

# Magnon scattering and transduction in Coulomb-coupled quantum Hall ferromagnets

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The magnetization field of a quantum Hall ferromagnet (QHFM) can host a variety of spin textures, including skyrmions and magnons. When projected into the lowest Landau level with  $\nu = 1$  filling, the topological (Pontryagin) charge density of the magnetization field is proportional to the electric charge density, allowing for long-range spin-spin interactions. Inspired by recent experimental developments that enable all-electrical magnon generation and detection, in this work we theoretically demonstrate two phenomena that can occur due to Coulomb interactions that are unique to QHFMs: magnons can scatter off of point charges at a distance, and skyrmions can act as *transmitters* and *receivers* for magnons to be transduced between separate layers of a bilayer QHFM. The latter Coulomb-mediated spin drag effect occurs at arbitrary distance and could facilitate long-range magnonics, such as detection of spin waves for future experiments in 2D materials.

## I. INTRODUCTION

Quantum Hall ferromagnets (QHFMs) [1] have come once again to the forefront of condensed matter research, thanks to experimental developments in 2D quantum materials. Due to the flat-band dispersion of the (idealized) Landau level, QHFMs conceptually serve as the simplest type of itinerant magnet (with a fully spin polarized, unentangled product ground state), but also provide a more general paradigm for interaction-induced spontaneous symmetry breaking in topological flat-band materials. In addition, QHFMs feature strong locking between charge and spin degrees of freedom, so that topological skyrmion spin configurations and propagating magnons carry electric monopole and dipole moments, respectively [1–6].

Recent attention has focused on elucidating the QHFM ground states and excitations in multivalley and/or multilayer graphene systems. Studies have addressed, for example, the realization and stability of skyrmions in low-energy Dirac Landau levels [7–14], as well as the nature of the  $\nu = 0$  insulating state of monolayer graphene [11, 15–19]. Another flurry of activity was triggered by the discovery of correlated states [20] and superconductivity [21–23] in flattened bands of twisted bilayer graphene (TBLG) [24]. These nearly flat bands can host Chern numbers [25, 26] and insulating QHFM analogs [20, 23, 27]. The observation of superconductivity in TBLG has supercharged the field of moiré materials, with some theoretical proposals linking this to the condensation of QHFM skyrmion pairs [26, 28].

Yet another important recent experiment demonstrated all-electrical generation and detection of magnons in the  $\nu = 1$  state of monolayer graphene [29]. Here, charge is injected between contact-induced *non-sample-spanning*  $\nu = 2$  edge states and the  $\nu = 1$  edge states that connect the source and drain contacts. For a bias exceeding the Zeeman energy gap, electrons tunnel between the  $\nu = 2$  and  $\nu = 1$  edges. This interedge tunneling induces a spin flip, emitting a magnon into the device (see Figure 1 in Ref. [29] for an illustration). The magnons are electrically

detected via the suppression of the Hall conductance from its quantized value, because magnon emission and absorption serves as a source of remote inelastic scattering between distant chiral edge states. Additional experiments exhibited similar phenomena in a quantum-Hall antiferromagnet [30]. Strong damping of magnon transmission at higher dopings away from  $\nu = 1$  was taken as evidence for the formation of a Skyrme crystal, whose phonons can dissipate the magnon energy [7, 12]. The electric dipole moment carried by magnons was probed via a related setup in [31]. Nonequilibrium magnon-skyrmion dressing was investigated in [14].

In this work, we theoretically explore novel magnonic phenomena enabled by electrical magnon generation, and facilitated by the strong spin-charge locking *unique* to QHFM devices. We simulate magnon injection into a spin SU(2) QHFM using semiclassical spin dynamics. We show that external electric charges deflect magnons, evidently due to the effective electric dipole moment carried by the latter. We further compare the results of the spin semiclassics to a second-order Born approximation for the effective magnonic Schrödinger equation, and find that these give similar results.

Second, we demonstrate a novel type of Coulomb-mediated *spin drag* that is directly enabled by skyrmion defects expected to be present near  $\nu = 1$  [9–11]. We consider a bilayer system of two QHFMs separated by an insulating barrier. We assume negligible tunneling, but take into account the Coulomb interactions between the spin textures in the two layers. We focus upon a geometry with a pair of skyrmions co-located in the top and bottom layers, pinned by an impurity potential. We show that magnons injected into the bottom layer are *transduced* into the top layer. The mechanism involves core undulations of the Coulomb-coupled skyrmions in the two layers, such that the top-layer skyrmion functions as a magnon emitter, see Figure 1 for an illustration of the setup. We show that experimentally realistic parameters should produce a detectable effect.

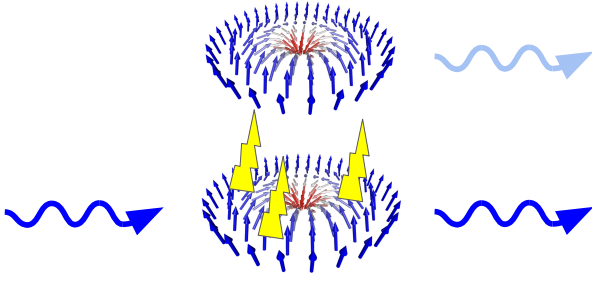


FIG. 1. Cartoon of Coulomb-mediated magnon-skyrmion-magnon transduction. Magnons injected in the lower layer induce undulations of the skyrmion core in that layer. These undulations couple via the Coulomb interaction to the noncollinear spin texture in the upper layer (associated e.g. to another proximal skyrmion). The induced core fluctuations in the top-layer core then function as an antenna, emitting directed magnon radiation into the upper layer.

### A. Outline

The rest of this paper is organized as follows. In Sec. II we define the model and describe the simulation method. Then we present our two main physics results, consisting of magnon-Coulomb charge scattering in a monolayer, as well as Coulomb-mediated magnon transduction (spin drag) between two monolayers separated by an insulating barrier. In Sec. III we provide details of the numerical implementation for the dynamics and observables. In Sec. IV, we consider single-layer magnon dynamics in more detail. In addition to magnon-charge scattering, we also benchmark our numerical methods by considering skyrmion-magnon scattering due to stiffness alone, previously considered in the context of ordinary chiral ferromagnets [32–35]. We consider future directions and conclude in Sec. V.

## II. MAIN RESULTS

### A. Model and equations of motion

An SU(2) QHFM in the lowest Landau level (LLL) at filling factor  $\nu = 1$  has the effective real-time action [1]

$$S_M = \int dt d^2\mathbf{r} \left[ -s n \dot{m}^i \mathcal{A}_i(\vec{m}) + \lambda(m^i m^i - 1) \right] \quad (1a)$$

$$+ \int dt d^2\mathbf{r} \left[ s n g \mu B^i m^i - \frac{\rho_s}{2} \partial_\alpha m^i \partial_\alpha m^i \right] \quad (1b)$$

$$+ \int dt d^2\mathbf{r} \left\{ \frac{e \varrho(\vec{m}) A^0}{2\epsilon} - \frac{e^2}{2\epsilon} \int d^2\mathbf{r}' \frac{\varrho[\vec{m}(t, \mathbf{r})] \varrho[\vec{m}(t, \mathbf{r}')] }{|\mathbf{r} - \mathbf{r}'|} \right\}. \quad (1c)$$

This is a ferromagnetic O(3)/O(2) nonlinear sigma model with magnetization field variable  $\vec{m}(t, \mathbf{r})$ . The Berry phase term  $\dot{m}^i \mathcal{A}_i(\vec{m})$  encodes the non-inertial dynamics of the

spins, while  $\lambda(t, \mathbf{r})$  is a Lagrange multiplier field enforcing the constraint  $\vec{m} \cdot \vec{m} = 1$ . Here  $s = 1/2$  and  $n = 1/2\pi l_B^2$  is the average electron density, with  $l_B = \sqrt{\hbar c/eB}$  the magnetic length. In Eq. (1b), the Zeeman coupling to the external magnetic field  $B^i$  is via  $g\mu$ , the Bohr magneton times the electron  $g$ -factor, while  $\rho_s$  denotes the spin stiffness coefficient (with units of energy). In Eq. (1c),  $e > 0$  is the magnitude of the electron charge,  $A^0(t, \mathbf{r})$  is the scalar electric potential, and  $\epsilon$  the dielectric constant of the material. Due to the LLL projection, the spin texture couples to electric charge through the topological Pontryagin density

$$\varrho(\vec{m}) \equiv \frac{1}{4\pi} \vec{m} \cdot (\partial_x \vec{m} \times \partial_y \vec{m}). \quad (2)$$

We denote 3D vectors with an overarrow and their components with Latin indices, and 2D vectors using boldface with components having Greek indices, e.g.  $\vec{m}(t, \mathbf{r})$  and  $\partial_\mu m^i(t, \mathbf{r})$  ( $\mu \in \{x, y\}$ ,  $i \in \{1, 2, 3\}$ ).

From this, we can derive the semiclassical equations of motion for the magnetization field:

$$\frac{d\vec{m}}{dt} = \vec{m} \times \vec{\mathcal{B}}_{\text{eff}}, \quad (3a)$$

$$\vec{\mathcal{B}}_{\text{eff}} = g\mu B \hat{z} + \frac{\rho_s}{sn} \nabla^2 \vec{m} \quad (3b)$$

$$- \frac{e}{4\pi sn} [\epsilon^{\mu\nu} \vec{m} \times \partial_\nu \vec{m}] E_\mu^{\text{ext}} + \frac{e}{4\pi sn} [\epsilon^{\mu\nu} \vec{m} \times \partial_\nu \vec{m}] \left\{ \frac{e}{\epsilon} \int d^2\mathbf{r}' \frac{(\mathbf{r} - \mathbf{r}')_\mu}{|\mathbf{r} - \mathbf{r}'|^3} \varrho[\vec{m}(t, \mathbf{r}')] \right\}, \quad (3c)$$

where  $E_\mu^{\text{ext}} = -\partial_\mu A^0(t, \mathbf{r})$  is the in-plane external electric field and  $\epsilon^{\mu\nu}$  is the Levi-Civita tensor.

We solve the semiclassical equations (3) via 4th-order Runge-Kutta method for spins in a finite  $L \times L$  sample, with lattice spacing set by  $l_B$ . Forced and absorbing boundary conditions are employed to inject and “detect” magnons, respectively. In the rest of this section, we describe the main setup and results of this work, including estimates for dependence upon certain parameters relevant to experimental detection of the predicted effects.

Our numerical results show evidence of two novel phenomena in LLL QHFMs: magnon electrodynamics in ferromagnets, and Coulomb-facilitated, skyrmion-mediated spin-drag between layers. In the first case, the system is initialized to a ferromagnetic texture and a point charge placed near the center but above the plane of the QHFM. Plane magnons are driven up from the bottom row and collected by an absorbing boundary layer in the top rows. The magnons scatter from the electric field despite carrying no net electrical charge, see Figure 2. This interaction occurs due to the *dipole moment* of the propagating magnons, proportional but perpendicular to the conserved U(1) spin current [2, 3]. In the second case, the system is extended to two layers that are coupled via the Coulomb interaction, each initialized with a stable skyrmion texture, and magnons are driven across the skyrmion in one

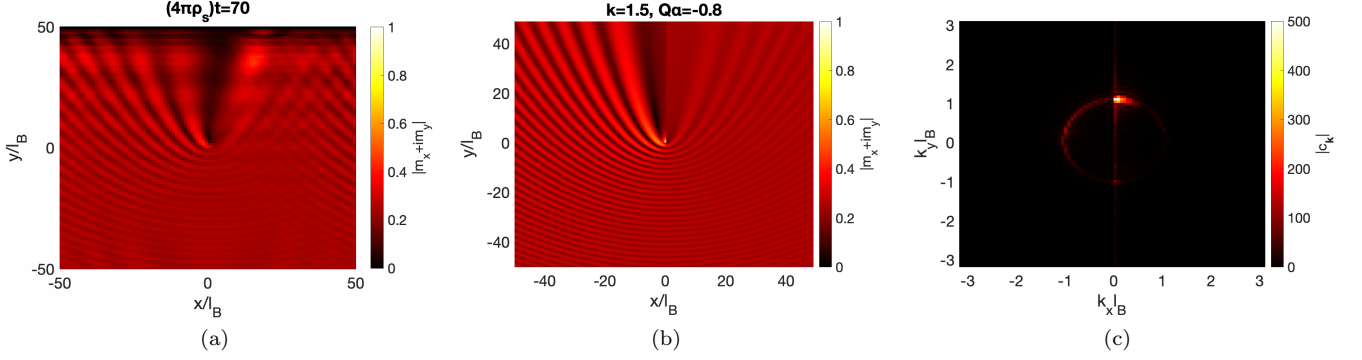


FIG. 2. (Color online) Plane magnons scatter off of the electrical field due to a point charge located at  $(x, y, z) = (0, 0, l_\perp)$ . Despite carrying zero net electric charge, magnons interact with electric fields due to the effective electrical dipole moment proportional to the U(1) spin current, Eq. (5). (a) Snapshot of a numerical simulation. The color indicates the magnitude of the deviation of the spin field from the ferromagnetic ground state  $|\psi(x, y)| \equiv \sqrt{m_x^2 + m_y^2}$  after plane magnons are driven into the sample from the bottom. The scattering arises due to the electric field of a point charge placed at a distance  $l_\perp = 2$  above the plane. The magnon has wavevector  $k = 1.5708$  and the magnitude of the point charge is  $Q = -9$ . Here all distances and inverse wavenumbers are measured in units of the magnetic length  $l_B = 1$ , and charge in units of the electron charge  $e > 0$ . (b) Analytical result via the second Born approximation for the magnon scattering off of a point charge at  $(x, y, z) = (0, 0, 0)$ , with  $k = 1.5$  and  $Q\alpha = -0.8$ , where  $\alpha$  is defined via Eq. (4). (c)  $k$ -space distribution of magnons at end of simulation. Here,  $c_{\mathbf{k}} = \sum_{\mathbf{r}} e^{-2\pi i \mathbf{k} \cdot \mathbf{r}/N} \psi(\mathbf{r})$  is the discrete Fourier transform for the mode  $\mathbf{k}$  of the magnon field  $\psi = m_x + im_y$ .

of the layers. We quantify the “spin drag” due to the Coulomb-mediated *transduction* of magnon energy from the driven layer to the “receiver” layer, see Figures 3 and 4.

In order to discuss these results in more detail, we first summarize the key parameters for the calculations. After discretizing the spin equations of motion (EOM) in space ( $N \times N$  lattice sites,  $N = L/l_B$ ) and time, we recast them in terms of a dimensionless time variable  $\tau$  and three dimensionless parameters,

$$\tau \equiv (4\pi\rho_s)t, \quad b \equiv \left(\frac{1}{4\pi\rho_s}\right)g\mu B, \quad (4a)$$

$$\mathcal{E} \equiv \left(\frac{1}{4\pi\rho_s}\right)\epsilon l_B \mathbf{E}^{ext}, \quad \alpha \equiv \left(\frac{1}{4\pi\rho_s}\right)\frac{e^2}{\epsilon l_B}. \quad (4b)$$

Here  $b$  and  $\mathcal{E}$  denote the dimensionless out-of-plane Zeeman and in-plane external electric fields, respectively. The stiffness determines the timescale, so it can be tuned by scaling  $\tau$ . Physically, the spin stiffness arises from Coulomb exchange and takes a value  $\rho_s \propto e^2/\epsilon l_B$ , which is the same energy scale for the residual Coulomb interaction at integer LLL filling. Therefore the interaction parameter  $\alpha$  is actually an order-one number, independent of  $e$  and  $l_B$ . Eq. (7) details the implementation of these parameters in the discretized EOM.

### B. Magnon-Coulomb charge scattering

Figure 2 shows the results of a simulation in which magnons interact with a point charge. In a ferromagnetic texture with no skyrmion, plane magnons with topological

charge density  $\varrho(\vec{m}) = 0$  everywhere are driven from the bottom row and subjected to an electric field due to a point charge at the origin, held at a distance of  $l_\perp = 2$  magnetic lengths above the plane in the  $z$ -direction.

The injected magnons scatter off of the point charge, leading to the diffraction pattern shown in Figure 2(a). While non-topological spin textures carry zero electric charge, the LLL projection endows magnons with an effective dipole moment [2, 3],

$$\vec{d} = \frac{e}{4\pi} \vec{J} \times \hat{z}, \quad (5)$$

where  $\vec{J}$  denotes the U(1) spin current, see Eq. (16).

In this simulation, the initial texture is  $\vec{m}(t=0, \mathbf{r}) = \hat{z}$ , a pure ferromagnet. The parameters are  $N = 100$  and dimensionless Zeeman strength  $b = 0.13$ , while the magnon carries wavenumber  $k_y = 1.5708$ . Unless otherwise noted, we set the lattice spacing (magnetic length)  $l_B = 1$  and measure charges in units of  $e$ . The scalar electric potential is due to an unscreened point charge placed a distance  $l_\perp = 2$  above the plane of the sample, with charge  $Q = -9$ . We drive the bottom row of spins to precess about the  $\hat{z}$ -axis with frequency  $\Omega = b + 2.0$ , so as to induce a plane spin wave that traverses the sample from bottom to top. [Dimensionless frequency is measured with respect to the stiffness energy, defined in terms of the dimensionless time  $\tau$ :  $\Omega = 2\pi/\tau$ , see Eq. (4).] The stiffness (Laplacian) term in Eq. (3) has periodic boundary conditions in the  $x$ -direction so that the walls do not reflect the outgoing spin wave profile. Along the top edges in each layer, we add an absorbing boundary layer with an exponential damping profile so as to catch and dissipate the magnons with minimal reflection [36]. The damping itself is imple-

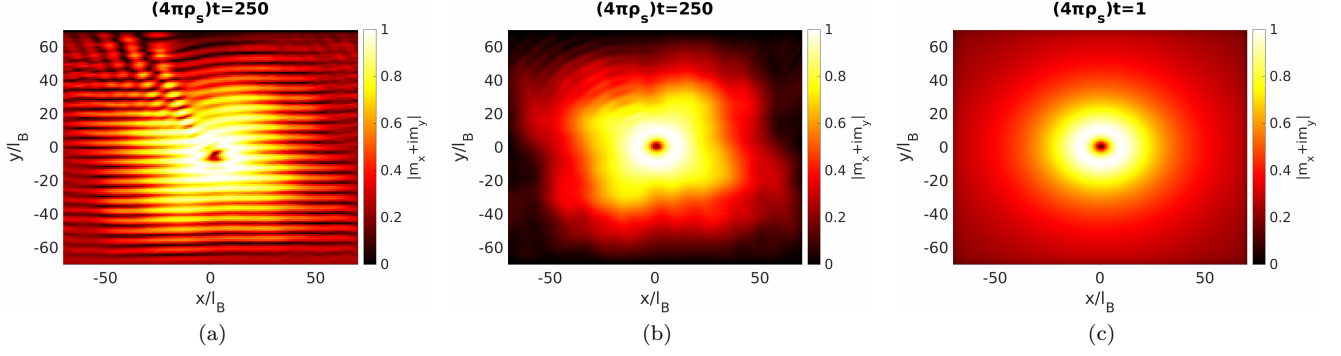


FIG. 3. Interlayer magnon transduction (“spin drag”) due to Coulomb-mediated skyrmion-skyrmion interactions. A cartoon of the setup is shown in Figure 1. We simulate two QHFM separated by small distance of  $0.1 l_B$ , each with a skyrmion defect of size  $\lambda = 5 l_B$ . The skyrmion size  $\lambda$ , defined in Appendix A [Eq. (A1)], is half of its radius. The defects are vertically stacked and pinned by an impurity potential. Plane-wave magnons are driven from the bottom of the spin field in layer 1. Magnon-skyrmion interactions in that layer produce a dynamic undulation in the skyrmion core, and this acts as a “transmitter” that (via Coulomb interaction) drives core undulations in the layer-2 skyrmion (the “receiver”). The latter produces directed magnon emission in the second layer. (a) Magnitude of the lateral magnetization field  $\sqrt{m_x^2 + m_y^2}$  in layer 1, in which magnons are driven from the bottom. (b) Magnitude of the lateral magnetization in layer 2, in which magnons can be seen emitted from the skyrmion, towards the upper-left of the magnetization field. Plots (a,b) are taken within a steady state at a late time in the simulation  $\tau = 250$ . Note that the anisotropy of the transduced spin texture (b) clearly reflects the magnon-skyrmion scattering occurring in the driven layer (a). (c) Initial stable skyrmion texture in each layer. All results have the *interlayer* “interaction strength”  $\alpha_{12} = 1$  [Eq. (4)].

mented via a Gilbert damping term in the equation of motion, see Eq. (11). Here, the damping profile is set to  $\gamma(\mathbf{r}) = \gamma_0 \exp[-(y - N)/2]$ , with  $\gamma_0 = 1.6$ .

For weak point charges, the outgoing waves appear mostly symmetrically distributed, but for stronger point charges, circular waves can be seen emitted towards the left or right, depending on the sign of the charge (negative or positive, respectively).

In the continuum, QHFM magnons can be interpreted as number-conserving, non-relativistic quantum particles subject to *synthetic* scalar and vector potentials due to the physical electric field  $\mathcal{E}$  [Eq. (4)]. The effective single-particle Hamiltonian is

$$\hat{h} = b + \left[ \hat{\mathbf{p}} - \mathcal{A}(\hat{\mathbf{r}}) \right]^2 + \mathcal{V}(\hat{\mathbf{r}}), \quad (6a)$$

$$\mathcal{A}^\beta(\mathbf{r}) \equiv -\frac{\epsilon^{\beta\alpha}}{2} \mathcal{E}^\alpha(\mathbf{r}), \quad (6b)$$

$$\mathcal{V}(\mathbf{r}) \equiv -\mathcal{A}^2(\mathbf{r}). \quad (6c)$$

Using the second Born approximation for plane-magnon scattering with this Hamiltonian yields Figure 2(b), which closely resembles the real-space pattern of our numerical results. Its angular form factor also resembles the  $k$ -space distribution shown in Figure 2(c). We note that the numerical and analytical results shown in Figs. 2(a,b) are not expected to produce identical results, because the second Born approximation is simple to evaluate only for a point charge located in the plane of the magnet, whereas the simulation has  $l_\perp = 2$  (to prevent a short-range divergence due to the Coulomb potential). Our simulation techniques also produce reliable results for magnon-skyrmion scattering, which can be used to benchmark our code and

compare to existing findings, see Figure 6. We discuss these results in detail in Section IV.

We also observe qualitatively similar Coulombic magnon scattering when simulating a bilayer of two QHFM, separated by a thin insulating layer that prevents tunneling between the layers but facilitates inter-layer Coulomb interactions. When magnons are driven in one layer in the ferromagnetic ground state, and the second layer contains a single skyrmion, magnons in the first layer scatter off of the skyrmion at a distance due to the electric field it produces. Therefore, to experimentally verify magnon electrodynamics, one can either apply a nonuniform external electric field or simply dope a nearby layer with electrons or holes. The dipolar character of magnons in QHFM was recently confirmed by interferometric measurements [31].

### C. Spin drag via Coulomb-mediated magnon-skyrmion transduction

Figure 3 shows the results of a simulation in which magnons interact across layers via a Coulomb-mediated, interskyrmion interaction. The setup is depicted in Figure 1. We consider two QHFM separated by a small insulating spacer layer, with collocated skyrmions in each. In Figure 3, the skyrmions are directly stacked one on top of the other, and held in place via an impurity potential. Plane magnons are driven across the skyrmion in layer 1, which induces undulations in its core. These are transduced via the Coulomb interaction to the skyrmion in the second layer, which re-emits the magnons there. The

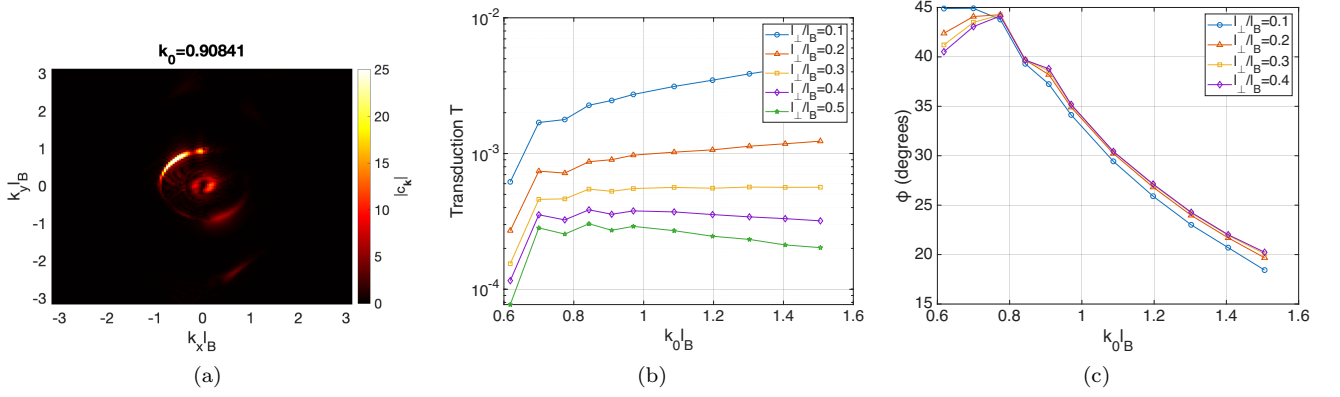


FIG. 4. Parameter dependence of Coulomb-mediated spin-drag simulations I. (a) Magnitude of the difference in the  $k$ -space distribution of magnons in the receiving layer at the end of a driven simulation versus undriven simulation. The arc shows that transduced magnons in the second layer share the wavenumber  $k_0 \approx 0.90841 l_B^{-1}$  of the skyrmion-scattered magnons in the driving layer (not shown). (b) Transduction ratio  $T$  [Eq. (14)] versus driven magnon wavenumber  $k_0$  and interlayer spacing  $l_\perp$  for various simulations, with skyrmion size  $\lambda = 5 l_B$ , interaction parameter  $\alpha = 1$ , and Gaussian pinning potential in each layer. Note that the  $k$ -values deviate by about 5% from the continuum quadratic dispersion for high  $k$ , since they obey the lattice magnon dispersion relation Eq. (10). (c) Average magnon scattering angle  $\phi$  in layer 2 versus driven magnon wavenumber  $k_0$  and interlayer spacing  $l_\perp$  for various simulations, with skyrmion size  $\lambda = 5 l_B$ , interaction parameter  $\alpha = 1$ , and Gaussian pinning potential in each layer. The scattering profile of transduced magnons always resembles that of the driving layer, see Figs. 3(a,b), independent of the spacing. This shows that its behavior under different  $k_0$  is similar to the single-layer case for the magnon-skyrmion scattering shown in Sec. IV

presence of a noncollinear spin texture in both layers is necessary for Coulomb-mediated transduction, since the Pontryagin density in Eqs. (2) and (1) must be nonzero in both layers. Thus Coulomb interactions here facilitate a novel type of “spin drag” effect, with skyrmion textures functioning as the intermediary.

Each layer initially has a bare skyrmion background texture with  $Q_{\text{top}} = -1$ . Such a profile minimizes the stiffness energy and represents a static solution to the spin equations of motion in the absence of Coulomb self-interaction and external effects (static in a rotating frame in the presence of Zeeman) [1, 37]. In layer 1 (the “transmitting layer”), magnons with wavenumber  $k_0$  are driven from the bottom row and caught by an absorbing boundary layer (ABL). Magnons generated in layer 2 by spin drag are also dissipated by an ABL in that layer.

In order to ensure that the topological charge distribution stays static in the presence of Coulomb interactions and magnon scattering, we apply an external electric pinning potential to counter the inter-skyrmion repulsion and drift from scattering [32]. We incorporate an external pinning electric field  $\mathcal{E}(x, y)$  [Eq. (4)] with a preselected (e.g. Gaussian) potential and anneal the initial skyrmion textures with damping. In some cases, we choose an external electric field that is exactly negative the initial internal field, and which keeps the charge distribution in each skyrmion static, making annealing unnecessary. For computational efficiency, we also set the *intra-layer* interaction parameters  $\alpha_1 = \alpha_2 = 0$ , and retain only the interlayer parameter  $\alpha_{12} = 1$ . The effects of intralayer Coulomb  $\alpha_1$  and  $\alpha_2$  are mostly negated by the pinning

potential. The other parameters in this case are  $N = 140$ , and  $b = 0.13$ , while the driven magnon wavenumber is  $k_0 \approx 0.908 l_B^{-1}$ . A damping parameter  $\gamma_0 = 1.6$  is used to absorb the magnons at the top edge of each layer, see Eq. (11).

In layer 1, the magnons incident on the skyrmion scatter off at an angle due to the skyrmion Hall effect [33], as shown in Figure 3(a). An otherwise free skyrmion would also tend to drift down and to the right due to momentum transfer from the magnon pressure, but in this case the pinning potential holds it in place.

The novel effect of spin drag is visible in layer 2, Figure 3(b). When the driven magnons cross over the skyrmion in layer 1, they induce magnons which emanate from the skyrmion in layer 2. We quantify this interlayer magnon transduction through the absorption coefficient  $T$ . By measuring the average power due to damping by the ABL in each layer across a time interval near the end of each simulation, we can compare the outgoing forward magnon flux of each layer.  $T$  is the ratio of the magnon power in layer 2 to the total magnon power of both layers, see Eq. (14) for a precise definition.

Our results imply that skyrmions in coupled QHFM can act as media for magnon transduction, or spin drag, between layers. Figure 4(b) shows that for a minimal interlayer separation  $l_\perp = 0.1 l_B$ , which corresponds to an experimentally realizable separation of 2.6 nm [38] at  $B = 1$  Tesla, the transduction ratio  $T$  increases with the driven magnon wavenumber  $k_0$ , showing no saturation up to wavenumbers of order  $\pi/l_B$ . By contrast, larger separations produce a decreasing  $T$  without a pronounced

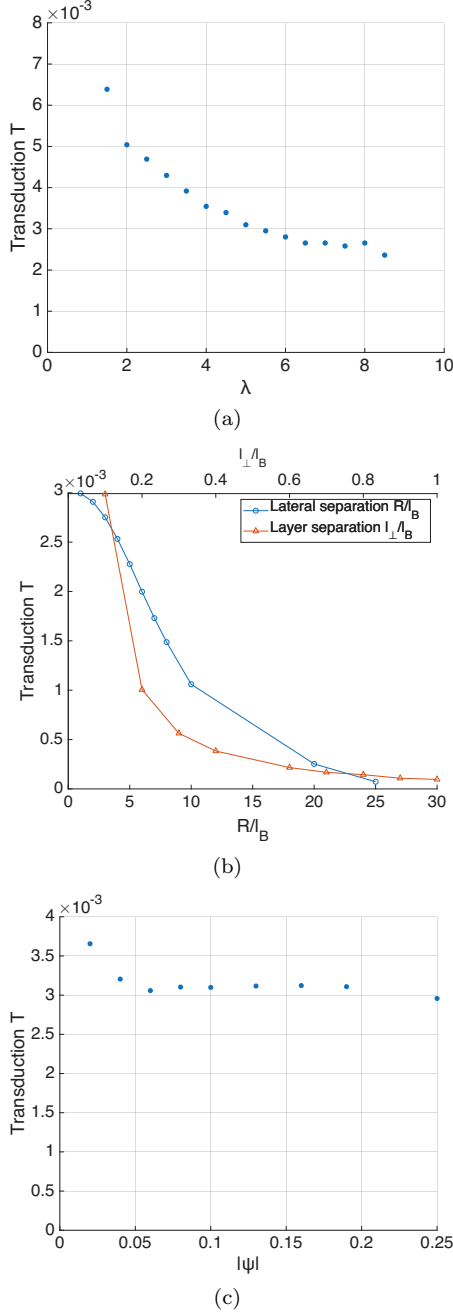


FIG. 5. Parameter dependence of Coulomb-mediated spin-drag simulations II. (a) Transduction ratio  $T$  versus skyrmion size  $\lambda$ . Smaller core sizes enhance the Coulomb interaction, producing larger transduction ratios. (b) Transduction ratio  $T$  versus lateral interlayer skyrmion core separation  $R = \sqrt{l_\perp^2 + l_{\text{offset}}^2}$  and interlayer spacing  $l_\perp$ , showing a roughly inverse relation  $T \sim 1/R$  for large distances. (c) Transduction ratio  $T$  versus incident magnon amplitude  $|\psi| = \sqrt{m_x^2 + m_y^2}$ . Note that the ratio is approximately independent of perturbation magnitude, suggesting that transduction occurs as an effective electrodynamic linear response of the composite two-skyrmion, two-layer system. The interlayer interaction parameter  $\alpha_{12} = 1$  and the driven magnon wavenumber is  $k_0 \approx 1.047 l_B^{-1}$ .

peak. The absence of a clear resonance is in part due to the geometry of the detection setup that we employ here: transduced magnons are detected only along the upper boundary of the second (receiver) layer. As shown in Figs. 4(c) and 7, smaller wavenumbers produce more diffuse, circular scattering in the first layer, and this is correspondingly transduced to the second. An alternative measure of transduced energy shown in Figure 9 indicates a more pronounced decrease with increasing wavenumbers for all but the smallest separation between layers.

It may also be that there exists a resonant frequency for spin drag when the wavelength  $\lambda = 2\pi/k_0$  is comparable to the size of the skyrmion, as is the case for the magnon Hall effect in reviewed in Sec. IV. For our simulations at very low  $k_0$ , the scattered and transduced waves involve strong core-mode undulations of the skyrmions that preferentially emit circular waves rather than preserving the magnons' forward momentum in each layer.

Figure 5(a,b) shows that the transduction  $T$  is optimized when the skyrmions are smaller and directly above each other, as the electric charge density and core-core Coulomb energy is increased. At the same time, spin drag is observed at arbitrary range: the layers can be separated by any finite distance, and the skyrmions themselves can be offset laterally with respect to each other. The transduction coefficient  $T$  appears to vary with inverse distance  $1/R$  for large  $R$  in each case, as shown in Figure 5(b), though this point-charge ansatz breaks down when the cores are laterally overlapping. Finally, Figure 5(c) shows that the transduction is unaffected by the strength of the layer-1 injected magnon's incoming amplitude; roughly the same fraction of the energy is transferred, independent of the amplitude. This suggests that the Coulomb-mediated spin-drag phenomenon studied here arises from an effective electrodynamic linear response in the two-layer, two-skyrmion system.

### 1. Estimation of effect size in experiment

To first order, the Coulomb-mediated spin drag phenomenon should scale linearly with the number of skyrmion pairs between layers. A particular simulation gave  $T \approx 0.00025$  for a pair of skyrmions with size  $\lambda \approx 5 l_B$ , layer separation  $l_\perp = 0.1 l_B$ , and lateral offset  $R = 20 l_B$ , with driven magnons of wavenumber  $k_0 \approx 1.047 l_B^{-1}$ . As such, our lower estimate for  $T$  in a real bilayer sample with  $\nu = 1 \pm 0.072$  and  $B = 1$  Tesla is  $3 \times 10^{-9}/\text{nm}^2$ , or  $T \sim O(10^{-2})$  for a sample of area  $9 \mu\text{m}^2$ .

Such a phenomenon could be verified experimentally using a bilayer version of the setup employed in [29]. Note that the phenomenon of spin drag requires non-trivial topological charge in both layers, created by doping each layer with electrons or holes. In our simulations, if the second layer is simply in the ferromagnetic ground state, we observe no transduction or interaction whatsoever. The spin drag effect is maximized for the smallest separation

between layers as shown in Figure 4(b), but the layers should not be too close together so as to enable significant interlayer tunneling [39, 40].

### III. NUMERICAL IMPLEMENTATION AND PARAMETERS

In this section, we summarize the numerical implementation of the semiclassical spin dynamics leading to the main results presented in Sec. II. The setup is discussed in the context of the bilayer spin drag calculations, although the same equations are employed for the single-layer magnon electrodynamics (Figure 2) and magnon-skyrmion scattering results (Figure 6).

In our simulation, we adapt the equations of motion [Eq. (3)] to use numerical derivatives in position and time:

$$\frac{\Delta \vec{m}}{\Delta \tau} = \vec{m} \times \vec{\mathcal{B}}_{\text{eff}} \quad (7a)$$

$$\begin{aligned} \vec{\mathcal{B}}_{\text{eff}} = & b\hat{\mathbf{z}} + (\vec{m}_{\mathbf{s}+\hat{\mathbf{x}}} + \vec{m}_{\mathbf{s}-\hat{\mathbf{x}}} + \vec{m}_{\mathbf{s}+\hat{\mathbf{y}}} + \vec{m}_{\mathbf{s}-\hat{\mathbf{y}}}) \\ & - 4\pi \left( \frac{\delta \varrho_{\mathbf{s}}}{\delta \vec{m}} V_{\mathbf{s}} + \frac{\delta \varrho_{\mathbf{s}-\hat{\mathbf{x}}}}{\delta \vec{m}} V_{\mathbf{s}-\hat{\mathbf{x}}} \right) \\ & - 4\pi \left( \frac{\delta \varrho_{\mathbf{s}-\hat{\mathbf{y}}}}{\delta \vec{m}} V_{\mathbf{s}-\hat{\mathbf{y}}} + \frac{\delta \varrho_{\mathbf{s}-\hat{\mathbf{x}}-\hat{\mathbf{y}}}}{\delta \vec{m}} V_{\mathbf{s}-\hat{\mathbf{x}}-\hat{\mathbf{y}}} \right) \end{aligned} \quad (7b)$$

Here we employ the dimensionless parameterization introduced in Eq. (4); the lattice spacing in position is the magnetic length  $l_B$ .

The continuous magnetization field in position space  $\vec{m}(\mathbf{r})$  is discretized over a square lattice, with lattice vector  $\mathbf{r} = l_B \mathbf{s}$ . Here  $[s_x, s_y]$  denote the integer coordinates of a lattice site. In Eq. (7),

$$\varrho_{\mathbf{s}} \equiv l_B^2 \varrho[\vec{m}(\tau, \mathbf{r} = l_B \mathbf{s})] \quad (8)$$

is the discretized Pontryagin density, not to be confused with the stiffness constant  $\rho_s$  used to set the energy units [Eq. (4)]. By replacing the triple product with a solid angle calculation on the unit sphere [41], we can find the charge density exactly in the discretized system. We choose a gauge where the scalar potential at a plaquette to the top-right of site  $\mathbf{s}$  is

$$V_{\mathbf{s}} = -\boldsymbol{\mathcal{E}} \cdot \mathbf{s} + \alpha \sum_{\mathbf{s}' \neq \mathbf{s}} \frac{\varrho_{\mathbf{s}'}}{|\mathbf{s} - \mathbf{s}'|} \quad (9)$$

and sum the electric contributions due to all four plaquettes neighboring the site. Time evolution is carried out via the fourth-order Runge-Kutta method.

All magnons in our discretized code obey the dispersion:

$$\Omega(\mathbf{k}) = b + 4 - 2 \cos(k_x l_B) - 2 \cos(k_y l_B), \quad (10)$$

where frequency  $\Omega$  has units of inverse dimensionless time  $\tau$ . This only approximates the continuum quadratic dispersion reasonably for low  $\mathbf{k}$ , so we only use low-energy magnons in the simulations.

Note that as this solution is energy conserving, the spins follow a path along an equipotential in the presence of the effective magnetic field rather than canting to align with it. Similarly, the charge tends to move perpendicularly to the effective electric field at constant velocity rather than along it. To locally minimize the energy, we can add a dissipative term to the equations of motion:

$$\frac{d\vec{m}}{d\tau} = \vec{m} \times \vec{\mathcal{B}}_{\text{eff}} - \gamma \vec{m} \times (\vec{m} \times \vec{\mathcal{B}}_{\text{eff}}), \quad (11)$$

where  $\gamma$  is the damping strength. This form can be derived from the Landau-Lifshitz-Gilbert equation, which has been shown to phenomenologically describe dissipative effects in micromagnetics [42].

The system's change in total energy over time can be found with

$$\frac{dE}{d\tau} = \frac{d\vec{m}}{d\tau} \cdot \vec{\mathcal{B}}_{\text{eff}}. \quad (12)$$

Using the LLG equation (11), we can find the change in energy specifically due to dissipative effects:

$$\frac{dE}{d\tau} = -\gamma \left[ (\vec{\mathcal{B}}_{\text{eff}} \cdot \vec{m})^2 - |\vec{\mathcal{B}}_{\text{eff}}|^2 \right] \quad (13)$$

From this, by measuring the power of dissipation due to the absorbing boundary layers (ABLs) of each layer separately, we can compare the intensities of magnons scattered from the skyrmions into each layer, specifically from the driving layer (layer 1) to the receiving layer (layer 2). We define the transduction ( $T$ ) and transmission ( $P$ ) coefficients as follows,

$$T = \left( \frac{dE_{L2}}{d\tau} \right) / \left( \frac{dE}{d\tau} \right), \quad (14a)$$

$$P = 1 - T, \quad (14b)$$

where  $E_{L2}$  denotes the energy absorbed in layer 2, while  $E$  is the total energy absorbed in both layers. However, since the skyrmion texture is not completely localized, the ABL in each layer can absorb energy from the tails of the skyrmion configurations, even if the texture is first subject to annealing (damping everywhere). As such, to isolate the power due to magnons alone in each simulation, we subtract the power absorbed from an identical simulation *without* driven spins.

### IV. SINGLE-LAYER DYNAMICS

In this section we describe the analytical approach to magnon-charge scattering. We also present additional numerical results for magnon-skyrmion scattering in both the presence and absence of Coulomb interactions. The latter are used to benchmark our numerics against previous studies.

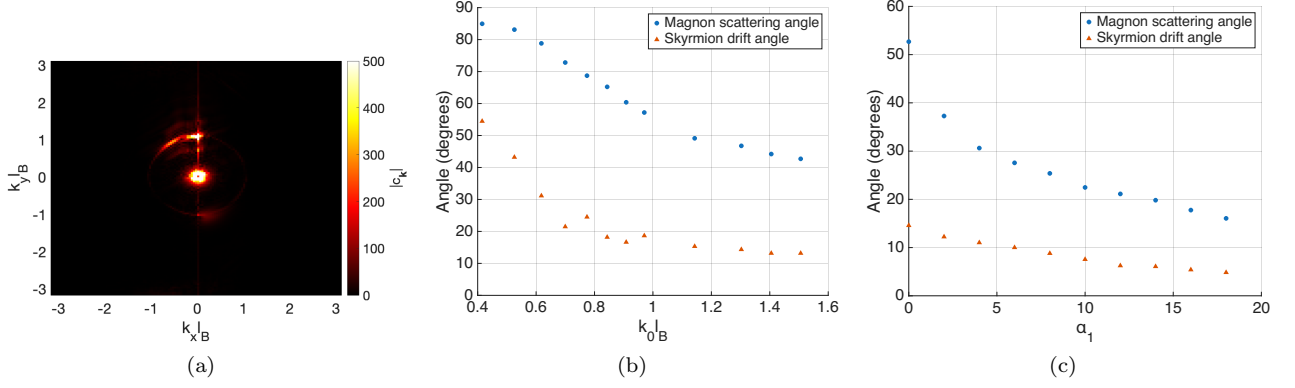


FIG. 6. Single-layer magnon-skyrmion scattering simulations. Panels (a) and (c) include Coulomb effects, while these are neglected in panel (b). Results are presented for a skyrmion of size  $\lambda \approx 6 l_B$ . (a)  $k$ -space distribution of scattered magnons at end of a simulation, with Coulomb interaction parameter  $\alpha = 2$  and incident magnon wavenumber  $k_0 = 1.0472 l_B^{-1}$ . (b) Magnon scattering angle and skyrmion drift angle versus magnon driving wavevector  $k_0$ , with  $\alpha = 0$ . Here we neglect Coulomb effects; results are presented to benchmark our numerics for spin-stiffness mediated interactions [32, 33]. (c) Magnon scattering angle and skyrmion drift angle versus Coulomb strength  $\alpha$ , with  $k_0 = 1.0472 l_B^{-1}$ . The main effect of Coulomb interactions is to bring both the magnon scattering angle and skyrmion recoil directions closer to the vertical incident line.

### A. Magnon-point charge interactions

We can expand Eq. (1) to second order in magnon fluctuations around a ferromagnetic ground state, leading to the action

$$S_\Pi \simeq \int dt d^2\mathbf{r} \left\{ \bar{\Pi} [i s n \partial_t + \rho_s \nabla^2 + s n g \mu B] \Pi \right\} \quad (15a)$$

$$+ \int dt d^2\mathbf{r} \vec{E} \cdot \left( \frac{e}{4\pi} \vec{J} \times \hat{z} \right), \quad (15b)$$

where  $\Pi \equiv \frac{1}{\sqrt{2}} (m^x + i m^y)$  is the complex boson field operator for magnons, and

$$\vec{J} \equiv -\frac{i}{2} \bar{\Pi} \left( \vec{\nabla} - \overleftarrow{\nabla} \right) \Pi \quad (16)$$

is the Noether current associated to U(1) rotational invariance around the  $z$ -axis. Note that as the electrical potential term now manifests as a coupling between the *electric field* and the *magnon current* (both time-reversal even, polar-vector quantities), rather than the topological charge. Magnons carry an effective electric dipole moment density  $\vec{d} \equiv \frac{e}{4\pi} \vec{J} \times \hat{z}$  and can scatter off of non-uniform electric fields despite possessing zero net Pontryagin (electric) charge.

Rescaled in terms of our dimensionless simulation parameters and with lattice spacing  $l_B = 1$ , an effective single-particle Hamiltonian can be extracted from the action:

$$S_\Pi = \frac{1}{4\pi} \int_{\tau, \mathbf{r}} \bar{\Pi} \left( i \partial_\tau - \hat{h} \right) \Pi \quad (17a)$$

$$\hat{h} = b - \nabla^2 - \frac{i \epsilon^{\beta\alpha}}{2} [\partial_\beta \mathcal{E}^\alpha(\hat{\mathbf{r}}) + \mathcal{E}^\alpha(\hat{\mathbf{r}}) \partial_\beta] \quad (17b)$$

$$= b + \frac{1}{2\mathcal{M}} \left[ \hat{\mathbf{P}} - \mathcal{A}(\hat{\mathbf{r}}) \right]^2 + \mathcal{V}(\hat{\mathbf{r}}), \quad (17c)$$

where  $\mathcal{M} = 1/2$  is the magnon mass,  $\hat{\mathbf{P}} = -i\nabla$  is the magnon momentum operator, and  $\mathcal{A}(\mathbf{r})$ ,  $\mathcal{V}(\mathbf{r})$  are the synthetic vector and scalar potentials for the magnon due to the physical electric field  $\mathcal{E}(\mathbf{r})$ , defined via Eq. (6).

To find the wave function of a magnon in position space subject to the perturbing Hamiltonian  $\hat{h}_1 \equiv -\left[ \hat{\mathbf{P}} \cdot \mathcal{A}(\hat{\mathbf{r}}) + \mathcal{A}(\hat{\mathbf{r}}) \cdot \hat{\mathbf{P}} \right]$ , we must solve the Schrödinger equation:

$$\{(\nabla^2 + \omega) - i[\nabla \cdot \mathcal{A}(\hat{\mathbf{r}}) + \mathcal{A}(\hat{\mathbf{r}}) \cdot \nabla]\} \psi = 0, \quad (18)$$

where  $\omega = k^2 + i\eta$  is the on-shell frequency of an unperturbed magnon, and the energy shift  $b$  is ignored. A formal solution is

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + 2i \int d^2\mathbf{r}' G(\omega, \mathbf{r} - \mathbf{r}') [\mathcal{A}(\mathbf{r}') \cdot \nabla'] \psi(\mathbf{r}'), \quad (19)$$

where  $\psi_0(\mathbf{r})$  denotes a solution to the unperturbed problem, and

$$G(\omega, \mathbf{r}) \equiv \frac{1}{4i} H_0^{(1)}(r\sqrt{\omega}) \quad (20)$$

is the retarded free-space Green's function for 2+1-D ferromagnetic outgoing magnons. In the above,  $H_0^{(1)}$  is the Hankel function of the first kind, and we have used the fact that  $\nabla \cdot \mathcal{A} = 0$  in this case.

The first Born approximation for the scattered wave amplitude  $\delta\psi_B^{(1)}(\mathbf{r})$  obtains by replacing  $\psi(\mathbf{r}')$  on the right-hand side (RHS) of Eq. (19) by the free-propagating wave  $\psi_0(\mathbf{r}')$ , while the second Born amplitude  $\delta\psi_B^{(2)}$  replaces  $\psi \rightarrow \delta\psi_B^{(1)}$  on the RHS of Eq. (19). Together, these give

$$\psi(\mathbf{r}) \simeq \psi_0(\mathbf{r}) + \delta\psi_B^{(1)}(\mathbf{r}) + \delta\psi_B^{(2)}(\mathbf{r}) + O(kQ\alpha)^3, \quad (21a)$$

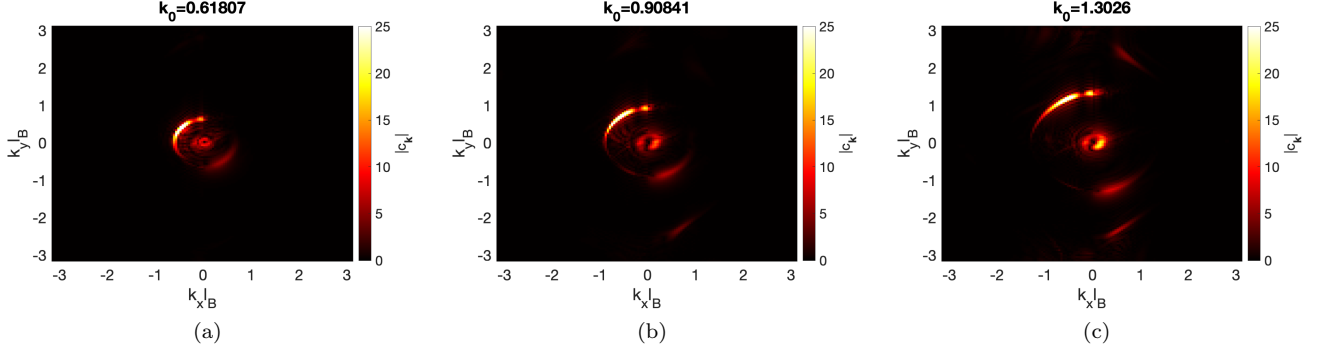


FIG. 7. Coulomb-mediated spin-drag simulations III. The plots depict the  $k$ -space distribution of magnons in the receiving layer at the end of a driven simulation, with skyrmion size  $\lambda = 5 l_B$ , interaction parameter  $\alpha = 1$ , interlayer separation  $l_\perp = 0.1 l_B$ , and Gaussian pinning potential in each layer. As in Figure 4(a), the results of the undriven second layer (with stable skyrmion texture) are subtracted here to highlight the spin drag. Results are depicted for three different layer-one driven magnon wave vectors  $k_0$ . These panels show that the emitted magnons give a more diffuse, circular scattering pattern at smaller wave numbers. This is partly responsible for the absence of a clear resonance in the transduction versus  $k_0$  plot, Figure 4(b).

where the on-shell amplitude is

$$\psi_0(\mathbf{r}) = e^{ikr \sin \phi}, \quad (21b)$$

$$\delta\psi_B^{(1)}(\mathbf{r}) = -\frac{\pi k Q \alpha}{2\sqrt{2}} \frac{\cos \phi}{\sqrt{1 - \sin \phi}} H_0^{(1)}(kr), \quad (21c)$$

$$\delta\psi_B^{(2)}(\mathbf{r}) = -\frac{\pi(kQ\alpha)^2}{16} f(\phi) H_0^{(1)}(kr) \quad (21d)$$

$$f(\phi) = \int_0^{2\pi} d\phi' \sqrt{[1 - \cos(\phi - \phi')]^3 [1 - \sin(\phi')].} \quad (21e)$$

Here we assume an incident plane wave state  $\psi_0 = e^{iky}$ . We note that the scattering from a charge in the plane of the magnon produces a non-analytic form factor already in the first-Born approximation, Eq. (21c). The magnitude of the wavefunction in Eq. (21) is visualized in Figure 2(b), for  $k = 1.5$  and  $Q\alpha = -0.4$ .

This expansion is valid for  $kQ\alpha \ll 1$ , and in this realm the model clearly resembles our numerical findings. The first-order term in  $kQ\alpha$  modifies the plane wave such that the wave-function magnitude varies across space in a distribution similar to Figure 2(a). The second-order correction  $\delta\psi_B^{(2)}(\mathbf{r})$  (significant for strong point charges) magnifies the outgoing Hankel function to one side, corresponding with the sign of the charge.

## B. Magnon-skyrmion interactions

In chiral magnets, magnons are known to scatter when passing over a skyrmion background texture, and to impart a recoil drift velocity to the impacted skyrmion [32]. In this subsection, we specialize our code to the single-layer case to demonstrate that this phenomenon also exists in QHFM.

### 1. Magnon and skyrmion Hall angle vs $k$

It has been shown numerically [32] that in a ferromagnet with Dzyaloshinskii-Moriya interactions, the magnon scattering angle and skyrmion drift angle are maximized when the incident magnon wavelength equals the skyrmion size. In addition, due to conservation of momentum, the magnon scattering angle should always be twice the skyrmion drift angle. In our case, it is difficult to judge the deflection angle at this resonant frequency, as the incident magnons excite internal modes within the skyrmion that emit circular waves. However, the skyrmion drift angle is maximized at this frequency, and the conservation ratio roughly holds for higher magnon wavenumbers, as shown in Figure 6(b).

### 2. Magnon and skyrmion Hall angle vs $\alpha$

We also vary the Coulomb strength  $\alpha$  in these single-layer scattering simulations to discern the influence of this interaction unique to QHFM. The skyrmion is first left to expand and equilibrate, and then magnons are driven. We find that increasing the Coulomb interaction parameter  $\alpha$  slightly decreases both the scattering angle and the skyrmion drift angle, as shown in Figure 6(c). However, large values of  $\alpha$  produce unreliable results, as they allow for magnon-magnon interactions that produce a variety of different wavenumbers. The angle variation could be influenced by the skyrmion's size varying with  $\alpha$ , but earlier results from Sec. II show that magnons interact with the electric field a skyrmion produces as well as its texture, which is non-negligible here.

## V. CONCLUSION

Using semiclassical simulations, we have examined two effects that arise from Coulomb interactions in the magnetization dynamics of QHFM. First, we demonstrated magnon deflection by an electric charge, Figure 2. This arises due to the magnon electric dipole moment, which is proportional but perpendicular to the spin current.

Second, we observe a Coulomb-mediated “spin drag” effect between disconnected, adjacent layers, Figs. 1 and 3–5. The effect occurs due to Coulomb-coupling between undulations in the topological textures of both layers, induced by magnon injection in one layer and observed as magnon generation in the other. The degree of power transduction between layers was quantified in terms of skyrmion densities achievable by small doping away from  $\nu = 1$ , see Sec. IIC 1.

Multiple avenues exist for further studies. First, these ideas can be generalized to other types of quantum Hall magnets with higher symmetries and more complicated defects, e.g.  $SU(N)$  skyrmions [43]. Second, one can also explore the dynamics of magnetic textures in QHFM analogs reported in moiré materials [20, 23, 27]. Another direction is to incorporate quantum fluctuations, which have been neglected in this work.

## ACKNOWLEDGMENTS

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### Appendix A: Skyrmion texture parameterization

Retaining only the stiffness interaction [i.e., setting the Coulomb interaction parameter  $\alpha = 0$ , Eq. (4)], the magnetization field for a stable skyrmion with  $Q_{\text{top}} = -1$  can be written as [37]

$$\vec{m}(r, \phi) = \left[ \frac{4\lambda r \cos \phi}{r^2 + 4\lambda^2}, \frac{4\lambda r \sin \phi}{r^2 + 4\lambda^2}, \frac{r^2 - 4\lambda^2}{r^2 + 4\lambda^2} \right], \quad (\text{A1})$$

where  $(r, \phi)$  are polar coordinates in the sample plane, the core is centered at the origin, and  $\lambda$  denotes half of the skyrmion radius. In our spin simulations, the addition of Coulomb interactions to such a bare skyrmion profile makes the skyrmion expand a bit before stabilizing.

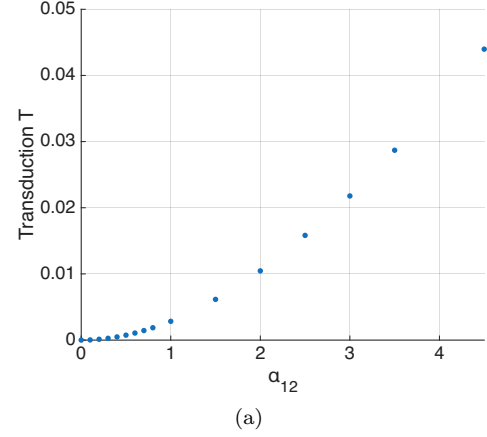


FIG. 8. Transduction ratio  $T$  vs varied interlayer Coulomb strength  $\alpha_{12}$ , with skyrmion size  $\lambda = 5 l_B$ , constant Gaussian pinning potential in each layer, driven magnon wavenumber  $k_0 \approx 1.047 l_B^{-1}$ , and interlayer spacing  $l_\perp = 0.1 l_B$ .

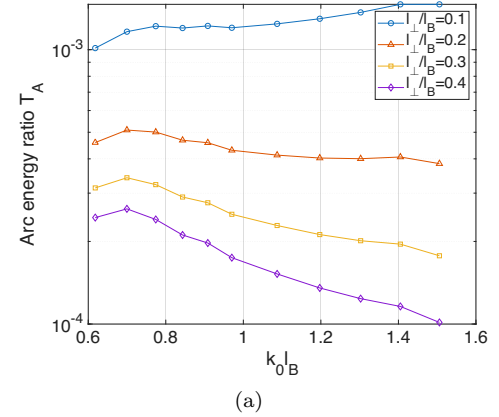


FIG. 9. Transduction ratio  $T_A$  (captured by arc-energy ratio, see Appendix B) versus driven magnon wavenumber  $k_0$  and interlayer spacing  $l_\perp$  for various simulations, with skyrmion size  $\lambda = 5 l_B$ , Coulomb strength  $\alpha = 1$ , and Gaussian pinning potential in each layer.

### Appendix B: Supplemental graphs

Figure 7 shows the magnitude of the  $k$ -space distribution of magnons in the second (“receiving”) layer at the end of a transduction simulation, for varied driving wavenumbers  $k_0$  [see Figure 4(b)]. The result of a “base case” without driven magnons is subtracted to graphically reduce the strong standing modes of the skyrmion, so that the weak transduced magnon modes are relatively brighter. The modes seen near the origin are due to the phase shift from the skyrmion drifting and deforming. We can see that the higher-momentum driven magnons glance off of the skyrmion at a smaller angle, but with a similar profile.

Figure 8 shows the transduction ratio  $T$  resulting from simulations with varied interlayer Coulomb strength  $\alpha_{12}$ .

The graph suggests a nonlinear relationship, perhaps that the interlayer magnon interaction term is second-order in  $\alpha$ . In reality, this parameter is a fixed number of order one, because the spin stiffness is itself mediated by Coulomb exchange.

Figure 9 depicts an alternative means of quantifying interlayer magnon transduction, compare to the fixed detector geometry assumed in Figure 4(b). Here, the transduction ratio  $T_A$  is calculated in a different manner so as to account for all outgoing magnon modes equally. Whereas  $T$  [Figure 4(b)] is the fraction of the power caught

by the ABL in layer 2, favoring forward momentum as an experimental setup might,  $T_A$  [Figure 9] is the fraction of the energy in layer 2 obtained by summing the energy contributions of every mode on the  $|\mathbf{k}| = k_0$  arc in  $k$ -space within  $-\pi/4 < \phi < 3\pi/4$ , each contribution of which is proportional to its amplitude squared:  $E(\mathbf{k}) \sim |c_{\mathbf{k}}|^2$ . This is intended to more accurately describe low- $k_0$  cases where the scattering profile is mostly circular to the left [Figure 7], but the graph shows a mostly similar result: a positive  $T_A$  vs  $k_0$  correlation for low interlayer spacing and a negative one for most others.

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