

Modeling nonlinear scales for dynamical dark energy cosmologies with COLA

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Upcoming galaxy surveys will bring a wealth of information about the clustering of matter at small scales, but modeling small-scale structure beyond Λ CDM remains computationally challenging. While accurate N -body emulators exist to model the matter power spectrum for Λ CDM and some limited extensions, it's unfeasible to generate N -body simulation suites for all candidate models. Motivated by recent hints of an evolving dark energy equation of state from galaxy surveys, we assess the viability of employing the COmoving Lagrangian Acceleration (COLA) method to generate simulation suites assuming the w_0w_a dynamical dark energy model. Following up on our previous work, we combine COLA simulations with an existing high-precision Λ CDM emulator to extend its predictions into new regions of parameter space. We assess the precision of our emulator at the level of the matter power spectrum, finding that our emulator can reproduce the nonlinear boosts from EUCLIDEMULATOR2 at less than 2% error. Moreover, we perform an analysis of a simulated cosmic shear survey akin to future data from the Legacy Survey of Space and Time (LSST) first year of observations, assessing the differences in parameter constraints between our COLA-based emulator and the benchmark emulator. We find our emulator to be in excellent agreement with the benchmark, achieving less than 0.3σ shifts in cosmological parameters across multiple fiducial cosmologies, and a 7D figure of bias of less than 0.35. We further compare our emulator's performance to a commonly used approach: assuming the Λ CDM boost can be employed for extended parameter spaces without modification. We find that our emulator yields a significantly smaller $\Delta\chi^2$ distribution, parameter constraint biases, and a more accurate figure of merit compared to this second approach. These results demonstrate that COLA emulators provide a computationally efficient and physically motivated path forward for modeling nonlinear structure in extended cosmologies, offering a practical alternative to full N -body suites in the era of precision cosmology.

I. INTRODUCTION

In recent decades, galaxy surveys have been able to map the large-scale structure of the Universe with great precision, being as competitive as CMB surveys. Current photometric surveys, such as DES [1–10], KiDS [11–23], HSC [24–29], and spectroscopic surveys, such as DESI [30–36], BOSS/eBOSS [37–50], and WiggleZ [51–55], have placed percent-level constraints on Λ CDM parameters, demonstrating the success of the theoretical model in describing independent datasets. However, in recent years, tensions in the cosmological parameter constraints have begun to arise with the increase in precision. Notably, most galaxy surveys report a mildly lower value for the structure growth parameter S_8 than those from CMB measurements (see *e.g.* [56]). Moreover, recent findings from the DESI collaboration, as well as Type Ia supernovae datasets such as Pantheon+, Union3, and DESY5 [30, 34, 35, 57–59] favor an evolution of the dark energy equation of state. Whether these results truly indicate new physics is still under debate [60–64].

Forthcoming Stage-IV galaxy surveys, such as the Vera Rubin Observatory's LSST [65], Euclid [66], SphereX [67], and Roman [68], will enable higher-precision measurements, especially at small scales where non-linearities in the matter density field become increasingly sizable [69], and will be decisive for investigating dark energy dynamics. Analyses of galaxy surveys rely on a key theoretical prediction: the matter power spectrum, as galaxies are biased tracers of the underlying matter density field. At large scales and early

times, the power spectrum can be quickly and accurately computed using Einstein-Boltzmann solvers such as CAMB [70] and CLASS [71, 72]. However, for small scales and low redshifts, linear perturbation theory breaks down, and accurately modeling non-linearities becomes a task of central importance.

Although N -body simulations offer the most accurate predictions in the nonlinear regime, they are computationally expensive — each one demanding tens of thousands of CPU hours [73–75]. As a result, integrating simulations into Bayesian analyses is a prohibitive task, as theoretical predictions must be provided for $O(10^5 - 10^6)$ points in the parameter space. To address this issue, machine learning emulators trained on N -body simulations have been developed for Λ CDM and simple, widely adopted extensions such as the phenomenological w_0w_a CDM [76, 77] parametrization for dark energy, as well as massive neutrinos with their total mass as a free parameter. Examples include EUCLIDEMULATOR2 [78], BACCO [79], COSMICEMU [80], the Dark Quest emulator [81, 82], the CSST Emulator [83–85], AEMULUS [86–89], among others. At the same time, due to the sheer amount of candidate cosmological models and the high cost of running N -body simulations, emulators for broader extensions of Λ CDM are still scarce: examples are emulators to specific modified gravity theories (*e.g.* [90–93]) where we have full N -Body simulations [94], as well as some hydrodynamical emulators.

A viable alternative to using full N -body simulations for constructing matter power spectrum emulators is to use well-established approximate methods for extended cosmological models. These methods reduce the computational complexity

of running hundreds of simulations to train emulators, at the cost of losing accuracy in deep non-linear scales. One compelling approach is to use the COrning Lagrangian Acceleration (COLA) method, which combines Lagrangian Perturbation Theory with a particle-mesh (PM) [95] evolution scheme to approximate N -body results while being cheaper than the usual N -body methods by 1-2 orders of magnitude [96, 97]. Emulators created using pure COLA simulations are prone to small-scale inaccuracies when compared directly to their N -body counterparts. To mitigate this effect, the work of [98] introduces an approach that combines COLA simulations with predictions from high-accuracy Λ CDM emulators or full N -body results, leveraging COLA's reduced computational cost and dramatically increasing small-scale accuracy simultaneously. Our previous work [99] has validated this approach, creating an emulator for the matter power spectrum of COLA simulations under the w CDM cosmological model, and testing it in a mock Stage-IV cosmic shear analysis. As such, we aim to demonstrate here that the hybrid approach of COLA-based emulators combined with high-resolution Λ CDM emulators can provide unbiased cosmological parameter constraints when compared to full N -body methods.

In this work, we now present a final validation of our COLA-based emulators on the w_0w_a CDM cosmological model, where the dark energy equation of state evolves linearly with the scale factor. This parametrization represents the most widely used and general extension of Λ CDM for which high-accuracy emulators currently exist, and remains central to ongoing investigations into dynamical dark energy [30, 34]. We extend our machine learning pipeline, combining COLA simulations with Λ CDM emulators, to predict the nonlinear matter spectrum across the w_0w_a CDM parameter space. We train a simple neural network to emulate the nonlinear correction factor (*i.e* the boost) from COLA, correcting for small-scale inaccuracies by referencing boosts from a high-fidelity Λ CDM emulator. This hybrid approach enables fast predictions across an extended cosmological parameter space while maintaining consistency with N -body precision. To validate our emulator in a cosmological inference setting, we perform a simulated cosmic shear analysis using survey specifications consistent with LSST's first year of observations (LSST-Y1) [65]. We compare parameter constraints derived using both our pipeline and a benchmark N -body emulator, chosen as EUCLIDEMULATOR2, quantifying their disagreement with standard tension metrics [100–102].

Additionally, we also benchmark our emulator against a widely used approximation method in beyond- Λ CDM analyses for models without dedicated nonlinear simulations [103–108]: projecting the nonlinear boost from the nearest Λ CDM cosmology. This projection method assumes that nonlinear corrections calibrated in Λ CDM remain valid in nearby extended cosmologies, providing a computationally inexpensive workaround but at the cost of uncontrolled systematics. In contrast, we demonstrate that for dynamical dark energy models, we find that the Λ CDM projection approach may introduce significant deviations in both goodness-of-fit and parameter constraints. Meanwhile, our COLA-based emulator reproduces the predictions of high-precision N -body emulators without

bias.

This paper is organized as follows: Section II describes the COLA simulations, cosmological parameters, simulation output processing, emulator construction, and validation; in Section III, we present our LSST-Y1 simulated cosmic shear analysis and the tension metrics used to assess their differences; in Section IV, we present and discuss the results of the LSST-Y1 simulated analysis; finally, we conclude in Section V.

II. COLA EMULATOR

A. COLA Simulation Suite

The COrning Lagrangian Acceleration (COLA) algorithm [96] is a fast approximate method for N -body simulations, wherein particles evolve in a frame comoving with trajectories calculated using Lagrangian Perturbation Theory (LPT), most commonly 2nd order Lagrangian perturbation theory (2LPT). For small scales, the method computes the force by using a Particle-Mesh (PM) algorithm, where the residual displacements not captured by LPT are added to the trajectories.

COLA has been shown to agree with full N -body simulations, at the level of the power spectrum at up to $k \sim 1 h/\text{Mpc}$, as well as when predicting ratios of the modified gravity power spectrum and the Λ CDM one, the so-called boost function in modified gravity [98, 109–111]. Despite being 1-2 orders of magnitude faster than a full N -body run, the computational cost of these approximations is still too high for direct use in the $O(10^6)$ computations of the matter power spectrum required for Monte Carlo searches. A practical alternative is to use a fixed set of COLA simulations to train emulators for the matter power spectrum, enabling efficient interpolation across cosmological parameter space. Our previous work demonstrated this approach for w CDM [99]; here, we extend it to w_0w_a CDM and evaluate its performance relative to the benchmark EUCLIDEMULATOR2 [78], which achieves $\lesssim 1\%$ precision for w_0w_a CDM + $\sum m_\nu$ up to $k = 10 h/\text{Mpc}^{-1}$ and $z \lesssim 3$.

1. Simulation Settings

We use the COLA algorithm as implemented in the public FML¹ code. Each simulation is performed in a box of size $L = 1024 h^{-1} \text{Mpc}$, populated with $N_{\text{part}} = 1024^3$ particles, initialized at $z_{\text{ini}} = 19$, and evolved over 51 time steps chosen to maintain a uniform time resolution of $\Delta a \approx 0.02$. The force grid uses $N_{\text{mesh}} = 2048^3$ cells, and the power spectra are calculated on-the-fly using a $N_{\text{pk-mesh}}^3 = 1024^3$ grid. Therefore, the corresponding Nyquist frequency is $k_{\text{Nyq}} = \pi N_{\text{pk-mesh}}/L = \pi h/\text{Mpc}$. To avoid aliasing, we

¹ <https://github.com/HAWinther/FML>

restrict our analysis to $k \leq k_{\text{Nyq}}$ [112]. Our choices are based on Reference [98] and are validated therein.

Initial conditions are generated using 2LPT, and we employ the forward approach [113] for our simulations. We provide the linear transfer functions of matter density, velocity, and relativistic species' densities at each time step using CLASS² in synchronous gauge, and convert to the N -body gauge [114–117] in COLA. Our simulations were run in the Seawulf³ cluster. With these settings, and using 128 cores, one COLA $w_0 w_a$ simulation takes approximately 40 minutes to finish, and requires a total RAM of approximately 950 GB.

To suppress sample variance from finite box effects at large scales ($k \approx 1/L$), we use the pairing-and-fixing method [118], in which we generate Gaussian random field modes with a fixed amplitude, $\delta_{i,\text{lin}}$, but with phase shifts of π with respect to one another. The initial overdensity fields are sampled as

$$\delta_{i,\text{lin}} = \sqrt{P_i} e^{i\theta_i}. \quad (1)$$

where θ_i is a random phase and P_i the initial power spectrum. Averaging over each pair, we find that the result substantially suppresses the effects of cosmic variance. This strategy was chosen for this work following [99] and [78].

2. Definition of the Parameter Space

We consider the cosmological $w_0 w_a$ CDM model, where the dark energy equation of state is parametrized as

$$w(a) = w_0 + w_a(1 - a), \quad (2)$$

with a being the scale factor, and w_0 and w_a control the present-day value and time derivative of the dark energy equation of state, respectively. The Λ CDM model is recovered in the limit $w_0 = -1$ and $w_a = 0$. The free cosmological parameters are:

- Ω_m , the total baryon density,
- Ω_b , the total matter density,
- $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, the dimensionless Hubble parameter,
- A_s , the amplitude of initial scalar fluctuations,
- n_s , the scalar spectral index,
- w_0 and w_a , the dark energy equation of state parameters.

We fix the summed neutrino masses to the minimum value allowed by neutrino oscillation experiments, $\sum m_\nu = 0.058$ eV, assuming three degenerate massive species [119, 120].

The parameter space boundaries are described in Table I, set to match those of EUCLIDEMULATOR2, which we have adopted

	Ω_m	Ω_b	n_s	$A_s \times 10^9$	h	w_0	w_a
Min	0.24	0.04	0.92	1.7	0.61	-1.3	-0.7
Max	0.40	0.06	1.00	2.5	0.73	-0.7	0.5
Center	0.319	0.05	0.96	2.1	0.67	-1	0

TABLE I. Parameter space validity bounds of our COLA-based emulator. The training set is drawn from a slightly bigger hypercube, where each dimension is stretched by 10% in each direction (e.g. $0.224 < \Omega_m < 0.416$). We also define a center cosmology chosen to agree with the EUCLIDEMULATOR2 reference cosmology [78].

as our benchmark for comparison. We emphasize that this choice is arbitrary, and our methodology is agnostic to the benchmark we have chosen. To improve model performance near parameter space edges, our COLA training simulations are sampled from an expanded box where each parameter interval has been stretched symmetrically by 10%. Cosmologies within this volume are selected using Latin hypercube sampling to ensure uniform coverage for training and validation.

B. Emulation of COLA Boosts

1. Emulator Prototypes with HALOFIT

Our goal is for the emulation error (*i.e.*, the error in recovering COLA boosts from a predetermined test set excluded from training) to be significantly smaller than the intrinsic COLA approximation error relative to full N -body simulations. To determine optimal hyperparameters, such as training set size and emulator architecture, we perform mock tests using HALOFIT [121] boosts. We generated training datasets with $N_{\text{train}} \in [500, 600, 700, 800, 1000]$ HALOFIT boosts and a test dataset with $N_{\text{test}} = 200$. We found that 600 simulations were sufficient to achieve $\sim 0.1\%$ error at $k = 1 \text{ h/Mpc}$; conservatively, we adopt $N_{\text{train}} = 700$ and $N_{\text{test}} = 200$ for our COLA emulator.

2. Post-processing the Simulation Boosts

We define the nonlinear boost as

$$B^X(k, z|\theta) \equiv \frac{P^X(k, z|\theta)}{P^L(k, z|\theta)}, \quad (3)$$

where θ refers to a point in the $w_0 w_a$ CDM parameter space, $P^X(k, z|\theta)$ is the matter power spectrum for cosmology θ , either linear (denoted P^L), or calculated using COLA or another N -body method (generically denoted P^X). Prior to computing B^{COLA} , we subtract the shot noise power spectrum, $P_{\text{SN}} = (L/N_{\text{part}})^3 = 1(\text{Mpc}/h)^3$, from P^{COLA} . At high redshift, $z > 1.182$, aliasing of the k modes near the Nyquist frequency leads to a power spectrum less than the shot noise for some simulations, and the subtraction would lead to unphysical negative values [113]; for these redshifts,

² https://github.com/lesgourg/class_public

³ <https://rci.stonybrook.edu/HPC>

we choose to cut the scales at half of the Nyquist frequency, $k_{z>1.182} \leq (\pi/2) h/\text{Mpc}$, following our procedure in [99] (also see [112]).

We then perform several transformations to optimize the inputs and outputs of our emulator. For instance, machine learning techniques are known to perform poorly if the features span several orders of magnitude. To stabilize the following procedures, we normalize the cosmological parameters θ to $[-1, 1]$ according to

$$\theta_N = -1 + 2 \frac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}}, \quad (4)$$

where minimum and maximum values correspond to the training set boundaries, *i.e.*, stretching the intervals of Table I by 10% in each direction. Furthermore, we standardize the boosts using

$$B_N^{\text{COLA}}(k, z|\theta) = \frac{B^{\text{COLA}}(k, z|\theta) - \bar{B}^{\text{COLA}}(k, z)}{\sigma_B(k, z)}, \quad (5)$$

where $B_N^{\text{COLA}}(k, z|\theta)$ is the normalized COLA boost, $\bar{B}(k, z)$ is the average of all boosts in the training set, and $\sigma_B(k, z)$ is their standard deviation. We then perform a Principal Component Analysis (PCA) decomposition of the COLA boosts using `SCIKIT-LEARN` [122] to reduce dimensionality. We retain $N_{\text{PC}} = 15$ components, which are sufficient to recover the test set boosts to within 0.2%.

3. Neural Network Emulator

After post-processing, we train our emulator with the normalized cosmological parameters as input features and the principal components as targets. We use a fully connected neural network with three hidden layers, each with 1024 neurons, with a mean squared error loss function,

$$\mathcal{L} = \sum_{i=1}^{N_{\text{train}}} \sum_{j=1}^{N_{\text{PC}}} (\alpha_j^{i,\text{train}} - \alpha_j^{i,\text{pred}})^2, \quad (6)$$

where $\alpha_j^{i,\text{train}}$ is the j -th principal component coefficient of the i -th cosmology in the training set, and $\alpha_j^{i,\text{pred}}$ the corresponding prediction. We use the parametric activation function [123, 124]

$$y_n^{m+1} = \left[\gamma_n^m + (1 - \gamma_n^m) \frac{1}{1 + e^{-\beta_n^m y_n^m}} \right] \tilde{y}_n^m, \quad (7)$$

where y_n^{m+1} is the value of the n -th neuron of the $(m+1)$ -th layer, $\tilde{y}_n^m$ the n -th neuron from the $(m+1)$ -th layer after the application of weights and biases, and γ_n^m and β_n^m are parameters of the activation function that can be back-propagated during training. We use the `ADAM` [125] optimizer to train the model parameters.

4. Boost Errors

We perform a series of accuracy checks on the emulator outputs. First, to assess the accuracy of our neural network, we compare the emulator's predictions for test set cosmologies, unseen in the training procedure, against the actual COLA simulations. The relative errors are shown in the first panel of Figure 1. At $k = 1 h/\text{Mpc}$, 90% of the test set cosmologies have an emulation error within 0.1%. Comparing these direct emulation errors to the errors on the corrected boosts $\tilde{B}^{\text{COLA}}(k, z)$ (second panel), we note that the errors between the two differ by an order of magnitude. This indicates that, in the context of comparing COLA with high-precision simulations, the COLA emulator faithfully reproduces its simulations, and differences between the emulators can be attributed to the COLA approximation rather than the performance of the machine learning model.

As per [98], COLA simulations increasingly lose power⁴ at progressively smaller scales, leading to typical errors of 10% at $k = 1 h/\text{Mpc}$ for raw COLA boosts B^{COLA} , as defined in Equation 3. However, this power loss is cosmology-independent. Our previous work [99] showed that the best technique to build a COLA-based emulator is to leverage existing high-precision emulators in ΛCDM , using COLA only to extend the results into new dimensions, *i.e.*, extra model parameters. This idea is encoded in the following expression for the nonlinear boost,

$$\tilde{B}^{\text{COLA}}(k, z|\theta) = B^{\text{N-body}}(k, z|\theta_p) \times \frac{B^{\text{COLA}}(k, z|\theta)}{B^{\text{COLA}}(k, z|\theta_p)}, \quad (8)$$

where θ_p is the projection of θ in the ΛCDM subspace and $B^{\text{N-body}}$ is the nonlinear boost obtained from our benchmark N -body prescription. We choose `EUCLIDEMULATOR2` as the base N -body prescription.

The second panel of Figure 1 shows the relative difference between $\tilde{B}(k, z)$, calculated using Equation 8, and the benchmark emulator predictions, $B^{\text{EE2}}(k, z)$, for all test set cosmologies. At $k = 1 h/\text{Mpc}$, 90% of cosmologies have emulation errors within 2%, with 50% of cosmologies contained well within 1%. This demonstrates that our method successfully mitigates the accumulation of errors typical of COLA simulations in the nonlinear regime, allowing us to generate accurate predictions across our target k range. Furthermore, the cosmologies with larger errors are those with higher values of $w_0 + w_a$, a region of the parameter space excluded by current data.

Finally, we consider a third nonlinear prescription: using the nonlinear boost from `EUCLIDEMULATOR2` in the ΛCDM subspace; this approach will be denoted as $\text{EE2 } \Lambda\text{CDM}$. For this purpose, we compute the nonlinear boosts for $w_0 w_a \text{CDM}$ cosmologies using `EUCLIDEMULATOR2`, setting $w_0 = -1$ and $w_a = 0$. The third panel of Figure 1 shows relative errors

⁴ This loss of power is well known to PM N -body codes, which fail to resolve the internal dynamics of halos. This trend of losing power starts roughly at a scale at which the pure 1-halo term of the halo model would dominate the power spectrum.

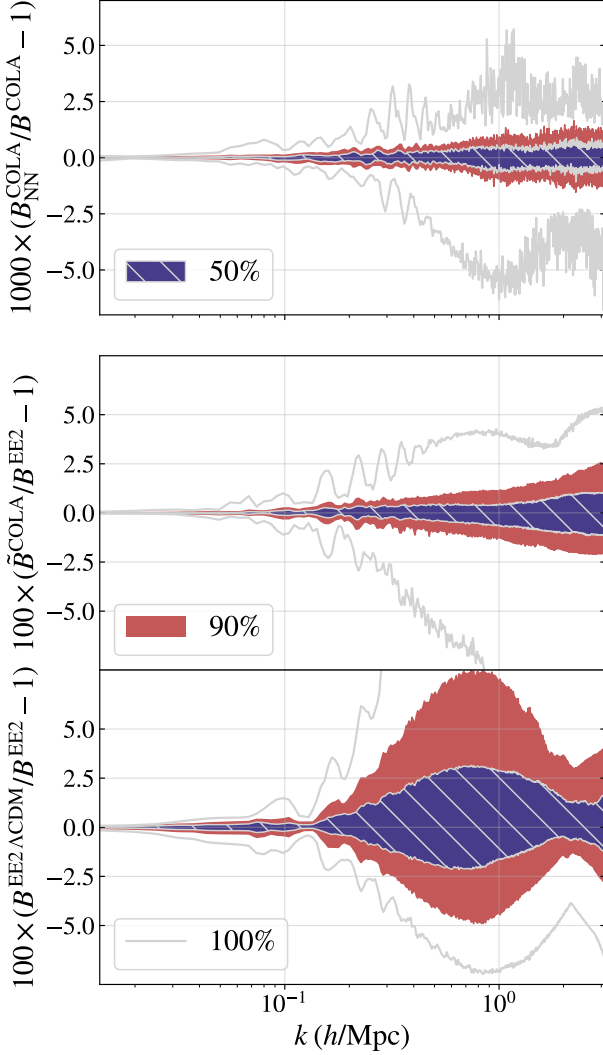


FIG. 1. **From top to bottom:** 1) Relative errors between the COLA boosts B^{COLA} predicted by the emulator versus those obtained from the test set simulations. 2) Relative errors between $\tilde{B}^{\text{COLA}}$ (see Equation 8) and the boosts from EE2. 3) Relative errors between $B^{\text{EE2 ACDM}}$ and EE2. Colors in all panels denote the percentile of cosmologies around the mean: blue contours enclose 50% of cosmologies, red contours enclose 90% of cosmologies, and the outer gray lines enclose 100% of cosmologies. All panels show results for $z = 0$, see Appendix A for the equivalent plots at higher redshifts.

between $B^{\text{EE2 ACDM}}$ and the actual EUCLIDEMULATOR2 boosts. The 90% percentile shows errors of the order of 7.5% at $k = 1$ h/Mpc, significantly worse than the COLA errors of the panel above.

These results support the viability of our emulator for parameter inference in $w_0w_a\text{CDM}$, as the emulated boost $\tilde{B}(k, z)$ agrees with our N -body proxy at a level suitable for upcoming precision cosmology experiments [126] while requiring significantly less computational expense compared to traditional N -body methods. In the following, we investigate how the differences shown in Figure 1 impact the parameter constraints from simulated cosmic shear analysis.

Parameter	Fiducial	Prior
Survey specifications		
Area	12300 deg ²	—
Shape noise per component	0.26	—
$n_{\text{eff}}^{\text{sources}}$	11.2 arcmin ⁻²	—
Photometric redshift offsets		
$\Delta z_{\text{source}}^i$	0	$\mathcal{N}[0, 0.002]$
Intrinsic alignment (NLA)		
a_1	0.7	$\mathcal{U}[-5, 5]$
η_1	-1.7	$\mathcal{U}[-5, 5]$
Shear calibration		
m^i	0	$\mathcal{N}[0, 0.005]$

TABLE II. Mock survey specifications for our simulated analysis, and nuisance parameter priors. $\mathcal{U}[a, b]$ denotes an uniform distribution with edges $[a, b]$, while $\mathcal{N}[a, b]$ denotes a Gaussian distribution with mean a and standard deviation b . Tomographic bin indices are denoted by i , and all our priors are the same for all bins.

III. ANALYSIS OF LSST-Y1 SIMULATED DATA

A. Simulating Cosmic Shear Data

We simulate cosmic shear observations based on LSST-Y1, following the methodology of [127, 128] and detailed in [99]. Survey specifications, source galaxy redshift distributions, and nuisance parameter priors are taken from the LSST DESC Science Requirements Document [65], and summarized in Table II. The redshift distribution is modeled as a Smail distribution convolved with a Gaussian uncertainty $0.02(1+z)$ and divided into five tomographic bins with equal galaxy number densities.

The cosmic shear two-point correlation functions $\xi_{\pm}^{ij}(\theta)$ are computed by first evaluating in Fourier space, $C_{\kappa\kappa}^{ij}(\ell)$, using the nonlinear matter power spectrum via the Limber approximation, then transforming to real space via the analytic functions in Appendix A of [6]. We compute $\xi_{\pm}^{ij}$ in 26 logarithmically spaced angular bins between 2.5 and 900 arcmin, averaging over each bin. We include standard self-calibrating systematics in our computation of $\xi_{\pm}$ — photometric redshift uncertainties, multiplicative shear calibration, and the non-linear alignment (NLA) model of intrinsic galaxy alignments (see, e.g., [129, 130]).

Likelihood analyses are performed using COCOA, the COBAYA-COSMOLIKE Joint Architecture⁵ [131–133]. Linear power spectra are computed with CAMB [134, 135], and nonlinear corrections are applied using either $\tilde{B}$ (Eq. 8), $B^{\text{EE2 ACDM}}$, or EUCLIDEMULATOR2. We use MCMC sampling to explore the parameter space and assess convergence using the Gelman–Rubin criterion ($|R - 1| < 0.01$) [136].

⁵ <https://github.com/CosmoLike/cocoa>

θ	Ω_m	$10^9 A_s$	n_s	w_0	w_a
$\theta^\uparrow$	0.36	2.3	0.98	-0.85	0.25
$\theta^\downarrow$	0.28	1.9	0.94	-1.15	-0.35

TABLE III. "High" ($\uparrow$) and "low" ($\downarrow$) cosmological parameter values used to construct the fiducial cosmologies of our analyses. In this notation, fiducial cosmologies are labeled in the text by the parameters shifted from the central values listed in Table I.

$\langle z \rangle$	0.33	0.54	0.74	1.01	1.62
Cutoff					
Cutoff 1	1.4	1.1	0.9	0.9	0.8
Cutoff 2	2.9	2.2	1.9	1.7	1.6
Cutoff 3	5.7	4.3	3.8	3.4	3.3

TABLE IV. Approximate scales in wavenumber k , measured in $h\text{Mpc}^{-1}$, for each galaxy source bin that correspond to the angular cutoffs in ξ_+ tested in our LSST-like cosmic shear analysis. The scales are approximated by computing the angular diameter distance to the mean redshift of each bin and converting to a wavenumber.

To evaluate the emulator's accuracy in the parameter space, we define 29 fiducial cosmologies within the emulator's validity range. One data vector is generated at the center cosmology shown in Table I. We then vary cosmological parameters to their intermediate "low" ($\downarrow$) and "high" ($\uparrow$) values, shown in Table III. We define four cosmologies by varying both w_0 and w_a for each combination of their low and high values, keeping other parameters fixed at their central values. We further define 24 fiducial cosmologies varying w_0 , w_a , and one of Ω_m , A_s , or n_s , using all combinations of their low and high values. All data vectors are generated using EUCLIDEMULATOR2 as the nonlinear prescription, and the covariance is computed analytically with CosmoCov [127]. We do not include a Gaussian noise realization in the fiducial data vectors.

To mitigate biases from emulator inaccuracies on small scales, which may degrade the goodness-of-fit, we apply three angular scale cuts (C1-C3) following [99], removing $\xi_\pm$ measurements below a minimum angular separation, $\theta_{\min}$. The corresponding wavenumbers are shown in Table IV; cuts are more aggressive for ξ_- due to its sensitivity to smaller scales.

Finally, to compute the cosmic shear integrals beyond the emulator's k -range, we extrapolate $\log(\tilde{B}(k))$ versus $\log(k)$ using a linear fit for both COLA and EE2 emulators. A Savitsky-Golay filter (order 1, window length 5) is applied to the last entries of the $\tilde{B}^{\text{COLA}}$ vector to suppress noise before extrapolation.

B. Quantifying discrepancies

To quantify deviations between parameter constraints from the LSST-Y1 simulated analyses using the nonlinear prescriptions X and Y , we first evaluate the one-dimensional bias for

key parameters Ω_m , S_8 , w_0 , and w_a , defined as

$$\frac{\Delta\theta_i}{\sigma_{\theta_i}} = \frac{\langle\theta_i\rangle_X - \langle\theta_i\rangle_Y}{\sqrt{\sigma_{\theta_i,X}^2 + \sigma_{\theta_i,Y}^2}}, \quad (9)$$

where θ_i denotes one of Ω_m , S_8 , w , or w_a , and $\langle\theta_i\rangle$ and $\sigma_{\theta_i}^2$ are, respectively, the sample mean and sample variance from MCMC posteriors.

To capture parameter correlations, we also compute the Figure of Bias (FoB), a multivariate generalization of the 1D bias defined by

$$\text{FoB}(\boldsymbol{\theta}) = [\Delta\langle\boldsymbol{\theta}\rangle^T \cdot (C_{\text{COLA}} + C_{\text{EE2}})^{-1} \cdot \Delta\langle\boldsymbol{\theta}\rangle]^{1/2}, \quad (10)$$

where $\boldsymbol{\theta}$ denotes a vector of cosmological parameters, $\Delta\langle\boldsymbol{\theta}\rangle = \langle\boldsymbol{\theta}\rangle_{\text{COLA}} - \langle\boldsymbol{\theta}\rangle_{\text{EE2}}$ is the difference in sample means, and C_X denotes the parameter covariance matrices for prescription X . We choose to calculate the FoB in selected 2D planes: $\Omega_m \times S_8$, $\Omega_m \times w_0$ and $\Omega_m \times w_a$. Furthermore, we also calculate the FoB in the seven cosmological parameters. A bias of less than 0.3 is considered negligible [137].

Changing the nonlinear modeling of the cosmic shear data vector may lead to underestimating or overestimating cosmological parameters, compared to a fiducial model. The strength of the constraints is measured by the Figure of Merit (FoM) statistic, defined as

$$\text{FoM} = \alpha \det(C)^{-1/2}, \quad (11)$$

where C is the covariance matrix of cosmological parameters obtained from the MCMC, and α is a prefactor that depends on the desired limits (*i.e.*, 1σ or 2σ) and the number of parameters considered [138]. We report the FoM ratio between the COLA and benchmark analyses.

To assess whether a nonlinear prescription X may degrade the goodness-of-fit when compared to the benchmark emulator, we compute the quantity

$$\Delta\chi^2 = (\mathbf{t}_X - \mathbf{t}_{\text{EE2}})^T \cdot C_{\text{data}}^{-1} \cdot (\mathbf{t}_X - \mathbf{t}_{\text{EE2}}), \quad (12)$$

where $\mathbf{t}_X$ is the cosmic shear theory prediction calculated using the nonlinear prescription X and C_{data} is the data covariance matrix. We compute this quantity for random points across the parameter space.

IV. RESULTS FOR LSST-Y1 SIMULATED ANALYSIS

To evaluate the accuracy of our emulator in practice, we begin by examining cosmological parameter constraints across three nonlinear prescriptions: the EUCLIDEMULATOR2 benchmark, our COLA-based emulator, and EE2 ΛCDM (*i.e.*, using the boost from the projected ΛCDM cosmology). Figure 2 shows 1D and 2D posterior contours (68% and 95%) for the parameters Ω_m , S_8 , w_0 and the sum $w_0 + w_a$, assuming the central cosmology (see Table I) as the fiducial. Since the constraints on w_a are weak, we focus on $w_0 + w_a$ as the more informative parameter combination.

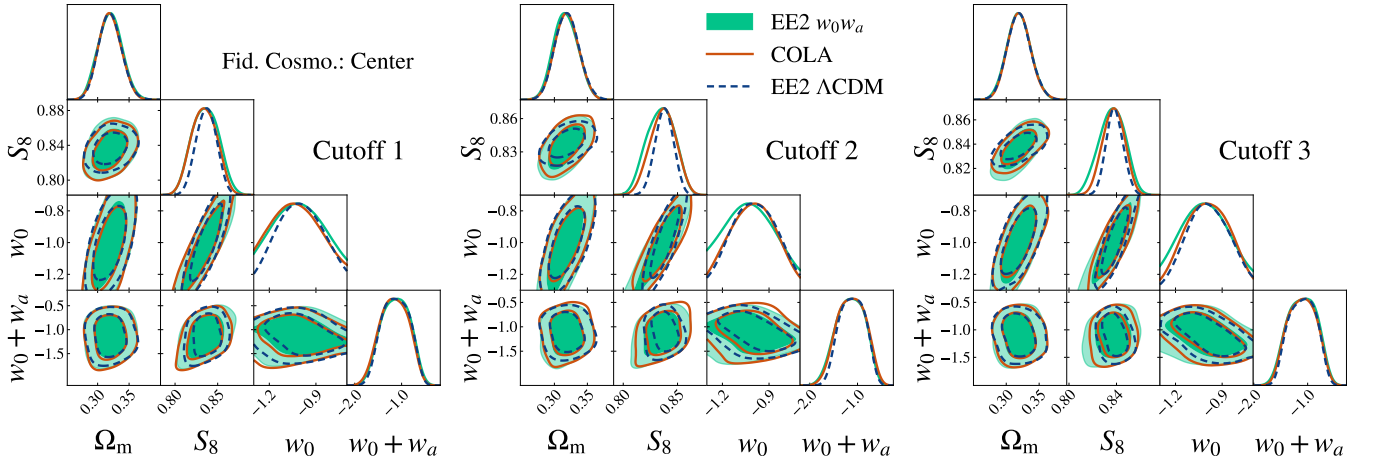


FIG. 2. Cosmological parameter constraints (68% and 95%) from the LSST-Y1 simulated analyses assuming the center cosmology from Table I as the fiducial. Green-filled contours denote constraints obtained using EUCLIDEMULATOR2 as the nonlinear prescription, orange dashed and dotted contours use our COLA emulator, and blue dashed contours use EE2 Λ CDM prescription. The left, middle, and right panels show constraints using the angular cutoffs C1, C2, and C3, respectively. We observe no shifts between the analyses; however, using cutoffs C2 and C3, the constraints obtained using COLA are slightly tighter than those found with EE2 for S_8 and w_0 . For EE2 Λ CDM, this effect is amplified.

Because our fiducial cosmology is in the Λ CDM subspace, where all prescriptions are equivalent, we observe no significant biases between the different constraints, and all posteriors peak at the same point. However, an indication of failure outside Λ CDM is observed in the size of the error bars. Employing the most conservative cutoff (C1), the COLA and EUCLIDEMULATOR2 contours are nearly indistinguishable. At the same time, those of EE2 Λ CDM are significantly smaller than the "true" error bars of EE2, especially at the parameters S_8 and w_0 , positively correlated. These results suggest that, in the context of LSST-Y1 cosmic shear, COLA emulators are equivalent to high-precision N -body emulators up to $k = 1 h/\text{Mpc}$.

Figure 1 indicates that, as we advance towards smaller angular scales, the disagreements in the boost predictions between COLA and EUCLIDEMULATOR2 quickly increase. For Cutoffs 2 and 3, the constraints are still in excellent agreement, but COLA yields slightly tighter error bars on S_8 and w_0 ; this effect is much more pronounced in the case of EE2 Λ CDM. The 1D marginalized constraints under Cutoff 2 are:

- EUCLIDEMULATOR2: $S_8 = 0.835 \pm 0.013$, $w_0 = -1.02^{+0.15}_{-0.18}$, $w_0 + w_a = -1.15^{+0.32}_{-0.28}$;
- COLA: $S_8 = 0.837 \pm 0.012$, $w_0 = -1.00 \pm 0.14$, $w_0 + w_a = -1.13 \pm 0.29$;
- EE2 Λ CDM: $S_8 = 0.838 \pm 0.008$, $w_0 = -1.00 \pm 0.14$, $w_0 + w_a = -1.11 \pm 0.26$.

Using Cutoff 3, the constraints are:

- EUCLIDEMULATOR2: $S_8 = 0.836^{+0.012}_{-0.010}$, $w_0 = -1.01 \pm 0.14$, $w_0 + w_a = -1.10 \pm 0.26$;
- COLA: $S_8 = 0.838 \pm 0.009$, $w_0 = -1.00 \pm 0.13$, $w_0 + w_a = -1.10 \pm 0.26$;
- EE2 Λ CDM: $S_8 = 0.838 \pm 0.007$, $w_0 = -0.99 \pm 0.13$, $w_0 + w_a = -1.09 \pm 0.24$.

To quantify the overestimation in S_8 and w_0 , we compute the figure of merit (FoM) in the $S_8 \times w_0$ plane. Assuming Cutoff 2, relative to EUCLIDEMULATOR2, the COLA emulator increases the FoM by 8%, indicating slightly tighter constraints, whereas the projected EE2 Λ CDM boost inflates the FoM by approximately 47%. For the most "aggressive" Cutoff 3, the FoM obtained with COLA is 19% bigger than that of EE2, while EE2 Λ CDM increases the FoM by 58%. Remarkably, in terms of figure of merit, the COLA emulator performs better at Cutoff 3 than EE2 Λ CDM at Cutoff 1, where the FoM is increased by 28%. From Figure 2, we observe that most of the disagreement between COLA and EUCLIDEMULATOR2 lies in low w_0 and low S_8 values, two parameters that are positively correlated in the analysis. This region of the parameter space is excluded by low-redshift geometric data from type-Ia supernovae [58, 139] and BAO [30, 34]. Therefore, we expect that this disagreement would not affect constraints obtained from the combination of LSST cosmic shear data with supernovae and BAO distance measurements.

Figure 3 further investigates the disagreements between COLA and EUCLIDEMULATOR2, showing histograms of $\Delta\chi^2$ (see Equation 12) for COLA and EE2 Λ CDM compared to EUCLIDEMULATOR2, obtained from 10,000 cosmologies sampled randomly from the emulation box (see Table I). Across all cutoffs, the EE2 Λ CDM prescription yields $\Delta\chi^2$ distributions that are substantially broader than those from our COLA emulator. As such, the use of COLA simulations can significantly improve the consistency with the high-precision benchmark, while the projection approach fails to capture the nonlinear corrections required by dynamical dark energy models, leading to degraded fits and potentially biased constraints. In Figure 4, we investigate how these $\Delta\chi^2$ values distribute around cosmological space; we find a clear trend of higher values of $\Delta\chi^2$ correlated with higher values of Ω_m and σ_8 .

To assess the robustness of our results beyond the Λ CDM

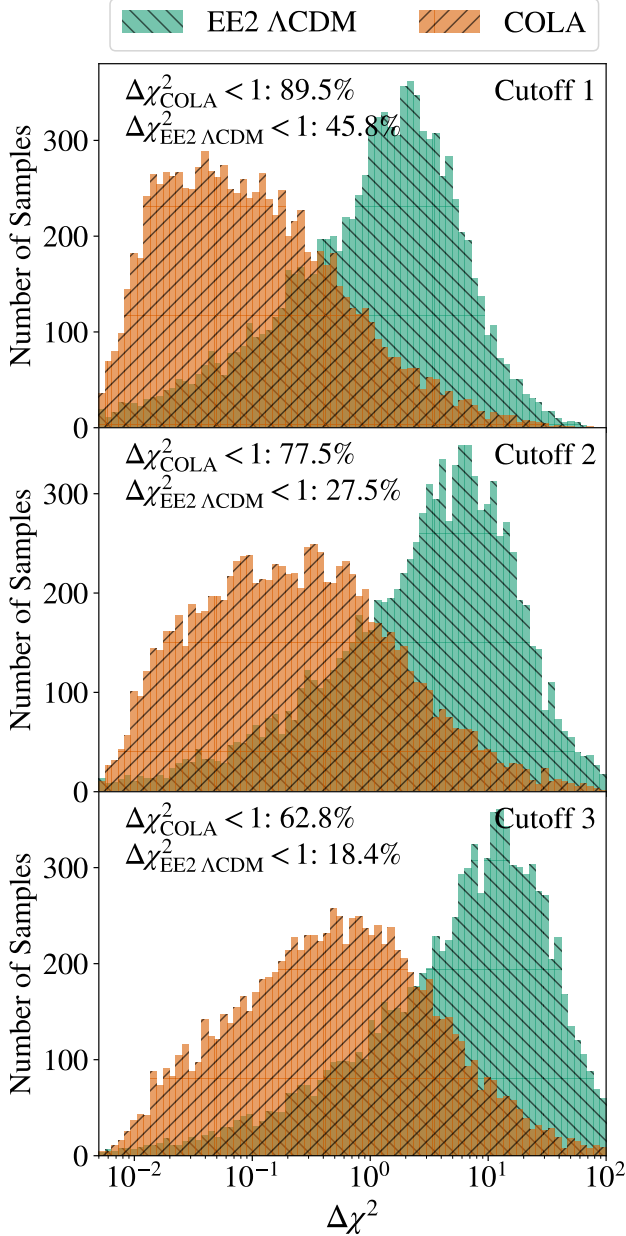


FIG. 3. Histograms of $\Delta\chi^2_X = (\mathbf{t}_X - \mathbf{t}_{\text{EE2}})^T \cdot \mathbf{C}_{\text{data}}^{-1} \cdot (\mathbf{t}_X - \mathbf{t}_{\text{EE2}})$ for prescription $X \in \{\text{COLA}, \text{EE2 } \Lambda\text{CDM}\}$ compared to EE2 at the full $w_0 w_a$ cosmology, with random samples drawn from the prior. The top, middle, and bottom panels show results for angular cut-offs C1, C2, and C3, respectively. The distribution of $\Delta\chi^2$ values demonstrates an order of magnitude difference between theory predictions calculated using our COLA method compared to the traditional EE2 ΛCDM approach, showing an improved fit from modeling the extended parameters using COLA.

subspace, we repeat our analysis using the fiducials described in Section II, all of them with $w_0 \neq -1$ and $w_a \neq 0$. We find that the biases may increase as we shift w_0 and w_a in the same direction, either higher or lower, in the parameter space of Table I. This is due to a known geometrical degeneracy along $w_0 + w_a$, which can suppress modeling systematics

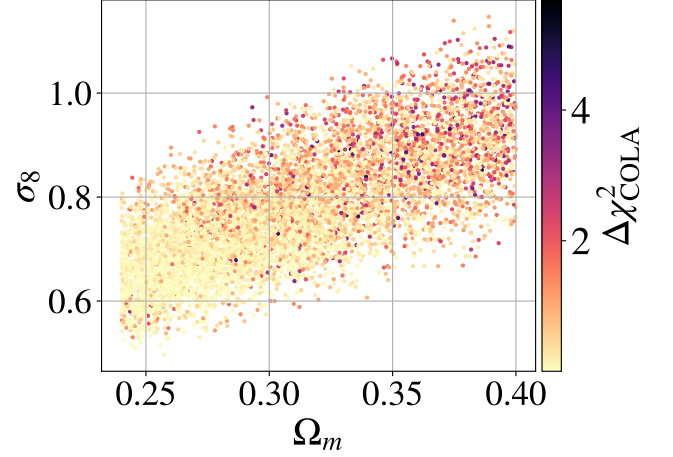


FIG. 4. Spatial distribution of $\Delta\chi^2$ in the $\Omega_m \times \sigma_8$ plane. We observe higher values of $\Delta\chi^2$ at higher values of Ω_m and σ_8 .

when $w_0 + w_a \approx 1$. Figure 5 illustrates the results for a fiducial with $w_0^\uparrow$ and $w_a^\uparrow$ (see Table III), keeping the other parameters fixed to their central values. In this case, we see a remarkable agreement between COLA and EE2 across all angular cut-offs. We focus on Cutoff 3, which yields the marginalized 1D constraints:

- EUCLIDEMULATOR2: $S_8 = 0.772^{+0.008}_{-0.006}$, $w_0 > -0.882$, $w_0 + w_a = -0.76^{+0.26}_{-0.15}$,
- COLA: $S_8 = 0.770^{+0.008}_{-0.007}$, $w_0 > -0.900$, $w_0 + w_a = -0.79^{+0.26}_{-0.17}$,
- EE2 ΛCDM : $S_8 = 0.762 \pm 0.006$, $w_0 = -0.879^{+0.140}_{-0.088}$, $w_0 + w_a = -0.82^{+0.26}_{-0.17}$.

In this case, the EE2 ΛCDM projection induces substantial biases in S_8 , shifting up by nearly 1σ . By comparison, the COLA emulator remains consistent to within 0.25 standard deviations. This trend extends to the multidimensional figures of bias: COLA has a 7D figure of bias of $\text{FoB}_{7\text{D}} = 0.27$ compared to the benchmark, while the projected EE2 ΛCDM reaches $\text{FoB}_{7\text{D}} = 1.04$.

Unlike the parameter constraints assuming the center cosmology from Table I as the fiducial, we see in Figure 5 that the posteriors calculated using our COLA-based emulator now closely track those generated using our N -body proxy, rather than overestimating S_8 and w_0 in any substantive way. We find that in switching from the center cosmology to $w_0^\uparrow$ and $w_a^\uparrow$, the relative ratio between FoMs of COLA and EE2 in the $S_8 \times w_0$ plane is 0.97, while the same ratio is 1.21 for EE2 ΛCDM .

To investigate whether our COLA emulator can provide unbiased constraints with FOMs similar to EE2 across the parameter space, Figure 6 shows the 1D biases of Equation 9 for Ω_m , S_8 , w_0 and w_a between the COLA emulator and our baseline EE2 for all scale cuts and all of the 29 cosmologies outlined in Section II. The fiducial cosmologies are listed in increasing order of their associated σ_8 values. All 1D biases are within

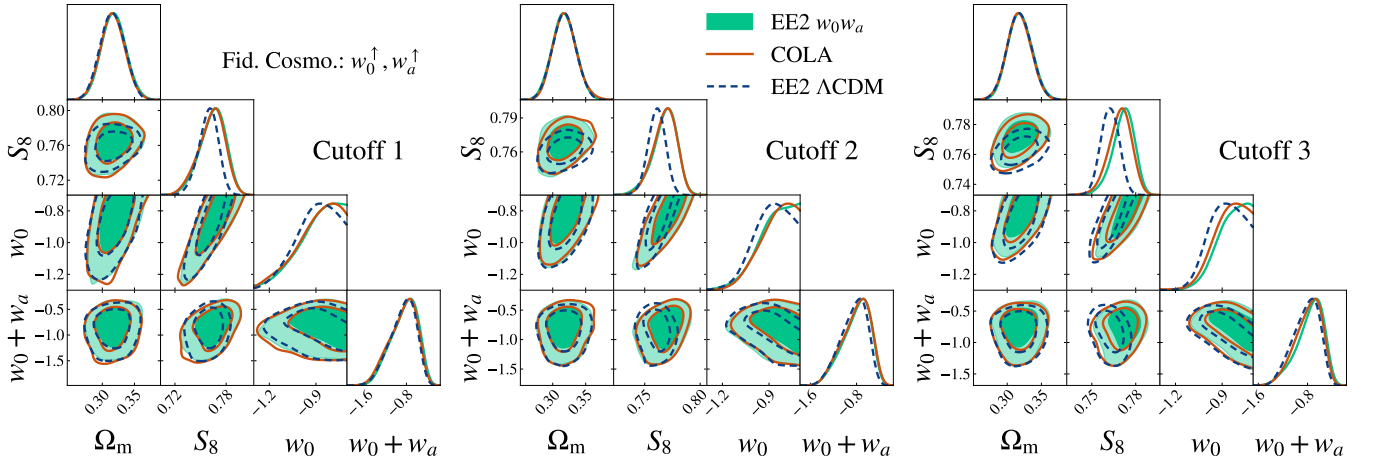


FIG. 5. Cosmological parameter constraints (68% and 95%) from the LSST-Y1 simulated analyses assuming a fiducial cosmology with $w_0^\uparrow$ and $w_a^\uparrow$ (see Table III), keeping the other parameters at their central values. Green-filled contours denote constraints obtained using EUCLIDEMULATOR2 as the nonlinear prescription, orange dashed and dotted contours use our COLA emulator, and blue dashed contours use EE2 Λ CDM prescription. The left, middle, and right panels show constraints using the angular cutoffs C1, C2, and C3, respectively. In this case, the EE2 Λ CDM prescription can provide significant biases in S_8 , which are not present when using COLA.

0.3σ , even for cosmologies with higher values of σ_8 and Ω_m , where Figure 4 shows our emulator performs worse. We computed the 7D figure of bias in cosmological parameters for all fiducial cosmologies using Cutoff 3, finding a maximum value of $\text{FoB}_{7D} = 0.35$ at the cosmology $\Omega_m^\uparrow, w_0^\uparrow, w_a^\uparrow$. We conclude that, for all scale cuts considered in this work, our emulator succeeds in providing unbiased constraints on cosmological parameters when compared to a high-precision N -body emulator in the context of dynamical dark energy models, even with significant variations in cosmological parameters and extreme values of σ_8 . Further, Figure 6 indicates our measure of the relative tightness of the parameter constraints in the $S_8 \times w$ plane compared to EE2, $\text{FOM}_{S_8 \times w}^{\text{COLA}} / \text{FOM}_{S_8 \times w}^{\text{EE2}}$, assuming Cutoff 3. The highest ratio is 1.19 for the center cosmology, shown in Figure 2. The same remark from before applies to other fiducial cosmologies: the disagreement between COLA and EE2 is driven mainly by low values of w_0 and S_8 , a region of the parameter space disallowed by low-redshift distance measurements.

V. CONCLUSION

In light of recent hints of dynamical dark energy, constraining its equation of state has become a task of central importance. While geometrical probes, such as BAO and supernovae measurements, are the primary probes of late dark energy behavior, galaxy surveys can also probe dark energy dynamics through their effects on the growth of large-scale structure. Extracting robust constraints from these surveys requires accurate modeling of nonlinear gravitational effects in the matter power spectrum, which becomes increasingly challenging in extended cosmologies where $w(z)$ departs from a cosmological constant. While accurate modeling can be achieved with N -body simulations, their high computational cost limits

their applicability across the myriad candidate dynamical dark energy models (*e.g.*, quintessence), and, further, for models that directly impact the growth of matter perturbations beyond modifications in the Universe’s expansion. In this context, COLA is a fast approximate alternative to N -body simulations, which, when appropriately corrected, presents an avenue for constructing accurate emulators for the nonlinear matter power spectrum at a fraction of the computational cost.

In this work, we have built an emulator for the nonlinear boost assuming the $w_0 w_a$ CDM cosmological model using a suite of 1400 COLA simulations, two for each of the 700 cosmologies in the training set to account for pairing-and-fixing. To evaluate the accuracy of the neural network, we ran a pair of simulations for each of the 200 cosmologies in the test set. The total computational cost of all simulations is estimated at 153.600 CPU-hours. A simple connected neural network with a trainable activation function can reproduce the test set boosts at 0.1% error. The computational cost of the simulation suite could potentially be lowered by using an alternative sampling algorithm for the training set cosmologies: the Sobol sequence [140], which has been shown to improve emulation errors compared to Latin hypercube sampling [83].

We have compared our COLA emulator to a benchmark N -body emulator, chosen as EUCLIDEMULATOR2. We test an additional nonlinear prescription common to analyses of extended cosmological models without N -body simulations: using N -body boosts at the projected Λ CDM cosmology (*i.e.*, setting $w_0 = -1$ and $w_a = 0$), an approach we denote as EE2 Λ CDM. We compare nonlinear models in two manners: at the boost level, shown in Figure 1, and at the level of a simulated cosmic shear analysis akin to LSST-Y1, assuming EUCLIDEMULATOR2 as the true nonlinear model. In the data analysis, to account for possible variations in the cosmological parameters when beyond- Λ CDM models are analyzed, we define fiducial cosmologies scattered across the parameter

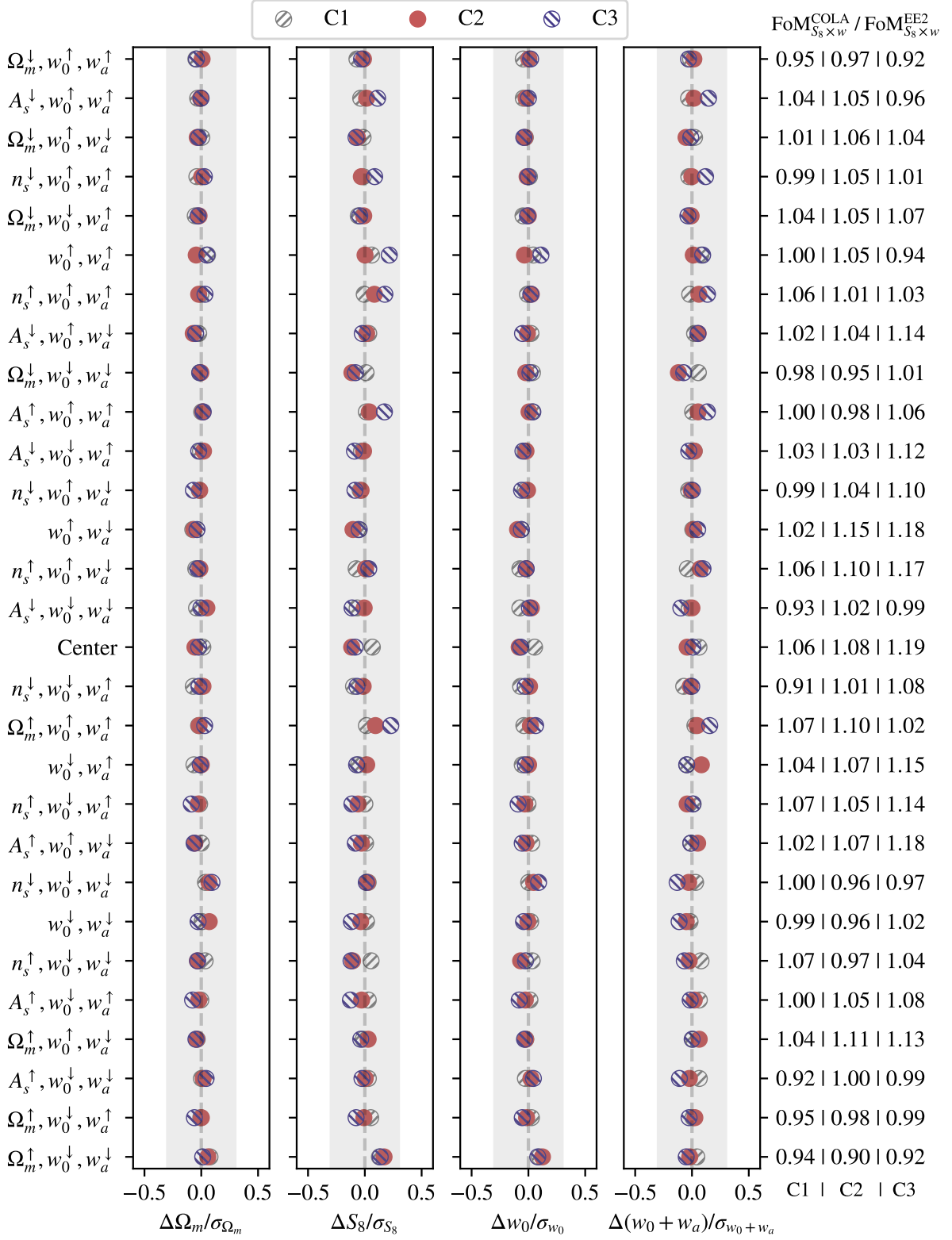


FIG. 6. One-dimensional biases from Equation 9, shown for the parameters Ω_m , S_8 , w_0 and w_a , sorted in increasing order of their associated σ_8 values. On the right-hand side, we show the ratios in figure of merit (see Equation 11) in the $S_8 \times w_0$ plane between the analyses using COLA and EE2, for the three angular cutoffs. The gray bands represent 0.3σ bias.

space, including outside Λ CDM, with significant variations in Ω_m and σ_8 . We have assessed the goodness-of-fit degradation, shown in Figure 3, and parameter constraint biases, shown in Figures 2, 5 and 6. We find that, at the boost level, our emulator can reproduce the benchmark emulator results with less than 2% error at $k = 1 h/\text{Mpc}$, while EE2 Λ CDM produces 7.5% errors at the same scales. As for the simulated analysis, we find that our COLA-based emulator can provide unbiased constraints compared to EUCLIDEMULATOR2: all 1D biases are well within 0.3σ , even for the most aggressive angular cutoffs and exotic fiducial cosmologies with extreme values of Ω_m , σ_8 , or outside Λ CDM. Furthermore, all 7D figures of bias are below 0.35. At the precision level expected for the first year of LSST observations, the COLA emulator yields constraints equivalent to those obtained using EUCLIDEMULATOR2 for scales up to $k \approx 3 h/\text{Mpc}$, comparable to our Cutoff 2.

Our results demonstrate that COLA, when combined with an accurate Λ CDM reference, offers a viable and flexible framework for extending nonlinear modeling to dynamical dark energy and other beyond- Λ CDM scenarios. We emphasize that, while we use EUCLIDEMULATOR2 as the "baseline" Λ CDM emulator in Equation 8, any other emulator could be used to provide Λ CDM boosts. Moreover, our methodology can be applied to more exotic models that also modify the growth of structure directly, such as modified gravity or coupled dark energy [105]. We also remark that there are avenues to improve our methodology. One example is the choice of "reference" cosmology. Equation 8 uses the projected Λ CDM cosmology because, geometrically, it is the closest cosmology; however, a possible choice that improves accuracy is to use a w CDM cosmology with the same value of σ_8 , akin to what is done in the

Casarini [141] prescription. Moreover, COLA can be combined with other analytical or semi-analytical prescriptions, such as the one proposed in [142], improving its accuracy at small scales.

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Appendix A: Simulations and Emulator Accuracy for Higher Redshifts

In this Appendix, we show relative errors from the simulations and the emulator for higher redshifts, ensuring that our discussion in Section II still holds. The results are presented in Figure 7. We find that for $z \leq 3$, the $z = 0$ results still hold in that at $k = 1 \, h/\text{Mpc}$, 90% of cosmologies have emulation errors within 2% on $\tilde{B}(k, z)$, and that 50% of cosmologies are contained well within 1%. COLA emulators trained at different redshifts demonstrate no notable change in the fidelity of predictions across the range $0 \leq z \leq 3$ (right panels), even though the highest 10% of raw emulation errors (left panels) and COLA boost errors (middle panels) grow with increasing z .

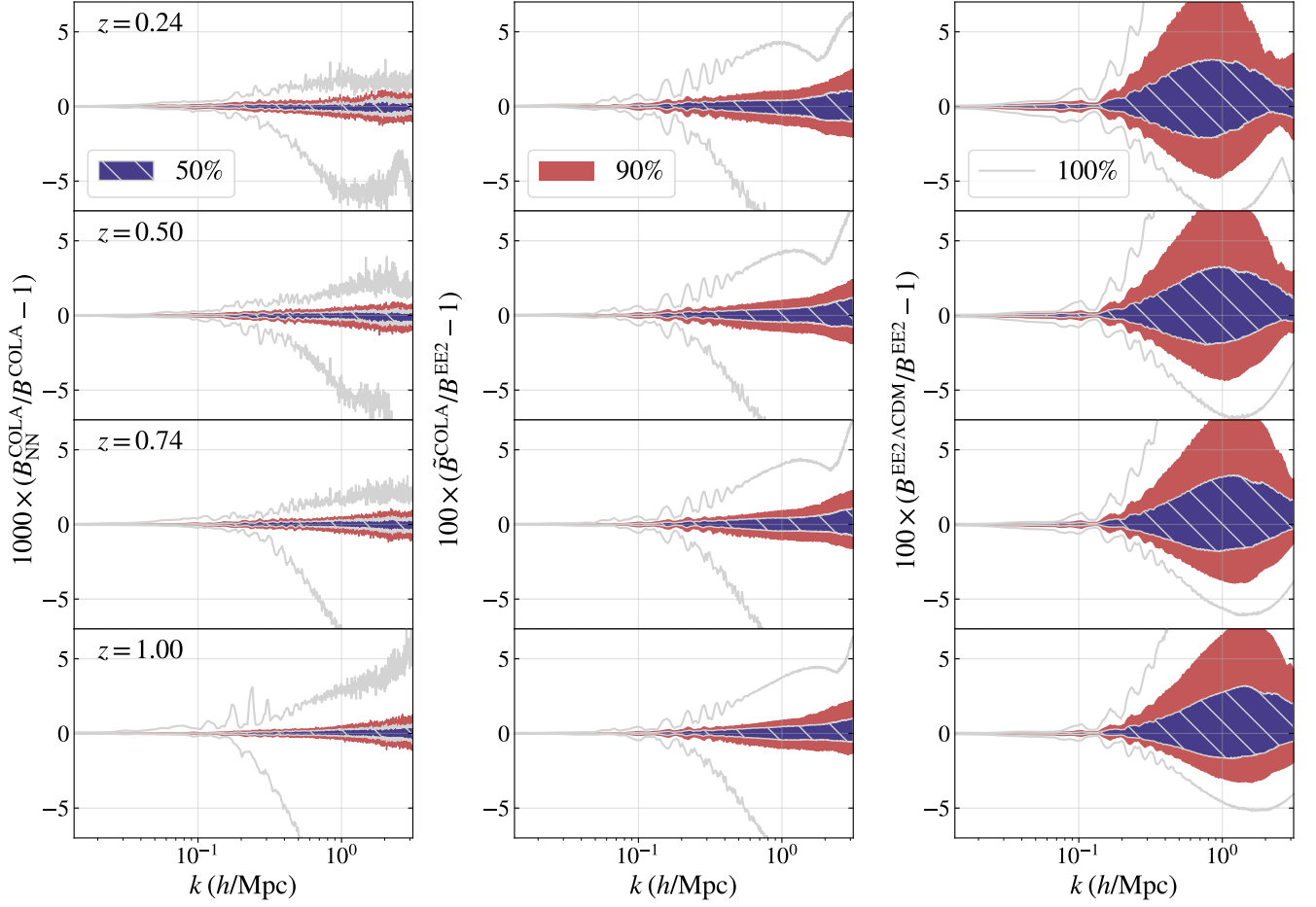


FIG. 7. **Left panel:** Relative errors between the COLA boosts B^{COLA} predicted by the emulator versus those obtained from the test set simulations. **Middle panel:** Relative errors between $\tilde{B}^{\text{COLA}}$ (see Equation 8) versus the boosts from EUCLIDEMULATOR2. **Right panel:** Relative errors between $B^{\text{EE2} \Lambda \text{CDM}}$ and EE2. Each row denotes a different redshift. Colors in all panels denote the percentile of cosmologies around the mean: blue contours enclose 50% of cosmologies, red contours enclose 90% of cosmologies, and the outer gray lines enclose 100% of cosmologies.