Co-Investment under Revenue Uncertainty Based on Stochastic Coalitional Game Theory

Amal Sakr[1*], Andrea Araldo[1], Tijani Chahed[1], Daniel Kofman[2]

[1*]Télécom SudParis, Institut Polytechnique de Paris, Palaiseau, France.
[2]Télécom Paris, Institut Polytechnique de Paris, Palaiseau, France.

*Corresponding author(s). E-mail(s): amal.sakr@telecom-sudparis.eu;
Contributing authors: andrea.araldo@telecom-sudparis.eu;
tijani.chahed@telecom-sudparis.eu; daniel.kofman@telecom-paristech.fr;

Abstract

The introduction of new services, such as Mobile Edge Computing (MEC), requires a massive investment that cannot be assumed by a single stakeholder, for instance the Infrastructure Provider (*InP*). Service Providers (SPs) however also have an interest in the deployment of such services. We hence propose a co-investment scheme in which all stakeholders, i.e., the *InP* and the SPs, form the so-called grand coalition composed of all the stakeholders with the aim of sharing costs and revenues and maximizing their payoffs. The challenge comes from the fact that future revenues are uncertain. We devise in this case a novel stochastic coalitional game formulation which builds upon robust game theory and derive a lower bound on the probability of the stability of the grand coalition, wherein no player can be better off outside of it. In the presence of some correlated fluctuations of revenues however, stability can be too conservative. In this case, we make use also of profitability, in which payoffs of players are non-negative, as a necessary condition for co-investment. The proposed framework is showcased for MEC deployment, where computational resources need to be deployed in nodes at the edge of a telecommunication network. Numerical results show high lower bound on the probability of stability when the SPs' revenues are of similar magnitude and the investment period is sufficiently long, even with high levels of uncertainty. In the case where revenues are highly variable however, the lower bound on stability can be trivially low whereas co-investment is still profitable.

Keywords: Co-investment, Uncertainty, Coalitional game, Stability, Profitability, Edge Computing

1 Introduction

Several examples exist of technologies that, from a purely technical point of view, would be ready to emerge or scale up, but whose development is hindered by a big economic barrier. This is the case of technologies requiring the deployment of a massive infrastructure, for which massive investments need to be engaged upfront, and whose precise future profits remain uncertain. We call Infrastructure Provider (InP) the actor responsible for deploying, maintaining and operating the physical infrastructure. We consider that, on top of such an infrastructure, third-party Service Providers (SPs) run their applications and collect revenues from end users.

An example of this setting is the electric vehicle charging infrastructure (Badia et al. 2019), where the electric grid operator, acting as the InP, is tasked with deploying the grid infrastructure, e.g., bringing high-capacity electricity lines to road networks. On top of such grid infrastructure, electric vehicle charging station providers, which take the role of SPs, can operate their business and get revenues from end-users (drivers) (Fredriksson et al. 2019).

Another example, which is also the one we consider in the numerical results, involves the deployment and operation of Edge Computing (EC) (Satyanarayanan 2017). EC is a computing architecture that decentralizes data processing by bringing it to the edge of the network, e.g., base stations (Shi et al. 2016). This proximity to end-users reduces latency, improves bandwidth efficiency and enables real-time data processing and analytics (Mohan et al. 2020). EC brings new revenue opportunities: third-party SPs, e.g., augmented or virtual reality applications (Ong and Nee 2013), (Reif and Walch 2008), automated car function providers (Faisal et al. 2019), (Parekh et al. 2022), can run low-latency applications on the edge nodes, benefiting from proximity to end-users, and collect revenues from them. In the EC context, the InP is the network operator (NO), which is the only entity owning the nodes at the edge. Any SP, willing to run its services at the edge has to pass through a NO. This is thus an unprecedented opportunity for NOs, who could capitalize on their privileged position of exclusive owners of edge nodes (Khan et al. 2019). However, deploying and maintaining EC infrastructure at network edges is highly costly (Santoyo-González and Cervelló-Pastor 2018). This is because EC requires establishing and managing distributed computing resources across multiple locations. Setting up these infrastructures demands significant investment in hardware, networking, and ongoing maintenance, making large-scale implementation financially challenging. As a result, EC has not yet been deployed at scale in today's network.

In the cases we consider in this paper, the InP is reluctant to bear the significant capital (CAPEX) and operational expenditures (OPEX) alone. This reluctance is compounded by the market dynamics, where the revenues generated from the edge services tend to flow to SPs, which directly interact with end users. Consequently, the InP often faces challenges in capturing a share of the profits from these services, particularly in highly competitive markets. To remove this economic barrier, we propose a co-investment scheme in which players (or stakeholders), i.e., one InP and several SPs, co-invest to share costs and revenues. By joining a coalition, they aim to maximize their payoff by deploying infrastructure with the optimal amount of resources and allocating it dynamically among players, based on how much revenue they could get by using the allocated resources. In this scheme, SPs are strongly incentivized to contribute to infrastructure costs, as it enables them to provide high-quality or innovative services, attract new users, improve satisfaction, and thereby increase revenue.

The challenge is whether there is an opportunity for co-investment between the stakeholders under uncertainty. To do so, we first tackle the question of whether this co-investment is the most profitable investment that each stakeholder can make. Determining this is highly complex due to the uncertainty of revenues, which depend themselves on the stochasticity of end-user demand. Demand is influenced by external factors such as market trends, technological shifts, and competitive dynamics, making future revenues inherently difficult to predict. Even small deviations from the expected demand can significantly alter potential revenues and, consequently, the stakeholders' payoffs, making it difficult to determine whether co-investment is indeed the most profitable option for each of them. To address this challenge, we cast the problem as a novel stochastic coalitional game, extending the principles of Robust Coalitional Game Theory (RCGT) (Doan and Nguyen 2014). In its basic form, RCGT requires coalitions to be formed and remain stable, i.e., no subset of players would be better off outside the coalition, even in the worst-case scenario. While this approach ensures robustness, it is overly restrictive in the context of co-investment, where risk is an inherent feature of the investments. Requiring no risk, as in the original RCGT framework, would effectively preclude any meaningful investment opportunities. To address this limitation, we reformulate RCGT to allow for an acceptable level of risk, and derive a lower bound on the probability that the grand coalition, composed of all the players, is stable, i.e., no subset of players is better off outside of the grand coalition.

Second, in the case where revenues are highly variable, which stems from possible correlations in time between demand fluctuations, even this probabilistic approach to stability can yield trivially low value for the lower bound. In this case, profitability, defined as the condition where no player's individual payoff is negative, presents a more significant measure for the opportunity of co-investment. This condition resembles the concept of satisficing (Simon 1997), where a decision is acceptable if it meets a given threshold, when that threshold is set to zero. In our case however, we determine the infrastructure plan (resources and shares) by maximizing total payoffs, not through a satisficing search. Profitability here reflects satisficing in outcome, but not in the decision process.

Our contributions are:

- A co-investment framework under uncertainty: we introduce a co-investment model where an *InP* and multiple SPs form a coalition to share the costs and the revenues under uncertainty. The uncertainty we consider in this paper derives from uncertain user demand, which in turn causes stochastic fluctuations in revenues.
- A lower bound on grand coalition stability: we establish a probabilistic guarantee that the grand coalition is stable despite revenue fluctuations, and we prove this result within a stochastic coalitional game. We also define and formulate the probability that the co-investment is profitable.
- Application to Edge Computing infrastructure: we apply the proposed framework to an EC setting, where an *InP* and several SPs cooperate to deploy and maintain

edge infrastructure over a defined investment period. Costs and revenues are shared fairly using the Shapley value.

We show numerically that when the investment period is sufficiently long (e.g., 5 years) and the SPs are of similar size, the lower bound on the probability of stability reaches 1. In cases where this lower bound is uninformative, the probability of profitable co-investment is also computed and found to be high.

The rest of this paper is as follows. Section 2 reviews related work. Section 3 delves into the co-investment scheme and coalitional game model. Section 4 provides results in the context of EC. Section 5 presents the conclusion and perspectives.

2 Related work

As our work consists of modeling co-investment using coalitional game theory with application to EC deployment, our related work is categorized into three sub-sections: co-investment strategies, coalitional game theory and the economic aspects of EC.

2.1 Co-investment strategies

Cooperation and infrastructure sharing among industry stakeholders have become essential to address the high costs of infrastructure deployment. The Connected Collaborative Computing Network (Union 2024) emphasizes the importance of cooperation across stakeholders (e.g., telecom, cloud) to reduce investment burdens.

Co-investment agreements have been studied under various structural and strategic assumptions. The author in (Jeanjean 2022) formalizes co-investment as a form of network sharing, where two *InP*s co-invest in infrastructure, including passive and active components. The results show that co-investment and infrastructure sharing reduce costs, boost investment and improve consumer welfare. In contrast, our work focuses on co-investment between heterogeneous players, an *InP* and multiple SPs.

Strategic misalignment in co-investment is addressed in (López et al. 2022), where incumbents may increase reported deployment costs to deter entry, harming welfare. Our model instead emphasizes fairness and mutual benefit, ensuring transparency and aligning incentives so that all stakeholders are motivated to join the co-investment.

Infrastructure sharing under uncertainty and regulation is also studied in (Inderst and Peitz 2014), where one firm invests in infrastructure and another decides later whether to access it, based on regulatory pricing and realized demand. However, we model pre-investment cooperation, where all stakeholders commit upfront to sharing the deployment and maintenance costs. Resources are then dynamically allocated among SPs based on their individual needs at each point in the investment period.

The impact of uncertainty on co-investment decisions is studied in (Bourreau et al. 2021), which analyses how demand uncertainty affects the timing of investment between incumbents and entrants in network infrastructure. The authors show that allowing entrants to co-invest ex-post, after demand is revealed, can distort investment incentives and reduce network coverage. To address this, they propose combining ex-ante co-investment options, before demand is known, with ex-post risk premia. While their model focuses on bilateral co-investment, our work extends the analysis to coalitions involving heterogeneous players, one InP and multiple SPs, using a stochastic coalitional game framework. Our setting aligned with the ex-ante case, where all players commit to co-investment before demand is realized, based on expectations.

The study in (Azevedo and Paxson 2018) focuses on investment timing in a competitive duopoly under both revenue and cost uncertainty. Similarly, (Kauppinen et al. 2018) examines a firm's sequential investment decisions, considering whether to initiate, suspend, or continue investment in response to uncertain future revenues and costs. However, our work considers only revenue uncertainty, with fixed timing, as stakeholders commit at the start of the co-investment strategy.

Cost, profit, and effort allocation mechanisms are examined in (Bhaskaran and Krishnan 2009) using a bargaining-based approach. This work focuses on incentive alignment and efficient cooperation between two firms. In contrast, we adopt the Shapley value in a stochastic environment to share payoff based on marginal contributions, ensuring fairness across players under uncertainty.

The work in (He et al. 2024) proposes a cooperative framework for infrastructure co-investment using Nash bargaining to share benefits, assuming predictable demand. While effective in structured settings, the model does not address demand uncertainty or the challenge of sustaining cooperation under risk. Our framework accounts for stochastic demand and offers probabilistic guarantees on cooperation profitability.

2.2 Coalitional game theory under uncertainty

Several studies have explored cooperative game theory frameworks to analyze investment decisions and coalition stability under uncertainty. For instance, the work in (Lin et al. 2021) investigates how independent stakeholders working on a shared project can cooperate by reallocating their resources to speed up completion and earn extra rewards. They apply cooperative game theory to ensure the resulting gains are fairly shared, mainly through Shapley value and proportional sharing rules. While their setting is deterministic and focused on time savings in project delivery, the spirit of cooperation and fairness they promote aligns closely with our goal. We extend these ideas to a stochastic setting, where investment decisions and resource sharing must account for uncertain demand and fluctuating revenues.

In (Ketelaars et al. 2023), cooperative game theory is combined with real options analysis to address timing and coalition stability under uncertainty. The authors propose a proportional investment scheme to maximize project value while keeping investment timing stable. Their approach accounts for strategic deferral and uses option value to represent flexibility. In contrast, our framework assumes full upfront investment by all players and models uncertainty directly using a stochastic coalitional game, without relying on real options or proportional mechanisms.

The work in (Kiedanski et al. 2020) extends coalitional games to model cooperative investment, incorporating stochastic load profiles, to minimize expected costs. It proves the existence of a non-empty core, the set of all possible distributions of total profit where no subset of players would be better off forming their own coalition. However, the formulation is tailored to linear cost minimization, whereas our setting involves nonlinear, concave revenue maximization based on dynamic resource allocation.

The concept of the expected value core is proposed in (Pantazis et al. 2023), ensuring stability in expectation under probabilistic distributions. Similarly, the study in (Akkarajitsakul et al. 2011) applies a Bayesian coalitional game to model cooperation under uncertainty, where players form coalitions based on expected payoffs derived from beliefs about others' behavior. However, in a co-investment context, players require stronger guarantees than stability in expectation. We instead introduce a nominal game, an expected version of the stochastic game, as an intermediate step, and we prove that the grand coalition is stable in this setting. Since this is not sufficient on its own, we further derive analytical lower bounds on the probability that the grand coalition is stable under stochastic demand.

To handle uncertainty robustly, (Raja and Grammatico 2021) introduces the robust core, defined as the intersection of instantaneous deterministic cores across discrete time slots. While this approach ensures stability at each time slot under bounded uncertainty, our framework differs by modeling a unified coalitional game over the entire investment horizon, incorporating dynamic resource allocation across time.

A related approach is proposed in (Borrero et al. 2016), where the authors study coalitional games under uncertainty by introducing several stability concepts. One of their key contributions is the common core, which requires the existence of a single weighting over scenarios such that all coalitions are stable under the corresponding weighted game. Although their framework allows for partial information about these weights, it still relies on the existence of such a weighting to assess stability. In contrast, our model makes no assumptions about scenario weights. Instead, we work with a bounded set of realizations and evaluate coalition stability uniformly over that set.

The Robust Coalitional Game Theory (RCGT) model in (Doan and Nguyen 2014) ensures grand coalition stability by evaluating payoffs under worst-case realizations. However, this strict robustness limits its practical applicability in uncertain co-investment contexts. We extend this work by replacing worst-case guarantees with probabilistic ones, computing a lower bound on the probability of stability that accounts for stochasticity in demand and revenue.

2.3 Economic aspects of Edge Computing

The economic viability of EC depends on efficient resource allocation, cost-sharing mechanisms, and investment models that account for uncertainty. Several studies focus on resource rental models. In (Li et al. 2020) and (Xu et al. 2017), SPs rent computational resources from InPs, based on historical demand or latency-aware workload optimization. While these works efficiently allocate existing resources, they assume the infrastructure is pre-deployed and entirely funded by the InPs. In contrast, our model targets co-investment, where InP and SPs jointly deploy and maintain infrastructure, shifting the focus to long-term profit sharing. Moreover, unlike (Xu et al. 2017), we explicitly model demand uncertainty in coalition formation.

Deployment under demand uncertainty is studied in (Nguyen et al. 2021; Chen 2023), where an optimization framework is proposed to handle uncertain demand or user mobility. The model optimizes edge server locations and capacities before demand realization. However, both works assume unilateral decision-making by either a single SP (Nguyen et al. 2021) or a single InP (Chen 2023), with SPs treated as passive users. Our approach differs by modeling a cooperative strategy where both *InP* and SPs actively co-invest and share decision-making responsibilities under uncertainty.

Cost-efficient infrastructure deployment is also addressed in (Zhao et al. 2018), which minimizes the number of edge nodes required for Internet of Things service coverage. However, this work neither considers which entities invest in infrastructure nor supports cost sharing among parties. Similarly, the work in (Cong et al. 2022) proposes a cooperative model where base stations share resources via overlapping placement. Yet, the model does not clarify whether cooperation spans multiple *InP*s or is internal to a single operator, nor does it consider joint financial investment.

A co-investment strategy is proposed in (Patanè et al. 2023), where an *InP* and multiple SPs deploy edge infrastructure. However, the model assumes static resource allocation and known user demands. Our work extends this by incorporating dynamic resource allocation based on fluctuating demand and providing probabilistic guarantees on co-investment profitability under uncertainty.

A preliminary version of this work has been presented in (Sakr et al. 2025) where we propose co-investment between the *InP* and SPs under uncertainty, for the case where the stochastic fluctuations in the revenues are uncorrelated. In this work however we tackle the case where they are correlated and establish a necessary condition for the profitability of co-investment.

3 Co-investment model

In this section, we first provide a schematic overview of the proposed co-investment strategy (Section 3.1). We then model the co-investment within a stochastic framework (Section 3.2) and introduce the stochastic coalitional game that represents it (Section 3.3). Next, we define the nominal game (Section 3.4), a deterministic version of the stochastic game, and establish key properties necessary for analyzing co-investment within the stochastic setting. After that, we present the main result of the paper: a lower bound on the probability that the grand coalition is stable under uncertainty (Section 3.5), followed by the probability that the co-investment is profitable for all players (Section 3.6). Finally, we apply the Shapley value concept to fairly distribute costs and revenues among the players (Section 3.7).

3.1 High-level description of the co-investment plan

First, players combine their funds into a common wallet to deploy and maintain the infrastructure, consisting of a certain amount of resources, such as computational resources, with capacity C, over an investment period I. These resources are then dynamically partitioned among SPs in a manner that we will explicit in the sequel. Each SP receives a stochastic load of requests from its end users and collects revenues by serving those requests. Revenues increase with load and with the amount of resources allocated to the SP. All revenues are then redistributed among the players. Note that, although the *InP* is the direct responsible for the installation and the maintenance of the infrastructure, it does not serve any user requests, and thus it does not collect any revenues from users. However, since no resources can be installed without it, the *InP* benefits from the redistribution of the revenues from SPs.

Table 1: Notation

Notation	Definition
t	Specific time slot in the set $\mathcal{T}$ (5)
$\mathcal{T}$	Set of time slots in the investment period I (5)
I	Investment period (1)
C	Computational resources (e.g., CPU) (1)
$Cost(I, C)$	Deployment and maintenance costs over the investment period (1)
h_i^t	Resources allocated for SP i at time slot t (4)
$\vec{h}^t$	Resources allocated to players at time slot t (5)
$\{\vec{h}^t\}_{t\in\mathcal{T}}$	Set of resource allocation vectors (5)
$C^*_{\mathcal{S}}, h^{t*}_{i,\mathcal{S}}$	Results of optimization problem (16)-(18)
$C^*_{\mathcal{S}}$	Optimal capacity that coalition $\mathcal{S}$ would install (Definition 5)
$h^{t*}_{i,S}$	Optimal resources allocated to SP i at time slot t within coalition $\mathcal{S}$ (Definition 5)
$u^t_{i,\omega}(h^t_i)$	Revenues collected by player i from end-users, in time slot t in realization ω (4)
$\bar{u}^t_i(h^t_i)$	Expected value of the quantity above (12)
$v_\omega(\mathcal{S})$	Coalition payoff in realization ω for coalition $\mathcal{S}$ (8)
$\bar{v}(\mathcal{S})$	Expected coalition payoff for $\mathcal{S}$ over all realizations (14)
$z_{i,\omega,S}$	Deviation of player i's revenue from expected value in realization ω (24)
$\hat{\delta}$	Uncertainty threshold (Definition 15)
range(u^t_i)	Range of uncertainty in utility of player i at time slot t (32)
$l^t_{i,\omega}$	Amount of requests for SP i in time slot t in realization ω (46)
$\bar{l}^t_i$	Expected amount of requests for SP i in time slot t (48)

We aim to answer the following questions:

1. Can we ensure, with high probability, that there is an opportunity for co-investment despite the uncertainty about the future loads?
2. How to decide the amount of resources that should be installed and the dynamic resource sharing strategy?
3. How to distribute fairly costs and revenues among players?

To answer Question 1, we formulate a novel stochastic game theoretical framework, suited to the particular case of co-investment, where a big cost must be borne at the beginning (to install resources), and then revenues over the investment period are uncertain. To answer Question 2, we resort to convex optimization to determine the optimal amount of resources to be purchased as well as its optimal dynamic sharing between players. As for Question 3, we compute a redistribution of costs and revenues based on the Shapley value.

Table 1 summarizes the frequently used notations in this paper.

3.2 Stochastic modeling of co-investment

Let $\mathcal{N}$ be the set of players composed of one Infrastructure Provider (InP) and N Service Providers (SPs). Let I be the investment period (e.g., $I = 5$ years). We discretize time I in set $\mathcal{T}$ of time slots, having each length Δ. At each time slot $t \in \mathcal{T}$, SP i is allocated a quantity h^t_i of infrastructure resources. Exploiting the allocated resources, SP i collects at each time slot $t \in \mathcal{T}$ revenue $u^t_i(h^t_i)$ (which we also call utility), which is a random function of the amount of resources h^t_i. The more the allocated resources, the more revenues will be collected. Revenue $u_i^t(h_i^t)$ is a random function in the sense that, for any value h_i^t, revenue $u_i^t(h_i^t)$ is a random variable defined on probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the sample space, $\mathcal{F}$ is the set of events and $\mathbb{P}$ is the probability measure (Ghahramani 2024). This randomness arises because revenue $u_i^t(h_i^t)$ depends on the traffic load received by SP i, which is inherently uncertain and varies over time. Let $u_{i,\omega}^t(h_i^t), \omega \in \Omega$, denote a realization of random variable $u_i^t(h_i^t)$.

A coalition is a subset $\mathcal{S} \subseteq \mathcal{N}$ of players. If a coalition is formed, its participants decide, altogether, the amount C of infrastructure resources to be deployed. The coalition incurs an upfront cost $\text{Cost}(I, C)$, which incorporates the cost of purchase and of maintenance of the resources over the investment period I. To cover this cost, each player $i \in \mathcal{S}$ contributes an initial payment p_i. The contributions must collectively cover the entire cost incurred by the coalition, that is:

$$\sum_{i \in \mathcal{S}} p_i = \text{Cost}(I, C) \tag{1}$$

$\text{Cost}(I, C)$ is a deterministic function that increases with C (the more the purchased resources, the higher the cost) and with I (the longer the period of guaranteed maintenance, the higher the cost)[1]. We make the following reasonable assumption:

$$\text{Cost}(I, C) = 0 \text{ if } C = 0 \tag{2}$$

At each time slot t, let us denote by $\vec{h}^t = \{h_i^t\}_{i \in \mathcal{N}}$ the vector of resources allocated to each player. With total amount C of resources, we have $\vec{h}^t \cdot \vec{1} = \sum_{i \in \mathcal{N}} h_i^t \leq C, \forall t \in \mathcal{T}$. Let $\{\vec{h}^t\}_{t \in \mathcal{T}}$ be a sequence of such vectors over time slots. Note that while all players participate in the co-investment, only SPs interact with end-users and generate revenue by serving requests. The *InP* is responsible for installing and maintaining the infrastructure but does not serve any user requests directly. Motivated by this distinction, we make the following assumption:

Assumption 1 *The InP does not receive any infrastructure resources during the investment period. That is,*

$$h_{InP}^t = 0, \quad \forall t \in \mathcal{T} \tag{3}$$

To make our analysis realistic, we consider the following assumption:

Assumption 2 *Utilities are non negative. Moreover, player $i \in \mathcal{N}$ does not collect revenues from the use of the infrastructure, when the share of resources it gets is 0, i.e.:*

$$u_{i,\omega}^t(h_i^t) \geq 0, \quad \forall h_i^t \geq 0; \quad u_{i,\omega}^t(h_i^t) = 0, \ \textit{if } h_i^t = 0, \quad \forall i \in \mathcal{N}, \ t \in \mathcal{T}, \ \omega \in \Omega \tag{4}$$

The following proposition restricts our attention to reasonable cases:

[1] In reality, the maintenance cost may be stochastic, but it can be assumed to be deterministic in case the coalition (or *InP* on its behalf) signs a contract with a resource vendor and pays upfront to that vendor the price of resources and a maintenance service guaranteed over period I.

Proposition 3 *In the coalition, the InP is a veto player, i.e., a coalition without it is pointless. Moreover, since we assume only SPs directly collect revenues by serving the end-users via the infrastructure, then the set of all SPs, considered altogether, is a veto player as well.*

The proof is provided in Appendix A.

The payoff of a coalition, i.e., how much it is worth in terms of net benefit, is given by the following two definitions.

Definition 4 *If coalition $\mathcal{S} \subseteq \mathcal{N}$ installs an amount C of resources and allocates them among players applying allocation sequence $\{\vec{h}^t\}_{t\in\mathcal{T}}$, the* value *of coalition $\mathcal{S}$ under $\{\vec{h}^t\}_{t\in\mathcal{T}}$, denoted by $v\left(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}}\right)$, is the payoff that it obtains, computed as the sum of collected revenues minus the cost, i.e.,*

$$v\left(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}}\right) = \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{S}} u_i^t(h_i^t) - Cost(I, C) \tag{5}$$

The quantity above is a random variable, as it depends on random variables $u_i^t(h_i^t), \forall i \in \mathcal{N}, t \in \mathcal{T}$. Its realizations are:

$$v_\omega\left(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}}\right) = \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{S}} u_{i,\omega}^t(h_i^t) - Cost(I, C), \qquad \forall \omega \in \Omega \tag{6}$$

Definition 5 *Let us assume that there is a criterion that associates, to each coalition $\mathcal{S} \subseteq \mathcal{N}$, an amount of resources $C_\mathcal{S}^*$ to be installed and an allocation $\{\vec{h}_\mathcal{S}^{t*}\}_{t\in\mathcal{T}} = \left\{\{h_{i,\mathcal{S}}^{t*}\}_{i\in\mathcal{S}}\right\}_{t\in\mathcal{T}}$ (this criterion will be specified in Assumption 10). We define the payoff (or value) of coalition $\mathcal{S}$ as the following random variable:*

$$v(\mathcal{S}) \triangleq v\left(\mathcal{S}, C_\mathcal{S}^*, \{\vec{h}_\mathcal{S}^{t*}\}_{t\in\mathcal{T}}\right) \tag{7}$$

Its realizations are:

$$v_\omega(\mathcal{S}) \triangleq v_\omega\left(\mathcal{S}, C_\mathcal{S}^*, \{\vec{h}_\mathcal{S}^{t*}\}_{t\in\mathcal{T}}\right), \qquad \forall \omega \in \Omega \tag{8}$$

Set function $v(\cdot) : 2^\mathcal{N} \to \mathbb{R}$, where $2^\mathcal{S}$ is the set of all possible subsets of $\mathcal{N}$, is called value function. *Observe that function $v(\cdot)$ is a random function.*

The main question we aim to answer is whether there is an opportunity for co-investment. This is challenging due to two factors: the long investment period, and uncertainty in revenue fluctuations over time. Since payoffs are realized at the end of the period, they depend on cumulative random variations, making it infeasible to track all possible outcomes explicitly. To address this challenge, we model the co-investment scheme as a stochastic coalitional game and analyze first whether the grand coalition can be formed and remains stable under uncertainty. The players are willing to co-invest if the grand coalition is stable. In a second step, we study the case where uncertainty results from correlated fluctuations in revenues, and where the notion of stability becomes too conservative. Hence, we resort to the profitability condition.

3.3 Model of the stochastic coalitional game

We borrow the following definition of stochastic game from (Charnes and Granot 1976, Definition 1) and we add the concept of restriction of the stochastic game. Such a restriction is useful, since we will be able to prove some important properties for large enough restrictions, but not for all the possible realizations of the stochastic game.

Definition 6 *A stochastic game is pair $(\mathcal{N}, \mathcal{V})$, where $\mathcal{N}$ is the set of players and $\mathcal{V}$, called* uncertainty set, *is the set of realizations of random function $v_\omega(\cdot)$:*

$$\mathcal{V} \coloneqq \{v_\omega(\cdot) : 2^{\mathcal{N}} \to \mathbb{R} | \omega \in \Omega\} \tag{9}$$

For any subset $\Omega' \subseteq \Omega$ of the sample space, we define a restriction $(\mathcal{N}, \mathcal{V}(\Omega'))$ of the original game as a new stochastic game, with the same set of players and the set of possible value functions defined as:

$$\mathcal{V}(\Omega') \coloneqq \{v_\omega(\cdot) : 2^{\mathcal{N}} \to \mathbb{R} | \omega \in \Omega'\} \tag{10}$$

The challenge is to evaluate the stability of the grand coalition under uncertainty. To show this, we need to show that a set of "good" allocations of payoff among players exist. The following definition from (Doan and Nguyen 2014, Definition 4) formalizes what are the "good" payoff allocations.

Definition 7 *The robust core of the stochastic game $(\mathcal{N}, \mathcal{V})$ is:*

$$rcore(\mathcal{N}, \mathcal{V}) \coloneqq \left\{ \vec{y} = (y_1, \ldots, y_{|\mathcal{N}|}) \in \mathbb{R}^{\mathcal{N}} \;\middle|\; \begin{array}{l} \underbrace{\sum_{i=1}^{|\mathcal{N}|} y_i = 1,}_{efficiency} \\ \underbrace{\sum_{i \in \mathcal{S}} v_\omega(\mathcal{N}) \cdot y_i \geq v_\omega(\mathcal{S}),}_{stability} \\ \forall v_\omega \in \mathcal{V}, \forall \mathcal{S} \subseteq \mathcal{N} \end{array} \right\} \tag{11}$$

Allocations $\vec{y}$ in the robust core are ways in which the payoff of the grand coalition can be shared among players. Each such allocation is efficient, in the sense that 100% of the payoff is divided among the players. They are stable, in the sense that, considering any subset $\mathcal{S} \subseteq \mathcal{N}$ of players (including singleton subsets), players in $\mathcal{S}$ are always better off joining the grand coalition, rather than forming a coalition between themselves, no matter the realization of payoff. This is why we define the grand coalition as *stable*, if it respects the following definition.

Definition 8 *Given a stochastic game $(\mathcal{N}, \mathcal{V})$, the grand coalition is stable if and only if $rcore(\mathcal{N}, \mathcal{V}) \neq \emptyset$, i.e., if and only if there exists at least one allocation of the payoff that is efficient and stable.*

Stability of the grand coalition $\mathcal{N}$ is obtained through non emptiness of the robust core. The model in (Doan and Nguyen 2014) however is based on worst-case assumption for the uncertainty. Instead, we aim to derive conditions for the stability of the grand coalition based on the subset of realizations for which this condition holds. We do so by introducing a probability measure for the existence of the grand coalition.

Definition 9 *Given a stochastic game $(\mathcal{N}, \mathcal{V})$, the grand coalition is stable with probability at least ν^{LB} if and only if there exists a set of realizations Ω' such that:*

- $v_\omega(\mathcal{N}) > 0, \forall \omega \in \Omega'$,
- *The grand coalition is stable in game* $\left(\mathcal{N}, \underbrace{\mathcal{V}(\Omega')}_{(10)}\right)$ *(which we call* restriction*), i.e.,* $rcore(\mathcal{N}, \mathcal{V}(\Omega')) \neq \emptyset$ *and*
- $\mathbb{P}(\Omega') = \nu^{LB}$

The main result we will prove in this paper (Theorem 17) is that the grand coalition is stable with probability at least $\nu^{\mathrm{LB}} > 0$, which we can bring close to 1 under some conditions. To prove this result, we need first to analyze the so-called "nominal game", which is a deterministic version of the aforementioned stochastic game.

3.4 Nominal game

The nominal version of our game is constructed by reasoning in terms of expected values. Let:

$$\bar{u}_i^t(h_i^t) \triangleq \mathbb{E}_\omega[u_{i,\omega}^t(h_i^t)] \tag{12}$$

be the expected utility collected at time slot t by player i, if it is allocated resources h_i^t. The expected payoff of coalition $\mathcal{S}$, when it installs an amount $\mathcal{C}$ of resources which are dynamically shared by SPs following $\{\vec{h}^t\}_{t\in\mathcal{T}}$, is

$$\begin{aligned}\bar{v}(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}}) &\triangleq \mathbb{E}_\omega\left[v_\omega(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}})\right] \\ &\overset{(6)(12)}{=} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{S}} \bar{u}_i^t(h_i^t) - Cost(I, C)\end{aligned} \tag{13}$$

By considering $C_{\mathcal{S}}^*$ and $\{\vec{h}_{\mathcal{S}}^{t*}\}_{t\in\mathcal{T}}$ from Definition 5, the nominal value function of coalition $\mathcal{S}$ is given by:

$$\begin{aligned}\bar{v}(\mathcal{S}) &\overset{(8)}{\triangleq} \mathbb{E}_\omega\left[v_\omega(\mathcal{S})\right] \\ &= \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{S}} \bar{u}_i^t(h_{i,\mathcal{S}}^{t*}) - Cost(I, C_{\mathcal{S}}^*)\end{aligned} \tag{14}$$

We now introduce the nominal game.

$$
\begin{aligned}
&\text{Nominal game: } (\mathcal{N}, \underbrace{\bar{v}}_{(14)}),\\
&\qquad \text{where } \mathcal{N} \text{ is the same set of players as before and}\\
&\qquad \bar{v} : 2^{\mathcal{N}} \to \mathbb{R} \text{ is the value function.}
\end{aligned} \tag{15}
$$

This game follows the standard definition of coalitional game theory (Osborne and Rubinstein 1994, Definition 257.1).

We have previously introduced in Definition 5, that once a coalition $\mathcal{S}$ is formed, its players collectively decide on an amount of resources $C^*_{\mathcal{S}}$ to install and a sequence of dynamic resource allocations $\{\vec{h}^{t*}_{\mathcal{S}}\}_{t\in\mathcal{T}}$. The following assumption states the practical criterion adopted in this paper to determine the amount of resources to deploy and how to allocate them among the coalition's players.

Assumption 10 *Given a coalition $\mathcal{S} \subseteq \mathcal{N}$, the InP precomputes amount $C^*_{\mathcal{S}}$ of resources that would be installed and vectors $\{\vec{h}^{t*}_{\mathcal{S}}\}_{t\in\mathcal{T}}$ with which resources would be dynamically shared, by solving the following optimization problem:*

$$C^*_{\mathcal{S}}, \left\{\vec{h}^{t*}_{\mathcal{S}}\right\}_{t\in\mathcal{T}} = \underset{C,\left\{\vec{h}\right\}_{t\in\mathcal{T}}}{\operatorname{argmax}} \ \bar{v}(\mathcal{S}, C, \{\vec{h}^t\}_{t\in\mathcal{T}}) \tag{16}$$

$$\textit{such that} \quad \sum_{i\in\mathcal{S}} h^t_i \leq C, \quad \forall t \in \mathcal{T} \tag{17}$$

$$h^t_i \geq 0, \quad \forall i \in \mathcal{S},\ t \in \mathcal{T} \tag{18}$$

This problem falls under stochastic programming (Birge and Louveaux 2011), as it involves optimizing the expected payoff under uncertainty, assuming a known and uniform distribution over demand realizations.

In the following proposition, we formally define how the quantities $C^*_{\mathcal{S}}$ and $\{\vec{h}^{t*}_{\mathcal{S}}\}_{t\in\mathcal{T}}$ are derived from the optimal solution (see Assumption 10), depending on whether the *InP* is part of the coalition $\mathcal{S}$.

Proposition 11 *Let $\mathcal{S} \subseteq \mathcal{N}$ be a coalition. The optimal resource capacity $C^*_{\mathcal{S}}$ and the dynamic allocations $h^{t*}_{i,\mathcal{S}}$ for all $i \in \mathcal{S}$, $t \in \mathcal{T}$, are determined as follows:*

- *If the InP does not belong to coalition $\mathcal{S}$, then no resources are installed, i.e.:*

$$C^*_{\mathcal{S}} = 0, \qquad h^{t*}_{i,\mathcal{S}} = 0, \qquad\qquad \forall i \in \mathcal{S},\ \forall t \in \mathcal{T} \tag{19}$$

- *If the InP belongs to coalition $\mathcal{S}$, then resources $C^*_{\mathcal{S}}$ can be potentially installed and used by players in $\mathcal{S}$, according to the dynamic shares precomputed in (16).*

See Appendix B for the proof of Proposition 11.

The following proposition ensures that $C^*_{\mathcal{S}}$ and $\{\vec{h}^{t*}_{\mathcal{S}}\}_{t\in\mathcal{T}}$ are uniquely defined for every coalition $\mathcal{S} \subseteq \mathcal{N}$, and can be computed in polynomial time (Boyd and Vandenberghe 2004, Section 11).

Proposition 12 *Problem (16)-(18) is convex, under the following assumptions:*

1. *Function $h^t_i \to \bar{u}^t_i(h^t_i)$ is increasing concave, $\forall i \in \mathcal{N}, t \in \mathcal{T}$.*
2. *Function $C \to Cost(I, C)$ is convex.*

As a result, the optimal resources can be determined analytically by solving the Karush-Kuhn-Tucker (KKT) conditions (Boyd and Vandenberghe 2004, Section 5.5.3).

The proof of Proposition 12 is provided in Appendix C.

A consequence of Proposition 12 is the following Theorem.

Theorem 13 *Nominal game $(\mathcal{N}, \bar{v})$ ((15)) is convex.*

Proof sketch We prove that the nominal game $(\mathcal{N}, \bar{v})$ is convex by showing that its value function $\bar{v}(\cdot)$ is supermodular (Shapley 1971, Theorem 3). Specifically, we verify that the marginal contribution of any player increases with the size of the coalition. For any coalitions $\mathcal{R} \subseteq \mathcal{S} \subseteq \mathcal{N}$ and any player $j \in \mathcal{N} \setminus \mathcal{S}$, the marginal contribution of j to a coalition $\mathcal{S}$ is defined as:

$$\Delta_j(\mathcal{S}) \triangleq \bar{v}(\mathcal{S} \cup \{j\}) - \bar{v}(\mathcal{S}) \tag{20}$$

and supermodularity requires that:

$$\Delta_j(\mathcal{S}) \geq \Delta_j(\mathcal{R}) \tag{21}$$

The challenge is that $\bar{v}(\mathcal{S})$ is the result of a convex optimization problem (Assumption 10) that changes with the coalition $\mathcal{S}$, so marginal contributions cannot be compared directly. To overcome this, we construct a feasible (but not necessary optimal) allocation for the larger coalition $\mathcal{S}\cup\{j\}$: player j is given the same resource shares as in $\mathcal{R}\cup\{j\}$, and each player $i \in \mathcal{S}$ is given at least as much as in the optimal solution of $\mathcal{S}$. This allocation yields a feasible value $\bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big)$ satisfying:

$$\begin{aligned} &\bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big) \leq \bar{v}(\mathcal{S} \cup \{j\}) \\ \text{and}\quad &\bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big) - \bar{v}(\mathcal{S}) \geq \Delta_j(\mathcal{R}) \end{aligned} \tag{22}$$

which implies that $\Delta_j(\mathcal{S}) \geq \Delta_j(\mathcal{R})$. This proves supermodularity and hence convexity of the game. The full proof with case analysis is provided in Appendix D. □

Theorem 13 implies that the grand coalition $\mathcal{N}$ in the nominal game is stable (Saad et al. 2009), i.e., no subset of players would be better off forming a coalition different than the grand coalition. This follows from the fact that, if the game is convex, then there exists at least one allocation $\vec{x}$ of grand coalition payoff among players that is efficient and stable (efficiency and stability are as defined in Definition 7) (Shapley 1971, Theorem 4). The existence of such an allocation $\vec{x}$ is equivalent to stating that the core is non-empty, where the core is defined as (Maschler et al. 1979, Definition 2.1)):

$$core(\mathcal{N}, \bar{v}) := \left\{ \vec{x} \in \mathbb{R}^{\mathcal{N}} \;\middle|\; \sum_{i \in \mathcal{N}} x_i = \bar{v}(\mathcal{N}),\ \sum_{i \in \mathcal{S}} x_i \geq \bar{v}(\mathcal{S}), \right.$$
$$\left. \forall \mathcal{S} \subsetneq \mathcal{N} \right\} \neq \emptyset \tag{23}$$

3.5 Stability of the grand coalition under uncertainty

After determining the optimal resources $C^*_{\mathcal{S}}$ to deploy and the corresponding dynamic allocations $\{\vec{h}^{t*}_{\mathcal{S}}\}_{t \in \mathcal{T}}$ based on expected values, we must acknowledge that due to the stochasticity in the load, the actual revenue collected by the SP could deviate from the expected one. We introduce the following definition to quantify this deviation:

Definition 14 *The revenue deviation collected by SP i over time in coalition $\mathcal{S} \subseteq \mathcal{N}$ in realization $\omega \in \Omega$ is defined as:*

$$z_{i,\omega,\mathcal{S}} = \sum_{t \in \mathcal{T}} \left(u^t_{i,\omega}(h^{t*}_{i,\mathcal{S}}) - \bar{u}^t_i(h^{t*}_{i,\mathcal{S}}) \right) \tag{24}$$

Due to (19) and Assumption 2, $z_{i,\omega,\mathcal{S}} = 0$ if $InP \notin \mathcal{S}$.

The stability of the grand coalition $\mathcal{N}$ in the stochastic game depends on whether $z_{i,\omega,\mathcal{N}}$ (24) is within a certain bound, $\forall i \in \mathcal{N}, \forall \omega \in \Omega$ (Doan and Nguyen 2014, Theorem 2). The bound on the probability that the grand coalition is stable can be computed based on the following definition, applicable to each player.

Definition 15 *The uncertainty bound of player $i \in \mathcal{N}$ within coalition $\mathcal{N}$ quantifies the likelihood that the revenue deviation $z_{i,\omega,\mathcal{N}}$ remains within a predefined threshold $\hat{\delta}$. Formally, it is given by:*

$$\mathbb{P}^{LB}_{i,\mathcal{N}} \triangleq \mathbb{P}(|z_{i,\omega,\mathcal{N}}| < \hat{\delta}) \tag{25}$$

where $\hat{\delta} > 0$ is designed as a restrictive threshold that ensures both efficiency and stability within the grand coalition and is defined as:

$$\hat{\delta} \triangleq \min \left(\frac{\bar{v}(\mathcal{N})}{|\mathcal{N}|}, \frac{\hat{\sigma}(\mathcal{N}, \bar{v})}{\max\limits_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(|\mathcal{S}| + (|\mathcal{N}| - 2|\mathcal{S}|) \cdot y_{\hat{\sigma}}(\mathcal{S}) \right)} \right) \tag{26}$$

To better understand this bound, let us break it down:

1. The first term serves as a uniform upper bound on the allowed deviation per player. It ensures that no individual perturbation exceeds the average share of the grand coalition's value. It also ensures the efficiency condition, because $\bar{v}(\mathcal{N})$ is fully distributed among the players in the grand coalition $\mathcal{N}$.

2. The second term quantifies how much better off players are within the grand coalition compared to any possible subset, by incorporating the stability value $\hat{\sigma}(\mathcal{N}, \bar{v})$. The denominator normalizes this effect by considering both coalition size and the extent of payoff variability. We define the stability value $\hat{\sigma}(\mathcal{N}, \bar{v})$ as follows:

$$\hat{\sigma}(\mathcal{N}, \bar{v}) \triangleq \min_{\mathcal{S} \subsetneq \mathcal{N}} \left(\sum_{i \in \mathcal{S}} \bar{x}_i - \bar{v}(\mathcal{S}) \right) \tag{27}$$

where $\bar{x}_i$ is the expected Shapley value of player i, obtained by distributing the expected payoff $\bar{v}(\mathcal{N})$ (14) among the players within the nominal game $(\mathcal{N}, \bar{v})$. According to Theorem 13 and (Shapley 1971, Theorem 7), the Shapley value lies in the core of the nominal game, ensuring both efficiency and stability. The expected Shapley value assigned to player i is given by:

$$\bar{x}_i \triangleq \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{i\}} \frac{|\mathcal{S}|!(|\mathcal{N}| - |\mathcal{S}| - 1)!}{|\mathcal{N}|!} \Delta_i(\mathcal{S}) \tag{28}$$

where $\Delta_i(\mathcal{S})$ is the marginal contribution of player i to coalition $\mathcal{S}$, as computed in (20). We now explain the intuition behind $\hat{\sigma}(\mathcal{N}, \bar{v})$ (27):

(a) If the sum of expected Shapley values of coalition $\mathcal{S}$ exceeds its coalition payoff $\bar{v}(\mathcal{S})$, then the players in $\mathcal{S}$ have a stronger incentive to remain in the grand coalition $\mathcal{N}$.

(b) By minimizing over all coalitions $\mathcal{S}$, this function identifies the weakest coalition configuration, ensuring that the uncertainty bound accounts for the most unstable scenario.

To quantify how the payoff of coalition $\mathcal{S}$ compares to its Shapley value share in the grand coalition $\mathcal{N}$, we introduce the normalized payoff $y_{\hat{\sigma}}(\mathcal{S})$:

$$y_{\hat{\sigma}}(\mathcal{S}) \triangleq \begin{cases} \frac{\bar{v}(\mathcal{S}) + \hat{\sigma}(\mathcal{N}, \bar{v})}{\bar{v}(\mathcal{N})} & \text{for all } \mathcal{S} \subsetneq \mathcal{N}, \\ 1 & \text{for } \mathcal{S} = \mathcal{N} \end{cases} \tag{29}$$

If a coalition $\mathcal{S}$ has a coalition payoff $\bar{v}(\mathcal{S})$ significantly greater than its Shapley value share in $\mathcal{N}$, its players might have an incentive to leave and form their own coalition, potentially reducing the stability of the grand coalition.

We include the following Lemma to justify our use of $\hat{\delta}$, showing that it remains safely below the threshold used in earlier work and reinforces the soundness of our approach.

Lemma 16 *The threshold $\hat{\delta}$ (26) is tighter than the one presented in (Doan and Nguyen 2014, Theorem 2), i.e,*

$$\hat{\delta} < \delta \tag{30}$$

where δ denotes the threshold introduced in (Doan and Nguyen 2014, Theorem 2).

The proof of Lemma 16 is provided in Appendix E.

We now present the main result of our work, which relies on two levels of independence. First, we assume that for each SP i, the utilities $u_i^t(\cdot)$ are independent across time slots $t \in \mathcal{T}$. This temporal independence is required to apply Hoeffding's inequality (Boucheron et al. 2013, Theorem 2.8), which provides a bound on the deviation of the total revenue from the expected value (24) (bound on the sum of utility fluctuations over time). Second, we assume that utilities of the SPs are independent from each other. This is justified by the fact that SPs often serve distinct user populations and handle traffic from unrelated applications. Consequently, the uncertainties influencing their revenues arise from separate sources. This cross-player independence enables us to express the joint probability that all revenue deviations (24) remain within a threshold $\hat{\delta}$ (26) as a product over individual probabilities, which is a key step in deriving the final bound on co-investment profitability.

Theorem 17 *Let us assume that $u_i^t(\cdot)$ are independent random functions and that they are bounded, $\forall i \in \mathcal{N}, t \in \mathcal{T}$. In stochastic game $(\mathcal{N}, \mathcal{V})$, the grand coalition is stable with probability at least ν^{LB} (see Definition 9), where:*

$$\nu^{LB} \triangleq \prod_{i\in\mathcal{N}} \max\left(1 - 2\exp\frac{-2\hat{\delta}^2}{\sum_{t\in\mathcal{T}}(range(u_i^t))^2}, 0\right) \tag{31}$$

where:

$$range(u_i^t) \triangleq \max_{\omega\in\Omega} u_{i,\omega}^{t,\max}(h_{i,\mathcal{N}}^{t*}) - \min_{\omega\in\Omega} u_{i,\omega}^{t,\max}(h_{i,\mathcal{N}}^{t*}) \tag{32}$$

represents the range of uncertainty of revenues collected by player $i \in \mathcal{N}$ at time slot $t \in \mathcal{T}$.

Proof We first find uncertainty bounds $\mathbb{P}_{i,\mathcal{N}}^{\text{LB}}, \forall i \in \mathcal{N}$, according to Definition 15. Since the *InP* does not directly interact with end users, it does not receive any resource allocation, i.e., $h_{InP,\mathcal{N}}^{t*} = 0$ for all $t \in \mathcal{T}$ (3), as stated in Assumption 1. Consequently, via (4) and (24), we have $z_{InP,\omega,\mathcal{N}} = 0, \forall\omega \in \Omega$, which implies that $\mathbb{P}_{InP,\mathcal{N}}^{\text{LB}} = 1$ (25). Let us now consider $i \neq$ *InP*. By assumption, the utility functions $u_i^t(\cdot)$ are independent across time slots $t \in \mathcal{T}$ for each player i, and the random variables $u_{i,\omega}^t(h_{i,\mathcal{N}}^{t*})$ are bounded. It follows that the deviations $u_{i,\omega}^t(h_{i,\mathcal{N}}^{t*}) - \bar{u}_i^t(h_{i,\mathcal{N}}^{t*})$ are independent, zero-mean, and bounded for each t. Therefore, revenue deviation $z_{i,\omega,\mathcal{N}}$ (24) satisfies the conditions for applying Hoeffding's inequality (Boucheron et al. 2013, Theorem 2.8): namely, it is a sum of independent, bounded random variables. The probability that $|z_{i,\omega,\mathcal{N}}|$ exceeds a given threshold $\hat{\delta}$ (26) is thus bounded as:

$$\mathbb{P}(|z_{i,\omega,\mathcal{N}}| \geq \hat{\delta}) \leq \min\left(2\exp\frac{-2\hat{\delta}^2}{\sum_{t\in\mathcal{T}}(\text{range}(u_i^t))^2}, 1\right), \qquad \forall i \in \mathcal{N} \tag{33}$$

Since values $h_{i,\mathcal{N}}^{t*}$ have been precomputed (Assumption 10), random variables $z_{i,\omega,\mathcal{N}}$ are statistically independent among players. Therefore:

$$\mathbb{P}(|z_{i,\omega,\mathcal{N}}| < \hat{\delta}, \forall i \in \mathcal{N}) = \prod_{i\in\mathcal{N}} \left(1 - \mathbb{P}(|z_{i,\omega,\mathcal{N}}| \geq \hat{\delta})\right) \tag{34}$$

Let $\Omega' \subseteq \Omega$ be the set of realizations in which all players' revenue deviations remain within the threshold $\hat{\delta}$, that is,

$$\Omega' \triangleq \left\{\omega \in \Omega : |z_{i,\omega,\mathcal{N}}| < \hat{\delta},\ \forall i \in \mathcal{N}\right\} \tag{35}$$

and applying the independence of the random variables $z_{i,\omega,\mathcal{N}}$ across players, we can write:

$$\mathbb{P}(\Omega') = \prod_{i \in \mathcal{N}} \mathbb{P}(|z_{i,\omega,\mathcal{N}}| < \hat{\delta}) = \prod_{i \in \mathcal{N}} \left(1 - \mathbb{P}(|z_{i,\omega,\mathcal{N}}| \geq \hat{\delta})\right) \tag{36}$$

Then, using the upper bound from Hoeffding's inequality (33), we obtain the following lower bound:

$$\mathbb{P}(\Omega') \geq \prod_{i \in \mathcal{N}} \left(1 - \min\left(2\exp\left(\frac{-2\hat{\delta}^2}{\sum_{t \in \mathcal{T}} (\text{range}(u_i^t))^2}\right), 1\right)\right) \tag{37}$$

If, for all realizations, we have that $z_{i,\omega,\mathcal{N}} < \hat{\delta}$, then the grand coalition is stable in the stochastic game (Doan and Nguyen 2014, Theorem 2). This condition is verified for all realizations in Ω'. Therefore, we can affirm that the grand coalition $\mathcal{N}$ is stable in stochastic game $(\mathcal{N}, \mathcal{V}(\Omega'))$ (according to Definition 9). Thanks to (37), we obtain the theorem. □

3.6 Profitability of the co-investment under uncertainty

Theorem 17 provides a lower bound on the probability of grand coalition stability under the assumption that revenues are uncorrelated and bounded. When revenues are correlated, one can instead apply Chebyshev's inequality (Feller 1991). However, in the presence of strong temporal correlations, Chebyshev's bound may become too loose to be informative. This raises a key question: Can co-investment remain profitable for all players despite high revenue variability?

Answering this question requires verifying a necessary condition, namely, whether the co-investment yields a non-negative payoff for each player. Specifically, we must evaluate whether the individual payoff $x_{i,\omega}$ for each player $i \in \mathcal{N}$, as well as the realized coalition payoff $v_\omega(\mathcal{S})$, for all $\omega \in \Omega$ and all $\mathcal{S} \subseteq \mathcal{N}$, is non-negative. To formalize what we mean by a profitable co-investment, we introduce the following definition.

Definition 18 *We say that the co-investment of a coalition $\mathcal{S} \subseteq \mathcal{N}$ is profitable for all its players with probability at least $\nu_{\mathcal{S}}^{LB} \in [0,1]$, if the probability that every player $i \in \mathcal{S}$ receives a non-negative payoff is at least $\nu_{\mathcal{S}}^{LB}$, i.e.,*

$$\mathbb{P}\left(x_{i,\omega} \geq 0, \forall i \in \mathcal{S}\right) \geq \nu_{\mathcal{S}}^{LB} \tag{38}$$

where $x_{i,\omega}$ denotes the random payoff of player i in realization ω, resulting from the co-investment by coalition $\mathcal{S}$.

We now formalize the relationship between the stability of the grand coalition and the profitability of the co-investment. By definition, if the grand coalition is stable, no player is better off not being part of the grand coalition, which implies that each player is receiving a non negative payoff. Therefore, stability guarantees profitability for all players. This is captured in the following proposition.

Proposition 19 *Assume that the realized value of the grand coalition is strictly positive for all realizations, i.e., $v_\omega(\mathcal{N}) > 0, \forall \omega \in \Omega$. Then, if the grand coalition $\mathcal{N}$ is stable in the stochastic game $(\mathcal{N}, \mathcal{V})$, the co-investment of the grand coalition is profitable for all players with probability 1.*

We now show, in the following Proposition and Corollary, that profitability alone does not guarantee stability: even when all players benefit, the grand coalition may still fail to hold together.

Proposition 20 *The probability that the co-investment is profitable for all players is larger than or equal to the probability that the grand coalition is stable, i.e.,*

$$\mathbb{P}\left(x_{i,\omega} \geq 0,\ \forall i \in \mathcal{N}\right) \geq \mathbb{P}\left(\textit{Grand coalition is stable}\right) \tag{39}$$

Corollary 21 *The lower bound on the probability that the grand coalition is stable, denoted ν^{LB}, also serves as a lower bound on the probability that the co-investment is profitable for all players. That is,*

$$\mathbb{P}\left(x_{i,\omega} \geq 0,\ \forall i \in \mathcal{N}\right) \geq \nu^{LB} \tag{40}$$

The proofs of Proposition 19, Proposition 20, and Corollary 21 are provided in Appendix F, Appendix G and Appendix H, respectively.

3.7 Sharing the payoff among players

After optimizing the capacity and the shares, and maximizing the total payoff of the game, the proposed co-investment scheme distributes the payoff among players. Stochastic game $(\mathcal{N}, \mathcal{V})$ presents uncertain payoffs (8). At the end of the investment period I, all payoffs become known and they must be redistributed in a fair way. Since we deal with monetary rewards, fairness (each player receives payoff proportional to their contribution) and efficiency (the coalition payoff is fully distributed) must be ensured. A solution concept such as the nucleolus has been widely used in cooperative game theory. The nucleolus provides a unique allocation that minimizes the maximum dissatisfaction among coalitions (Schmeidler 1969). However, its notion of fairness is based on dissatisfaction minimization rather than on individual contribution, which is less aligned with our objective. In addition, it requires solving a series of lexicographic optimization problems, which can become computationally intensive, especially in large-scale settings. One alternative is the Banzhaf value (Van der Laan and Van den Brink 2002), but it does not ensure efficiency, i.e., it does not ensure that all payoff is redistributed, which is essential in economic settings like ours.

In contrast, the Shapley value (Shapley 1971) is the only solution that satisfies efficiency, symmetry (players with equal contributions receive equal payoffs), additivity, and the null player property (Saad et al. 2009), ensuring that each player receives their rightful share based on their contribution, without arbitrary advantage or loss. Although the Shapley value poses computational challenges, due to the need to consider all possible coalition permutations, it has a well defined closed form expression. Moreover, the Shapley value aligns more closely with the fairness objective of our analysis, which is based on players' marginal contributions.

We adopt the stochastic formulation of the Shapley value proposed by (Ma et al. 2008; Chau et al. 2023), which extends the classic formulation (Shapley 1971, (19)) by computing the value for each realization $\omega \in \Omega$.

We define the amount of payoff that should go to player i in realization ω, after the game ends, as:

$$x_{i,\omega} \triangleq \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{i\}} \frac{|\mathcal{S}|!(|\mathcal{N}| - |\mathcal{S}| - 1)!}{|\mathcal{N}|!} \Delta_{i,\omega}(\mathcal{S}) \tag{41}$$

where $\Delta_{i,\omega}(\mathcal{S})$ is the marginal contribution of player i to coalition $\mathcal{S}$ in realization ω. It can be calculated as follows:

$$\Delta_{i,\omega}(\mathcal{S}) \triangleq v_\omega(\mathcal{S} \cup \{i\}) - v_\omega(\mathcal{S}) \tag{42}$$

where $v_\omega(.)$ is calculated as in (8).

Observe that Shapley value $x_{i,\omega}$ is thus a random variable $\forall i \in \mathcal{N}$. At the end of the game, the actual realization $\omega \in \Omega$ that occurred becomes known, and revenues are redistributed so that the payoff of each player is $x_{i,\omega}$. Recall that each player i contributes an initial payment p_i at the start of the game, such that the total cost is fully covered by the coalition (1). While these payments may differ across players, the scheme ensures that all contributions are fairly balanced through the final reward distribution. This reward, denoted as $r_{i,\omega}$, represents the actual amount received by player i in realization ω at the end of the game. It reflects both the payoff determined by the Shapley value and the return on investment:

$$r_{i,\omega} \triangleq x_{i,\omega} + p_i \quad \forall \omega \in \Omega, i \in \mathcal{S} \tag{43}$$

An important property of the reward distribution (43) is that it ensures budget balance: the total amount redistributed to the players exactly matches the total collected utility. In other words, the mechanism guarantees that no more is distributed than what has actually been earned. This property is stated in the following Lemma.

Lemma 22 *The sum of the individual rewards $r_{i,\omega}$ distributed to players $i \in \mathcal{S}$ equals the total utility collected across all time slots in realization ω:*

$$\sum_{i \in \mathcal{S}} r_{i,\omega} = \sum_{i \in \mathcal{S}} \sum_{t \in \mathcal{T}} u^t_{i,\omega}(h^{t*}_{i,\mathcal{S}}) \tag{44}$$

The proof of this Lemma can be found in Appendix I.

4 Application to Edge computing (EC)

We now consider a particular investment setting, namely the one of EC, where the *InP* is a NO, such as Comcast or Orange, and SPs are actors receiving requests from users, processing them and replying to them. Examples of SPs are Netflix, Youtube or new companies providing new generation services, such as augmented or virtual reality.

4.1 Settings

We aim to capture a realistic and analytically tractable expression for the total cost $Cost(I, C)$ incurred when installing computational capacity C in an edge node (e.g., a cellular network base station) and maintaining it over an investment period I. This cost should reflect two main components: (i) the upfront cost of installing resources, which increases with the total installed capacity, and (ii) the operational cost of maintaining these resources throughout the investment period, which increases with both the amount of capacity and the duration of the investment. A natural formulation that meets these requirements is the following:

$$Cost(I, C) = d \cdot C + d' \cdot I \cdot C \tag{45}$$

where d represents the price per unit of computational capacity, and d' represents the maintenance price per unit of time and unit of capacity.

To realistically model the revenue generated from services provided to end users at the edge, the utility function to be adopted must meet the following criteria: (i) monotonically increasing in the allocated resources, as more resources lead to higher revenue, (ii) exhibit diminishing returns, as the marginal benefit of additional resources decreases beyond a certain point due to capacity saturation or limited user demand, (iii) load-dependent, meaning that the generated utility scales with the amount of user traffic an SP receives, and (4) bounded, capturing economic limitations (e.g., no infinite revenue). We do not explicit in this work the quality of service of admitted end users, we assume that it is fulfilled using the quantity of resources allocated to the SPs. The following utility function satisfies all of the above properties:

$$u_{i,\omega}^t = \beta_i \cdot l_{i,\omega}^t \cdot (1 - \exp^{-\xi \cdot h_i^t}), \quad \forall \omega \in \Omega, t \in \mathcal{T}, i \in \mathcal{N} \tag{46}$$

where $l_{i,\omega}^t$ is the load of end-user requests received by SP i in realization $\omega \in \Omega$, parameter ξ represents the diminishing return parameter and β_i, which we call benefit factor, can be interpreted as the revenue SP i gets from serving one unit of load.

The load $l_{i,\omega}^t$ represents the amount of requests collected by SP i at time slot t in realization ω and is given by:

$$l_{i,\omega}^t \triangleq \lambda_{i,\omega}^t \cdot \Delta \tag{47}$$

where $\lambda_{i,\omega}^t$ is the request rate and Δ is the duration of time slot.

Rate $\lambda_{i,\omega}^t$ (and thus load $l_{i,\omega}^t$) may change from realization $\omega \in \Omega$ to another. It is thus a random variable. Therefore, the stochasticity of the utilities in our numerical results derives from the stochasticity of the user request rate, as shown in (46) and (47)[2]. The expected amount of requests received by SP i at time slot t is defined as follows:

$$\bar{l}_i^t \triangleq \mathbb{E}_\omega[l_{i,\omega}^t] \overset{(47)}{=} \bar{\lambda}_i^t \cdot \Delta \tag{48}$$

where $\bar{\lambda}_i^t$ denotes the expected request rate, defined as:

$$\bar{\lambda}_i^t \triangleq \mathbb{E}_\omega \left[\lambda_{i,\omega}^t\right] \tag{49}$$

[2]Note that, the theory developed in the previous section holds as well if the sources of stochasticity are others, e.g., imperfect knowledge of benefit factors.

To model the load $l_{i,\omega}^t$ (47) of each SP, we aim for a formulation that satisfies the following criteria: (i) periodic, to reflect the regular temporal cycles observed in network traffic (e.g., day–night patterns in user activity), (ii) random, as service demand includes fluctuations caused by unpredictable user behavior, and (iii) tunable, meaning the model allows control over the influence of randomness versus expected trends, enabling flexible simulation and analysis. To showcase the generality of the proposed co-investment scheme, we consider two settings, with fundamentally different characterizations of uncertainty. In the first setting (section 4.1.1), the stochastic process underlying $l_{i,\omega}^t$ is unknown. What is known is just that it has finite and known upper and lower bounds and that values at different timeslots are independent. In the second setting (section 4.1.2), we instead allow rates to be unbounded and correlated from a timeslot to another. Although this case does not comply with the hypothesis of Theorem 17, we will show numerically that the proposed co-investment scheme is still profitable for all players with high probability. In both cases, the uncertainty captured in $l_{i,\omega}^t$ corresponds to randomness, in the sense defined by (Gendreau 2023, Page 285), which reflects inherent, statistically characterizable although with possible unknown fluctuations in the process.

4.1.1 Traffic load with bounded fluctuations

In this setting, the load satisfies the three desired criteria mentioned above (Section 4.1) through a bounded stochastic model with values that are independent across time. The request rate of an SP determines the number of user requests received per time unit. For example, with a typical 5G mean capacity of 1 Gbps (Xu et al. 2020) and an average request size of $r_s = 100$ Kbits, the mean service rate (s_r) corresponds to approximately 10,000 requests per second. In our setup, we set the mean request rate (α_r) to 20% of s_r (Peng et al. 2015).

Empirical observations across communication networks reveal daily periodic patterns driven by temporal cycles (Akgül et al. 2010) (Shi and Li 2018). We define the expected request rate $\bar{\lambda}_i^t$ (49) of SP i at time slot t using the model proposed by large network operators (Vela et al. 2016) to capture periodic fluctuations:

$$\bar{\lambda}_i^t = a_{0,i} + \sum_{k=1}^{K} a_{k,i} \cdot \sin\left(2k\pi \cdot \frac{t - t_{k,i}}{T}\right) \tag{50}$$

where $a_{k,i}$ and $t_{k,i}$ represent the amplitude and phase shift of the kth sinusoidal component, respectively. These coefficients are chosen such that $\bar{\lambda}_i^t$ fluctuates around α_r, e.g., $\bar{\lambda}_i^t \in [0.1\alpha_r,\ 1.5\alpha_r]$.

We consider the actual request rate $\lambda_{i,\omega}^t$ of SP i at time slot t in realization ω to fluctuate around $\bar{\lambda}_i^t$, i.e.,

$$\lambda_{i,\omega}^t \in \left[(1-\sigma)\bar{\lambda}_i^t,\ (1+\sigma)\bar{\lambda}_i^t\right] \tag{51}$$

where $\sigma \in [0, 1]$ quantifies the level of uncertainty.

The amount of requests received by SP i at time slot t in realization ω, given by (47), is directly tied to $\lambda_{i,\omega}^t$ and inherits the same bounded fluctuation (51). It satisfies:

$$l_{i,\omega}^t \in \left[(1-\sigma)\bar{l}_i^t,\ (1+\sigma)\bar{l}_i^t\right], \quad \forall\omega\in\Omega,\ t\in\mathcal{T},\ i\in\mathcal{N} \tag{52}$$

where $\bar{l}_i^t$ is the expected amount of requests (48).

4.1.2 Traffic load with correlated fluctuations

Another formulation of $l_{i,\omega}^t$ is based on an alternative expression of $\lambda_{i,\omega}^t$, where request rates follow an unbounded and temporally correlated stochastic process. This model continues to satisfy the three desired criteria (Section 4.1). We define the request rate as follows:

$$\lambda_{i,\omega}^t = d_i^t \cdot \left[(1-\alpha)\cdot\mathbb{E}_\omega\left[S_{i,\omega}^t\right] + \alpha\cdot S_{i,\omega}^t\right] \tag{53}$$

where $S_{i,\omega}^t$ is a non-negative stochastic process (to ensure that the request rate $\lambda_{i,\omega}^t$ remains positive), and $\alpha\in[0,1]$ serves as a tunable parameter that quantifies the level of uncertainty introduced in the model. The term d_i^t captures the periodic trend, assumed to be known, and follows a sinusoidal form as:

$$d_i^t = a_{i,0} + \sum_{k=1}^{K} a_{i,k}\cdot\sin\left(2k\pi\frac{t-t_{i,k}}{T}\right) \tag{54}$$

where $a_{i,k}$ and $t_{i,k}$ represent the amplitude and phase shift of the kth components in the sinusoidal function.

Internet traffic is known to exhibit two fundamental characteristics: self similarity and long range dependence (Karagiannis et al. 2004; Lee and Fapojuwo 2005) ("A large number of studies have shown that the network traffic possesses self-similarity and long-term dependence properties" (Bi et al. 2021)). Self-similarity refers to the persistence of traffic patterns across different time scales, while long-range dependence implies strong temporal correlations that decay slowly over time. It is important to clarify that long range dependence does not mean a direct, memory like dependency on individual past events. Rather, it reflects persistent patterns and correlations over long timescales. A common cause is the presence of bursty, high-variance applications such as video streaming, social media, or large file transfers. For example, when a video goes viral, it can continue to generate significant traffic days or even months after its initial release, as users repeatedly share and engage with it over time. Another source of long-range dependence is aggregated user behavior. When millions of users follow regular usage patterns such as daily, weekly, or seasonal routines, these behaviors create persistent statistical patterns in the traffic. These patterns appear as long term correlations in measurements. To capture these elements, we model stochastic process $S_{i,\omega}^t$ contained in (53) as:

$$S_{i,\omega}^t \triangleq \max(0, f_{i,\omega}^t) \tag{55}$$

where $f_{i,\omega}^t$ is a fractional Brownian motion (fBm) with parameter H (Mandelbrot and Van Ness 1968), (Mishura 2008). This choice is motivated by the fact that fbm effectively captures the above characteristics of internet traffic, making it a natural tool for modeling it (Dieker 2004). The behavior of fBm is governed by the Hurst parameter $H \in (0,1)$, which controls the correlation structure of the process. When $H > 0.5$, the process exhibits long-range dependence and positive correlation between increments. When $H = 0.5$, fBm reduces to classical Brownian motion with independent increments. When $H < 0.5$, the process exhibits anti-persistence, meaning that an increase in the process is more likely to be followed by a decrease, and vice versa, indicating a negative correlation between increments. The $\max(\cdot)$ in (55) ensures that $S^t_{i,\omega}$ is always non-negative and thus remains consistent with its interpretation as a request rate component, while preserving the essential temporal structure captured by fBm.

The following Lemma provides a closed-form expression for $\bar{l}^t_i$ (48), which is essential for deriving the amount of resources to be installed and their dynamic allocation among SPs (Assumption 10).

Lemma 23 *The expected amount of requests* (48) *received by SP i at time slot t is given by:*

$$\bar{l}^t_i = d^t_i \cdot \frac{t^H}{\sqrt{2\pi}} \cdot \Delta \tag{56}$$

The proof of Lemma 23 can be found in Appendix J.

4.2 Characterization of the resource sharing

In our model, we consider an *InP* and several SPs. The following proposition allows computing resource sharing, according to Assumption 10, in closed form (see Proposition 12). Its proof (in Appendix K) relies on the convexity of (16) (18). The notation below is compliant with Table 1 and Table 2.

Proposition 24 *Given a coalition $\mathcal{S}$, let $\mathcal{S}' \subseteq \mathcal{S} \setminus \{InP\}$ denote the subset of SPs with strictly positive load, i.e., those i for which $\bar{l}^t_i > 0$ for all $t \in \mathcal{T}$. The resources installed and their allocation are as follows:*

- *Case 1:*
  If $\mathcal{S}' \neq \emptyset$, the total installed capacity is given by:

$$C^*_{\mathcal{S}} = \frac{|\mathcal{S}'|}{\xi} \ln \left(\frac{\sum_{t \in \mathcal{T}} \left(\prod_{i \in \mathcal{S}'} \xi \cdot \beta_i \cdot \bar{l}^t_i \right)^{\frac{1}{|\mathcal{S}'|}}}{d + d' \cdot I} \right) \tag{57}$$

  and the share of the capacity $C^*_{\mathcal{S}}$ allocated to each SP $i \in \mathcal{S}'$ is:

$$h^{t*}_{i,\mathcal{S}} = \frac{C^*_{\mathcal{S}}}{|\mathcal{S}'|} + \frac{1}{\xi} \ln \left(\frac{\beta_i \cdot \bar{l}^t_i}{\left(\prod_{j \in \mathcal{S}'} \beta_j \cdot \bar{l}^t_j \right)^{\frac{1}{|\mathcal{S}'|}}} \right) \tag{58}$$

Table 2: Settings

Functions	Symbols	Values
Investment period	$I = \|\mathcal{T}\| \cdot \Delta$	5 years
Price per vcore (45)	d	10.94 \$/vcore (Microsoft Azure)
Price per hour for one virtual CPU (vcore) (45)	d'	16.25 \$/hour/vcore (Microsoft Azure)
Benefit factor (46)	$\beta_i = \beta$	6×10^{-6} \$/req (Amazon Web Services 2024)
Diminishing return (46)	$\xi_i = \xi$	0.03
Time slot length (47)	Δ	1 hour
Number of time slots per day (periodicity of load) (50)	T	24 time slots
Parameter controlling the level of uncertainty in the uncorrelated load (51)	σ	$[0, 1]$
Parameter controlling the level of uncertainty in the correlated load (53)	α	$[0, 1]$
Hurst parameter (56)	H	0.7

- *Case 2:*
  If $\mathcal{S}' = \emptyset$ (i.e., all SPs have zero load), then no capacity is installed and no resources are allocated:

$$C^*_{\mathcal{S}} = 0, \quad h^{t*}_{i,\mathcal{S}} = 0, \; \forall\, i \in \mathcal{S}, \; t \in \mathcal{T} \tag{59}$$

We now show the following proposition:

Proposition 25 *From Proposition 24, the resources $h^{t*}_{i,\mathcal{S}}, \forall t \in \mathcal{T}$ allocated to SP i are strictly increasing with respect to its expected load $\bar{l}^t_i$ and its revenue per unit of load β_i, for all cases where $\bar{l}^t_i > 0$ and $\beta_i > 0$.*

The proof is provided in Appendix L.

We now evaluate our model using synthetic parameters, which were carefully selected to reflect a range of realistic EC scenarios, as detailed in Table 2.

4.3 Evaluation of the traffic load with bounded fluctuations

We start by evaluating the first traffic model introduced in Section 4.1.1.

4.3.1 Illustration of traffic load realizations

Fig. 1 illustrates the traffic load corresponding to one SP over a five-day period, where the x-axis represents time in days and the y-axis denotes the load in requests per second. The red solid line represents the expected load, which follows a clear periodic pattern, reflecting cyclic daily variations in traffic demand. The dashed lines correspond to multiple stochastic load realizations, which exhibit significant variability around the expected load. In some instances, the stochastic realizations reach the double of the expected load, while in others, they drop close to zero, highlighting the unpredictable nature of traffic fluctuations. Additionally, the stochastic realizations are uncorrelated, meaning that each trajectory evolves independently without following a common trend beyond the underlying periodicity. This variability suggests the presence of a compensation effect, where high and low deviations cancel each other out over time, leading to an overall balance despite short-term fluctuations.

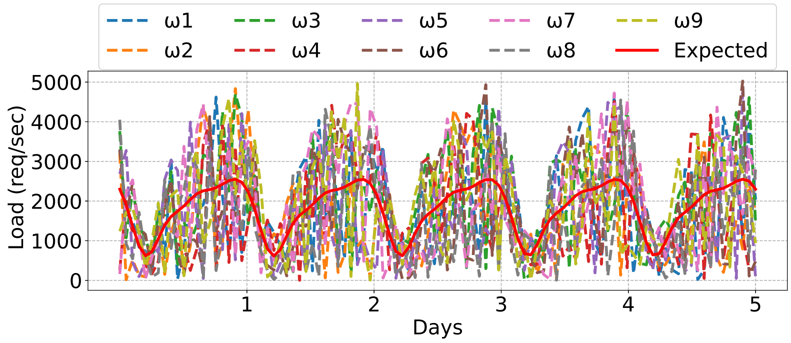


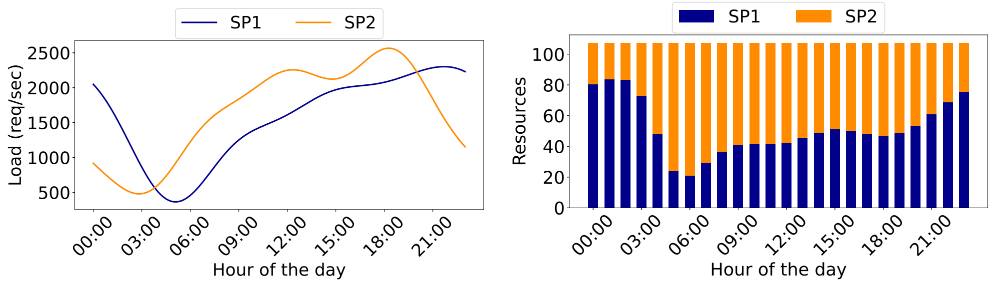
Fig. 1: Traffic load with bounded fluctuations over five days within the investment period

4.3.2 Dynamic resource sharing

We first consider a coalition $\mathcal{N}$ including one *InP* and two SPs. Fig. 2 illustrates the expected traffic load (2a) and the corresponding dynamic resource allocation (2b) over a one-day period within the investment period. As shown in Fig. 2a, the traffic load patterns of SP1 and SP2 reflect the distinct characteristics of the areas they serve. SP1 exhibits a low traffic load during the early morning hours, followed by a steady increase throughout the day, peaking in the evening. This pattern is typical of residential areas, where usage ramps up as people return home and engage in online activities like streaming and gaming. In contrast, SP2 shows lower traffic in the early morning, followed by a rapid rise during the daytime, peaking in the afternoon and declining after business hours. This trend resembles business or commercial usage patterns, where activity is driven by daytime operations. To accommodate these variations, the resource allocation strategy dynamically adjusts to them, as shown in Fig. 2b. During high-demand periods for SP1, a greater share of resources is allocated to it, whereas SP2 receives a larger share when SP1's demand decreases. This dynamic approach ensures optimal utilization of available resources, improving efficiency and performance for both SPs.

4.3.3 Impact of coalition size and player heterogeneity on stability

We analyze different coalitions $\mathcal{N}$, each including one *InP* along with 2, 3, 4, or 5 SPs. Among the SPs, SP1, SP2, and SP3 have comparable traffic loads, SP4 has high traffic load, and SP5 has low traffic load. Fig. 3 presents the lower bound ν^{LB} on the probability that the grand coalition is stable (Theorem 17), across different levels of uncertainty σ. In the extreme case, $\sigma = 1$ means that the uncertainty in the load is equal to 100% of its expected value, indicating the maximum range of unknown requests, i.e., $[0, 2\bar{l}_i^t]$ (52). We observe that coalitions composed of SPs with comparable traffic loads, such as SP1, SP2, SP3, and even SP4, achieve a lower bound equal to 1, indicating stable cooperation across all uncertainty levels σ. Expanding the coalition always increases the total coalition payoff. When the added SPs have comparable or higher traffic levels, this increase is sufficient to ensure that all players remain in the



(a) Expected load over one day

(b) Share of resources

Fig. 2: Expected traffic load and the corresponding resource allocation for a typical day during the investment period

grand coalition, resulting in a lower bound of 1. However, when a low traffic load SP, like SP5, is included, as in coalition $\{1, 2, 3, 5\}$, the lower bound on stability decreases significantly, and drops to zero beyond moderate values of σ (e.g., when $\sigma = 0.5$). This suggests that the coalition becomes less attractive for at least some players. Although some stability may still be achievable under low uncertainty, the disparity in traffic patterns reduces the incentive of players to form and stay in the grand coalition and thus co-invest. This effect is most pronounced in the full coalition $\{1, 2, 3, 4, 5\}$, where the lower bound remains zero across all uncertainty levels σ, except when $\sigma = 0.001$. Despite the potential for a higher coalition payoff in larger coalitions, the inclusion of highly heterogeneous SPs, especially those with low traffic load, leads to instability. When individual profiles are not aligned due to unequal contribution or risk exposure, players are unlikely to form the coalition.

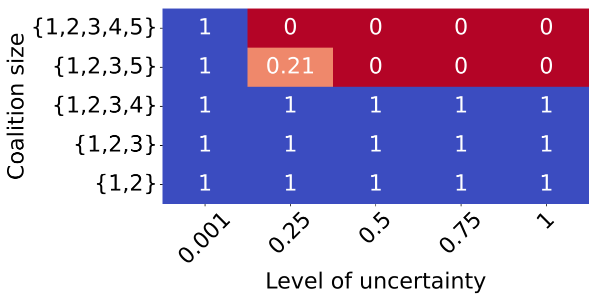


Fig. 3: Lower bound on the probability that the the grand coalition is stable under different levels of uncertainty σ and varying coalition sizes

We now consider the coalition $\mathcal{N}$ formed by one *InP* and the set $\{1, 2, 3, 4, 5\}$ of SPs. Table 3 shows the individual lower-bound probabilities $\mathbb{P}_i^{\text{LB}}$, $\forall i \in \mathcal{N}$ (25), which represent the probability that each player has an incentive to form and stay in the grand coalition $\mathcal{N}$ across different values of σ. It also shows ν^{LB} (Theorem 17), the lower-bound probability for the stability of the grand coalition $\mathcal{N}$. We observe from the table that SP5, the smallest player with the lowest traffic load, is highly motivated to join the coalition, as it benefits from partnering with bigger players regardless of uncertainty. In contrast, SP4, the biggest player with the highest traffic load, is more hesitant and only finds co-investment profitable when uncertainty is negligible ($\sigma = 0.001$), highlighting its risk-averse nature. SP1, SP2, and SP3, which have comparable medium traffic loads, exhibit similar behaviors: their lower-bound probability of stability drops rapidly, reaching zero shortly after $\sigma = 0.25$. Note that $\mathbb{P}_{\text{InP}}^{\text{LB}} = 1$ for all values of σ. This indicates that the *InP* always benefits from the co-investment, since the coalition cannot form without it (as established in Proposition 3).

Table 3: Lower-bound probabilities for individual players and the grand coalition $\mathcal{N}$ across different levels of uncertainty σ

σ	Lower bound probability of stability					
	$\mathbb{P}_{\text{SP1}}^{\text{LB}}$	$\mathbb{P}_{\text{SP2}}^{\text{LB}}$	$\mathbb{P}_{\text{SP3}}^{\text{LB}}$	$\mathbb{P}_{\text{SP4}}^{\text{LB}}$	$\mathbb{P}_{\text{SP5}}^{\text{LB}}$	ν^{LB}
0.001	1	1	1	1	1	1
0.25	0.31	0.20	0.30	0	1	0
0.5	0	0	0	0	0.99	0

4.4 Evaluation of the traffic load with correlated fluctuations

We now evaluate the second traffic model introduced in Section 4.1.2.

4.4.1 Illustration of stochastic realizations

Fig. 4 illustrates the stochastic process $S_{i,\omega}^{t}$ (55) corresponding to one SP in an EC environment over a five-day period. The expected load (red line) follows a gradual increase, reflecting the assumption that EC is a newly introduced technology. Initially, there is little to no traffic, but as the system is adopted, more applications offload workloads to edge nodes, leading to a steady rise in demand. However, the stochastic realizations ($\omega 1$ to $\omega 9$) exhibit significant divergence from the expected trend, demonstrating highly unpredictable traffic behavior. Some realizations remain close to zero throughout the period, indicating scenarios where edge adoption is minimal or delayed. Others experience sharp increases, suggesting rapid deployment and high usage of edge resources, while some trajectories exhibit fluctuations, either rising and falling repeatedly or showing a delayed start followed by a sudden increase or decline.

4.4.2 Outcome of the stochastic game

We provide in Fig. 5 a detailed analysis of revenue, payment and payoff redistributions using the Shapley value (Section 3.7) across different players (*InP*, SP1, SP2) under varying levels of uncertainty represented by α. In the case of $\alpha = 1$, the traffic load depends entirely on the stochastic realization, with no contribution from its expected

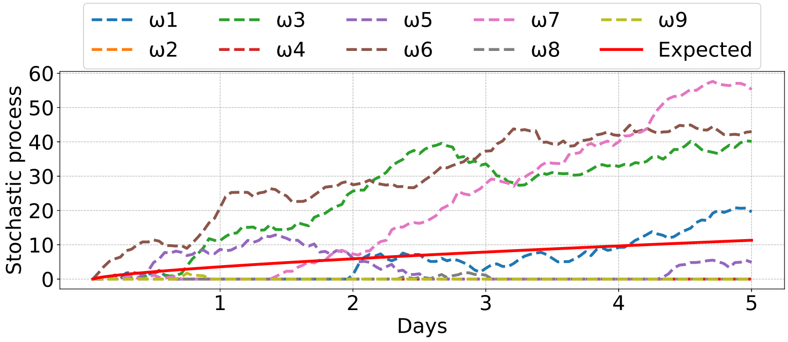


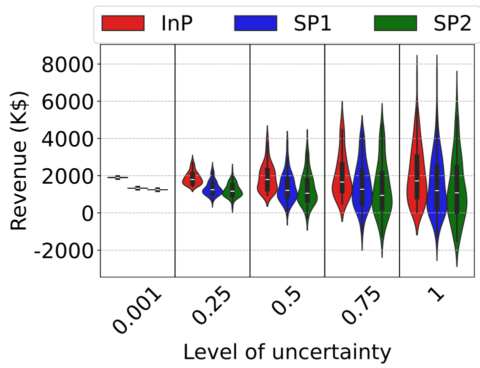
Fig. 4: Stochastic realizations used to model traffic load variations over five days within the investment period

value (according to (53)). The figure shows outcome redistributions across multiple realizations ω. To better understand the impact of uncertainty, we present in Fig. 5a, Fig.5b and Fig.5c violin plots of these redistributions. The width of each violin reflects the density of the data, where wider sections indicate a higher concentration of values. The length of the violin represents the variability of the distribution.

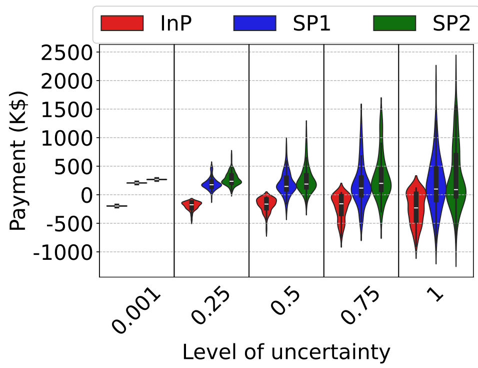
Starting with revenue redistribution (Fig. 5a) (43), we observe that at low values of α, the violins are narrow and short, with a compact interquartile range (IQR), represented by the black box. This box represents the middle 50% of the data, ranging from the first quartile (25th percentile) to the third quartile (75th percentile), while the white line inside the box indicates the median revenue. As α increases, the violin plots widen indicating that revenue becomes dispersed and unpredictable. Additionally, the elongation of the upper tail further indicates that while higher uncertainty may provide opportunities for greater revenues, the lower tail suggests the possibility of lower revenue, raising the risk of extreme outcomes. This effect is particularly evident for SP1 and SP2, as they are affected by fluctuating demand and service usage. The InP's revenues remain stable and do not experience significant drops. It is important to note that this revenue is not intrinsic, it is a revenue that arises from the redistribution of the total revenue generated by the SPs across all players, including the InP itself. Interestingly, negative revenues also appear under high uncertainty, meaning that for some realizations, players consume more resources than the value they generate.

A similar pattern is observed in the payment redistribution (Fig. 5b), where at low α values (e.g., $\alpha = 0.25$), payments remain tightly clustered, as seen in the small IQR and high central density. However, as uncertainty grows, the IQR expands, and the violin plots flatten, meaning payments are no longer concentrated around a single expected value but are spread across a broader range. The increased width of the violins at high α values (e.g., $\alpha = 1$), particularly for SP1 and SP2, indicates that payments fluctuate significantly under uncertain conditions, leading to a higher financial risk and variability for these players. This suggests that in some cases, SPs may pay significantly more or less than expected, reinforcing their exposure to financial uncertainty. Notably, we observe negative payments, particularly for the InP and occasionally for SP1 and SP2. This might seem counterintuitive, but this makes sense under Shapley value redistribution. Payments are calculated based on payoffs and revenues redistribution, not just raw cost. When a player has a high payoff because they are essential to the coalition or contribute significantly, they may receive money from others to reflect their role. For the *InP*, this often occurs because it acts as a veto player (Proposition 3). For SPs, it may happen because they generate the revenue through user services, making them key contributors to the coalition's payoff.

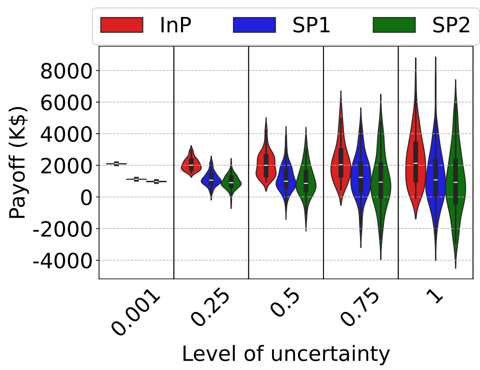
These dynamics come together in the payoff redistribution (41). We show in Fig. 5c that at low α, all players enjoy stable and positive payoffs, meaning they recover more than they pay. But under higher uncertainty (e.g., $\alpha = 1$), the payoff redistributions widen, particularly for SP1 and SP2. In these cases, not only does variability increase, but some players experience negative payoffs, meaning they end up paying more than they earn overall. This highlights a key risk of operating under high uncertainty: while some scenarios may lead to high profits, others can result in financial losses.



(a) Revenue redistribution



(b) Payment redistribution



(c) Payoff redistribution

Fig. 5: Violin plots of revenue, payment, and payoff redistributions for *InP*, SP1, and SP2 under varying levels of uncertainty α, using 100 generated realizations

Table 4: Individual probability of profitable co-investment for each player and for the coalition $\mathcal{N}$ as a whole across different levels of uncertainty α

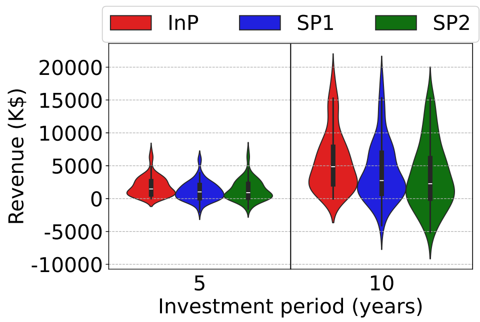
α	Probability of profitable co-investment			
	$\tilde{\mathbb{P}}_{InP}$	$\tilde{\mathbb{P}}_{\mathrm{SP1}}$	$\tilde{\mathbb{P}}_{\mathrm{SP2}}$	$\tilde{\mathbb{P}}_{\mathcal{N}}$
0.001	1	1	1	1
0.25	1	1	0.97	0.97
0.5	1	0.95	0.90	0.85
0.75	1	0.83	0.72	0.59
1	0.94	0.77	0.71	0.51

4.4.3 profitability of the co-investment under uncertainty

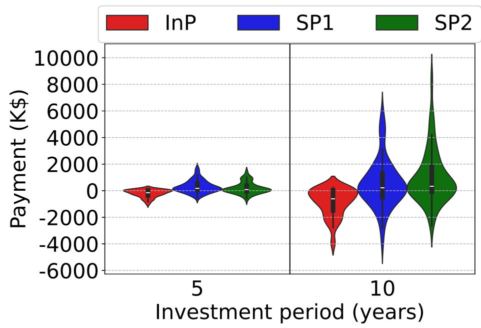
We initially attempted analytically to estimate a lower bound on the probability that the grand coalition is stable using Chebyshev's inequality, but the result was overly conservative and therefore not useful. Instead, we estimate the probability of profitable co-investment empirically, across multiple realizations ω, to assess its profitability according to Definition 18. We show in Table 4 how the probability of profitable co-investment for all players varies with different α values. The probability $\tilde{\mathbb{P}}$ represents the empirical Monte Carlo probability, where profitability is determined by directly evaluating the probability that revenue exceeds payment. This makes it the most economically meaningful and realistic measure of positive payoffs. We observe that the probability of profitable co-investment decreases as uncertainty increases, with the SPs becoming increasingly exposed to risk. At low uncertainty levels (e.g, $\alpha = 0.001$ and $\alpha = 0.25$), all players, including the coalition as a whole, are almost guaranteed to make profit. However, as uncertainty grows, the gap widens: SPs see their chances of profitability fall significantly, while the *InP* maintains a high probability throughout. This reflects the *InP*'s role as a veto player, since the infrastructure cannot be deployed without it, it faces less risk when joining the coalition.

4.4.4 Effect of the investment period on the outcome of the game

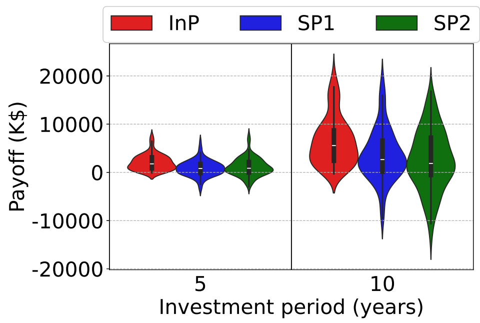
Fig. 6 presents the impact of increasing the investment period I (from 5 to 10 years) on revenue, payment and payoff redistributions for each player when $\alpha = 1$. As shown in Fig. 6a, the revenue redistribution per player increases with longer investment periods, reflecting the natural increase in cumulative earnings over time. However, the wider and more elongated violin plots indicate that uncertainty in revenue grows. This means that while financial gains increase, the risk of fluctuations in earnings also rises. The payment redistribution per player, observed in Fig. 6b, shows that longer investment periods require larger financial contributions from all players. We observe that SP1 and SP2 experience the highest increase in payment, whereas the *InP* sees a gradual rise. This difference is likely because the *InP* plays a veto role, whereas the SPs' contributions depend more on service demand and market fluctuations. Fig. 6c shows the payoff redistribution per player. It reveals that all players benefit from longer investment periods, with the *InP* achieving the highest payoff. SP1 and SP2 also see significant growth in their payoffs. Overall, our findings suggest that extending the investment period is generally beneficial, leading to higher cumulative revenues, greater payoffs, and increased payments. However, it also amplifies uncertainty, particularly for SPs.



(a) Revenue redistribution



(b) Payment redistribution



(c) Payoff redistribution

Fig. 6: Effect of investment period length on revenue, payment, and payoff redistribution, based on Shapley values with $\alpha = 1$ and 100 generated realizations

4.4.5 Impact of investment period on co-investment profitability

We show in Fig. 7 how the probability that the co-investment is profitable evolves with different investment periods I and uncertainty levels α. The results indicate that as the investment period increases, profitability improves significantly. For low uncertainty level ($\alpha \leq 0.25$), profitability remains guaranteed for both investment periods (5 and 10 years). However, as uncertainty increases, shorter investment periods become less profitable than longer ones. The increase in profitability with longer investment periods is primarily due to the accumulation of revenues over time, which allows the players to better absorb market fluctuations and operational costs. A longer time horizon enables co-investors to amortize their fixed investment costs over a greater number of operational years, reducing the impact of upfront capital expenditures on overall financial performance. Moreover, demand growth over time contributes to increased revenue, further improving investment viability. Importantly, uncertainty does not increase proportionally with the investment period, meaning that while revenues grow, the relative level of risk remains manageable.

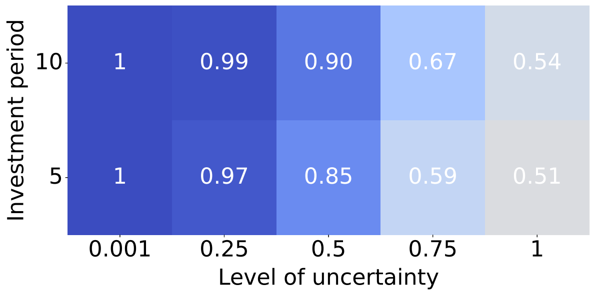
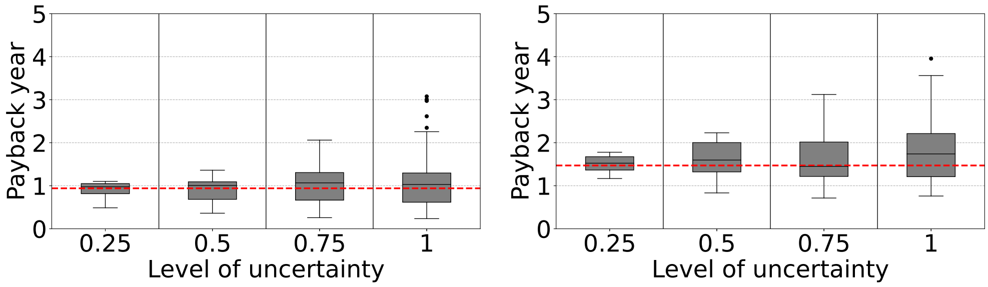


Fig. 7: Probability that co-investment is profitable for different investment periods across different levels of uncertainty α

4.4.6 Impact of investment period and uncertainty on payback year

At the start of the game, all players make an initial investment to purchase a package that includes computational resources and maintenance. A key financial question is when do players recover their investment? This is determined by the payback year, the point at which the total payoff surpasses the initial investment costs for a given investment period I. To track this, we proceed as follows: the realized coalition payoff $v_\omega(\mathcal{N})$ (8) is typically calculated over the full investment period I. We now compute this payoff over different periods $Z \leq I$, in order to identify when the payoff becomes positive. From this moment onward, all additional earnings represent pure profit. Fig. 8 presents this analysis using box plots. Each box plot summarizes the variability of outcomes: the box captures the IQR, representing the middle 50% of observations, while whiskers extend to the most extreme values within 1.5 times the IQR; points beyond the whiskers are outliers caused by extreme randomness. Fig. 8a and Fig. 8b illustrate the distribution of payback years under varying investment periods ($I = 5$ and $I = 10$ years) and uncertainty levels ($\alpha = 0.25, 0.5, 0.75, 1$). The red line in each figure represents the expected payback year, which serves as a reference point for understanding deviations due to stochastic effects. However, when uncertainty is introduced, the actual payback year can deviate from this expected value. For a 5-year investment period, we observe in Fig. 8a that at low α values (e.g., $\alpha = 0.25$), the payback year is highly concentrated around its expected value. This means that investment recovery time is predictable. However, as α value increases ($\alpha = 0.25$), the IQR widens and outliers become frequent, reflecting greater variability in recovery time. In these cases, the payback year may occur earlier or later than expected, depending on the realized stochastic fluctuations. Similarly, for a longer investment period ($I = 10$ years), we show in Fig. 8b that extending the investment period leads to an earlier payback year in relative terms. This makes longer investment horizons more attractive. This effect is most evident at low α values, where payoff accumulation follows the expected trend. Additionally, as α value increases, the payback year becomes broader, reinforcing the idea that greater uncertainty makes it more difficult to predict the exact payback year.



(a) $I = 5$ years

(b) $I = 10$ years

Fig. 8: Distribution of the payback year for different investment periods and levels of uncertainty α. The red line represents the expected payback year

4.4.7 Impact of the Hurst parameter on co-investment profitability

The Hurst parameter, denoted as H, is a key characteristic of fractional Brownian motion and is used in our model to represent the degree of long-range dependence in traffic load fluctuations (see Section 4.1.2). Fig. 9 illustrates its impact on the probability that the co-investment is profitable under different levels of uncertainty (α values). We show that, when $H = 0.7$, where traffic patterns exhibit strong long-term dependence and persistency, profitability remains high even as uncertainty increases. At $\alpha = 0.75$, the probability remains above 0.5, and even at $\alpha = 1$, there is still a chance of profitability. This suggests that when traffic fluctuations are positively correlated over time, payoff is enhanced, reducing the impact of short-term fluctuations that could negatively impact investment profitability. On the other hand, when $H = 0.5$, traffic follows an uncorrelated pattern with no long-range dependence. In this case, profitability follows a similar trend as in the case of positive correlation but declines more rapidly as α increases. However, the situation changes for $H = 0.3$, where traffic exhibits negative correlation, an increase in demand is often followed by a drop, and vice versa. We show that the probability of profitability drops to 0.5 for small α values. This suggests that in bursty and highly fluctuating traffic conditions, payoff is significantly compromised, making co-investment more risky.

5 Conclusion

We proposed a co-investment scheme between an *InP* and multiple SPs to support the deployment and maintenance of EC infrastructure under uncertain traffic loads

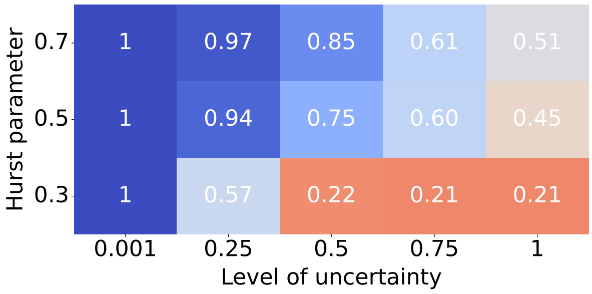


Fig. 9: Probability that co-investment is profitable under different Hurst parameter values and levels of uncertainty α

and hence revenues. We modeled the problem using stochastic coalitional game theory, introducing a framework to assess whether all players benefit from joining the grand coalition. In the case of uncorrelated traffic, we derive a theoretical lower bound on the probability that the grand coalition is stable. Our results show that this lower bound is high when the SPs' loads are of similar magnitude and the investment period is sufficiently long to smooth out uncertainty. For correlated traffic, modeled using fractional Brownian motion, we compute the empirical probability of profitability through simulations. The results indicate a high probability of profitability even under a high level of uncertainty. The *InP*, as a veto player, faces minimal risk due to its important role in infrastructure deployment. Overall, the proposed strategy enables fair, and efficient infrastructure sharing, even in the presence of significant demand uncertainty.

Our model can be extended to a dynamic framework. In a dynamic setting, players may join or leave the grand coalition over time, based on individual incentives or evolving traffic load. Such an extension would require modeling coalition formation as a multi-stage game, where payoff redistribution mechanisms ensure fairness and stability at each stage. This constitutes our future work perspective.

Acknowledgements. This work was supported by the ANR under the France 2030 program, grant NF-NAI: ANR-22-PEFT-0003.

Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

Appendix A Proof of Proposition 3

The *InP* is responsible for installing the infrastructure. Without the *InP*, there is no infrastructure, and thus no revenues can be collected, due to Assumption 2. Similarly, if no SP joins a coalition, no revenues are collected from end-users, according to Assumption 1, and thus such a coalition would be pointless.

Appendix B Proof of Proposition 11

If the *InP* does not belong to coalition $\mathcal{S}$, then no resources can be installed, as only the *InP* is responsible for infrastructure deployment (see Proposition 3). In this case, the total installed capacity is zero and no player receives any allocation, which directly leads to (19).

If the *InP* is included in $\mathcal{S}$, then the coalition can install resources $C^*_{\mathcal{S}}$ and apply the optimal dynamic allocation $\{h^{t*}_{i,\mathcal{S}}\}$ computed from the solution of the optimization problem (16). In this case, the installed capacity and the shares allocated to players in $\mathcal{S}$ are equal to those determined by the global optimization.

Appendix C Proof of Proposition 12

We prove that the optimization problem (16)-(18) is convex by showing that: (i) the objective function is concave (since the problem is a maximization), and (ii) the feasible set defined by the constraints is convex.

From (13), the objective function in (16) consists of two parts: the sum of utility functions and a cost function:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}\bar{u}_i^t(h_i^t) - \mathit{Cost}(I, C) \tag{C1}$$

Each utility function $\bar{u}_i^t(\cdot)$ is concave by assumption. Therefore, their sum is concave as well (Boyd and Vandenberghe 2004, Section 3.2). The cost function $\mathit{Cost}(I, C)$ is convex in C. To express it in terms of the decision variables $\{h_i^t\}$, we use the affine mapping:

$$C = \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} h_i^t \tag{C2}$$

Since the composition of a convex, non-decreasing function with an affine function preserves convexity (Roberts and Varberg 1974, Page 2), it follows that $\mathit{Cost}(I, C)$ is convex in $\{h_i^t\}$. Therefore, the objective function (16), which is a concave term (sum of utilities) minus a convex term (cost), is concave overall (thus, (i) holds). The constraints (17)-(18) are affine, so the feasible set is convex (Boyd and Vandenberghe 2004, Section 4.2.1) (thus (ii) holds). Thus, the entire optimization problem is convex. Therefore, the Karush-Kuhn-Tucker (KKT) conditions can be applied to determine the optimal solution (Boyd and Vandenberghe 2004, Section 5.5.3).

Appendix D Proof of Theorem 13

(Shapley 1971, (3)) shows that a game is convex if and only if its value function is supermodular (Shapley 1971, (6)). This means that, considering two coalitions $\mathcal{S}$ and $\mathcal{R}$ such that $\mathcal{R} \subseteq \mathcal{S} \subseteq \mathcal{N}$, the marginal contribution of any player when added to the bigger coalition $\mathcal{S}$ is at least equal to when it is added to the smaller coalition $\mathcal{R}$. This property is important, as it shows that the incentive in joining the coalition increases with the size of the coalition, which makes the grand coalition appealing for players.

More formally, let us define the marginal contribution of player j to coalition $\mathcal{S}$ as:

$$\Delta_j(\mathcal{S}) \triangleq \bar{v}(\mathcal{S} \cup j) - \bar{v}(\mathcal{S}) \tag{D3}$$

The game is supermodular (and thus convex) when

$$\Delta_j(\mathcal{S}) \geq \Delta_j(\mathcal{R}), \qquad \forall \mathcal{R} \subseteq \mathcal{S} \subseteq N, j \in \mathcal{N} \setminus \mathcal{S} \tag{D4}$$

Thanks to Proposition 11 and Assumption 2 and (2), (14) becomes:

$$\bar{v}(\mathcal{S}) = \begin{cases} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{S}} \bar{u}_i^t(h_{i,\mathcal{S}}^{t*}) - Cost(I, C_{\mathcal{S}}^*), & \text{if } InP \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases} \tag{D5}$$

We first show that the nominal value function $\bar{v}(\cdot)$ is monotonically non-decreasing, i.e., for all coalitions $\mathcal{R} \subseteq \mathcal{S} \subseteq \mathcal{N}$, it holds that:

$$\bar{v}(\mathcal{R}) \leq \bar{v}(\mathcal{S}) \tag{D6}$$

To prove (D6), we distinguish three cases based on the presence of the *InP* in the coalitions:

- Suppose $InP \notin \mathcal{R}$. Since $\mathcal{R} \subseteq \mathcal{S}$, it follows that $InP \notin \mathcal{S}$ as well. Then, via (D5), we have $\bar{v}(\mathcal{R}) = \bar{v}(\mathcal{S}) = 0$, so (D6) holds.
- Suppose $InP \notin \mathcal{R}$ but $InP \in \mathcal{S}$. Then, again via (D5), we have $\bar{v}(\mathcal{R}) = 0$, while $\bar{v}(\mathcal{S}) \geq 0$. The inequality in (D6) is trivially satisfied.
- Suppose $InP \in \mathcal{R} \subseteq \mathcal{S}$. In this case, define a suboptimal solution for coalition $\mathcal{S}$ by using the optimal capacity $C_{\mathcal{R}}^*$ and resource allocation $\{h_{i,\mathcal{R}}^{t*}\}_{t\in\mathcal{T}}$ computed for $\mathcal{R}$, via (16)–(18), for all players in $\mathcal{R}$, and assigning zero resources to all players in $\mathcal{S}\setminus\mathcal{R}$. Let the value of this suboptimal solution be denoted by $\bar{v}_{\text{sub}}(\mathcal{S})$. By construction, this gives:
$$\bar{v}_{\text{sub}}(\mathcal{S}) = \bar{v}(\mathcal{R}) \tag{D7}$$
Since $\bar{v}(\mathcal{S})$ is the optimal value over all feasible allocations for coalition $\mathcal{S}$, and the allocation used to define $\bar{v}_{\text{sub}}(\mathcal{S})$ is one such feasible solution, we have:
$$\bar{v}(\mathcal{S}) \geq \bar{v}_{\text{sub}}(\mathcal{S}) = \bar{v}(\mathcal{R}) \tag{D8}$$

Therefore, the value function $\bar{v}(\cdot)$ is monotonically non-decreasing ((D6) holds).

We show that (D4) holds, by distinguishing all possible cases. Let us take $j \notin \mathcal{S}$.

- Suppose $j \neq InP$, i.e., j is a SP. There can be the following two subcases:
  - Suppose $InP \notin \mathcal{R}$. Then (D5) implies that $\bar{v}(\mathcal{R}) = \bar{v}(\mathcal{R} \cup \{j\}) = \Delta_j(\mathcal{R}) = 0$. On the other hand, since $\bar{v}(\mathcal{S}) \leq \bar{v}(\mathcal{S} \cup \{j\})$ (by the monotonicity of $\bar{v}(\cdot)$ from (D6)), it follows that $\Delta_j(\mathcal{S}) = \bar{v}(\mathcal{S} \cup \{j\}) - \bar{v}(\mathcal{S}) \geq 0$. Thus, condition (D4) holds.

– Suppose $InP \in \mathcal{R}$. This implies that $InP \in \mathcal{S}$. Then, via applying Assumption 10, the value function $\bar{v}(\mathcal{R} \cup j)$ is obtained from $\bar{v}(\mathcal{R})$ by adjusting the total capacity $C^*_{\mathcal{R}}$ by the following amount (which can be positive or negative or zero):

$$\Delta C^*_{\mathcal{R},j} = C^*_{\mathcal{R}\cup\{j\}} - C^*_{\mathcal{R}} \tag{D9}$$

This change in capacity comes at an associated additional cost $\Delta\text{Cost}(I, \Delta C^*_{\mathcal{R},j})$, which can be positive or negative or zero. It is defined as:

$$\Delta\text{Cost}(I, \Delta C^*_{\mathcal{R},j}) = \text{Cost}(I, C^*_{\mathcal{R}\cup\{j\}}) - \text{Cost}(I, C^*_{\mathcal{R}}) \tag{D10}$$

Player j receives a resource share $h^{t*}_{j,\mathcal{R}\cup\{j\}}$, allowing it to generate utility $u^t_j(h^{t*}_{j,\mathcal{R}\cup\{j\}})$ at each time slot $t \in \mathcal{T}$.
For each existing player $i \in \mathcal{R}$, the utility at time slot t is affected by the new resource allocation, resulting in a utility change:

$$\Delta u^t_i = u^t_i(h^{t*}_{i,\mathcal{R}\cup\{j\}}) - u^t_i(h^{t*}_{i,\mathcal{R}}) \tag{D11}$$

Such changes in utility can be positive or negative or zero. Summing over all players in $\mathcal{R}$ and across all time slots, the total utility change for the coalition is defined as:

$$\Delta u_{\mathcal{R}} = \sum_{i\in\mathcal{R}} \sum_{t\in\mathcal{T}} \Delta u^t_i \tag{D12}$$

Thus, the resulting coalition value is:

$$\bar{v}(\mathcal{R} \cup j) = \bar{v}(\mathcal{R}) + \Delta\text{Cost}(I, \Delta C^*_{\mathcal{R}\cup j - \mathcal{R}}) + \Delta u_{\mathcal{R}} + \sum_{t\in\mathcal{T}} u^t_j(h^{*t}_{j,\mathcal{R}\cup j}) \tag{D13}$$

This implies:

$$\begin{aligned} \Delta_j(\mathcal{R}) &\triangleq \bar{v}(\mathcal{R} \cup \{j\}) - \bar{v}(\mathcal{R}) \\ &= \Delta\text{Cost}(I, \Delta C^*_{\mathcal{R},j}) + \Delta u_{\mathcal{R}} + \sum_{t\in\mathcal{T}} u^t_j(h^{t*}_{j,\mathcal{R}\cup\{j\}}) \end{aligned} \tag{D14}$$

We now consider the larger coalition $\mathcal{S} \cup \{j\}$ and fix a feasible (but not necessarily optimal) capacity $\tilde{C}_{\mathcal{S}\cup\{j\}}$ and allocation $\{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\}_{i\in\mathcal{S}\cup\{j\}, t\in\mathcal{T}}$ such that

$$\sum_{i\in\mathcal{S}\cup\{j\}} \tilde{h}^t_{i,\mathcal{S}\cup\{j\}} \le \tilde{C}_{\mathcal{S}\cup\{j\}}, \quad \forall t \in \mathcal{T} \tag{D15}$$

The improvement brought by player j with this possibly suboptimal solution is, obviously, a lower bound of the improvement obtained with the optimal solution. However, by appropriately constructing such a suboptimal solution of (16)–(18), we will show that it already brings an improvement larger than $\Delta_j(\mathcal{R})$. This will ensure, as we will show in (D24), that $\Delta_j(\mathcal{S}) \ge \Delta_j(\mathcal{R})$.

We define the change in total capacity due to the addition of player j as:

$$\Delta\tilde{C}_{\mathcal{S},j} = \tilde{C}_{\mathcal{S}\cup\{j\}} - C^*_{\mathcal{S}} \tag{D16}$$

We deliberately fix the value of $\tilde{C}_{\mathcal{S}\cup\{j\}}$ large enough so that:

$$\Delta\tilde{C}_{\mathcal{S},j} \geq \Delta C^*_{\mathcal{R},j} \tag{D17}$$

We assign player j the same resource share as in $\mathcal{R}\cup\{j\}$:

$$\tilde{h}^t_{j,\mathcal{S}\cup\{j\}} = h^{t*}_{j,\mathcal{R}\cup\{j\}}, \quad \forall t \in \mathcal{T} \tag{D18}$$

and we impose that the shares of existing players in $\mathcal{S}$ are increased:

$$\tilde{h}^t_{i,\mathcal{S}\cup\{j\}} \geq h^{t*}_{i,\mathcal{S}}, \quad \forall i \in \mathcal{S},\ t \in \mathcal{T} \tag{D19}$$

We construct this allocation (D19) in order to ensure that the total utility change $\Delta u_{\mathcal{S}}$ meets the following condition:

$$\Delta u_{\mathcal{S}} \geq \Delta u_{\mathcal{R}} \tag{D20}$$

where

$$\Delta u_{\mathcal{S}} = \sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}} \Big(u^t_i(\tilde{h}^t_{i,\mathcal{S}\cup\{j\}}) - u^t_i(h^{t*}_{i,\mathcal{S}}) \Big) \tag{D21}$$

We denote the nominal value of this feasible allocation by $\bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \big\{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\big\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big)$, consistent with (13), and, similarly to (D13), we compute

$$\begin{aligned} \bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \big\{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\big\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big) &= \bar{v}(\mathcal{S}) + \Delta\mathrm{Cost}(I, \Delta\tilde{C}_{\mathcal{S},j}) \\ &\quad + \Delta u_{\mathcal{S}} + \sum_{t\in\mathcal{T}} u^t_j(h^{t*}_{j,\mathcal{R}\cup\{j\}}) \end{aligned} \tag{D22}$$

Since $\bar{v}\Big(S\cup\{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \big\{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\big\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big)$ corresponds to a feasible solution that satisfies the capacity constraint, it holds that:

$$\bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \big\{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\big\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big) \leq \bar{v}(\mathcal{S}\cup\{j\}) \tag{D23}$$

where the right-hand side represents the optimal nominal value.

Finally, we obtain:

$$\begin{aligned}
\Delta_j(\mathcal{S}) &\triangleq \bar{v}(\mathcal{S} \cup \{j\}) - \bar{v}(\mathcal{S}) \\
&\overset{(D23)}{\geq} \bar{v}\Big(S \cup \{j\}, \tilde{C}_{\mathcal{S}\cup\{j\}}, \{\tilde{h}_{i,\mathcal{S}\cup\{j\}}\}_{i\in\mathcal{S}\cup\{j\},\, t\in\mathcal{T}}\Big) - \bar{v}(\mathcal{S}) \\
&\overset{(D22)}{=} \Delta\mathrm{Cost}(I, \Delta\tilde{C}_{\mathcal{S},j}) + \Delta u_{\mathcal{S}} + \sum_{t\in\mathcal{T}} u_j^t(h_{j,\mathcal{R}\cup\{j\}}^{t*}) \\
&\overset{(D17),(D20)}{\geq} \Delta\mathrm{Cost}(I, \Delta C_{\mathcal{R},j}^{*}) + \Delta u_{\mathcal{R}} + \sum_{t\in\mathcal{T}} u_j^t(h_{j,\mathcal{R}\cup\{j\}}^{t*}) \\
&\overset{(D14)}{=} \bar{v}(\mathcal{R} \cup \{j\}) - \bar{v}(\mathcal{R}) \triangleq \Delta_j(\mathcal{R})
\end{aligned} \tag{D24}$$

Therefore (D4) holds.

- Suppose that $j = InP$, then:

$$\begin{aligned}
\Delta_j(\mathcal{S}) = \bar{v}(\mathcal{S} \cup \{j\}) - \bar{v}(\mathcal{S}) &= \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{S}\cup\{j\}} \bar{u}_i^t(h_{i,\mathcal{S}\cup\{j\}}^{t*}) - Cost(I, C_{\mathcal{S}\cup\{j\}}^{*}) - 0 \\
&\geq \bar{v}(\mathcal{R} \cup \{j\}) - \bar{v}(\mathcal{R}) = \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{R}\cup\{j\}} \bar{u}_i^t(h_{i,\mathcal{R}\cup\{j\}}^{t*}) - Cost(I, C_{\mathcal{R}\cup\{j\}}^{*}) - 0 \\
&= \Delta_j(\mathcal{R}),
\end{aligned} \tag{D25}$$

  This inequality holds because $\bar{v}(\mathcal{S} \cup \{j\}) \geq \bar{v}(\mathcal{R} \cup \{j\})$, due to the monotonicity property established at the beginning of the proof (D6).

The inequality (D4) holds in all cases, and thus the Theorem is proved.

Appendix E Proof of Lemma 16

The stability value $\sigma(\mathcal{N}, \bar{v})$ is given by the following optimization problem [Page 7](Doan and Nguyen 2014): Given a parameter $\epsilon \geq 0$

$$\sigma(\mathcal{N}, \bar{v}) = \max_{\mathbf{x},\epsilon} -\epsilon \tag{E26}$$

subject to:

$$\sum_{i\in\mathcal{N}} x_i = \bar{v}(\mathcal{N}) \tag{E27}$$

$$\sum_{i\in\mathcal{S}} x_i - \bar{v}(\mathcal{S}) \geq -\epsilon \quad \forall \mathcal{S} \subsetneq \mathcal{N}, \mathcal{S} \neq \emptyset \tag{E28}$$

Here, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ represents a payoff vector, where x_i is the allocation assigned to player i.

Constraint (E28) can be rewritten as:

$$-\epsilon \leq \min_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(\sum_{i \in \mathcal{S}} x_i - \bar{v}(\mathcal{S}) \right) \tag{E29}$$

Since the objective is to maximize $-\epsilon$ ((E26)), the optimal value is obtained when the inequality in (E29) holds with equality, which leads to the following reformulation of (E26)-(E28):

$$\sigma(\mathcal{N}, \bar{v}) = \max_{\mathbf{x}} \min_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(\sum_{i \in \mathcal{S}} x_i - \bar{v}(\mathcal{S}) \right) \tag{E30}$$

subject to (E27), where we maximize $\sigma(\mathcal{N}, \bar{v})$ by choosing the best vector $\mathbf{x}$, ensuring that the minimum over all coalitions $\mathcal{S}$ is as large as possible.

Now, we define:

$$\hat{\sigma}(\mathcal{N}, \bar{v}) \triangleq \min_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(\sum_{i \in \mathcal{S}} \bar{x}_i - \bar{v}(\mathcal{S}) \right) \tag{E31}$$

where $\bar{x}_i$ is the expected Shapley value of player i (see (28)). It follows directly from the definition (E31) that:

$$\hat{\sigma}(\mathcal{N}, \bar{v}) \leq \sigma(\mathcal{N}, \bar{v}) \tag{E32}$$

We then define:

$$y_{\hat{\sigma}}(\mathcal{S}) \triangleq \begin{cases} \frac{\bar{v}(\mathcal{S}) + \hat{\sigma}(\mathcal{N}, \bar{v})}{\bar{v}(\mathcal{N})}, & \text{if } \mathcal{S} \subsetneq \mathcal{N} \\ 1, & \text{if } \mathcal{S} = \mathcal{N} \end{cases} \tag{E33}$$

and

$$\hat{\delta} \triangleq \min \left(\frac{\bar{v}(\mathcal{N})}{|\mathcal{N}|}, \frac{\hat{\sigma}(\mathcal{N}, \bar{v})}{\max\limits_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} (|\mathcal{S}| + (|\mathcal{N}| - 2|\mathcal{S}|) \cdot y_{\hat{\sigma}}(\mathcal{S}))} \right) \tag{E34}$$

Similarly, from (Doan and Nguyen 2014, Theorem 2), $y_{\sigma}(\mathcal{S})$ is defined as:

$$y_{\sigma}(\mathcal{S}) \triangleq \begin{cases} \frac{\bar{v}(\mathcal{S}) + \sigma(\mathcal{N}, \bar{v})}{\bar{v}(\mathcal{N})}, & \text{if } \mathcal{S} \subsetneq \mathcal{N} \\ 1, & \text{if } \mathcal{S} = \mathcal{N} \end{cases} \tag{E35}$$

and

$$\delta \triangleq \min \left(\frac{\bar{v}(\mathcal{N})}{|\mathcal{N}|}, \frac{\sigma(\mathcal{N}, \bar{v})}{\max\limits_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} (|\mathcal{S}| + (|\mathcal{N}| - 2|\mathcal{S}|) \cdot y_{\sigma}(\mathcal{S}))} \right) \tag{E36}$$

From the definition of δ in (E36), and by directly substituting the expression for $y_{\sigma}(\mathcal{S})$ from (E35), we define the function:

$$f(\sigma(\mathcal{N}, \bar{v})) = \frac{\sigma(\mathcal{N}, \bar{v})}{\max\limits_{\substack{\mathcal{S} \subsetneq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(|\mathcal{S}| + (|\mathcal{N}| - 2|\mathcal{S}|) \cdot \left(\frac{\bar{v}(\mathcal{S}) + \sigma(\mathcal{N}, \bar{v})}{\bar{v}(\mathcal{N})} \right) \right)} \tag{E37}$$

Dividing both the numerator and denominator in (E37) by $\sigma(\mathcal{N}, \bar{v}) > 0$, we obtain:

$$f(\sigma(\mathcal{N}, \bar{v})) = \frac{1}{\max\limits_{\substack{\mathcal{S} \subseteq \mathcal{N} \\ \mathcal{S} \neq \emptyset}} \left(\frac{|\mathcal{S}|}{\sigma(\mathcal{N}, \bar{v})} + \frac{(|\mathcal{N}| - 2|\mathcal{S}|)}{\bar{v}(\mathcal{N})} \cdot \left(\frac{\bar{v}(\mathcal{S})}{\sigma(\mathcal{N}, \bar{v})} + 1 \right) \right)} \tag{E38}$$

It is clear that the denominator in (E38) decreases as $\sigma(\mathcal{N}, \bar{v})$ increases, implying (E37) is increasing. Therefore, since $\hat{\sigma} \leq \sigma$ (E32), we conclude:

$$\hat{\delta} \leq \delta \tag{E39}$$

Thus, the lemma is proved.

Appendix F Proof of Proposition 19

If the grand coalition $\mathcal{N}$ is stable in the stochastic game $(\mathcal{N}, \mathcal{V})$, by Definition 7, this means that there exists an allocation policy $\vec{y} = (y_i)_{i \in \mathcal{N}}$ such that:

$$\sum_{i \in \mathcal{N}} y_i = 1 \quad , \qquad \sum_{i \in \mathcal{S}} v_\omega(\mathcal{N}) \cdot y_i \geq v_\omega(\mathcal{S}), \quad \forall \omega \in \Omega, \mathcal{S} \subseteq \mathcal{N} \tag{F40}$$

We define the individual payoff of player i in realization ω as:

$$x_{i,\omega} \triangleq y_i \cdot v_\omega(\mathcal{N}), \quad \forall i \in \mathcal{N} \tag{F41}$$

Since the robust core conditions apply to all coalitions, including singleton coalitions $\{i\} \subseteq \mathcal{N}$, it follows that:

$$y_i \geq \frac{v_\omega(\{i\})}{v_\omega(\mathcal{N})}, \quad \forall i \in \mathcal{N} \tag{F42}$$

In our case, for any player $i \in \mathcal{N}$, the singleton payoff $v_\omega(\{i\})$ is zero; this holds for both the *InP* and the SPs, as established in Proposition 3. Moreover, since $v_\omega(\mathcal{N}) > 0$ (by assumption), it follows from (F42) that $y_i \geq 0$ for all $i \in \mathcal{N}$. Therefore, we obtain:

$$x_{i,\omega} \triangleq y_i \cdot v_\omega(\mathcal{N}) \geq 0, \quad \forall \omega \in \Omega, i \in \mathcal{N} \tag{F43}$$

Thus, the co-investment is profitable for all players in every realization $\omega \in \Omega$ (see Definition 18), i.e.,

$$\mathbb{P}(\ x_{i,\omega} \geq 0, \forall i \in \mathcal{N}) = 1 \tag{F44}$$

which proves the Proposition.

Appendix G Proof of Proposition 20

Let $\Omega_{\text{stable}} \subseteq \Omega$ be the set of realizations in which the grand coalition is stable, i.e., where there exists a payoff allocation policy $\vec{y}$ satisfying the robust core conditions (Definition 7).

By Proposition 19 , if $\omega \in \Omega_{\text{stable}}$, then $x_{i,\omega} \geq 0$ (F41) for all $i \in \mathcal{N}$. Thus:

$$\Omega_{\text{stable}} \subseteq \Omega_{\text{profitable}} \coloneqq \{\omega \in \Omega \mid \forall i \in \mathcal{N},\ x_{i,\omega} \geq 0\} \tag{G45}$$

Taking the probability of both sets gives:

$$\mathbb{P}(\Omega_{\text{stable}}) \leq \mathbb{P}(\Omega_{\text{profitable}}) \tag{G46}$$

which proves the claim.

Appendix H Proof of Corollary 21

By definition, ν^{LB} is a lower bound on the probability that the grand coalition is stable, i.e,

$$\mathbb{P}\left(\text{Grand coalition is stable}\right) \geq \nu^{\text{LB}} \tag{H47}$$

From Proposition 20, we have:

$$\mathbb{P}\left(x_{i,\omega} \geq 0,\ \forall i \in \mathcal{N}\right) \geq \mathbb{P}\left(\text{Grand coalition is stable}\right) \tag{H48}$$

Combining (H47) and (H48) yields:

$$\mathbb{P}\left(x_{i,\omega} \geq 0,\ \forall i \in \mathcal{N}\right) \geq \nu^{\text{LB}} \tag{H49}$$

which proves the corollary.

Appendix I Proof of Lemma 22

We start from the definition of the reward for each player $i \in \mathcal{S}$ (43):

$$r_{i,\omega} = x_{i,\omega} + p_i \tag{I50}$$

Summing over all players $i \in \mathcal{S}$, we get:

$$\sum_{i\in\mathcal{S}} r_{i,\omega} = \sum_{i\in\mathcal{S}} x_{i,\omega} + \sum_{i\in\mathcal{S}} p_i \tag{I51}$$

By definition of the Shapley value (Section 3.7), the total payoff distributed to players satisfies:

$$\sum_{i\in\mathcal{S}} x_{i,\omega} = v_\omega(\mathcal{S}) \overset{(8),(6)}{=} \sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}} u^t_{i,\omega}(h^{t*}_{i,\mathcal{S}}) - \text{Cost}(I, C^*_{\mathcal{S}}), \quad \forall\omega\in\Omega \tag{I52}$$

From (1), we also have:

$$\sum_{i\in\mathcal{S}} p_i = \text{Cost}(I, C^*_{\mathcal{S}}) \tag{I53}$$

Substituting (I53) into (I51):

$$\sum_{i\in\mathcal{S}} r_{i,\omega} = \left(\sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}} u^t_{i,\omega}(h^{t*}_{i,\mathcal{S}}) - \text{Cost}(I, C^*_{\mathcal{S}})\right) + \text{Cost}(I, C^*_{\mathcal{S}}) \tag{I54}$$

Therefore:

$$\sum_{i\in\mathcal{S}} r_{i,\omega} = \sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}} u^t_{i,\omega}(h^{t*}_{i,\mathcal{S}}) \tag{I55}$$

This completes the proof.

Appendix J Proof of Lemma 23

The expected amount of requests (48) received by SP i at time slot t is given by:

$$\bar{l}^t_i \triangleq \bar{\lambda}^t_i \cdot \Delta \tag{J56}$$

where

$$\bar{\lambda}^t_i \overset{(49)}{\triangleq} \mathbb{E}_\omega[\lambda^t_{i,\omega}] \overset{(53)}{=} d^t_i \cdot \left[(1-\alpha)\cdot \mathbb{E}_\omega\left[S^t_{i,\omega}\right] + \alpha \cdot \mathbb{E}_\omega\left[S^t_{i,\omega}\right]\right] = d^t_i \cdot \mathbb{E}_\omega\left[S^t_{i,\omega}\right] \tag{J57}$$

We now compute the expected value of the stochastic process $S^t_{i,\omega}$, which is defined in (55) as $S^t_{i,\omega} \triangleq \max(0, f^t_{i,\omega})$.

The term $f^t_{i,\omega}$ is a fractional Brownian motion, as described in (Section 4.1.2). The mean of $f^t_{i,\omega}$ is zero, and its standard deviation is given by $\sigma^t_i = t^H$ (Dieker 2004, Page 6).

The expected value of $S^t_{i,\omega}$ is:

$$\mathbb{E}[S^t_{i,\omega}] = \int_0^\infty x \cdot pdf_{f^t_{i,\omega}}(x)\, dx \tag{J58}$$

where x is the possible values that $f^t_{i,\omega}$ can take. Since $S^t_{i,\omega} \triangleq \max(0, f^t_{i,\omega})$, only the non-negative values of $f^t_{i,\omega}$ contribute to the expectation. Therefore, the integral is taken over $x \geq 0$, which accounts for all non-negative realizations of $f^t_{i,\omega}$.

Since $f^t_{i,\omega}$ is Gaussian at each time slot t (Enriquez 2004, Section 1), the probability density function $pdf_{f^t_{i,\omega}}$ is given by (Bertsekas and Tsitsiklis 2008, Section 3.3):

$$pdf_{f^t_{i,\omega}}(x) = \frac{1}{\sqrt{2\pi}\sigma^t_i} \exp\left(-\frac{x^2}{2(\sigma^t_i)^2}\right) \tag{J59}$$

Substituting (J59) into (J58):

$$\begin{aligned}
\mathbb{E}[S_{i,\omega}^t] &= \int_0^\infty x \, \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \exp\!\Big(-\tfrac{x^2}{2\,(\sigma_i^t)^2}\Big)\, dx \\
&= \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \int_0^\infty x\ \exp\!\Big(-\tfrac{x^2}{2\,(\sigma_i^t)^2}\Big)\, dx \\
&\text{Let } \alpha = \frac{1}{2\,(\sigma_i^t)^2} \quad \text{Then:} \\
&= \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \int_0^\infty x\ \exp\!\big(-\alpha\, x^2\big)\, dx \\
&\text{Here, make the change of variable } t = x^2 \\
&\therefore \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \int_0^\infty x\, e^{-\alpha x^2}\, dx \;=\; \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \times \frac{1}{2\,\alpha} \\
&= \frac{1}{\sqrt{2\pi}\,\sigma_i^t} \times \frac{1}{2 \times \frac{1}{2\,(\sigma_i^t)^2}} \;=\; \frac{\sigma_i^t}{\sqrt{2\pi}}
\end{aligned} \tag{J60}$$

Substituting (J60) into (J57), and subsequently (J57) into (J56), yields the result stated in the Lemma.

Appendix K Proof of Proposition 24

K.1 Case 1:

Starting from the optimization problem (16)–(18), and substituting each term with its expression from (13),(12),(46), and (45), we obtain the following optimization problem:

$$\begin{aligned}
\max_{\{\vec{h}^t\}_{t\in\mathcal{T}},\, C\geq 0} \quad & \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{S}} \beta_i \bar{l}_i^t \left(1 - e^{-\xi h_i^t}\right) - (d + d' \cdot I) \cdot C \\
\text{s.t.} \quad & \sum_{i\in\mathcal{S}} h_i^t \leq C, \quad \forall t \in \mathcal{T} \\
& h_i^t \geq 0, \quad \forall i \in \mathcal{S},\, t \in \mathcal{T} \\
& C > 0
\end{aligned} \tag{K61}$$

According to Assumption 1, the InP does not interact directly with end users; therefore, its load is $\bar{l}_{InP}^t = 0$, and no resources are allocated to the InP, i.e., $h_{InP,\mathcal{S}}^{t*} = 0, \quad \forall t \in \mathcal{T}$ (3).

Let $\mathcal{S}' \subseteq \mathcal{S} \setminus \{InP\}$ denote the subset of SPs with strictly positive load, i.e., $\bar{l}_i^t > 0$ for all $t \in \mathcal{T}$. The capacity and allocation depend only on this active subset.

We define the Lagrangian function as follows:

$$\mathcal{L} = \sum_{t \in \mathcal{T}} \left[\sum_{i \in \mathcal{S}'} \beta_i \bar{l}_i^t \left(1 - e^{-\xi h_i^t}\right) - \lambda_t \left(\sum_{i \in \mathcal{S}'} h_i^t - C \right) \right] - (d + d' \cdot I) \cdot C \tag{K62}$$

Here, λ_t is the Lagrange multiplier associated with the capacity constraint at each time slot $t \in \mathcal{T}$.

We take the partial derivatives of (K62) with respect to h_i^t and C.

- Partial derivative with respect to h_i^t:

$$\frac{\partial \mathcal{L}}{\partial h_i^t} = \beta_i \bar{l}_i^t \cdot \xi e^{-\xi h_i^t} - \lambda_t \tag{K63}$$

Setting the derivative in (K63) to zero for optimality:

$$\beta_i \bar{l}_i^t \cdot \xi e^{-\xi h_i^t} = \lambda_t \tag{K64}$$

Solving for h_i^t:

$$h_{i,\mathcal{S}}^{t*} = \frac{1}{\xi} \ln \left(\frac{\xi \beta_i \bar{l}_i^t}{\lambda_t} \right) \tag{K65}$$

- Partial derivative with respect to C:

$$\frac{\partial \mathcal{L}}{\partial C} = \sum_{t \in \mathcal{T}} \lambda_t - (d + d' \cdot I) \tag{K66}$$

Setting the derivative in (K66) to zero for optimality:

$$\sum_{t \in \mathcal{T}} \lambda_t = d + d' \cdot I \tag{K67}$$

We substitute the optimal $h_{i,\mathcal{S}}^{t*}$ from (K65) into the constraint in (K61):

$$\sum_{i \in \mathcal{S}'} h_{i,\mathcal{S}}^{t*} = \sum_{i \in \mathcal{S}'} \frac{1}{\xi} \ln \left(\frac{\xi \beta_i \bar{l}_i^t}{\lambda_t} \right) = C, \quad \forall t \in \mathcal{T} \tag{K68}$$

This gives:

$$\frac{1}{\xi} \left(\sum_{i \in \mathcal{S}'} \ln(\xi \beta_i \bar{l}_i^t) - |\mathcal{S}'| \ln \lambda_t \right) = C \tag{K69}$$

where $|\mathcal{S}'|$ is the number of SPs in the coalition $\mathcal{S}$.

Then, solving (K69) for λ_t:

$$\ln \lambda_t = \frac{1}{|\mathcal{S}'|} \sum_{i \in \mathcal{S}'} \ln(\xi \beta_i \bar{l}_i^t) - \frac{\xi C}{|\mathcal{S}'|} \quad \Rightarrow \quad \lambda_t = \left(\prod_{i \in \mathcal{S}'} \xi \beta_i \bar{l}_i^t \right)^{\frac{1}{|\mathcal{S}'|}} \cdot e^{-\frac{\xi C}{|\mathcal{S}'|}} \tag{K70}$$

Substituting (K70) into (K67):

$$\sum_{t\in\mathcal{T}}\left(\prod_{i\in\mathcal{S}'}\xi\beta_i\bar{l}_i^t\right)^{\frac{1}{|\mathcal{S}'|}}\cdot e^{-\frac{\xi C}{|\mathcal{S}'|}}=d+d'\cdot I \tag{K71}$$

Solving (K71) for C:

$$e^{-\frac{\xi C}{|\mathcal{S}'|}}=\frac{d+d'\cdot I}{\sum_{t\in\mathcal{T}}\left(\prod_{i\in\mathcal{S}'}\xi\beta_i\bar{l}_i^t\right)^{\frac{1}{|\mathcal{S}'|}}}\quad\Rightarrow\quad C_{\mathcal{S}}^*=\frac{|\mathcal{S}'|}{\xi}\ln\left(\frac{\sum_{t\in\mathcal{T}}\left(\prod_{i\in\mathcal{S}'}\xi\beta_i\bar{l}_i^t\right)^{\frac{1}{|\mathcal{S}'|}}}{d+d'\cdot I}\right) \tag{K72}$$

Substitute (K70) and (K72) into (K65):

$$h_{i,\mathcal{S}}^{t*}=\frac{1}{\xi}\ln\left(\frac{\xi\beta_i\bar{l}_i^t}{\lambda_t}\right)=\frac{1}{\xi}\ln\left(\frac{\xi\beta_i\bar{l}_i^t}{\left(\prod_{j\in\mathcal{S}'}\xi\beta_j\bar{l}_j^t\right)^{\frac{1}{|\mathcal{S}'|}}\cdot e^{-\frac{\xi C_{\mathcal{S}}^*}{|\mathcal{S}'|}}}\right) \tag{K73}$$

$$=\frac{C_{\mathcal{S}}^*}{|\mathcal{S}'|}+\frac{1}{\xi}\ln\left(\frac{\beta_i\bar{l}_i^t}{\left(\prod_{j\in\mathcal{S}'}\beta_j\bar{l}_j^t\right)^{\frac{1}{|\mathcal{S}'|}}}\right) \tag{K74}$$

K.1.1 Special case: Only one SP has non-zero load

If $|\mathcal{S}'|=1$, let the single active SP be denoted by i. In this case, the optimization problem simplifies, since only one SP contributes to utility and receives allocation. The optimization problem becomes:

$$\begin{aligned}\max_{\{h_i^t\}_{t\in\mathcal{T}},C\geq 0}\quad & \sum_{t\in\mathcal{T}}\beta_i\bar{l}_i^t\left(1-e^{-\xi h_i^t}\right)-(d+d'\cdot I)\cdot C\\ \text{s.t.}\quad & h_i^t\leq C,\quad \forall t\in\mathcal{T}\\ & h_i^t\geq 0,\quad \forall t\in\mathcal{T}\\ & C>0\end{aligned} \tag{K75}$$

The Lagrangian becomes:

$$\mathcal{L}=\sum_{t\in\mathcal{T}}\left[\beta_i\bar{l}_i^t\left(1-e^{-\xi h_i^t}\right)-\lambda_t\left(h_i^t-C\right)\right]-(d+d'\cdot I)\cdot C \tag{K76}$$

We take the partial derivatives of (K76) with respect to h_i^t and C.

- Partial derivative with respect to h_i^t:

$$\frac{\partial\mathcal{L}}{\partial h_i^t}=\beta_i\bar{l}_i^t\cdot\xi e^{-\xi h_i^t}-\lambda_t \tag{K77}$$

Setting the derivative in (K77) to zero for optimality:

$$\beta_i \bar{l}_i^t \cdot \xi e^{-\xi h_i^t} = \lambda_t \tag{K78}$$

Solving for h_i^t:

$$h_{i,\mathcal{S}}^{t*} = \frac{1}{\xi} \ln\left(\frac{\xi \beta_i \bar{l}_i^t}{\lambda_t}\right) \tag{K79}$$

- Partial derivative with respect to C:

$$\frac{\partial \mathcal{L}}{\partial C} = \sum_{t \in \mathcal{T}} \lambda_t - (d + d' \cdot I) \tag{K80}$$

Setting the derivative in (K80) to zero for optimality:

$$\sum_{t \in \mathcal{T}} \lambda_t = d + d' \cdot I \tag{K81}$$

We now use the fact that $|\mathcal{S}'| = 1$, so the constraint in (K75) implies:

$$h_{i,\mathcal{S}}^{t*} = C, \quad \forall t \in \mathcal{T} \tag{K82}$$

Substitute (K82) into (K78):

$$\lambda_t = \xi \beta_i \bar{l}_i^t \cdot e^{-\xi C} \tag{K83}$$

Substituting (K83) into (K81):

$$\sum_{t \in \mathcal{T}} \xi \beta_i \bar{l}_i^t \cdot e^{-\xi C} = d + d' \cdot I \tag{K84}$$

Solving (K84) for C:

$$e^{-\xi C} = \frac{d + d' \cdot I}{\sum_{t \in \mathcal{T}} \xi \beta_i \bar{l}_i^t} \quad \Rightarrow \quad C_{\mathcal{S}}^* = \frac{1}{\xi} \ln\left(\frac{\sum_{t \in \mathcal{T}} \xi \beta_i \bar{l}_i^t}{d + d' \cdot I}\right) \tag{K85}$$

Using (K82) and (K85), the final allocation is:

$$h_{i,\mathcal{S}}^{t*} = C_{\mathcal{S}}^* = \frac{1}{\xi} \ln\left(\frac{\sum_{t \in \mathcal{T}} \xi \beta_i \bar{l}_i^t}{d + d' \cdot I}\right) \tag{K86}$$

K.2 Case 2:

From the utility function (46) and (12), if $\bar{l}_i^t = 0$, then $\bar{u}_i^t(h_i^t) = 0$ for any h_i^t. Since no utility is generated, the value of the objective function (16) is zero regardless of the installed capacity. Therefore, the optimizer will choose $C_{\mathcal{S}}^* = 0$ and allocates no resources ($h_{i,\mathcal{S}}^{t*} = 0, \forall t \in \mathcal{T}$) to avoid unnecessary costs.

Appendix L Proof of Proposition 25

- Case 1:
  The allocation given by (58) can be rewritten as:

$$h_{i,\mathcal{S}}^{t*} = \frac{C_{\mathcal{S}}^*}{|\mathcal{S}'|} + \frac{1}{\xi}\left(\ln(\beta_i \cdot \bar{l}_i^t) - \frac{1}{|\mathcal{S}'|}\sum_{j\in\mathcal{S}'}\ln(\beta_j \cdot \bar{l}_j^t)\right) \tag{L87}$$

  Taking the partial derivatives of (L87):

$$\frac{\partial h_{i,\mathcal{S}}^{t*}}{\partial \beta_i} = \frac{1}{\xi}\left(\frac{1}{\beta_i} - \frac{1}{|\mathcal{S}'|}\cdot\frac{1}{\beta_i}\right) = \frac{1}{\xi}\left(1 - \frac{1}{|\mathcal{S}'|}\right)\cdot\frac{1}{\beta_i} > 0 \quad \text{for } \beta_i > 0 \tag{L88}$$

$$\frac{\partial h_{i,\mathcal{S}}^{t*}}{\partial \bar{l}_i^t} = \frac{1}{\xi}\left(\frac{1}{\bar{l}_i^t} - \frac{1}{|\mathcal{S}'|}\cdot\frac{1}{\bar{l}_i^t}\right) = \frac{1}{\xi}\left(1 - \frac{1}{|\mathcal{S}'|}\right)\cdot\frac{1}{\bar{l}_i^t} > 0 \quad \text{for } \bar{l}_i^t > 0 \tag{L89}$$

  So $h_{i,\mathcal{S}}^{t*}$ is strictly increasing in both β_i and $\bar{l}_i^t$.
  For the special case discussed in K.1.1, we take the partial derivatives for (K86):

$$\frac{\partial h_{i,\mathcal{S}}^{t*}}{\partial \beta_i} = \frac{1}{\xi}\cdot\frac{\sum_{t\in\mathcal{T}}\bar{l}_i^t}{\sum_{t\in\mathcal{T}}\beta_i\cdot\bar{l}_i^t} = \frac{1}{\xi\cdot\beta_i} > 0 \tag{L90}$$

$$\frac{\partial h_{i,\mathcal{S}}^{t*}}{\partial \bar{l}_i^t} = \frac{1}{\xi}\cdot\frac{\xi\cdot\beta_i}{\sum_{t'\in\mathcal{T}}\xi\cdot\beta_i\cdot\bar{l}_i^{t'}} = \frac{1}{\xi}\cdot\frac{1}{\sum_{t'\in\mathcal{T}}\bar{l}_i^{t'}} > 0 \tag{L91}$$

  So again, $h_{i,\mathcal{S}}^{t*}$ increases strictly with β_i and $\bar{l}_i^t$.
- Case 3:
  If $\bar{l}_i^t = 0$, then $h_{i,\mathcal{S}}^{t*} = 0$ (59), regardless of β_i, so the proposition holds trivially.

References

Akgül, T., Baykut, S., Erol-Kantarci, M., Oktug, S.F.: Periodicity-based anomalies in self-similar network traffic flow measurements. IEEE Transactions on Instrumentation and Measurement **60**(4), 1358–1366 (2010)

Akkarajitsakul, K., Hossain, E., Niyato, D.: Coalition-based cooperative packet delivery under uncertainty: A dynamic bayesian coalitional game. IEEE Transactions on Mobile Computing **12**(2), 371–385 (2011)

Amazon Web Services: AWS Lambda Pricing. https://aws.amazon.com/fr/lambda/pricing/. Accessed: 2024-07-18 (2024)

Azevedo, A., Paxson, D.: Rivalry and uncertainty in complementary investments with dynamic market sharing. Annals of Operations Research **271**, 319–355 (2018)

Badia, B., Berry, R.A., Wei, E.: Investment in ev charging spots for parking. IEEE Transactions on Network Science and Engineering **7**(2), 650–661 (2019)

Bourreau, M., Cambini, C., Hoernig, S., Vogelsang, I.: Co-investment, uncertainty, and opportunism: ex-ante and ex-post remedies. Information Economics and Policy **56**, 100913 (2021)

Borrero, D.V., Hinojosa, M.A., Mármol, A.M.: Stable solutions for multiple scenario cost allocation games with partial information. Annals of Operations Research **245**(1), 209–226 (2016)

Bhaskaran, S.R., Krishnan, V.: Effort, revenue, and cost sharing mechanisms for collaborative new product development. Management Science **55**(7), 1152–1169 (2009)

Birge, J.R., Louveaux, F.: Introduction to Stochastic Programming. Springer, New York (2011)

Boucheron, S., Lugosi, G., Massart, P.: Concentration Inequalities - A Nonasymptotic Theory of Independence. Oxford University Press, Oxford (2013). https://doi.org/10.1093/ACPROF:OSO/9780199535255.001.0001 . https://doi.org/10.1093/acprof:oso/9780199535255.001.0001

Bertsekas, D., Tsitsiklis, J.N.: Introduction to Probability vol. 1. Athena Scientific, Belmont, MA (2008)

Boyd, S., Vandenberghe, L.: Convex Optimization. Cambridge university press, Cambridge (2004)

Bi, J., Zhang, X., Yuan, H., Zhang, J., Zhou, M.: A hybrid prediction method for realistic network traffic with temporal convolutional network and lstm. IEEE Transactions on Automation Science and Engineering **19**(3), 1869–1879 (2021)

Charnes, A., Granot, D.: Coalitional and chance-constrained solutions to n-person games. i: the prior satisficing nucleolus. SIAM Journal on Applied Mathematics **31**(2), 358–367 (1976)

Chen, X.: Building mobile edge infrastructure in an uncertain environment. In: 2023 IEEE Virtual Conference on Communications (VCC), pp. 127–132 (2023). IEEE

Chau, S.L., Muandet, K., Sejdinovic, D.: Explaining the uncertain: Stochastic shapley values for gaussian process models. arXiv preprint arXiv:2305.15167 (2023)

Cong, R., Zhao, Z., Zhang, L., Min, G.: Coopedge: cost-effective server deployment for cooperative multi-access edge computing. In: 2022 19th Annual IEEE International Conference on Sensing, Communication, and Networking (SECON), pp. 208–216 (2022). IEEE

Dieker, T.: Simulation of fractional brownian motion. PhD thesis, Masters Thesis, Department of Mathematical Sciences, University of Twente ... (2004)

Doan, X.V., Nguyen, T.-D.: Robust stable payoff distribution in stochastic cooperative games. arXiv preprint arXiv:1403.5906 (2014)

Enriquez, N.: A simple construction of the fractional brownian motion. Stochastic Processes and their Applications **109**(2), 203–223 (2004)

Fredriksson, H., Dahl, M., Holmgren, J.: Optimal placement of charging stations for electric vehicles in large-scale transportation networks. Procedia computer science **160**, 77–84 (2019)

Feller, W.: An Introduction to Probability Theory and Its Applications, Volume 2 vol. 2. John Wiley & Sons, New York (1991)

Faisal, A., Kamruzzaman, M., Yigitcanlar, T., Currie, G.: Understanding autonomous vehicles. Journal of transport and land use **12**(1), 45–72 (2019)

Gendreau, M.: Network design with applications to transportation and logistics (2023)

Ghahramani, S.: Fundamentals of Probability. CRC Press, Boca Raton, FL (2024)

He, M., Censi, A., Frazzoli, E., Zardini, G.: Co-investment with payoff sharing benefit operators and users in network design. arXiv preprint arXiv:2409.19409 (2024)

Inderst, R., Peitz, M.: Investment under uncertainty and regulation of new access networks. Information Economics and Policy **26**, 28–41 (2014) https://doi.org/10.1016/j.infoecopol.2013.11.001

Jeanjean, F.: Co-investment in the sharing of telecommunications infrastructures. Theoretical Economics Letters **12**(5), 1297–1314 (2022)

Khan, W.Z., Ahmed, E., Hakak, S., Yaqoob, I., Ahmed, A.: Edge computing: A survey. Future Generation Computer Systems **97**, 219–235 (2019)

Ketelaars, M., Borm, P., Kort, P.: Dynamic stability of cooperative investment under uncertainty. Technical report, Tilburg University, School of Economics and Management (2023)

Karagiannis, T., Molle, M., Faloutsos, M.: Long-range dependence ten years of internet traffic modeling. IEEE internet computing **8**(5), 57–64 (2004)

Kiedanski, D., Orda, A., Kofman, D.: Discrete and stochastic coalitional storage games. In: Proceedings of the Eleventh ACM International Conference on Future Energy Systems, pp. 351–362 (2020)

Kauppinen, L., Siddiqui, A.S., Salo, A.: Investing in time-to-build projects with uncertain revenues and costs: A real options approach. IEEE Transactions on Engineering Management **65**(3), 448–459 (2018)

Lee, I.W., Fapojuwo, A.O.: Stochastic processes for computer network traffic modeling. Computer communications **29**(1), 1–23 (2005)

López, Á.L., Manganelli, A.-G., Martín-Rodríguez, M.: Co-investment deterrence. Economics Letters **211**, 110263 (2022)

Li, L., Shi, D., Hou, R., Li, X., Wang, J., Li, H., Pan, M.: Data-driven optimization for cooperative edge service provisioning with demand uncertainty. IEEE Internet of Things Journal **8**(6), 4317–4328 (2020)

Lin, X., Zhou, J., Zhang, L., Zeng, Y.: Revenue sharing for resource reallocation among project activity contractors. Annals of Operations Research **301**, 121–141 (2021)

Mohan, N., Corneo, L., Zavodovski, A., Bayhan, S., Wong, W., Kangasharju, J.: Pruning edge research with latency shears. In: Proceedings of the 19th ACM Workshop on Hot Topics in Networks, pp. 182–189 (2020)

Ma, Y., Gao, Z., Li, W., Jiang, N., Guo, L., *et al.*: The shapley value for stochastic cooperative game. Modern Applied Science **2**(4), 1–76 (2008)

Microsoft Azure: Pricing - Azure Stack Edge. https://azure.microsoft.com/en-us/pricing/details/azure-stack/edge/. Accessed: 2025-01-09

Mishura, Y.: Stochastic Calculus for Fractional Brownian Motion and Related Processes. Springer, Berlin (2008)

Maschler, M., Peleg, B., Shapley, L.S.: Geometric properties of the kernel, nucleolus, and related solution concepts. Mathematics of operations research **4**(4), 303–338 (1979)

Mandelbrot, B.B., Van Ness, J.W.: Fractional brownian motions, fractional noises and applications. SIAM review **10**(4), 422–437 (1968)

Nguyen, D.T., Nguyen, H.T., Trieu, N., Bhargava, V.K.: Two-stage robust edge service placement and sizing under demand uncertainty. IEEE Internet of Things Journal **9**(2), 1560–1574 (2021)

Ong, S.K., Nee, A.Y.C.: Virtual and Augmented Reality Applications in Manufacturing. Springer, London (2013)

Osborne, M.J., Rubinstein, A.: A Course in Game Theory. The MIT Press, Cambridge, USA (1994). electronic edition

Patanè, R., Araldo, A., Chahed, T., Kiedanski, D., Kofman, D.: Coalitional game-theoretical approach to coinvestment with application to edge computing. In: 2023 IEEE 20th Consumer Communications & Networking Conference (CCNC), pp. 517–522 (2023). IEEE

Peng, Z., Cui, D., Zuo, J., Li, Q., Xu, B., Lin, W.: Random task scheduling scheme based on reinforcement learning in cloud computing. Cluster computing **18**, 1595–1607 (2015)

Pantazis, G., Franci, B., Grammatico, S., Margellos, K.: Distributionally robust stability of payoff allocations in stochastic coalitional games. In: 2023 62nd IEEE Conference on Decision and Control (CDC), pp. 7617–7622 (2023). IEEE

Parekh, D., Poddar, N., Rajpurkar, A., Chahal, M., Kumar, N., Joshi, G.P., Cho, W.: A review on autonomous vehicles: Progress, methods and challenges. Electronics **11**(14), 2162 (2022)

Raja, A.A., Grammatico, S.: Payoff distribution in robust coalitional games on time-varying networks. IEEE Transactions on Control of Network Systems **9**(1), 511–520 (2021)

Roberts, A.W., Varberg, D.E.: Convex Functions: Convex Functions vol. 57. Academic Press, New York (1974)

Reif, R., Walch, D.: Augmented & virtual reality applications in the field of logistics. The Visual Computer **24**, 987–994 (2008)

Sakr, A., Araldo, A., Chahed, T., Patanè, R., Kofman, D.: Co-investment under uncertainty: Coalitional game formulation and application to edge computing. In: IEEE International Conference on Communications (ICC) (2025)

Satyanarayanan, M.: The emergence of edge computing. Computer **50**(1), 30–39 (2017)

Schmeidler, D.: The nucleolus of a characteristic function game. SIAM Journal on applied mathematics **17**(6), 1163–1170 (1969)

Shi, W., Cao, J., Zhang, Q., Li, Y., Xu, L.: Edge computing: Vision and challenges. IEEE internet of things journal **3**(5), 637–646 (2016)

Santoyo-González, A., Cervelló-Pastor, C.: Edge nodes infrastructure placement parameters for 5g networks. In: 2018 IEEE Conference on Standards for Communications and Networking (CSCN), pp. 1–6 (2018). IEEE

Shapley, L.S.: Cores of convex games. International journal of game theory **1**, 11–26 (1971)

Saad, W., Han, Z., Debbah, M., Hjorungnes, A., Basar, T.: Coalitional game theory for communication networks. Ieee signal processing magazine **26**(5), 77–97 (2009)

Simon, H.A.: Models of Bounded Rationality: Empirically Grounded Economic Reason vol. 3. MIT press, Cambridge, MA (1997)

Shi, H., Li, Y.: Discovering periodic patterns for large scale mobile traffic data: Method and applications. IEEE Transactions on Mobile Computing **17**(10), 2266–2278 (2018)

Union, E.: How to master europe's digital infrastructure needs. Technical report, European Commission (2024). Accessed: January 19, 2025. https://digital-strategy.ec.europa.eu/en/library/white-paper-how-master-europes-digital-infrastructure-n

Van der Laan, G., Van den Brink, R.: A banzhaf share function for cooperative games in coalition structure. Theory and Decision **53**, 61–86 (2002)

Vela, A.P., Vía, A., Morales, F., Ruiz, M., Velasco, L.: Traffic generation for telecom cloud-based simulation. In: 2016 18th International Conference on Transparent Optical Networks (ICTON), pp. 1–4 (2016). IEEE

Xu, J., Palanisamy, B., Ludwig, H., Wang, Q.: Zenith: Utility-aware resource allocation for edge computing. In: 2017 IEEE International Conference on Edge Computing (EDGE), pp. 47–54 (2017). IEEE

Xu, D., Zhou, A., Zhang, X., Wang, G., Liu, X., An, C., Shi, Y., Liu, L., Ma, H.: Understanding operational 5g: A first measurement study on its coverage, performance and energy consumption. In: Proceedings of the Annual Conference of the ACM Special Interest Group on Data Communication on the Applications, Technologies, Architectures, and Protocols for Computer Communication, pp. 479–494 (2020)

Zhao, Z., Min, G., Gao, W., Wu, Y., Duan, H., Ni, Q.: Deploying edge computing nodes for large-scale iot: A diversity aware approach. IEEE Internet of Things Journal **5**(5), 3606–3614 (2018)