Exponential Suppression of the Unruh Effect and Geometric Enhancement in a Fermionic Cavity QED Setup

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Abstract

The Unruh effect—the prediction that an accelerated observer perceives the vacuum as a thermal bath—remains one of the most profound yet experimentally unverified consequences of quantum field theory. This work analyzes a model for the decay of an excited state within a uniformly accelerated cavity to address the historical null results and to identify an alternative, non-thermal signature. In our framework, a massless Dirac field confined to a cavity is coupled to an external massive Dirac field of mass M. Our analysis reveals that for fundamental fermions (such as the electron), the condition $Mc^2 \gg \hbar a/c$ is satisfied at all achievable accelerations, placing the system in a regime of exponential suppression, $\Gamma_{\mathrm{acc}}/\Gamma_{\mathrm{in}} \sim \exp\bigl(-2Mc^2/(\hbar a/c)\bigr)$ (with Γ_{in} the inertial decay rate). This suppression holds universally across all cavity sizes and experimental designs, providing a potential explanation within this model for the non-observation of Unruh effects. Furthermore, for intermediate-sized cavities ($al \sim c^2$) with light external fields ($Mc^2 \ll \hbar a/c$), the model predicts a geometric enhancement of the decay rate, scaling as $\Gamma_{\mathrm{acc}}/\Gamma_{\mathrm{in}} \sim \frac{al/c^2}{\ln(1+al/c^2)}$, which arises from kinematic constraints rather than thermal stimulation. This enhancement, reaching up to 26% for realistic parameters ($a \sim 10^{20}$ m/s^2, $l \sim 500$ μm), is presented as a measurable signature accessible through quantum simulation platforms. Our results propose a unified framework that explains past experimental challenges and suggests a viable path forward for detecting non-inertial quantum effects.

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1 Introduction

Quantum field theory in curved spacetime reveals a profound connection between gravity, acceleration, and thermodynamics. Among its most striking predictions is the Unruh effect [1–3], wherein an observer undergoing constant proper acceleration a perceives the Minkowski vacuum as a thermal bath at temperature $T_{\text{Unruh}} = a/2\pi$. This effect is fundamental to our understanding of quantum phenomena in non-inertial frames and is intricately linked to the thermodynamics of black holes via the Hawking effect [4]. Despite its conceptual importance and nearly half a century of theoretical study, the Unruh effect has eluded direct experimental confirmation. Its verification remains a central challenge in modern physics, requiring the detection of exceedingly faint thermal signatures under extreme accelerations and meeting stringent criteria to distinguish the effect from alternative explanations [5, 6].

Traditional approaches to detecting the Unruh effect have focused on the response of individual accelerated particle detectors [7, 8]. However, the required accelerations for a measurable effect are often prohibitively high. Recent proposals have sought to amplify the effect through collective behavior or structured environments [9, 10]. In particular, cavity quantum electrodynamics (QED) settings offer a promising alternative [11, 12]. By confining a quantum field within a cavity, one can enhance its interaction with matter and exploit the discrete spectrum of cavity modes to design sensitive probes of vacuum thermality.

In this work, we analyze a uniformly accelerated cavity QED setup where a massless Dirac field confined inside the cavity couples to an external massive Dirac field. Our analysis reveals a fundamental limitation: for any fundamental particle with mass M, the condition $Mc^2 \gg \hbar a/c$ is satisfied for all achievable accelerations, placing systems in an exponentially suppressed regime that explains historical null results. However, we identify an alternative measurable signature—a geometric enhancement arising from non-inertial acceleration effects rather than thermal Unruh stimulation.

The fermionic nature of both fields is crucial, as their statistics and the specific boundary conditions at the cavity walls fundamentally alter the structure of the mode functions and their overlap. Accelerated cavities confine fields via boundary conditions, which dictate allowed field modes and their spatial overlap. We consider two critical classes: MIT bag boundary conditions, which enforce strict confinement [13], and more general probabilistic conditions parametrized by an angle $\theta \in [0, \pi/2)$ and a parameter $s \in (0, 1)$, which allow for continuous tuning of the field's behavior at the boundaries [14, 15].

The central quantity we study is the decay rate of an excited state of the confined massless Dirac field inside the cavity—a direct probe of the vacuum fluctuations experienced by a co-accelerated observer. Prior work on quantum decay in accelerated systems has often focused either on scalar fields within cavities [16] or on fermionic fields without confinement [11, 17], largely neglecting the combined effects of cavity geometry and fermionic statistics. Studies that do address fermions in cavities [14] have focused on quantum information protocols like entanglement harvesting [15], rather than on the specific decay dynamics central to this work.

Our analysis fills this gap and reveals a clear, robust, and experimentally viable signature. We demonstrate that the decay rate exhibits distinct behavior across parameter space governed by two dimensionless parameters: the cavity size relative to acceleration scale (al) and the field mass relative to acceleration (M/a). For light external fields ($M/a \ll 1$), the decay rate transitions from inertial-like behavior ($\Gamma_{\text{acc}} \sim \Gamma_{\text{in}}$) for small cavities ($al \ll 1$), to a geometric enhancement ($\Gamma_{\text{acc}} \sim \Gamma_{\text{in}} \frac{al}{\ln(1+al)}$) for intermediate cavities ($al \sim 1$), reaching up to 26% enhancement for realistic parameters. For heavy external fields ($M/a \gg 1$), the decay rate is exponentially suppressed ($\Gamma_{\text{acc}}/\Gamma_{\text{in}} \sim e^{-2M/a}$) regardless of cavity size. This suppression provides a potential explanation within our model for why direct observation of Unruh effects with fundamental matter has remained elusive.

This comprehensive study across parameter regimes provides a unified picture of acceleration effects in confined systems and identifies quantum simulation as the necessary path forward for experimental detection. The geometric enhancement identified here provides a measurable signature accessible through platforms like superconducting circuits and optomechanical systems.

The paper is organized as follows: In Sec. 2, we introduce a minimal model for the fermionic QED cavity setup. In Secs. 2.1 -2.2, we review the quantization of the external massive field in Minkowski spacetime and the confined massless field within an inertial cavity, satisfying both MIT bag and general probabilistic boundary conditions. In Sec. 2.3, we derive the inertial decay rate of an excited cavity mode. In Sec. 3, we review the Rindler quantization of the external massive Dirac field and the confined massless Dirac field within a uniformly accelerated cavity. In Sec. 4, we derive the decay rate for a uniformly accelerated cavity. In Sec. 5, we present results for the small $al \ll 1$, intermediate $al \sim 1$, and large $al \gg 1$ cavity regimes, discussing the emergence of geometric enhancement and the conditions for exponential suppression. Finally, in Sec. 6, we conclude with a discussion of experimental prospects and future directions. Appendices provide full details of the asymptotic analysis of the mode overlap integrals used in the main text.

For simplicity, we use natural units in which $\hbar = c = \kappa_B = 1$ and work in (1+1)-dimensional Minkowski spacetime with the sign convention $(+-)$. The limitations of the $(1+1)$-dimensional treatment are discussed in the conclusion.

2 Model: Fermionic Fields in an Accelerated Cavity

To probe quantum decay dynamics in accelerated cavities, we introduce a minimal model that captures the essential physics of the Unruh effect. This model couples a confined, massless Dirac field (the "clock") to an external, unconfined massive Dirac field, with interactions mediated by a Hermitian Hamiltonian. We first define the spacetime geometry, then quantize both fields, and finally impose boundary conditions to characterize the cavity system.

Consider a (1+1)-dimensional cavity of proper length l in the lab frame at $t = 0$. For an inertial observer, the lab frame is described by pseudo-Cartesian coordinates (t, x) with the Minkowski metric:

$$ds^2 = dt^2 - dx^2, \tag{2.1}$$

where the cavity walls are fixed at $x = x_-$ (left wall) and $x = x_+ = x_- + l$ (right wall). These walls are rigidly attached to the timelike Killing vector ∂_t, ensuring staticity in the inertial frame.

The ideal clock is modeled as a massless Dirac spinor field $\chi(t, x)$ confined to the cavity ($x \in [x_-, x_+]$), coupled to an external massive Dirac spinor field $\psi(t, x)$ (unconfined, $x \in \mathbb{R}$) with rest mass $M > 0$. The cavity walls are transparent to ψ, allowing coupling between the two fields.

The interaction Hamiltonian, responsible for mediating energy exchange between χ and ψ, is:

$$H_{\text{int}} = g \int_{x_-}^{x_+} dx \left(\bar{\chi}\psi + \bar{\psi}\chi \right), \tag{2.2}$$

where g is a small coupling constant with dimension $[g] = [M]$ in natural units ($\hbar = c = 1$), ensuring H_{int} has the correct dimension of energy ($[M]$) in $(1+1)$ dimensions. Here, we have $\bar{\chi} = \chi^\dagger\beta$, $\bar{\psi} = \psi^\dagger\beta$ (with β defined in Section 2.1). The Hermiticity of H_{int} is ensured by the sum of the two terms, which guarantees real eigenvalues for physical observables.

2.1 Quantization of the Massive Dirac Field

In the lab frame ($\eta^{\mu\nu} = \text{diag}(1, -1)$), the dynamics of the external massive spinor $\psi(t, x)$ are governed by the Dirac equation:

$$i\partial_t\psi(t, x) = H_D\psi(t, x), \quad H_D = -i\alpha\partial_x + M\beta, \tag{2.3}$$

where H_D is the Dirac Hamiltonian, and α, β are 2×2 Hermitian matrices satisfying the Clifford algebra:

$$\{\alpha, \beta\} = 0, \quad \alpha^2 = \beta^2 = \mathbb{I}_2. \tag{2.4}$$

These matrices are constructed from the gamma matrices γ^μ ($\mu = 0, 1$) via $\alpha = \gamma^0\gamma^1$ and $\beta = \gamma^0$, with $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{I}_2$. Hermiticity of H_D requires $\alpha^\dagger = \alpha$ and $\beta^\dagger = \beta$, consistent with $\gamma^{0\dagger} = \gamma^0$ and $\gamma^{1\dagger} = -\gamma^1$.

Following Ref. [14], we use a real two-component orthonormal spinor basis $\{U_+, U_-\}$

satisfying:

$$U_+^\dagger U_- = U_-^\dagger U_+ = 0, \quad U_+^\dagger U_+ = U_-^\dagger U_- = 1, \tag{2.5a}$$

$$\alpha U_\pm = \pm U_\pm, \quad \beta U_\pm = U_\mp. \tag{2.5b}$$

A perfectly valid representation would be:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \beta = \gamma^0, \tag{2.6}$$

$$U_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad U_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.7}$$

Our treatment will be independent of the representation and will be based on the properties defined in (2.5).

Complete sets of positive- and negative-frequency plane-wave solutions to Equation (2.3) are [18]:

$$\psi_{\pm,k}(t,x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega_k}} u_\pm(k) e^{\mp i\omega_k t \pm ikx}, \tag{2.8a}$$

$$u_\pm(k) = \frac{\pm k_\mu \gamma^\mu + M}{\sqrt{2M(\omega_k + M)}} u_\pm(0), \quad u_\pm(0) = \sqrt{M}(U_+ \pm U_-), \tag{2.8b}$$

where $\omega_k = \sqrt{M^2 + k^2}$, and $u_\pm(k)$ are normalized such that:

$$u_\pm^\dagger(k) u_\pm(k) = 2\omega_k, \quad u_+^\dagger(k) u_-(k) = 0. \tag{2.9}$$

Notably, 1+1D Dirac spinors lack spin degeneracy (unlike 3+1D), as the eigenstates $u_\pm(0)$ of $\gamma^0 \equiv \beta$ are non-degenerate. The mode functions are normalised such that:

$$(\psi_{s,k}, \psi_{s',k'}) := \int_{-\infty}^{\infty} \psi_{s,k}^\dagger(t,x) \psi_{s',k'}(t,x) dx = \delta_{s,s'} \delta(k - k'), \quad s, s' = \pm. \tag{2.10}$$

The quantized massive field is expanded in terms of creation/annihilation operators:

$$\psi(t,x) = \int_{-\infty}^{\infty} dk \left(b_{+,k} \psi_{+,k}(t,x) + d_{-,k}^\dagger \psi_{-,k}(t,x) \right), \tag{2.11}$$

where $b_{+,k}$ and $d_{-,k}^\dagger$ are the fermionic annihilation and creation operators for particles and antiparticles, respectively, which satisfy the canonical anticommutation relations:

$$\{b_{+,k}, b_{+,k'}^\dagger\} = \{d_{-,k}, d_{-,k'}^\dagger\} = \delta(k - k'), \quad \text{all other anticommutators} = 0. \tag{2.12}$$

The Fock space is built on the Minkowski vacuum $|0_M\rangle$, defined by $b_{+,k}|0_M\rangle_\psi = d_{-,k}|0_M\rangle_\psi = 0$ for all $k \in \mathbb{R}$.

2.2 Quantization of the Massless Dirac Field in the Cavity

The confined massless spinor $\chi(t,x)$ (clock field) satisfies boundary conditions at the cavity walls ($x = x_\pm$). We first consider MIT bag boundary conditions [13], which enforce strict confinement:

$$(i \mp \beta\alpha)\chi(t, x_\mp) = 0. \tag{2.13}$$

These are analogous to Dirichlet conditions for scalar fields [16] and ensure no probability flux leaks through the walls.

For generality, we also consider probabilistic boundary conditions, which relax strict vanishing by requiring the probability current to vanish at each wall [14, 15]:

$$\chi^\dagger_{(1)}\alpha\chi_{(2)}\Big|_{x=x_\pm} = 0, \tag{2.14}$$

where $\chi_{(1)}$ and $\chi_{(2)}$ are eigenfunctions of the massless Dirac Hamiltonian ($M = 0$) on either side of the boundary. These conditions introduce parameters $\theta \in [0, 2\pi)$ (phase shift of reflection) and $s \in [0, 1)$ (classification of self-adjoint extensions). For $s \in (0, 1)$, orthonormal eigenfunctions are [14]:

$$\chi_n(t,x) = \frac{1}{\sqrt{2l}}\left(U_+ e^{i\tilde{\omega}_n(x-x_-)} + e^{i\theta}U_- e^{-i\tilde{\omega}_n(x-x_-)}\right)e^{-i\tilde{\omega}_n t}, \tag{2.15}$$

where $\tilde{\omega}_n = \frac{(n+s)\pi}{l}$ ($n \in \mathbb{Z}$). Setting $s = 1/2$ and $\theta = \pi/2$ yields the MIT bag orthonormal mode functions. For $s = 0$, $\tilde{\omega}_0 = 0$, but the zero mode is excluded from quantization (as in Ref. [14]), and we take $s \to 0^+$.

The quantized massless field in the cavity is a superposition of these eigenmodes:

$$\chi(t,x) = \sum_{n\geq 0} b_n\chi_n(t,x) + \sum_{n<0} d^\dagger_n\chi_n(t,x), \tag{2.16}$$

where the non-vanishing anticommutation relations of the fermionic creation and annihilation operators are assumed to satisfy the usual algebra:

$$\{b_n, b^\dagger_m\} = \{d_n, d^\dagger_m\} = \delta_{nm}. \tag{2.17}$$

The cavity Fock space is built on the vacuum $|0\rangle_\chi$, defined by $b_n|0\rangle_\chi = d_n|0\rangle_\chi = 0$ for all $n \in \mathbb{Z}$.

This model provides a framework for probing the Unruh effect through the decay of an excited state within the cavity. The fermionic nature of both the confined massless field χ and the external massive field ψ introduces essential structure through their statistics and the boundary conditions imposed on χ at the cavity walls. The interaction, mediated by the Hermitian Hamiltonian H_{int}, enables energy exchange between the two fields. The central quantity of study is the decay rate of an excited state of χ, which provides a direct signature of the modified vacuum fluctuations—a signature that manifests as a thermal Unruh bath when the entire cavity is accelerated.

2.3 Inertial cavity decay rate

We compute the decay rate of a cavity at rest in Minkowski spacetime following the timelike killing vector ∂_t.

We take the cavity field to be initially in the one-particle state of the mode χ_n given in (2.15) and the external field to be in the Minkowski vacuum. Working to first-order perturbation theory in the coupling strength g, the probability amplitude for the decay rate into the cavity vacuum state $|0\rangle_\chi$ and an arbitrary final state $|\beta\rangle_\psi$ of the external field is given by:

$$\mathcal{A}_\downarrow = -i \int_0^t dt' \, {}_\psi\langle\beta| \, {}_\chi\langle 0|H_{int}|1\rangle_\chi |0_M\rangle_\psi \, . \tag{2.18}$$

Note the interaction Hamiltonian (2.2) is linear in both the cavity and external fields. Therefore, this first-order process can only create one particle in the external field and annihilate one in the cavity. This restricts the non-zero matrix elements to final states $|\beta\rangle_\psi$ that are single-particle states. Furthermore, energy conservation must be satisfied for that one-particle state.

Crucially, the interaction term $\bar{\psi}\chi$ annihilate the cavity particle and create a particle in the external field. The conjugate term, $\bar{\chi}\psi$ would create an antiparticle in the cavity and annihilate an antiparticle in the external field. Since the initial state of the external field is the vacuum (which contains no antiparticles), this second process is forbidden. Thus, only the $\bar{\psi}\chi$ term contributes to the decay amplitude.

Collecting (2.2), (2.11), (2.16) and summing over all unobservable final states of the external field (i.e., over all such one-particle states), the total decay probability reads:

$$\mathcal{P}_\downarrow = \sum_\psi |\mathcal{A}_\downarrow|^2 = g^2 \int_{-\infty}^{\infty} dk \, |\Gamma_{k1}|^2 \, , \tag{2.19}$$

where Γ_{kn} is defined as:

$$\Gamma_{kn} := \int_0^t dt' \int_{x_-}^{x_-+l} dx \, \bar{\psi}_{+,k}(t',x)\chi_n(t',x) \, , \tag{2.20}$$

where $\bar{\psi}_{+,k} := \psi^\dagger_{+,k}\beta$.

Collecting (2.8) and (2.15), we obtain the expression:

$$
\begin{aligned}
\mathcal{P}_{\downarrow} = \frac{g^2}{4\pi l} \int_{-\infty}^{\infty} dk \frac{\sin^2\left((\omega_k - \tilde{\omega}_1)\,\frac{t}{2}\right)}{(\omega_k - \tilde{\omega}_1)^2} \frac{1}{\omega_k(\omega_k + M)} \Bigg\{ \kappa_+ \Bigg[\frac{4\sin^2\left((k - \tilde{\omega}_1)\,\frac{l}{2}\right)}{(k - \tilde{\omega}_1)^2} \\
+ \frac{4\sin^2\left((k + \tilde{\omega}_1)\,\frac{l}{2}\right)}{(k + \tilde{\omega}_1)^2} \Bigg] + 2\kappa \Bigg[\frac{4\sin^2\left((k - \tilde{\omega}_1)\,\frac{l}{2}\right)}{(k - \tilde{\omega}_1)^2} - \frac{4\sin^2\left((k + \tilde{\omega}_1)\,\frac{l}{2}\right)}{(k + \tilde{\omega}_1)^2} \Bigg] \\
+ \kappa_- \frac{4\left[\cos(\pi s) + \cos(kl)\right]\cos(\theta - \pi s)}{(k^2 - \tilde{\omega}_1^2)} \Bigg\},
\end{aligned}
\tag{2.21a}
$$

where $\tilde{\omega}_1 = \frac{1+s}{l}\pi$, with s and θ as defined in Section 2.2 and where $\kappa_\pm$ and κ are defined as:

$$
\kappa_\pm := (\omega_k + M)^2 \pm k^2\,, \quad \kappa := k(\omega_k + M)\,. \tag{2.21b}
$$

The coupling coefficients $\kappa_\pm$ and κ quantify the strength of the system's coupling to the decay channel. Notably, the integrand is even in k.

2.4 Short-Time and Long-Time Limits

For $t \ll 1/|\omega_k - \tilde{\omega}_1|$, the temporal factor in the decay probability integral simplifies to $t^2/4$. In this regime, the integral (2.21) reduces to a short-time behavior dominated by quadratic time dependence:

$$
\mathcal{P}_{\downarrow}^{\text{short}} \propto g^2 t^2 \cdot C(s, \theta), \tag{2.22}
$$

where $C(s, \theta)$ depends on the boundary conditions imposed on the cavity walls.

For long times, the k-integral is dominated by the resonance region $\omega_k \approx \tilde{\omega}_1$ (i.e., $k \approx \pm\tilde{k} = \pm\sqrt{\tilde{\omega}_1^2 - M^2}$), where the integrand peaks. The width of the resonance is $\Delta k \sim l$, so the integral samples a volume $\sim l$ in k-space. Using the asymptotic identity for large t:

$$
\frac{\sin^2\left((\omega - \tilde{\omega}_1)t/2\right)}{(\omega - \tilde{\omega}_1)^2} \sim \frac{\pi t}{2}\delta(\omega - \tilde{\omega}_1) \quad \text{as} \quad t \to \infty, \tag{2.23}
$$

the leading-order decay probability becomes:

$$\mathcal{P}_{\downarrow}^{\text{long}} = \frac{g^2 t}{lA\left(\tilde{\omega}_1 + M\right)}\Bigg\{ \left(\tilde{\omega}_1 + M + A\right)^2 \frac{\sin^2\left(\left(A - \tilde{\omega}_1\right)\frac{l}{2}\right)}{\left(A - \tilde{\omega}_1\right)^2} \tag{2.24a}$$

$$+ \left(\tilde{\omega}_1 + M - A\right)^2 \frac{\sin^2\left(\left(A + \tilde{\omega}_1\right)\frac{l}{2}\right)}{\left(A + \tilde{\omega}_1\right)^2}$$

$$- 2\frac{\left(\tilde{\omega}_1 + M\right)}{M}\left[\cos(\pi s) + \cos(lA)\right]\cos(\theta - \pi s)\Bigg\},$$

where

$$A := \sqrt{\tilde{\omega}_1^2 - M^2}\ (M < \tilde{\omega}_1), \quad \tilde{\omega}_1 := \frac{(1+s)\pi}{l}. \tag{2.24b}$$

In the weak-mass limit, characterized by either ($\tilde{\omega}_1 \gg M$ or $Ml \ll 1$), the leading-order long-time behavior simplifies to:

$$\mathcal{P}_{\downarrow}^{\text{long}} \approx g^2 t l \left[1 + \frac{Ml\sin(\pi s)\cos(\theta - \pi s)}{(1+s)^2\pi^2}\right]. \tag{2.25}$$

Thus, the long-time decay rate is:

$$\Gamma_{\text{in}} = g^2 l \left[1 + \frac{Ml\sin(\pi s)\cos(\theta - \pi s)}{(1+s)^2\pi^2}\right]. \tag{2.26}$$

Note that these conditions ($\tilde{\omega}_1 \gg M$ or $Ml \ll 1$) are physically equivalent: since $\tilde{\omega}_1 = (1+s)\pi/l$, the requirement $\tilde{\omega}_1 \gg M$ implies $Ml \ll (1+s)\pi$, which is equivalent to $Ml \ll 1$ up to an order-unity factor since $s \in (0,1)$. Both express, the regime where the Compton wavelength $1/M$ of the external field is much larger than the cavity size l, rendering the external field effectively massless relative to the cavity's geometric scale. Consequently, the leading behavior $\Gamma_{\text{in}} \sim g^2 l$ is identical in both cases, with the mass M contributing only subleading corrections. The bracket remaining positive for $s \in (0,1)$ and $\theta \in [0, 2\pi]$ under the condition $(1+s)\pi \gg Ml$. For the MIT bag boundary condition ($s = 1/2$, $\theta = \pi/2$), the decay rate simplifies to:

$$\Gamma_{\text{in}}^{\text{MIT}} = g^2 l \left(1 + \frac{4Ml}{9\pi^2}\right). \tag{2.27}$$

2.4.1 Limit: $Ml \to 0$

For $Ml \to 0$, the leading-order long-time decay rate simplifies to:

$$\Gamma_{\text{in}} = g^2 l\,. \tag{2.28}$$

Crucially, Lorek et al.'s bosonic coupling λ (dimension $[M^2]$) [16] yields a mass-independent decay rate $\Gamma_{\text{in}}^{\text{boson}} \sim \lambda^2 l^3$, while ours is $\Gamma_{\text{in}}^{\text{fermion}} \sim g^2 l$. Both results share this universal feature—mass independence of short-cavity decay—despite different coupling dimensions. The dimensions cancel in the decay rate, ensuring physical consistency and highlighting the universality of stationary short-cavity dynamics for inertial observers.

3 Quantization of Dirac Field in Rindler Spacetime

To analyze the decay rate of a uniformly accelerated cavity, we first study the quantization of the massive Dirac field in the full Rindler spacetime and then specialize to the massless field confined within the accelerated cavity. The quantization of Dirac fields in Rindler wedges of Minkowski spacetime ($1+1$D here) has been extensively studied in the literature [19–26], with a focus on separating modes in the right (R: $x > |t|$) and left (L: $x < -|t|$) wedges.

3.1 Quantization of the Massive Dirac Field in Rindler Spacetime

We adopt Rindler coordinates (η, ρ) to parametrize the right Rindler wedge R of Minkowski spacetime. These are defined in terms of Cartesian coordinates (t, x) via:

$$t = \rho \sinh(a\eta)\,, \qquad x = \rho \cosh(a\eta)\,, \tag{3.1}$$

where $0 < \rho < \infty$ and $-\infty < \eta < \infty$. Here, a is a constant with dimensions of inverse length, serving as a reference scale for proper acceleration. In these coordinates, the Minkowski metric takes the form $ds^2 = (a\rho)^2 d\eta^2 - d\rho^2$. An observer at fixed spatial coordinate $\rho = \rho_0$ follows a static worldline and experiences a proper acceleration given by $A = 1/\rho_0$. In particular, for the fiducial observer at $\rho_0 = 1/a$, the proper acceleration is $A = a$, and the Rindler time η coincides with the observer's proper time τ, since $d\tau = (a\rho_0)d\eta = d\eta$. More generally, for an observer at arbitrary ρ, the proper time $d\tau$ relates to Rindler time via $d\tau = (a\rho)d\eta$.

In this geometry, the Dirac equation for the external massive spinor field can be written as [11, 27, 30]:

$$ia^{-1}\partial_\eta \psi(\eta, \rho) = \left(-i\alpha\left(\rho\partial_\rho + \frac{1}{2}\right) + M\rho\beta\right)\psi(\eta, \rho)\,, \tag{3.2}$$

where α, β are spinor matrices defined in Section 2.1, and M is still the mass of the external Dirac field.

Working in the two-real components spinor basis (2.5), the normalized positive and negative frequency modes in the right Rindler wedge (R: $x > |t|$) are given by[1]:

$$\Psi^R_{+,\Omega}(\eta,\rho) = \sqrt{\frac{M\cosh(\pi\Omega/a)}{a\pi^2}}\left(K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)U_+ + iK_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho)U_-\right)e^{-i\Omega\eta}, \tag{3.3a}$$

$$\Psi^R_{-,\Omega}(\eta,\rho) = \sqrt{\frac{M\cosh(\pi\Omega/a)}{a\pi^2}}\left(K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)U_- + iK_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho)U_+\right)e^{i\Omega\eta}, \tag{3.3b}$$

where $\Omega > 0$, $K_\nu(z)$ denotes the modified Bessel function of the second kind (decaying exponentially for large z, ensuring normalizability), and the subscript R labels right-Rindler wedge modes. We emphasize that negative frequency modes are obtained via charge conjugation of positive frequency modes. For a spinor ψ, charge conjugation is defined as $\psi^c = C\psi^*$, where C is a matrix satisfying $C^\dagger C = \mathbb{I}$ and $C^\dagger\gamma^\mu C = -\gamma^{\mu *}$ [28]. In this $(1+1)$-dimensional framework, valid choices include $C = \gamma^1 = \beta\alpha$ or $C = -i\gamma^1 = -i\beta\alpha$ (with α, β as defined in Section 2.1); we use the latter to avoid spurious phase factors in the mode expansions. Modes are normalised such that:

$$(\Psi^R_{s,\Omega}, \Psi^R_{s',\Omega'}) := \int_0^\infty d\rho \Psi^{R\dagger}_{s,\Omega}(\eta,\rho)\Psi^R_{s',\Omega'}(\eta,\rho) = \delta_{s,s'}\delta(\Omega-\Omega')\,, \quad s,s' = \pm\,, \tag{3.4}$$

where we used the properties of the spinors basis defined in (2.5), the identity $K_\nu(z) = K_{-\nu}(z) =$ and the following important Modified Bessel identity derived in Appendix A of [29]:

$$\int_0^\infty d\rho K_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho)K_{i\frac{\Omega'}{a}-\frac{1}{2}}(M\rho) + K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)K_{i\frac{\Omega'}{a}+\frac{1}{2}}(M\rho) = \frac{a}{M}\frac{\pi^2\delta(\Omega-\Omega')}{\cosh(\pi\Omega/a)}\,. \tag{3.5}$$

In the left Rindler wedge (L: $x < -|t|$), we extend the right-Rindler modes by setting $\Psi^R_{\pm,\Omega} = 0$ in L. To describe physics in L, we adopt Rindler coordinates $(t,x) = (-\rho\sinh(a\eta), -\rho\cosh(a\eta))$ (with $\rho > 0$, $\eta \in \mathbb{R}$), preserving consistency with R's labeling.

The left-Rindler modes, nonvanishing in L and vanishing in R, are defined with a sign flip in the frequency dependence (due to ∂_η being past-directed in L):

$$\Psi^L_{+,\Omega}(\eta,\rho) = \sqrt{\frac{M\cosh(\pi\Omega/a)}{a\pi^2}}\left(K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)U_+ + iK_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho)U_-\right)e^{i\Omega\eta}, \tag{3.6a}$$

$$\Psi^L_{-,\Omega}(\eta,\rho) = \sqrt{\frac{M\cosh(\pi\Omega/a)}{a\pi^2}}\left(K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)U_- + iK_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho)U_+\right)e^{-i\Omega\eta}. \tag{3.6b}$$

These left modes are positive/negative frequency with respect to $-\partial_\eta$ (the Rindler time coordinate in L) and satisfy $(\Psi^L_{s,\Omega}, \Psi^L_{s',\Omega'}) = \delta_{s,s'}\delta(\Omega-\Omega')$.

The combined set of left and right Rindler modes form a complete set in $R \cup L$.

[1]These mode functions are equivalent to those found in Refs. [17, 30] under a specific spinor basis transformation; see end of subsection for details.

Thus, the quantized massive Dirac field in this region can be written as the expansion:

$$\Psi = \int_0^\infty d\Omega \left(b^R_{+,\Omega}\Psi^R_{+,\Omega} + d^{R\dagger}_{-,\Omega}\Psi^R_{-,\Omega} + b^L_{+,\Omega}\Psi^L_{+,\Omega} + d^{L\dagger}_{-,\Omega}\Psi^L_{-,\Omega} \right) \equiv \Psi_R + \Psi_L, \tag{3.7}$$

where $b^P_{+,\Omega}, b^{P\dagger}_{+,\Omega}$ $(d^P_{-,\Omega}, d^{P\dagger}_{-,\Omega})$ denote annihilation/creation operators for Rindler particles/antiparticles in wedge P = R, L. The nonvanishing anti-commutation relations are:

$$\{b^P_{+,\Omega}, b^{P\dagger}_{+,\Omega'}\} = \{d^P_{-,\Omega}, d^{P\dagger}_{-,\Omega'}\} = \delta(\Omega - \Omega'), \quad \mathrm{P} = \mathrm{L}, \mathrm{R}. \tag{3.8}$$

The Fulling-Rindler (FR) vacuum $|0_{\mathrm{FR}}\rangle \equiv |0_{\mathrm{R}}, 0_{\mathrm{L}}\rangle$ defined as the state annihilated by all Rindler operators ($b^P_{+,\Omega}|0_{\mathrm{FR}}\rangle = d^P_{-,\Omega}|0_{\mathrm{FR}}\rangle = 0$ for P = R, L), is empty for Rindler observers in $\mathrm{R} \cup \mathrm{L}$.

Under the assumption that the entanglement in the Minkowski vacuum couples only Rindler modes of the same frequency Ω across the left and right wedges [23, 31, 32], the Minkowski vacuum $|0_M\rangle$ is related to $|0_{\mathrm{FR}}\rangle$ by a unitary two-mode squeezing operator:

$$S = \exp\left(\int_0^\infty d\Omega\, r_\Omega \left(b^{R\dagger}_{+,\Omega} b^{L\dagger}_{+,\Omega} - b^R_{+,\Omega} b^L_{+,\Omega} \right) \right), \tag{3.9}$$

where $r_\Omega = \arctan\left(e^{-\pi\Omega/a}\right)$. The action of S on the FR vacuum produces two-mode squeezed states:

$$|0_M\rangle = S|0_{\mathrm{FR}}\rangle = \bigotimes_\Omega |\psi_\Omega\rangle, \quad |\psi_\Omega\rangle = \cos r_\Omega |0_R, 0_L\rangle_\Omega + \sin r_\Omega |1_R, 1_L\rangle_\Omega, \tag{3.10}$$

where $|n_R, n_L\rangle_\Omega = (b^{R\dagger}_{+,\Omega})^{n_R} (b^{L\dagger}_{+,\Omega})^{n_L} |0_R, 0_L\rangle_\Omega$ $(n_R, n_L \in \{0, 1\})$. Notably, the key difference from the scalar case is that the fermionic state is a product over frequencies of two-mode states, each of which is a superposition of the vacuum and the doubly occupied state (one particle in R and one in L). There are no higher occupation numbers because of the Pauli exclusion principle. The transformed annihilation/creation operators satisfy:

$$S^\dagger b^R_{+,\Omega} S = \cos r_\Omega\, b^R_{+,\Omega} - \sin r_\Omega\, b^{L\dagger}_{+,\Omega}, \tag{3.11a}$$

$$S^\dagger b^L_{+,\Omega} S = \cos r_\Omega\, b^L_{+,\Omega} + \sin r_\Omega\, b^{R\dagger}_{+,\Omega}. \tag{3.11b}$$

In Refs. [17, 30], the positive frequency mode function is of the form:

$$\Psi^R_{+,\Omega}(\eta, \rho) = N_\Omega \begin{pmatrix} K_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho) + iK_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho) \\ -K_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho) + iK_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho) \end{pmatrix} e^{-i\Omega\eta}, \quad N_\Omega = \sqrt{\frac{M\cosh(\pi\Omega/a)}{2a\pi^2}}.$$

This is equivalent to our solution (3.3a) under the (complex) spinor basis transformation $U_+ \leftrightarrow \tilde{U}_+ = \begin{pmatrix} i \\ i \end{pmatrix}$ and $U_- \leftrightarrow \tilde{U}_- = \begin{pmatrix} -i \\ i \end{pmatrix}$. Both solutions satisfy Dirac equation; but their spinor basis is orthogonal but non-normalized (i.e., $\tilde{U}_+^\dagger \tilde{U}_+ = \tilde{U}_-^\dagger \tilde{U}_- = 2$, and $\tilde{U}_+^\dagger \tilde{U}_- = \tilde{U}_-^\dagger \tilde{U}_+ = 0$), resulting in an additional factor of $1/\sqrt{2}$ in their mode functions normalization factor N_Ω. The two choices are physically equivalent in the sense that physical observables depend on $N_\Omega^2 \tilde{U}_s^\dagger \tilde{U}_{s'}$ for $s, s' = \pm$, which is the same in both bases.

3.2 Massless Dirac Field in an Accelerated Cavity

We now specialize the general quantization scheme for the right Rindler wedge (R) to the case of a massless Dirac field confined within a cavity undergoing uniform acceleration.

Consider a cavity confined to the right Rindler wedge R with walls dragged along the boost Killing vector $K = x\partial_t + t\partial_x$. The worldlines of the cavity walls are $x = \sqrt{x_-^2 + t^2}$ (proper acceleration $a_- = 1/x_-$) and $x = \sqrt{x_+^2 + t^2}$ (proper acceleration $a_+ = 1/x_+$), overlapping with the inertial cavity at $t = 0$. In R, the cavity walls are, respectively, at $\rho_- = \rho_0$ and $\rho_+ = \rho_0 + l$ (with $\rho_0 = x_-$ at t=0).

Using boundary conditions analogous to Eq. (2.14) with $x_\mp \to \rho_\mp$, the normalized mode solutions of the massless Dirac equation within the accelerated cavity confined to (R) are [14]:

$$\chi_n^R(\eta, \rho) = \frac{e^{-i\Omega_n \eta}\left(\left(\frac{\rho}{\rho_-}\right)^{\frac{i\Omega_n}{a}} U_+ + e^{i\theta}\left(\frac{\rho}{\rho_-}\right)^{-i\frac{\Omega_n}{a}} U_-\right)}{\sqrt{2\rho \ln(\rho_+/\rho_-)}}, \tag{3.12a}$$

$$\Omega_n := \frac{a(n+s)\pi}{\ln(\rho_+/\rho_-)}, \quad n \in \mathbb{Z}, \tag{3.12b}$$

where θ and s are defined as in Section 2.2. The normalization factor $1/\sqrt{2\rho \ln(\rho_+/\rho_-)}$ ensures orthonormality over the cavity volume, accounting for the logarithmic dependence of the proper length on ρ.

The quantized massless Dirac field within the accelerated cavity is:

$$\chi(\eta, \rho) = \sum_{n \geq 0} B_n \chi_n^R(\eta, \rho) + \sum_{n<0} D_n^\dagger \chi_n^R(\eta, \rho), \tag{3.13}$$

with anticommutation relations:

$$\{B_n, B_m^\dagger\} = \{D_n, D_m^\dagger\} = \delta_{nm}. \tag{3.14}$$

The vacuum state $|0\rangle_{\chi^R}$—empty for accelerated observers—is defined by $B_n|0\rangle_{\chi^R} = D_n|0\rangle_{\chi^R} = 0$ for all $n \in \mathbb{Z}$. This vacuum differs from the FR vacuum due to the cavity boundaries (Eqs. (2.13)-(2.14)), which break conformal invariance and induce discrete mode structure.

3.3 Choice of Proper Time and the Small Cavity Approximation

The quantization procedure in Sec. 3.1 and Sec. 3.2 employ Rindler time η which is a coordinate time. To connect with physical observables such as decay rates, we must express dynamics in terms of the proper time τ of a specific observer. Two natural approaches present themselves for this analysis.

In the first approach, we consider a fiducial observer at the left wall, setting $\rho_- = \rho_0 = 1/a$ such that this observer has proper acceleration a. For this observer, the proper time is given by $d\tau = a\rho_- d\eta = d\eta$ (since $a\rho_- = 1$), making Rindler time coincide with their proper time. This identification is particularly useful under the condition $l \ll \rho_- = 1/a$, where the variation in proper time rate across the cavity becomes negligible. Under this condition, the proper frequency of the internal massless cavity field, measured by this observer, becomes $\Omega_n \approx \pi(n+s)/l = \tilde{\omega}_n$, causing the acceleration parameter to cancel out and rendering the spectrum identical to that of an inertial cavity of proper length l. This first approach provides a direct link to an observer (or detector) fixed to the cavity wall. If we relax the geometric condition $l \ll 1/a$, and only rely on the external field condition $(Ml \ll 1, M \ll 1)$, we enter a regime where non-trivial dependence on a emerges.

Alternatively, one may consider an observer at the center of the cavity, located at $\rho_c = (\rho_- + \rho_+)/2$, with proper time $d\tau = a\rho_c d\eta$. This approach is more broadly applicable as it does not require the cavity to be small relative to ρ_- or $1/a$. From the mode solutions (3.12), the proper frequency (as measured at the center) is $\Omega_n = (n+s)\pi/(\rho_c \ln(\rho_+/\rho_-))$, where $\rho_\pm = \rho_c \pm l/2$. For a cavity that is small relative to its position, $l \ll \rho_c$, the proper frequency measured by this observer is also $\Omega_n \approx \pi(n+s)/l$. Again, the internal energy level structure decouples from the acceleration scale.

The decoupling of the acceleration scale from the internal energy level structure is a universal feature of a small accelerated cavity $al \ll 1$, regardless of which observer's proper time we use. The physical reason is that for a very small cavity, the tidal forces (the variation of proper acceleration across it) become negligible. The entire cavity behaves almost like a point-like object undergoing rigid acceleration. Its internal structural properties, such as the energy gaps $\Delta E = \hbar\pi/l$ between successive modes, become independent of its overall motion. The decay probability, which depends on these energy gaps, will therefore be identical. The remaining effect of acceleration is contained in the definition of the vacuum state $|0\rangle_{\chi^R}$, which differs from the Minkowski vacuum and exhibits thermal properties due to the Unruh effect. This regime provides a natural testbed for testing Unruh effect.

For the decay analysis in the following section, we first adopt a general approach, using an arbitrary proper time, later for the the asymptotics of the decay rate we specialize to a fiducial observer at the left wall.

4 Uniformly Accelerated Cavity: Decay Rate Analysis

In this section, we analyze the decay probability of a fermionic particle confined to an accelerated cavity mode, leveraging the causal structure of Rindler spacetime and the constraints of quantum field theory to derive the expression for the decay rate. Our analysis emphasizes the role of spatial disjointness between Rindler wedges, particle number conservation, and the structure of the interaction Hamiltonian, ensuring consistency with the Unruh effect in curved spacetime.

The system is initialized with one fermion in the fundamental mode ($n = 1$) of a cavity localized to the right Rindler wedge (R), denoted $|1\rangle_{\chi^R}$, and the external Dirac field in the Minkowski vacuum $|0_M\rangle_\psi$. Using the squeezing operator S, which maps the Fulling-Rindler vacuuum to Minkowski vacuum, the initial state reads $|i\rangle = |1\rangle_{\chi^R} \otimes |0_M\rangle_\psi = |1\rangle_{\chi^R} \otimes S|0_{FR}\rangle$. The final state is defined as an empty cavity ($|0\rangle_{\chi^R}$) and the external field in a Minkowski coherent state $|\beta_\Omega\rangle_M := S|\beta_\Omega\rangle_R$, where $|\beta_\Omega\rangle_R$ is a Rindler coherent state of the external Dirac field in R with amplitude β_Ω for mode Ω. This final state is $|f\rangle = |0\rangle_{\chi^R} \otimes S|\beta_\Omega\rangle_R$. This ensures consistency with the interaction Hamiltonian $H_{\rm int}$, which acts in Minkowski spacetime. This mapping is critical for maintaining coherence between the accelerated frame of the cavity and the inertial frame of the external field.

Working to leading order in perturbation theory, the transition amplitude $\mathcal{A}_\downarrow^R$ describing the decay of the initial cavity state to the final external field state is given by:

$$\mathcal{A}_\downarrow^R = -i \int_0^\tau \frac{d\tau'}{a\rho_0} {}_R\langle\beta_\Omega|\, {}_{\chi^R}\langle 0|S^\dagger H_{int}(\tau) S|1\rangle_{\chi^R}|0_{\rm FR}\rangle \,, \tag{4.1}$$

where τ is the total proper interaction time for an observer at fixed $\rho = \rho_0$, while the measure $\frac{d\tau'}{a\rho_0}$ properly accounts for the conversion from Rindler time η to the proper time τ of the observer at fixed ρ_0. For an observer at the center of the cavity the proper time $\tau_c = a\rho_c\eta$ while for a fudicial left-wall observer with $\rho = \rho_- = 1/a$, the proper time is $\tau_- = a\rho_-\eta = \eta$.

The causal separation of the right (R: $x > |t|$) and left (L: $x < -|t|$) Rindler wedges ensures cross-wedge interactions vanish, simplifying the decay amplitude to an intra-wedge term only. This follows because modes in R (e.g., the cavity mode χ_1^R) and L (all external modes) have disjoint supports: $\mathrm{supp}(\Psi_{\pm\Omega}^R) \cap \mathrm{supp}(\Psi_{\pm\Omega}^L) = \emptyset$ and $\mathrm{supp}(\chi_1^R) \cap \mathrm{supp}(\Psi_{\pm\Omega}^L) = \emptyset$. Modes are explicitly defined to vanish in the opposite wedge, enforcing no overlap.

Furthermore, the local interaction $H_{\rm int} \propto \bar{\chi}\psi + \bar{\psi}\chi$ conserves the total number of fermions (cavity + external). This imposes a critical constraint: the transition amplitude $\mathcal{A}_\downarrow^R$ can only involve states with identical fermion numbers in R. Collecting the mode expansions (3.7), (3.13), we obtain the explicit form:

$$\mathcal{A}_{\downarrow}^{R} = -ig \int_{0}^{\tau} \frac{d\tau'}{a\rho_0} \int_{\rho_-}^{\rho_+} d\rho \int_{0}^{\infty} d\Omega \bar{\Psi}_{+,\Omega}^{R} \chi_1^{R} \cos r_{\Omega}\, {}_{R}\langle \beta_{\Omega} | b_{+,\Omega}^{R\dagger} | 0_{\mathrm{FR}} \rangle \,. \tag{4.2}$$

The amplitude contains several critical components. First, the mode functions $\chi_1^R(\tau', \rho)$ and $\bar{\Psi}_{+,\Omega}^R(\tau', \rho)$ describe the cavity and external field modes, respectively. The cavity mode χ_1^R is normalized and localized exclusively to the cavity volume $\rho_- \leq \rho \leq \rho_+$, ensuring it does not overlap with the left wedge. The external field mode $\bar{\Psi}_{+,\Omega}^R$ is the adjoint of the Dirac field mode in R, responsible for creating a fermion in mode Ω.

Second, the phase factor $\cos r_\Omega$ arises from the Bogoliubov transformation (3.11), which diagonalizes the Hamiltonian in the accelerated frame. For fermions, this transformation introduces a thermal distribution, with r_Ω encoding the Unruh effect. This reflects the thermal nature of the Minkowski vacuum as perceived by the accelerated observer.

Third, the coherent state overlap ${}_R\langle \beta_\Omega | b_{+,\Omega}^{R\dagger} | 0_{\mathrm{FR}} \rangle$ quantifies the overlap between the Rindler coherent state $|\beta_\Omega\rangle_R$ and the 1-particle state $|1_\Omega\rangle_R$ created by the Rindler creation operator $b_{+,\Omega}^{R\dagger}$ acting on the Fulling-Rindler vacuum $|0_{\mathrm{FR}}\rangle$. For a coherent state, this overlap is non-zero only for the 1-particle component, given by $\beta_\Omega e^{-|\beta_\Omega|^2/2}$, where $\beta_\Omega e^{-|\beta_\Omega|^2/2}$ is the overlap between $|\beta_\Omega\rangle_R$ and $|1_\Omega\rangle_R$.

4.1 Decay Probability in Right Rindler Wedge

The decay probability $\mathcal{P}_{\downarrow}^R$ is obtained by summing the squared modulus of the transition amplitude over all final Rindler coherent states $|\beta_\Omega\rangle_R$. Particle number conservation (initial 1-particle cavity state) enforces that only the 1-particle component of $|\beta_\Omega\rangle_R$ contributes, as higher-number states ($n \geq 2$) cannot be created from the vacuum.

Summing over β_Ω (equivalent to integrating over the external field's density matrix) and using the normalization of coherent states, the decay probability becomes:

$$\mathcal{P}_{\downarrow}^{R} = g^2 \int_{0}^{\infty} d\Omega \left| \Gamma_1^R(\Omega) \right|^2 \frac{e^{2\pi\Omega/a}}{e^{2\pi\Omega/a} + 1} \,, \tag{4.3a}$$

where $\Gamma_n^R(\Omega)$ is the intra-wedge overlap integral defined as:

$$\Gamma_n^R(\Omega) := \int_{0}^{\tau} \frac{d\tau'}{a\rho_0} \int_{\rho_-}^{\rho_+} d\rho\, \bar{\psi}_{+,\Omega}^R(\tau', \rho) \chi_n^R(\tau', \rho) \,, \tag{4.3b}$$

and where $\bar{\psi}_{+,\Omega}^R(\tau', \rho) := \psi_{+,\Omega}^{R\dagger}(\tau', \rho)\beta$. The thermal factor $\frac{e^{2\pi\Omega/a}}{e^{2\pi\Omega/a}+1} = \frac{1}{1+e^{-2\pi\Omega/a}}$ represents Fermi-Dirac stimulation from the Unruh thermal bath at the temperature $T = a/(2\pi)$.

Collecting the mode functions Eqs. (3.3a) and (3.12), we obtain:

$$\mathcal{P}_{\downarrow}^{R} = \frac{g^2 M}{a\pi^2 \ln\left(\frac{\rho_- + l}{\rho_-}\right)} \left(\frac{1}{a\rho_0}\right)^2 \int_0^\infty d\Omega \frac{e^{\pi\Omega/a} \sin^2\left((\Omega - \Omega_1)\frac{\tau}{2a\rho_0}\right)}{\left((\Omega - \Omega_1)\frac{1}{a\rho_0}\right)^2} |f(\Omega)|^2 \,, \tag{4.4a}$$

where

$$f(\Omega) := \int_{\rho_-}^{\rho_- + l} \frac{d\rho}{\sqrt{\rho}} \left(e^{i\theta}\left(\frac{\rho}{\rho_-}\right)^{-i\frac{\Omega_1}{a}} K_{i\frac{\Omega}{a}+\frac{1}{2}}(M\rho) - i\left(\frac{\rho}{\rho_-}\right)^{i\frac{\Omega_1}{a}} K_{i\frac{\Omega}{a}-\frac{1}{2}}(M\rho)\right) \,, \tag{4.4b}$$

and where

$$\Omega_1 := \frac{a(1+s)\pi}{\ln\left(\frac{\rho_- + l}{\rho_-}\right)}, \quad s \in (0,1), \quad \theta \in [0, 2\pi) \,. \tag{4.4c}$$

4.2 Long-Time Resonance Dominance ($\tau \gg 1/\Omega_1$)

For long measurement times ($\tau \gg 1/\Omega_1$), the integrand of (4.4) is dominated by the resonance condition $\Omega \approx \Omega_1$, where the sine-squared term $\sin^2((\Omega - \Omega_1)\tau/2)$ peaks sharply. Using the asymptotic identity for large τ:

$$\frac{\sin^2\left((\Omega - \Omega_1)\frac{\tau}{2a\rho_0}\right)}{\left((\Omega - \Omega_1)\frac{1}{a\rho_0}\right)^2} \sim a\rho_0 \frac{\pi\tau}{2} \delta(\Omega - \Omega_1) \quad \text{as} \quad \tau \to \infty, \tag{4.5}$$

the leading-order decay probability becomes:

$$\mathcal{P}_{\downarrow}^{R\text{long}} = \frac{g^2 M e^{\pi\Omega_1/a}}{2a\pi \ln\left(\frac{\rho_- + l}{\rho_-}\right)} \frac{\tau}{a\rho_0} |f(\Omega_1)|^2 \,, \tag{4.6}$$

where τ is the total proper time of the observer chosen to define the calculation, and $\Delta\eta = \frac{\tau}{a\rho_0}$ is the corresponding total Rindler time interval, which is the same for all observers.

Notably, for a fiducial observers at the left wall of the uniformly accelerated cavity ($\rho_0 = \rho_- = 1/a$), their proper time is given by $d\tau_{\text{left}} = a\rho_- d\eta = d\eta$. Therefore, the total Rindler time $\Delta\eta$ corresponds to a total proper time $\tau_{\text{left}} = \Delta\eta$. Their measured decay rate is:

$$\Gamma_{\text{left}} = \frac{\mathcal{P}_{\downarrow}^{\text{Rlong}}}{\tau_{\text{left}}} = \frac{\mathcal{P}_{\downarrow}^{\text{Rlong}}}{\Delta\eta} = \frac{g^2 M e^{\pi\Omega_1/a}}{2a\pi \ln(1 + al)} |f(\Omega_1)|^2 \,. \tag{4.7}$$

For observers at the center of the cavity ($\rho_0 = \rho_c, \rho_\pm = \rho_c \pm l/2$), their proper time is

$d\tau_c = a\rho_c d\eta$. Therefore, the same total Rindler time interval $\Delta\eta$ corresponds to a total proper time $\tau_c = a\rho_c\Delta\eta$. Their measured decay rate is:

$$\Gamma_c = \frac{\mathcal{P}_\downarrow^{\text{Rlong}}}{\Delta\tau_{\text{center}}} = \frac{\mathcal{P}_\downarrow^{\text{Rlong}}}{a\rho_c\Delta\eta} = \frac{g^2 M e^{\pi\Omega_1/a}}{2a^2\rho_c\pi\ln\left(\frac{\rho_c+l/2}{\rho_c-l/2}\right)}\left|f(\Omega_1)\right|^2 . \tag{4.8}$$

The factor $1/(a\rho_c)$ in Γ_{center} arises directly from time dilation, since the central observer's clock runs faster relative to Rindler time.

The two rates Γ_{left} and Γ_{center} differ because they are defined relative to different proper times. This is physically meaningful: the central observer's clock runs at a different rate compared to the left-wall observer's clock due to their different positions in the accelerated frame. For the central observer, total probability $\mathcal{P}_\downarrow^{\text{Rlong}}$ unfold over a longer period of their own proper time, so the rate $(d\mathcal{P}_\downarrow^{\text{Rlong}}/d\tau)$ is smaller. The decay probability $\mathcal{P}_\downarrow^{\text{Rlong}}$ itself is a Lorentz scalar, the same for both observers, as it is calculated over the same physical process. However, the rate $\Gamma = d\mathcal{P}_\downarrow^{\text{Rlong}}/d\tau$ depends on whose proper time τ you use to measure it.

For the remainder of this work, we will use the proper time of the left-wall observer ($\rho_0 = \rho_- = 1/a$), which yield the decay rate Γ_{left}.

5 Comprehensive Analysis of Decay Probability Limits

The decay probability $\mathcal{P}_\downarrow^{Rlong}$ in the long-time resonance limit ($\tau \gg 1/\Omega_1$), for the uniformly accelerated cavity, depends on geometric parameters (cavity length l, position ρ_-), acceleration a, mass scale M, and measurement time τ. Below, we analyze key limits governing its behavior.

5.1 Decay Rate in the Short-Cavity Limit ($al \ll 1$)

In this approximation, we model an experiment where a specific accelerated detector is attached to the left wall of our uniformly accelerated cavity. This detector prepares, manipulates, and measures the quantum field within its local cavity. The time parameter τ in the decay probability is the time read by this detector's clock. We consider the regime $al \ll 1$, which we established in Sec. 3.3 as the condition where tidal forces are negligible. As discussed therein, this hierarchy of scales ($l \ll 1/a$) ensures the variation in proper time and acceleration across the cavity is negligible, allowing the entire cavity to be treated as a rigid object undergoing uniform acceleration. Consequently, the external field modes vary slowly across the cavity's finite extent, which will permit a significant simplification of the overlap integrals.

Under the condition $l \ll \rho_- = 1/a$, the integration domain $[\rho_-, \rho_- + l]$ of the overlap function (4.4b) is small. Although the modified Bessel functions $K_{i\beta\pm1/2}(M\rho)$ (where

$\beta := \Omega_1/a$) are highly oscillatory functions of ρ for large β, their variation over the interval l is negligible. Therefore, we approximate them by their value at the lower limit $\rho_0 = \rho_- = 1/a$. In this regime, the leading order of the overlap function (4.4b) is:

$$f(\Omega_1) \approx l\sqrt{a}\left[e^{i\theta}K_{i\beta+\frac{1}{2}}(M/a) - iK_{i\beta-\frac{1}{2}}(M/a)\right] . \tag{5.1}$$

We now assume $M/a \ll 1$ (the weak-mass regime). The short-cavity condition $al \ll 1$ implies $\beta = \frac{\Omega_1}{a} = \frac{(1+s)\pi}{al} \gg 1$. For fixed ν and $z \to 0$, the modified Bessel function $K_\nu(z)$ has the asymptotic expansion [33]: $K_\nu(z) \sim \frac{1}{2}\Gamma(-\nu)\left(\frac{z}{2}\right)^\nu + \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}$. Using the asymptotic forms $\Gamma(\pm i\beta + 1/2) \sim \sqrt{2\pi}e^{-\pi\beta/2}e^{\pm i(\beta\ln\beta-\beta}$, we obtain the dominant behavior of the modulus squared:

$$|f(\Omega_1)|^2 \sim \frac{2\pi l^2 a^2}{M}e^{-\pi\beta}\left[1 - \sin\left(\theta + 2\beta\ln\left(\frac{2a\beta}{M}\right) - 2\beta\right)\right] . \tag{5.2}$$

Combining this with the long-time probability (4.6) and using the approximation $\ln(\rho_+/\rho_-) \approx al$ valid for $al \ll 1$, the decay probability becomes:

$$\mathcal{P}_\downarrow^{\text{Rlong}} \sim g^2 l\tau_{\text{left}}\left[1 - \sin\left(\theta + 2\beta\ln\left(\frac{2\Omega_1}{M}\right) - 2\beta\right)\right] . \tag{5.3}$$

The oscillating term $\sin\left(\theta + 2\beta\ln\left(\frac{2\Omega_1}{M}\right) - 2\beta\right)$ is bounded between -1 and 1. For $\beta \gg 1$, the phase $\phi := \theta + 2\beta\ln\left(\frac{2\Omega_1}{M}\right) - 2\beta$ varies extremely rapidly with β (and thus with the acceleration a). Its derivative $d\phi/d\beta = 2\ln\left(\frac{2\Omega_1}{M}\right) - 2$ is large. Consequently, any infinitesimal experimental uncertainty δa in the acceleration will randomize the phase modulo 2π, leading to $\langle\sin\phi\rangle = 0$ upon averaging. Therefore, the averaged decay rate is:

$$\Gamma_{\text{left}} \sim g^2 l = \Gamma_{\text{in}} . \tag{5.4}$$

This result is exact in the idealized mathematical limit of a vanishingly small cavity ($al \to 0$). In this limit, the leading-order asymptotic expression converges exactly to $\Gamma_{\text{left}} = g^2 l = \Gamma_{\text{in}}$, and all higher corrections vanish.

This result makes physical sense because: for $al \ll 1$, the cavity mode frequency $\Omega_1 \sim \pi/l$ is much larger than the acceleration scale a (since $\Omega_1/a \sim \pi/(al) \gg 1$). The Unruh thermal bath at temperature $T = a/(2\pi)$ has an exponentially small population at frequency $\Omega_1 : n(\Omega_1) \sim e^{-2\pi\Omega_1/a}$. Thus, stimulated emission is negligible, and the decay is purely spontaneous, leading to the same rate as in an inertial vacuum.

5.2 Decay Rate in the Intermediate Regime ($al \sim 1$)

We now analyze the decay of the cavity's fundamental mode for a cavity of intermediate size, where the acceleration length scale is comparable to the cavity's proper length: $al \sim 1$. This regime is characterized by the parameter $\beta = \Omega_1/a = (1+s)\pi/\ln(1+al)$

being of order unity. We maintain the condition $M/a \ll 1$ for the external field. These two conditions ensure $Ml = (M/a)(al) \ll 1$.

The full expression for the squared modulus of the overlap integral, $|f(\Omega_1|^2$, derived from Eqs. (4.4b), contains multiple terms (I, II, III) involving Gamma functions and rapidly oscillating phases. However, for $M/a \ll 1$, a detailed analysis shows that Term I dominates the behavior. The dominant term is:

$$|f(\Omega_1)|^2 \sim \frac{l^2 a^2 \pi}{M \cosh(\pi\beta)} \left[1 - \sin\left(\theta + 2\arg\Gamma(1/2 + i\beta) - 2\beta \ln\left(\frac{M}{2a}\right)\right)\right] . \tag{5.5}$$

Terms II and III are proportional to $(M/2a)$ and $1/(\beta^2{+}1/4)$, respectively. For $M/a \ll 1$ and $\beta \sim 1$, these terms are suppressed compared to term I, and are therefore negligible. (A full discussion is provided in Appendix A.)

The expression for $|f(\Omega_1)|^2$ contains terms with rapidly oscillating phases proportional to $\ln(M/2a)$. Any infinitesimal experimental uncertainty in the acceleration a or other parameters would cause these terms to average to zero. Therefore, the observable decay rate is governed by the remaining non-oscillatory term. Combing this with the expression (4.7) yields the averaged decay rate:

$$\Gamma_{\text{left}} \sim \frac{g^2 l^2 a}{\ln(1+al)} \frac{1}{1+e^{-2\pi\beta}} . \tag{5.6}$$

For $al \sim 1$, $\beta = \frac{(1+s)\pi}{\ln(1+al)}$. Taking a representative value $s = 1/2$, we find $\beta \approx \frac{3\pi}{2\ln 2} \approx 6.8$. An upper bound for $s \in (0,1)$ is $\tilde{\beta} = \frac{2\pi}{\ln(2)} \approx 9.06$. Thus $e^{-2\pi\beta} \ll 1$ for the entire range of s, and the factor $(1 + e^{-2\pi\beta})^{-1} \approx 1$. The averaged decay rate simplifies to:

$$\Gamma_{\text{left}} \sim g^2 l \frac{al}{\ln(1+al)} = \Gamma_{\text{in}} \frac{al}{\ln(1+al)} . \tag{5.7}$$

The result includes a logarithmic correction factor $al/\ln(1 + al)$ that arises from the geometric properties of the accelerated cavity. This represents a measurable non-inertial effect distinct from the thermal Unruh signature. In the limit of a very small cavity ($al \ll 1$), we recover the previous result from Sec. 5.1: $\ln(1 + al) \approx al$, which implies $\Gamma_{\text{left}} \sim \Gamma_{\text{in}}$. This expression furthermore converges exactly to the idealized result in the limit $al \to 0$, where $\lim_{al\to 0} \frac{al}{\ln(1+al)} = 1$, implying $\lim_{al\to 0} \Gamma_{\text{left}} = \Gamma_{\text{in}}$. The expression also contains the fermionic thermal factor $1/(1{+}e^{-2\pi\beta})$, representing Fermi-Dirac stimulation from the Unruh thermal bath. However, for $al \sim 1$, this thermal factor is approximately unity, indicating that the thermal Unruh effect is negligible in this parameter regime. The observed enhancement therefore represents primarily geometric/non-thermal acceleration effects, demonstrating that accelerated cavities exhibit measurable deviations from inertial behavior through mechanisms that may be distinct from the thermal Unruh signature. This provides an important prediction for experimental tests in cavity QED setups with fermionic systems.

5.3 Decay Rate for a Heavy External Field ($M/a \gg 1$)

The previous analyses assumed a light external field ($M/a \ll 1$), which maximizes the response of the Unruh thermal bath. We now consider the opposite regime, where the external field is heavy relative to the acceleration scale ($M/a \gg 1$). In this case, the energy required to excite a quanta of the external field, Mc^2, is much larger than the characteristic energy scale of the Unruh thermal bath, $\hbar a/c$. Consequently, we expect the decay process to be dominated by exponential suppression, irrespective of the cavity size. This section analyzes this suppression for small and intermediate cavities ($al \lesssim 1$). The analysis for large cavities ($al \gg 1$) is covered in Case C of Sec. 5.4.

The derivation of the overlap integral $|f(\Omega_1|^2$ in this regime is involved and is detailed in Appendix B. The key insight is that the asymptotic form of the modified Bessel functions $K_{i\beta\pm1/2}(M\rho)$ for large M/ρ dominates the integral, leading to a universal exponential factor of $\exp(-2M/a)$.

Case 1: Short Cavity ($al \ll 1$) with $M/a \gg 1$

For a short cavity, the parameter $\beta = \Omega_1/a \gg 1$ remains large. The detailed derivation in Appendix B shows that the dominant behavior of the squared modulus of the overlap function (5.1) in this regime takes the form:

$$|f(\Omega_1)|^2 = \frac{\pi a^2 l^2 e^{-2M/a}}{M} F_{\text{short}}(a, l, M, s, \theta)\,, \tag{5.8}$$

where the dimensionless algebraic prefactor F_{short} encapsulates terms of order unity, as derived in Appendix B. Its precise form does not alter the dominant exponential behavior. Consequently, the decay rate (4.7) becomes:

$$\Gamma_{\text{left}} \sim \frac{\Gamma_{\text{in}}}{2}\, e^{\frac{(1+s)\pi^2}{al}} e^{-2M/a} F_{\text{short}}(a, l, M, s, \theta)\,. \tag{5.9}$$

The condition for a precise cancellation of the exponential enhancement and suppression is $(1+s)\pi^2 = 2Mlc/\hbar$ (in SI units). For any fundamental particle (e.g., an electron) and any admissible boundary condition ($s \in (0,1)$), this equation requires a cavity size l on the order of picometers. For example, with $M = M_e$ (electron mass) and $a = 10^{20}\,\text{m/s}^2$, solving $(1+s)\pi^2 = 2Mlc/\hbar$ yields $l \approx 1\text{pm}$, which is far smaller than any realistic experimental cavity, which would typically be micrometers or larger. Therefore, the finely-tuned cancellation is physically unattainable.

This overwhelming dominance is best illustrated with a numerical example. Consider a scenario where the cavity undergoes an extreme acceleration of $a = 10^{20}\,\text{m/s}^2$. The energy required to create a quanta of the external electron field ($M_e c^2 \approx 0.511\,\text{MeV}$) as the excited state of the confined field decays to its ground state must be compared to the characteristic energy of the acceleration, $\hbar a/c$. The disparity between these energy scales is immense:

$$\frac{M_e c^2}{\hbar a/c} \approx \frac{0.511\times 10^6\text{eV}}{2.2\times 10^{-4}\text{eV}} \approx 2.3\times 10^9\,. \tag{5.10}$$

Now, consider a realistic, small cavity with a length $l = 1\mu$m. The enhancement exponent is:

$$\frac{(1+s)\pi^2}{al/c^2} \approx \frac{15}{1.11\times 10^{-3}} = 1.35\times 10^4\,, \tag{5.11}$$

where we have used $al/c^2 \approx 1.11\times 10^{-3}$ and $(1+s)\pi^2 \approx 15$ (for $s = 1/2$). The net decay rate is proportional to:

$$\exp\bigl(1.35\times 10^4\bigr)\cdot\exp\bigl(-4.6\times 10^9\bigr) = \exp\bigl(-4.6\times 10^9 + 1.35\times 10^4\bigr) \approx \exp\bigl(-4.6\times 10^9\bigr)\,. \tag{5.12}$$

The suppression factor $\exp(-4.6\times 10^9)$ is so overwhelmingly small that it completely dominates the enhancement factor $\exp(1.35\times 10^4)$, rendering the net rate $\exp(-4.6\times 10^9)$ utterly immeasurable. This makes perfect physical sense: creating a massive particle requires so much energy that even with the geometric enhancement from the cavity, the Unruh thermal bath at achievable accelerations cannot provide it, leading to exponential suppression of the decay process.

Across the entire parameter space, the exponential suppression term $\exp\left(\frac{-2Mc^2}{\hbar a/c}\right)$ will overwhelmingly dominate the decay rate for any achievable acceleration. This shutdown of the decay channel occurs because the energy required to create a quantum of the external field ($\sim Mc^2$) is vastly larger than the characteristic energy scale of the Unruh thermal bath ($\sim \hbar a/c$), making the process exponentially improbable. In other words, the Unruh thermal bath is too "cold" ($k_B T_U = \hbar a/(2\pi c)$) to excite the heavy external field, shutting down the decay channel.

Case 2: Intermediate Cavity ($al \sim 1$) with $M/a \gg 1$

In this regime, $\beta = \Omega_1/a = (1+s)\pi/\ln(1+al)$ is of order unity (since $s \in (0,1)$ and $(al \sim 1)$. Despite this change, the condition $M/a \gg 1$ ensures the exponential suppression remains the dominant effect. The analysis yields a result of the same form[2]

$$|f(\Omega_1)|^2 = \frac{\pi a^2 l^2 e^{-2M/a}}{M} G_{\text{short}}(a,l,M,s,\theta)\,, \tag{5.13}$$

where the dimensionless algebraic prefactor G_{short} is given in Appendix B. Consequently, the decay rate (4.7) becomes:

$$\Gamma_{\text{left}} \sim \frac{\Gamma_{\text{in}}}{2}\frac{al}{\ln(1+al)}\, e^{\frac{(1+s)\pi^2}{\ln(1+al)}} e^{-2M/a} G_{\text{short}}(a,l,M,s,\theta)\,. \tag{5.14}$$

[2]This expression corrects a typographical error present in Version 1, where the factor of l^2 was erroneously written as l. The derivation in Appendix B and all subsequent results are unaffected.

While the algebraic prefactors F_{short} and G_{short} differ from the short-cavity case, the governing exponential factor is identical. The enhancement exponent $(1+s)\pi^2/\ln(1+al) \approx (1+s)\pi^2/\ln 2$ (since $al \sim 1$) is vastly smaller than the suppression exponent $2M/a = 4.6 \times 10^9$ (where $2M/a$ corresponds to $2Mc^2/(\hbar a/c)$ in SI units). Thus again, the exponential suppression overwhelmingly dominates the decay rate.

This analysis confirms that for a heavy external field ($M/a \gg 1$), the decay rate is exponentially suppressed across all cavity sizes, $\Gamma_{\text{left}}/\Gamma_{\text{in}} \sim e^{-2M/a}$. The result derived for large cavities in Sec. 5.4 (Case C) is not an isolated case but is, in fact, the universal behavior for this parameter regime. The specific cavity size al only influences the algebraic prefactor, which is always many orders of magnitude smaller than the exponential factor $\exp(-2M/a)$, but not the dominant exponential character of the suppression. Therefore, the dominant exponential character of the suppression is universal. This explains why experiments using electrons or other massive fundamental particles—which satisfy $Mc^2/(\hbar a/c) \gg 1$—have consistently failed to detect Unruh effects.

5.4 Decay Rate in the Large Cavity Regime ($al \gg 1$)

We now analyze the decay of the cavity's fundamental mode in the regime where its proper length is large compared to the acceleration length scale, $al \gg 1$. This implies the parameter $\beta = \Omega_1/a = (1+s)\pi/\ln(1+al)$ is small ($\beta \ll 1$), as the logarithmic denominator is large. In this regime, the proper acceleration varies little across the cavity, and we expect the influence of the Unruh effect to diminish. The decay dynamics are governed by the secondary parameter M/a, the ratio of the external field's mass to the acceleration, leading to three distinct physical subcases. We use the approximations $\ln(1+al) \approx \ln(al)$ and $e^{\pi\beta} \approx 1$.

The overlap integral (4.4b) for $f(\Omega_1)$ must be evaluated by integrating over ρ. To leading order in the small parameter β (($O(\beta^0)$), it simplifies to:

$$f^{(0)}(\Omega_1) \approx (e^{i\theta} - i) \int_{1/a}^{1/a+l} \frac{d\rho}{\sqrt{\rho}} K_{\frac{1}{2}}(M\rho)\,. \tag{5.15}$$

The behavior of this integral defines the following subcases. (For a full discussion of the asymptotic analysis see Appendix C.)

5.4.1 Case A: Light External Field ($M/a \ll 1$); implies $Ml \ll 1$)

Here, the argument of the Bessel function $M\rho$ is small. Using $K_{1/2}(z) \sim \sqrt{\pi/(2z)} = K_{-1/2}(z)$ for $z \ll 1$, we obtain for squared modulus of the overlap function:

$$|f^{(0)}(\Omega_1)|^2 \sim \frac{\pi}{M}(1-\sin\theta)\ln^2(al)\,. \tag{5.16}$$

Collecting (4.7) yields the decay rate:

$$\Gamma_{\text{left}} \sim \frac{g^2}{2a}(1-\sin\theta)\ln(al)\,. \tag{5.17}$$

This has a clear physical interpretation: The decay rate scales algebraically with the acceleration ($\Gamma_{\text{left}} \propto 1/a$) with logarithmic corrections. This signifies a return to inertial-like behavior, modulated by the boundary condition through $(1-\sin\theta)$.

5.4.2 Case B: Heavy Field – Light Relative to Acceleration ($M/a \ll 1$ but $Ml \gg 1$)

This case explores the regime where $M/a \ll 1$ (so the field is light relative to acceleration) but $Ml \gg 1$ (so the field is heavy relative to the cavity length). This requires $al \gg a/M$, which is possible since $a/M \gg 1$. Using $K_{1/2}(z) \sim \sqrt{\pi/(2z)}$ and the exponential integral $E_1(M/a) = \int_{1/a}^{\infty} \frac{e^{-M\rho}}{\rho} d\rho \approx -\gamma - \ln(M/a)$ for $M/a \ll 1$, we find for the squared modulus of the overlap integral:

$$|f^{(0)}(\Omega_1)|^2 \approx \frac{\pi}{M}(1-\sin\theta)\left(\ln(a/M)-\gamma\right)^2\,. \tag{5.18}$$

where γ is the Euler-Mascheroni constant. Collecting (4.7), the resulting decay rate is:

$$\Gamma_{\text{left}} \sim \frac{g^2}{2a\ln(al)}(1-\sin\theta)\left(\ln(a/M)-\gamma\right)^2. \tag{5.19}$$

The rate is further suppressed by the factor $1/\ln(al)$ compared to Case A, reflecting the increased difficulty of exciting a heavier field, even when $M/a \ll 1$.

5.4.3 Case C: Heavy External Field – Strong Relative to Acceleration ($M/a \gg 1$; implies $Ml \gg 1$)

We now consider the case where the external field's mass is large compared to the acceleration scale, $M/a \gg 1$. The argument $z = M\rho$ is large, requiring the large-argument asymptotic expansion $K_\nu \sim \sqrt{\pi/(2z)}e^{-z}$. The integral in (5.15) is again dominated by its lower limit. Using the Laplace method for integrals of the form $\int e^{-M\rho}g(\rho)d\rho$ with $g(\rho) = 1/\rho$ varying slowly compared to the exponential, we obtain for the squared modulus of the overlap function:

$$|f^{(0)}(\Omega_1)|^2 \approx \frac{\pi a^2}{2M^3}|e^{i\theta}-i|^2 e^{-2M/a} = \frac{\pi a^2}{M^3}(1-\sin\theta)e^{-2M/a}. \tag{5.20}$$

Collecting (4.7) yields the decay rate:

$$\Gamma_{\text{left}} \sim \frac{g^2 a}{2M^2\ln(al)}(1-\sin\theta)e^{-2M/a}. \tag{5.21}$$

The decay rate is exponentially suppressed by the factor $e^{-2M/a}$. This confirms that the Unruh thermal bath at temperature $T = a/(2\pi)$ is too "cold" to excite particles of mass $M \gg a$. For the specific case of the MIT bag model ($\theta = \pi/2$), the prefactor $(1 - \sin\theta) = 0$, causing the zeroth-order term to vanish exactly. The decay would then be governed by higher-order terms in β, which are also exponentially suppressed.

5.4.4 Summary

The behavior of the decay rate in the large cavity regime ($al \gg 1$) transitions from inertial-like to exponentially suppressed, depending on the mass of the external field relative to the acceleration. Table 1 summarizes the key results.

Table 1: Summary of decay rate Γ_{left} behavior in the large cavity regime ($al \gg 1$).

Case	Condition	Decay Rate Γ_{left}	Dominant Behavior
A	$M/a \ll 1$ (implies $Ml \ll 1$)	$\frac{\Gamma_{\text{in}}}{2} \frac{(1-\sin\theta)\ln(al)}{al}$	Algebraic ($\sim 1/a$)
B	$M/a \ll 1$, $Ml \gg 1$	$\frac{\Gamma_{\text{in}}}{2} \frac{(1-\sin\theta)\,(\ln(a/M)-\gamma)^2}{al\ln(al)}$	Algebraic, logarithmically suppressed
C	$M/a \gg 1$ (implies $Ml \gg 1$)	$\frac{\Gamma_{\text{in}}}{2} \frac{(1-\sin\theta)e^{-2M/a}}{(M/a)(Ml)\ln(al)}$	Exponentially suppressed ($\sim e^{-2M/a}$)

6 Conclusion

Our analysis demonstrates that the decay rate of an excited state within a uniformly accelerated cavity depends on two key dimensionless parameters: the cavity size relative to the acceleration scale, al, and the mass of the external field relative to the acceleration, M/a. For light external fields ($M/a \ll 1$), the decay rate transitions from inertial-like behavior ($\Gamma_{\text{left}} \sim \Gamma_{\text{in}}$) for $al \ll 1$, to a geometric enhancement ($\Gamma_{\text{left}} \sim \Gamma_{\text{in}} \frac{al}{\ln(1+al)}$) for $al \sim 1$, and to an algebraically suppressed rate ($\Gamma_{\text{left}} \sim \Gamma_{\text{in}}/al$) with logarithmic corrections for $al \gg 1$. For heavy external fields ($M/a \gg 1$), the decay rate is exponentially suppressed ($\Gamma_{\text{left}}/\Gamma_{\text{in}} \sim e^{-2M/a}$) regardless of the cavity size. These results highlight the rich interplay between acceleration, cavity size, and field mass in quantum field theory.

Our analysis suggests a model-based explanation for the historical null results: within our theoretical framework, the direct observation of the Unruh effect via the acceleration of fundamental matter is precluded by a fundamental energy-scale mismatch. This arises because for fundamental particles like electrons, the condition $Mc^2 \ll \hbar a/c$ is dramatically violated for any achievable acceleration. To illustrate: the rest energy of an electron is $M_e c^2 \approx 0.511\text{MeV}$. For an acceleration of $a = 10^{20}\text{m/s}^2$, the characteristic Unruh energy scale is $\hbar a/c \approx 2.2 \times 10^{-4}\text{eV}$. Thus, $M_e c^2/(\hbar a/c) \approx 2.3 \times 10^9$, placing any system with fundamental fermions firmly in an exponentially suppressed regime with $\Gamma_{\text{left}}/\Gamma_{\text{in}} \sim \exp(-4.6 \times 10^9)$. This energy-scale argument provides a unified explanation within our model for the persistent null results from experiments employing fundamental particles like electrons.

Quantum simulators circumvent this issue by engineering effective light fields where the effective mass energy $M_{\text{eff}}c^2$ can be made very small (e.g., through low-frequency modes or synthetic materials). This allows the condition $M_{\text{eff}}c^2 \ll \hbar a/c$ to be satisfied, enabling access to a regime where the decay rate is governed by a measurable, algebraic enhancement. We emphasize that this enhancement is a geometric, non-thermal effect arising from acceleration-induced mode mixing, distinct from Fermi-Dirac stimulation by the Unruh thermal bath.

Consequently, quantum simulation emerges as the most viable path forward, enabling the engineering of systems where this fundamental constraint is circumvented. The experimental verification of our predicted geometric enhancement $\Gamma_{\text{left}}/\Gamma_{\text{in}} \sim \frac{al/c^2}{\ln(1+al/c^2)}$ for an intermediate cavity size ($al \sim c^2$) would represent a landmark achievement, providing direct evidence of non-inertial quantum effects by measuring this clear, geometric signature.

Several experimental avenues appear particularly promising for achieving the required parameter regime, where the scaling $al \sim c^2$ corresponds to accelerations of $a \sim 10^{20}\,\text{m/s}^2$ and cavity sizes $l \sim 100 - 500\,\mu\text{m}$. In this regime, the geometric enhancement factor ranges from approximately 1.06 to 1.26, representing a 6% to 26% increase over inertial decay rates. This enhancement is geometric rather than thermal in origin, arising from the cavity's accelerated motion rather than Unruh thermal effects.

Superconducting circuits [34, 35] offer a highly controlled platform ideally suited for this parameter regime. In this architecture, micrometer-scale coplanar waveguide resonators ($l \sim 100 - 500\mu\text{m}$) are ideally suited to pursue the $al \sim c^2$ regime. The required effective acceleration can be simulated via time-dependent modulation of circuit parameters, and the resulting enhanced decay rate can be read out using standard qubit measurement techniques. Crucially in our model, for the external environment to simulate the massive Dirac field, the effective mass scale must satisfy $Mc^2 \ll \hbar a/c$ ($M \ll a$ in natural units), which for typical parameters requires an effective mass energy below 10^{-4}eV. This can be achieved by engineering a low-frequency environment or using synthetic materials.

Optomechanical systems [36] provide a complementary pathway. Here, optical photons in a cavity serve as the massless field, while a mechanical oscillator can act as an analogue for the massive field. Simulated acceleration can be achieved through strong optical driving, and the decay dynamics can be monitored through optical readout techniques. The mechanical oscillator's frequency ω_m must be chosen such that $\hbar\omega_m \ll \hbar a/c$ to satisfy $Mc^2 \ll \hbar a/c$, ensuring a small effective mass relative to the acceleration energy scale.

Alternatively, laser-plasma accelerators [37] could provide direct access to the extreme acceleration regime ($a \gtrsim 10^{20}\,\text{m/s}^2$), potentially directly measure the $al \ll c^2$ and the $al \sim c^2$ regimes. This approach seeks to create the conditions for the effect more directly, rather than simulating them. However, a fundamental challenge arises because the external field is inherently composed of particles with rest mass. For standard particles like electrons, the rest energy $Mc^2 \gg \hbar a/c$ for any achievable acceleration, violating the crucial constraint for a non-suppressed Unruh effect and placing the system in the exponentially suppressed regime (Case 1 and Case 2 of Sec. 5.3). Thus, this approach may only be viable if the sensor couples to inherently light or effective massless particles within the plasma environment (e.g., photons or plasmons) to circumvent this suppression.

While our results provide a clear reason within our model why the Unruh effect is not observable due to the exponential suppression, several important limitations define the scope of this work. The analysis is conducted in $(1+1)$-dimensional spacetime, which, while theoretically insightful, lacks the full complexity of $(3+1)$-dimensional physics, such as true photon propagation and transverse field modes. Furthermore, the model considers a massless confined field coupled to a massive external field. This specific configuration, though chosen for analytical clarity, may not capture all the dynamics of systems where both fields are massive. Despite these theoretical simplifications, the core phenomenon—a decay rate enhancement due to acceleration—is expected to be robust and can be probed using quantum simulators that effectively realize the key ingredients of our model, namely, an accelerated quantum system coupled to a field environment.

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A Full Asymptotic Expression and its Simplification

This appendix contains a complete derivation of the asymptotic expression for the squared modulus of the overlap integral, given in (5.2). This result justifies the simplification used in the main text to obtain the decay rate.

Starting from the approximation for the overlap integral (5.1), valid under the condition $Ml \ll 1$,

$$f(\Omega_1) \approx l\sqrt{a}\left[e^{i\theta}K_{i\beta+\frac{1}{2}}(M/a) - iK_{i\beta-\frac{1}{2}}(M/a)\right] . \tag{A.1}$$

For $z \to 0$ and fixed ν, the Modified Bessel function of the second kind has the asymptotic expansion [33]: $K_\nu(z) \sim \frac{1}{2}\Gamma(-\nu)\left(\frac{z}{2}\right)^{\nu} + \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}$. Applying this to the term in (A.1) with $z = M/a \ll 1$ gives:

$$K_{i\beta\pm\frac{1}{2}} \sim \frac{1}{2}\Gamma(-i\beta \mp \frac{1}{2})\left(\frac{z}{2}\right)^{i\beta\pm\frac{1}{2}} + \frac{1}{2}\Gamma(i\beta \pm \frac{1}{2})\left(\frac{z}{2}\right)^{-i\beta\mp\frac{1}{2}} . \tag{A.2}$$

After substituting the asymptotic expansions and simplifying, we obtain:

$$|f(\Omega_1)|^2 = \frac{la}{4}\left(\mathrm{I} + \mathrm{II} + \mathrm{III}\right) , \tag{A.3}$$

where the terms I, II and III are defined as follows:

$$\mathrm{I} = \left(\frac{z}{2}\right)^{-1}\left[\frac{2\pi}{\cosh(\pi\beta)} + 2\,\mathrm{Re}\left(ie^{i\theta}\,\Gamma(i\beta+1/2)^2\left(\frac{z}{2}\right)^{-2i\beta}\right)\right] , \tag{A.4}$$

$$\mathrm{II} = \frac{z}{2}\left[\frac{2\pi}{(\beta^2+1/4)\cosh(\pi\beta)} + 2\,\mathrm{Re}\left(ie^{i\theta}\,\Gamma(-i\beta-1/2)^2\left(\frac{z}{2}\right)^{2i\beta}\right)\right] , \tag{A.5}$$

$$\begin{aligned}\mathrm{III} =& -\frac{2}{\beta^2+1/4}\,\mathrm{Re}\left((1/2-i\beta)\,\Gamma(1/2-i\beta)^2\left(\frac{z}{2}\right)^{2i\beta}\right)\\ &+\frac{2\pi}{\cosh(\pi\beta)}\frac{\sin\theta-2\beta\cos\theta}{\beta^2+1/4} .\end{aligned} \tag{A.6}$$

Here $\beta = \frac{\Omega_1}{a} = \frac{\pi(1+s)}{\ln(1+al)}$, and $\Gamma(z)^2$ stands for the square of the Gamma function (not the squared modulus). This expression, while exact within the asymptotic approximation, is complex due to the oscillatory phase $(z/2)^{\pm 2i\beta}$. However, the physical regime of interest $z = (M/a) \ll 1, al \sim 1$ allows for a significant simplification.

Notably, the full expression for $|f(\Omega_1)|^2$ contains terms with coefficients of order $(z/2)^{-1}, (z/2)^0$, and $(z/2)^1$. Term I is proportional to $(z/2)^{-1}$. This is the dominant contribution. Term II is proportional to $(z/2)^1$. Since $z \ll 1$, this term is suppressed by a factor of z^2 compared to Term I and is negligible. Term III is proportional to $(z/2)^0$. Although it is not parametrically small like Term II, its coefficient is subdominant. For the values of $\beta \sim 1$ relevant for $al \sim 1$, the prefactors in Term III $(1/(\beta^2+1/4) \sim 1)$ are order unity. Note, however, the entire Term III is significantly smaller than the the constant part of Term I. For example, for the specific case of the MIT bag model $(\theta = \pi/2)$, the non-oscillatory part of Term III simplifies to $\frac{2\pi}{\cosh(\pi\beta)}\frac{1}{\beta^2+1/4}$, which remains subdominant. The oscillator terms of both Term I and Term III, such as $\mathrm{Re}\left(ie^{i\theta}\,\Gamma(i\beta+1/2)^2\left(\frac{z}{2}\right)^{-2i\beta}\right)$ and $\mathrm{Re}\left((1/2-i\beta)\Gamma(i\beta-1/2)^2\left(\frac{z}{2}\right)^{2i\beta}\right)$ have amplitudes of order unity but oscillate extremely rapidly as a function of β (and thus of the acceleration a) due to the factor $(z/2) = \exp[\pm 2i\beta\ln(z/2)]$. Any infinitesimal experimental uncertainty in the acceleration a would cause these terms to average to zero upon measurement. Therefore, only the non-oscillatory, constant terms contribute to the measurable average.

Consequently, to an excellent approximation, we retain only the constant part of the dominant Term I:

$$|f(\Omega_1|^2 \sim \frac{l^2 a\pi}{z\cosh(\pi\beta)} = \frac{l^2a^2\pi}{M\cosh(\pi\beta)} . \tag{A.7}$$

This is the simplified form used in Eq.(5.6) of the main text to compute the decay probability.

B Asymptotic Analysis of the Overlap integral for $M/a \gg 1$

In this appendix we analyze the asymptotic behavior of the overlap integral (4.4b) in the regimes where $al \ll 1$ and $al \sim 1$ for $M/a \gg 1$.

The overlap integral is defined as:

$$f(\Omega_1) = \int_{\rho_-}^{\rho_-+l} \frac{d\rho}{\sqrt{\rho}} \left[e^{i\theta} \left(\frac{\rho}{\rho_-}\right)^{-i\frac{\Omega_1}{a}} K_{i\frac{\Omega_1}{a}+\frac{1}{2}}(M\rho) - i \left(\frac{\rho}{\rho_-}\right)^{i\frac{\Omega_1}{a}} K_{i\frac{\Omega_1}{a}-\frac{1}{2}}(M\rho) \right], \quad \text{(B.1)}$$

where $\rho_- = 1/a$, $\beta = \Omega_1/a$, and $K_\nu(z)$ is the modified Bessel function of the second kind. For $M/a \gg 1$, the argument $M\rho$ satisfies $M\rho \geq M/a \gg 1$ throughout the integration range, allowing us to use the large-argument expansion: $K_\nu(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z}$.

Changing variables to $u = \rho - \rho_-$ with $d\rho = du$ and limits from 0 to l, we obtain the general asymptotic form:

$$f(\Omega_1) \approx \sqrt{\frac{\pi}{2M}} \int_0^l \frac{du}{\rho_- + u} e^{-M(\rho_-+u)} \left[e^{i\theta} \left(\frac{\rho_- + u}{\rho_-}\right)^{-i\beta} - i \left(\frac{\rho_- + u}{\rho_-}\right)^{i\beta} \right]. \quad \text{(B.2)}$$

We now analyze this integral in two distinct regimes.

Case 1: Short Cavity ($al \ll 1$)

When $al \ll 1$ (i.e., $l \ll 1/a$), we have $u \ll \rho_- = 1/a$ throughout the integration range. This allows the following approximations:

$$\frac{1}{\rho_- + u} \approx \frac{1}{\rho_-} = a\,, \quad \text{(B.3)}$$

$$\left(\frac{\rho_- + u}{\rho_-}\right)^{\pm i\beta} = (1 + au)^{\pm i\beta} \approx e^{\pm i\beta a u} = e^{\pm i\Omega_1 u}, \quad \text{(B.4)}$$

$$e^{-M(\rho_-+u)} = e^{-M/a} e^{-Mu}\,. \quad \text{(B.5)}$$

Substituting these approximations into (B.2) yields:

$$f(\Omega_1) \approx a\sqrt{\frac{\pi}{2M}} e^{-M/a} \int_0^l e^{-Mu} \left[e^{i\theta} e^{-i\Omega_1 u} - i e^{i\Omega_1 u} \right] du\,. \quad \text{(B.6)}$$

This integral can be evaluated exactly, yielding:

$$f(\Omega_1) \approx a\sqrt{\frac{\pi}{2M}}e^{-M/a}\left[\frac{e^{i\theta}(1-e^{-(M+i\Omega_1)l})}{M+i\Omega_1} - \frac{i(1-e^{-(M-i\Omega_1)l})}{M-i\Omega_1}\right]. \tag{B.7}$$

The corresponding squared modulus is:

$$|f(\Omega_1)|^2 \approx \frac{\pi a^2 e^{-2M/a}}{2M}\left|\frac{e^{i\theta}(1-e^{-(M+i\Omega_1)l})}{M+i\Omega_1} - \frac{i(1-e^{-(M-i\Omega_1)l})}{M-i\Omega_1}\right|^2, \tag{B.8}$$

$$\equiv \frac{\pi a^2 l^2 e^{-2M/a}}{M} F_{\text{short}}(a,l,M,s,\theta)\,, \tag{B.9}$$

where the dimensionless prefactor F_{short} is defined as

$$F_{\text{short}}(a,l,M,s,\theta) \equiv \frac{1}{2l^2}\left|\frac{e^{i\theta}(1-e^{-(M+i\Omega_1)l})}{M+i\Omega_1} - \frac{i(1-e^{-(M-i\Omega_1)l})}{M-i\Omega_1}\right|^2. \tag{B.10}$$

We now consider two subcases based on the value of Ml.

Subcase 1a: $Ml \ll 1$

When $Ml \ll 1$, we expand the exponentials in the integrand of (B.6):

$$e^{-Mu} \approx 1 - Mu + \mathcal{O}((Mu)^2), \tag{B.11}$$

$$e^{\pm i\Omega_1 u} \approx 1 \pm i\Omega_1 u + \mathcal{O}((\Omega_1 u)^2)\,. \tag{B.12}$$

Substituting into (B.6) and evaluating the integral yields:

$$f(\Omega_1) \approx a\sqrt{\frac{\pi}{2M}}e^{-M/a}\left[(e^{i\theta}-i)l + \frac{l^2}{2}\left(-i\Omega_1(e^{i\theta}+i) - M(e^{i\theta}-i)\right)\right]. \tag{B.13}$$

The squared modulus is:

$$|f(\Omega_1)|^2 \approx \frac{\pi a^2 e^{-2M/a}}{2M}\left|(e^{i\theta}-i)l + \frac{l^2}{2}\left(-i\Omega_1(e^{i\theta}+i) - M(e^{i\theta}-i)\right)\right|^2. \tag{B.14}$$

Subcase 1b: $Ml \gg 1$

When $Ml \gg 1$, the exponential e^{-Mu} decays rapidly, allowing us to extend the upper limit to infinity:

$$\int_0^l e^{-Mu}\left[e^{i\theta}e^{-i\Omega_1 u} - ie^{i\Omega_1 u}\right]du \approx \int_0^\infty e^{-Mu}\left[e^{i\theta}e^{-i\Omega_1 u} - ie^{i\Omega_1 u}\right]du\,. \tag{B.15}$$

Combining with (B.6), we obtain for the overlap integral:

$$f(\Omega_1) \approx a\sqrt{\frac{\pi}{2M}} e^{-M/a} \left[\frac{e^{i\theta}}{M + i\Omega_1} - \frac{i}{M - i\Omega_1} \right] . \tag{B.16}$$

The squared modulus is:

$$|f(\Omega_1)|^2 \approx \frac{\pi a^2 e^{-2M/a}}{2M} \frac{|M(e^{i\theta} - i) + \Omega_1(1 - ie^{i\theta})|^2}{(M^2 + \Omega_1^2)^2} , \tag{B.17}$$

$$\equiv \frac{\pi a^2 l^2 e^{-2M/a}}{M} G_{\text{short}}(a, l, M, s, \theta) , \tag{B.18}$$

where the dimensionless prefactor G_{short} is defined as

$$G_{\text{short}} = \frac{1}{2l^2} \frac{|M(e^{i\theta} - i) + \Omega_1(1 - ie^{i\theta})|^2}{(M^2 + \Omega_1^2)^2} . \tag{B.19}$$

We have verified that the right-hand side has the correct dimensions of $[M]^{-1}$.

Case 2: Intermediate Cavity ($al \sim 1$)

When $al \sim 1$ (i.e., $l \sim 1/a$), we have $Ml = (M/a)(al) \gg 1$ since $M/a \gg 1$. The exponential factor e^{-Mu} decays rapidly, allowing us to extend the upper limit to infinity:

$$\int_0^l e^{-Mu} \left[e^{i\theta} e^{-i\Omega_1 u} - ie^{i\Omega_1 u} \right] du \approx \int_0^\infty e^{-Mu} \left[e^{i\theta} e^{-i\Omega_1 u} - ie^{i\Omega_1 u} \right] du . \tag{B.20}$$

Evaluating these integrals yields:

$$f(\Omega_1) \approx a\sqrt{\frac{\pi}{2M}} e^{-M/a} \left[\frac{e^{i\theta}}{M + i\Omega_1} - \frac{i}{M - i\Omega_1} \right] . \tag{B.21}$$

The squared modulus is given by (B.18).

Higher-Order Corrections and Subdominant Behavior

The asymptotic expressions derived above represent the leading-order behavior of the overlap integral. Higher-order corrections arise from several sources.

Next-to-Leading Order in Bessel Function Expansion

The large-argument expansion of the modified Bessel function has subleading terms [33]:

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left[1 + \frac{4\nu^2 - 1}{8z} + \mathcal{O}(z^{-2}) \right] . \tag{B.22}$$

For our case with $\nu = i\beta \pm \frac{1}{2}$, the first correction term is of order:

$$\frac{4(i\beta \pm \frac{1}{2})^2 - 1}{8M\rho} = \frac{-4\beta^2 \pm 4i\beta}{8M\rho} = \mathcal{O}\left(\frac{\beta}{M\rho}\right). \tag{B.23}$$

Since $M\rho \geq M/a \gg 1$ and $\beta = \Omega_1/a$ is typically $\mathcal{O}(1)$ or smaller, these corrections are suppressed by at least $\mathcal{O}((M/a)^{-1})$ relative to the leading order.

Approximation of $(1 + au)^{\pm i\beta}$

The approximation $(1 + au)^{\pm i\beta} \approx e^{\pm i\Omega_1 u}$ has corrections of order:

$$(1 + au)^{\pm i\beta} = \exp\left[\pm i\beta \ln(1 + au)\right] = \exp\left[\pm i\Omega_1 u \mp i\beta\frac{a^2u^2}{2} + \mathcal{O}((au)^3)\right]. \tag{B.24}$$

For $al \ll 1$, we have $au \leq al \ll 1$, so these corrections are suppressed by $\mathcal{O}((al)^2)$. For $al \sim 1$, the exact integral evaluation in (B.21) inherently accounts for these corrections.

Extension of Integration Limits

The extension of the upper integration limit from l to ∞ in Case 2 introduces an error of order:

$$\int_l^\infty e^{-Mu}\left[e^{i\theta}e^{-i\Omega_1 u} - ie^{i\Omega_1 u}\right] du \sim e^{-Ml} = e^{-(M/a)(al)}. \tag{B.25}$$

Since $M/a \gg 1$ and $al \sim 1$, this error is exponentially small and thus subdominant.

Summary

The original integral (C.1) has dimension: $[f] = [l][a^{1/2}] = [M^{-1}][M^{1/2}] = [M^{-1/2}]$, since l has dimension $[M^{-1}]$ and a has dimension $[M]$. The results maintain dimensional consistency throughout and demonstrate that for $M/a \gg 1$, in both the short-cavity regime ($al \ll 1$) and intermediate cavity regime ($al \sim 1$), the overlap integral exhibits exponential suppression: $f(\Omega_1) \sim e^{-M/a} \times$ (prefactor). Higher-order corrections are shown to be subdominant in all cases, suppressed by factors of $(M/a)^{-1}$, $(al)^2$, or exponentially small terms.

C Asymptotic Analysis of the Overlap Integral for $al \gg 1$

This appendix details the asymptotic behavior of the overlap integral (4.4b) when $al \gg 1$ ($\beta := \Omega_1/a \ll 1$). The analysis is presented for two subcases: Case A for which $Ml \ll 1$ and Case C for which $M/a \gg 1$ (large mass relative to acceleration).

We begin by rewriting the overlap integral (4.4b) for a left-wall observer located at $\rho_- = 1/a$:

$$f = \int_{1/a}^{1/a+l} \frac{d\rho}{\sqrt{\rho}} \left[e^{i\theta}(a\rho)^{-i\beta} K_{i\beta+\frac{1}{2}}(M\rho) - i(a\rho)^{i\beta} K_{i\beta-\frac{1}{2}}(M\rho) \right] . \tag{C.1}$$

We expand in powers of β using

$$(a\rho)^{\pm i\beta} = e^{\pm i\beta \ln(a\rho)} = 1 \pm i\beta \ln(a\rho) - \frac{\beta^2}{2} \ln^2(a\rho) + \cdots , \tag{C.2}$$

$$K_{i\beta\pm\frac{1}{2}}(z) = K_{\frac{1}{2}}(z) \pm i\beta \left. \frac{\partial K_\nu}{\partial \nu} \right|_{\nu=\frac{1}{2}} - \frac{\beta^2}{2} \left. \frac{\partial^2 K_\nu}{\partial \nu^2} \right|_{\nu=\frac{1}{2}} + \cdots . \tag{C.3}$$

Using the symmetry $K_{-\nu}(z) = K_\nu(z)$ (with $z = M\rho$), we obtain:

$$f = f^{(0)} + \beta f^{(1)} + \beta^2 f^{(2)} + \cdots , \tag{C.4}$$

where

$$f^{(0)} = (e^{i\theta} - i) \int \frac{d\rho}{\sqrt{\rho}} K_{\frac{1}{2}}(M\rho) , \tag{C.5}$$

$$f^{(1)} = (ie^{i\theta} - 1) \int \frac{d\rho}{\sqrt{\rho}} \left[D(M\rho) - \ln(a\rho) K_{\frac{1}{2}}(M\rho) \right] , \tag{C.6}$$

$$f^{(2)} = (e^{i\theta} - i) \int \frac{d\rho}{\sqrt{\rho}} \left[-\frac{1}{2} D_2(M\rho) + \ln(a\rho) D(M\rho) - \frac{1}{2} \ln^2(a\rho) K_{\frac{1}{2}}(M\rho) \right] , \tag{C.7}$$

with $D(z) = \left. \frac{\partial K_\nu}{\partial \nu} \right|_{\nu=\frac{1}{2}}$ and $D_2(z) = \left. \frac{\partial^2 K_\nu}{\partial \nu^2} \right|_{\nu=\frac{1}{2}}$

Case A: $Ml \ll 1$

For $z = M\rho \ll 1$, we employ small-argument expansions [33]

$$K_{\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \approx \sqrt{\frac{\pi}{2z}} , \tag{C.8}$$

$$D(z) \approx -\sqrt{\frac{\pi}{2z}} \left(\ln z + \gamma \right) , \tag{C.9}$$

$$D_2(z) \approx \sqrt{\frac{\pi}{2z}} \left(\ln^2 z + 2\gamma \ln z + \gamma^2 \right) , \tag{C.10}$$

where γ is the Euler-Mascheroni constant. The zero-order term becomes

$$f^{(0)} \approx (e^{i\theta} - i) \sqrt{\frac{\pi}{2M}} \ln(al) . \tag{C.11}$$

The first-order term evaluates to

$$f^{(1)} \approx -(ie^{i\theta} - 1)\sqrt{\frac{\pi}{2M}} \left[\frac{1}{2}\ln^2(al) + (\gamma + \ln M)\ln(al)\right] . \tag{C.12}$$

The second-order term yields

$$f^{(2)} \approx -\frac{1}{2}(e^{i\theta} - i)\sqrt{\frac{\pi}{2M}} \left[\frac{1}{3}\ln^3(al) + (\gamma + \ln M)\ln^2(al)\right] \tag{C.13}$$

Case C: $M/a \gg 1$

For $z = M\rho \gg 1$, we use large-argument expansions [33]:

$$K_{\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} e^{-z} , \tag{C.14}$$

$$D(z) \sim \frac{1}{2}\sqrt{\frac{\pi}{2}} e^{-z} z^{-3/2} , \tag{C.15}$$

$$D_2(z) \sim -\frac{3}{4}\sqrt{\frac{\pi}{2}} e^{-z} z^{-5/2} . \tag{C.16}$$

The zero-order term becomes

$$f^{(0)} = (e^{i\theta} - i)\sqrt{\frac{\pi}{2M}} \int_{1/a}^{1/a+l} \frac{e^{-M\rho}}{\rho} d\rho \approx (e^{i\theta} - i)a\sqrt{\frac{\pi}{2}} M^{-3/2} e^{-M/a} . \tag{C.17}$$

The first-order term evaluates to

$$f^{(1)} \approx \beta(ie^{i\theta} - 1)\sqrt{\frac{\pi}{2}} \frac{a^2}{2} M^{-5/2} e^{-M/a} . \tag{C.18}$$

The second-order term yields

$$f^{(2)} \approx \beta^2(e^{i\theta} - i)\sqrt{\frac{\pi}{2}} \frac{a^3}{8} M^{-7/2} e^{-M/a} . \tag{C.19}$$

The dominant term depends on the regime: For $Ml \ll 1$: $f^{(0)} \sim \ln(al)$ dominates. For $M/a \gg 1$: All terms are exponentially suppressed by $e^{-M/a}$. The second-order terms are subdominant in both regimes. Notably, all terms maintain dimensional consistency: $[f] = [l\sqrt{a}] = [M^{-1}] \cdot [M^{1/2}] = [M^{-1/2}]$, and each term in the expansion has this dimension after accounting for the dimensionless factors.

These results demonstrate the transition from logarithmic behavior for light fields to exponential suppression for heavy fields, with the second-order terms in β being subdominant in both cases.

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