

Updated Cosmological Constraints from 2D BAO Measurements: A New Compilation and Comparison with DESI DR2

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We investigate and update observational constraints on cosmological parameters within the Λ CDM and dynamical dark energy frameworks, using a new compilation of the transverse (or 2D) BAO data, measurements that provide a relatively model-independent estimate of the BAO angular scale at a given redshift. Firstly, we assess the consistency of this compilation with CMB-Planck data and recent BAO results from the DESI collaboration. After confirming minimal tension with CMB data, we perform a series of joint analyses combining CMB data with the 2D BAO compilation, as well as with several recent Type Ia supernova (SNIa) samples. In all cases, we compare the constraining power of the 2D BAO data with that of DESI DR2 samples. Our results indicate that combining 2D BAO with CMB and SNIa data provides observational constraints that are competitive with those obtained using DESI DR2. Although the precision of DESI DR2 results remains higher, as expected due to the more accurate 3D measurements, the 2D BAO compilation yields strong constraints. For example, in the Λ CDM context, we find $H_0 = 68.16^{+0.41}_{-0.37}$ km s⁻¹ Mpc⁻¹ (CMB + 2D BAO) and $\Sigma m_\nu < 0.081$ eV (95% CL). These results are comparable to analogous analyses using DESI DR2. Several other cases are analyzed and presented in the main text. Due to these results, we conclude that this new 2D BAO compilation is both robust and competitive in constraining cosmological parameters, and, importantly, it does not exhibit significant tension with CMB measurements.

Keywords: Baryon Acoustic Oscillations, Cosmological parameters, Dark Energy

I. INTRODUCTION

The standard cosmological model, the flat Λ CDM, built upon general relativity with a positive cosmological constant and cold dark matter, has successfully explained a broad range of cosmological observations over the past two decades. Nevertheless, as astronomical data have become increasingly precise and diverse, a growing number of measurements appear difficult to reconcile within the standard paradigm, placing the Λ CDM cosmology at a potential crossroads. The most striking example is the current tension in the determination of the Hubble constant, H_0 . Under the Λ CDM framework, the analysis of Planck CMB data [1] yields $H_0 = 67.36 \pm 0.54$ km s⁻¹ Mpc⁻¹, whereas a model-independent local determination from the SH0ES collaboration, based on long-period Cepheid variables, finds $H_0 = 73.18 \pm 0.88$ km s⁻¹ Mpc⁻¹ [2]. These estimates differ by approximately 6σ . The lower value inferred from the Planck-CMB data is, however, in excellent agreement with the joint constraints from Baryon Acoustic Oscillations (BAO) and Big Bang Nucleosynthesis (BBN) [3, 4], as well as with results from other CMB experiments such as ACT-DR6 and SPT [5, 6]. Several additional, though

less statistically significant, tensions have also been reported in recent years (see [7] for a comprehensive review). Taken together, these tensions—originating from independent and complementary data sets—have motivated extensive investigations into extensions of the standard Λ CDM model [7].

On the other hand, recent observations from the Dark Energy Spectroscopic Instrument (DESI) have placed this assumption under increasing scrutiny [8]. By measuring BAO in an unprecedented sample of more than 14 million galaxies and quasars, DESI second data release (DR2) reports statistically significant deviations from the Λ CDM predictions, with tensions reaching up to 4.2σ , depending on the sample considered [8]. These findings may represent a turning point, suggesting that dark energy (DE) might not be well described by a simple cosmological constant. Moreover, several independent analyses have confirmed significant deviations from the Λ CDM framework and proposed new cosmological tests to further probe it in light of BAO DESI-DR2 samples [9–57], with some alternative cosmological models providing a better fit than Λ CDM to the data by up to 5σ [58].

The growing number of reported tensions with the Λ CDM model, now also emerging in traditional three-dimensional (3D) BAO measurements, has intensified the need to explore complementary late-time probes capable of testing the standard cosmological model in a minimally correlated and model-independent way. In this context,

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there is an ongoing debate regarding whether the θ_{BAO} ¹ measurements themselves show a tension with ΛCDM (see, e.g., [59–62]).

The BAO phenomenon can be studied without assuming a fiducial cosmology by employing the transverse BAO approach, also known as the two-dimensional (2D) BAO. An important advantage of this method is that it does not depend on a fiducial cosmology to calculate three-dimensional comoving distances between pairs of cosmic objects, because it analyzes data distributed in spherical shells of redshift thickness Δz , and find the BAO signal considering only the angular correlations between pairs. The redshift shells used in the analysis must be carefully chosen: if they are too thick, projection effects smooth and shift the BAO signal; if they are too thin, the number density of tracers may become insufficient to yield an adequate signal-to-noise ratio.

In a deep astronomical survey covering a wide sky area, the observed volume can be divided into several disjoint redshift shells to avoid correlations between adjacent bins. The analysis of these shells provides the angular scale of the 2D BAO signal, $\theta_{\text{BAO}}(z)$, at each redshift z , or equivalently, measurements of the angular diameter distance $D_A(z)$, with the comoving sound horizon scale r_d serving as a standard ruler. Due to the weakly model-dependent nature of 2D BAO measurements, the set $\{\theta_{\text{BAO}}(z)\}$ offers a powerful and independent means to test both the standard ΛCDM cosmology and alternative models, as well as to perform cross-comparisons among different observational datasets. Such measurements have been widely employed in recent literature across a broad range of observational tests and analyses [63–78].

In this work, we investigate observational constraints on cosmological parameters by combining 17 measurements of the 2D BAO scale with state-of-the-art Planck CMB data and several recent Type Ia supernova (SNIa) samples. Our main objectives can be summarized as follows:

- Provide an updated compilation of 2D BAO measurements for the community. This compilation is minimally consistent with CMB data, allowing joint analyses of CMB and 2D BAO datasets from a general observational perspective.
- Update observational constraints within the ΛCDM framework and its minimal extensions, such as the CPL parametrization [79, 80], by combining 2D BAO and CMB measurements with the most recent SNIa data, including PantheonPlus, DES Y5, and Union 3.0.

- Compare the constraining power of cosmological parameters of this 2D BAO compilation with that of the BAO DESI DR2 collaboration.

These objectives represent novel results and perspectives within the field and can serve as a reference for researchers working on cosmological parameter estimation who are interested in incorporating 2D BAO measurements into their analyses. Our results provide a robust and timely update on the use of 2D BAO measurements within a broader cosmological context.

This work is organized as follows. In Section II, we present a new compilation of 2D BAO measurements, along with the complementary datasets that will be used in our analysis. The theoretical methodology is also briefly described in this section. In Section III, we present and discuss our main results. Finally, in Section IV, we summarize our conclusions and outline prospects for future work.

II. THE DATA SET AND METHODOLOGY

The large-scale structure of the Universe is well described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, which assumes a homogeneous and isotropic spacetime. In comoving coordinates, the line element can be written as

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (1)$$

where $a(t)$ is the scale factor and K is the spatial Gaussian curvature ($K = 0, > 0, < 0$ for Euclidean, spherical, and hyperbolic spatial geometries, respectively).

Assuming that General Relativity accurately describes the dynamics of the cosmic expansion, the evolution of the Hubble parameter $H(z) \equiv \dot{a}/a$ is governed by the Friedmann equation, which can be expressed as

$$\frac{H(z)}{H_0} = \left[(\Omega_b + \Omega_{\text{cdm}})(1+z)^3 + \Omega_\gamma(1+z)^4 + \Omega_K(1+z)^2 + \Omega_\nu \frac{\rho_\nu(z)}{\rho_{\nu,0}} + \Omega_{\text{DE}} \frac{\rho_{\text{DE}}(z)}{\rho_{\text{DE},0}} \right]^{1/2}. \quad (2)$$

Here, Ω_b , Ω_c , Ω_γ , Ω_K , Ω_ν , and Ω_{DE} represent the present-day fractional energy densities of baryons, cold dark matter, radiation, curvature, neutrinos, and dark energy, respectively.

In the ΛCDM framework, dark energy is modeled as a cosmological constant, Λ , with an energy density $c^2\rho_{\text{DE}}$ that does not vary with time or spatial position. More generally, if dark energy is characterized by an equation-of-state parameter

$$w(z) \equiv \frac{P(z)}{c^2\rho_{\text{DE}}(z)}, \quad (3)$$

¹ The quantity θ_{BAO} corresponds to the transverse BAO measurement derived from the two-point angular correlation function between pairs of cosmic tracers, which is used to identify and measure the BAO angular scale at an effective redshift.

where $P(z)$ denotes its pressure, its energy density evolves according to

$$\frac{\rho_{\text{DE}}(z)}{\rho_{\text{DE},0}} = \exp \left[3 \int_0^z \frac{1 + w(z')}{1 + z'} dz' \right]. \quad (4)$$

For a constant w , Eq. (4) simplifies to $\rho_{\text{DE}}(z)/\rho_{\text{DE},0} = (1+z)^{3(1+w)}$, with $w = -1$ corresponding to a true cosmological constant. A widely used parametrization of w as a function of the scale factor $a(z) = (1+z)^{-1}$ is the CPL model [79, 80]

$$w(a) = w_0 + w_a(1 - a), \quad (5)$$

which evolves from $w \simeq w_0 + w_a$ at early times to w_0 today. This form captures the behavior of many physically motivated dark energy models [81], though more intricate evolution patterns are possible. Cosmological models that combine cold dark matter with the parametrization in Eq. (5) are typically referred to as $w_0w_a\text{CDM}$. In this work, we adopt the same standard notation.

This parametrization provides a consistent evolution of w across the entire history of the universe, preventing unphysical divergences both at early times ($z \rightarrow \infty$, where $w \rightarrow w_0 + w_a$) and in the distant future ($z \rightarrow -1$, where $w \rightarrow w_0$). A notable property of the CPL model is that it reduces to the standard ΛCDM scenario when $w_0 = -1$ and $w_a = 0$, allowing for straightforward and systematic tests of the cosmological constant within a broader dynamical dark energy context.

More broadly, a variety of other two-parameter dark energy models based on w_0 and w_a have been proposed in the literature [82–85]. Nevertheless, from the viewpoint of observational constraints, these alternatives produce essentially identical predictions [21, 86–88]. Therefore, for the objectives of this study, the CPL parametrization is fully adequate, as it encapsulates the key phenomenology while avoiding superfluous parametric complexity.

The theoretical scenarios explored in this work were implemented within the `CLASS` Boltzmann solver [89], and we employed the `MontePython` sampler [90, 91] to perform Monte Carlo analyses based on Markov Chain Monte Carlo (MCMC) methods.

The main cosmological parameters sampled in this study include the physical baryon density $\omega_b = \Omega_b h^2$, the physical cold dark matter density $\omega_c = \Omega_c h^2$, the angular size of the sound horizon at recombination θ_s , the reionization optical depth τ , the scalar amplitude of primordial perturbations A_s , the scalar spectral index n_s , and the sum of the neutrino masses Σm_ν . In addition to these standard parameters, we also varied w_0 and w_a within the framework of the $w_0w_a\text{CDM}$ model defined above. Convergence of all chains was verified using the Gelman–Rubin diagnostic [92], requiring that $R - 1 \leq 10^{-2}$.

The statistical analysis was performed with the `GetDist` package², which we used to extract numerical

results, including one-dimensional posterior distributions and two-dimensional marginalized probability contours. In what follows, we describe the likelihood functions and methodology adopted throughout this work.

- *Transverse Baryon Acoustic Oscillations:* We use transverse BAO measurements, $\{\theta_{\text{BAO}}(z)\}$, from a set of cosmological distance measures obtained through a methodology that weakly depends on fiducial cosmology [93, 94]. Our analysis employs 17 measurements of θ_{BAO} : 14 data points³ listed in Table I of Ref. [95], covering the redshift range $0.35 \leq z \leq 0.63$, together with three additional independent measurements at $z = 2.225$ [96], $z = 0.11$ [97], and $z = 1.725$ [98], the latter being presented in a companion work.

These 2D BAO measurements were obtained by identifying the transverse BAO signature in the two-point angular correlation function of pairs of cosmic objects, such as quasars or galaxies. In this framework, the BAO signal emerges as an excess probability at a characteristic angular scale, reflecting the preferred separation imprinted by acoustic waves in the early Universe.

For a successful detection of the transverse BAO signal, three key factors must be taken into account [99]:

- (i) the number density of the cosmic objects;
- (ii) the surveyed area on the sky; and
- (iii) precise redshift measurements, which are essential to accurately define the sample of cosmic objects within the thin redshift bin under analysis.

Once these observational factors are accounted for, a measurement of the BAO angular scale at a given redshift z provides a direct estimate of the angular diameter distance through

$$D_A(z) = \frac{r_d}{(1+z)\theta_{\text{BAO}}(z)}, \quad (6)$$

where r_d denotes the comoving sound horizon at the baryon drag epoch. It is therefore crucial to have a robust estimate of r_d to accurately infer the function $D_A(z)$ from the measured $\theta_{\text{BAO}}(z)$ data.

In this work, we unify the previously described θ_{BAO} measurements into a new compilation, providing a more comprehensive dataset for cosmological analyses. It is worth emphasizing that

² <https://github.com/cmbant/getdist>

³ We also account for the correlation between these measurements due to the overlap of their redshift bins. The corresponding covariance matrix is presented in Table II of Ref. [95].

this new compilation differs significantly from the one presented in [63], in terms of the included datasets and the methodology used to analyze the transverse BAO measurements. As a result, this novel compilation provides greater statistical leverage and robustness in constraining cosmological parameters, especially in high-redshift regimes and model-independent analyses. The resulting dataset of these measurements is commonly referred to as **2D BAO**.

- **Baryon Acoustic Oscillations (DESI-DR2):** We employ BAO measurements from the second data release of the DESI survey, which include observations from galaxies, quasars [8], and Lyman- α tracers [100]. These measurements, detailed in Table IV of Ref. [8], cover the effective redshift range $0.295 \leq z \leq 2.330$, divided into nine bins. The BAO constraints are expressed in terms of the transverse comoving distance D_M/r_d , the Hubble distance D_H/r_d , and the angle-averaged distance D_V/r_d , all normalized to the comoving sound horizon at the drag epoch, r_d . We also take into account the correlation structure among these quantities through the cross-correlation coefficients $r_{V,M/H}$ and $r_{M,H}$, which describe the covariance between different BAO measurements. This dataset is referred to as **DESI-DR2**.
- **Cosmic Microwave Background:** We use the temperature and polarization anisotropy power spectra of the CMB measured by the *Planck* satellite [1], along with their cross-spectra from the 2018 legacy data release. Specifically, we adopt the high- ℓ **Planck** likelihood for TT ($30 \leq \ell \leq 2508$), TE, and EE ($30 \leq \ell \leq 1996$), as well as the low- ℓ TT-only ($2 \leq \ell \leq 29$) and EE-only ($2 \leq \ell \leq 29$) **SimAll** likelihoods [101]. In addition, we include the CMB lensing reconstruction based on the temperature four-point correlation function [102]. This dataset is referred to as **CMB**.
- **Type Ia Supernovae (SN Ia):** We employ the following recent SN Ia samples:
 - (i) **PantheonPlus:** The PantheonPlus sample [103] provides distance-modulus measurements from 1701 light curves of 1550 distinct SN Ia events, spanning the redshift range $0.01 \leq z \leq 2.26$. This dataset is referred to as **PP**.
 - (ii) **Union 3.0:** The Union 3.0 compilation [104] consists of 2087 SN Ia in the redshift range $0.001 < z < 2.260$, with 1363 objects overlapping with the PantheonPlus sample. This dataset, referred to as **Union3**, adopts a Bayesian hierarchical modeling approach to address systematic uncertainties and measurement errors. This dataset is referred to as **Union3**.

- (iii) **DESY5:** The Dark Energy Survey Year 5 (DESY5) sample [105] includes 1635 photometrically classified SN Ia with redshifts $0.1 < z < 1.3$, along with 194 low-redshift SN Ia ($0.025 < z < 0.1$) shared with the PantheonPlus compilation. This dataset is referred to as **DESY5**.

Finally, we note that throughout our analysis we employ state-of-the-art assumptions for Big Bang Nucleosynthesis (BBN), which is particularly sensitive to constraints on the physical baryon density, $\omega_b = \Omega_b h^2$. In particular, we use the BBN likelihood that incorporates the most precise measurements of primordial light element abundances currently available: the helium mass fraction, Y_P , as reported in [106], and the deuterium-to-hydrogen ratio, $y_{DP} = 10^5 n_D/n_H$, from [107]. As is standard practice, BBN information is included in all analyses where CMB data are unavailable.

III. MAIN RESULTS

In this section, we present the main results of our analysis, derived from a series of statistical studies using the datasets introduced in the previous section. We discuss the constraints obtained for different cosmological models, highlight their implications, and compare them where relevant. A detailed discussion of the results, including potential systematic effects and consistency checks, follows.

A. Constraints in Λ CDM

We start by examining the constraints on the minimal Λ CDM parameters obtained from different datasets independently. The matter density parameter, Ω_m , is found to be $0.421^{+0.073}_{-0.10}$ from 2D BAO, $\Omega_m = 0.2974 \pm 0.0086$ with DESI-DR2, and $\Omega_m = 0.3143 \pm 0.0076$ from the CMB. The 2D BAO analysis provides broader uncertainties, but its results are consistent within 1σ with the more precise DESI-DR2 and CMB measurements. DESI-DR2 and CMB yield tighter constraints, reflecting the higher statistical power of these datasets.

Similarly, the derived combination $r_d h$ takes values of $r_d h = 98.1^{+2.9}_{-2.6}$ Mpc for 2D BAO, $r_d h = 101.56 \pm 0.73$ Mpc with DESI-DR2, and $r_d h = 99.21 \pm 0.95$ Mpc from the CMB. While 2D BAO provides slightly lower central values and broader uncertainties, all results remain compatible within their respective errors. These comparisons highlight the complementarity of 2D BAO, DESI-DR2, and CMB datasets in constraining the standard Λ CDM model, establishing a robust baseline for analyses of extended cosmological scenarios in the following sections. *Therefore, from a statistical perspective, we find no significant tensions among these main datasets.*

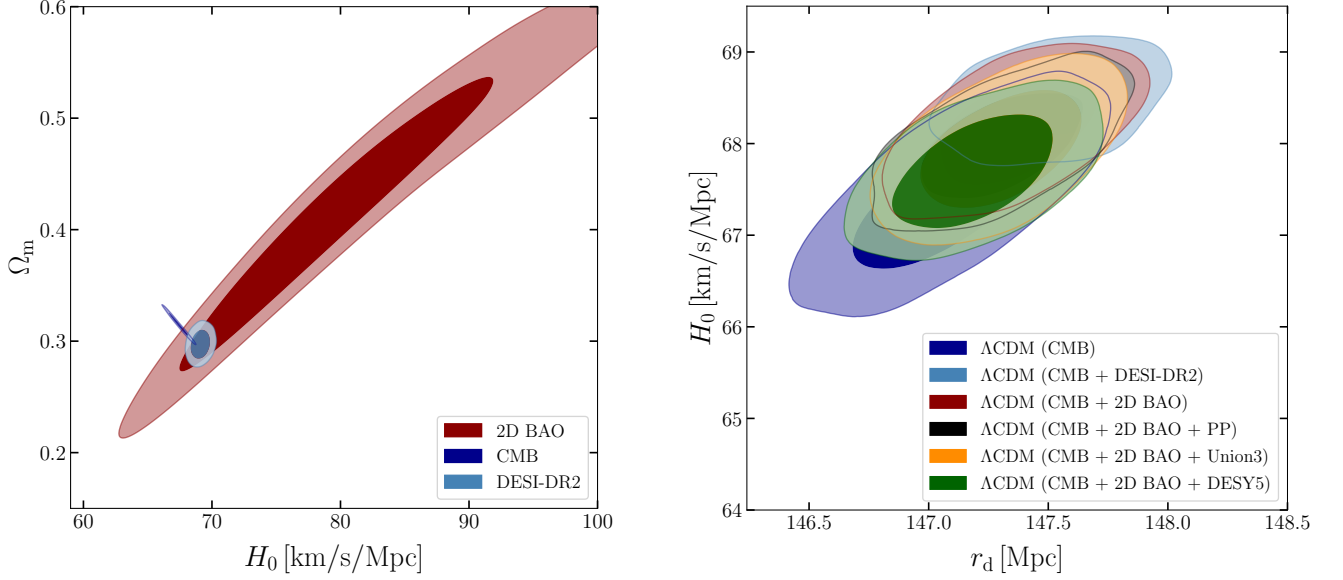


FIG. 1. Left panel: Two-dimensional confidence contours at the 68% and 95% confidence levels for the parameters Ω_m and H_0 , obtained from different data combinations as indicated in the legend, within the framework of the Λ CDM model. Right panel: Same as in the left panel, but for several joint analyses combining CMB data with the 2D BAO samples.

TABLE I. Summary table of cosmological parameter constraints showing 68% confidence levels (CL) for parameters of the Λ CDM model, obtained using our 2D BAO compilation and DESI-DR2 data in joint analyses with CMB measurements. We present both cases where the sum of neutrino masses is fixed and where it is allowed to vary freely, corresponding to the respective joint analyses. This allows a direct assessment of the impact of neutrino mass assumptions on the derived cosmological parameters. We quote 68% confidence levels for all parameters, except for the neutrino mass in eV units, for which we report the 95% upper limit.

Parameter	CMB	CMB + 2D BAO	CMB + DESI-DR2	CMB + 2D BAO + Σm_ν	CMB + DESI-DR2 + Σm_ν
$10^2 \omega_b$	2.239 ± 0.015	2.251 ± 0.013	$2.256^{+0.014}_{-0.012}$	2.250 ± 0.014	2.254 ± 0.013
ω_{cdm}	0.1199 ± 0.0012	0.11836 ± 0.00087	0.11760 ± 0.00065	0.11846 ± 0.00091	0.11792 ± 0.00071
$100 \theta_s$	1.04188 ± 0.00029	1.04204 ± 0.00028	1.04210 ± 0.00028	1.04203 ± 0.00027	1.04206 ± 0.00028
$\ln(10^{10} A_s)$	3.046 ± 0.015	$3.051^{+0.013}_{-0.015}$	3.055 ± 0.014	3.049 ± 0.015	3.051 ± 0.015
n_s	$0.9659^{+0.0044}_{-0.0039}$	0.9695 ± 0.0037	0.9714 ± 0.0035	0.9694 ± 0.0036	0.9708 ± 0.0035
τ_{reio}	0.0547 ± 0.0077	$0.0587^{+0.0064}_{-0.0079}$	0.0612 ± 0.0072	$0.0579^{+0.0070}_{-0.0082}$	0.0595 ± 0.0077
Σm_ν [eV]	—	—	—	< 0.0810	< 0.0698
H_0 [$\text{km s}^{-1} \text{Mpc}^{-1}$]	67.45 ± 0.55	$68.16^{+0.41}_{-0.37}$	68.49 ± 0.29	68.29 ± 0.47	68.61 ± 0.31
σ_8	0.8113 ± 0.0060	$0.8091^{+0.0055}_{-0.0063}$	0.8083 ± 0.0059	$0.813^{+0.011}_{-0.0067}$	$0.8149^{+0.0089}_{-0.0071}$
Ω_m	0.3143 ± 0.0076	$0.3047^{+0.0048}_{-0.0054}$	0.3002 ± 0.0037	0.3033 ± 0.0058	0.2991 ± 0.0038
r_d [Mpc]	147.09 ± 0.27	147.36 ± 0.22	147.52 ± 0.20	147.36 ± 0.23	147.46 ± 0.20

Figure 1 (left panel) shows the parameter space of Ω_m versus H_0 with 1σ and 2σ confidence level (CL) regions for the analyses discussed above. The left panel illustrates that the 2D BAO error contours are highly degenerate. Consequently, when analyzed in isolation, 2D BAO measurements are insufficient to tightly constrain key cosmological parameters, including H_0 , r_d , and Ω_m . This highlights the importance of combining 2D BAO with other datasets, such as CMB, to break parameter degeneracies and obtain precise cosmological constraints.

In this sense, we combine 2D BAO data with CMB measurements to break the parameter degeneracies present in the BAO-only analyses. The joint 2D BAO + CMB analysis significantly improves the constraints on key cosmological parameters. For example, the Hubble constant is found to be $H_0 = 68.16^{+0.41}_{-0.37} \text{ km s}^{-1} \text{Mpc}^{-1}$, compared to $H_0 = 67.45 \pm 0.55 \text{ km s}^{-1} \text{Mpc}^{-1}$ from CMB alone and $H_0 = 68.49 \pm 0.29 \text{ km s}^{-1} \text{Mpc}^{-1}$ when combining CMB with DESI-DR2. Similarly, the matter density parameter tightens to $\Omega_m = 0.3047^{+0.0048}_{-0.0054}$, com-

TABLE II. Summary table of cosmological parameter constraints showing 68% CL for parameters of the Λ CDM model, obtained from joint analyses of our 2D BAO compilation with different Type Ia supernova (SNIa) samples. These results highlight the impact of including SNIa data on the precision of cosmological parameters and the complementarity with 2D BAO measurements.

Parameter	CMB + 2D BAO + PP	CMB + 2D BAO + Union3	CMB + 2D BAO + DESY5
$10^2 \omega_b$	2.248 ± 0.013	2.247 ± 0.014	2.243 ± 0.014
ω_{cdm}	0.11874 ± 0.00089	0.11880 ± 0.00088	0.11937 ± 0.00089
$100 \theta_s$	1.04201 ± 0.00029	$1.04198^{+0.00029}_{-0.00025}$	1.04196 ± 0.00029
$\ln(10^{10} A_s)$	$3.049^{+0.012}_{-0.016}$	3.046 ± 0.015	3.047 ± 0.014
n_s	$0.9687^{+0.0035}_{-0.0039}$	0.9686 ± 0.0037	0.9672 ± 0.0036
τ_{reio}	$0.0577^{+0.0059}_{-0.0080}$	0.0566 ± 0.0076	$0.0561^{+0.0068}_{-0.0076}$
H_0 [km s $^{-1}$ Mpc $^{-1}$]	67.98 ± 0.39	67.93 ± 0.42	67.69 ± 0.40
σ_8	0.8095 ± 0.0060	0.8085 ± 0.0059	0.8106 ± 0.0058
Ω_m	0.3070 ± 0.0052	0.3076 ± 0.0054	0.3109 ± 0.0054
r_d [Mpc]	147.30 ± 0.23	147.30 ± 0.22	147.19 ± 0.22

pared with $\Omega_m = 0.3143 \pm 0.0076$ from CMB only and $\Omega_m = 0.3002 \pm 0.0037$ for CMB + DESI-DR2.

The combination also provides precise measurements of the sound horizon, with $r_d = 147.36 \pm 0.22$ Mpc for CMB + 2D BAO, and slightly higher values for CMB + DESI-DR2 (147.52 ± 0.20 Mpc). Other parameters, such as σ_8 and the spectral index n_s , are also marginally shifted but remain consistent with the CMB-only constraints [1].

All main statistical results are summarized in Table I. Comparing the combinations, we see that both CMB + 2D BAO and CMB + DESI-DR2 significantly improve parameter constraints relative to CMB alone. However, the combination with DESI-DR2 consistently provides tighter bounds. For instance, the Hubble constant is measured as $H_0 = 68.16^{+0.41}_{-0.37}$ km s $^{-1}$ Mpc $^{-1}$ for CMB + 2D BAO, while CMB + DESI-DR2 achieves $H_0 = 68.49 \pm 0.29$ km s $^{-1}$ Mpc $^{-1}$. These comparisons highlight that while 2D BAO data help break degeneracies and improve constraints beyond the CMB alone, the DESI-DR2 measurements provide even stronger links to the fundamental parameters, due to their higher statistical precision and coverage. Overall, the table illustrates the complementarity and relative constraining power of these different dataset combinations within the Λ CDM framework.

Currently, various SNIa samples have been compiled and have consistently provided strong observational constraints on cosmological parameters. Following the state of the art in cosmological analyses, we adopt multiple SNIa datasets, as defined in Section II, and incorporate them in a joint analysis with 2D BAO measurements. This combined approach allows us to exploit the complementary constraining power of SNIa luminosity distances

and 2D BAO scales, improving parameter determinations beyond what is possible with either dataset alone.

Table II summarizes the main results in Λ CDM analyses. When using the Pantheon+ (PP) sample, the Hubble constant is measured as $H_0 = 67.98 \pm 0.39$ km s $^{-1}$ Mpc $^{-1}$, slightly higher than the value obtained with the Union3 compilation ($H_0 = 67.93 \pm 0.42$ km s $^{-1}$ Mpc $^{-1}$) and the DESY5 sample ($H_0 = 67.69 \pm 0.40$ km s $^{-1}$ Mpc $^{-1}$). Similarly, the matter density parameter, Ω_m , shows minor variations: $\Omega_m = 0.3070 \pm 0.0052$ for PP, $\Omega_m = 0.3076 \pm 0.0054$ for Union3, and $\Omega_m = 0.3109 \pm 0.0054$ for DESY5.

Other derived parameters, such as σ_8 and the sound horizon r_d , remain stable across the different SNIa datasets, with σ_8 ranging from 0.8085 to 0.8106 and $r_d \approx 147.2$ – 147.3 Mpc. These results indicate that while the choice of SNIa compilation slightly shifts the central values, the overall constraints remain robust. The combination of 2D BAO with CMB and SNIa consistently improves the precision of cosmological parameters compared to CMB + 2D BAO alone, demonstrating the complementary role of SNIa data in breaking parameter degeneracies and refining measurements within the Λ CDM framework.

Examining the Hubble constant values in Table II, we note that all three joint analyses (CMB + 2D BAO + SNIa) yield H_0 values in the range 67.69–67.98 km s $^{-1}$ Mpc $^{-1}$, which remain in close agreement with the Planck CMB measurements. These values are systematically lower than the local distance ladder determinations, such as those from the SH0ES collaboration [2] ($H_0 \sim 73$ km s $^{-1}$ Mpc $^{-1}$), highlighting the persistent tension between early- and late-Universe probes. Although the inclusion of different SNIa samples slightly

shifts the central value — DESY5 leading to the lowest H_0 and Pantheon+ the highest — the changes are within the 1σ uncertainties and do not alleviate the tension. This emphasizes that, within the Λ CDM framework, the combined CMB + 2D BAO + SNIa datasets reinforce the lower H_0 values preferred by early-Universe measurements, underscoring the need for either new physics or systematic re-evaluations to reconcile the discrepancy.

B. Constraints in w_0w_a CDM

Table III summarizes the constraints on cosmological parameters for the w_0w_a CDM model, obtained from a variety of dataset combinations. The analyses mirror those performed for the standard Λ CDM model, allowing for a direct comparison between the two scenarios.

Focusing first on the dark energy equation-of-state parameters, we note that the constraints on w_0 and w_a vary significantly depending on the dataset used. When using only CMB data, the uncertainties are relatively large ($w_0 = -1.10 \pm 0.43$, $w_a < 0.0362$), reflecting the limited ability of CMB alone to constrain dynamical dark energy. Including 2D BAO measurements or DESI-DR2 data substantially improves the constraints, driving w_0 closer to -0.4 to -0.9 and providing meaningful bounds on w_a . These trends demonstrate the importance of combining CMB with BAO data to break parameter degeneracies and tighten constraints. Although these shifts are not yet statistically significant enough to claim a detection of deviation, they indicate the sensitivity of combined datasets to departures from a pure cosmological constant. Figure 2 (left panel) displays the parameter space in the w_0 - w_a plane for the CMB + 2D BAO and CMB + DESI DR2 combinations, allowing a direct comparison of their constraining power. As previously discussed, the CMB + 2D BAO data yield highly degenerate constraints, whereas the inclusion of DESI DR2 measurements significantly improves parameter localization. The resulting contours for CMB + DESI DR2 indicate a moderate statistical preference for the w_0w_a CDM model, with a confidence level exceeding 2σ . This behavior, however, is consistent with several previous studies reported in the literature (e.g., [8, 86]).

The inferred value of the Hubble constant, H_0 , shows a clear dependence on the choice of dataset. The CMB-only analysis yields a relatively high value, $H_0 = 76.6 \pm 8.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while the inclusion of 2D BAO data lowers both the central value and its associated uncertainty, reaching $H_0 \simeq 69.1^{+3.4}_{-4.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the CMB + 2D BAO combination. A similar trend is observed for σ_8 , indicating a better-constrained picture of structure formation when multiple probes are combined.

Other parameters, including the baryon and cold dark matter densities (ω_b , ω_{cdm}), the sound horizon at the drag epoch r_d , and the scalar spectral index n_s , remain largely stable across the different dataset combinations, indicating that these quantities are already tightly con-

strained by the CMB data only.

Similar as done for Λ CDM model, Table IV summarizes the cosmological constraints obtained from the combination of CMB + 2D BAO data with PP, the Union3 supernova sample, and the DES-Y5 dataset. Overall, all combinations deliver mutually consistent results, indicating excellent internal concordance among current cosmological probes. The baryon and cold dark matter physical densities, ω_b and ω_{cdm} , are tightly constrained and show no significant dependence on the chosen dataset. The same holds for the sound horizon at the drag epoch, r_d , which remains stable around 147.1 Mpc across all combinations. This confirms that the background cosmology at early times is robustly described within the standard framework, and any deviations from Λ CDM primarily arise from the late-time evolution of dark energy.

The dark energy equation-of-state parameters exhibit mild but coherent deviations from the cosmological constant scenario. In particular, we find $w_0 = -0.834 \pm 0.062$ and $w_a = -0.71 \pm 0.26$ for the CMB + 2D BAO + PP combination. When supernova datasets are included, the constraints shift toward more negative values of w_a , yielding $w_0 = -0.69 \pm 0.10$, $w_a = -1.13 \pm 0.38$ for CMB + 2D BAO + Union3, and $w_0 = -0.672 \pm 0.071$, $w_a = -1.20^{+0.33}_{-0.28}$ for CMB + 2D BAO + DES-Y5. This trend can be visually appreciated in the right panel of Figure 2 shows that the inclusion of SNe datasets slightly favors more dynamical dark energy models. For these joint analyses, we find the quantitative deviation from Λ CDM to be: 1.9σ for CMB + 2D BAO + PP, 2.9σ for CMB + 2D BAO + Union3, and 4.4σ for CMB + 2D BAO + DESY5.

For comparison, the DESI Collaboration (DR2) reports similar trends in their joint analyses:

$$\begin{aligned} w_0 &= -0.838 \pm 0.055, & w_a &= -0.62^{+0.22}_{-0.19}, \\ w_0 &= -0.667 \pm 0.088, & w_a &= -1.09^{+0.31}_{-0.27}, \\ w_0 &= -0.752 \pm 0.057, & w_a &= -0.86^{+0.23}_{-0.20}, \end{aligned}$$

from (CMB + DESI DR2 + PP), (CMB + DESI DR2 + Union3.0) and (CMB + DESI DR2 + DES-Y5), respectively.

Overall, our results are consistent with the DESI findings, reinforcing the indication of deviations from a pure cosmological constant behavior ($w_0 = -1$, $w_a = 0$). More importantly, from a comparative perspective, we can conclude that replacing the 2D BAO compilation with DESI DR2 leads to similarly robust observational constraints in joint analyses. In other words, both datasets provide competitive and complementary results when combined with other cosmological probes.

The derived Hubble constant also shows a modest dependence on the adopted supernova dataset. We obtain $H_0 = 68.0 \pm 0.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from the CMB + 2D BAO + PP combination, and slightly lower values of $H_0 \simeq 66.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ when Union3 or DES-Y5 are included. These results remain consistent with the Planck Λ CDM estimate and do not indicate any allevi-

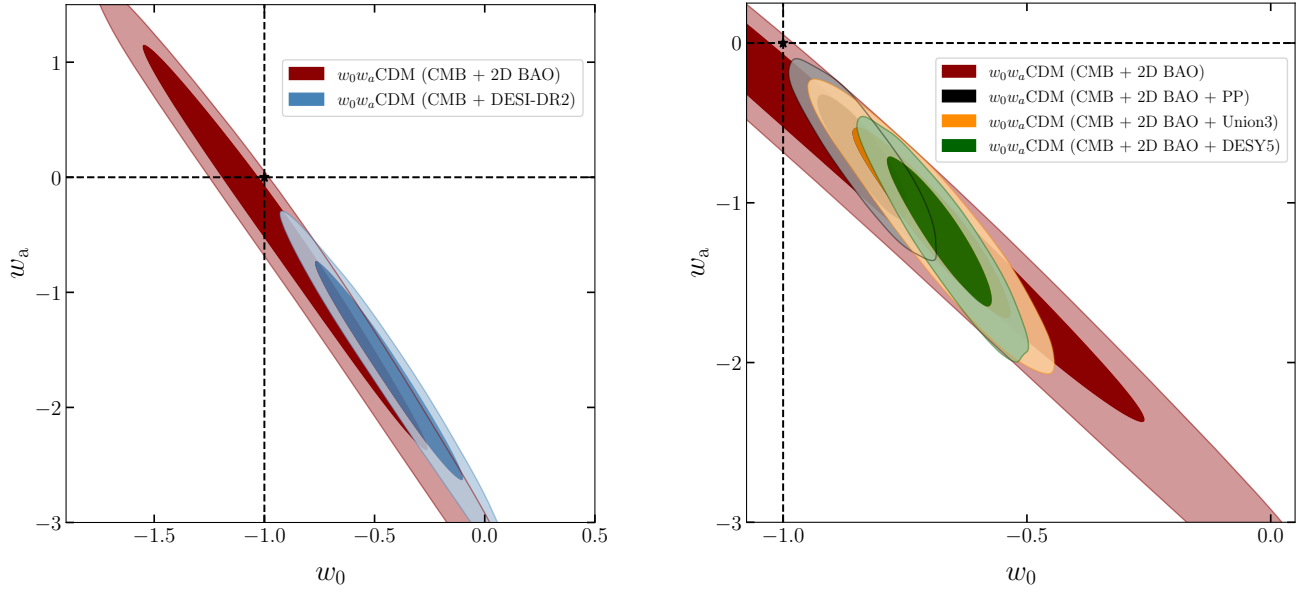


FIG. 2. Left panel: Two-dimensional confidence contours at the 68% and 95% confidence levels for the parameters w_0 and w_a , obtained from different data combinations as indicated in the legend. Right panel: Same as in the left panel, but for several joint analyses combining CMB data with the 2D BAO samples.

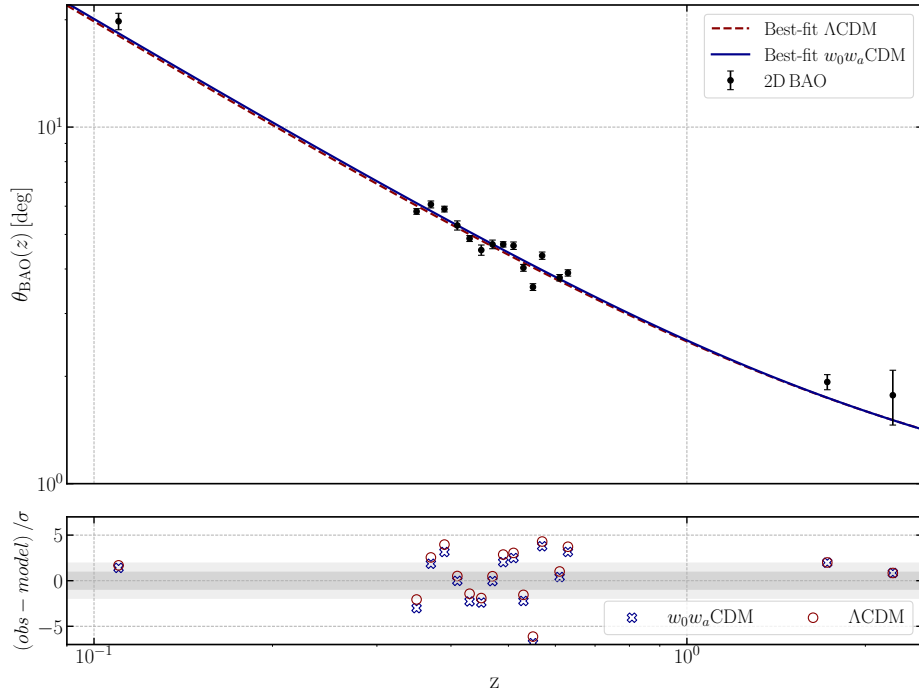


FIG. 3. Best-fit models for the rescaled distance-redshift relation, $\theta_{\text{BAO}}(z)$, in the $w_0w_a\text{CDM}$ (solid line) and ΛCDM (dashed line) frameworks, derived from the combined analysis of CMB and DESI DR2 data. The black dots correspond to the BAO measurement included in the study, as indicated in the legend. The vertical bars represent the 1σ observational uncertainties. *Bottom panel:* Differences between the theoretical predictions and the observed BAO values, expressed in units of their respective observational errors.

TABLE III. Same as Table I, but for the dynamical dark energy model w_0w_a CDM. We quote 68% confidence levels for all parameters, except for the neutrino mass in eV units, for which we report the 95% upper limit. The last row reports the difference in minimum chi-square value, $\Delta\chi^2_{\min} = \chi^2_{\min}(w_0w_a\text{CDM}) - \chi^2_{\min}(\Lambda\text{CDM})$, where negative values indicate a better fit of the w_0w_a CDM model compared to the Λ CDM model.

Parameter	CMB	CMB + 2D BAO	CMB + DESI-DR2	CMB + 2D BAO + Σm_ν	CMB + DESI-DR2 + Σm_ν
$10^2\omega_b$	2.242 ± 0.016	2.241 ± 0.014	2.241 ± 0.014	2.236 ± 0.015	2.240 ± 0.014
ω_{cdm}	$0.1194^{+0.0012}_{-0.0014}$	0.1199 ± 0.0011	$0.11965^{+0.00097}_{-0.00084}$	$0.1199^{+0.0012}_{-0.0010}$	0.11951 ± 0.00094
$100\theta_s$	1.04194 ± 0.00030	1.04189 ± 0.00028	1.04191 ± 0.00028	$1.04291^{+0.00055}_{-0.0015}$	1.04194 ± 0.00030
$\ln(10^{10}A_s)$	3.041 ± 0.015	3.043 ± 0.013	$3.042^{+0.013}_{-0.014}$	3.045 ± 0.015	3.043 ± 0.015
n_s	0.9672 ± 0.0043	0.9658 ± 0.0040	0.9664 ± 0.0037	0.9651 ± 0.0043	0.9665 ± 0.0037
τ_{reio}	0.0534 ± 0.0077	0.0537 ± 0.0069	$0.0535^{+0.0064}_{-0.0077}$	0.0546 ± 0.0076	0.0541 ± 0.0077
w_0	-1.10 ± 0.43	-0.91 ± 0.39	-0.44 ± 0.21	$-0.78^{+0.29}_{-0.38}$	$-0.42^{+0.24}_{-0.20}$
w_a	< 0.0362	$-0.6^{+1.3}_{-1.0}$	-1.67 ± 0.59	$-1.13^{+1.1}_{-0.96}$	$-1.75^{+0.63}_{-0.74}$
Σm_ν [eV]	—	—	—	< 0.372	< 0.186
H_0 [km s $^{-1}$ Mpc $^{-1}$]	76.6 ± 8.4	$69.1^{+3.4}_{-4.0}$	$63.8^{+1.6}_{-2.1}$	68.1 ± 2.8	$63.8^{+1.7}_{-2.2}$
σ_8	$0.887^{+0.087}_{-0.077}$	$0.827^{+0.030}_{-0.034}$	$0.782^{+0.015}_{-0.017}$	0.807 ± 0.027	0.777 ± 0.021
Ω_m	$0.252^{+0.067}_{-0.076}$	0.302 ± 0.031	0.351 ± 0.021	$0.312^{+0.021}_{-0.030}$	0.352 ± 0.022
r_d [Mpc]	147.20 ± 0.28	147.08 ± 0.26	147.14 ± 0.22	$147.05^{+0.26}_{-0.29}$	147.17 ± 0.22
$\Delta\chi^2_{\min}$	-4.38	-2.62	-9.20	-1.44	-2.90

TABLE IV. Same as Table II, but for the dynamical dark energy model w_0w_a CDM. The last row reports the difference in minimum chi-square value, $\Delta\chi^2_{\min} = \chi^2_{\min}(w_0w_a\text{CDM}) - \chi^2_{\min}(\Lambda\text{CDM})$, where negative values indicate a better fit of the w_0w_a CDM model compared to the Λ CDM model.

Parameter	CMB + 2D BAO + PP	CMB + 2D BAO + Union3	CMB + 2D BAO + DESY5
$10^2\omega_b$	$2.241^{+0.016}_{-0.014}$	$2.240^{+0.013}_{-0.015}$	2.240 ± 0.015
ω_{cdm}	0.1196 ± 0.0011	0.1197 ± 0.0011	0.1198 ± 0.0011
$100\theta_s$	1.04189 ± 0.00030	1.04188 ± 0.00029	1.04189 ± 0.00029
$\ln(10^{10}A_s)$	3.043 ± 0.015	3.040 ± 0.014	3.040 ± 0.015
n_s	0.9665 ± 0.0040	0.9664 ± 0.0039	0.9662 ± 0.0040
τ_{reio}	0.0538 ± 0.0076	0.0525 ± 0.0075	0.0523 ± 0.0076
w_0	-0.834 ± 0.062	-0.69 ± 0.10	-0.672 ± 0.071
w_a	-0.71 ± 0.26	-1.13 ± 0.38	$-1.20^{+0.33}_{-0.28}$
H_0 [km s $^{-1}$ Mpc $^{-1}$]	68.03 ± 0.64	66.86 ± 0.86	66.81 ± 0.61
σ_8	0.817 ± 0.010	0.808 ± 0.011	0.8082 ± 0.0096
Ω_m	0.3084 ± 0.0060	0.3195 ± 0.0085	0.3200 ± 0.0063
r_d [Mpc]	147.14 ± 0.25	147.13 ± 0.25	147.12 ± 0.25
$\Delta\chi^2_{\min}$	-5.86	-11.34	-22.60

ation of the Hubble tension. Similarly, the amplitude of matter fluctuations, σ_8 , and the total matter density, Ω_m , exhibit stable and well-constrained values across all datasets, pointing toward a coherent picture of structure formation.

Figure 3 shows the theoretical prediction for the angu-

lar BAO scale, θ_{BAO} , in comparison with the corresponding observational measurements from the sample used in this work. The solid and dashed curves represent the best-fit predictions for the Λ CDM and w_0w_a CDM models, respectively. As can be seen, the theoretical curves provide an excellent description of the data across the en-

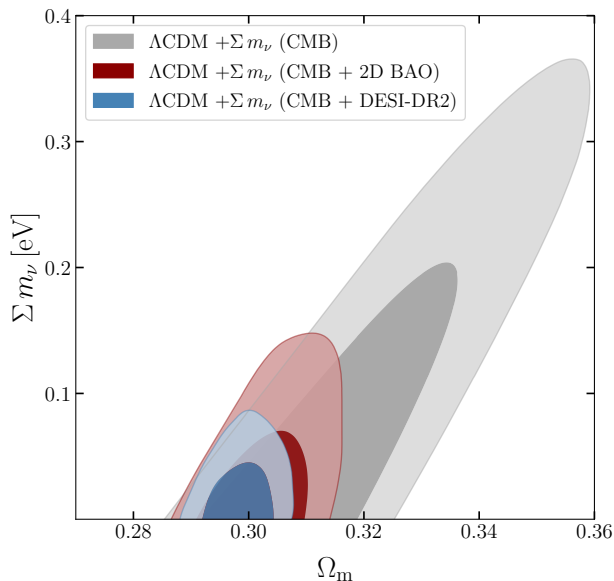


FIG. 4. The 68% and 95% confidence regions for the sum of neutrino masses, Σm_ν , and the matter density parameter, Ω_m , within the Λ CDM framework. The analysis assumes a prior of $\Sigma m_\nu > 0$ eV. CMB-only constraints reveal a strong positive correlation between Σm_ν and Ω_m . Both 2D BAO and DESI BAO data, while largely insensitive to the neutrino mass, provide tight measurements of Ω_m , effectively breaking the geometric degeneracy and resulting in a more stringent upper limit on Σm_ν .

tire redshift range. This comparison highlights the sensitivity of the 2D BAO measurements to potential departures from the cosmological constant, thus providing a powerful test of dark energy dynamics.

C. Constraints on the Neutrino Mass

Cosmological observations offer one of the most promising avenues to constrain the absolute neutrino mass scale (see, e.g., [108–112] for a review), providing a complementary approach to laboratory measurements from oscillation and beta-decay experiments. This sensitivity arises because massive neutrinos leave subtle but measurable imprints on various cosmological observables, including the expansion history, the growth of large-scale structure, and the suppression of power on small scales. Nevertheless, the precise determination of the total neutrino mass depends critically on the cosmological model assumed in the analysis. State-of-the-art datasets have been extensively employed to investigate neutrino properties, particularly using BAO measurements from DESI DR2, as discussed in, e.g., [50, 113–119].

In our baseline framework, as outlined in the previous subsection, we adopt a total mass for the three active neutrino species fixed at the minimum value allowed by neutrino oscillation experiments, $\Sigma m_\nu = 0.06$ eV, con-

sistent with the normal mass ordering scenario. In this fiducial configuration, the neutrino sector is modeled as one massive and two effectively massless states, which is adequate for most standard cosmological analyses.

To explore possible deviations and assess the robustness of our results, we extend our parameter space by allowing Σm_ν to vary freely. Throughout this section, we adopt a model in which the three neutrino mass eigenstates are assumed to be degenerate in mass. This approximation reproduces with high accuracy the cosmological effects of both the normal and inverted mass orderings, and it has been shown to recover the correct total mass value in the case of a positive detection, without introducing significant bias [108]. A uniform physical prior $\Sigma m_\nu > 0$ eV is imposed, ensuring the exploration of physically meaningful regions of parameter space.

In this subsection, we present updated results obtained using the 2D BAO compilation, combined with several complementary datasets discussed previously. We emphasize that these results are new in their own right, independent of the main objective, which is to analyze the updated 2D BAO dataset. Tables I and III summarize our constraints on the neutrino mass for the Λ CDM and $w_0 w_a$ CDM models, respectively.

Allowing the sum of neutrino masses, Σm_ν , to vary freely leads to tight upper limits from our main joint analyses. Using the combination CMB + 2D BAO, we obtain $\Sigma m_\nu < 0.081$ eV (95% CL), while the CMB + DESI DR2 analysis yields an even stronger constraint of $\Sigma m_\nu < 0.0698$ eV (95% CL). The inclusion of large-scale-structure information from either 2D BAO or DESI DR2 significantly enhances the sensitivity to Σm_ν compared to CMB-only analyses, mainly by breaking degeneracies between the neutrino mass and matter density parameters. The slightly tighter bound obtained with DESI DR2 reflects its higher precision in measuring the distance–redshift relation, confirming that both datasets provide robust and complementary constraints on the total neutrino mass. Importantly, the inclusion of the 2D BAO compilation delivers competitive and reliable constraints, demonstrating that this dataset constitutes a robust and independent probe for testing neutrino mass effects in cosmology.

Figure 4 displays the parameter space in the Ω_m – Σm_ν plane for the CMB, CMB + 2D BAO, and CMB + DESI DR2 analyses. It is evident that the inclusion of BAO data significantly tightens the upper limits on Σm_ν , with the strongest constraints obtained when either 2D BAO or DESI DR2 measurements are added. As discussed above for other cosmological parameters, we observe a similar trend for the neutrino mass: the DESI DR2 dataset provides more stringent bounds, reflecting the higher precision of its distance–redshift measurements. This comparison highlights the complementary role of different BAO compilations and demonstrates that both 2D BAO and DESI DR2 contribute robust and independent constraints on the total neutrino mass.

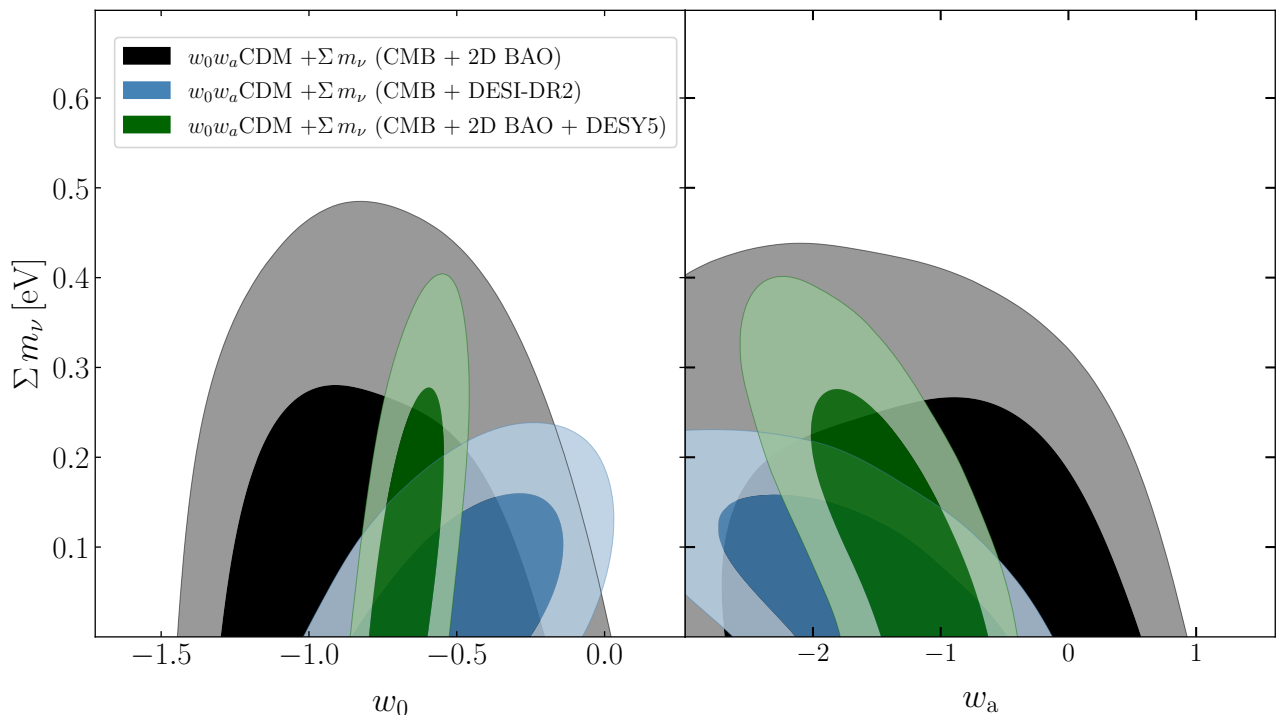


FIG. 5. Limits on the sum of neutrino masses within the w_0w_a CDM framework. The shaded regions represent the 68% and 95% credible intervals of the posterior distribution, assuming a prior of $\Sigma m_\nu > 0$ eV in all cases.

Neutrino mass constraints in the w_0w_a CDM model. To robustly constrain the sum of neutrino masses, Σm_ν , we combine CMB and 2D BAO measurements with different supernova samples. Allowing Σm_ν to vary freely results in upper limits that are somewhat weaker than those obtained in the Λ CDM case, reflecting the additional freedom introduced by the dynamical dark energy parameters. From the joint analyses, we find $\Sigma m_\nu < 0.290$ eV (95% CL) for CMB + 2D BAO + PP, while including supernova datasets slightly relaxes the constraint: $\Sigma m_\nu < 0.323$ eV for CMB + 2D BAO + Union3 and $\Sigma m_\nu < 0.332$ eV for CMB + 2D BAO + DES-Y5. This trend is expected, as the additional dark energy parameters (w_0 , w_a) partially degenerate with Σm_ν , reducing the constraining power of the data. Nevertheless, all limits remain well below 0.35 eV, demonstrating that the combination of CMB, BAO, and supernova measurements continues to provide a robust upper bound on the total neutrino mass even in the context of dynamical dark energy models.

Figure 5 shows the parameter space of Σm_ν versus w_0 and Σm_ν versus w_a for several of the joint analyses discussed above. These plots illustrate the statistical correlations between the neutrino mass and the dark energy parameters, as well as how different combinations of datasets constrain Σm_ν within this extended parameter space.

Overall, in this subsection, we show that combining

2D BAO measurements with CMB data and other geometric probes, such as SN Ia, can provide robust constraints on the neutrino mass. These constraints are competitive in precision with those obtained from DESI DR2 measurements. Accordingly, 2D BAO can be regarded as a complementary probe for studying neutrino properties.

IV. FINAL REMARKS

In this work, we have presented updated cosmological constraints using the 2D BAO compilation in combination with complementary datasets, including CMB, BAO DESI DR2, and various supernova samples, considering both the Λ CDM and w_0w_a CDM frameworks. For the Λ CDM model, the combination of CMB and 2D BAO data provides robust constraints on standard cosmological parameters and a stringent upper limit on the sum of neutrino masses, $\Sigma m_\nu < 0.081$ eV (95% CL). Incorporating DESI DR2 measurements further tightens this bound to $\Sigma m_\nu < 0.070$ eV, highlighting the high precision of DESI in constraining the matter density parameter and breaking the geometric degeneracy present in CMB-only analyses. In the context of dynamical dark energy models, w_0w_a CDM, allowing the dark energy equation-of-state parameters to vary introduces additional degeneracies with Σm_ν , leading to slightly weaker upper limits: $\Sigma m_\nu < 0.290$ eV for CMB + 2D BAO + PP, and $\Sigma m_\nu < 0.332$ eV when including

supernova datasets. Nevertheless, these constraints remain competitive, demonstrating that both 2D BAO and DESI DR2 provide robust and complementary probes of neutrino mass even in the extended parameter space of dynamical dark energy.

Ultimately, our results show that current BAO measurements, particularly when combined with CMB and supernova data, offer powerful constraints on cosmological parameters, including the sum of neutrino masses. Both 2D BAO and DESI DR2 datasets yield consistent and competitive results, reinforcing their crucial role in precision cosmology and in testing extensions beyond the minimal Λ CDM scenario.

In addition to the results presented here, we have shown that this new 2D BAO compilation is both robust and competitive in constraining cosmological parameters. Moreover, it can be combined with CMB-Planck data and supernova samples without introducing significant tensions among the datasets. Looking ahead, future observations of θ_{BAO} from public galaxy and quasar catalogs from DESI and Euclid will be crucial. Such measurements will allow for minimal model-independent tests and will provide a valuable resource for the broader cosmology community.

DATA AVAILABILITY

The datasets and products used in this research, including Boltzmann codes and likelihoods, will be made available upon reasonable request to the corresponding author following the publication of this article.

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