High-Quality Axion Models with the Anomalous $U(1)_X$ Gauge Symmetry

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We propose the generic high-quality axion models with anomalous $U(1)_X$ gauge symmetry and vector-like particles. We briefly review the gauge anomaly cancellations via the Green-Schwarz mechanism, study the breaking of the $U(1)_X$ gauge symmetry, as well as derive the Nambu-Goldstone boson, Peccei-Quinn (PQ) axion, and axion decay constant in general. The high-dimensional operators, which break the $U(1)_{PQ}$ global symmetry, have dimension eleven or higher due to the anomalous $U(1)_X$ gauge symmetry, and thus the axion quality problem is solved. In particular, unlike the high-quality axion models with anomaly free $U(1)$ gauge symmetry, we only need to introduce two pairs of vector-like particles. To be concrete, we present three specific models with two pairs of vector-like particles. We show that gauge anomalies in all three models can be canceled via the Green-Schwarz mechanism. To achieve gauge coupling unification, we need to introduce additional vector-like particles only in Model I. We find that gauge coupling unification is achieved at the unification scale around 10^{16} GeV with a relative error of less than 1%. Notably, gauge coupling unification in Model II is achieved naturally with the smallest relative error of 0.1%.

INTRODUCTION

As we know, there exists a topological term for the Quantum Chromodynamics (QCD) theory in the Standard Model (SM) as follows

$$\mathcal{L} \supset \frac{g_3^2}{16\pi^2}\,\overline{\theta}\,\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) \,, \tag{1}$$

where g_3 is the strong gauge coupling, $G_{\mu\nu}$ is the gluon field strength, $\tilde{G}^{a,\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}$ is its dual. In particular, $\overline{\theta}$ is a dimensionless parameter, and it is the sum of the original θ and the contributions from the quark mass matrices. This topological term violates the CP symmetry, and can generate the neutron electric dipole moment at the order of $d_n \sim (10^{-16}\,\overline{\theta})\,e$-cm [1]. The current experimental bound on d_n is $|d_n| \leq 1.8\times 10^{-26}\,e$-cm [2], and thus gives a stringent constraint on $|\overline{\theta}| \leq 10^{-10}$. This fine-tuning problem is called the strong CP problem, which might indicate the new physics beyond the SM.

There are a few solutions to the strong CP problem, and the natural solution is the Peccei-Quinn (PQ) mechanism [3]. In this mechanism, we introduce a global $U(1)_{PQ}$ symmetry, which is anomalous under $SU(3)_C$ and can be broken by the instanton effect. Because the peudo Nambu-Goldstone boson (pNGB) from the $U(1)_{PQ}$ symmetry breaking cancels the strong CP phase $\overline{\theta}$ dynamically, the strong CP problem is solved elegantly [4, 5].

The original Weinberg–Wilczek axion [4, 5] has already been excluded by various experiments, in particular by the non-observation of the rare decay $K \to \pi + a$, for recent review, see Ref. [6]. To evade the experimental bounds, two viable "invisible" axion models has been proposed: (1) The Kim–Shifman–Vainshtein–Zakharov (KSVZ) axion model, which introduces a SM singlet and a pair of extra vector-like quarks that carry $U(1)_{PQ}$ charges while the SM fermions and Higgs fields are neutral under $U(1)_{PQ}$ symmetry [7, 8]; (2) The Dine–Fischler–Srednicki–Zhitnitskii (DFSZ) axion model, in which a SM singlet and one pair of Higgs doublets are introduced, and the SM fermions and Higgs fields are charged under $U(1)_{PQ}$ symmetry [9, 10]. Interestingly, the QCD axion emerges as a compelling cold dark matter candidate, and it yields the correct relic density if its decay constant f_a is about 10^{11} GeV.

However, it is well-known that all the global symmetries in the nature can be violated by the quantum gravity effects, for example, black holes and worm holes. Thus, the $U(1)_{PQ}$ symmetry can be broken by the non-renormalizable high-dimensional operators suppressed by the reduced Planck scale. The $\overline{\theta}$ parameter will be shifted away from zero significantly, and then the strong CP problem cannot be explained, unless the coefficients of these non-renormalizable operators, which violate the $U(1)_{\rm PQ}$ symmetry, are extremely small, or the dimensions of these non-renormalizable operators are large. To be concrete, in order to preserve the solution to the strong CP problem, one obtains that the coefficients of the dimension-five operators should be smaller than 10^{-50} [11, 12], or the dimensions of the non-renormalizable operators are larger than or equal to 11 for $f_a \geq 10^{11}$ [13]. So there exists the axion quality problem due to the severe fine-tuning [14–17].

Various mechanisms, which can solve the axion quality problem, have been proposed: the accidental $U(1)_{\rm PQ}$ symmetries arising from the gauged $U(1)$ symmetries [11, 12, 16, 18, 19] or non-Abelian gauge symmetries [20, 21], the composite axions [13, 22–28], the discrete gauged PQ symmetries from anomalous $U(1)_A$ gauge theory in string theory [29, 30], the cancellations of θ terms due to mirror symmetry [31, 32], the non-linearly realized discrete symmetries due to the N copies of the SM [33, 34], the QCD axion from the five-dimensional parity odd gauge field [35], as well as the string axions [36], etc.

For the solutions to the axion quality problem via the accidental $U(1)_{\rm PQ}$ symmetries arising from the gauged $U(1)$ symmetries [11, 12, 16, 18, 19], we need to introduce eleven pairs of vector-like particles for the renormalizable theory, or equivalently introduce eleven pairs of vector-like particles for the non-renormalizable theory which arises from the renormaliable theory, where we count both the vector-like particles in the high-dimensional operators as well as the vector-like particles which are integrated out. To be concrete, to have dimension n operators $XF\overline{XF}\Phi^{n-3}/M_*^{n-4}$ with $n\geq 4$ from the renormalizable theory, we need to introduce $n-3$ pair of vector-like particles. It is trivial for $n=4$, and has been proved for $n=8$ explicitly in Ref. [12].

In this paper, to construct the high quality axion model with less number of the vector-like particles, we consider the anomalous $U(1)_X$ gauge symmetry whose gauge anomalies are canceled by the Green-Schwarz mechanism [37]. First, we propose the generic high-quality axion models with anomalous $U(1)_X$ gauge symmetry and vector-like particles. We briefly review the gauge anomaly cancellations via the Green-Schwarz mechanism, study the breaking of the $U(1)_X$ gauge symmetry, as well as derive the Nambu-Goldstone boson, Peccei-Quinn (PQ) axion, and axion decay constant in general. The high-dimensional operators, which break the $U(1)_{PQ}$ global symmetry, have dimension eleven or higher due to the anomalous $U(1)_X$ gauge symmetry, and thus the axion quality problem is solved. Second, we propose three models. In Model I, we introduce two pairs of the vector-like particles which belong to the fundamental and anti-fundamental representations of $SU(5)$ model. We cancel the gauge anomalies via the Green-Schwarz mechanism. To achieve gauge coupling unification, we introduce the TeV-scale vector-like particles. Also, we discuss the variation of this model. In Model II, we introduce two pair of vector-like particles which belong to the anti-symmetric representation and its Hermitian conjugate of the flipped $SU(5)$ model. The gauge anomalies can be canceled by Green-Schwarz mechanism. In particular, the gauge coupling unification can be achieved naturally. By the way, the SM singlets in the anti-symmetric representation and its Hermitian conjugate can be considered as the right-handed neutrinos, or we do not need to introduce them. In Model III, we introduce two pairs of vector-like particles: the quantum numbers for one pair of vector-like particles are the same as the quark doublet and its Hermitian conjugate, and the quantum numbers for the other pair of vector-like particles are the same as the righ-handed down-type quark and its Hermitian conjugate. Also, the gauge anomalies can be canceled by Green-Schwarz mechanism. Moreover, these vector-like particles can obtain the masses from the renormalizable Yukawa couplings. Interestingly, if these vector-like particles obtain the masses from the non-renormalizable dimension-five Yukawa couplings, their masses can be around TeV scale. We show that we can achieve the gauge coupling unification as well.

THE GENERIC HIGH QUALITY AXION MODELS WITH ANOMALOUS $U(1)_X$ GAUGE SYMMETRY

First, let us introduce our convention, which is similar to the supersymmetric SM, i.e., all the chiral fermions in the left-handed format. We introduce the vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugates. Their quantum numbers under $SU(3)_C\times SU(2)_L\times U(1)_Y$ and their contributions to one-loop beta functions, $\Delta b\equiv(\Delta b_1,\Delta b_2,\Delta b_3)$ are [38]

$$
\begin{aligned}
&XQ+\overline{XQ}=(\mathbf{3},\mathbf{2},\mathbf{\frac{1}{6}})+(\mathbf{\bar{3}},\mathbf{2},-\mathbf{\frac{1}{6}})\,,\quad \Delta b=(\frac{2}{15},2,\frac{4}{3})\,;\\
&XU+\overline{XU}=(\mathbf{3},\mathbf{1},\mathbf{\frac{2}{3}})+(\mathbf{\bar{3}},\mathbf{1},-\mathbf{\frac{2}{3}})\,,\quad \Delta b=(\frac{16}{15},0,\frac{2}{3})\,;\\
&XD+\overline{XD}=(\mathbf{3},\mathbf{1},-\mathbf{\frac{1}{3}})+(\mathbf{\bar{3}},\mathbf{1},\mathbf{\frac{1}{3}})\,,\quad \Delta b=(\frac{4}{15},0,\frac{2}{3})\,;\\
&XL+\overline{XL}=(\mathbf{1},\mathbf{2},\mathbf{\frac{1}{2}})+(\mathbf{1},\mathbf{2},-\mathbf{\frac{1}{2}})\,,\quad \Delta b=(\frac{6}{15},\frac{2}{3},0)\,;\\
&XE+\overline{XE}=(\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},-\mathbf{1})\,,\quad \Delta b=(\frac{12}{15},0,0)\,,\\
&XN+\overline{XN}=(\mathbf{1},\mathbf{1},\mathbf{0})+(\mathbf{1},\mathbf{1},\mathbf{0})\,,\quad \Delta b=(0,0,0)\,.\\
&XG=(\mathbf{8},\mathbf{1},\mathbf{0})\,,\quad \Delta b=(0,0,2)\,;\\
&XW=(\mathbf{1},\mathbf{3},\mathbf{0})\,,\quad \Delta b=(0,\frac{4}{3},0)\,;\\
&XB=(\mathbf{1},\mathbf{1},\mathbf{0})\,,\quad \Delta b=(0,0,0)\,.
\end{aligned}
$$

We define

$$
\begin{aligned}
&XF\equiv XD+\overline{XL}\ ,\ \overline{XF}\equiv\overline{XD}+XL\,;\\
&XT\equiv XQ+\overline{XD}+\overline{XN}\ ,\ \overline{XT}\equiv\overline{XQ}+XD+XN\,.
\end{aligned}
$$

Thus, XF and $\overline{XF}$ belong to the fundamental and anti-fundamental representations of $SU(5)$ model, and XT and $\overline{XT}$ belong to the anti-symmetric representation and its Hermitian conjugate of flipped $SU(5)$ model. Their quantum numbers under $SU(5)$/flipped $SU(5)$ models, and their contributions to one-loop beta functions, $\Delta b\equiv$

Particles	$\boldsymbol{U(1)_X}$	$\boldsymbol{U(1)_{PQ}}$
$(Xf, \overline{Xf})$	p	q
$(Xf', \overline{Xf'})$	$-q$	p
Φ_1	$q_1 = -2p$	$-2q$
Φ_2	$q_2 = 2q$	$-2p$

TABLE I. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields. In particular, p and q are coprime positive integers.

$(\Delta b_1, \Delta b_2, \Delta b_3)$ are

$$XF + \overline{XF} = \mathbf{5} + \bar{\mathbf{5}}\,, \quad \Delta b = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3})\,;$$

$$XT + \overline{XT} = (\mathbf{10}, \mathbf{1}) + (\overline{\mathbf{10}}, -1)\,, \quad \Delta b = (\frac{2}{5}, 2, 2)\,.$$

In general, we introduce two pairs of vector-like particles $(Xf, \overline{Xf})$ and $(Xf', \overline{Xf'})$. To realize the high quality axions and cancel the gauge anomalies via Green-Schwarz mechanism, $(Xf, \overline{Xf})$ and $(Xf', \overline{Xf'})$ cannot be singlets under $SU(3)_C \times SU(2)_L$ and are neutral under $U(1)_Y$. To break the $U(1)_X$ gauge symmetry, we introduce two Higgs fields Φ_1 and Φ_2. We assume that the $U(1)_X$ charges for $Xf/\overline{Xf}$, $Xf'/\overline{Xf'}$, Φ_1 and Φ_2 are p, $-q$, q_1, and q_2. For renormalizable theory, we have $q_1 = \pm 2p$, and $q_2 = \pm 2q$. While for the non-renormalizable theory with dimension-five operators, we have $q_1 = \pm p$, and $q_2 = \pm q$. To be concrete, we further assume that p and q are positive integers and coprime, and $q_1 = -2p$, and $q_2 = 2q$. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields are given in Table I. The discussions for the other values of p, $-q$, q_1, and q_2 are similar.

It is easy to show that the $\left[U(1)_Y U(1)_X^2\right]$ gauge anomaly is canceled, and we have the gauge anomalies $A_1 \equiv \left[U(1)_Y^2 U(1)_X\right]$, $A_2 \equiv \left[SU(2)_L^2 U(1)_X\right]$, $A_3 \equiv \left[SU(3)_C^2 U(1)_X\right]$, $A_{1X} \equiv \left[U(1)_X^3\right]$, and gravity anomaly $A_{\text{gravity}} \equiv \left[\text{Gravity}^2 U(1)_X\right]$. These anonalies can be canceled via the Green-Schwarz mechanism.

In weakly coupled heterotic string model building, in general we have an anomalous $U(1)_X$ gauge symmetry with its anomalies canceled by the Green–Schwarz mechanism [37]. For Type II orientifold string model building, we have more than one anomalous $U(1)_X$ gauge symmetries whose anomalies can be cancelled by the generalized Green–Schwarz mechanism [39]. The Green–Schwarz anomaly cancellation conditions from an effective theory point of view are [40–42]

$$\frac{A_i}{k_i} = \frac{A_{1X}}{k_{1X}} = \frac{A_{\text{gravity}}}{12} = \delta_{GS}\ , \tag{2}$$

where $i = 1,\ 2,\ 3$. Also, k_i and k_{1X} are the levels of the corresponding Kac–Moody algebra, and δ_{GS} is a constant which is not specified by low-energy theory alone. For a non-Abelian group, k_i is a positive integer, while for the $U(1)$ gauge symmetry, k_i needs not be an integer. To be concrete, k_1 and k_{1X} are positive real numbers, and k_2 and k_3 are positive integers. Thus, the anomaly cancellation conditions for A_1 and A_{1X} are not very useful from an effective low energy theory point of view. Also, the anomalies A_{1X} and A_{gravity} have an arbitrariness due to the normalization of $U(1)_X$, and they can be easily canceled by introducing SM singlet fermions which are charged under $U(1)_X$. Thus, we shall not consider them here. From the four-dimensional effective field theory point of view, we can cancel the gauge anomalies $A_1 \equiv \left[U(1)_Y^2 U(1)_X\right]$, $A_2 \equiv \left[SU(2)_L^2 U(1)_X\right]$, $A_3 \equiv \left[SU(3)_C^2 U(1)_X\right]$, $A_{1X} \equiv \left[U(1)_X^3\right]$, and gravity anomaly $A_{\text{gravity}} \equiv \left[\text{Gravity}^2 U(1)_X\right]$ by introducing two-form fields and its dual singlet scalars. By the way, all the other gauge anomalies such as $G_i G_j G_k$ and $[U(1)_X]^2 \times G_i$ must vanish, and we can easily confirm that these conditions are satisfied.

The Lagrangian for the Yukawa Couplings is

$$-\mathcal{L} = y_{Xf} Xf \overline{Xf} \Phi_1 + y_{Xf'} Xf' \overline{Xf'} \Phi_2\ . \tag{3}$$

And the Higgs potential at the renormalizable level is

$$\begin{aligned} V &= M_1^2 |\Phi_1|^2 + M_2^2 |\Phi_2|^2 + \frac{\lambda_1}{4} |\Phi_1|^4 + \frac{\lambda_2}{4} |\Phi_2|^4 \\ &\quad + \lambda_{12} |\Phi_1|^2 |\Phi_2|^2\ . \end{aligned} \tag{4}$$

After Φ_i obtain the Vacuum Expectation Values (VEVs), we parametrize Φ_i as

$$\Phi_i = \frac{1}{\sqrt{2}} \left(\phi_i + f_i\right) e^{i b_i / f_i}\ , \tag{5}$$

where ϕ_i are the radial components, b_i are the axial components, and f_i are the VEVs of Φ_i. The domains of b_i/f_i are $[0, 2\pi)$.

For the axial components, the $U(1)_X$ gauge symmetry becomes shift symmetry, and the transformations of axial components are

$$b_i \to b_i + q_i f_i \epsilon\ . \tag{6}$$

For the Higgs potential in Eq. (4), we have two global symmetries from the phases of Φ_i: one is the $U(1)_X$ gauge symmetry, and the other is the $U(1)_{PQ}$ symmetry. Let us study it in details. From the kinetic terms of Φ_i, we obtain

$$\begin{aligned} \mathcal{L} &= |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 \\ &\supset \frac{1}{2} \partial^\mu b_1 \partial_\mu b_1 + \frac{1}{2} \partial^\mu b_2 \partial_\mu b_2 \\ &\quad - \left(q_1 f_1 \partial^\mu b_1 + q_2 f_2 \partial^\mu b_2\right) g_X A_{X\mu} \\ &\quad + \frac{1}{2} g_X^2 \left(q_1^2 f_1^2 + q_2^2 f_2^2\right) A_X^\mu A_{X\mu}\ , \end{aligned} \tag{7}$$

where $A_{X\mu}$ is gauge field and g_X is gauge coupling of the $U(1)_X$ gauge theory. The mass of the $U(1)_X$ gauge boson is

$$M_X = g_X^2 \left(q_1^2 f_1^2 + q_2^2 f_2^2\right)\ . \tag{8}$$

We redefine the axial components as

$$\begin{pmatrix} a \\ G \end{pmatrix} = \frac{1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} . \quad (9)$$

From Eq.(7), we obtain that the axial field G is the Nambu-Goldstone boson for the $U(1)_X$ gauge symmetry breaking, while the gauge invariant axial field a is the PQ axion.

Due to the QCD anomaly, we obtain the anomalous couplings for b_i

$$\mathcal{L} = \frac{g_3^2}{32\pi^2} \left(\frac{b_1}{f_1} + \frac{b_2}{f_2} \right) G\tilde{G} . \quad (10)$$

And then we obtain the axion decay constant

$$f_a = \frac{f_1 f_2 \sqrt{p^2 f_1^2 + q^2 f_2^2}}{p f_1^2 + q f_2^2} . \quad (11)$$

Because $U(1)_X$ is anomalous gauge symmetry, the decay constants in our models are different from those in Refs. [11, 12, 18, 19]. Of course, if we consider the anomaly free $U(1)_X$ gauge symmetry, for example, we introduce q pairs of vector-like particles $(Xf, \overline{Xf})$, and p pairs of the vector-like particles $(Xf', \overline{Xf'})$, we obtain the same axion decay constant as in Refs. [11, 12, 18, 19]

$$f_a = \frac{f_1 f_2}{\sqrt{p^2 f_1^2 + q^2 f_2^2}} . \quad (12)$$

It is well-known that quantum gravitational effects, which are associated with black holes, worm holes, etc., are believed to violate all the global symmetries, while they respect all the gauge symmetries [43–45]. The non-renormalizable operators with the lowest dimension for Higgs fields Φ_1 and Φ_2, which preserve the $U(1)_X$ gauge symmetry whike violate the $U(1)_{PQ}$ symmetry, are

$$\mathcal{L} \supset \frac{1}{p!q!} \frac{\Phi_1^q \Phi_2^p}{M_{\rm Pl}^{p+q-4}} + {\rm H.C.} . \quad (13)$$

Thus, we obtain $p + q \geq 11$. For example, we can have

$$\begin{aligned}(p, q) \; = \; & (1, 10), \; (2, 9), \; (3, 8), \; (4, 7), \; (5, 6), \; (6, 5), \\ & (7, 4), \; (8, 3), \; (9, 2), \; (10, 1) . \end{aligned} \quad (14)$$

MODEL I WITH VECTOR-LIKE PARTICLES $(XF, \overline{XF})$ AND $(XF', \overline{XF'})$

In Model I, we introduce two paris of vector-like particles $(XF, \overline{XF})$ and $(XF', \overline{XF'})$, as well as the Higgs fields Φ_1 and Φ_2. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields are given in Table II.

Particles	$U(1)_X$	$U(1)_{PQ}$
$(XF, \overline{XF})$	p	q
$(XF', \overline{XF'})$	$-q$	p
Φ_1	$-2p$	$-2q$
Φ_2	$2q$	$-2p$

TABLE II. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields in Model I. In particular, p and q are coprime positive integers.

First, we discuss the cancellation conditions for the gauge anomalies via the Green-Schwarz mechanism. The gauge anomalies in Model I are

$$A_1 = A_2 = A_3 = p - q . \quad (15)$$

Thus, we obtain

$$k_1 = k_2 = k_3 = p - q . \quad (16)$$

Second, to achieve the gauge coupling unification, we introduce XB, XW, XG, and $(XL, \overline{XL})$ around the TeV scale. Because the vector-like particles $(XF, \overline{XF})$ and $(XF', \overline{XF'})$ form the complete $SU(5)$ multiplets, they do not affect the gauge coupling unification. For simplicity, we assume their masses around 10^{11} GeV via the renormalizable Yukawa couplings. Following the convention in Refs. [46–50], we numerically solved the two-loop renormalization group equations (RGEs) for the gauge couplings by setting the central value of top quark pole mass as $m_t = 173.34 \pm 0.27(\text{stat}) \pm 0.71(\text{syst})$ GeV [51] and taking the strong coupling constant at reference Z boson mass as $\alpha_3(M_Z^2) = 0.1179 \pm 0.0009$ [52]. The evolution of the two-loop gauge couplings is presented in Fig. 1. The unification condition is defined as $\alpha_{\rm GUT}^{-1} \equiv \alpha_1^{-1} = (\alpha_2^{-1} + \alpha_3^{-1})/2$ and the unification is achieved at the GUT scale $M_{\rm GUT} = 1.9 \times 10^{16}$ GeV. The relative error $\Delta \equiv |\alpha_1^{-1} - \alpha_2^{-1}|/\alpha_1^{-1}$ quantifies the fractional deviation of $\alpha_{2,3}$ from α_1. Evidently, TeV-scale vector-like particles (XB, XW, XG, and $(XL, \overline{XL})$) must be incorporated to mitigate this relative error, and these particles exhibit a small mass splitting. Specifically, when XW is introduced at 800 GeV with the remaining particles introduced at 5 TeV, the unification is achieved with a relative error of 0.9%. However, if the mass of both XW and $(XL, \overline{XL})$ are adjusted to $M_{XW} = M_{(XL, \overline{XL})} = 800$ GeV, while XB and XG remain fixed at $M_{XB} = M_{XG} = 5$ TeV, the relative error is further reduced to 0.5%. For the remaining vector-like particles $(XF, \overline{XF})$ and $(XF', \overline{XF'})$, the impact of their masses on the relative error is negligible. Consequently, within the framework of this model, the masses of these particles are set to $M_{(XF, \overline{XF})} = M_{(XF', \overline{XF'})} = 10^{11}$ GeV.

Third, if we do not consider gauge coupling unification, for simplicity, we can introduce vector-like particles $(XD', \overline{XD'})$ instead of $(XF', \overline{XF'})$. And then the gauge

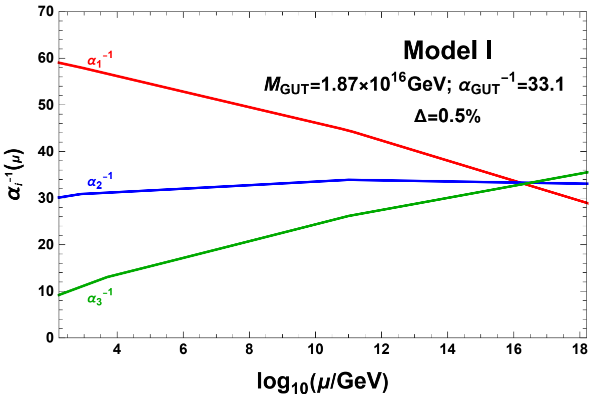

FIG. 1. The evolution of two-loop gauge couplings in the Model I with vector-like particles $(XF, \overline{XF})$ and $(XF', \overline{XF'})$. The masses of the vector-like particles are set as follows: $M_{XW} = M_{(XL,\overline{XL})} = 800$ GeV for XW and $(XL, \overline{XL})$, $M_{XB} = M_{XG} = 5$ TeV for XB and XG, and $M_{(XF,\overline{XF})} = M_{(XF',\overline{XF'})} = 10^{11}$ GeV for $(XF, \overline{XF})$ and $(XF', \overline{XF'})$.

Particles	$U(1)_X$	$U(1)_{PQ}$
$(XT, \overline{XT})$	p	q
$(XT', \overline{XT'})$	$-q$	p
Φ_1	$-2p$	$-2q$
Φ_2	$2q$	$-2p$

TABLE III. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields in Model II. In particular, p and q are coprime positive integers.

anomalies are

$$A_1 = p - \frac{2}{5}q \ , \quad A_2 = p \ , \quad A_3 = p - q \ . \tag{17}$$

Thus, we obtain

$$k_1 = p - \frac{2}{5}q \ , \quad k_2 = p \ , \quad k_3 = p - q \ . \tag{18}$$

MODEL II WITH VECTOR-LIKE PARTICLES $(XT, \overline{XT})$ AND $(XT', \overline{XT'})$

In Model II, we introduce two paris of vector-like particles $(XT, \overline{XT})$ and $(XT', \overline{XT'})$, as well as the Higgs fields Φ_1 and Φ_2. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields are given in Table III.

First, we discuss the cancellation conditions for the gauge anomalies via the Green-Schwarz mechanism. The gauge anomalies in Model II are

$$5A_1 = A_2 = A_3 = 3(p - q) \ . \tag{19}$$

Thus, we obtain

$$5k_1 = k_2 = k_3 = 3(p - q) \ . \tag{20}$$

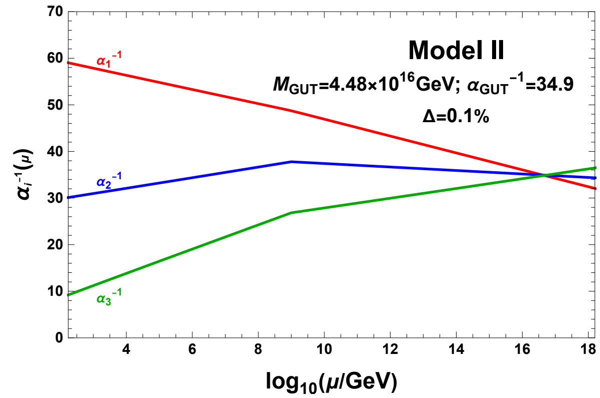

FIG. 2. The evolution of two-loop gauge couplings in the Model II with vector-like particles $(XT, \overline{XT})$ and $(XT', \overline{XT'})$. The masses of these particles are set as $M_{(XT,\overline{XT})} = M_{(XT',\overline{XT'})} = 10^9$ GeV.

Second, we assume the Yukawa couplings y_{xf} and y'_{xf} in Eq. (3) are of $\mathcal{O}(10^{-2})$, and then the vector-like particles $(XT, \overline{XT})$ and $(XT', \overline{XT'})$ have masses around 10^9 GeV. Interestingly, we can achieve gauge coupling unification without introducing new particles. In Fig. 2, after numerically solving the two-loop RGEs using input parameters consistent with earlier analyses, unification is achieved at $M_{\rm GUT} = 4.5 \times 10^{16}$ GeV with a mere 0.1% relative error. Notably, the relative error Δ of this model is substantially lower than the 0.5%-0.9% relative errors observed in the other two Models. When the vector-like particles $(XT, \overline{XT})$ and $(XT', \overline{XT'})$ are introduced at 10^9 GeV, the RGE evolution curves of the gauge couplings exhibit a characteristic deflection. This deflection arises from the non-trivial beta function contributions of these particles. Unlike the TeV-scale particles $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$ in Model III, a key advantage of this model is that the minimal relative error can still be achieved without the need for mass splitting.

MODEL III WITH VECTOR-LIKE PARTICLES $(XQ, \overline{XQ})$ AND $(XD', \overline{XD'})$

In Model III, we introduce two paris of vector-like particles $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$, as well as the Higgs fields Φ_1 and Φ_2. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields are given in Table IV. For the renormalizabe model, we have $q_1 = -2p$ and $q_2 = -2q$. While for the non-renormalizable model, we have $q_1 = -p$ and $q_2 = -q$.

First, we discuss the cancellation conditions for the gauge anomalies via the Green-Schwarz mechanism. The gauge anomalies in Model III are

$$A_1 = \frac{1}{5}p + \frac{2}{5}q \ , \quad A_2 = 3p \ , \quad A_3 = 2p + q \ . \tag{21}$$

Particles	$U(1)_X$	$U(1)_{PQ}$
$(XQ, \overline{XQ})$	p	$-q$
$(XD', \overline{XD'})$	q	p
Φ_1	q_1	$-q_2$
Φ_2	q_2	q_1

TABLE IV. The $U(1)_X \times U(1)_{PQ}$ quantum numbers for vector-like particles and Higgs fields in Model III. In particular, p and q are coprime positive integers.

Thus, we obtain

$$k_1 = \frac{1}{5}p + \frac{2}{5}q \ , \quad k_2 = 3p \ , \quad k_3 = 2p + q \ . \qquad (22)$$

Second, for the renormalizable Yukawa couplings, we obtain that the vector-like particles $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$ have masses around 10^{11} GeV. In addition, the Lagrangian for the non-renormalizable Yukawa couplings is

$$-\mathcal{L} = \frac{1}{2M_{\rm Pl}} y_{XQ} XQ\overline{XQ}\Phi_1^2 + \frac{1}{2M_{\rm Pl}} y_{XD'} XD'\overline{XD'}\Phi_2^2 \ (23)$$

where $M_{\rm Pl}$ is the reduced Planck scale. Thus, we obtain that the vector-like particles $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$ have masses around TeV scale. In this case, we can achieve the gauge coupling unification at $M_{\rm GUT} = 2.14 \times 10^{16}$ GeV with a 0.7% relative error. In Fig. 3, the vector-like particles are introduced at $M_{(XQ,\overline{XQ})} = 1.5$ TeV and $M_{(XD',\overline{XD'})} = 5.5$ TeV. From the beta functions of these particles, $(XQ, \overline{XQ})$ modifies the RGE evolutionary trends of α_1^{-1}, α_2^{-1} and α_3^{-1}, while $(XD', \overline{XD'})$ alters the RGE trends of α_1^{-1} and α_3^{-1}. The contribution of these two pairs of vector-like particles to α_1^{-1} is smaller than those to α_2^{-1} and α_3^{-1}. Moreover, the mass splitting between $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$ enables targeted mitigation of the relative error Δ. As Δ quantifies the fractional separation of α_2 and α_3 from α_1, the relative error decreases with the increasing mass of $(XD', \overline{XD'})$ and decreasing mass of $(XQ, \overline{XQ})$. For instance, when $(XD', \overline{XD'})$ has mass 10^5 GeV and $(XQ, \overline{XQ})$ has mass 1.5 TeV, the error is reduced to 0.1%. However, $(XQ, \overline{XQ})$ may not satisfy the LHC search constraints.

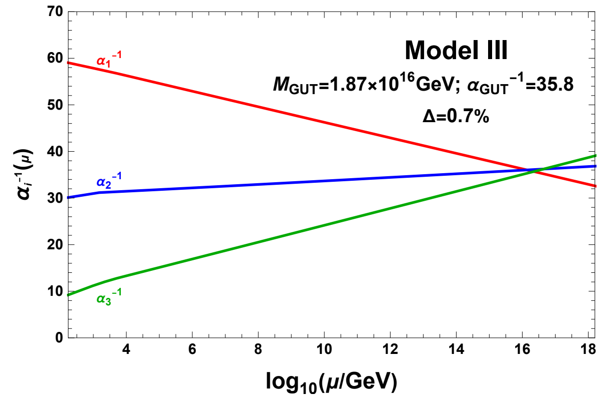

FIG. 3. The evolution of two-loop gauge couplings in the Model III with vector-like particles $(XQ, \overline{XQ})$ and $(XD', \overline{XD'})$. The masses of the vector-like particles are set as follows: $M_{(XQ,\overline{XQ})} = 1.5$ TeV for $(XQ, \overline{XQ})$, and $M_{(XD',\overline{XD'})} = 5.5$ TeV for $(XD', \overline{XD'})$.

CONCLUSIONS

We constructed the generic high-quality axion models with anomalous $U(1)_X$ gauge symmetry and vector-like particles. First, we briefly reviewed the gauge anomaly cancellations via the Green-Schwarz mechanism, discussed the $U(1)_X$ gauge symmetry breaking, as well as calculated the Nambu-Goldstone boson, PQ axion, and axion decay constant in general. Because of the anomalous $U(1)_X$ gauge symmetry, we showed that the high-dimensional operators, which break the $U(1)_{PQ}$ global symmetry, can have dimension eleven or higher. Therefore, the axion quality problem is solved. Interestingly, we only need to introduce two pairs of vector-like particles. To be concrete, we presented three specific models with two pairs of vector-like particles: In Model I, these vector-like particles belong to the fundamental and anti-fundamental representations of the $SU(5)$ model; in Model II, they belong to the anti-symmetric representation and its Hermitian conjugate of the flipped $SU(5)$ model; in Model III, one pair of vector-like particles has the same quantum numbers as the quark doublet and its Hermitian conjugate, while the other pair has the same quantum numbers as the right-handed down-type quark and its Hermitian conjugate. We showed that gauge anomalies in all three models can be canceled via the Green-Schwarz mechanism. To achieve gauge coupling unification, we need to introduce additional vector-like particles in Model I. We found that gauge coupling unification is achieved at the unification scale around 10^{16} GeV with a relative error of less than 1%. In particular, gauge coupling unification in Model II is achieved naturally with the smallest relative error of 0.1%.

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