterogeneity present in different spatial positions within individual core samples [29].

The statistical convergence analysis revealed that while larger sub-images achieve improved mean porosity convergence, they simultaneously reduce porosity variability within individual samples, presenting a critical consideration for porosity-conditioned GANs that require diverse training distributions. To address this trade-off, a porosity standard deviation threshold of  $\sigma = 0.06$  was established through systematic analysis of the convergence

behavior across all samples. At  $480 \times 480$  pixels, this threshold was achieved across all geological formations, providing an optimal balance between statistical representativeness and porosity diversity necessary for effective conditional generation (Figure 2).

This selection ensures that the extracted training patches capture sufficient spatial heterogeneity for pore network characterization while enabling subsequent generation of representative images conditioned on measured core sample porosity values. The multi-image approach accounts for natural spatial variability within individual core samples, addressing both statistical convergence requirements and the preservation of intra-sample geological heterogeneity essential for practical applications in subsurface characterization.

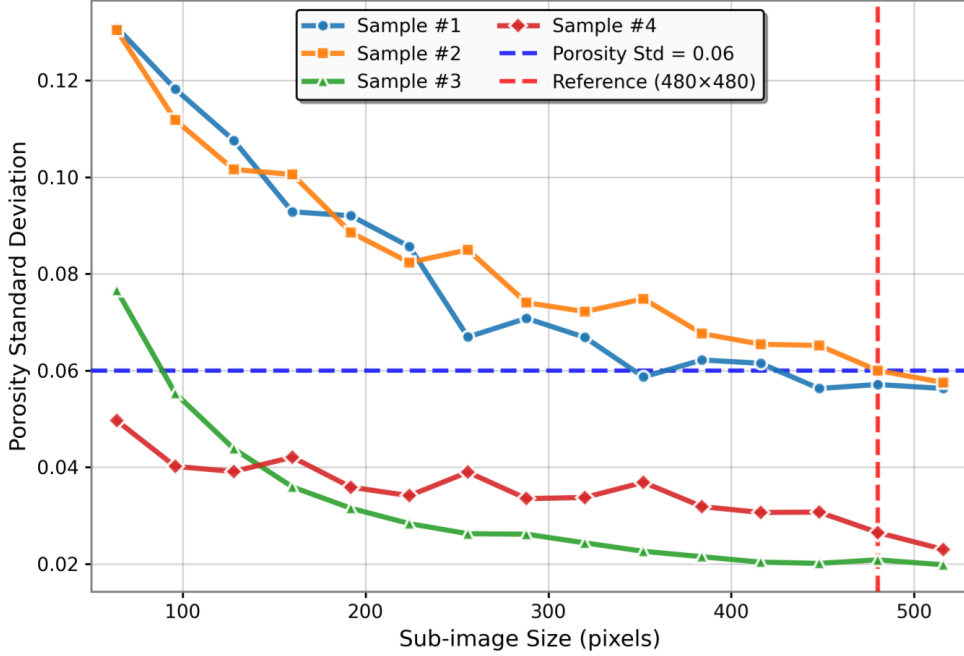


Figure 2: Representative Elementary Volume (REV) analysis demonstrating porosity standard deviation analysis showing variability reduction with increasing sub-image size for four carbonate samples, with the established threshold ( $\sigma = 0.06$ ) for maintaining adequate variability in cGAN training. The selected  $480 \times 480$  pixel size (red dashed line) optimizes both statistical representativeness and porosity diversity essential for effective conditional generation.

### 2.1.1. Automated Porosity Quantification via Enhanced U-Net

Accurate porosity quantification forms the foundation of our conditional generation framework, requiring precise segmentation to distinguish pore spaces from solid matrix in thin-section images to establish reliable porosity labels for training. To achieve this, we employed an enhanced U-Net architecture, a convolutional neural network originally developed for biomedical image segmentation [30]. U-Net’s encoder-decoder structure with skip connections preserves fine-grained spatial information while capturing semantic context, making it particularly suitable for pore segmentation where precise boundary delineation is critical. The architecture’s symmetric design allows it to work effectively with limited training data by leveraging both contracting (encoder) and expanding (decoder) paths to learn robust feature representations.

We enhanced the standard U-Net by incorporating attention gates [31] and deep supervision mechanisms to improve performance on complex carbonate pore structures. The attention gates selectively emphasize relevant features at skip connections, improving boundary delineation in complex pore structures characteristic of carbonate formations where boundaries between pore and solid phases can be indistinct due to partial dolomitization and varying crystal sizes. The network processes  $480 \times 480 \times 3$  RGB thin section images to produce corresponding binary masks, where the pore spaces are distinguished from the solid matrix material.

Training utilized a hybrid loss function combining Dice coefficient and binary cross-entropy (BCE) with equal weighting (0.5:0.5), addressing the inherent class imbalance between pore and solid phases while ensuring both accurate region overlap and pixel-wise classification. The segmentation model achieved high performance across

the test dataset of 938 samples, with mean Dice coefficient of  $0.952 \pm 0.041$ , IoU of  $0.910 \pm 0.071$ , and accuracy exceeding  $0.956 \pm 0.032$ . Porosity values calculated from the predicted masks demonstrated strong agreement with ground truth measurements, exhibiting mean absolute error of 0.018, confirming the reliability of automated porosity quantification. This pre-trained segmentation model served dual roles in our framework, calculating porosity labels for training data preparation and validating the porosity of generated images, ensuring consistency throughout the conditional generation pipeline.

Following automated porosity quantification, the data preparation process involved extracting patches of size  $480 \times 480$  pixels from the original images across the four samples (Table 1). For each sample, we independently analyzed and categorized the extracted patches into 10 distinct porosity classes based on their specific porosity distribution (Table 2), using the porosity values computed by the U-Net segmentation model.

Table 2: Porosity value ranges for each class from different samples.

| Sample No. | Porosity Class  |                 |                 |                 |                 |                 |                 |                 |                 |                 |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | 0               | 1               | 2               | 3               | 4               | 5               | 6               | 7               | 8               | 9               |
| 1          | 0.0488 - 0.0727 | 0.0727 - 0.0962 | 0.0963 - 0.1197 | 0.1200 - 0.1433 | 0.1437 - 0.1668 | 0.1672 - 0.1899 | 0.1911 - 0.2137 | 0.2140 - 0.2371 | 0.2377 - 0.2609 | 0.2615 - 0.2867 |
| 2          | 0.0867 - 0.1103 | 0.1103 - 0.1339 | 0.1349 - 0.1574 | 0.1578 - 0.1805 | 0.1812 - 0.2041 | 0.2057 - 0.2284 | 0.2287 - 0.2518 | 0.2526 - 0.2756 | 0.2757 - 0.2993 | 0.3001 - 0.3229 |
| 3          | 0.0424 - 0.0495 | 0.0497 - 0.0567 | 0.0568 - 0.0639 | 0.0640 - 0.0711 | 0.0712 - 0.0783 | 0.0784 - 0.0852 | 0.0857 - 0.0927 | 0.0927 - 0.0997 | 0.0998 - 0.1063 | 0.1072 - 0.1142 |
| 4          | 0.0561 - 0.0650 | 0.0652 - 0.0740 | 0.0740 - 0.0829 | 0.0830 - 0.0918 | 0.0919 - 0.1005 | 0.1009 - 0.1095 | -               | 0.1192 - 0.1276 | 0.1277 - 0.1365 | 0.1367 - 0.1461 |

Addressing the inherent unbalanced data distribution challenges in each sample dataset, we implemented a balancing strategy to ensure consistent model training across all porosity classes. Our analysis revealed significant variations in the number of samples across different porosity classes within each sample (Figure 3.a), which could lead to biased learning toward overrepresented classes.

To create balanced datasets, we established a target of 160 images per porosity class for each sample, strategically excluding classes with fewer than 20 samples to prevent overfitting. For underrepresented classes with 20-160 samples, we employed data augmentation through geometric transformations (flips and rotations) and minor noise additions ( $\pm 2$  pixel values) to create realistic variations while preserving pore structure characteristics. This approach transformed the initially skewed distributions into uniform datasets of approximately 1,600 images per sample, with each viable porosity class containing exactly 160 images (Figure 3.b). This balancing strategy was crucial for stable cGAN training, ensuring equal exposure to all porosity values and preventing the model from memorizing specific examples rather than learning generalizable patterns.

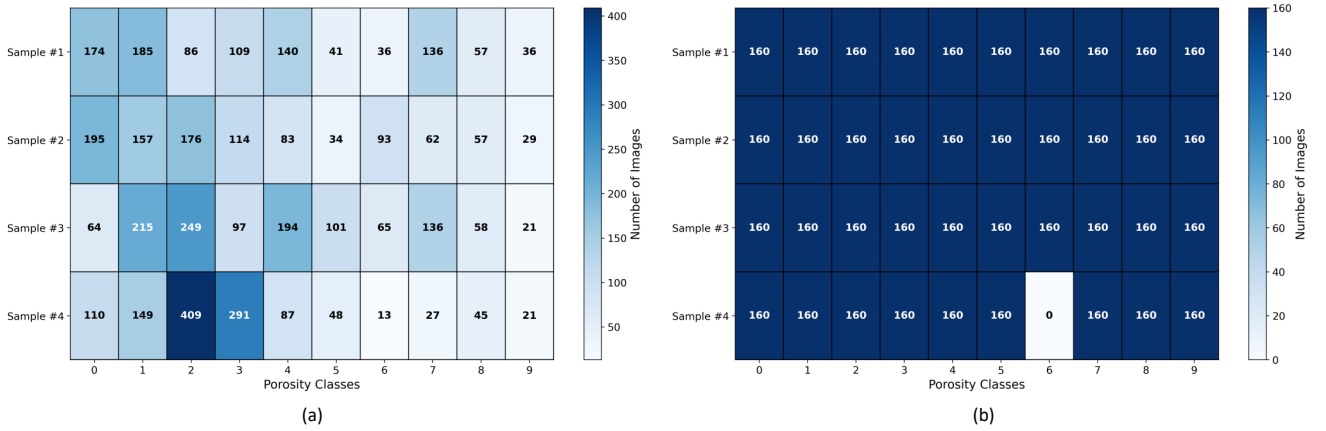


Figure 3: Distribution of sub-images across sample and porosity class combinations: (a) Initial unbalanced distribution showing significant variations in sub-image counts across different porosity classes within each sample, and (b) Balanced distribution after data augmentation, with each viable porosity class containing exactly 160 images per class.

Figure 4 presents a comprehensive visualization of thin section images organized by sample and porosity class. Each cell in the visualization displays a representative example from the corresponding sample-porosity class, illustrating the structural heterogeneity present in our dataset, with notable variations in pore characteristics both across different samples and within the porosity spectrum at each depth.

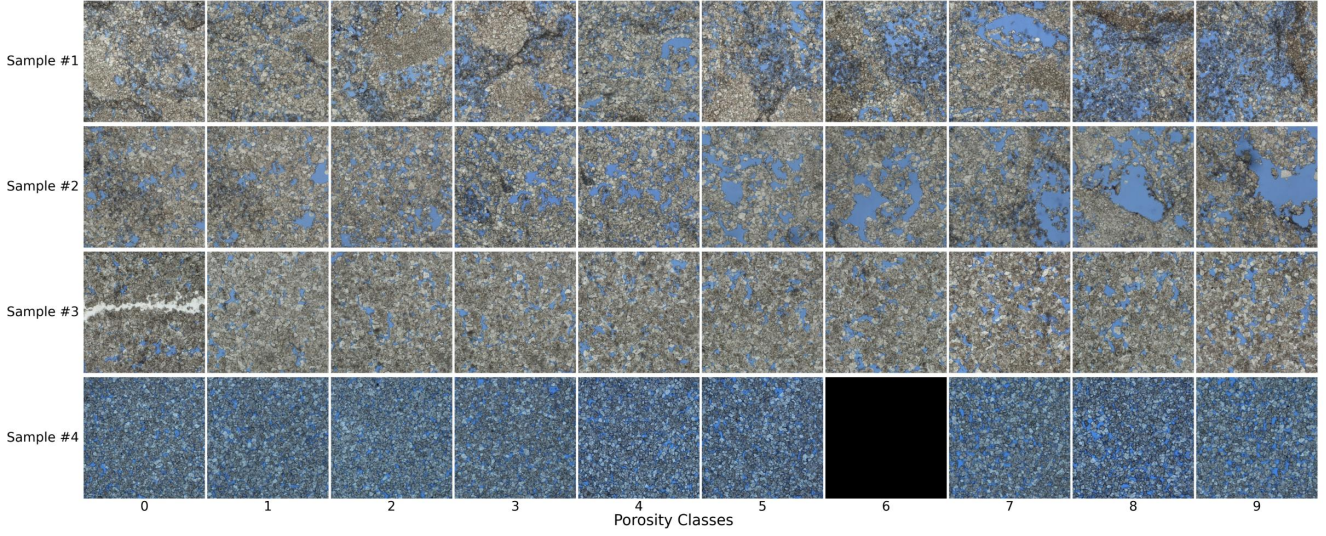


Figure 4: Visualization of extracted sub-images ( $480 \times 480$  pixels) across different samples and porosity classes. Each row represents a different sample in the carbonate formation (Sample 1 to Sample 4), while columns represent porosity classes (0-9).

## 2.2. Theoretical Framework

### 2.2.1. Generative Adversarial Networks (GANs)

Generative Adversarial Neural Networks (GANs), introduced by Goodfellow et al. [17], represent a revolutionary framework for generative modeling. Inspired by game theory's zero-sum games, GANs pit two neural networks against each other in an adversarial training process, a generator network (G) and a discriminator network (D). The generator learns to create synthetic samples that mimic the underlying data distribution, while the discriminator acts as a classifier, attempting to distinguish between real and generated samples. This competition drives both networks to improve until reaching a Nash equilibrium, where neither network can unilaterally gain an advantage.

In the GAN architecture, the generator takes random noise ( $z$ ) as input and transforms it into synthetic samples ( $G(z)$ ) that aim to match the real data distribution ( $P_{data}$ ). The discriminator evaluates each sample, outputting a probability between 0 and 1 to indicate its assessment of the sample's authenticity. A visual representation of this architecture is provided in Figure 5.

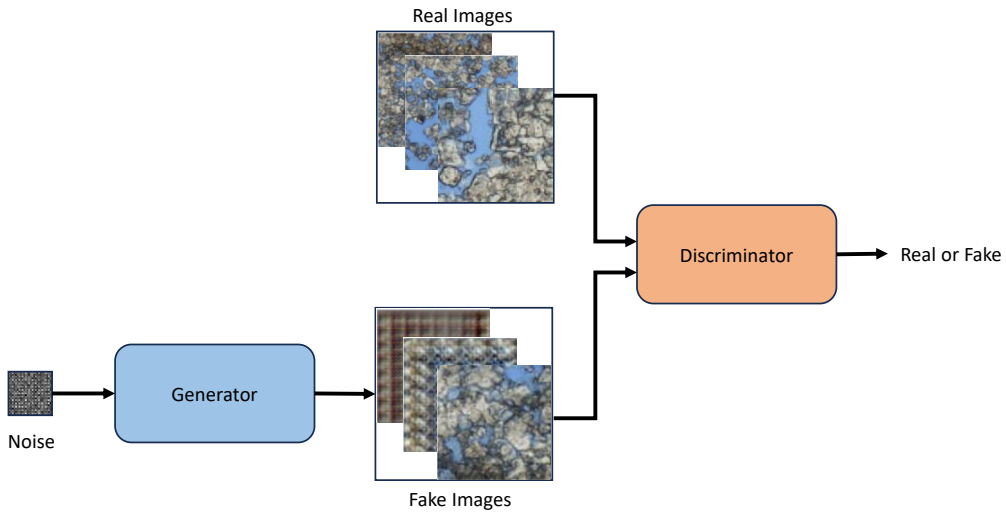


Figure 5: Schematic architecture of a standard Generative Adversarial Network (GAN) showing the adversarial training process. The generator network transforms random noise vectors ( $z$ ) into synthetic porous media images, while the discriminator network evaluates both real training images and generator-produced fake images to distinguish their authenticity.

In the original formulation, both networks are typically implemented as multilayer perceptrons (MLPs), forming what is known as adversarial networks. This architecture leverages well-established training techniques, including backpropagation for gradient computation and dropout for regularization. The generator’s MLP transforms random noise through multiple layers to produce synthetic samples, while the discriminator’s MLP processes inputs to output classification probabilities. The training process can be mathematically expressed as a minimization-maximization (min-max) optimization problem:

$$L_{\text{adv}}(\theta, \omega) = \mathbb{E}_{x \sim p_{\text{data}}(x)} [\log(D_{\theta}(x))] + \mathbb{E}_{z \sim p_z(z)} [\log(1 - D_{\theta}(G_{\omega}(z)))] \quad (1)$$

Where  $x$  denotes real data samples,  $p_{\text{data}}(x)$  represents the underlying probability distribution of this real data,  $D_{\theta}(x)$  signifies the probability assigned by the discriminator (with parameters  $\theta$ ) to a data point  $x$  originating from the real data distribution,  $z$  represents the random noise input (often called a latent vector), typically containing values between 0 and 1,  $p_z(z)$  denotes the probability distribution of this noise.

The generator, parameterized by  $\omega$ , transforms noise into synthetic data, denoted by  $G_{\omega}(z)$ , the notations  $E_{x \sim p_{\text{data}}(x)}$  and  $E_{z \sim p_z(z)}$  represent mathematical expectations. The first refers to the expectation when the training data  $x$  for the discriminator aligns with the real data distribution  $p_{\text{data}}$ , and the second signifies the expectation when the generator’s input vector  $z$  conforms to the chosen noise distribution  $p_z$ .

The training objective revolves around the adversarial loss term  $L_{\text{adv}}$ . The discriminator aims to maximize this term by outputting 1 for real data and 0 for generated samples, while the generator seeks to minimize it by producing samples that fool the discriminator.

After successful training, generating new samples becomes straightforward, requiring only forward propagation through the generator network with random noise as input. This efficient sampling procedure eliminates the need for complex inference methods or iterative processes like Markov chains, which are often required in other generative models [17].

### 2.2.2. Conditional Generative Adversarial Neural Networks (cGANs)

Traditional generative models lack output control, so the GAN framework extends to conditional GANs (cGANs) that incorporate additional input parameters to guide generation and produce samples with specified properties [18].

The complete cGAN workflow incorporates automated porosity extraction through the enhanced U-Net alongside the adversarial training process (Figure 6). Both the generator and discriminator are implemented as fully convolutional neural networks to effectively process spatial image data. The generator  $G$  processes both random noise  $z$  and the condition vectors (porosity values extracted via U-Net and depth labels). Subsequently, the discriminator  $D$  evaluates the authenticity of samples by considering both the input images (real or generated) and their corresponding conditional vectors, thereby maintaining consistency between the porosity quantification and generation processes throughout the network.

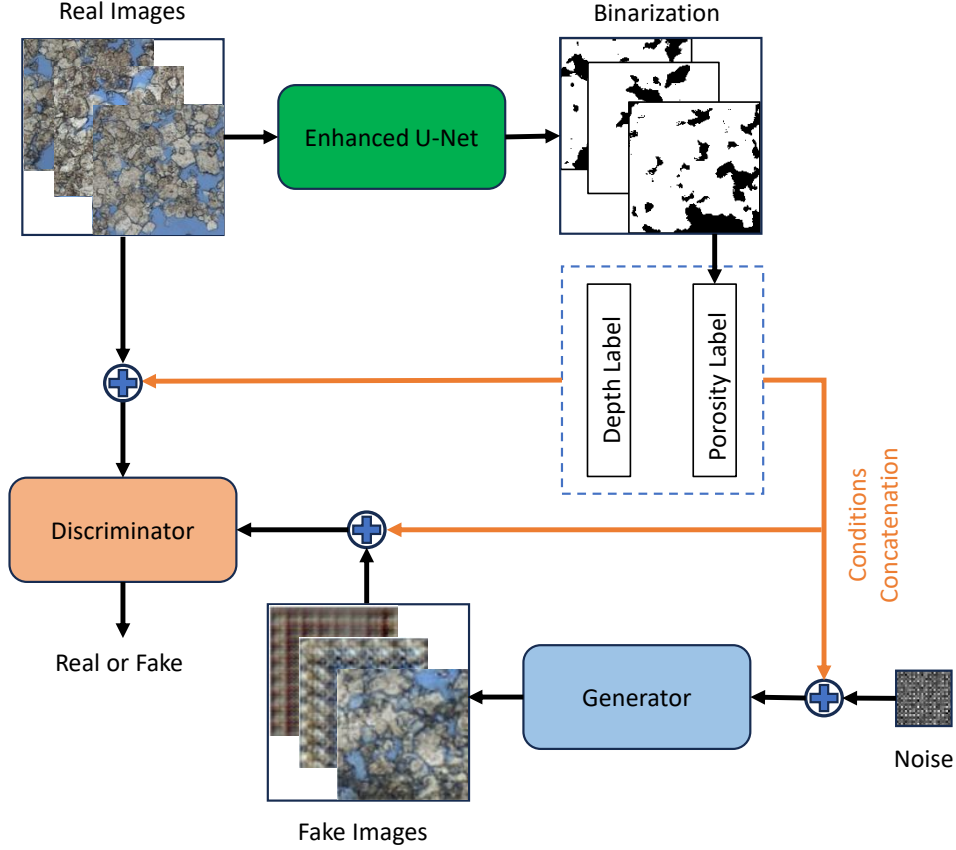


Figure 6: Complete workflow of the multi-conditional GAN architecture for porous media generation. Real thin section images are processed through an Enhanced U-Net for binarization to extract porosity labels, which along with depth labels serve as conditional inputs. The Generator network receives random noise combined with these conditional data (porosity and depth labels) to produce synthetic porous media images. The Discriminator network evaluates both real images and generator-produced images, each paired with their corresponding conditional data (porosity and depth labels), to determine authenticity. The enhanced U-Net ensures consistent porosity quantification throughout the training and generation process.

The adversarial loss function for the cGAN becomes:

$$L_{adv}(\theta, \omega) = \mathbb{E}_{x \sim p_{data}(x)} [\log D_{\theta}(x|c)] + \mathbb{E}_{z \sim p_z(z)} [\log(1 - D_{\theta}(G_{\omega}(z|c)))] \quad (2)$$

where  $c$  represents the conditional parameters.

For this application, the networks are conditioned simultaneously on both porosity ( $\phi$ ) and sample depths. The unified multi-conditional architecture enables the model to learn both universal pore network principles and depth-specific geological characteristics within a single framework. This approach allows the network to generate images that capture the unique structural patterns characteristic of particular depths while maintaining precise control over porosity parameters. The methodology is detailed in Algorithm 1 and Algorithm 2, describing the unified training approach that processes data from all depths simultaneously.

The preprocessing phase (Algorithm 1) involves extracting patches ( $480 \times 480$  pixels) from petrography images, calculating relevant porosity using pre-trained U-Net, and balancing the dataset across different porosity classes for each sample using the data augmentation techniques described in Section 2.1.1. This systematic preprocessing ensures that each depth-specific dataset is properly prepared for training with balanced representation across all viable porosity classes.

The training process (Algorithm 2) adapts the standard GAN training procedure to incorporate dual conditioning on porosity and sample depth. The algorithm iteratively trains the generator and discriminator networks using minibatch stochastic gradient descent on the unified dataset combining all depths. During each iteration, the networks process real and generated images along with their corresponding conditioning vectors (porosity values and depth labels), updating parameters by minimizing their respective loss functions as detailed below.

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**Algorithm 1** Data Preprocessing for Multi-Conditional Porous Media Analysis

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```
1: procedure PREPROCESS DATASET
2:   Load petrography images from all depths
3:   for each depth subset do
4:     for each original image do
5:       Extract patches of size  $480 \times 480$  pixels
6:       for each patch do
7:         Calculate porosity using pre-trained Enhanced U-Net
8:         Assign depth label to patch
9:       end for
10:    end for
11:    Categorize patches into  $n\_classes\_porosity$  based on porosity values
12:    Balance dataset by data augmentation for underrepresented classes
13:  end for
14:  Combine all depth datasets into unified training set
15:  Create dual conditioning vectors (porosity, depth)
16: end procedure
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**Algorithm 2** Multi-Conditional GAN Training

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```
1: procedure TRAIN MULTI-CONDITIONAL GAN
2:   Initialize generator and discriminator networks
3:   Load unified dataset from all depths
4:   for number of training epochs do
5:     for number of batches per epoch do
6:       Sample minibatch of  $m/2$  real patches  $\{x^{(1)}, \dots, x^{(m/2)}\}$  from unified dataset
7:       Get corresponding porosity values  $\{\phi^{(1)}, \dots, \phi^{(m/2)}\}$ 
8:       Get corresponding depth labels  $\{d^{(1)}, \dots, d^{(m/2)}\}$ 
9:       Sample minibatch of  $m/2$  noise samples  $\{z^{(1)}, \dots, z^{(m/2)}\}$  from  $p_z(z)$ 
10:      Generate fake samples:  $\tilde{x}^{(i)} = G(z^{(i)}, \phi^{(i)}, d^{(i)})$  ( $i = 1 : m/2$ )
11:      Update discriminator by minimizing:
12:       $L_D = -\frac{1}{m/2} \sum_{i=1}^{m/2} [\log D(x^{(i)}, \phi^{(i)}, d^{(i)}) + \log(1 - D(\tilde{x}^{(i)}, \phi^{(i)}, d^{(i)}))]$  (based on Eq. 2)
13:      Sample minibatch of  $m$  noise samples  $\{z^{(1)}, \dots, z^{(m)}\}$  from  $p_z(z)$ 
14:      Sample minibatch of  $m$  porosity values  $\{\phi^{(1)}, \dots, \phi^{(m)}\}$ 
15:      Sample minibatch of  $m$  depth labels  $\{d^{(1)}, \dots, d^{(m)}\}$ 
16:      Update generator by minimizing:
17:       $L_G = -\frac{1}{m} \sum_{i=1}^m \log(D(G(z^{(i)}, \phi^{(i)}, d^{(i)}), \phi^{(i)}, d^{(i)}))$  (based on Eq. 2)
18:    end for
19:  end for
20:  Evaluate generator on test porosity-depth combinations
21:  Save trained unified model
22: end procedure
```

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The architectures of the generator and discriminator networks in the developed cGAN model are detailed in Tables 3 and 4, respectively. Both networks are implemented as fully convolutional neural networks with conditional label inputs. The generator’s architecture (Table 3 and Figure 7.A) processes three inputs comprising a 100-dimensional latent vector ( $z$ ) of uniform random values, continuous porosity ( $\phi$ ) labels, and discrete depth labels for multi-conditional generation.

The generator architecture processes multiple input streams. A 100-dimensional noise vector is transformed through a dense layer to produce 512 feature channels, which are then concatenated with depth conditions (4-dimensional one-hot encoding for the four depths, where each depth is represented as a binary vector) and porosity

Table 3: Multi-Condition Generator Architecture

| Layer Type                                                                                   | Output Shape                              | Filters             | Kernel/Stride                         | Activation                   |
|----------------------------------------------------------------------------------------------|-------------------------------------------|---------------------|---------------------------------------|------------------------------|
| Input (Noise + Conditions*)                                                                  | $30 \times 30 \times 517$                 | —                   | —                                     | —                            |
| Conv2DTranspose Block 1                                                                      | $30 \times 30 \times 256$                 | 256                 | $3 \times 3 / 1 \times 1$             | LeakyReLU                    |
| Conv2DTranspose Block 2-5                                                                    | $60 \times 60 \rightarrow 480 \times 480$ | $128 \rightarrow 3$ | $3 \times 3, 5 \times 5 / 2 \times 2$ | LeakyReLU $\rightarrow$ tanh |
| * Conditions: Depth $d$ (4-dim one-hot) + Porosity $\phi$ (continuous), spatially replicated |                                           |                     |                                       |                              |

conditions (1-dimensional continuous values). This concatenation results in 517 total channels ( $512 + 4 + 1 = 517$ ) at the initial  $30 \times 30$  spatial resolution. The one-hot depth vectors and porosity values are spatially replicated across all spatial locations to maintain consistent dimensionality for the subsequent transposed convolutional layers.

Table 4: Multi-Condition Discriminator Architecture

| Layer Type                                                                                 | Output Shape                                | Filters              | Kernel/Stride             | Activation | Dropout |
|--------------------------------------------------------------------------------------------|---------------------------------------------|----------------------|---------------------------|------------|---------|
| Input (Image + Conditions*)                                                                | $480 \times 480 \times 8$                   | —                    | —                         | —          | —       |
| Conv2D Block 1-2                                                                           | $240 \times 240 \rightarrow 120 \times 120$ | $64 \rightarrow 128$ | $5 \times 5 / 2 \times 2$ | LeakyReLU  | 0.3     |
| Conv2D Block 3-5                                                                           | $60 \times 60 \rightarrow 15 \times 15$     | 256                  | $3 \times 3 / 2 \times 2$ | LeakyReLU  | 0.3     |
| Flatten + Dense                                                                            | 1                                           | —                    | —                         | Sigmoid    | —       |
| * Conditions: Depth $d$ (4-dim) + Porosity $\phi$ (1-dim) concatenated with image channels |                                             |                      |                           |            |         |

The discriminator receives  $480 \times 480 \times 8$  input channels comprising the 3-channel RGB image concatenated with spatially replicated condition information consisting of 4 channels for depth (one-hot encoded) and 1 channel for porosity values ( $3 + 4 + 1 = 8$  total channels), as illustrated in Table 4 and Figure 7.b.

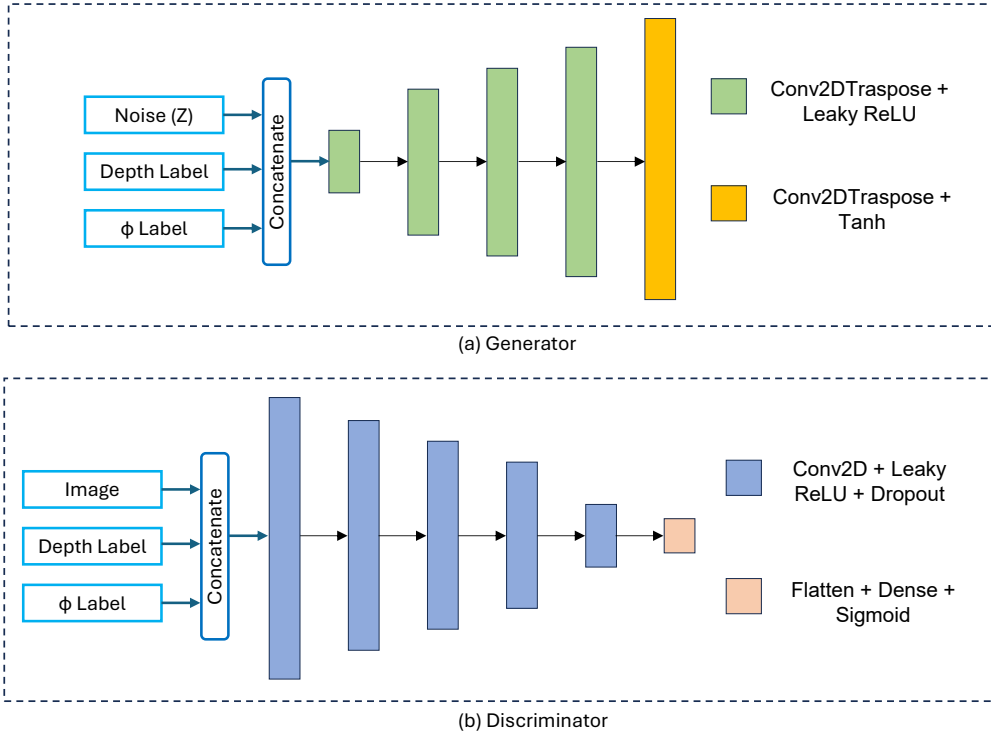


Figure 7: Detailed network architectures of the cGAN components: (a) Conditional Generator processes a 100-dimensional noise vector ( $z$ ) through a dense layer, combines it with tiled porosity labels ( $\phi$ ), then applies five transposed convolutional layers with LeakyReLU activations and final tanh activation to produce  $480 \times 480 \times 3$  RGB porous media images. (b) Conditional Discriminator concatenates input images with porosity condition maps and processes them through five convolutional layers with downsampling, dropout regularization (0.3), and LeakyReLU activations, followed by flattening and sigmoid activation for binary authenticity classification.

The generator processes the latent vector alongside the porosity condition values through a series of transposed

convolutional layers, progressively upsampling to produce the final  $480 \times 480 \times 3$  output images. Similarly, the discriminator evaluates the input images (real or generated) along with both porosity and depth conditioning information through convolutional layers with downsampling, ultimately producing a binary classification output.

Binary Cross Entropy (BCE) loss is utilized for training both networks, providing an appropriate objective function for the discriminator’s binary classification task and the generator’s goal of producing convincing samples, following the standard adversarial training approach established by Goodfellow et al. [17, 32]. The discriminator aims to maximize the BCE loss by correctly identifying real and fake samples (Equation 1), while the generator aims to minimize this loss by creating samples that the discriminator misclassifies as real. For conditional generation, this extends to the conditional adversarial loss (Equation 2).

### 2.3. Baseline Architectures for Comparative Analysis

To validate the architectural design choices of our multi-conditional GAN framework, we implemented two alternative architectures with systematically reduced parameter counts while maintaining identical training conditions and conditioning mechanisms. This comparative analysis follows established practices in deep learning research for demonstrating the relationship between model capacity and performance in complex generative tasks [10], enabling quantitative assessment of architectural necessity versus potential over-engineering.

The baseline architectures were designed with distinct parameter reduction strategies to explore different approaches to computational efficiency. Model A implements architectural innovation through increased depth while reducing width, whereas Model B employs proportional scaling reduction across all layers. All models utilize identical dual-conditioning mechanisms (porosity and depth), training hyperparameters (200 epochs, batch size 16, polynomial learning rate decay from  $2.0 \times 10^{-4}$  to  $2.0 \times 10^{-6}$ ), and data preprocessing pipelines to ensure fair comparative evaluation.

#### 2.3.1. Model A: Depth-Optimized Architecture (24% Parameter Reduction)

Model A explores whether architectural sophistication can compensate for reduced parameters through increased depth rather than width reduction. This approach maintains feature learning capacity through additional processing layers while reducing the initial feature space, resulting in approximately 38M parameters with a 24% reduction from the original 50M parameter architecture.

The generator architecture (Table 5) reduces the initial dense transformation from 512 to 384 feature channels while introducing an additional transposed convolutional layer for enhanced feature refinement. The six-layer structure includes an initial feature refinement layer at  $30 \times 30$  resolution, followed by progressive upsampling through [256, 128, 96, 64, 32, 3] channels, enabling more sophisticated feature transformation despite reduced initial capacity.

Table 5: Model A Generator Architecture (Depth-Optimized)

| Layer Type                                                                                 | Output Shape                              | Filters             | Kernel/Stride                         | Activation                   |
|--------------------------------------------------------------------------------------------|-------------------------------------------|---------------------|---------------------------------------|------------------------------|
| Input (Noise + Conditions)                                                                 | $30 \times 30 \times 389$                 | –                   | –                                     | –                            |
| Conv2DTranspose Block 1                                                                    | $30 \times 30 \times 256$                 | 256                 | $3 \times 3 / 1 \times 1$             | LeakyReLU                    |
| Conv2DTranspose Blocks 2-6                                                                 | $60 \times 60 \rightarrow 480 \times 480$ | $128 \rightarrow 3$ | $3 \times 3, 5 \times 5 / 2 \times 2$ | LeakyReLU $\rightarrow$ tanh |
| <i>Key Innovation: Additional refinement layer + reduced initial capacity (384 vs 512)</i> |                                           |                     |                                       |                              |

The discriminator architecture implements moderate filter reduction [56, 112, 224, 224, 224] compared to the original [64, 128, 256, 256, 256], maintaining discrimination capability while achieving computational efficiency. This balanced approach tests whether architectural innovation can maintain quality with moderate parameter reduction.

#### 2.3.2. Model B: Proportional Reduction Architecture (50% Parameter Reduction)

Model B implements systematic parameter reduction through proportional scaling of all network components, resulting in approximately 25M parameters with a 50% reduction from the original architecture. This approach maintains identical architectural depth while testing the lower bounds of parameter requirements for effective geological feature learning.

Table 6: Model B Generator Architecture (Proportional Reduction)

| Layer Type                                                        | Output Shape                              | Filters            | Kernel/Stride                         | Activation                   |
|-------------------------------------------------------------------|-------------------------------------------|--------------------|---------------------------------------|------------------------------|
| Input (Noise + Conditions)                                        | $30 \times 30 \times 261$                 | –                  | –                                     | –                            |
| Conv2DTranspose Block 1                                           | $30 \times 30 \times 192$                 | 192                | $3 \times 3 / 1 \times 1$             | LeakyReLU                    |
| Conv2DTranspose Blocks 2-5                                        | $60 \times 60 \rightarrow 480 \times 480$ | $96 \rightarrow 3$ | $3 \times 3, 5 \times 5 / 2 \times 2$ | LeakyReLU $\rightarrow$ tanh |
| <i>Strategy: Proportional scaling - all layers reduced by 50%</i> |                                           |                    |                                       |                              |

Table 7: Model B Discriminator Architecture (Proportional Reduction)

| Layer Type                                                                                                              | Output Shape                                | Filters             | Kernel/Stride             | Activation | Dropout |
|-------------------------------------------------------------------------------------------------------------------------|---------------------------------------------|---------------------|---------------------------|------------|---------|
| Input (Image + Conditions*)                                                                                             | $480 \times 480 \times 8$                   | –                   | –                         | –          | –       |
| Conv2D Block 1-2                                                                                                        | $240 \times 240 \rightarrow 120 \times 120$ | $48 \rightarrow 96$ | $5 \times 5 / 2 \times 2$ | LeakyReLU  | 0.3     |
| Conv2D Block 3-5                                                                                                        | $60 \times 60 \rightarrow 15 \times 15$     | 192                 | $3 \times 3 / 2 \times 2$ | LeakyReLU  | 0.3     |
| Flatten + Dense                                                                                                         | 1                                           | –                   | –                         | Sigmoid    | –       |
| <i>* Conditions: Depth <math>d</math> (4-dim) + Porosity <math>\phi</math> (1-dim) concatenated with image channels</i> |                                             |                     |                           |            |         |

Model B processes the 100-dimensional noise vector through a dense layer producing 256 feature channels (vs. 512 in original), with subsequent layers scaled proportionally as [192, 96, 48, 24, 3]. The discriminator employs correspondingly reduced configurations [48, 96, 192, 192, 192], testing whether the complex geological modeling task can be accomplished with substantially reduced network capacity.

### 2.3.3. Comparative Training Protocol

All three architectures underwent identical training protocols using the unified multi-conditional dataset across all four geological depths. Training used Adam optimization with  $\beta_1 = 0.5$ ,  $\beta_2 = 0.999$ , and polynomial learning rate decay from  $2.0 \times 10^{-4}$  to  $2.0 \times 10^{-6}$  for both generator and discriminator networks. Binary cross-entropy loss functions were utilized following standard adversarial training procedures.

The comparative evaluation focuses on examining (1) porosity control accuracy through correlation analysis, (2) visual quality assessment of generated geological structures, and (3) morphological preservation of pore network characteristics. All experiments were conducted on the University of Leeds Aire High Performance Computing (HPC) facility, utilizing NVIDIA A2 GPU with 15 GB memory, AMD EPYC 9354P 32-Core Processor, and TensorFlow 2.6.0 framework.

This systematic comparison enables quantitative assessment of the relationship between architectural complexity and geological modeling performance, providing insights into the computational requirements for effective subsurface characterization while validating the design choices implemented in the original architecture.

## 3. Results and Discussion

### 3.1. Model Training and Performance

#### 3.1.1. Training Convergence and Loss Evolution

The model was trained for 200 epochs using the unified approach described in Section 2.2.2. This approach eliminates the need for multiple training runs while enabling the network to capture both universal pore network principles and depth-specific geological characteristics within a single framework.

The training dynamics of the multi-conditional GAN model were monitored through multiple diagnostic metrics to assess convergence behavior and porosity control accuracy. Figure 8 presents an integrated analysis of the training process, incorporating adversarial loss evolution and porosity prediction accuracy across all four depths simultaneously.

The multi-conditional model shows characteristic GAN training dynamics (Figure 8a) with the generator loss starting at approximately 4.0 and the discriminator loss beginning around 1.6. The generator loss (red) demonstrates an overall downward trajectory throughout the 200 epochs, showing rapid initial decline in the first 25

epochs followed by continued descent with periodic variations. The discriminator loss (blue) exhibits more pronounced variability with notable fluctuations, particularly between epochs 75-150, while maintaining an overall declining trend throughout the training process.

The training demonstrates stable convergence with both networks successfully learning their respective objectives across all four geological formations simultaneously. The generator’s progressive loss reduction indicates continuous improvement in generating realistic porous media images, while the discriminator maintains its classification capability throughout training. The oscillatory patterns observed, especially in the discriminator loss, are characteristic of adversarial training dynamics, indicating active learning rather than premature convergence. By the final epochs, both losses stabilize at low values, suggesting the networks have reached a stable state suitable for generating geologically authentic images across all depth conditions, as confirmed by the comprehensive validation metrics presented in subsequent sections.

This stable adversarial training provides the foundation for effective porosity control across diverse geological formations. The porosity control analysis (Figure 8b) reveals the progressive effectiveness of the dual-conditional mechanism. Each sample demonstrates distinct convergence patterns toward their respective target values, with Sample 1 ( $\phi = 0.2450$ , blue), Sample 2 ( $\phi = 0.1810$ , orange), Sample 3 ( $\phi = 0.0567$ , green), and Sample 4 ( $\phi = 0.0710$ , red).

The convergence behavior exhibits formation-specific characteristics that reflect the underlying geological complexity. Samples 1 and 2, representing higher porosity grainstones, demonstrate robust convergence toward their respective targets with characteristic oscillatory patterns typical of adversarial training dynamics. In contrast, Samples 3 and 4, representing lower porosity crystalline formations, exhibit more gradual convergence patterns, with Sample 3 showing the most challenging control behavior due to its fine-grained crystalline structure and anhydrite inclusions, while Sample 4 demonstrates steady oscillations around its target throughout training.

The simultaneous convergence across this wide porosity range (0.0567 to 0.2450) validates the model’s capability to learn complex porosity-depth relationships within a unified framework. By the final epochs, all samples achieve relative stability near their respective targets, demonstrating successful integration of both porosity and depth conditioning information into the generation process.

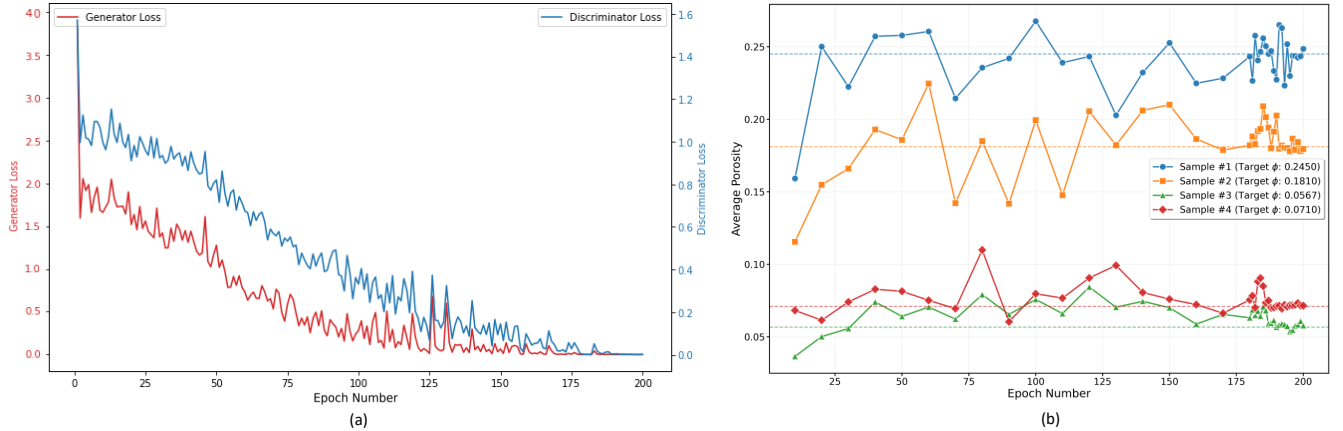


Figure 8: Comprehensive training analysis for the multi-conditional GAN model over 200 epochs: (a) Evolution of generator loss (red) and discriminator loss (blue) showing characteristic adversarial dynamics with convergence toward zero, indicating successful unified training across all depth conditions, and (b) Porosity control tracking for all four samples simultaneously, showing convergence toward respective target values: Sample 1 ( $\phi = 0.2450$ ), Sample 2 ( $\phi = 0.1810$ ), Sample 3 ( $\phi = 0.0567$ ), and Sample 4 ( $\phi = 0.0710$ ). Dashed horizontal lines represent target porosity values, demonstrating the model’s ability to learn dual-conditional control (porosity and depth) across diverse geological formations within a single unified framework.

### 3.1.2. Porosity Control Accuracy

Following completion of the 200-epoch training process, we conducted correlation analysis between target porosity values used as conditional input and observed porosity values in generated images. For the multi-conditional model, we generated a total of 100 synthetic images distributed across all four samples, with target porosity values

uniformly distributed within the observed range of each corresponding sample in the training dataset. The target sample conditions were randomly selected to ensure representative sampling, and the observed porosity of each generated image was calculated using the same pre-trained U-Net segmentation model employed during the data preparation phase.

Figure 9 presents the correlation analysis results. The plot displays the expected porosity values on the x-axis and corresponding observed porosity values on the y-axis, with data points color-coded by sample. The ideal 1:1 correlation is represented by a dashed line, and the coefficient of determination ( $R^2$ ) provides quantitative measures of correlation strength and prediction accuracy.

The results reveal strong correlation with an overall  $R^2$  value of 0.95, demonstrating effective porosity control across diverse geological formations. Sample-specific analysis shows consistent accuracy with Mean Absolute Errors of 0.0105, 0.0197, 0.0101, and 0.0099 for Samples 1-4 respectively, indicating reliable porosity prediction across different geological contexts.

This exceptional porosity control capability reflects the model’s ability to learn and maintain fabric-appropriate pore morphologies across the full spectrum of carbonate textures. The visual progression from lower porosity crystalline fabrics (Samples 3 and 4) to higher porosity grainstone fabrics (Samples 1 and 2) demonstrates the model’s capacity to generate corresponding changes in pore network architecture while maintaining geological realism.

The unified training approach enables the model to learn consistent porosity-generation relationships while preserving sample-specific geological characteristics, successfully handling the varying complexity of pore structures from dual-scale porosity typical of dolomitized grainstones to smaller, uniformly distributed pore spaces characteristic of crystalline formations [33]. This validates the multi-conditional GAN approach for subsurface characterization applications requiring specific porosity targets.

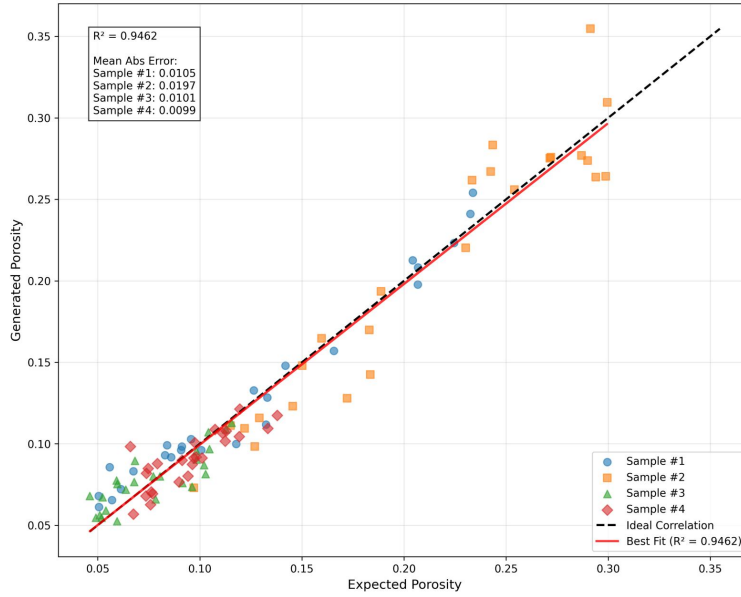


Figure 9: Correlation between expected (target) porosity values and generated porosity values in images from the multi-conditional GAN model across all four samples ( $R^2 = 0.9462$ ). The plot includes data points from all samples: Sample 1 (blue circles), Sample 2 (orange squares), Sample 3 (green triangles), and Sample 4 (red diamonds), with sample-specific Mean Absolute Errors of 0.0105, 0.0197, 0.0101, and 0.0099 respectively. The unified model demonstrates effective porosity control across different geological formations. The dashed black line represents ideal 1:1 correlation, while the solid red line shows the best fit.

### 3.1.3. Visual Quality Assessment and Geological Feature Validation

The visual quality of generated images provides critical validation of the cGAN model’s ability to capture and reproduce depth-specific geological characteristics. This comprehensive assessment evaluates the model’s performance across all four carbonate samples, examining both overall visual fidelity and detailed geological feature preservation.

Figures 10 to 13 present systematic comparisons between training samples and synthetically generated images

for the four carbonate formations studied. Each figure displays original training images in the top row with corresponding synthetic generations in the middle and bottom rows, all labeled with their respective porosity values to enable direct quantitative comparison. These visual comparisons reveal the model’s ability to generate realistic porous media images while maintaining precise porosity control across the full range of geological formations encountered in the carbonate sequence.

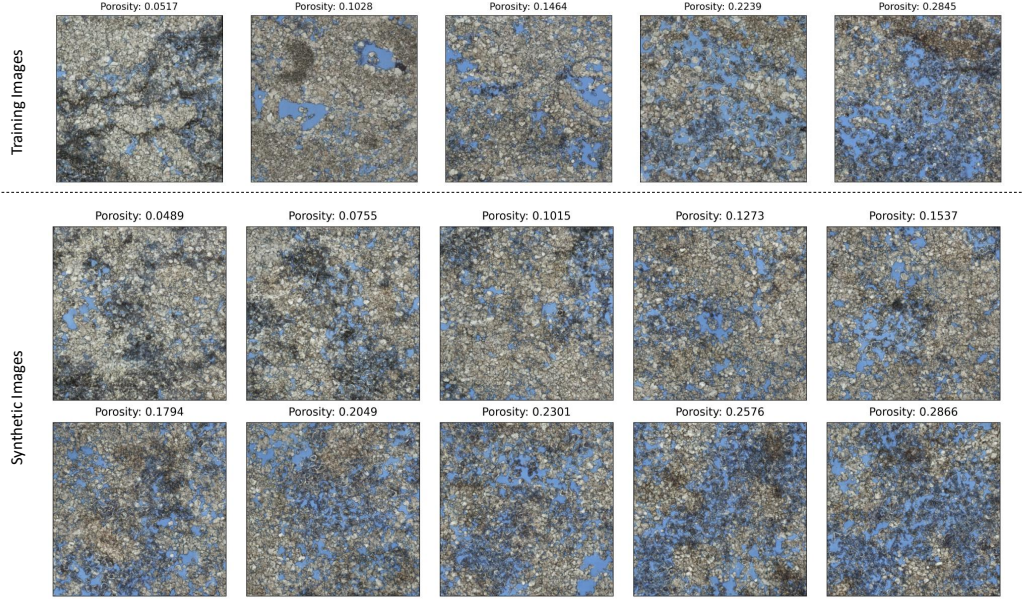


Figure 10: Comparison between training images (top row) and synthetic images (middle and bottom rows) generated by the cGAN model for Sample 1. Each image is labeled with its corresponding porosity value, demonstrating the model’s ability to generate realistic porous structures with varying porosity levels.

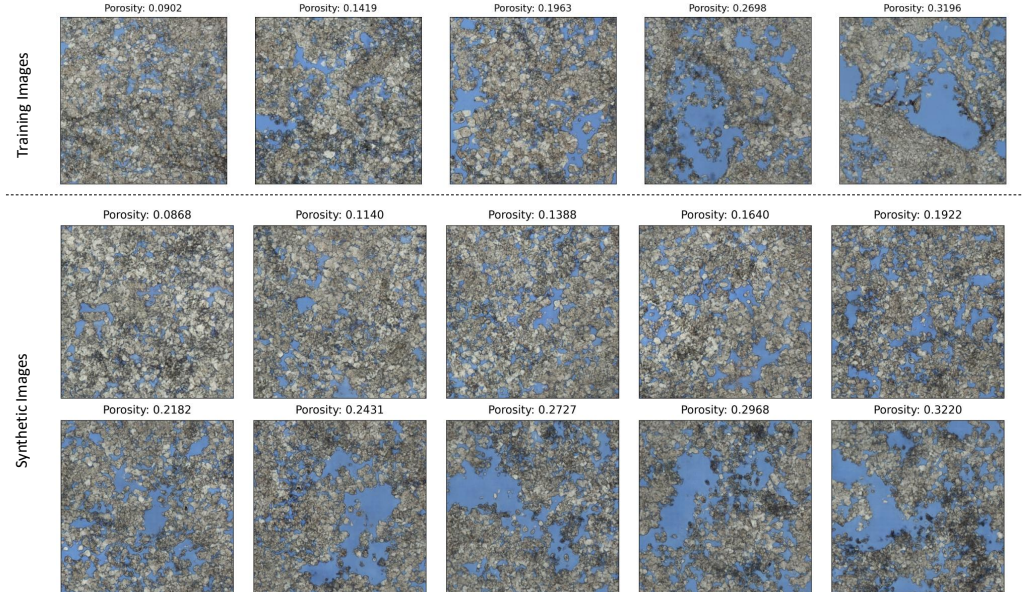


Figure 11: Comparison between training images (top row) and synthetic images (middle and bottom rows) generated by the cGAN model for Sample 2. The synthetic images demonstrate the model’s capability to reproduce the characteristic pore morphology specific to this depth, with appropriate porosity values ranging from 0.0867 to 0.3229.

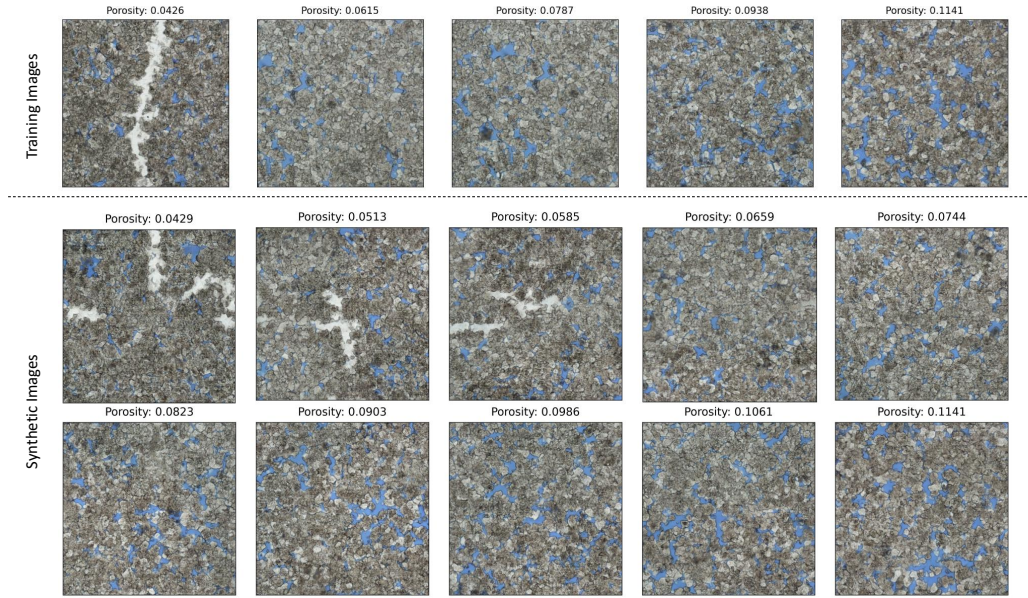


Figure 12: Comparison between training images (top row) and synthetic images (middle and bottom rows) generated by the cGAN model for Sample 3. Note the distinctive white mineral patches and inclusions in both training and synthetic images, representing different mineral compositions characteristic of this specific depth. Porosity values range from 0.0424 to 0.1142.

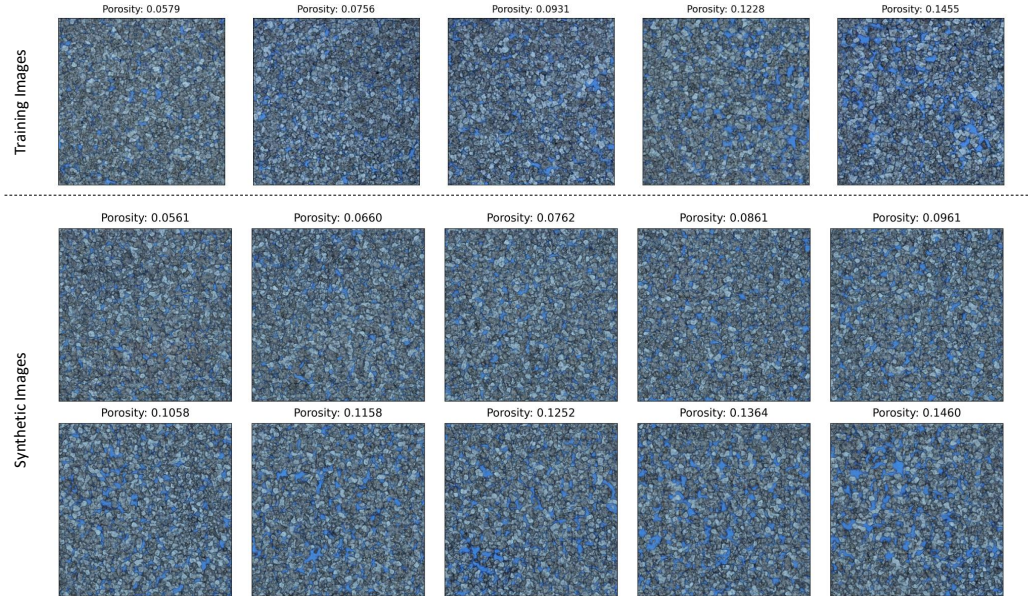


Figure 13: Comparison between training images (top row) and synthetic images (middle and bottom rows) generated by the cGAN model for Sample 4. The model accurately reproduces the fine-grained texture and uniform pore distribution patterns specific to this depth, with porosity values ranging from 0.0561 to 0.1461.

Visual inspection of these generated images reveals successful capture of distinctive geological characteristics unique to each formation depth. Sample 1 at 1879.50 m demonstrates the model's capability to reproduce grainstone fabric with characteristic interparticle-intercrystalline pore types. The synthetic images accurately capture well-connected pore networks typical of grainstones, representing the dual porosity system where both primary interparticle and secondary intercrystalline porosity contribute to the formation's flow characteristics [25]. This grainstone interval exhibits the complex pore architecture resulting from partial dolomitization that preserves original depositional textures while creating secondary porosity.

The enhanced pore connectivity characteristic of Sample 2 at 1881.90 m is faithfully reproduced in the generated images, which successfully capture the extensive interparticle-intercrystalline pore system typical of dolomitized

grainstones that have undergone fabric-preserving dolomitization [34]. The model maintains the delicate balance between preserving original grainstone textures and representing the enhanced connectivity developed through diagenetic processes.

In contrast to the grainstone fabrics, Sample 3 at 1918.50 m represents crystalline fabric with intercrystalline pore types in dolomite-anhydrite lithology. The synthetic images demonstrate the model’s ability to accurately reproduce the distinctive white anhydrite mineral patches, which reflect sulfate precipitation processes typical of evaporitic carbonate sequences [35]. The porosity in these samples occurs exclusively as narrow intercrystalline channels between crystal faces, a characteristic the model captures with high precision. Sample 4 at 1943.50 m displays pure dolomite with crystalline fabric and intercrystalline pore types, where the model accurately reproduces the fine-grained crystalline texture and uniform small-scale intercrystalline porosity typical of recrystallized carbonates [36].

To further validate the model’s capability to capture formation-specific geological features at a detailed level, comparative analysis was performed on two representative samples exhibiting contrasting carbonate fabrics. Figure 14 presents annotated comparisons between real and generated images for Sample 1, representing grainstone fabric, and Sample 3, representing crystalline fabric with anhydrite inclusions. While only these two samples are presented in detail for brevity, similar successful feature reproduction was observed in the generated images for Samples 2 and 4, which also accurately mimicked their respective geological characteristics.

This detailed examination reveals the model’s ability to reproduce not only overall textural patterns but also specific mineralogical and pore network features critical for accurate subsurface characterization. The grainstone fabric of Sample 1 (Fig. 14a-b) exhibits clearly visible individual dolomite grain boundaries and angular crystal morphologies in both real and generated images. The dual pore system, comprising larger interparticle porosity between grains and smaller intraparticle porosity within individual grains, is faithfully reproduced in the synthetic generation. The generated image maintains distinct grain boundaries, preserves angular crystal edges, and accurately replicates the dual porosity distribution pattern observed in the real sample, demonstrating the model’s understanding of complex grainstone pore architectures.

Sample 3 (Fig. 14c-d) presents a contrasting crystalline fabric where distinctive white non-porous anhydrite patches appear clearly against the darker dolomite matrix. The sharp boundaries between dolomite and anhydrite minerals are preserved in the generated images, along with the fine-grained texture created by individual dolomite crystals. The porosity, occurring exclusively as narrow intercrystalline channels between crystal faces, is accurately replicated with correct spatial distribution and connectivity patterns. The successful preservation of anhydrite patch morphology and distribution demonstrates the model’s capability to learn and reproduce complex mineral assemblages beyond simple pore-solid relationships.

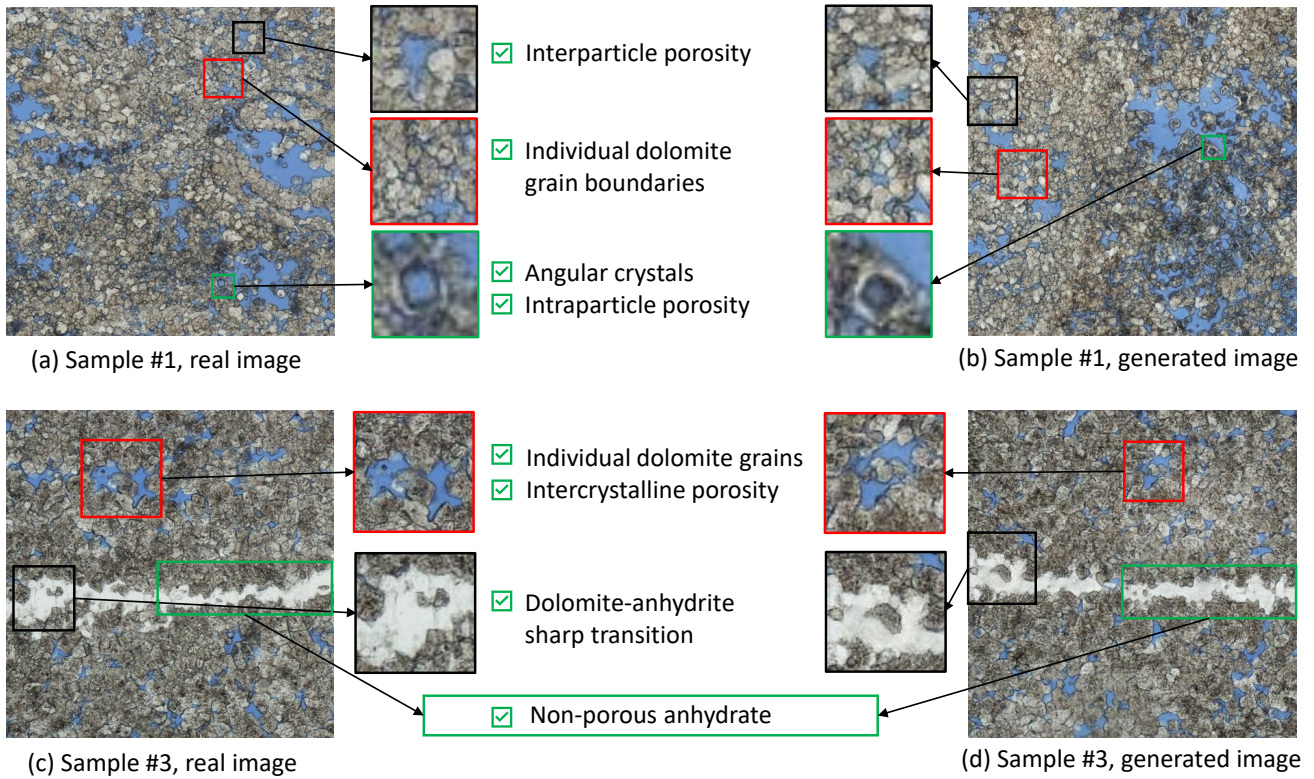


Figure 14: Detailed geological feature comparison between real and generated thin section images for representative carbonate formations. (a) Sample 1 real image showing grainstone fabric with labeled interparticle porosity, individual dolomite grain boundaries, angular crystals, and intraparticle porosity. (b) Sample 1 generated image demonstrating successful reproduction of grainstone characteristics. (c) Sample 3 real image displaying crystalline fabric with annotated non-porous anhydrite patches, dolomite-anhydrite boundaries, individual dolomite grains, and intercrystalline porosity. (d) Sample 3 generated image accurately replicating crystalline texture with anhydrite inclusions. Blue regions indicate pore spaces filled with epoxy resin.

These comprehensive visual assessments, from overall formation characteristics to detailed mineralogical features, demonstrate that the multi-conditional GAN framework successfully captures the full spectrum of geological complexity present in the carbonate sequence. The model’s ability to reproduce features ranging from the coarse-grained, dual-porosity system of grainstone fabrics to the fine-grained, anhydrite-bearing texture of crystalline formations validates its capability to learn and generate formation-specific characteristics essential for accurate subsurface representation. This visual fidelity, combined with precise porosity control, establishes the framework as a powerful tool for generating geologically authentic pore-scale images that preserve the complex relationships between depositional environment, diagenetic processes, and resulting pore network architectures.

#### 3.1.4. Architecture Comparison (Full vs. Baseline Models)

To validate the necessity of our original architectural choices, we conducted comprehensive comparative analysis between the original model (50M parameters) and two systematically reduced alternatives, namely Model A with depth-optimization strategy (38M parameters, 24% reduction) and Model B with proportional scaling approach (25M parameters, 50% reduction). This analysis evaluates the fundamental trade-off between computational efficiency and geological modeling quality across identical training conditions.

Figure 15 shows generated images for Sample 1 at porosity values of 0.082, 0.133, and 0.177. The original model (top row) produces geologically realistic images with well-defined pore structures and authentic mineral distributions. Model A (middle row) maintains recognizable geological features but exhibits repetitive pattern generation, as highlighted by red rectangles in the figure, demonstrating areas where identical textural motifs are artificially replicated across the image. Repetition of patterns reduces textural authenticity and compromises geological realism while retaining basic structural characteristics. Model B (bottom row) demonstrates complete failure in geological feature reconstruction, producing images lacking authentic geological structure and realistic

pore networks.

Quantitative analysis reveals progressive performance degradation. Porosity control accuracy decreased from  $R^2 = 0.94$  (original) to  $R^2 = 0.84$  (Model A) and  $R^2 = 0.51$  (Model B), representing 11% and 46% degradation respectively. Mean Absolute Errors confirm this trend, with Model A achieving reasonable accuracy across samples (MAE: 0.0132-0.0219) while Model B shows substantially higher errors (MAE: 0.0246-0.1069).

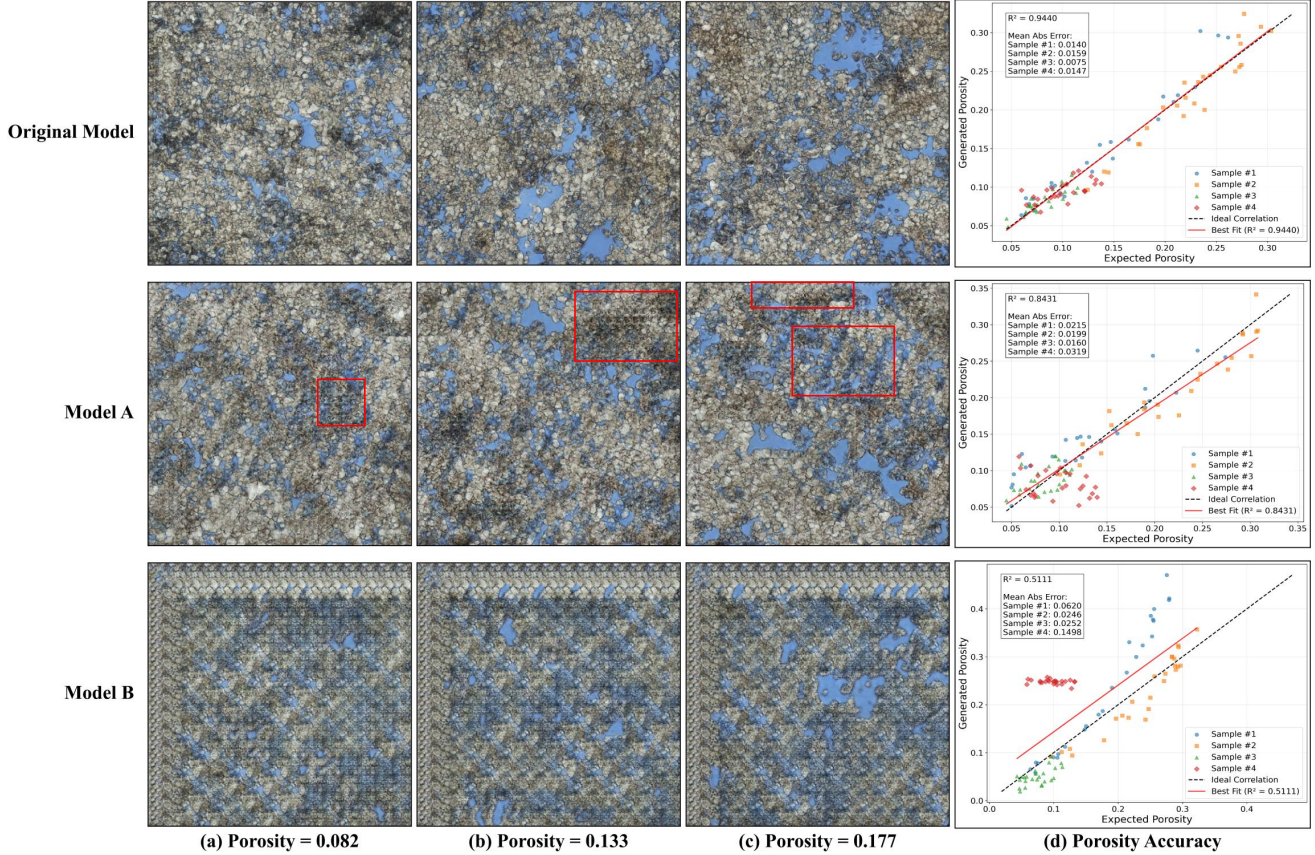


Figure 15: Architectural comparison for Sample 1: Original model (top, 50M parameters), Model A (middle, 38M parameters, 24% reduction), and Model B (bottom, 25M parameters, 50% reduction). Columns 1-3 show generated images at target porosity values of 0.082, 0.133, and 0.177. Red rectangles in Model A images highlight regions of repetitive pattern artifacts resulting from reduced model capacity. Column 4 presents porosity control accuracy with  $R^2$  values: Original ( $R^2 = 0.94$ ), Model A ( $R^2 = 0.84$ ), and Model B ( $R^2 = 0.51$ ). Progressive quality degradation with parameter reduction validates the necessity of original architectural design, demonstrating clear efficiency-quality trade-offs in geological modeling applications.

Results demonstrate that the original architecture represents optimal design rather than over-engineering. Model A approaches the practical limit for acceptable geological modeling, while model B falls below the minimum threshold for subsurface characterization applications. The clear performance degradation with parameter reduction validates the necessity of substantial network capacity for effective dual-conditional geological image generation, establishing practical boundaries for computational efficiency in geological modeling tasks.

### 3.2. Quantitative Validation and Morphological Preservation

Morphological analysis evaluated the preservation of critical pore network characteristics between real and synthetic images across porosity levels. This quantitative assessment employed three fundamental morphological parameters, including average pore radius, specific surface area, and tortuosity, which are essential descriptors of pore network geometry and connectivity that directly influence fluid flow behavior in porous media [37]. For comprehensive yet focused validation, we present detailed morphological analysis for two representative samples. Sample 2 demonstrating optimal model performance with grainstone characteristics, and Sample 3 representing the most challenging geological formation with crystalline fabric and anhydrite inclusions (Table 1). The remaining

samples (1 and 4) also demonstrated successful morphological preservation with Cohen’s  $d$  values within acceptable geological tolerances, confirming model effectiveness across the complete range of carbonate formations studied.

Statistical significance was assessed using Kolmogorov-Smirnov (KS) tests for distribution comparisons and Student’s  $t$ -tests for mean comparisons. Throughout this manuscript, statistical significance is indicated using superscript notation: \*\*\* for  $p < 0.001$ , \*\* for  $p < 0.01$ , \* for  $p < 0.05$ , and <sup>ns</sup> for  $p > 0.05$  (non-significant). Effect sizes were quantified using Cohen’s  $d$ , where  $d < 0.2$  indicates negligible effect,  $0.2 \leq d < 0.5$  small effect,  $0.5 \leq d < 0.8$  medium effect, and  $d \geq 0.8$  large effect.

Statistical analysis reveals excellent agreement between real and generated images across all morphological parameters for the selected representative depths. The cGAN model demonstrate remarkable ability to preserve fundamental pore network characteristics while maintaining geological realism, with Cohen’s  $d$  values ranging from  $0.01^{\text{ns}}$  to  $1.38^*$ , where most values remain within moderate geological tolerances [38]. Figure 16 presents comprehensive morphological validation results for Samples 2 and 3 across all three parameters, exemplifying the range of model performance from optimal capabilities to geologically challenging formations.

Generated images successfully maintain pore size distributions across all porosity levels. For Sample 2 (Fig. 16.a), average pore radius values demonstrate exceptional preservation ranging from  $17.98 \mu\text{m}$  to  $24.75 \mu\text{m}$  (real) with generated values from  $18.12 \mu\text{m}$  to  $23.60 \mu\text{m}$ , achieving near-perfect alignment particularly in medium porosity ranges ( $20.96 \mu\text{m}$  vs  $20.93 \mu\text{m}$ ) with no significant differences detected (KS-test:  $p = 0.602^{\text{ns}}$ ,  $t$ -test:  $p = 0.821^{\text{ns}}$ ) and negligible effect size (Cohen’s  $d = 0.09^{\text{ns}}$ ). Sample 3 (Fig. 16.b), shows consistent reproduction with some formation-specific challenges, particularly at low porosity levels where complex pore structures yield Cohen’s  $d = 1.38^*$ , reflecting geological complexity rather than systematic model limitations.

Surface area preservation demonstrates the models’ capability to reproduce reactive surface characteristics essential for chemical transport processes [39]. Sample 2 (Fig. 16.c) exhibits outstanding preservation across all porosity categories, with exceptional agreement in medium porosity ranges ( $2.57 \times 10^{-5}$  vs  $2.55 \times 10^{-5} 1/\mu\text{m}$ ) showing no significant differences (Cohen’s  $d = 0.09^{\text{ns}}$ ) and excellent correspondence in high porosity samples ( $2.92 \times 10^{-5}$  vs  $2.94 \times 10^{-5} 1/\mu\text{m}$ ) with Cohen’s  $d = 0.08^{\text{ns}}$ . Sample 3 (Fig. 16.d) demonstrates good overall preservation with excellent calibration in medium porosity ranges ( $1.56 \times 10^{-5}$  vs  $1.56 \times 10^{-5} 1/\mu\text{m}$ , Cohen’s  $d = 0.03^{\text{ns}}$ ), validating accurate representation of fluid-rock interaction potential.

All models successfully capture the expected inverse relationship between porosity and tortuosity, confirming that models learn underlying physical principles governing pore network connectivity rather than merely reproducing visual patterns [40]. Sample 2 (Fig. 16.e) shows exceptional preservation with virtually identical distributions at high porosity ( $1.4090$  vs  $1.4096$ , Cohen’s  $d = 0.01^{\text{ns}}$ ) and outstanding performance across all porosity levels. Sample 3 (Fig. 16.f) maintains systematic porosity-dependent behavior with values decreasing from  $1.61$  at low porosity to  $1.53$  at high porosity, though showing greater variability at low porosity (Cohen’s  $d = 1.36^*$ ) reflecting the complex connectivity patterns characteristic of crystalline formations.

The performance differential between samples validates the models’ sensitivity to geological complexity. Sample 2 consistently demonstrates superior preservation due to its well-developed grainstone pore networks, while Sample 3 represents systematic challenges posed by fine-grained crystalline structures with anhydrite inclusions. These variations demonstrate the model’s geological authenticity rather than systematic limitations, as performance correlates directly with established geological complexity rankings.

Morphological validation across representative formations reveals consistent trends validating fundamental physics captured by multi-conditional GAN model, demonstrating successful learning of both universal pore network principles and formation-specific geological characteristics. Both representative samples exhibit expected physical relationships: pore radius increases with porosity, specific surface area shows systematic porosity-dependence, and tortuosity exhibits inverse porosity relationships ( $1.61$ - $1.41$  at low to high porosity for Sample 2;  $1.61$ - $1.53$  for Sample 3), confirming capture of fundamental physics rather than visual patterns alone.

Each sample maintains unique morphological signatures reflecting geological heterogeneity. Sample 2 exhibits well-developed pore networks with larger pore radii (up to  $24.75 \mu\text{m}$ ) and higher specific surface areas, while Sample 3 displays compressed parameter ranges with fine-grained characteristics reflecting its crystalline fabric and anhydrite composition. This preservation of interrelated parameters validates capture of fundamental pore network relationships essential for fluid flow predictions.

These morphological results demonstrate that cGAN-generated images preserve fundamental characteristics of

real porous media across three critical dimensions. The accurate reproduction of average pore radius, specific surface area, and tortuosity confirms preservation of essential pore network characteristics that govern subsurface fluid flow. Particularly noteworthy is the successful reproduction of inverse porosity-tortuosity relationships across all samples, which validates that the model captures connectivity principles rather than merely replicating visual patterns. The systematic preservation of surface area relationships further ensures accurate representation of fluid-rock interactions across different formation types, a critical requirement for reactive transport modeling applications.

The performance differential between samples provides additional validation of the model’s geological authenticity. Sample 2 consistently demonstrates superior morphological preservation due to its well-developed grainstone pore networks, while Sample 3 presents systematic challenges arising from its fine-grained crystalline structure with anhydrite inclusions. Rather than indicating model limitations, these variations correlate directly with established geological complexity rankings, demonstrating the model’s sensitivity to formation-specific characteristics. This differential performance actually strengthens confidence in the model’s ability to distinguish between geological fabrics of varying complexity.

Furthermore, both representative samples exhibit the expected physical relationships that characterize natural porous media. Pore radius increases systematically with porosity, specific surface area shows consistent porosity-dependence, and tortuosity maintains its characteristic inverse relationship with porosity, decreasing from 1.61-1.41 for Sample 2 and 1.61-1.53 for Sample 3 as porosity increases from low to high values. This preservation of interrelated parameters confirms that the model has learned fundamental pore network relationships essential for accurate fluid flow predictions, rather than simply memorizing training patterns. The consistency of these physical relationships across different geological formations further validates the model’s capability to generate geologically meaningful representations suitable for subsurface characterization applications.

Multi-metric analysis employing Kolmogorov-Smirnov tests, t-tests, and Cohen’s d calculations confirms that while some statistically significant differences exist, they represent natural geological variability rather than systematic model limitations [41]. The combination of distributional similarity assessment and mean difference analysis ensures comprehensive evaluation across diverse geological formations, establishing the cGAN framework’s capability to preserve essential pore network characteristics while maintaining geological authenticity.

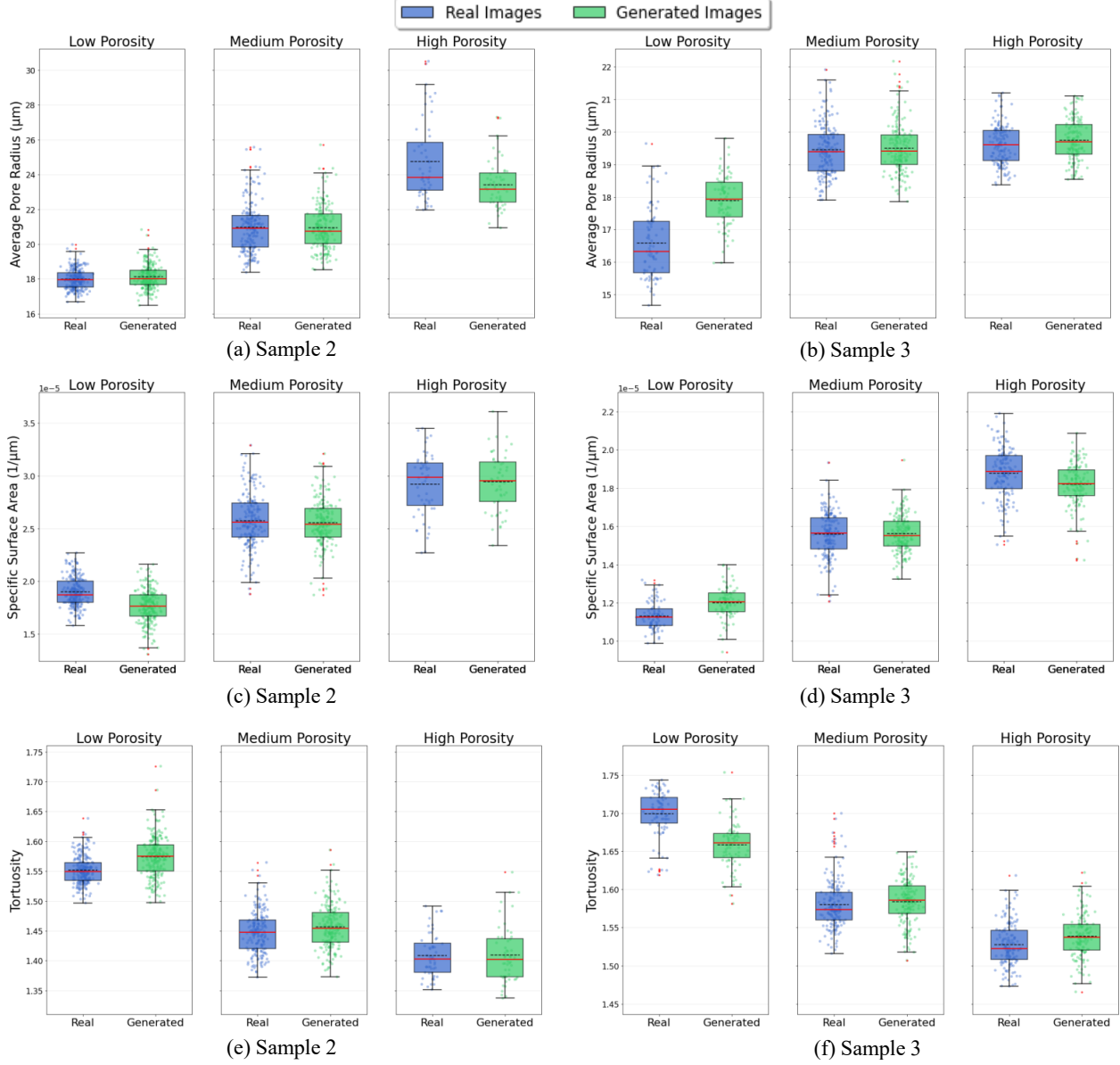


Figure 16: Comprehensive morphological validation comparing real (blue) and generated (green) images for representative Samples 2 and 3 across three critical parameters: (a,b) Average pore radius analysis showing preserved pore size distributions with values ranging from 17.72-24.75  $\mu\text{m}$ , (c,d) Specific surface area analysis demonstrating maintained reactive surface characteristics ( $1.58 \times 10^{-5}$  to  $3.44 \times 10^{-5}$   $1/\mu\text{m}$ ), and (e,f) Tortuosity analysis confirming systematic porosity-dependent behavior with inverse relationships from 1.61-1.66 (low porosity) to 1.40-1.47 (high porosity). Left column shows Sample 2 (1881.90m), right column shows Sample 3 (1918.50m). Cohen's d values range from 0.03<sup>ns</sup>-0.94<sup>ns</sup> across all parameters, with most showing non-significant differences and preservation of essential pore network characteristics within geological tolerances.

### 3.3. Practical Application Results

#### 3.3.1. Core Sample-Based Representative Image Generation

A practical application of our cGAN framework is the ability to generate representative porous media images that optimally match measured petrophysical properties from core samples. This capability addresses the challenge of obtaining truly representative pore-scale images, as natural spatial variability within core samples often results in images that deviate significantly from bulk formation properties. Through controlled generation with target property specifications, the framework can produce synthetic images that more closely approximate desired property values compared to randomly extracted sub-images from the same formation. Additionally, this approach can address fundamental challenges in subsurface characterization where direct imaging data is unavailable but petro-

physical properties have been measured through well logging or other indirect methods, enabling the generation of geologically consistent pore-scale representations that match the measured formation characteristics.

To demonstrate this practical application, we utilized the porosity and permeability measurements from the core samples as detailed in Table 1. The methodology involved two parallel approaches for comparative analysis. For the real samples, in each original sample images we took several random sub-images ( $480 \times 480$  pixels), and then assessed their porosity and permeability based on the methods used in this article. For generating representative images based on the trained cGAN model, we used the dual-constraint error system (Equation 4) to find the most representative image in terms of porosity and permeability.

### 3.3.2. Validation Framework for Multi-Property Representativeness

To quantitatively assess the representativeness of generated images beyond porosity control, we developed a validation framework incorporating both porosity and permeability characteristics. An empirical permeability prediction model was established (Equation 3) using 60 core samples from carbonate formations, incorporating porosity and weighted mean throat radius from MICP data:

$$K = 1.3049 \exp(1.7432\phi R_{th}) \quad (3)$$

where  $K$  is permeability (mD),  $\phi$  is porosity (dimensionless), and  $R_{th}$  is the weighted mean throat radius ( $\mu\text{m}$ ). This exponential relationship achieved  $R^2 = 0.80$ , reflecting the non-linear relationship between pore network connectivity and fluid transport consistent with established percolation theory [42].

The validation approach involved parallel processing of real and generated image sets through identical procedures, which include binarization using the pre-trained U-Net segmentation model, throat radius estimation through morphological analysis by Deepore model [43], and permeability prediction using the empirical relationship (Equation 3). A dual-constraint optimization framework was developed to systematically identify the most representative generated images. The framework employs a normalized error function that quantifies the deviation from the properties of the target core sample:

$$E = w_\phi \times \frac{|\phi_{target} - \phi_{calculated}|}{\phi_{target}} + w_K \times \frac{|K_{target} - K_{calculated}|}{K_{target}} \quad (4)$$

where  $\phi_{target}$  and  $K_{target}$  represent the measured core sample porosity and permeability values, respectively;  $\phi_{calculated}$  and  $K_{calculated}$  are the computed values from image analysis; and  $w_\phi$  and  $w_K$  are weighting factors. Equal weighting ( $w_\phi = w_K = 0.5$ ) was applied to balance optimization between both properties, ensuring neither parameter dominates the selection process.

The error function yields dimensionless values ranging from 0 (perfect match) to higher positive values indicating increasing deviation from target core properties. For each target condition, 100 candidate images were generated and evaluated, with the image achieving the minimum error selected as the most representative sample. This systematic approach enables objective identification of the most representative generated images based on quantitative property matching rather than subjective visual assessment, thereby enhancing the scientific rigor of the validation process.

### 3.3.3. Multi-Property Performance Analysis

Table 8 presents a comprehensive comparison of porosity, permeability, and dual-constraint error performance across all samples, quantitatively demonstrating the effectiveness of the cGAN generation and scoring-based selection methodology compared to random sub-image extraction for obtaining representative images. Generated images consistently demonstrate superior performance over randomly extracted real sub-images across all geological formations, achieving substantially lower dual-constraint errors ranging from 1.9% to 11.3%, compared to 36.4% to 578.0% for real sub-images.

Sample 1 demonstrates excellent representativeness through the scoring-based selection approach. The generated images achieved mean porosity (15.6%) closely matching the core sample value (15.73%), representing only a 0.8% deviation from the target, while real sub-images showed greater deviation (8.5%) with mean porosity of 14.4% compared to the core sample porosity. For permeability, the generated images (33.8 mD) demonstrated high accuracy compared to the core sample target (33.64 mD), while real sub-images exhibited significant scatter with

higher mean values (86.0 mD). This resulted in a dual-constraint error of only 1.9% for generated images compared to 115.0% for real sub-images.

Sample 2 exhibited the most significant performance improvements, with generated images achieving mean porosity (20.5%) closer to the core sample (24.77%) compared to real sub-images (17.5%). The permeability results were particularly remarkable, with generated images (184.1 mD) achieving excellent agreement with core sample values (181.44 mD), substantially outperforming the extreme scatter observed in real sub-images (2511.8 mD). This resulted in a dual-constraint error reduction from 578.0% (real sub-images) to 11.3% (generated images).

Sample 3 represents the most challenging geological formation characterized by crystalline fabric with anhydrite inclusions. Generated images achieved mean porosity of 10.3% compared to the core sample value of 10.58%, demonstrating high accuracy, while real sub-images showed significant deviation (7.3%) from the target. For permeability, generated images (13.8 mD) closely approximated the core sample value (13.39 mD), while real sub-images showed lower values (8.3 mD). The dual-constraint error improved from 36.4% (real sub-images) to 3.4% (generated images).

Sample 4 continued the trend of improved generated image performance, with generated images achieving porosity (11.4%) closer to the core sample target (13.32%) compared to real sub-images (8.7%). For permeability, generated images (12.6 mD) provided better target approximation (12.09 mD) compared to real sub-images (8.7 mD), resulting in a dual-constraint error reduction from 40.5% to 10.7%.

Table 8: Comprehensive comparison of porosity, permeability, and dual-constraint error performance between core samples, randomly extracted real sub-images, and scoring-optimized generated images.

| Sample                                                                                                                                                                                                                                                                                                                                | Depth<br>(m) | Core Sample   |           | Real Sub-images |           |              | Generated Images |           |              |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------|-----------|-----------------|-----------|--------------|------------------|-----------|--------------|
|                                                                                                                                                                                                                                                                                                                                       |              | $\phi$<br>(%) | K<br>(mD) | $\phi$<br>(%)   | K<br>(mD) | Error<br>(%) | $\phi$<br>(%)    | K<br>(mD) | Error<br>(%) |
| 1                                                                                                                                                                                                                                                                                                                                     | 1879.50      | 15.73         | 33.64     | 14.4            | 86.0      | 115.0        | 15.6             | 33.8      | 1.9          |
| 2                                                                                                                                                                                                                                                                                                                                     | 1881.90      | 24.77         | 181.44    | 17.5            | 2511.8    | 578.0        | 20.5             | 184.1     | 11.3         |
| 3                                                                                                                                                                                                                                                                                                                                     | 1918.50      | 10.58         | 13.39     | 7.3             | 8.3       | 36.4         | 10.3             | 13.8      | 3.4          |
| 4                                                                                                                                                                                                                                                                                                                                     | 1943.50      | 13.32         | 12.09     | 8.7             | 8.7       | 40.5         | 11.4             | 12.6      | 10.7         |
| $\phi$ = porosity, $K$ = permeability, Error = dual-constraint deviation error<br>Real sub-images represent mean values from multiple random extractions<br>Generated images represent mean values from dual-constraint scoring optimization<br>Lower error values indicate better representativeness relative to core sample targets |              |               |           |                 |           |              |                  |           |              |

Figure 17 presents distribution comparisons for both porosity and permeability properties for Samples 1 and 3 as representative examples, revealing that generated images consistently demonstrate tighter distributions around target values compared to real images. This demonstrates the cGAN model’s capability to generate geological candidates with controlled properties, enabling systematic identification of images that closely match target characteristics. In contrast, randomly extracted real sub-images exhibit broader distributions due to natural spatial variability within formations, where sub-image properties depend on the specific location sampled. The higher representativeness of generated samples highlights the cGAN model’s effectiveness in learning fundamental geological relationships and generating controlled variations that facilitate targeted property matching. This capability enables creation of controlled visual profiles of subsurface formations that are more representative of target conditions than traditional sampling approaches, thereby enhancing geological interpretation and fluid flow modeling efforts for carbon storage, groundwater management, and geothermal energy applications.

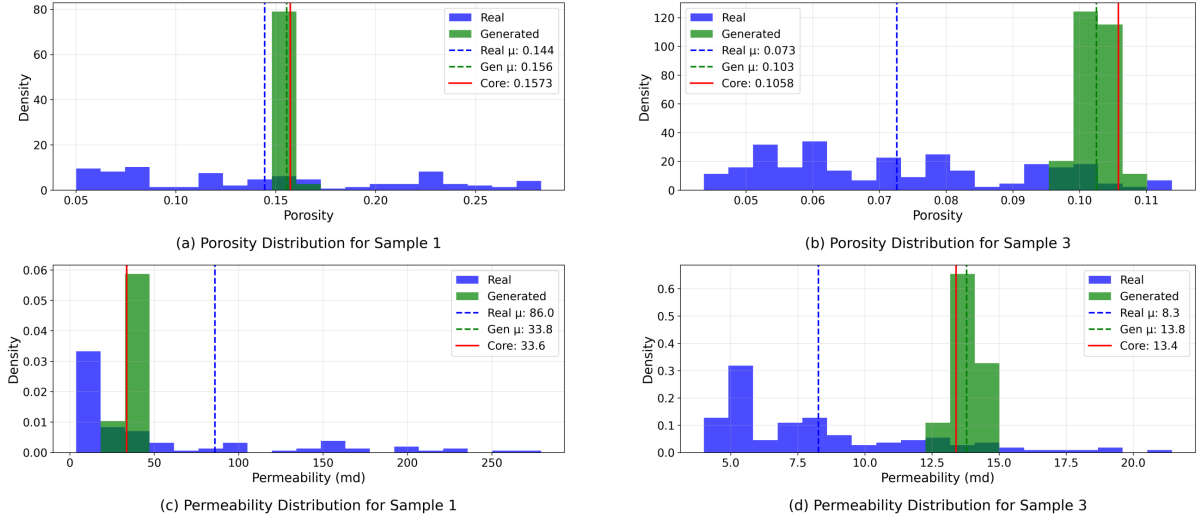


Figure 17: Porosity and permeability distribution comparison between real sub-images (blue) and generated images (green) for representative samples. (a) Porosity distribution for Sample 1 (1879.50 m), (b) porosity distribution for Sample 3 (1918.50 m), (c) permeability distribution for Sample 1, and (d) permeability distribution for Sample 3. Dashed lines show distribution means ( $\mu$ ), solid red lines indicate core sample targets. Generated images demonstrate tighter clustering around target values compared to randomly extracted real sub-images.

## 4. Conclusions

This study developed a unified multi-conditional GAN framework for generating representative pore-scale images with precisely controlled properties, addressing fundamental data scarcity challenges in subsurface characterization. The framework successfully learned complex porosity-depth-pore network relationships across four distinct carbonate formations (1879.50–1943.50 m) within a single model, eliminating the need for formation-specific models.

The model achieved excellent porosity control ( $R^2 = 0.95$ ) with mean absolute errors of 0.0099–0.0197 across all formations, from grainstone fabrics with interparticle-intercrystalline porosity to crystalline textures with anhydrite inclusions. Comprehensive validation confirmed preservation of critical morphological parameters (average pore radius, specific surface area, tortuosity) with statistical differences remaining within acceptable geological tolerances. The model preserved fundamental physical relationships, including inverse porosity-tortuosity correlations, validating capture of underlying pore network principles rather than mere visual replication.

Most significantly, generated images demonstrated better representativeness compared to randomly extracted real sub-images when evaluated against core sample properties. Dual-constraint error analysis revealed generated images achieved 1.9–11.3% deviation from target porosity-permeability combinations, compared to 36.4–578% for real sub-images. This improvement indicates the framework learned fundamental pore geometry-petrophysical property relationships, enabling generation of images that more consistently represent bulk formation characteristics than randomly extracted sub-images from naturally heterogeneous core samples.

Architectural comparison validated the necessity of the 50M-parameter design, with systematic performance degradation in reduced models (24% reduction approaching acceptable threshold; 50% reduction resulting in geological feature failure), confirming substantial network capacity requirements for effective dual-conditional generation.

The demonstrated performance improvements have significant implications for subsurface characterization workflows. By achieving property deviations an order of magnitude lower than conventional random sampling, the framework addresses a fundamental limitation in digital rock analysis where natural heterogeneity compromises sample representativeness. This capability becomes particularly valuable for sparse data scenarios common in deep subsurface investigations.

Future applications could extend this approach to other geological formations and integrate with existing reservoir modeling workflows, potentially revolutionizing how pore-scale heterogeneity is incorporated into field-scale simulations. The success across diverse carbonate fabrics suggests broader applicability to complex geological systems where traditional sampling strategies struggle to capture representative properties.

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## Data and Computer Code Availability

The computer codes and trained machine learning models developed in this study are available publicly on GitHub at: <https://github.com/AliSadeghkhani1990/PCP-GAN>. Part of the project image dataset is also available through the same repository for reproducibility purposes.

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