

JOINT CHANNEL AND CFO ESTIMATION FROM BEAM-SWEPT SYNCHRONIZATION SIGNAL UNDER STRONG INTER-CELL INTERFERENCE

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ABSTRACT

Complete awareness of the wireless environment, crucial for future intelligent networks, requires sensing all transmitted signals, not just the strongest. A fundamental barrier is estimating the target signal when it is buried under strong co-channel interference from other transmitters, a failure of which renders the signal unusable. This work proposes a maximum likelihood (ML)-based cross-preamble estimation framework that exploits carrier frequency offset (CFO) constancy across beam-swept synchronization signal (SS), coherently aggregating information across multiple observations to reinforce the desired signal against overwhelming interference. Cramer-Rao lower bound (CRLB) analysis and simulation demonstrate reliable estimation even when the signal is over a thousand times weaker than the interference. A low-altitude radio-map case study further verifies the framework's practical effectiveness.

Index Terms—CFO estimation, strong interference, beam-swept SS, multi-transmitter systems.

1. INTRODUCTION

The paradigm of wireless networks is evolving towards complex, multi-point architectures where awareness of the entire radio environment, not just the strongest signal, is critical [1–6]. For instance, cell-free systems require channel information from numerous access points to enable cooperation [1–3]. Likewise, high-resolution radio map construction needs to characterize all signals to provide a complete environmental picture [4–6]. A unifying theme across these systems is the necessity of processing weak signals in the presence of strong ones. This creates a challenge: weak signals of interest are often buried under severe co-channel interference, and the critical first step of CFO estimation fails, rendering them invisible to the network.

To estimate CFO, blind methods exploit inherent signal redundancy, such as the cyclic prefix (CP), for estimation

without dedicated overhead [7, 8]. However, their reliance on short data segments provides limited averaging gain, rendering them unreliable in low signal-to-interference-and-noise ratio (SINR) and strong interference conditions. Training-based methods offer improved robustness by utilizing known preambles or pilots as a clean reference [9–11]. However, practical synchronization signals are never perfectly orthogonal [12, 13], leading to cross-correlation leakage that allows strong interfering signals to corrupt the estimation of weaker ones. Even dedicated iterative techniques like successive interference cancellation (SIC) [14], designed specifically for this challenge, often fall short, as residual estimation errors from strong signals are typically still powerful enough to completely mask the weak signals.

We tackle this challenge by exploiting a structural property of modern training bursts. Within a short coherence block, multiple preambles exhibit heterogeneous per-sequence (beam-dependent) gains, while the CFO is common across the burst. For example, in 5G new radio (NR) SS block (SSB) structure [11, 15, 16]: each SSB has a distinct, beam-dependent power gain, yet all SSBs share a common CFO over a short duration (≈ 5 ms); in cell-free networks [1–3]: the same pilot from a user is received by multiple base stations (BSs) through different channels, so the received gains differ across sites, whereas the user-induced CFO is identical at all receivers. Our approach leverages this structure, proposing a cross-preamble estimation across the entire burst to reinforce the weak signal against strong co-channel interference.

This paper develops a principled framework for weak-signal synchronization. It establishes the performance limits, showing that it is not the instantaneous SINR that matters, but the aggregate SINR across all observations. Our key contributions are:

- We develop an ML-based cross-preamble CFO estimator that exploits a shared CFO across multiple preambles and, via CRLB analysis, show that the estimation error is inversely proportional to the aggregate SINR over all preambles and proportional to the CFO magnitude. Therefore, an iterative refinement algorithm, pre-compensation, and SIC, followed by re-estimation, is

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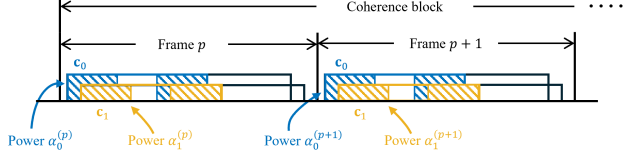


Fig. 1. Preamble Structure.

proposed to estimate the channel and CFO.

- Simulations demonstrate reliable CFO estimation down to -30 dB SINR and an approximate 6 dB improvement in channel estimation at low SINR. A low-altitude radio-map field test further confirms the framework's practical effectiveness.

2. SYSTEM MODEL

Consider a multi-transmitter system where a single receiver is within the coverage of K BSs, indexed by $\mathcal{K} \in \{0, 1, \dots, K-1\}$. Each BS periodically broadcasts a signal burst containing P distinct preamble frames, indexed by $\mathcal{P} \in \{0, 1, \dots, P-1\}$. All preamble frames contain two identical training sequences, as shown in Fig. 1. In addition, all preambles within one coherence block share the same timing and frequency offsets, while each preamble frame experiences a distinct channel.

2.1. Preamble Structure

Let $\mathcal{C}$ denote the set of all length- N training sequences. The preamble transmitted by the k th BS can be expressed as

$$\mathbf{c}_k = \mathbf{c}_{k,0} \oplus \mathbf{0} \oplus \mu_k \mathbf{c}_{k,1} \quad (1)$$

where $\mathbf{c}_{k,0}, \mathbf{c}_{k,1} \in \mathcal{C}$ are the selected training sequences; individual sequences may be reused across different BS, but the ordered pair $(\mathbf{c}_{k,0}, \mathbf{c}_{k,1})$, and thus the preamble $\mathbf{c}_k$ is unique to BS k ; $\mu_k \in \mathbb{R}^+$ is a scaling factor that represents the power difference between the two training sequences; and $\mathbf{0}$ is a zero-padding vector of fixed length τ_0 , corresponding to the timing offset between the two training signals.

The cross-correlation between two training sequences $\mathbf{c}_k, \mathbf{c}_l \in \mathcal{C}$ with timing offset τ and frequency offset ω is given by

$$r_{k,l}(\tau, \omega) \triangleq \frac{1}{N} \sum_{m=0}^{N-1} c_k[m] c_l^*[m-\tau] e^{-j\omega m}. \quad (2)$$

Because training sequences are not perfectly orthogonal, we model the cross-correlation for any mismatch ($k \neq l$) or nonzero delay ($\tau \neq 0$) as a zero-mean circular complex Gaussian with a variance that decays with the sequence length

N . For the matched case ($k = l$ and $\tau = 0$), the statistic reduces to the sequence's autocorrelation $r_k(\omega)$. As a result, the cross-correlation can be expressed as

$$r_{k,l}(\tau, \omega) = \begin{cases} r_k(\omega) & k = l, \tau = 0 \\ \mathcal{CN}(0, \sigma_c^2/N) & \text{otherwise} \end{cases} \quad (3)$$

where σ_c^2 captures sequence-family sidelobes and implementation impairments (e.g., filtering, residual multipath), and the magnitude of autocorrelation $|r_k(\omega)|$ follows a Dirichlet profile; $|r_k(0)| = 1$ and although oscillatory, its overall trend decreases as $|\omega|$ increases, e.g., for Zadoff–Chu sequences, $|r_k(\omega)| = |\sin(N\omega/2)/(N \sin(\omega/2))|$ [13].

2.2. Transmission Model

For the k th BS, denote τ_k and ω_k as the group timing and frequency offsets respectively, and denote $\alpha_k^{(p)}$ denote the complex channel gain in the i th frame. Then, the received signal at time m can be expressed as

$$y[m] = \sum_{k=0}^{K-1} \sum_{p=0}^{P-1} \alpha_k^{(p)} c_k[m - pN_f - \tau_k] e^{-j\omega_k m} + \nu[m] \quad (4)$$

for all $m \in \{0, 1, \dots, N_o - 1\}$, where N_f is the frame length and $\nu[m] \sim \mathcal{CN}(0, \sigma_n^2)$ denotes additive white Gaussian noise. The parameter N_o represents the total observation length under consideration. Without loss of generality, we assume that the received signal length is sufficiently large to accommodate all reference signals, and that training sequences from different frames do not overlap. Moreover, $c_k[m] = 0$ for $m \notin \{0, \dots, N-1\}$.

For notational convenience, we collect the received samples into a vector $\mathbf{y} \triangleq [y[0], y[1], \dots, y[N_o - 1]]^T$.

In this work, the initial tasks of active BS detection and delay estimation are assumed to be complete, consistent with established generalized likelihood ratio test (GLRT) methods [17]. The scope is therefore narrowed to estimating the remaining parameters for the active set $\mathcal{K}$: CFOs $\boldsymbol{\omega} \triangleq [\omega_k]_{k \in \mathcal{K}}$, scaling factors $\boldsymbol{\mu} \triangleq [\mu_k]_{k \in \mathcal{K}}$, and per-frame channel gains $\boldsymbol{\alpha} \triangleq [\alpha_k^{(p)}]_{k \in \mathcal{K}, p \in \mathcal{P}}$.

$$\underset{\boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\mu}}{\text{maximize}} \quad \mathbb{P}(\mathbf{y} | \boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\mu}).$$

This joint estimation problem is challenging due to strong parameter coupling, the inherent nonlinearity of CFO estimation, and imperfect orthogonality of the training sequences.

3. CROSS-PREAMBLE ESTIMATION

In this section, we first reformulate channel estimation as a linear problem and solve it via the classical linear least squares (LLS) estimator [18]. Second, we perform a single-preamble analysis and derive a separate CFO estimator. Then,

we extend the analysis across preambles, leveraging the burst-wise shared CFO to obtain a cross-preamble CFO estimator.

3.1. Channel Estimator

For each frame of the received signal, denoted as $\mathbf{y}^{(p)} \triangleq y[m + pN_f]_{m \in \{0,1,\dots,N_f-1\}} \in \mathbb{C}^{N_f \times 1}$, where $p \in \mathcal{P}$, then the p th received signal can be written as

$$\mathbf{y}^{(p)} = \underbrace{e^{j\omega(p-1)N_f} \boldsymbol{\Omega}(\omega) \odot \mathbf{C}(\tau)}_{\mathbf{A}^{(p)}} \boldsymbol{\alpha}^{(p)} + \mathbf{v}^{(p)}$$

where $\mathbf{C}(\tau) = [\mathbf{0}_{\tau \times 1}; \mathbf{c}_k; \mathbf{0}]_{k \in \mathcal{K}} \in \mathbb{C}^{N_f \times K}$, $\boldsymbol{\Omega}(\omega) = [\Omega_k(\omega_k)]_{k \in \mathcal{K}} \in \mathbb{C}^{N_f \times K}$, $\Omega_k(\omega_k) = [1; e^{j\omega_k}; \dots; e^{j\omega_k(N_f-1)}]$, $\boldsymbol{\alpha}^{(p)} = [\alpha_k^{(p)}]_{k \in \mathcal{K}} \in \mathbb{C}^{K \times 1}$, and $\mathbf{v}^{(p)} = [v[m]]_{m \in \{0,\dots,N_f-1\}} \in \mathbb{C}^{N_f \times 1}$.

As a result, the received signals can be expressed as

$$\mathbf{y} = \underbrace{\text{blkdiag}(\mathbf{A}^{(0)}, \dots, \mathbf{A}^{(P-1)})}_{\mathbf{A}(\tau, \omega)} \boldsymbol{\alpha} + \mathbf{v}.$$

Then, according to the LLS solution, the channel gain $\boldsymbol{\alpha}$ can be estimated by

$$\hat{\boldsymbol{\alpha}} = \mathbf{A}(\tau, \omega)^\dagger \mathbf{y}. \quad (5)$$

In addition, the received signals $\mathbf{y}$ can be expressed as

$$\mathbf{A}_0 \boldsymbol{\alpha} + \underbrace{[\mathbf{A}^{(0)} \text{diag}(\boldsymbol{\alpha}^{(0)}); \dots; \mathbf{A}^{(P-1)} \text{diag}(\boldsymbol{\alpha}^{(P-1)})]}_{\mathbf{B}} \boldsymbol{\mu} + \mathbf{v}.$$

where $\mathbf{A}_i = \text{blkdiag}(\mathbf{A}_i^{(0)}, \dots, \mathbf{A}_i^{(P-1)})$, $\mathbf{A}_i^{(p)} = e^{j\omega(p-1)N_f} \boldsymbol{\Omega}(\omega) \odot \mathbf{C}_i(\tau)$, $\mathbf{C}_i(\tau) = [\mathbf{0}_{(\tau+\tau_c i) \times 1}; \mathbf{c}_{k,i}; \mathbf{0}]_{k \in \mathcal{K}}$, and $\tau_c = N + \tau_0$. Then, according to the LLS solution, the scaling factor $\boldsymbol{\mu}$ can be estimated by

$$\hat{\boldsymbol{\mu}} = \mathbf{B}^\dagger (\mathbf{y} - \mathbf{A}_0). \quad (6)$$

3.2. Separate Analysis and Separate CFO Estimator

For an arbitrary frame p of the received signal, we define the cross-correlation between the received signal $\mathbf{y}$ and the two training sequences $\mathbf{c}_{k,0}$ and $\mathbf{c}_{k,1}$ as

$$r_{y,k}^{p,i}[\tau] \triangleq \frac{1}{N} \sum_{m=\tau}^{\tau+N-1} y[m + pN_f + i\tau_c] c_{k,i}^H[m - \tau]$$

where $k \in \mathcal{K}$, $p \in \mathcal{P}$, and $i \in \{0,1\}$.

Based on the cross-relation of the training sequences, the distribution of $r_{y,k}^{p,i}[\tau]$ is summarized as

Lemma 1. For any $k \in \mathcal{K}$, $p \in \mathcal{P}$, and $i \in \{0,1\}$, if $\tau = \tau_c$, the cross-correlation $r_{y,k}^{p,i}[\tau_k]$ follows

$$r_{y,k}^{p,i}[\tau_k] \sim \alpha_k^{(p)} r_k(\omega_k) (\mu_k e^{-j\omega_k \tau_c})^i + \mathcal{CN}(0, \sigma_{k,p,i}^2)$$

where the $\sigma_{k,p,i}^2$ is the averaged interference plus noise

$$\sigma_{k,p,i}^2 = \frac{\sum_{q \neq k, q \in \mathcal{K}} (\mu_k^i \alpha_q^{(p)} \sigma_c)^2 + \sigma_n^2}{N}.$$

Lemma 1 indicates that the ratio between the two correlation outputs, $r_{y,k}^{p,0}$ and $r_{y,k}^{p,1}$, reveals the exponential term associated with the CFO, which leads to the following separate CFO estimator:

$$\hat{\omega}_k^{\text{sep}} = -\frac{1}{\tau_c} \angle (\mathbb{E}[r_{y,k}[p,1]] / \mu_k / \mathbb{E}[r_{y,k}[p,0]]). \quad (7)$$

However, its variance depends on the per-sequence SINR $\delta_{k,p,i}^2$ and is easily overwhelmed by strong co-channel signals, which motivates the cross-preamble estimator below.

3.3. Cross-Preamble CFO Estimator

Let the set of correlation variables associated with CFO ω_k as $\mathbf{r}_{y,k} = [r_{y,k}^{p,i}[\tau_k]]_{p \in \mathcal{P}, i \in \{0,1\}}$, and define the cross-preamble likelihood as $\mathbb{P}(\mathbf{r}_{y,k}|\omega)$. Since preambles from different frames are non-overlapping, the correlation outputs are conditionally independent given ω_k , yielding

$$\mathbb{P}(\mathbf{r}_{y,k}|\omega) = \prod_{p \in \mathcal{P}} \prod_{i \in \{0,1\}} \mathbb{P}(r_{y,k}^{p,i}[\tau_k] | \omega_k).$$

According to Lemma 1, each correlation output $r_{y,k}^{p,i}[\tau_k]$ is Gaussian distributed, and thus the log-likelihood reduces to

$$\sum_{p \in \mathcal{P}} \sum_{i \in \{0,1\}} -\frac{|r_{y,k}^{p,i}[\tau_k] - \alpha_k^{(p)} r_k(\omega_k) (\mu_k e^{-j\omega_k \tau_c})^i|^2}{\sigma_{k,p,i}^2}. \quad (8)$$

We prove that the cross-preamble CFO estimator is shown in the following proposition.

Proposition 1. The cross-preamble ML CFO estimator is

$$\hat{\omega}_k^{\text{cross}} \triangleq \arg \max_{\omega_k} \mathbb{P}(\mathbf{r}_{y,k}|\omega) = \frac{1}{\tau_c} \angle \{\psi_1 \psi_0^H\} \quad (9)$$

where

$$\psi_0 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} r_{y,k}^{p,0}[\tau_k]^H}{\sigma_{k,p,0}^2}, \quad \psi_1 \triangleq \sum_{p \in \mathcal{P}} \frac{\alpha_k^{(p)} \mu_k r_{y,k}^{p,1}[\tau_k]^H}{\sigma_{k,p,1}^2}. \quad (10)$$

Proposition 2. The cross-preamble ML CFO estimator $\hat{\omega}_k^{\text{cross}} \stackrel{a}{\sim} \mathcal{N}(\omega, [\mathbf{I}^{-1}]_{11})$, where $[\mathbf{I}^{-1}]_{11}$ is CRLB

$$[\mathbf{I}^{-1}]_{1,1} = \frac{\Gamma_0^{-1} + \Gamma_1^{-1}}{2\tau_c^2 |r_k(\omega_k)|^2} \quad (11)$$

and Γ_0 and Γ_1 are the aggregated SINR of the two training signals

$$\Gamma_0 = \sum_{p \in \mathcal{P}} \frac{|\alpha_k^{(p)}|^2}{\sigma_{k,p,0}^2}, \quad \Gamma_1 = \sum_{p \in \mathcal{P}} \frac{|\mu_k \alpha_k^{(p)}|^2}{\sigma_{k,p,1}^2}. \quad (12)$$

Algorithm 1 Joint CFO and Channel Estimation Algorithm

- # **Initialization:** $\hat{\omega} \leftarrow 0$, $\hat{\alpha} \leftarrow 0$, $\hat{\mu} \leftarrow 1$, $\tilde{y} \leftarrow y$
- 1: Construct $\mathbf{A}$ based on $\hat{\omega}$ and $\hat{\mu}$ and update $\hat{\alpha}$ based on (5); Then, Construct $\mathbf{A}_0$ and $\mathbf{B}$ based on $\hat{\omega}$ and $\hat{\alpha}$ and update $\hat{\mu}$ based on (6).
 - 2: For each BS k , perform SIC and compensation: $\tilde{y}_k \leftarrow (y - \mathbf{A}(\hat{\tau}_{-k}, \hat{\omega}_{-k})\hat{\alpha}_{-k}) \odot \Omega_k(\omega_k)$. Then, estimate the remaining CFO δ_k based on the cross-preamble estimator (9), and update $\hat{\omega}_k \leftarrow \hat{\omega}_k + \delta_k$.
 - 3: Go to step 1) until $\sum_k |\delta_k| < \epsilon$.
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Our analysis of the CRLB reveals two key insights into CFO estimation. First, accurate CFO estimation for a weak signal is achievable. This follows from the estimation error being inversely proportional to the aggregate SINR across all beams (*i.e.*, $\Gamma_0^{-1} + \Gamma_1^{-1}$). In addition, a coarse-to-fine (iterative) refinement, pre-compensation, and SIC, followed by re-estimation, can progressively reduce the effective offset because the estimation variance increases with interference and increases with the magnitude of the CFO, $|\omega_k|$.

3.4. Joint CFO and Channel Estimation Algorithm

At first, we estimate channels based on (5) and (6). Then, for every k , we cancel the contributions of the other BSs, pre-compensate the CFO of BS k , and estimate its residual CFO increment δ_k . The to-be-estimated *residual* signal for BS k is

$$\tilde{y}_k = \left(y - \mathbf{A}(\hat{\tau}_{-k}, \hat{\omega}_{-k})\hat{\alpha}_{-k} \right) \odot \Omega_k(\hat{\omega}_k) \quad (13)$$

where “ $-k$ ” denotes the collection of parameters for all BSs except k (*i.e.*, the entries associated with BS k are set to zero). The residual CFO is then updated as $\hat{\omega}_k \leftarrow \hat{\omega}_k + \delta_k$. The iteration stops when the residual CFO corrections are small. Intuitively, improving the CFO estimate sharpens the matched preamble and reduces the bias/variance of the channel estimates; the refined channel estimates, in turn, yield smaller CFO increments. Under Proposition 2, this yields a monotone decrease of the residual CFO, and $\delta_k \rightarrow 0$.

4. SIMULATION RESULTS

We develop a simulation platform that generates 5G NR waveforms [15] and use it to assess the robustness of the proposed cross-preamble estimation framework. For comparison, we consider: i) CP-based blind estimator [8]; ii) Auto-correlation (separate) estimator that splits each training sequence into two segments and estimates CFO from their phase difference [11]; iii) Cross-correlation (separate) estimator as in (7); iv) Weighted average of separated estimated CFOs according to channel power.

Fig. 2 (a) illustrates the mean absolute error (MAE) over SINR. As expected, error decreases with SINR. The ranking

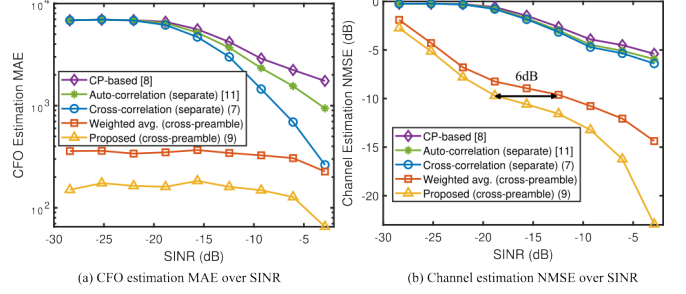


Fig. 2. CFO ω estimation MAE and channel α estimation NMSE over SINR, obtained from 200 Monte-Carlo simulations, with $K = 12$ BSs, and $P = 12$ preambles.

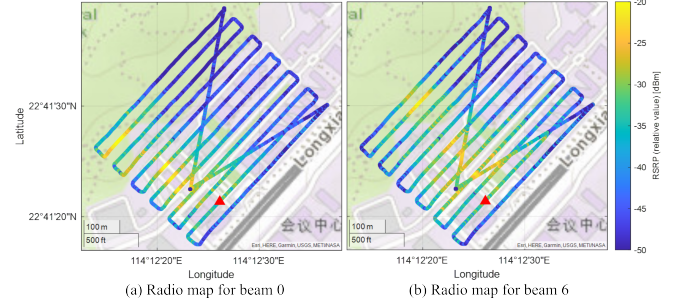


Fig. 3. Field test: Radio maps for two beams transmitted by a ground BS; the BS location is indicated by a red triangle.

is consistent with the length of the used reference sequence: cross-preamble $<$ separate $<$ CP-based. Cross-correlation outperforms auto-correlation because the separation τ_c between the two training sequences provides a larger phase baseline. The proposed cross-preamble estimator further improves accuracy by pooling information across frames under the shared-CFO constraint. Notably, the cross-preamble estimator at SINR = -30 dB still outperforms the weighted-average baseline operating at SINR = -5 dB confirming robustness in low-SINR regimes. Consequently, improving CFO accuracy reduces variance in channel estimation; the error (normalized mean squared error (NMSE)) versus SINR follows the same trend as in Fig. 2(b).

Fig. 3 shows the radio-map estimation for a low-altitude setting. The two clearly resolved (pointing in different directions) beams corroborate the effectiveness of the proposed framework in practice.

5. CONCLUSION

Leveraging CFO constancy within a coherence block, we propose an ML cross-preamble estimation framework that aggregates preamble correlations. Simulations show estimation gains exceeding 30 dB for CFO and 5 dB for the channel. Field tests demonstrate the ability to identify beams in low-altitude airspace.

6. REFERENCES

- [1] B. W. Zarikoff and J. K. Cavers, "Multiple frequency offset estimation for the downlink of coordinated MIMO systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 6, pp. 901–912, 2008.
- [2] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, 2010.
- [3] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 1834–1850, 2017.
- [4] S. J. Maeng, O. Ozdemir, I. Guvenc, M. L. Sichitiu, M. Mushi, and R. Dutta, "LTE I/Q data set for UAV propagation modeling, communication, and navigation research," *IEEE Commun. Mag.*, vol. 61, no. 9, pp. 90–96, 2023.
- [5] Y. Zeng, J. Chen, J. Xu, D. Wu, X. Xu, S. Jin, X. Gao, D. Gesbert, S. Cui, and R. Zhang, "A tutorial on environment-aware communications via channel knowledge map for 6G," *IEEE Commun. Surveys Tuts.*, vol. 26, no. 3, pp. 1478–1519, 2024.
- [6] H. Sun and J. Chen, "Integrated interpolation and block-term tensor decomposition for spectrum map construction," *IEEE Transactions on Signal Processing*, vol. 72, pp. 3896–3911, 2024.
- [7] T.-C. Lin and S.-M. Phoong, "A new cyclic-prefix based algorithm for blind CFO estimation in OFDM systems," *IEEE Trans. on Wireless Commun.*, vol. 15, no. 6, pp. 3995–4008, 2016.
- [8] S. Singh, S. Kumar, S. Majhi, U. Satija, and C. Yuen, "Blind carrier frequency offset estimation techniques for next-generation multicarrier communication systems: Challenges, comparative analysis, and future prospects," *IEEE Commun. Surveys Tuts.*, vol. 27, no. 1, pp. 1–36, 2025.
- [9] Y.-R. Tsai, H.-Y. Huang, Y.-C. Chen, and K.-J. Yang, "Simultaneous multiple carrier frequency offsets estimation for coordinated multi-point transmission in OFDM systems," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 9, pp. 4558–4568, 2013.
- [10] O. H. Salim, A. A. Nasir, H. Mehrpouyan, and W. Xiang, "Multi-relay communications in the presence of phase noise and carrier frequency offsets," *IEEE Trans. on Commun.*, vol. 65, no. 1, pp. 79–94, 2017.
- [11] R. Tuninato, D. G. Riviello, R. Garello, B. Melis, and R. Fantini, "A comprehensive study on the synchronization procedure in 5G NR with 3GPP-compliant link-level simulator," *EURASIP J. Wireless Com. Network*, vol. 2023, no. 1, p. 111.
- [12] P. Stoica and O. Besson, "Training sequence design for frequency offset and frequency-selective channel estimation," *IEEE Trans. on Commun.*, vol. 51, no. 11, pp. 1910–1917, 2003.
- [13] M. Hua, M. Wang, K. W. Yang, and K. J. Zou, "Analysis of the frequency offset effect on Zadoff-Chu sequence timing performance," *IEEE Trans. on Commun.*, vol. 62, no. 11, pp. 4024–4039, 2014.
- [14] M. Morelli, C.-C. J. Kuo, and M.-O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review," *Proc. IEEE*, vol. 95, no. 7, pp. 1394–1427, 2007.
- [15] 3rd Generation Partnership Project, "Technical Specification Group Radio Access Network; NR; Physical channels and modulation (Release 18)," 3GPP, TS 38.211, May 2024, v18.2.0.
- [16] S. Yoneda, M. Sawahashi, S. Nagata, and S. Suyama, "Prach transmission employing carrier frequency offset pre-compensation based on measurement at UE for NR uplink," *IEICE Trans. on Commun.*, vol. E108-B, no. 3, pp. 330–338, 2025.
- [17] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume II: Detection Theory*. Upper Saddle River, NJ, USA: Prentice Hall, 1998.
- [18] —, *Fundamentals of statistical signal processing: estimation theory*. Prentice-Hall, Inc., 1993.