

Electromagnetic Observables of Weakly Collisional Black Hole Accretion

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ABSTRACT

The black holes in the Event Horizon Telescope sources Messier 87* and Sagittarius A* (Sgr A*) are embedded in a hot, collisionless plasma that is fully described in kinetic theory yet is usually modeled as an ideal, magnetized fluid. In this Letter, we present results from a new set of weakly collisional fluid simulations in which leading order kinetic effects are modeled as viscosity and heat conduction. Consistent with earlier, lower-resolution studies, we find that overall flow dynamics remain very similar between ideal and non-ideal models. For the first time, we synthesize images and spectra of Sgr A* from weakly collisional models—assuming an isotropic, thermal population of electrons—and find that these remain largely indistinguishable from ideal fluid predictions. However, most weakly collisional models exhibit lower light curve variability, with all magnetically dominated models showing a small but systematic decrease in variability.

1. INTRODUCTION

In recent years, the Event Horizon Telescope (EHT) has produced high-resolution radio images ($\lambda \sim 1.3\text{mm}$) of the supermassive black hole at the center of the Milky Way [Event Horizon Telescope Collaboration et al. \(2022a,b,c,d,e,f, 2024a,b\)](#). These images reveal a ring-like structure with an ordered linear polarization pattern, consistent with synchrotron emission from relativistic plasma accreting onto the black hole. Interpretation of these observations relies on general relativistic magnetohydrodynamics (GRMHD) simulations of black hole accretion ([Narayan et al. 2022; Dhruv et al. 2025](#)). Synthetic images and spectra produced from these simulations, when compared with EHT data and observations of Sgr A* at other wavelengths, constrain the state of the accreting plasma and the spacetime in the vicinity of the black hole ([Event Horizon Telescope Collaboration et al. 2022e,f, 2024b](#)).

As part of the 2017 EHT campaign, the Atacama Large Millimeter/submillimeter Array (ALMA) recorded long-duration (3–10 hrs), high-cadence (4 s) 230 GHz light curves of the Galactic Center ([M. Wielgus et al. 2022](#)). The measured source variability, characterized by the modulation index σ/μ , was consistent with previous 230 GHz measurements of Sgr A*. However, most simulations in the EHT analysis exhibited a higher modulation index, meaning they were more variable than the actual source ([Event Horizon Telescope Collabora-](#)

[tion et al. 2022e](#)). One possible explanation for this discrepancy, the *variability crisis*, is missing physics in the numerical models.

In particular the simulations used to interpret EHT observations use an ideal GRMHD (IGRMHD) model, which treats the relativistic plasma as a fluid in local thermodynamic equilibrium ([Gammie et al. 2003; Mizuno & Rezzolla 2024](#)). Accretion flows surrounding low-luminosity active galactic nuclei (LLAGNs) like Sgr A* are, however, Coulomb-collisionless ([Mahadevan & Quataert 1997](#)), which opens the possibility that finite mean free path effects may alter both accretion dynamics and horizon-scale emission ([Galishnikova et al. 2023b,a](#)). For example, magnetic reconnection, a mechanism considered to explain high-energy flares observed in black hole accretion systems ([Nathanail et al. 2020; Ripperda et al. 2022; Hakobyan et al. 2023; von Fellenberg et al. 2025; Solanki et al. 2025; Sironi et al. 2025](#)), is slower in the collisional regime compared to the collisionless expectation. The hierarchy of scales motivates a kinetic treatment of the problem, such as particle-in-cell (PIC) methods, that resolve the relevant microscopic scales in the plasma. Indeed, recent multidimensional, global, kinetic studies have explored black hole magnetospheres in the force-free limit ([Parfrey et al. 2019; Crinquand et al. 2020, 2021, 2022; El Mellah et al. 2022, 2023](#)) and have also modeled black hole accretion ([Galishnikova et al. 2023b; Vos et al.](#)

2025). However, these simulations are too computationally expensive to permit the large-scale parameter surveys necessary for systematic comparison with observational data. Additionally, collective plasma phenomena such as pitch-angle scattering due to kinetic instabilities (see e.g., Bott et al. 2024 and references therein) and stochastic plasma echoes that stifle phase mixing in turbulent systems (Parker et al. 2016; Schekochihin et al. 2016; Meyrand et al. 2019) tend to make the plasma more fluid-like. In this work we model the plasma as a *weakly collisional*, or equivalently, a *dissipative* fluid, where non-ideal effects are introduced as deviations from thermodynamic equilibrium.

The theory of relativistic dissipative fluids is intricate, with some models prone to pathologies such as acausality and instability (Hiscock & Lindblom 1983, 1985, 1988; Garcia-Perciante et al. 2009; Denicol & Rischke 2021). Over the past few decades, some advancements were driven by the need to model quark-gluon plasma formed in relativistic heavy-ion collisions (Rocha et al. 2024; Shen & Yan 2020; Romatschke & Romatschke 2017; Bernhard et al. 2019; Noronha-Hostler et al. 2009; Heinz & Snellings 2013; Gale et al. 2013). The framework was also applied in cosmology, (Padmanabhan & Chitre 1987; Zakari & Jou 1993; Maartens 1995; Piatella et al. 2011; Brevik & Grøn 2013, 2014; Brevik et al. 2017), and more recently, to the study of ultradense matter formed during neutron stars mergers (e.g., Most et al. 2024, 2025), and the dynamics of radiatively-inefficient accretion disks around supermassive black holes (Chandra et al. 2015; Foucart et al. 2016, 2017).

In this Letter, we use the Extended GRMHD (EGRMHD) model of Chandra et al. (2015) in global 3D simulations of black hole accretion. EGRMHD modifies ideal GRMHD by including heat conduction along magnetic field lines and shear viscosity. The theory is causal and strongly hyperbolic (Cordeiro et al. 2024). This Letter goes beyond Foucart et al. (2017) by generating synthetic horizon-scale images and spectral energy distributions (SEDs) from a new set of high-resolution, long-duration EGRMHD simulations. By comparing images and SEDs with those from corresponding IGRMHD simulations, we evaluate the impact of dissipative physics on electromagnetic observables. We find that time-averaged images and spectra are almost unchanged, although simulations incorporating dissipative effects produce 230 GHz light curves with reduced variability on three-hour timescales.

2. METHODS

We have modified the ideal, GPU-enabled GRMHD code KHARMA (Prather 2024) to simulate weakly col-

lisional accretion onto a black hole. The code evolves two additional scalar variables: the scalar heat flux along the magnetic field, q , and the pressure anisotropy defined with respect to the local magnetic field ΔP ($\Delta P = P_{\perp} - P_{\parallel}$). These are defined as $q^{\mu} \equiv q\hat{b}^{\mu}$ and $\pi^{\mu\nu} \equiv -\Delta P(\hat{b}^{\mu}\hat{b}^{\nu} - \frac{1}{3}h^{\mu\nu})$, where q^{μ} is the heat flux four-vector, $\hat{b}^{\mu}$ is the unit magnetic field four-vector, $\pi^{\mu\nu}$ is the shear-stress tensor, and $h^{\mu\nu}$ is the projection tensor onto a spatial slice orthogonal to the fluid four-velocity u^{μ} . The evolution equations for q and ΔP include source terms with time derivatives, requiring a locally semi-implicit time-stepping scheme (Chandra et al. 2017; see Appendix A for details on the algorithm used in EGRMHD simulations and a suite of test problems validating the implementation in KHARMA). EGRMHD simulations are $\sim 10\times$ more expensive than their ideal counterparts.

Our simulations are initialized with a hydrostatic equilibrium torus solution (Fishbone & Moncrief 1976). The solution has two free parameters: the radius at the inner edge of the disk r_{in} and the pressure maximum radius r_{max} . We seed the torus with a poloidal magnetic field, and as the fluid accretes, the magnetic field is dragged along (consistent with Alfvén’s theorem, Alfvén 1942), causing magnetic flux to accumulate on the event horizon. The accumulated flux is characterized by the dimensionless flux $\phi_b \equiv \Phi_{\text{BH}}/(\dot{M}r_g^2c)^{1/2}$ (here Φ_{BH} is the net magnetic flux crossing one hemisphere of the event horizon; Tchekhovskoy et al. 2011).

The evolution exhibits two distinct states, depending on ϕ_b . In the magnetically-arrested state (MAD; Narayan et al. 2003; Igumenshchev et al. 2003; Tchekhovskoy et al. 2011) $\phi_b \sim 16$ and magnetic flux grows until it is large enough to halt accretion. Flux is then expelled in a violent eruption event, and flux accumulation begins again. In the standard and normal evolution state (SANE; Narayan et al. 2012; Sądowski et al. 2013) $\phi_b \ll 16$ and the magnetic field is comparatively weak and drives outward transport of angular momentum via the magnetorotational instability (MRI). Our initial magnetic field is expressed in terms of a vector potential A_{ϕ} , which is $\max[\rho/\rho_{\text{max}}(r/r_{\text{in}}\sin\theta)^3e^{-r/400} - 0.2, 0]$ for MAD models and $\max[\rho/\rho_{\text{max}} - 0.2, 0]$ for SANE models. Here, ρ is the fluid rest-mass density and ρ_{max} is the maximum density in the initial torus.

We consider four EGRMHD simulations: SANE and MAD at black hole spin $a_* = 0, 15/16$ ($a_* \equiv Jc/(GM^2)$; hereafter we adopt units such that $GM = c = 1$). We also conduct four otherwise identical IGRMHD simulations as controls. The governing equations are solved in modified spherical Kerr-Schild coordinates (FMKS; Wong et al. 2022) which concentrates grid zones at the

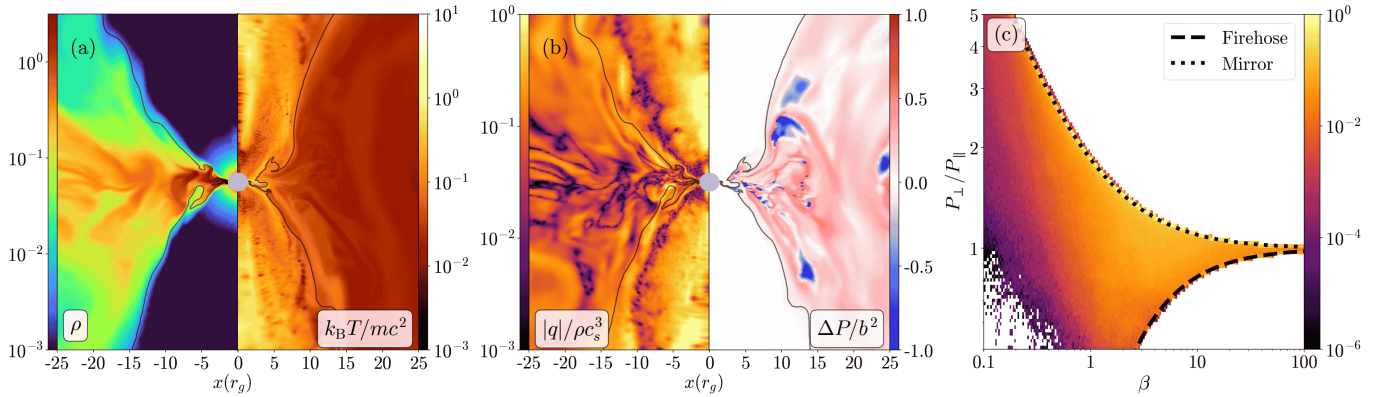


Figure 1. Snapshot from an EGRMHD, MAD, $a_* = 15/16$ simulation. (a) Poloidal (r, θ) slices of fluid rest-mass density (left) and dimensionless temperature (right). (b) Poloidal slices of heat flux normalized by the free-streaming value (left) and pressure anisotropy normalized by magnetic energy density (right). The black contour marks $\sigma = 1$, which separates the accretion disk from the magnetically dominated jet. (c) Mass-weighted distribution of pressure anisotropy as a function of β within $r \leq 20r_g$ for the snapshot shown in panels (a) and (b). The black dotted (dashed) lines indicate the mirror (firehose) instability threshold.

midplane close to the event horizon. The computational domain has $N_r \times N_\theta \times N_\phi = 384 \times 192 \times 192$ resolution elements. The grid extends radially from just inside the event horizon to $1000 r_g$, with $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. Here $r_g \equiv GM/c^2$ is the gravitational radius. Each simulation is evolved to $t = 30,000 t_g$ ($t_g \equiv r_g/c$), which is long enough to enable a comparison of model and observed light curve variability (Lee & Gammie 2022; Event Horizon Telescope Collaboration et al. 2022e; M. Wielgus et al. 2022).

Our GRMHD simulations evolve a single fluid, with electron temperature determined from a parameterized model in the radiative transfer calculations. We assume a thermal electron distribution function and prescribe the electron temperature using the “ R_{high} ” model (Mościbrodzka et al. 2016), which is motivated by models of kinetic dissipation of Alfvénic turbulence (Quataert 1998; Quataert & Gruzinov 1999; Howes 2010; Gammie 2025). To generate synthetic images we use the polarized ray-tracing code *ipole* (Mościbrodzka & Gammie 2018). To generate SEDs we use the Monte Carlo radiation transport code *igrmonty* (Dolence et al. 2009) which accounts for synchrotron, bremsstrahlung, and Compton scattering. The simulation suite is summarized in Table 1.

3. RESULTS

The initial state evolves due to (1) winding of initially radial field lines by differential rotation and (2) the MRI (Balbus & Hawley 1991; Hawley & Balbus 1991, 1992; Balbus & Hawley 1992; Hawley et al. 1995; Balbus & Hawley 1998). SANE models do not accumulate significant magnetic flux on the horizon and flow dynamics are governed primarily by fluid forces, with $\beta \sim 10$ in the disk ($\beta \equiv P_{\text{gas}}/P_{\text{mag}}$ is the ratio of fluid pressure

Table 1. Summary of GRMHD and GRRT parameters for EGRMHD and IGRMHD simulations. $\hat{\gamma}$ is the adiabatic index of the fluid^a; r_{in} (r_{max}) is the inner (pressure-maximum) radius of the initial torus. R_{high} is a free parameter in the emission model that sets the ion-to-electron temperature ratio; i is the inclination angle (angle between the line of sight and the black hole spin axis). All synthetic images have a $200\mu\text{as}$ field of view.

Flux	a_*	$\hat{\gamma}$	r_{in}	r_{max}	R_{high}	$i(^{\circ})$
MAD	0	13/9	20	41	1,10,40,160	10,30,...,90
MAD	15/16	13/9	20	41	1,10,40,160	10,30,...,90
SANE	0	4/3	10	20	1,10,40,160	10,30,...,90
SANE	15/16	4/3	10	20	1,10,40,160	10,30,...,90

^aThe adiabatic index was chosen to enable comparison with earlier EHT-related simulations (Wong et al. 2022; Dhruv et al. 2025); a better choice for two-temperature, collisionless accretion flows would be $\hat{\gamma}$ slightly less than $5/3$ (Gammie 2025; Chael 2025).

to magnetic pressure). MAD models accumulate significant magnetic flux, generating strong, ordered magnetic fields ($\beta \sim 1$ in the disk near the black hole), relativistic jets along the spin axis, and intermittent flux-eruption events (Tchekhovskoy et al. 2011; Ripperda et al. 2022; Chatterjee & Narayan 2022; Gelles et al. 2022). MAD models are favored for EHT sources (Event Horizon Telescope Collaboration et al. 2019, 2022e). These features of SANE and MAD models are also observed in our EGRMHD simulations.

The non-ideal fields are initialized to zero but evolve on the dynamical time $\tau_d \equiv (r^3/GM)^{1/2}$ toward their corresponding (covariant) Braginskii values (Braginskii 1965). The pressure anisotropy cannot grow unbounded and is limited by the onset of mirror (Hasegawa 1969; Southwood & Kivelson 1993; Kivelson & Southwood

1996; Kunz et al. 2014) and firehose (Rosenbluth & Longmire 1957; Chandrasekhar et al. 1958; Parker 1958; Gary et al. 1998; Bott et al. 2025) instabilities, which pitch-angle scatter the particles. The EGRMHD model incorporates this effect by increasing the effective scattering rate as the instability boundaries are approached. The effect is to confine ΔP within the mirror and firehose bounds, as suggested by PIC simulations (Kunz et al. 2014). The model incorporates a similar increase in scattering rate as the heat flux approaches the free-streaming value $q_{\text{max}} \simeq \rho c_s^3$ (c_s is the sound speed).

Figure 1 shows a snapshot of the MAD $a_* = 15/16$ simulation at $t \sim 30,000 t_g$. At this point a quasi-steady state is well established in the inner regions of the accretion flow. We find that the heat flux remains well below its free-streaming value near the disk midplane, with $q \lesssim 0.1 q_{\text{max}}$, and has a negligible impact on the flow's thermodynamics. An appreciable fraction of the disk mass lies near the mirror and firehose thresholds: $\Delta P > 0.99 \Delta P_{\text{mirror}}$ or $\Delta P > 0.99 \Delta P_{\text{firehose}}$, where $\Delta P_{\text{mirror}} \equiv b^2/2 \cdot (P_{\parallel}/P_{\perp})$ ($\sim b^2/2$ when $\beta \gg 1$), and $\Delta P_{\text{firehose}} \equiv -b^2$. Within the inner $20 r_g$, 40-45% of the plasma reaches the mirror threshold and 2-3% is at the firehose threshold. Panel (c) shows the mass-weighted distribution of pressure anisotropy in the $(\beta, P_{\perp}/P_{\parallel})$ plane. Notably, this tendency for saturation at the instability thresholds is more pronounced in SANE simulations where $\sim 65\%$ of the disk resides at mirror threshold and $\sim 10\%$ at the firehose threshold (see Appendix B for a more detailed comparison of SANE and MAD simulations).

We model mirror and firehose instabilities as mechanisms that regulate the growth of pressure anisotropy, motivated by solar wind measurements (Hellinger et al. 2006; Bale et al. 2009) which show that for $\Delta P > 0$ the plasma anisotropy is bound by the mirror instability threshold, and exceeds the predicted ion cyclotron (IC) threshold. However, this could be due to the assumption of a bi-Maxwellian plasma when calculating the IC threshold (Isenberg 2012; Isenberg et al. 2013). PIC simulations of magnetically-dominated ($\beta \lesssim 1$) electron-ion plasmas that are motivated by MAD accretion flows find that the anisotropy of each species in the saturated state is predominantly set by its respective cyclotron instability (Dhruv et al., in preparation). In a future study we will incorporate the IC threshold in our equations to study its potential importance on the flow dynamics.

Figure 2 compares the time-averaged structures of the MAD $a_* = 15/16$ IGRMHD and EGRMHD simulations. The models are remarkably similar, as shown in panel (a), which shows the azimuthally averaged profiles of ρ and plasma magnetization $\sigma \equiv b^2/\rho$ averaged

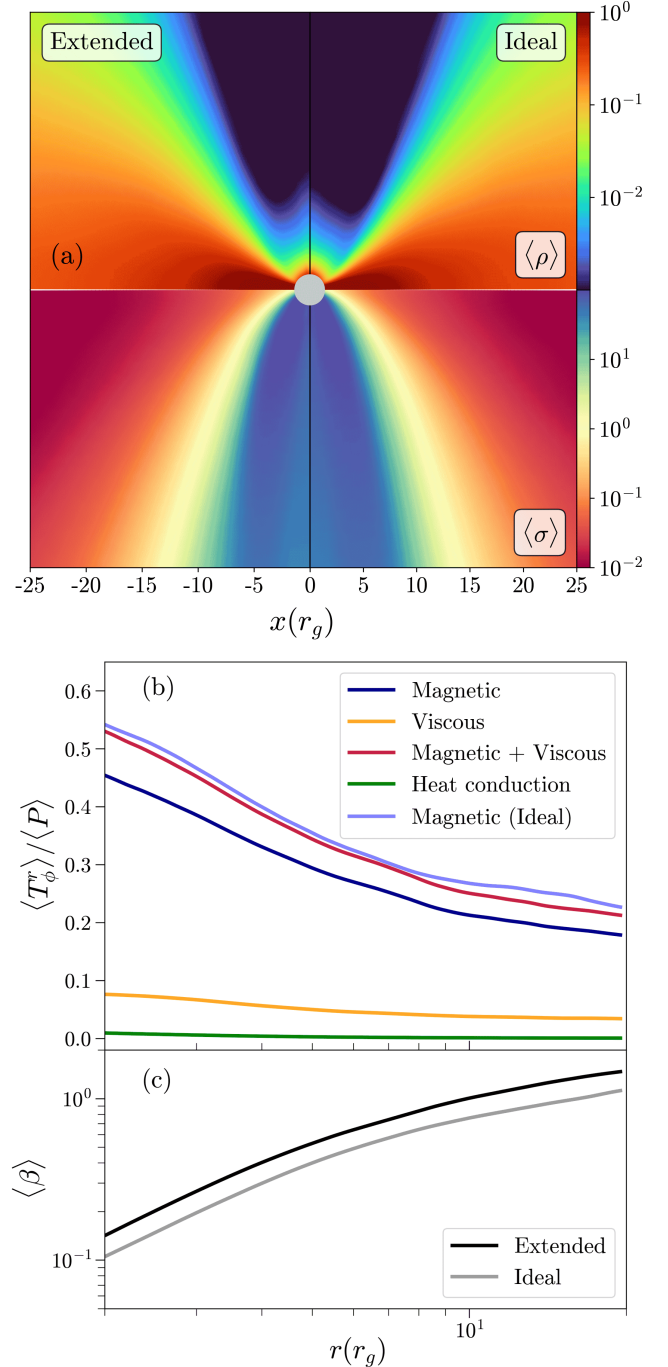


Figure 2. Comparison of time-averaged fluid quantities between Extended and Ideal simulations (MAD, $a_* = 15/16$). (a) The top row shows the time- and azimuthally-averaged rest-mass density, ρ , and the bottom row displays plasma magnetization, σ . The left column shows the Extended simulation and the right column shows the Ideal simulation. (b) Angular momentum transport in the disk: components of the density-weighted average $\langle T_\phi^r \rangle$ normalized by gas pressure $\langle P \rangle$ are plotted as a function of radius. (c) Radial profiles of β for Extended and Ideal simulations.

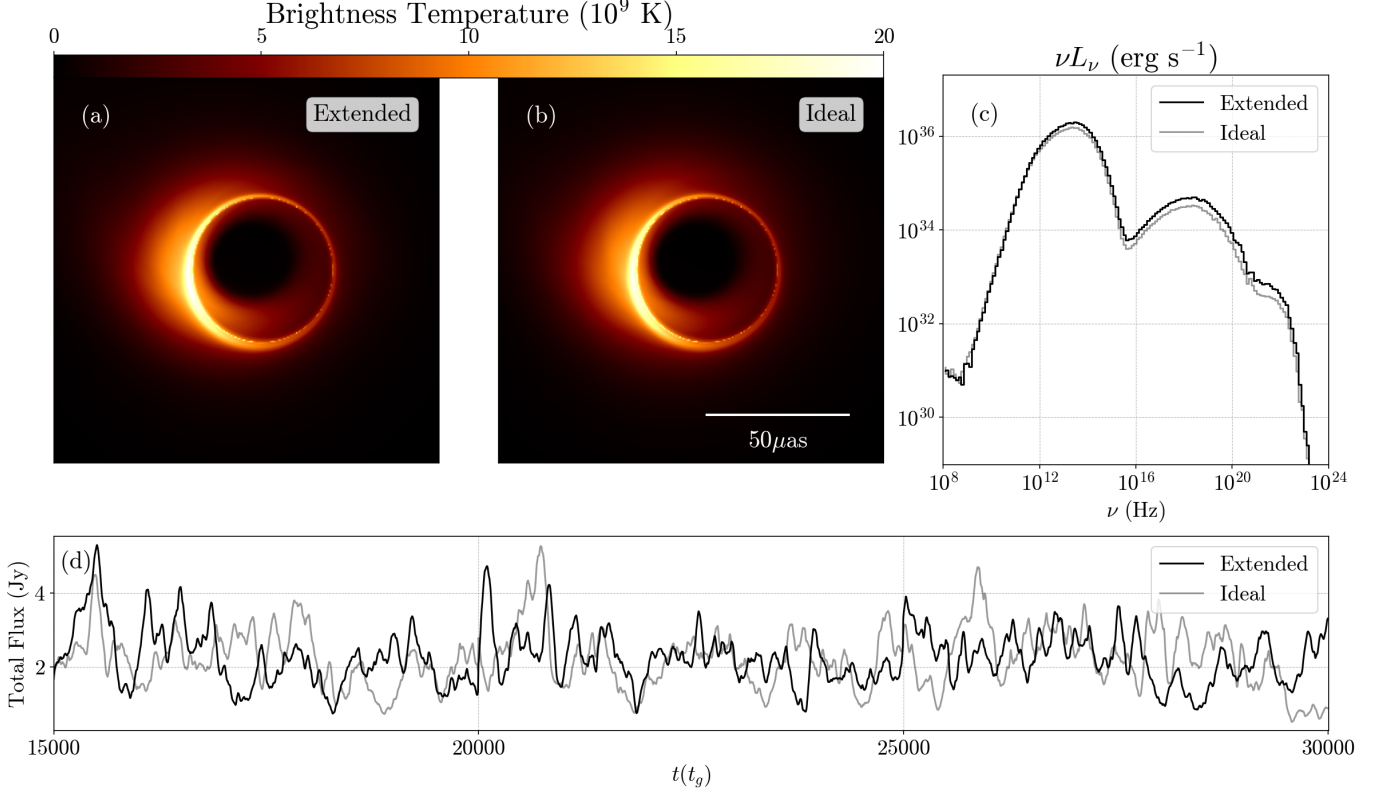


Figure 3. Electromagnetic observables for MAD $a_* = 15/16$ simulation with $R_{\text{high}} = 160$ at a viewing angle of 30° . (a) and (b) Time-averaged 230 GHz total intensity images for the Extended and Ideal GRMHD simulations, respectively. (c) Comparison of time-averaged spectra for the simulations shown in (a) and (b). (d) Light curves at 230 GHz over a duration of $\Delta t = 15,000 t_g$ (~ 84 hours for Sgr A*).

over 15,000 to 30,000 t_g . In MAD models strong magnetic fields govern the dynamics, suppressing the influence of viscous stresses and heat conduction. Although MAD models exhibit larger pressure anisotropy $\Delta P/P$ than SANE models (see Figure 8), the anisotropy remains small, on average, relative to magnetic pressure. The pressure anisotropy is equivalent to a viscosity, and its inclusion provides an additional mechanism for angular momentum transport through the shear stress $\sim -\Delta P \hat{b}^r \hat{b}_\phi$. In EGRMHD simulations the combined magnetic and viscous angular momentum flux is comparable to the magnetic flux in IGRMHD simulations (see panel (b) in Figure 2) and the EGRMHD disk is 20 – 30% less magnetized (see panel (c) in Figure 2). In summary, we find that the time-averaged structure of the accretion flow is remarkably similar in weakly collisional and ideal models, despite the rapid growth of pressure anisotropy.

Figure 3 shows electromagnetic observables for one of EHT’s preferred models of the Galactic center (Event Horizon Telescope Collaboration et al. 2022e)—MAD, $a_* = 15/16$, $R_{\text{high}} = 160$, and an inclination angle of 30° . Panels (a) and (b) plot total intensity images for

the EGRMHD and IGRMHD simulations, respectively, averaged over 5000 t_g , while panel (c) compares their SEDs. The time-averaged radiative signatures of the weakly collisional fluid models are nearly indistinguishable from those of the ideal models. We attribute this to the similarity in fluid structures between the two plasma models, which governs the ion-to-electron temperature ratio T_i/T_e for the chosen emission model, along with the assumption of an isotropic Maxwellian electron distribution function. This strong similarity in synthetic observables is a general feature of our models.

Submillimeter-wavelength observations of Sgr A* suggest that its light curve can be modeled as a red noise process on timescales ranging from minutes to a few hours (Dexter et al. 2014; B. Georgiev et al. 2022; M. Wielgus et al. 2022). EHT analyses of Sgr A* (Event Horizon Telescope Collaboration et al. 2022e) characterized the light curve variability using the three-hour modulation index M_3 (where $M_{\Delta t} \equiv \sigma_{\Delta t}/\mu_{\Delta t}$, $\sigma_{\Delta t}$ is the standard deviation measured over the interval Δt , and $\mu_{\Delta t}$ is the mean over the same interval). All ideal GRMHD MAD models, along with a significant fraction

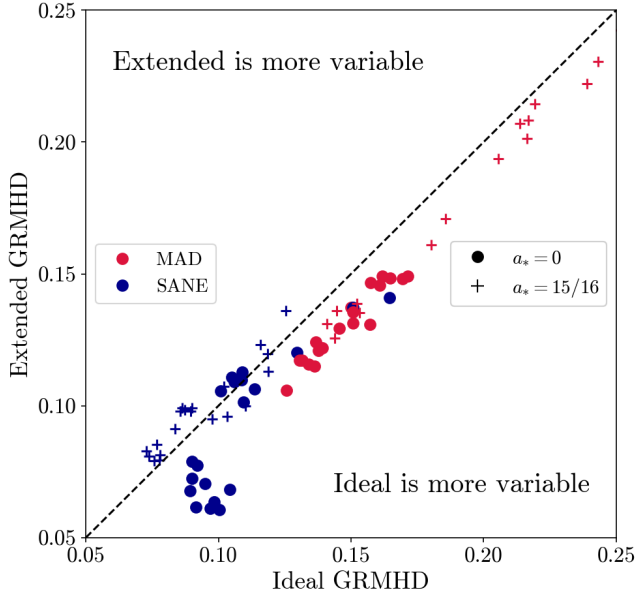


Figure 4. Three-hour modulation index for all models considered in this work. Marker color represents the magnetization state (MAD vs SANE), while marker shape indicates the black hole spin. Points below the dashed line represent models where the Extended GRMHD simulation produces 230 GHz light curves with lower variability than their Ideal counterparts.

of SANE models, were found to exhibit excess variability compared to observations.

Figure 4 highlights the variability trends in EGRMHD and IGRMHD simulations. We analyzed independent three-hour segments of the light curve over $15,000 t_g$ (from $t = 15,000$ to $30,000 t_g$; see panel (d) in Figure 3), corresponding to ~ 83 hours for Sgr A* and yielding a sample size of 27 intervals for the M_3 analysis. We find that $\approx 75\%$ of weakly collisional models (marginalized over all GRMHD and GRRT parameters) exhibit lower variability than their ideal counterparts. Although this reduction does not fully reconcile the discrepancy with observations (e.g., M. Wielgus et al. 2022 report $M_3 \in [0.024, 0.051]$ from April 5-10, 2017), it is notable that *all* MAD EGRMHD simulations show a systematic decrease in variability. The power spectral density (PSD) of the lightcurves indicates that both EGRMHD and IGRMHD simulations exhibit similar slopes at timescales $\lesssim 2$ hours. EGRMHD simulations with lower M_3 relative to the ideal case generally show reduced variability at timescales longer than ~ 1 hour.

Although variations in electron anisotropy might change light curve variability that is not possible here because we have assumed an isotropic electron distribution. The reduced variability of EGRMHD models must

therefore be caused by changes in the EGRMHD fluid evolution. This is plausibly explained by lower turbulent intensity in EGRMHD models, as evidenced by higher average β (see Figure 2): weaker field implies weaker velocity and density fluctuations, lowering light curve variability.

4. SUMMARY

We have studied the impact of low collisionality on the structure and observables associated with low luminosity black hole accretion. We used the Extended GRMHD model (Chandra et al. 2015), which incorporates leading order corrections to the ideal GRMHD model: pressure anisotropy, or equivalently viscosity, and heat conduction. We consider both MAD and SANE (strongly and weakly magnetized) models. We find the flow structure in the Extended and Ideal GRMHD models to be similar. The only significant difference we observe is a higher level of magnetization in the IGRMHD disk. These results are consistent with previous work (Foucart et al. 2017), which integrated the same physical model at lower numerical resolution (up to $6x$). Whether these findings persist at even higher numerical resolutions is an open question, as viscous shear stresses do seem to be sensitive to resolution in a local, nonrelativistic Braginskii model (Kempinski et al. 2019).

We have also presented the first event-horizon-scale images and spectra of weakly collisional accretion models of Sgr A*. We find 230 GHz lightcurves of Sgr A* from weakly collisional, magnetically dominated models to be less variable than their ideal counterparts, resulting in better agreement with observations. The time-averaged properties of the models' radiative signatures, however, strongly resemble corresponding IGRMHD simulations.

We attribute the similarity of weakly collisional and ideal models to (i) pressure-anisotropy limiters that model the effect of plasma instabilities, thereby limiting the influence of pressure anisotropy; and (ii) the use of a simple, isotropic, thermal electron distribution function in estimating the emergent radiation. For example, Salas et al. (2025) showed that radiative cooling in two-temperature models reduces light-curve variability. In addition, in magnetized, collisionless plasmas, electrons naturally develop anisotropies which are regulated by nonresonant instabilities such as mirror and firehose, and by resonant kinetic instabilities, e.g., whistler (Sudan 1963, 1965; Gladd 1983). Anisotropic eDFs can directly affect horizon-scale synthetic observables (Galishnikova et al. 2023a) and viscous stresses may be a dominant source for electron heating in collisionless disks (Sharma et al. 2007). Moreover, nonther-

mal processes in the accretion flow can generate power-law tails (see e.g., [Comisso & Sironi 2021, 2022; Comisso 2024](#)), which—together with electron anisotropy—may help explain limb-brightened jet images of M87* ([Tsuneto et al. 2025](#)).

Looking forward, it would be natural to extend the EGRMHD framework to a full two-fluid model that solves separate evolution equations for electrons and ions. The model would self-consistently predict electron energy density, heat flux, and pressure anisotropy. Any such model is likely to be very expensive, however, and if they are to be truly predictive then the closure relations (e.g. estimates for the viscosity and heat conductivity) would have to be calibrated by kinetic (PIC) simulations.

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APPENDIX

A. EXTENDED GRMHD IN KHARMA

In this section we briefly discuss the EGRMHD model, summarize its implementation in KHARMA, and present results from a suite of test problems that validate its numerical implementation.

A.1. Physical Model

The EGRMHD formalism is a single-fluid description of plasmas that satisfy the hierarchy of length scales $\rho_L \ll \lambda_{\text{mfp}} \ll r_g$, where ρ_L is the particle Larmor radius and λ_{mfp} is the collisional mean free path. This regime implies a collisional plasma with anisotropic transport along the local magnetic field. EGRMHD is a relativistic generalization of the Braginskii model ([Braginskii 1965](#)), replacing constitutive expressions for heat flux and pressure anisotropy with evolution equations,

$$\frac{dq}{d\tau} = -\frac{q - q_0}{\tau_R} - \frac{q}{2} \frac{d}{d\tau} \log\left(\frac{\tau_R}{\chi P^2}\right) \quad (\text{A1a})$$

$$\frac{d\Delta P}{d\tau} = -\frac{\Delta P - \Delta P_0}{\tau_R} - \frac{\Delta P}{2} \frac{d}{d\tau} \log\left(\frac{\tau_R}{\rho \nu P}\right). \quad (\text{A1b})$$

Here χ, ν are thermal and momentum diffusivity respectively, and q_0 and ΔP_0 are the covariant generalizations of the Braginskii heat flux and pressure anisotropy $q_0 \equiv -\rho \chi \hat{b}^\mu (\nabla_\mu \Theta + \Theta a_\mu)$ and $\Delta P_0 \equiv 3\rho \nu (\hat{b}^\mu \hat{b}^\nu \nabla_\mu u_\nu - 1/3 \cdot \nabla_\mu u^\mu)$. τ_R is a relaxation timescale that dictates

how quickly q and ΔP approach their respective Braginskii values. In the collisionless regime, τ_R may be interpreted as the effective mean free time due to wave-particle scattering. τ denotes the proper time, and the operator $d/d\tau \equiv u^\mu \nabla_\mu$ is the relativistic extension of the material (convective) derivative. The evolution equations for q and ΔP are derived from thermodynamic considerations using an Israel-Stewart-like approach ([Israel & Stewart 1979](#)), and are closed by constitutive relations for χ and ν from nonrelativistic collisional theory: $\chi = \phi c_s^2 \tau_R$, $\nu = \psi c_s^2 \tau_R$. Here, c_s is the relativistic sound speed, while ϕ and ψ are dimensionless constants of order unity. These parameters govern the influence of the dissipative terms and are chosen to ensure the model remains causal and stable. The EGRMHD stress-energy tensor is given by,

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu}, \quad (\text{A2})$$

where $T_{\text{ideal}}^{\mu\nu}$ is the IGRMHD stress-energy tensor. A detailed description of the EGRMHD model is provided in [Chandra et al. \(2015\)](#).

A.2. Numerical Implementation and Validation

KHARMA ([Prather 2024](#)) is an open-source C++17 rewrite of the `harm` algorithm ([Gammie et al. 2003](#)) designed to run efficiently on heterogeneous architectures. Originally designed for IGRMHD simulations of black

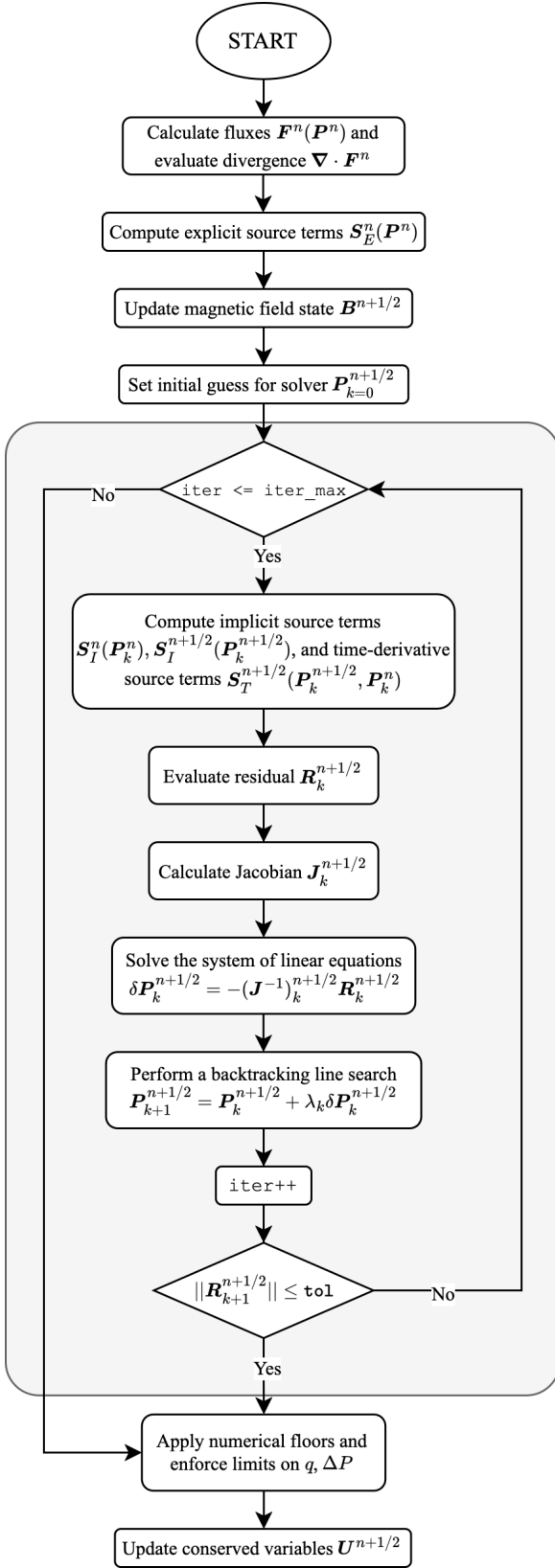


Figure 5. A schematic flowchart illustrating the sequence of operations during a half-step ($t^n \rightarrow t^{n+1/2}$) in the EGRMHD evolution. The gray box marks the **Implicit** kernel, which iteratively determines the next fluid state.

hole accretion, the code fosters extensibility through a package-based framework that simplifies the addition of new physics. Below, we describe our approach to incorporating EGRMHD in KHARMA. We adopt the **grim** algorithm as described in Chandra et al. (2017).

In addition to the eight evolution equations corresponding to the ideal MHD framework that evolve ρ , u^μ and $\mathbf{B}$, EGRMHD evolves q and ΔP . For numerical stability, the code evolves Equations A1a, A1b rescaled by ρ ,

$$\nabla_\mu(\tilde{q}u^\mu) = -\frac{\tilde{q} - \tilde{q}_0}{\tau_R} + \frac{\tilde{q}}{2}\nabla_\mu u^\mu \quad (\text{A3a})$$

$$\nabla_\mu(\Delta\tilde{P}u^\mu) = -\frac{\Delta\tilde{P} - \Delta\tilde{P}_0}{\tau_R} + \frac{\Delta\tilde{P}}{2}\nabla_\mu u^\mu, \quad (\text{A3b})$$

where $\tilde{q} = q(\tau_R/\rho\chi\Theta^2)^{1/2}$ and $\Delta\tilde{P} = \Delta P(\tau_R/\rho\nu\Theta)^{1/2}$. Equations A3a, A3b contain (i) stiff source terms, e.g., $\tilde{q}/\tau_R$ where τ_R can attain a very small value, and (ii) source terms with time derivatives such as $\tilde{q}_0 \sim \nabla_\mu \Theta$, necessitating a local semi-implicit solver. The explicit timestepping scheme for IGRMHD is replaced by a semi-implicit scheme where the fluid variables are updated via a seven-dimensional Newton-Raphson solve, while the magnetic field is updated explicitly.

Figure 5 depicts the algorithm during half-step $t^n \rightarrow t^{n+1/2}$. The initial sequence of operations—calculating the face-centered fluxes $\mathbf{F}^n(\mathbf{P}^n)$ and their divergence $\nabla \cdot \mathbf{F}^n$, evaluating explicit source terms $\mathbf{S}_E^n(\mathbf{P}^n)$, and updating the magnetic field primitives $\mathbf{B}^{n+1/2}$ —are identical to an explicit update. $\mathbf{P}^n$ represents the vector of primitive variables at timestep ‘ n ’. The gray box indicates the series of tasks within the **Implicit** kernel that solves a system of nonlinear equations for the fluid primitives. The initial guess for the solver ($k = 0$) is $\mathbf{P}^n$ along with the updated magnetic field primitives $\mathbf{B}^{n+1/2}$. The solver iteratively refines the estimate for $\mathbf{P}^{n+1/2}$ by finding the roots of the residual $\mathbf{R}^{n+1/2} \equiv (\mathbf{U}^{n+1/2} - \mathbf{U}^n)/(\Delta t/2) + \nabla \cdot \mathbf{F}^n - \mathbf{S}^n$ where $\mathbf{U}$ is the vector of conserved variables and $\mathbf{S}$ is the source term vector (it includes explicit $\mathbf{S}_E$, implicit $\mathbf{S}_I$ source terms and source terms that contain a time-derivative $\mathbf{S}_T$). This procedure is equivalent to solving the evolution equations. A backtracking line search is employed to ensure each iteration improves $\mathbf{P}^{n+1/2}$. Once the prescribed tolerance (**tol**) is reached or the solver exceeds the maximum iteration count (**iter_max**), the code exits the **Implicit** kernel. q and ΔP are adjusted to maintain $q < q_{\text{max}}$ and $\Delta P_{\text{firehose}} \leq \Delta P \leq \Delta P_{\text{mirror}}$. Finally, the half-step conserved variables $\mathbf{U}^{n+1/2}$ are computed. KHARMA then advances the solution from $t^n \rightarrow t^{n+1}$, employing half-step fluxes $\mathbf{F}^{n+1/2}$ and explicit source

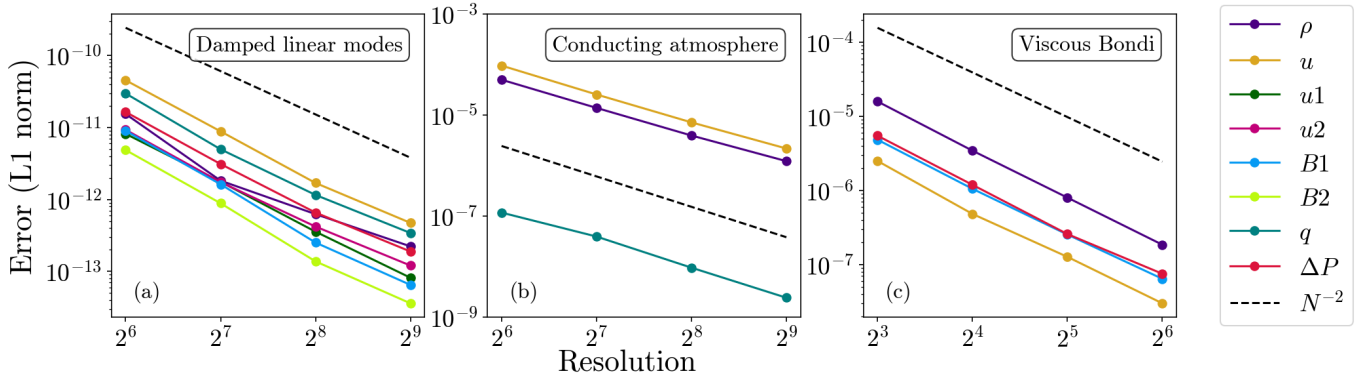


Figure 6. Convergence tests for EGRMHD test problems. (a) Damped linear waves in flat space. (b) Hydrostatic equilibrium in Schwarzschild geometry with a radial temperature gradient. (c) Spherical accretion with anisotropic viscosity, neglecting the backreaction of viscosity on the steady-state inflow.

terms $\mathbf{S}_E^{n+1/2}$, together with end-of-step source terms $\mathbf{S}_I^{n+1}$ and $\mathbf{S}_T^{n+1}$, to obtain the updated primitives $\mathbf{P}^{n+1}$.

We validate the EGRMHD implementation in KHARMA using a suite of test problems detailed in Chandra et al. (2017). Figure 6 shows convergence results from a representative subset of these tests, each probing distinct aspects of the numerical algorithm described in the previous section. The norm of the L_1 error decreases with resolution at the anticipated order of convergence.

B. COMPARISON BETWEEN SANE AND MAD EXTENDED GRMHD SIMULATIONS

The main text primarily focuses on magnetically dominated flows. Here we examine SANE simulations and compare the evolution of q and ΔP with that observed in MAD accretion.

Panel (a) in Figure 7 presents a poloidal snapshot from the SANE $a_* = 15/16$ simulation. The heat flux is negligible within the disk $q \lesssim 0.01 q_{\max}$, and is an order of magnitude lower than what is observed in MAD flows. This is evident in mass-weighted distribution profiles of $q/q_{\max}$ plotted in Figure 8 (dashed lines)—the normal-

ized heat flux has a smaller spread about zero for SANE models. This can be explained by noticing that a larger fraction of the disk mass saturates at one of the instability thresholds ($\sim 65\%$ attains ΔP_{mirror} and $\sim 10\%$ reaches $\Delta P_{\text{firehose}}$) compared to MAD simulations (see Figure 7 panel (b) and cf. Figure 1 (c) in main text). This leads to a suppression of τ_R and, hence, the target value q_0 because $q_0 \propto \chi \propto \tau_R$. Consequently, viscous stresses in SANE simulations contribute nearly half as much as magnetic stresses to angular momentum transport in the disk (Figure 7c). Although $\langle \Delta P \rangle / \langle b^2 \rangle$ is greater in the SANE case, the accretion disks are weakly magnetized, $\beta \gtrsim 10$, and as a result the deviations of fluid pressure from the ideal gas value are more strongly constrained by plasma instabilities. We see in Figure 8 that $\Delta P/P \lesssim 0.3$ in SANE models (dotted dark blue lines) while $\Delta P/P$ can exceed unity in MAD flows.

Finally, Figure 9 presents time-averaged electromagnetic observables for an EGRMHD SANE model and its ideal counterpart. As in the MAD case, the synthetic images of both plasma models are nearly identical. The SEDs also show close agreement, with the EGRMHD model exhibiting slightly higher power at $\nu \sim 10^{18}$ Hz, primarily due to increased Compton upscattering.

REFERENCES

- Alfvén, H. 1942, *Nature*, 150, 405, doi: [10.1038/150405d0](https://doi.org/10.1038/150405d0)
- B. Georgiev et al. 2022, *The Astrophysical Journal Letters*, 930, L20, doi: [10.3847/2041-8213/ac65eb](https://doi.org/10.3847/2041-8213/ac65eb)
- Balbus, S. A., & Hawley, J. F. 1991, *ApJ*, 376, 214, doi: [10.1086/170270](https://doi.org/10.1086/170270)
- . 1992, *ApJ*, 400, 610, doi: [10.1086/172022](https://doi.org/10.1086/172022)
- . 1998, *Reviews of Modern Physics*, 70, 1, doi: [10.1103/RevModPhys.70.1](https://doi.org/10.1103/RevModPhys.70.1)
- Bale, S. D., Kasper, J. C., Howes, G. G., et al. 2009, *PhRvL*, 103, 211101, doi: [10.1103/PhysRevLett.103.211101](https://doi.org/10.1103/PhysRevLett.103.211101)
- Bernhard, J. E., Moreland, J. S., & Bass, S. A. 2019, *Nature Physics*, 15, 1113, doi: [10.1038/s41567-019-0611-8](https://doi.org/10.1038/s41567-019-0611-8)
- Bott, A. F. A., Cowley, S. C., & Schekochihin, A. A. 2024, *Journal of Plasma Physics*, 90, 975900207, doi: [10.1017/S0022377824000308](https://doi.org/10.1017/S0022377824000308)

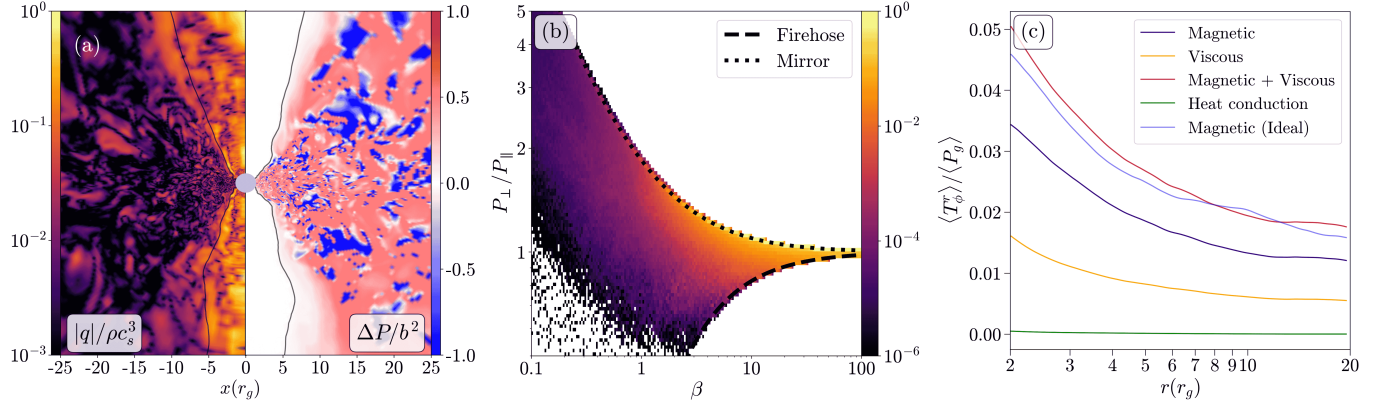


Figure 7. GRMHD quantities from the SANE $a_* = 15/16$ simulation. (a) Poloidal slices of the dissipative variables: heat flux normalized by the free-streaming value (left) and pressure anisotropy normalized by magnetic energy density (right). The black contour marks $\sigma = 1$. (b) Mass-weighted distribution of pressure anisotropy as a function of β within $r \leq 20r_g$. (c) Time-averaged radial profile of angular momentum transport in the disk.

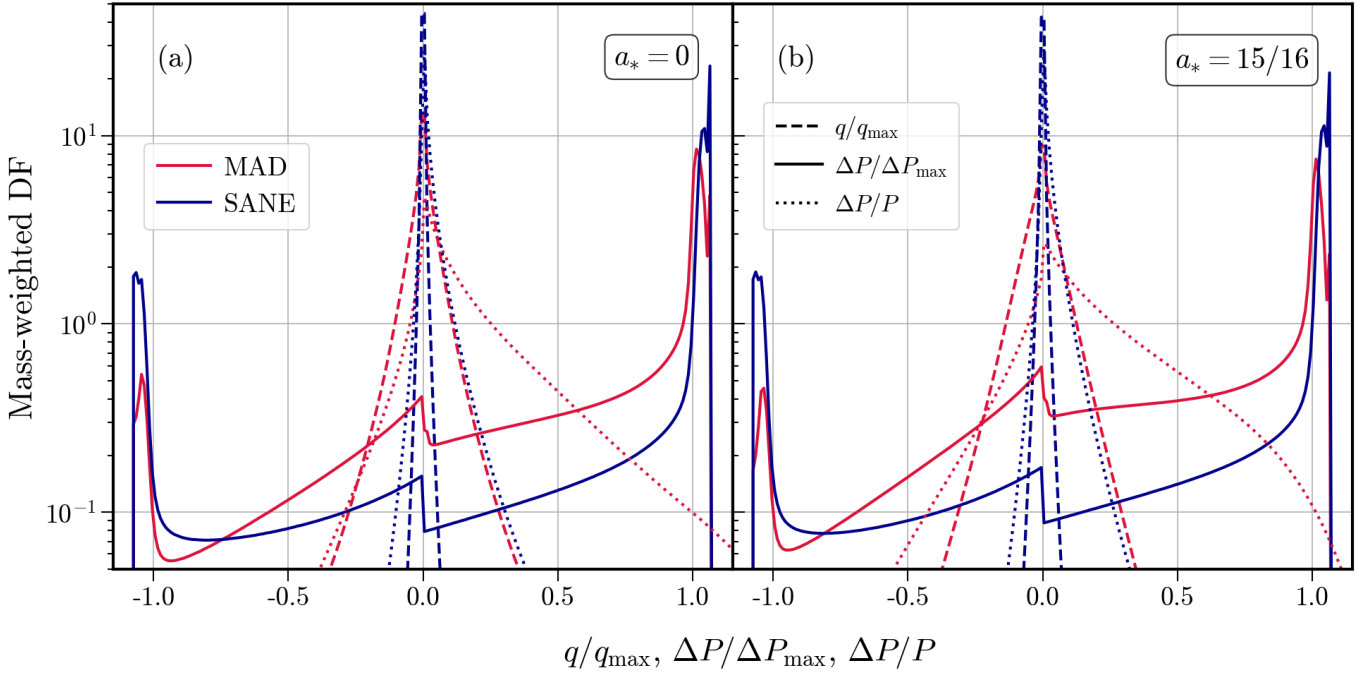


Figure 8. Time-averaged, mass-weighted distribution functions of $q/q_{\max}$, $\Delta P/\Delta P_{\max}$ and $\Delta P/P$ for (a) $a_* = 0$ and (b) $a_* = 15/16$ simulations. $\Delta P_{\max} = \Delta P_{\text{mirror}}$ if $\Delta P > 0$ and $\Delta P_{\max} = \Delta P_{\text{firehose}}$ if $\Delta P < 0$.

Bott, A. F. A., Kunz, M. W., Quataert, E., Squire, J., & Arzamasskiy, L. 2025, arXiv e-prints, arXiv:2501.13663, doi: [10.48550/arXiv.2501.13663](https://doi.org/10.48550/arXiv.2501.13663)

Braginskii, S. I. 1965, Reviews of Plasma Physics, 1, 205

Brevik, I., & Grøn, Ø. 2013, Astrophysics and Space Science, 347, 399, doi: [10.1007/s10509-013-1525-z](https://doi.org/10.1007/s10509-013-1525-z)

—. 2014, arXiv e-prints, arXiv:1409.8561, doi: [10.48550/arXiv.1409.8561](https://doi.org/10.48550/arXiv.1409.8561)

Brevik, I., Grøn, Ø., de Haro, J., Odintsov, S. D., & Saridakis, E. N. 2017, International Journal of Modern Physics D, 26, 1730024, doi: [10.1142/S0218271817300245](https://doi.org/10.1142/S0218271817300245)

Chael, A. 2025, MNRAS, 537, 2496, doi: [10.1093/mnras/staf200](https://doi.org/10.1093/mnras/staf200)

Chandra, M., Foucart, F., & Gammie, C. F. 2017, ApJ, 837, 92, doi: [10.3847/1538-4357/aa5f55](https://doi.org/10.3847/1538-4357/aa5f55)

Chandra, M., Gammie, C. F., Foucart, F., & Quataert, E. 2015, ApJ, 810, 162, doi: [10.1088/0004-637X/810/2/162](https://doi.org/10.1088/0004-637X/810/2/162)

Chandrasekhar, S., Kaufman, A. N., & Watson, K. M. 1958, Proceedings of the Royal Society of London Series A, 245, 435, doi: [10.1098/rspa.1958.0094](https://doi.org/10.1098/rspa.1958.0094)

Chatterjee, K., & Narayan, R. 2022, The Astrophysical Journal, 941, 30, doi: [10.3847/1538-4357/ac9d97](https://doi.org/10.3847/1538-4357/ac9d97)

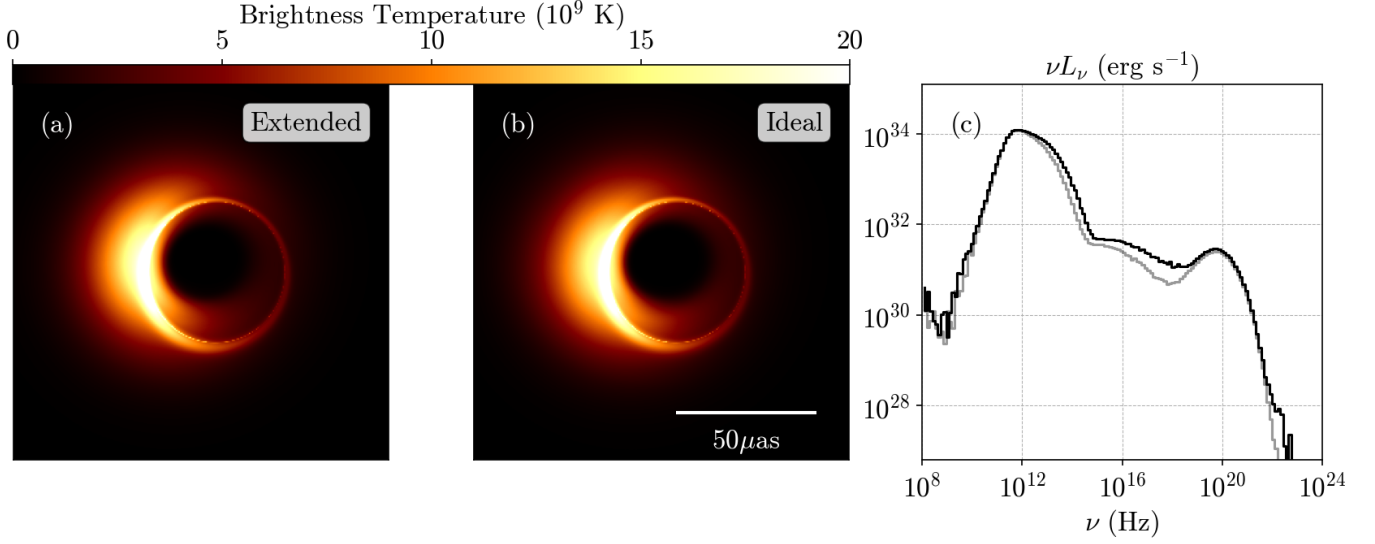


Figure 9. Electromagnetic observables for SANE $a_* = 15/16$ simulation with $R_{\text{high}} = 10$ at a viewing angle of 10° . (a) and (b) Time-averaged 230 GHz total intensity images for the Extended and Ideal GRMHD simulations, respectively. (c) Comparison of time-averaged spectra for the simulations shown in (a) and (b).

- Comisso, L. 2024, *The Astrophysical Journal*, 972, 9,
doi: [10.3847/1538-4357/ad51fe](https://doi.org/10.3847/1538-4357/ad51fe)
- Comisso, L., & Sironi, L. 2021, *PhRvL*, 127, 255102,
doi: [10.1103/PhysRevLett.127.255102](https://doi.org/10.1103/PhysRevLett.127.255102)
- . 2022, *The Astrophysical Journal Letters*, 936, L27,
doi: [10.3847/2041-8213/ac8422](https://doi.org/10.3847/2041-8213/ac8422)
- Cordeiro, I., Speranza, E., Ingles, K., Bemfica, F. S., & Noronha, J. 2024, *PhRvL*, 133, 091401,
doi: [10.1103/PhysRevLett.133.091401](https://doi.org/10.1103/PhysRevLett.133.091401)
- Crinquand, B., Cerutti, B., Dubus, G., Parfrey, K., & Philippov, A. 2021, *Astronomy & Astrophysics*, 650, A163, doi: [10.1051/0004-6361/202040158](https://doi.org/10.1051/0004-6361/202040158)
- . 2022, *PhRvL*, 129, 205101,
doi: [10.1103/PhysRevLett.129.205101](https://doi.org/10.1103/PhysRevLett.129.205101)
- Crinquand, B., Cerutti, B., Philippov, A., Parfrey, K., & Dubus, G. 2020, *PhRvL*, 124, 145101,
doi: [10.1103/PhysRevLett.124.145101](https://doi.org/10.1103/PhysRevLett.124.145101)
- Denicol, G. S., & Rischke, D. H. 2021, *Microscopic Foundations of Relativistic Fluid Dynamics*,
doi: [10.1007/978-3-030-82077-0](https://doi.org/10.1007/978-3-030-82077-0)
- Dexter, J., Kelly, B., Bower, G. C., et al. 2014, *Monthly Notices of the Royal Astronomical Society*, 442, 2797,
doi: [10.1093/mnras/stu1039](https://doi.org/10.1093/mnras/stu1039)
- Dhruv, V., Prather, B., Wong, G. N., & Gammie, C. F. 2025, *ApJS*, 277, 16, doi: [10.3847/1538-4365/adaea6](https://doi.org/10.3847/1538-4365/adaea6)
- Dolence, J. C., Gammie, C. F., Mościbrodzka, M., & Leung, P. K. 2009, *The Astrophysical Journal Supplement Series*, 184, 387, doi: [10.1088/0067-0049/184/2/387](https://doi.org/10.1088/0067-0049/184/2/387)
- El Mellah, I., Cerutti, B., & Crinquand, B. 2023, *Astronomy & Astrophysics*, 677, A67,
doi: [10.1051/0004-6361/202346781](https://doi.org/10.1051/0004-6361/202346781)
- El Mellah, I., Cerutti, B., Crinquand, B., & Parfrey, K. 2022, *Astronomy & Astrophysics*, 663, A169,
doi: [10.1051/0004-6361/202142847](https://doi.org/10.1051/0004-6361/202142847)
- Event Horizon Telescope Collaboration et al. 2019, *The Astrophysical Journal Letters*, 875, L5, (M87* Paper V),
doi: [10.3847/2041-8213/ab0f43](https://doi.org/10.3847/2041-8213/ab0f43)
- . 2022a, *The Astrophysical Journal Letters*, 930, L12,
doi: [10.3847/2041-8213/ac6674](https://doi.org/10.3847/2041-8213/ac6674)
- . 2022b, *The Astrophysical Journal Letters*, 930, L13,
doi: [10.3847/2041-8213/ac6675](https://doi.org/10.3847/2041-8213/ac6675)
- . 2022c, *The Astrophysical Journal Letters*, 930, L14,
doi: [10.3847/2041-8213/ac6429](https://doi.org/10.3847/2041-8213/ac6429)
- . 2022d, *The Astrophysical Journal Letters*, 930, L15,
doi: [10.3847/2041-8213/ac6736](https://doi.org/10.3847/2041-8213/ac6736)
- . 2022e, *The Astrophysical Journal Letters*, 930, L16,
doi: [10.3847/2041-8213/ac6672](https://doi.org/10.3847/2041-8213/ac6672)
- . 2022f, *The Astrophysical Journal Letters*, 930, L17,
doi: [10.3847/2041-8213/ac6756](https://doi.org/10.3847/2041-8213/ac6756)
- . 2024a, *The Astrophysical Journal Letters*, 964, L25,
doi: [10.3847/2041-8213/ad2df0](https://doi.org/10.3847/2041-8213/ad2df0)
- . 2024b, *Astrophys. J. Lett.*, 964, L26,
doi: [10.3847/2041-8213/ad2df1](https://doi.org/10.3847/2041-8213/ad2df1)
- Fishbone, L. G., & Moncrief, V. 1976, *The Astrophysical Journal*, 207, 962, doi: [10.1086/154565](https://doi.org/10.1086/154565)
- Foucart, F., Chandra, M., Gammie, C. F., & Quataert, E. 2016, *MNRAS*, 456, 1332, doi: [10.1093/mnras/stv2687](https://doi.org/10.1093/mnras/stv2687)

- Foucart, F., Chandra, M., Gammie, C. F., Quataert, E., & Tchekhovskoy, A. 2017, *MNRAS*, 470, 2240, doi: [10.1093/mnras/stx1368](https://doi.org/10.1093/mnras/stx1368)
- Gale, C., Jeon, S., & Schenke, B. 2013, *International Journal of Modern Physics A*, 28, 1340011, doi: [10.1142/S0217751X13400113](https://doi.org/10.1142/S0217751X13400113)
- Galishnikova, A., Philippov, A., & Quataert, E. 2023a, *ApJ*, 957, 103, doi: [10.3847/1538-4357/acfa77](https://doi.org/10.3847/1538-4357/acfa77)
- Galishnikova, A., Philippov, A., Quataert, E., et al. 2023b, *PhRvL*, 130, 115201, doi: [10.1103/PhysRevLett.130.115201](https://doi.org/10.1103/PhysRevLett.130.115201)
- Gammie, C. F. 2025, *ApJ*, 980, 193, doi: [10.3847/1538-4357/adaea3](https://doi.org/10.3847/1538-4357/adaea3)
- Gammie, C. F., McKinney, J. C., & Tóth, G. 2003, *The Astrophysical Journal*, 589, 444, doi: [10.1086/374594](https://doi.org/10.1086/374594)
- Garcia-Perciante, A. L., Garcia-Colin, L. S., & Sandoval-Villalazo, A. 2009, *General Relativity and Gravitation*, 41, 1645, doi: [10.1007/s10714-008-0735-0](https://doi.org/10.1007/s10714-008-0735-0)
- Gary, S. P., Li, H., O'Rourke, S., & Winske, D. 1998, *Journal of Geophysical Research*, 103, 14567, doi: [10.1029/98JA01174](https://doi.org/10.1029/98JA01174)
- Gelles, Z., Chatterjee, K., Johnson, M., Ripperda, B., & Liska, M. 2022, *Galaxies*, 10, 107, doi: [10.3390/galaxies10060107](https://doi.org/10.3390/galaxies10060107)
- Gladd, N. T. 1983, *Physics of Fluids*, 26, 974, doi: [10.1063/1.864249](https://doi.org/10.1063/1.864249)
- Hakobyan, H., Ripperda, B., & Philippov, A. A. 2023, *The Astrophysical Journal Letters*, 943, L29, doi: [10.3847/2041-8213/acb264](https://doi.org/10.3847/2041-8213/acb264)
- Hasegawa, A. 1969, *Physics of Fluids*, 12, 2642, doi: [10.1063/1.1692407](https://doi.org/10.1063/1.1692407)
- Hawley, J. F., & Balbus, S. A. 1991, *ApJ*, 376, 223, doi: [10.1086/170271](https://doi.org/10.1086/170271)
- . 1992, *ApJ*, 400, 595, doi: [10.1086/172021](https://doi.org/10.1086/172021)
- Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1995, *ApJ*, 440, 742, doi: [10.1086/175311](https://doi.org/10.1086/175311)
- Heinz, U., & Snellings, R. 2013, *Annual Review of Nuclear and Particle Science*, 63, 123, doi: [10.1146/annurev-nucl-102212-170540](https://doi.org/10.1146/annurev-nucl-102212-170540)
- Hellinger, P., Trávníček, P., Kasper, J. C., & Lazarus, A. J. 2006, *Geophys. Res. Lett.*, 33, L09101, doi: [10.1029/2006GL025925](https://doi.org/10.1029/2006GL025925)
- Hiscock, W. A., & Lindblom, L. 1983, *Annals of Physics*, 151, 466, doi: [10.1016/0003-4916\(83\)90288-9](https://doi.org/10.1016/0003-4916(83)90288-9)
- . 1985, *PhRvD*, 31, 725, doi: [10.1103/PhysRevD.31.725](https://doi.org/10.1103/PhysRevD.31.725)
- . 1988, *Physics Letters A*, 131, 509, doi: [10.1016/0375-9601\(88\)90679-2](https://doi.org/10.1016/0375-9601(88)90679-2)
- Howes, G. G. 2010, *MNRAS*, 409, L104, doi: [10.1111/j.1745-3933.2010.00958.x](https://doi.org/10.1111/j.1745-3933.2010.00958.x)
- Igumenshchev, I. V., Narayan, R., & Abramowicz, M. A. 2003, *The Astrophysical Journal*, 592, 1042, doi: [10.1086/375769](https://doi.org/10.1086/375769)
- Isenberg, P. A. 2012, *Physics of Plasmas*, 19, 032116, doi: [10.1063/1.3697721](https://doi.org/10.1063/1.3697721)
- Isenberg, P. A., Maruca, B. A., & Kasper, J. C. 2013, *ApJ*, 773, 164, doi: [10.1088/0004-637X/773/2/164](https://doi.org/10.1088/0004-637X/773/2/164)
- Israel, W., & Stewart, J. M. 1979, *Annals of Physics*, 118, 341, doi: [10.1016/0003-4916\(79\)90130-1](https://doi.org/10.1016/0003-4916(79)90130-1)
- Kempster, P., Quataert, E., Squire, J., & Kunz, M. W. 2019, *Monthly Notices of the Royal Astronomical Society*, 486, 4013, doi: [10.1093/mnras/stz1111](https://doi.org/10.1093/mnras/stz1111)
- Kivelson, M. G., & Southwood, D. J. 1996, *Journal of Geophysical Research*, 101, 17365, doi: [10.1029/96JA01407](https://doi.org/10.1029/96JA01407)
- Kunz, M. W., Schekochihin, A. A., & Stone, J. M. 2014, *Physical Review Letters*, 112, doi: [10.1103/physrevlett.112.205003](https://doi.org/10.1103/physrevlett.112.205003)
- Lee, D., & Gammie, C. F. 2022, *Research Notes of the American Astronomical Society*, 6, 279, doi: [10.3847/2515-5172/acade5](https://doi.org/10.3847/2515-5172/acade5)
- M. Wielgus et al. 2022, *The Astrophysical Journal Letters*, 930, L19, doi: [10.3847/2041-8213/ac6428](https://doi.org/10.3847/2041-8213/ac6428)
- Maartens, R. 1995, *Classical and Quantum Gravity*, 12, 1455, doi: [10.1088/0264-9381/12/6/011](https://doi.org/10.1088/0264-9381/12/6/011)
- Mahadevan, R., & Quataert, E. 1997, *ApJ*, 490, 605, doi: [10.1086/304908](https://doi.org/10.1086/304908)
- Meyrand, R., Kanekar, A., Dorland, W., & Schekochihin, A. A. 2019, *Proceedings of the National Academy of Science*, 116, 1185, doi: [10.1073/pnas.1813913116](https://doi.org/10.1073/pnas.1813913116)
- Mizuno, Y., & Rezzolla, L. 2024, *arXiv e-prints*, arXiv:2404.13824, doi: [10.48550/arXiv.2404.13824](https://doi.org/10.48550/arXiv.2404.13824)
- Mościbrodzka, M., Falcke, H., & Shiokawa, H. 2016, *Astronomy & Astrophysics*, 586, A38, doi: [10.1051/0004-6361/201526630](https://doi.org/10.1051/0004-6361/201526630)
- Mościbrodzka, M., & Gammie, C. F. 2018, *MNRAS*, 475, 43, doi: [10.1093/mnras/stx3162](https://doi.org/10.1093/mnras/stx3162)
- Most, E. R., Haber, A., Harris, S. P., et al. 2024, *The Astrophysical Journal Letters*, 967, L14, doi: [10.3847/2041-8213/ad454f](https://doi.org/10.3847/2041-8213/ad454f)
- Most, E. R., Peterson, J., Scurto, L., Pais, H., & Dexheimer, V. 2025, *ApJL*, 989, L29, doi: [10.3847/2041-8213/adf62d](https://doi.org/10.3847/2041-8213/adf62d)
- Narayan, R., Chael, A., Chatterjee, K., Ricarte, A., & Curd, B. 2022, *MNRAS*, 511, 3795, doi: [10.1093/mnras/stac285](https://doi.org/10.1093/mnras/stac285)
- Narayan, R., Igumenshchev, I. V., & Abramowicz, M. A. 2003, *Publications of the Astronomical Society of Japan*, 55, L69, doi: [10.1093/pasj/55.6.L69](https://doi.org/10.1093/pasj/55.6.L69)

- Narayan, R., Sądowski, A., Penna, R. F., & Kulkarni, A. K. 2012, *Monthly Notices of the Royal Astronomical Society*, 426, 3241, doi: [10.1111/j.1365-2966.2012.22002.x](https://doi.org/10.1111/j.1365-2966.2012.22002.x)
- Nathanail, A., Fromm, C. M., Porth, O., et al. 2020, *MNRAS*, 495, 1549, doi: [10.1093/mnras/staa1165](https://doi.org/10.1093/mnras/staa1165)
- Noronha-Hostler, J., Noronha, J., & Greiner, C. 2009, *PhRvL*, 103, 172302, doi: [10.1103/PhysRevLett.103.172302](https://doi.org/10.1103/PhysRevLett.103.172302)
- Padmanabhan, T., & Chitre, S. M. 1987, *Physics Letters A*, 120, 433, doi: [10.1016/0375-9601\(87\)90104-6](https://doi.org/10.1016/0375-9601(87)90104-6)
- Parfrey, K., Philippov, A., & Cerutti, B. 2019, *PhRvL*, 122, 035101, doi: [10.1103/PhysRevLett.122.035101](https://doi.org/10.1103/PhysRevLett.122.035101)
- Parker, E. N. 1958, *Physical Review*, 109, 1874, doi: [10.1103/PhysRev.109.1874](https://doi.org/10.1103/PhysRev.109.1874)
- Parker, J. T., Highcock, E. G., Schekochihin, A. A., & Dellar, P. J. 2016, *Physics of Plasmas*, 23, 070703, doi: [10.1063/1.4958954](https://doi.org/10.1063/1.4958954)
- Piattella, O. F., Fabris, J. C., & Zimdahl, W. 2011, *Journal of Cosmology and Astroparticle Physics*, 2011, 029, doi: [10.1088/1475-7516/2011/05/029](https://doi.org/10.1088/1475-7516/2011/05/029)
- Prather, B. S. 2024, *arXiv*, doi: [10.48550/arXiv.2408.01361](https://doi.org/10.48550/arXiv.2408.01361)
- Quataert, E. 1998, *ApJ*, 500, 978, doi: [10.1086/305770](https://doi.org/10.1086/305770)
- Quataert, E., & Gruzinov, A. 1999, *ApJ*, 520, 248, doi: [10.1086/307423](https://doi.org/10.1086/307423)
- Ripperda, B., Liska, M., Chatterjee, K., et al. 2022, *The Astrophysical Journal Letters*, 924, L32, doi: [10.3847/2041-8213/ac46a1](https://doi.org/10.3847/2041-8213/ac46a1)
- Rocha, G. S., Wagner, D., Denicol, G. S., Noronha, J., & Rischke, D. H. 2024, *Entropy*, 26, 189, doi: [10.3390/e26030189](https://doi.org/10.3390/e26030189)
- Romatschke, P., & Romatschke, U. 2017, *arXiv e-prints*, arXiv:1712.05815, doi: [10.48550/arXiv.1712.05815](https://doi.org/10.48550/arXiv.1712.05815)
- Rosenbluth, M. N., & Longmire, C. L. 1957, *Annals of Physics*, 1, 120, doi: [10.1016/0003-4916\(57\)90055-6](https://doi.org/10.1016/0003-4916(57)90055-6)
- Salas, L. D. S., Liska, M. T. P., Markoff, S. B., et al. 2025, *MNRAS*, 538, 698, doi: [10.1093/mnras/staf240](https://doi.org/10.1093/mnras/staf240)
- Schekochihin, A. A., Parker, J. T., Highcock, E. G., et al. 2016, *Journal of Plasma Physics*, 82, 905820212, doi: [10.1017/S0022377816000374](https://doi.org/10.1017/S0022377816000374)
- Sharma, P., Quataert, E., Hammett, G. W., & Stone, J. M. 2007, *The Astrophysical Journal*, 667, 714, doi: [10.1086/520800](https://doi.org/10.1086/520800)
- Shen, C., & Yan, L. 2020, *Nuclear Science and Techniques*, 31, 122, doi: [10.1007/s41365-020-00829-z](https://doi.org/10.1007/s41365-020-00829-z)
- Sironi, L., Uzdensky, D. A., & Giannios, D. 2025, *ARA&A*, 63, 127, doi: [10.1146/annurev-astro-020325-115713](https://doi.org/10.1146/annurev-astro-020325-115713)
- Solanki, S., Davelaar, J., Ripperda, B., & Philippov, A. 2025, *ApJ*, 985, 147, doi: [10.3847/1538-4357/adcb9](https://doi.org/10.3847/1538-4357/adcb9)
- Southwood, D. J., & Kivelson, M. G. 1993, *Journal of Geophysical Research*, 98, 9181, doi: [10.1029/92JA02837](https://doi.org/10.1029/92JA02837)
- Sudan, R. N. 1963, *Physics of Fluids*, 6, 57, doi: [10.1063/1.1724508](https://doi.org/10.1063/1.1724508)
- . 1965, *Physics of Fluids*, 8, 153, doi: [10.1063/1.1761082](https://doi.org/10.1063/1.1761082)
- Sądowski, A., Narayan, R., Penna, R., & Zhu, Y. 2013, *Monthly Notices of the Royal Astronomical Society*, 436, 3856, doi: [10.1093/mnras/stt1881](https://doi.org/10.1093/mnras/stt1881)
- Tchekhovskoy, A., Narayan, R., & McKinney, J. C. 2011, *Monthly Notices of the Royal Astronomical Society: Letters*, 418, L79, doi: [10.1111/j.1745-3933.2011.01147.x](https://doi.org/10.1111/j.1745-3933.2011.01147.x)
- Tsunetoe, Y., Pesce, D. W., Narayan, R., et al. 2025, *arXiv e-prints*, arXiv:2501.14862, <https://arxiv.org/abs/2501.14862>
- von Fellenberg, S. D., Roychowdhury, T., Michail, J. M., et al. 2025, *ApJL*, 979, L20, doi: [10.3847/2041-8213/ada3d2](https://doi.org/10.3847/2041-8213/ada3d2)
- Vos, J., Cerutti, B., Mościbrodzka, M., & Parfrey, K. 2025, *PhRvL*, 135, 015201, doi: [10.1103/5wwb-7t2n](https://doi.org/10.1103/5wwb-7t2n)
- Wilson, L. A. 2017, *Big Data Research*, 8, 57, doi: <https://doi.org/10.1016/j.bdr.2017.04.001>
- Wong, G. N., Prather, B. S., Dhruv, V., et al. 2022, *The Astrophysical Journal Supplement Series*, 259, 64, doi: [10.3847/1538-4365/ac582e](https://doi.org/10.3847/1538-4365/ac582e)
- Zakari, M., & Jou, D. 1993, *PhRvD*, 48, 1597, doi: [10.1103/PhysRevD.48.1597](https://doi.org/10.1103/PhysRevD.48.1597)