Fractional Aharonov-Bohm effect for retarded potentials

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It has been suggested that the magnetic Aharonov-Bohm effect can be interpreted equally well as being due to a phase shift associated with an electron in an interferometer enclosing a magnetic flux, or as a phase shift associated with the electrons in the solenoid that generates the field. Here the Aharonov-Bohm effect is derived using second-quantized field theory to describe all the electrons as well as the electromagnetic field in a consistent way. The results are in agreement with the usual expression for the Aharonov-Bohm effect when the retardation of the electromagnetic field is negligible, but they predict the possibility of a fractional phase shift when retardation effects are significant.

The magnetic Aharonov-Bohm (AB) effect [1–11] is of fundamental interest because it shows that the vector potential can have an observable effect on an electron that is traveling through an interferometer that encloses a magnetic flux, even though the electric and magnetic fields are zero along the path of the electron. This suggests that the vector potential may be more fundamental than the electric and magnetic fields. In response, it has been suggested that the magnetic AB effect can be interpreted equally well as a phase shift experienced by the electrons in the solenoid that generates the magnetic flux, where the electric field is not zero [10].

In this paper, second-quantized field theory will be used to describe all the electrons as well as the electromagnetic field in a consistent way. This approach allows the AB effect to be derived in a straightforward way using the Feynman propagator. The results are in agreement with the usual expression for the AB effect when the retardation of the electromagnetic field is negligible, but they predict the possibility of a fractional phase shift when retardation effects are significant. A fractional AB effect has previously been observed in solid-state systems for reasons that are unrelated to retardation [12–14].

The magnetic AB effect is illustrated schematically in Fig. 1. The wave function of an incident electron is split into two paths labeled 1 and 2 by a beam splitter or diffraction. The two paths enclose a solenoid S that generates a magnetic flux Φ_M inside the solenoid, with the electric field $\mathbf{E} = 0$ and the magnetic field $\mathbf{B} = 0$ outside of the solenoid. The vector potential $\mathbf{A}$ is nonzero along paths 1 and 2, which produces a phase difference of $\Delta\phi$. The probability of detecting the electron in the two output paths of the interferometer is proportional to $cos^2(\Delta\phi)$ or $sin^2(\Delta\phi)$.

The covariant form of the interaction Hamiltonian is given by

$$\hat{H}'(t) = -\int d^3\mathbf{r}\, \hat{A}^\mu(\mathbf{r},t)\hat{j}_\mu(\mathbf{r},t). \tag{1}$$

Here $\hat{A}^\mu$ is the second-quantized vector potential while $\hat{j}_\mu$ is the current operator for the electrons. The Coulomb potential $\hat{A}^0$ can be neglected here because the solenoid

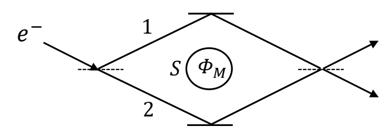

FIG. 1. Magnetic Aharonov-Bohm effect. The wave function of an incident electron (e^-) is split into two paths 1 and 2 by a beam splitter (dashed line). The vector potential from an enclosed solenoid S with magnetic flux Φ_M can change the relative phase between the two paths, even though the electric and magnetic fields are zero outside of the solenoid.

is uncharged, in which case Eq. (1) can be rewritten as

$$\hat{H}'(t) = -\int d^3\mathbf{r}\, \hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{j}}(\mathbf{r},t), \tag{2}$$

where $\hat{\mathbf{A}}(\mathbf{r},t)$ and $\hat{\mathbf{j}}(\mathbf{r},t)$ are three-vectors. For simplicity, natural units with $\hbar = c = 1$ will be used and the metric $g^{\mu\nu}$ will be chosen to have diagonal elements of (-1,1,1,1).

In the Schrodinger picture, the second-quantized vector potential is defined in the Lorentz gauge as [15]

$$\hat{A}^\mu(\mathbf{r},t) = \frac{1}{(2\pi)^3}\sum_\lambda \int \frac{d^3\mathbf{k}}{\sqrt{2\omega_k}}\left[\epsilon^\mu(\mathbf{k},\lambda)\hat{a}_\lambda(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}} + h.c.\right], \tag{3}$$

where $\hat{a}_\lambda(\mathbf{k})$ annihilates a photon of momentum $\mathbf{k}$ and polarization $\epsilon^\mu(\mathbf{k},\lambda)$, while $\omega_k = ck$.

Although the motion of the electrons is nonrelativistic, it is more straightforward to use the Dirac theory where the current operator is given by $\hat{j}^\mu = q\bar{\psi}\gamma^\mu\hat{\psi}$. Here $\hat{\psi}(\mathbf{r},t)$ is the second-quantized field operator for the electrons and the γ^μ are related to the Dirac matrices. This approach has the advantage that the current operator does not involve the vector potential and H' in Eq. (1) is exact.

Retardation effects due to the time dependence of the currents will play a major role in what follows. As a result, the interaction Hamiltonian will be time dependent and the time-evolution operator $\hat{U}(t_f,t_i)$ in the interaction picture is given by the Dyson formula [16]

$$\hat{U}(t_f,t_i) = \hat{T}\,exp\left[-i\int_{t_i}^{t_f}\hat{H}'(t)dt\right]. \tag{4}$$

Here $\hat{T}$ is the time-ordering operator that places operators with the earliest time to the right, while t_i and t_f are the initial and final times. Combining Eqs. (2) and (4) gives

$$\hat{U} = \hat{T}\, exp\left[i \int d^3\mathbf{r} \int_{t_i}^{t_f} dt\, \hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{j}}(\mathbf{r},t)\right]. \quad (5)$$

First consider the phase shift ϕ_1 that is generated along path 1. The interaction between the electrons is due to the exchange of virtual photons that are generated from the vacuum state of the field, which can be calculated as usual using perturbation theory [16–18]. The initial state of the system will be denoted by $|\Psi_0\rangle = |\Psi_e\rangle|0\rangle$, where $|\Psi_e\rangle$ is the initial state of the electrons and $|0\rangle$ is the vacuum state of the electromagnetic field.

The interaction will produce a final state $|\Psi_f\rangle = \hat{U}|\Psi_0\rangle$. There will be a small probability amplitude to produce a state that is orthogonal to $|\Psi_0\rangle$, which would reduce the visibility of the interference pattern, and a larger probability amplitude to remain in the initial state with an overall phase shift given by $|\Psi_f\rangle = exp(i\phi_1)|\Psi_0\rangle$. Thus the phase shift ϕ_1 is given by

$$e^{i\phi_1} = \langle\Psi_o|\hat{U}(\Delta t)|\Psi_0\rangle, \quad (6)$$

where $\Delta t = t_f - t_i$. Berry used a similar approach to derive his geometric phase [19, 20].

Expanding the exponential in Eq. (5) in a Taylor series through second order and inserting it into Eq. (6) gives

$$\begin{aligned} e^{i\phi_1} &= \langle\Psi_0|\Psi_0\rangle \\ &+ i\langle\Psi_0|\int d^3\mathbf{r}\int_{t_i}^{t_f} dt\,\hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{j}}(\mathbf{r},t)|\Psi_0\rangle \\ &- \frac{1}{2}\langle\Psi_0|\int d^3\mathbf{r}\int d^3\mathbf{r}'\int_{t_i}^{t_f} dt\int_{t_i}^{t_f} dt' \\ &\times\ \hat{T}\left[\hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{j}}(\mathbf{r},t)\,\hat{\mathbf{A}}(\mathbf{r}',t')\cdot\hat{\mathbf{j}}(\mathbf{r}',t')\right]|\Psi_0\rangle. \quad (7)\end{aligned}$$

The first term in Eq. (7) is unity and the second term is zero because $\langle 0|\hat{a}|0\rangle = \langle 0|\hat{a}^\dagger|0\rangle = 0$. It can be seen that the Taylor series expansion of the exponential has introduced a factor of $1/2$ in the final term, which is necessary to eliminate any double-counting of the interaction.

Consider the case in which $\mathbf{r}$ is located in one of the paths through the interferometer while $\mathbf{r}'$ is located in the solenoid. In that case, $\hat{\mathbf{j}}(\mathbf{r},t)$ and $\hat{\mathbf{j}}(\mathbf{r}',t')$ are independent operators and $[\hat{\mathbf{j}}(\mathbf{r},t),\hat{\mathbf{j}}(\mathbf{r}',t')] = 0$ in the limit of large separations. (This is equivalent to neglecting the Feynman propagator for an electron, which decreases exponentially with distance.) Thus the time ordering of the current operators is irrelevant and

$$\begin{aligned}&\hat{T}\left[\hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{j}}(\mathbf{r},t)\,\hat{\mathbf{A}}(\mathbf{r}',t')\cdot\hat{\mathbf{j}}(\mathbf{r}',t')\right] \\ &= \hat{j}^i(\mathbf{r},t)\hat{T}\left[\hat{A}_i(\mathbf{r},t)\hat{A}_k(\mathbf{r}',t')\right]\hat{j}^k(\mathbf{r}',t'). \quad (8)\end{aligned}$$

By definition, the Feynman propagator $D_{\mu\nu}$ for a photon is given by [15, 17, 18, 21–23]

$$D_{\mu\nu}(\mathbf{r},t;\mathbf{r}',t') = \langle 0|\hat{T}\left[\hat{A}_\mu(\mathbf{r},t)\hat{A}_\nu(\mathbf{r}',t')\right]|0\rangle. \quad (9)$$

Inserting Eqs. (8) and (9) into Eq. (7) gives

$$\begin{aligned} i\phi_1 &= -\frac{1}{2}\int d^3\mathbf{r}\int d^3\mathbf{r}'\int_{t_i}^{t_f} dt\int_{t_i}^{t_f} dt' \\ &\times\ \langle\Psi_e|\hat{j}^i(\mathbf{r},t)\,D_{ik}(\mathbf{r},t;\mathbf{r}',t')\hat{j}^k(\mathbf{r}',t')|\Psi_e\rangle. \quad (10)\end{aligned}$$

Here it has been assumed that the interaction is sufficiently weak that $exp(i\phi_1) \simeq 1 + i\phi_1$. The process can be divided into smaller time intervals if necessary in order to satisfy that approximation.

It will also be assumed that the interaction is sufficiently weak that it does not generate any significant correlation between the two currents. In that case,

$$\langle\hat{j}^i(\mathbf{r},t)\hat{j}^k(\mathbf{r}',t')\rangle = \langle\hat{j}^i(\mathbf{r},t)\rangle\,\langle\hat{j}^k(\mathbf{r}',t')\rangle. \quad (11)$$

The expectation value of the current will be denoted by $J^i(r,t) = \langle\Psi_e|\hat{j}^i(\mathbf{r},t)|\Psi_e\rangle$. Inserting this into Eq. (10) gives

$$\begin{aligned} i\phi_1 &= -\frac{1}{2}\int d^3\mathbf{r}\int d^3\mathbf{r}'\int_{t_i}^{t_f} dt\int_{t_i}^{t_f} dt' \\ &\times\ J^i(\mathbf{r},t)\,D_{ik}(\mathbf{r},t;\mathbf{r}',t')J^k(\mathbf{r}',t'). \quad (12)\end{aligned}$$

If the Coulomb potential had been included in H' in Eq. (2), then Eq. (12) could be written in the covariant form

$$i\phi_1 = -\frac{1}{2}\int d^4x\int d^4x'\,J^\mu(x)\,D_{\mu\nu}(x-x')J^\nu(x'), \quad (13)$$

where $x = (\mathbf{r},t)$.

The Feynman propagator is a function of $x - x'$ and it obeys the relation [24]

$$\begin{aligned} D_{\mu\nu}(x-x') &= \frac{1}{2}\left[\Delta^R{}_{\mu\nu}(x-x') + \Delta^A{}_{\mu\nu}(x-x')\right] \\ &+ \frac{i}{2}\Delta^1{}_{\mu\nu}(x-x'). \quad (14)\end{aligned}$$

The retarded propagator $\Delta^R{}_{\mu\nu}$ is related to the retarded Green's function $G^R{}_{\mu\nu}$ of classical electrodynamics by

$$\Delta^R{}_{\mu\nu}(x-x') = -iG^R{}_{\mu\nu}(x-x'). \quad (15)$$

A similar relation holds for the advanced propagator $\Delta^A{}_{\mu\nu}$. It can be shown that the function $\Delta^1{}_{\mu\nu}(x-x')$ does not contribute to the phase shift and it can be neglected here.

The integral over dt' in Eq. (12) can be divided into two intervals to give

$$\begin{aligned} &\int_{t_i}^{t_f} dt\int_{t_i}^{t_f} dt'\,J^i(\mathbf{r},t)\,D_{ik}(\mathbf{r},t;\mathbf{r}',t')J^k(\mathbf{r}',t') \\ &= \int_{t_i}^{t_f} dt\int_{t_i}^{t} dt'\,J^i(\mathbf{r},t)\,D_{ik}(\mathbf{r},t;\mathbf{r}',t')J^k(\mathbf{r}',t') \\ &+ \int_{t_i}^{t_f} dt\int_{t}^{t_f} dt'\,J^i(\mathbf{r},t)\,D_{ik}(\mathbf{r},t;\mathbf{r}',t')J^k(\mathbf{r}',t'). \quad (16)\end{aligned}$$

Interchanging the order of integration in the last term of Eq. (16) and making the change of variables $\mathbf{r}, t \leftrightarrow \mathbf{r}', t'$ gives

$$\int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' J^i(\mathbf{r},t)\, D_{ik}(\mathbf{r},t;\mathbf{r}',t') J^k(\mathbf{r}',t')$$
$$= \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt' J^i(\mathbf{r},t)\, \Delta^R_{ik}(\mathbf{r},t;\mathbf{r}',t') J^k(\mathbf{r}',t'), \quad (17)$$

where Eq. (14) was used along with the fact that $D_{\mu\nu}(\mathbf{r},t)$ is an even function of $\mathbf{r}$ and t. Inserting Eqs. (15) and (17) into Eq. (12) gives

$$\phi_1 = \frac{1}{2} \int d^3\mathbf{r} \int d^3\mathbf{r}' \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt' \times J^i(\mathbf{r},t)\, G^R{}_{ik}(\mathbf{r},t;\mathbf{r}',t') J^k(\mathbf{r}',t'). \quad (18)$$

Taking $t_i \to -\infty$, the integrals over $d^3\mathbf{r}'$ and dt' give

$$\int d^3\mathbf{r}' \int_{-\infty}^{t} dt' G^R{}_{ik}(\mathbf{r},t;\mathbf{r}',t') J^k(\mathbf{r}',t') = \mathcal{A}_i(\mathbf{r},t), \quad (19)$$

where $\boldsymbol{\mathcal{A}}(\mathbf{r},t)$ is the expectation value of the retarded vector potential produced by the current distributions. Combining this with Eq. (18) gives

$$\phi_1 = \frac{1}{2} \int d^3\mathbf{r} \int_{-\infty}^{t} dt\, \boldsymbol{J}(\mathbf{r},t) \cdot \boldsymbol{\mathcal{A}}(\mathbf{r},t). \quad (20)$$

The total current can be written in the form

$$\boldsymbol{J} = \boldsymbol{J}_e + \boldsymbol{J}_S, \quad (21)$$

where $\boldsymbol{J}_e$ is the expectation value of the current produced by the electron in the interferometer while $\boldsymbol{J}_S$ is the expectation value of the current in the solenoid. In the same way, the total vector potential can be written as

$$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}}_e + \boldsymbol{\mathcal{A}}_S, \quad (22)$$

where $\boldsymbol{\mathcal{A}}_e$ and $\boldsymbol{\mathcal{A}}_S$ are the expectation values of the retarded vector potentials produced by the electrons in the interferometer and in the solenoid, respectively. Inserting Eqs. (21) and (22) into Eq. (20) gives

$$\phi_1 = \frac{1}{2} \int d^3\mathbf{r}\, dt\, \left[\boldsymbol{J}_e \cdot \boldsymbol{\mathcal{A}}_S + \boldsymbol{J}_{\boldsymbol{S}} \cdot \boldsymbol{\mathcal{A}}_e\right], \quad (23)$$

where both vector potentials are retarded. Self-interaction terms such as $\boldsymbol{J}_e \cdot \boldsymbol{\mathcal{A}}_e$ have been omitted since they are the same along both paths and do not contribute to the phase difference.

For quasi-static fields where retardation effects are negligible, it can be shown that

$$\int d^3\mathbf{r}\, dt\, \boldsymbol{J}_S \cdot \boldsymbol{\mathcal{A}}_e = \int d^3\boldsymbol{r}\, dt\, \boldsymbol{J}_e \cdot \boldsymbol{\mathcal{A}}_S. \quad (24)$$

FIG. 2. Fractional Aharonov-Bohm effect. A source S of a static magnetic field consists of a constant current flowing in a circular loop. An electron passes through a distant interferometer, which encloses a magnetic flux from the source. The distance D between the source and the interferometer is assumed to be sufficiently large that the electron passes through the interferometer before its retarded vector potential can reach the source S. In that case, Eq. (28) predicts that a fractional Aharonov-Bohm phase shift will be observed.

In that case, Eq. (23) can be rewritten as

$$\phi_1 = \int d^3\mathbf{r}\, dt\, \boldsymbol{J}_e \cdot \boldsymbol{\mathcal{A}}_S. \quad (25)$$

If the electron in the interferometer is described by a localized wave packet, the spatial integral of the current density gives $q\mathbf{v}$ in the nonrelativistic limit, where $\mathbf{v}$ is the velocity. Taking the line integral around the two paths gives

$$\Delta\phi = \frac{q}{\hbar} \oint \boldsymbol{\mathcal{A}}_S \cdot\; d\mathbf{l}, \quad (26)$$

which is the usual expression for the AB effect. This is equivalent to quantizing only $\hat{\mathbf{j}}_e$ in a conventional approach based on a classical vector potential.

Eq. (24) could be used equally well to rewrite Eq. (23) as

$$\phi_1 = \int d^3\mathbf{r}\, dt\, \boldsymbol{J}_S \cdot \boldsymbol{\mathcal{A}}_e. \quad (27)$$

This shows that the phase shift could be obtained by quantizing the electrons in the solenoid instead in agreement with Ref. [10], provided that retardation effects are negligible.

The situation is more interesting when retardation effects are important, as is the case in the experiment illustrated in Fig. 2. A constant current in source S generates a static magnetic field. The distance D between the source and an electron interferometer is assumed to be sufficiently large that $c\Delta t << D$, where Δt is the transit time of an electron through the interferometer.

It will also be assumed that the current $\boldsymbol{J}_e$ in the interferometer is zero outside of the time interval Δt. As a result, the retarded vector potential $\boldsymbol{\mathcal{A}}_e(\mathbf{r},t)$ at the source S is zero throughout the entire process and the last term in Eq. (23) is zero. Reinserting factors of $\hbar$, the remaining term gives an observable phase difference between the two paths of

$$\Delta\phi = \frac{1}{2}\frac{q}{\hbar} \oint \boldsymbol{\mathcal{A}}_S \cdot\; d\mathbf{l}. \quad (28)$$

Eq. (28) predicts that a fractional AB effect can occur as a result of retardation effects.

The factor of 1/2 in Eq. (28) is a consequence of the fact that half of the phase shift is associated with the electron in the interferometer while the other half is associated with those in the source of the static magnetic field, as can be seen in Eq. (23). It may seem counterintuitive that the output of an electron interferometer can depend on the phase shift of the electrons in a distant source. This situation is analogous to the nonlocal interference between two entangled photons that pass through two distant interferometers, where the output of one interferometer depends on the phase setting in both interferometers [25–27]. The nonlocal nature of the overall phase of the combined system is the essential feature in both situations.

It may be possible to test these results using a superconducting quantum interference device (SQUID), which is a magnetometer with a loop geometry similar to Fig. 1 and a response time as short as $1 \mu s$ [28]. Light can travel a distance of 300 m during that time, and the separation between the source and the SQUID would have to be larger than that to eliminate the retarded vector potential at the source.

A number of earlier papers [29–33] also quantized the electromagnetic field using different techniques, but none of them predicted a fractional AB phase shift with the exception of Ref. [29]. The approach in that paper would have double-counted the interaction and given an incorrect result were it not for cancellation from a sign error.

In summary, second-quantized field theory was used to describe all the electrons as well as the electromagnetic field in a consistent way. The results predict that a fractional Aharonov-Bohm effect can occur when retardation effects are significant. These results may be relevant in high-precision applications of SQUID magnetometers, and they may lead to new insight into the nature of retarded electromagnetic interactions in general. An experiment of this kind could test quantum electrodynamics under conditions that are very different from the usual scattering experiments.

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