

Probing departures from Λ CDM by late-time datasets

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Observational data play a pivotal role in identifying cosmological models that are both theoretically consistent and empirically viable. In this work, we investigate whether the standard Λ CDM model exhibits significant departure with current late time datasets, including Cosmic Chronometers, Baryon Acoustic Oscillations from DESI DR2, and various Type Ia supernova compilations (Pantheon⁺, DES-SN5Y, Union3). We analyze several dynamical dark energy models, including ω CDM, $o\omega$ CDM, $\omega_0\omega_a$ CDM, Logarithmic, Exponential, JBP, BA, and GEDE. While CC + DESI DR2 data show mild deviations from Λ CDM ($\lesssim 2\sigma$), adding supernova samples (DES-SN5Y or Union3) increases deviations, with BA, JBP, and Logarithmic models reaching 3–3.5 σ , and CC + DESI DR2 + DES-SN5Y producing the largest deviations. We find consistent evidence for $\omega_0 > -1$ and $\omega_a < 0$ in all dark energy models, indicating that the cosmological constant faces a potential crisis and that dynamical dark energy models could provide a possible solution, characterized by a Quintom-B-type scenario. The Λ CDM model has long served as the cornerstone of modern cosmology, successfully shaping our understanding of the Universe from its earliest epochs to the present day. However, in light of DESI DR2 and other recent measurements, emerging cracks in this paradigm suggest that a complete understanding of the cosmos may require moving beyond the cosmological constant and exploring new physics governing the dark sector.

I. INTRODUCTION

The late-time accelerated expansion of the Universe remains one of the most profound mysteries in modern cosmology [1, 2]. Within the standard Lambda Cold Dark Matter (Λ CDM) framework, this phenomenon is attributed to a positive cosmological constant Λ with negative pressure, introduced in Einstein’s field equations of General Relativity [3]. Although Λ CDM provides an excellent fit to observational data, the cosmological constant itself is a phenomenological parameter lacking a fundamental theoretical explanation for its observed value (see [4–6] for early discussions). Furthermore, the magnitude of Λ required by current observations implies a cosmic coincidence, marking our epoch as a particularly special time in the evolution of the Universe [6–8]. In response to these conceptual challenges, numerous theoretical models have been proposed to explain cosmic acceleration through dynamical mecha-

nisms [7, 9–20]. These alternatives typically involve dynamical energy densities that closely mimic Λ . In order to navigate the range of modeling possibilities, we consider parameterizations of background level cosmological quantities. This includes models involving expansions, parameterizations or principal component analyses of the equation of state $w \equiv p/\rho$ of a DE fluid with pressure p and energy density ρ [15–17, 21].

In modern concordance cosmology, there are certain discrepancies attributed to data sets (for instance, the Hubble H_0 (about $> 4\sigma$), and $S8 = \sigma_8\sqrt{\Omega_m}/0.3$ (about 2–3 σ) parameters measurements.) [22–25]. The full-shape analyses of large-scale structure data indicate a more pronounced tension, reaching a significance level of at least 4.5 σ [26, 27], thereby suggesting a potential inconsistency between constraints derived from early and late time observations of the Universe.

Complementing these findings, early results from Planck 2013 [28], suggested $\omega = -1.13^{+0.13}_{-0.14}$, slightly favoring the phantom regime. Subsequent improvements in supernova calibration, in 2014 Joint Light-curve Analysis (JLA) dataset [29] reduced this discrepancy and combining JLA with Planck 2013 brought dark energy constraints in agreement with Λ CDM. Planck 2015 [30], using JLA as its default supernova dataset, confirmed this consistency, yielding $\omega = -1.006^{+0.085}_{-0.091}$.

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However, in 2022, the Pantheon⁺ supernova compilation [31] reported $\omega = -0.90 \pm 0.14$ (SN only), and $\omega = -0.978^{+0.024}_{-0.031}$ when combined with CMB and BAO data, consistent with Λ CDM within 2σ . The Union3 compilation [32] further supported this trend, indicating mild tension with Λ CDM at $1.7 - 2.6\sigma$ and favoring dynamical dark energy models with $\omega_0 > -1$ and $\omega_a < 0$.

Furthermore, in 2024, building on hints from the Pantheon⁺ and Union3 compilations, DESY5 found that best-fit values of ω were consistently slightly greater than -1 at more than the 1σ level, both for supernova data alone and combined with CMB, BAO, and 3×2 pt measurements, supporting a trend toward mildly dynamical dark energy. DESI's first-year BAO data [33] further strengthened this evidence, showing deviations from Λ CDM at $2.6\sigma - 3.9\sigma$ when combined with CMB, Pantheon+, Union3, and DESY5 datasets, favoring a dynamical dark energy scenario with $\omega_0 > -1$, $\omega_a < 0$, and $\omega_\omega + \omega_a < -1$ (Quintom-B). The DESI DR2 BAO data [34] alone excludes Λ CDM at 3.1σ , and up to 4.2σ when combined with other datasets, with improved precision over DR1. These findings have motivated further exploration, and the dynamical behavior of dark energy and various theoretical models have been extensively studied in light of the DESI DR1 and DESI DR2 results [35–41].

The persistent discrepancies observed across multiple cosmological datasets call into question the completeness of the Λ CDM model, potentially motivating refinements in modeling techniques or extensions to its theoretical foundation. A model-independent approach to probing such deviations involves the parameterization of the dark energy equation of state (EoS), $\omega(z)$. This study considers different parameterizations of the EoS, based on the Chevallier–Polarski–Linder (CPL) form [15], also known as parameterization $\omega_0\omega_a$. The CPL framework enables the investigation of deviations from a true cosmological constant, and when applied to combined analyses of CMB data and various Type Ia supernova datasets, it reveals a departure from $\omega = -1$.

In this paper, we compare the parameterized constraints in ω phenomenologically derived with different dataset compilations. The statistical significance of dynamical DE features can be revealed by such an analysis. Our paper is organized as follows. In Section II, we introduce the cosmological background equations and models. Section III details the core of this work with datasets and methodology using the Markov Chain Monte Carlo (MCMC) sampling against the recent DESI DR2 dataset, while section IV is dedicated to the discussion of results. In Section V, we draw the conclusions.

II. THE BACKGROUND COSMOLOGY

Under the two foundational conditions, i.e., spatial uniformity and directional symmetry of the cosmological principle, which has become a testable hypothesis supported by extensive observational evidence across cosmic scales (e.g., [42, 43]), the spacetime geometry of the Universe can be described by the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime metric,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1)$$

where r , θ , and ϕ denote the comoving spatial coordinates, and t represents cosmic time. The parameter k characterizes the spatial curvature of the three-dimensional geometry of the Universe. The evolution of the Universe is governed by the scale factor $a(t)$, which is a function of the energy densities and pressures of the components that fill the Universe. This time, evolution is formally governed by the two Friedmann equations, which follow from the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{\Lambda}{8\pi G} g_{\mu\nu} \right) \quad (2)$$

when applied to a FLRW spacetime:

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_i - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_i + 3p_i) + \frac{\Lambda}{3}, \quad (4)$$

Here, H denotes the Hubble expansion rate, Λ represents the cosmological constant, ρ_i is the aggregate energy density of all cosmic components, and p_i is the corresponding pressure. Although the cosmological constant Λ can be absorbed into the overall energy density, it is conventionally kept explicit to honor its historical introduction in Einstein's equations. In light of observations that confirm cosmic acceleration, Λ is now interpreted as the simplest form of dark energy.

Dark energy influences the expansion rate through two primary quantities: its current density parameter relative to the critical density, Ω_{de} , and its equation-of-state parameter, ω . The most straightforward assumption is that the equation-of-state parameter remains fixed in time. However, in the general case, ω may vary with cosmic time or redshift. The energy-momentum conservation equation

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0, \quad (5)$$

is not independent but follows directly from combining Eqs. (3) and (4). Substituting the solution of Eq. (5) that

represents the redshift evolution of the individual energy components into Eq. (3) yields the Hubble expansion function, which governs the dynamical evolution of the Universe:

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda f_{\text{DE}}(z) \right], \quad (6)$$

where $E(z) \equiv H^2(z)/H_0^2$ is the dimensionless Hubble function, H_0 is the present-day Hubble constant, and Ω_r , $\Omega_m = \Omega_b + \Omega_{\text{cdm}}$, Ω_k , and Ω_Λ are the density parameters of radiation, matter (baryonic plus cold dark matter), spatial curvature, and dark energy, respectively. Also, the function $f_{\text{DE}}(z)$ denotes the redshift dependence of the dark energy component and is defined as

$$f_{\text{DE}}(\equiv \rho_{\text{DE}}(z)/\rho_{\text{DE},0}) = \exp \left[3 \int_0^z \frac{1 + \omega(z')}{1 + z'} dz' \right]. \quad (7)$$

For a cosmological constant ($\omega = -1$), $f_{\text{DE}}(z) = 1$, and Eq. (6) reduces to the standard Λ CDM,

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda \right]. \quad (8)$$

In the special case of a spatially flat Universe ($\Omega_{k0} = 0$), Eq. (8) reduces to

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda \right]. \quad (9)$$

In the case of a constant ω , Eq. (7) simplifies to $(1+z)^{3(1+\omega)}$, and substituting this into Eq. (6) yields the corresponding Hubble function of the ω CDM model.

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda(1+z)^{3(1+\omega)} \right]. \quad (10)$$

Taking into account $\Omega_{k0} = 0$ in Eq. (10), it reduces to the spatially flat ω CDM model.

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+\omega)} \right]. \quad (11)$$

We also consider several dynamical dark energy models that correspond to different functional forms of $\omega(z)$, as summarized in Table I (2nd column). For each model, using Eq. (7), we can derive the corresponding form of $f_{\text{DE}}(z)$ (3rd column). These expressions for $f_{\text{DE}}(z)$ can then be substituted in Eq. (6) to obtain the corresponding Hubble function, which characterizes the expansion history for each model.

III. DATASETS AND METHODOLOGY

In our analysis, we use the `SimpleMC` cosmological inference code [55, 56] to estimate the posterior distributions of parameters for each cosmological model. In this code, we use the Metropolis–Hastings Markov Chain Monte Carlo (MCMC) algorithm [57], which allows efficient exploration of the parameter space. The convergence of Markov chains is tested using the Gelman–Rubin diagnostic ($R - 1$) [58], applying a strict threshold of $R - 1 < 0.03$. In our analysis, we used multiple datasets, with a focus on late-time observations, to determine the posterior distributions of the model parameters. The details of these datasets are provided below.

- **Cosmic Chronometers:** First, we consider Cosmic Chronometers (CC), massive, passively evolving galaxies with old stellar populations and low star formation, which allow direct measurements of the Hubble parameter $H(z)$ via the differential age technique [59]. These measurements are valuable for model independent studies of the Universe’s expansion history. For our analysis, we use 15 measurements from [60–62] instead of the 31 considered in [63], as only these consist of statistical and systematic part of the covariance matrix [64, 65]. While [63] only consider the statistical part only.
- **Baryon Acoustic Oscillations:** Next, we use recent baryon acoustic oscillation (BAO) measurements from over 14 million galaxies and quasars obtained by the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2) [34]. These measurements are obtained from different tracers, such as Bright Galaxy Sample (BGS), Luminous Red Galaxies (LRG1), LRG2, LRG3+Emission Line Galaxies (ELG1), ELG2, Quasars (QSO), and Lyman- α forests (Ly α); for further details, see Section 3 of [34]. Each tracer provides different BAO measurements expressed through various ratios: D_M/r_d , D_H/r_d , D_V/r_d , and D_M/D_H . Here, $D_H(z) = c/H(z)$ represents the Hubble distance, $D_M(z) = c \int_0^z \frac{dz'}{H(z')}$ represents the comoving angular diameter distance, and $D_V(z) \equiv [z D_M^2(z) D_H(z)]^{1/3}$ represents the volume-averaged distance. The r_d corresponds to the sound horizon in the drag epoch, which in a flat Λ CDM model takes the value $r_d = 147.09 \pm 0.2$ Mpc [66].
- **Type Ia supernovae :** Finally, we use three dif-

Parameterization	$\omega(z)$	$f_{DE}(z)$	Reference
$\omega_0\omega_a$ CDM	$\omega_0 + \frac{z}{1+z}\omega_a$	$(1+z)^{3(1+\omega_0+\omega_a)}e^{-\frac{3\omega_0z}{1+z}}$	[44, 45]
Logarithmic	$\omega_0 + \omega_a \log(1+z)$	$(1+z)^{3(1+\omega_0)}e^{\frac{3}{2}\omega_a(\log(1+z))^2}$	[46, 47]
Exponential	$\omega_0 + \omega_a \left(e^{\frac{z}{1+z}} - 1\right)$	$e^{\left[3\omega_a\left(\frac{-z}{1+z}\right)\right]}(1+z)^{3(1+\omega_0+\omega_a)}e^{\left[3\omega_a\left(\frac{1}{4(1+z)^2} + \frac{1}{2(1+z)} - \frac{3}{4}\right)\right]}(1+z)^{\frac{3}{2}\omega_a}$	[48–50]
JBP	$\omega_0 + \frac{z}{(1+z)^2}\omega_a$	$(1+z)^{3(1+\omega_0)}e^{\frac{3\omega_0z^2}{2(1+z)^2}}$	[51]
BA	$\omega_0 + \frac{z(1+z)}{1+z^2}\omega_a$	$(1+z)^{3(1+\omega_0)}(1+z^2)^{\frac{3\omega_a}{2}}$	[52]
GEDE	$-1 - \frac{\Delta}{3\ln(10)} \left[1 + \tanh \left(\Delta \log_{10} \left(\frac{1+z}{1+z_t} \right) \right) \right]$	$\left(\frac{1 - \tanh \left(\Delta \times \log_{10} \left(\frac{1+z}{1+z_t} \right) \right)}{1 + \tanh \left(\Delta \times \log_{10} (1+z_t) \right)} \right)$	[53, 54]

TABLE I: Dark energy parameterizations with their equations of state $\omega(z)$, evolution functions $f_{DE}(z)$

ferent supernova compilations. First, we consider the Pantheon⁺ sample [31], which comprises 1701 light curves from 1550 Type Ia Supernovae (SNe Ia). In our analysis, we exclude the light curves below $z < 0.01$, as these light curves are significantly affected by systematic uncertainties arising from peculiar velocities. Next, we use a sample of 1829 photometric light curves from the full five years of the Dark Energy Survey Supernova program (DES-SN5Y) [67]. This dataset includes 1,635 DES-discovered events along with 194 externally sourced low- z supernovae from CfA and CSP. Finally, we use the Union3 compilation [32], which comprises 2,087 SNe Ia, including 1,363 events overlapping with the Pantheon⁺ sample. For each sample, we marginalize the parameter $\mathcal{M}$; see Equations (A9–A12) in [68] for further details.

To evaluate and compare the statistical performance of each model, we compute the Bayesian evidence $\ln \mathcal{Z}$ using the MCEvidence framework [69]. This metric quantifies the goodness of fit while penalizing the complexity of the model. Model comparison is performed via the Bayes factor: $B_{ab} = \frac{\mathcal{Z}_a}{\mathcal{Z}_b}$, or equivalently, through the difference in logarithmic evidence: $c\Delta \ln \mathcal{Z} = \ln \mathcal{Z}_a - \ln \mathcal{Z}_b$. A higher value of $\ln \mathcal{Z}$ indicates stronger statistical support for the model, while a lower value suggests weaker support. The strength of evidence is interpreted using the Jeffreys scale:

- $|\Delta \ln \mathcal{Z}| < 1$: Inconclusive / Weak evidence,
- $1 \leq |\Delta \ln \mathcal{Z}| < 3$: Moderate evidence,
- $3 \leq |\Delta \ln \mathcal{Z}| < 5$: Strong evidence,
- $|\Delta \ln \mathcal{Z}| \geq 5$: Decisive evidence.

This method provides a robust approach to selecting the most suitable cosmological model. In our analy-

sis, we adopt several foundational assumptions. Specifically, for a dynamical dark energy model, we assume a flat scenario ($\Omega_k = 0$) and compute the present-day radiation density parameter as $\Omega_r = 2.469 \times 10^{-5} h^{-2} (1 + 0.2271 N_{\text{eff}})$ following [70], where $N_{\text{eff}} = 3.04$ is the standard effective number of relativistic species [71]. Under these assumptions, the dark energy density parameter is given by $\Omega_\Lambda = 1 - \Omega_r - \Omega_m - \Omega_k$. In the case of the flat Universe $\Omega_{k0} = 0$ this realization reduces to $\Omega_\Lambda = 1 - \Omega_r - \Omega_m$. As a result, both Ω_Λ and Ω_r are not treated as independent parameters, since they are fully determined by the remaining parameters. The priors chosen for these models are summarized in Table II.

IV. RESULTS

In cosmology, most anomalies such as the Hubble tension, the S_8 discrepancy, the M_B calibration offset, and the CMB lensing anomaly typically show deviations in the range of $2\text{--}4\sigma$. While not definitive, such levels are considered statistically significant and often prompt further investigation. Here, we carry out such an analysis by exploring a range of cosmological models and their compatibility with current observational datasets.

Table III shows the numerical values of the corresponding parameters for the Λ CDM, $\text{o}\Lambda$ CDM, ω CDM, $\omega_0\omega_a$ CDM, Logarithmic, Exponential, JBP, BA and GEDE models using MCMC analysis. Fig. 1 shows the corner plots of each cosmological model using combinations of DESI DR2, various SNe Ia compilations and CC measurements. The diagonal panels show the 1D marginalized posterior distributions for each parameter, while the off-diagonal panels show the 2D marginalized confidence contours at 68% and 95% confidence levels.

First, we also obtain numerical values obtained from the CC measurements alone. For the Λ CDM model,

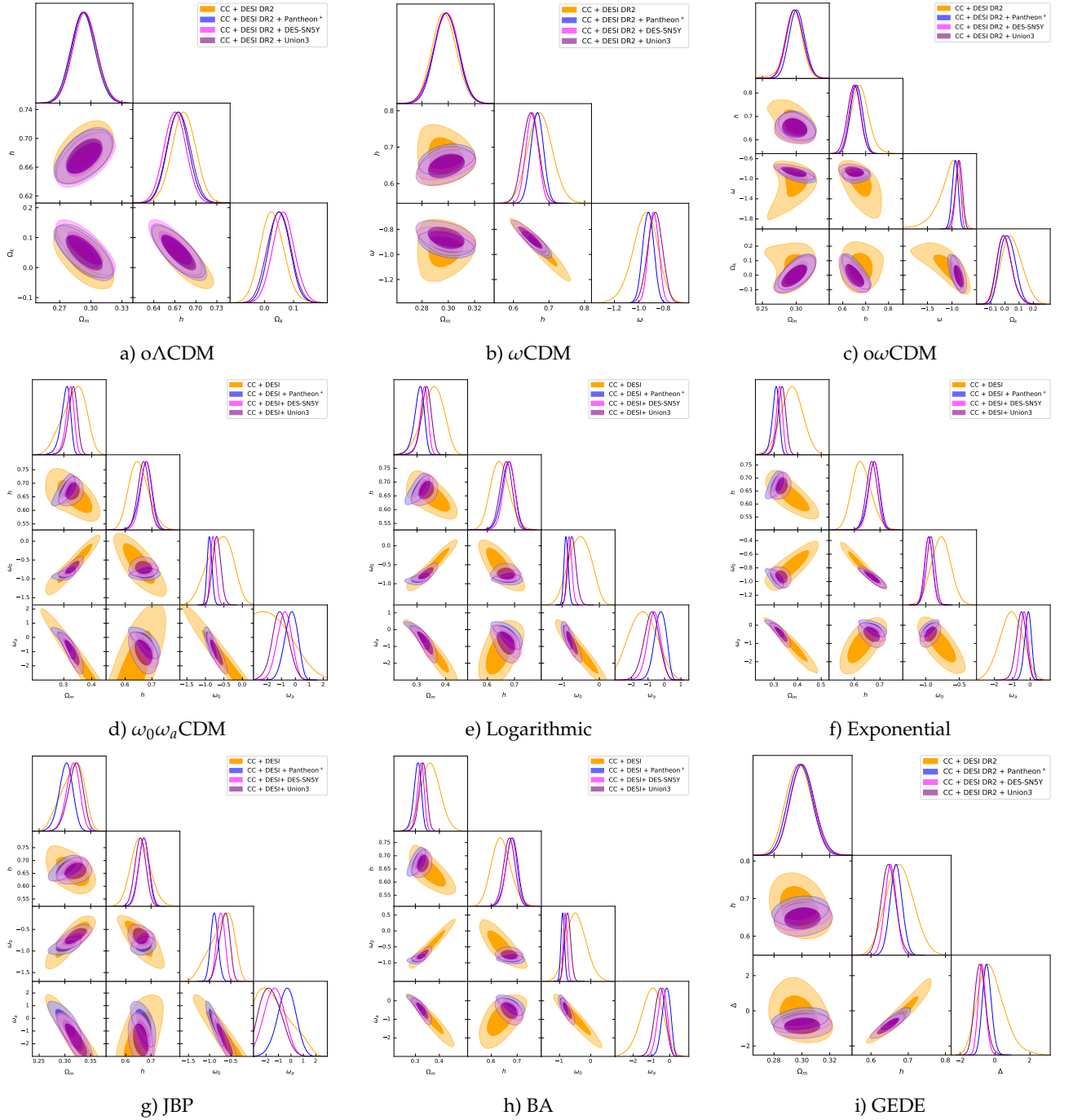


FIG. 1: The figure shows the corner plot of the $\text{o}\Lambda\text{CDM}$, ωCDM , $\text{o}\omega\text{CDM}$, $\omega_0\omega_a\text{CDM}$, Logarithmic, Exponential, JBP, BA, and GEDE models using DESI DR2 measurements in combination with the CMB and SNe Ia measurements, at the 68% (1σ) and 95% (2σ) confidence intervals.

we find $h = 0.662^{+0.053}_{-0.048}$ (hereafter referred to as the ΛCDM value), consistent [60–62]. Among its simple extensions, $\text{o}\Lambda\text{CDM}$ predicts $h = 0.674 \pm 0.053$, corresponding to a 0.24σ deviation from the ΛCDM value, while GEDE predicts $h = 0.678 \pm 0.053$, deviating by 0.32σ . ωCDM yields $h = 0.689^{+0.087}_{-0.097}$, showing a slight

tension of 0.53σ relative to the value of ΛCDM . In contrast, the $\text{o}\omega\text{CDM}$ model predicts $h = 0.716^{+0.077}_{-0.089}$, showing about a 1.07σ deviation from the ΛCDM value. Similarly, the $\omega_0\omega_a\text{CDM}$ model yields $h = 0.727 \pm 0.082$, corresponding to a 1.29σ deviation, while the JBP model gives $h = 0.728^{+0.084}_{-0.072}$, with a deviation 1.31σ . The BA

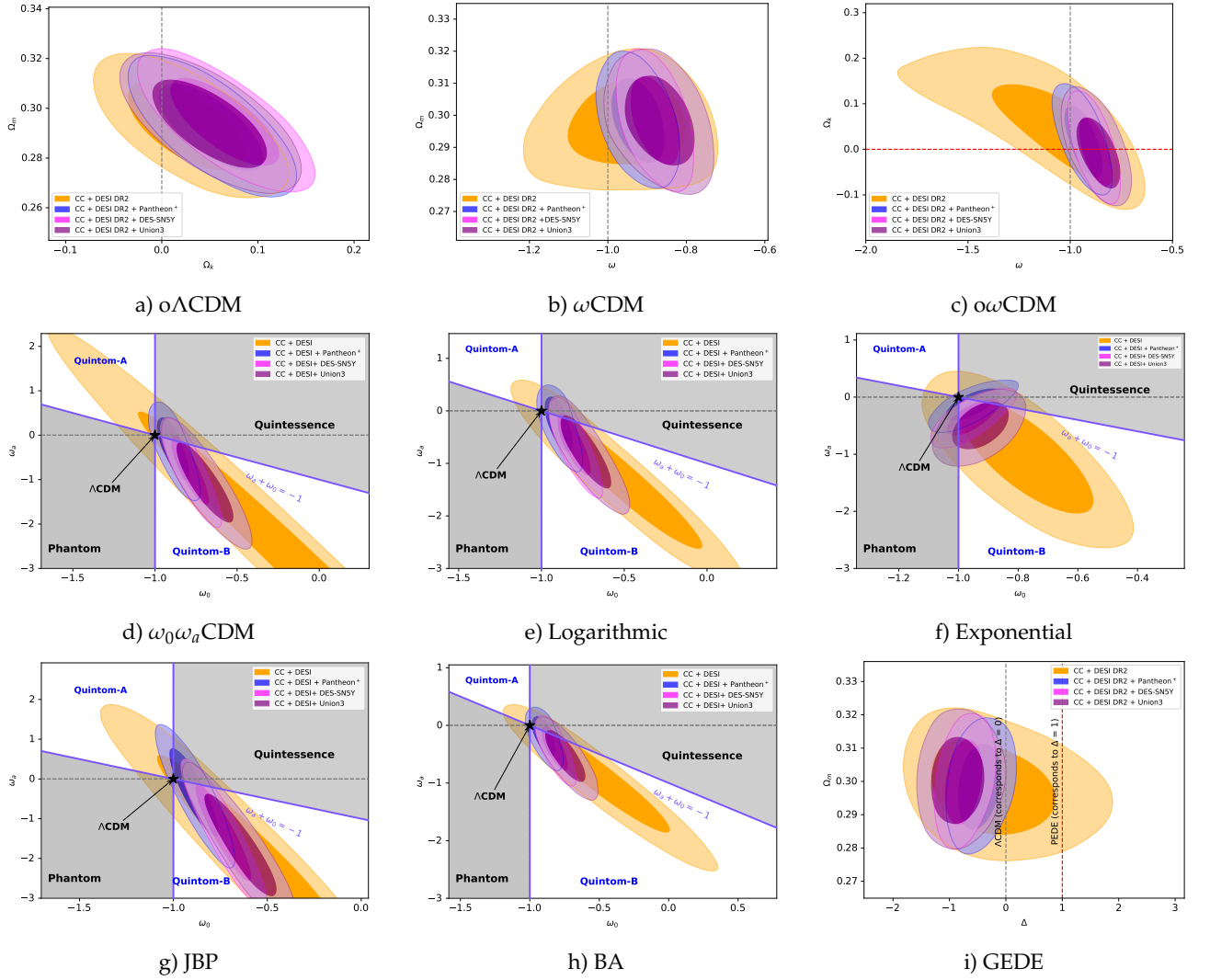


FIG. 2: The figure shows the posterior distributions of different planes of the $\text{o}\Lambda\text{CDM}$, ωCDM , $\text{o}\omega\text{CDM}$, $\omega_a\omega_0\text{CDM}$, Logarithmic, Exponential, JBP, BA, and GEDE models using DESI DR2 measurements in combination with the CMB and SNe Ia measurements, at the 68% (1σ) and 95% (2σ) confidence intervals.

parameterization predicts $h = 0.733 \pm 0.079$, marking the largest shift among these models at 1.41σ . Among other parameterizations, the Logarithmic model estimates $h = 0.731 \pm 0.081$ (1.37σ deviation), and the Exponential model provides the highest shift, $h = 0.736 \pm 0.079$, with a deviation 1.47σ relative to the ΛCDM value, although tensions remain below 1.5σ .

We further analyzed the deviations of the Hubble parameter h for each dark energy model relative to the ΛCDM model, incorporating the CC dataset along with DESI DR2 and various SNe Ia compilations. The tensions in h relative to ΛCDM vary between models and datasets. Models such as $\text{o}\Lambda\text{CDM}$ and $\text{o}\omega\text{CDM}$ exhibit mild to moderate tensions, with $\text{o}\Lambda\text{CDM}$ showing $[0.64, 1.13, 1.48, 1.06]\sigma$ and $\text{o}\omega\text{CDM}$

showing $[0.40, 1.11, 1.41, 1.41]\sigma$ for CC + DESI DR2, CC + DESI DR2 + Pantheon⁺, CC + DESI DR2 + DES-SN5Y, and CC + DESI DR2 + Union3, respectively. The ωCDM model shows negligible to moderate tensions, $[0.48, 1.22, 1.98, 1.78]\sigma$, while the $\omega_0\omega_a\text{CDM}$ model exhibits negligible to mild tensions, $[1.06, 0.60, 0.61, 0.95]\sigma$. Other models show a diverse behavior, with Logarithmic at $[1.25, 0.59, 0.56, 0.97]\sigma$, Exponential at $[1.64, 0.66, 0.72, 1.05]\sigma$, JBP at $[0.85, 0.86, 0.98, 1.42]\sigma$, BA at $[1.43, 0.53, 0.60, 0.98]\sigma$, and GEDE at $[0.40, 1.27, 2.11, 1.18]\sigma$. These results indicate negligible to moderate dataset-dependent variations in h across the considered dark energy models.

For the matter density Ω_m , the tensions relative to

Model	Parameter	Prior
Λ CDM	Ω_{m0}	$\mathcal{U}[0, 1]$
	$h = H_0/100$	$\mathcal{U}[0, 1]$
o Λ CDM	Ω_{k0}	$\mathcal{U}[-1, 1]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
ω CDM	ω_0	$\mathcal{U}[-3, 1]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
o ω CDM	ω_0	$\mathcal{U}[-3, 1]$
	Ω_{k0}	$\mathcal{U}[-1, 1]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
$\omega_0\omega_a$ CDM	ω_0	$\mathcal{U}[-3, 1]$
	ω_a	$\mathcal{U}[-3, 2]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
Logarithmic	ω_0	$\mathcal{U}[-3, 1]$
	ω_a	$\mathcal{U}[-3, 2]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
Exponential	ω_0	$\mathcal{U}[-3, 1]$
	ω_a	$\mathcal{U}[-3, 2]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
JBP	ω_0	$\mathcal{U}[-3, 1]$
	ω_a	$\mathcal{U}[-3, 2]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
BA	ω_0	$\mathcal{U}[-3, 1]$
	ω_a	$\mathcal{U}[-3, 2]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$
GEDE	Δ	$\mathcal{U}[-10, 10]$
	Ω_{m0}, h	$\mathcal{U}[0, 1]$

TABLE II: The parameters and their priors including uniform priors $\mathcal{U}$ and the reduced Hubble constant $h \equiv H_0/100$.

Λ CDM are as follows. Geometric models show negligible to mild tensions: o Λ CDM $[0.28, 0.83, 1.04, 0.66]\sigma$, o ω CDM $[0.07, 0.22, 0.76, 0.35]\sigma$. ω CDM shows negligible to mild tensions $[0.08, 0.53, 0.91, 0.42]\sigma$, while $\omega_0\omega_a$ CDM exhibits negligible to moderate deviations in some datasets $[0.95, 0.11, 0.72, 1.47]\sigma$. Logarithmic $[1.49, 0.11, 0.87, 1.46]\sigma$ and Exponential $[1.67, 0.06, 0.68, 1.53]\sigma$ show mild to moderate deviations relative to Λ CDM. BA $[1.62, 0.24, 0.74, 1.47]\sigma$ also exhibits mild to moderate tensions, whereas JBP $[0.74, 0.13, 0.39, 1.05]\sigma$ and GEDE $[0.08, 0.53, 0.80, 0.27]\sigma$ remain closer to the predictions of Λ CDM, indicating negligible to mild alignment in Ω_m .

Fig. 2 shows the different parameter planes of various cosmological models. These planes provide insights into the geometry of the Universe and the nature of dark energy. Figs. 2a, 2b, and 2c show the $\omega - \Omega_m$, $\omega - \Omega_k$ and $\Omega_m - \Omega_k$ planes for the o Λ CDM, ω CDM and o ω CDM models, respectively. It is important to note that the o Λ CDM model predicts $\Omega_k \approx 0$ for each combination of datasets. Similarly, the o ω CDM model predicts $\Omega_k \approx 0$ when considering CC + DESI DR2

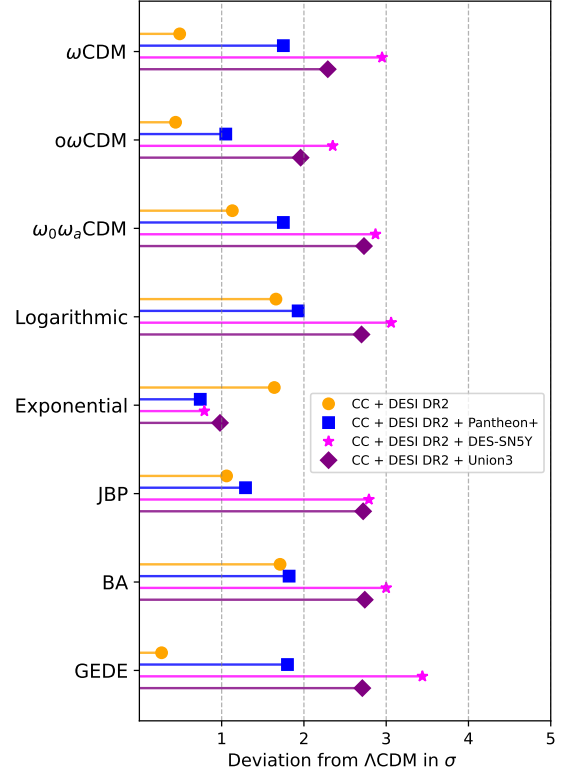


FIG. 3: The figure shows the σ deviation of each model from the Λ CDM model using different combinations of datasets with the DESI DR2 data.

and CC + DESI DR2 + Pantheon⁺, respectively. On the other hand, when we include DES-SN5Y and Union3, the prediction changes slightly to $\Omega_k \approx -0.002$. Indeed, these predictions show close agreement with those from WMAP ($-0.0179 < \Omega_k < 0.0081$, 95% CL) [72], BOOMERanG ($0.88 < \Omega_{M/R} + \Omega_\Lambda < 1.0081$, 95% CL) [73], and Planck ($\Omega_{M/R} + \Omega_\Lambda = 1.00 \pm 0.026$, 68% CL) [66], suggesting that $\Omega_k \approx 0$; $\Rightarrow k \approx 0$, which corresponds to a spatially flat Universe. Furthermore, we also observe that in the cases of the ω CDM and o ω CDM models, the predicted value of ω in each case, $\omega_0 > -1$, shows a deviation from $\omega = -1$. However, in the o ω CDM model, when we consider the combination CC + DESI DR2, we obtain $\omega_0 \approx -1$, which is close to the prediction Λ CDM. This indicates that when different calibrations for SNe Ia are added, we observe a deviation from the predictions of Λ CDM.

Figs. 2d, 2e, 2f, 2g, and 2h show the $\omega_0 - \omega_a$ planes for the $\omega_0\omega_a$ CDM, Logarithmic, Exponential, JBP, and BA models, respectively. These provide important insights into the nature of dark energy. Each model predicts $\omega_0 > -1$ and $\omega_a < 0$ for each combination of DESI DR2 datasets with CC and different calibrations

Parameter	Dataset	Λ CDM	ω CDM	ω CDM	ω CDM	$\omega_0\omega_a$ CDM	Logarithmic	Exponential	JBP	BA	GEDE
h	CC + DESI DR2	0.692 ± 0.01	0.683 ± 0.01	0.675 ± 0.034	$0.677^{+0.033}_{-0.040}$	$0.652^{+0.032}_{-0.041}$	$0.647^{+0.031}_{-0.038}$	$0.629^{+0.033}_{-0.041}$	$0.659^{+0.035}_{-0.040}$	$0.638^{+0.033}_{-0.040}$	0.677 ± 0.036
	CC + DESI DR2 + Pantheon ⁺	0.691 ± 0.01	0.675 ± 0.01	0.667 ± 0.017	0.660 ± 0.026	$0.676^{+0.025}_{-0.021}$	$0.676^{+0.025}_{-0.021}$	$0.675^{+0.024}_{-0.020}$	0.671 ± 0.021	$0.678^{+0.024}_{-0.021}$	0.667 ± 0.016
	CC + DESI DR2 + DES-SN5Y	0.691 ± 0.01	0.670 ± 0.01	0.652 ± 0.017	0.653 ± 0.025	$0.677^{+0.022}_{-0.019}$	$0.678^{+0.022}_{-0.020}$	$0.675^{+0.021}_{-0.019}$	0.670 ± 0.019	$0.677^{+0.022}_{-0.020}$	0.653 ± 0.015
	CC + DESI DR2 + Union3	0.691 ± 0.01	0.676 ± 0.01	0.648 ± 0.022	0.649 ± 0.028	0.668 ± 0.022	0.668 ± 0.022	0.668 ± 0.021	0.658 ± 0.021	0.669 ± 0.022	0.647 ± 0.036
Ω_m	CC + DESI DR2	0.297 ± 0.008	0.293 ± 0.012	0.296 ± 0.009	$0.296^{+0.014}_{-0.011}$	$0.337^{+0.048}_{-0.035}$	$0.354^{+0.042}_{-0.033}$	0.375 ± 0.046	$0.314^{+0.025}_{-0.018}$	0.363 ± 0.040	0.298 ± 0.009
	CC + DESI DR2 + Pantheon ⁺	0.304 ± 0.008	0.292 ± 0.012	0.298 ± 0.008	0.301 ± 0.011	$0.306^{+0.020}_{-0.013}$	$0.306^{+0.020}_{-0.013}$	$0.305^{+0.017}_{-0.012}$	$0.302^{+0.014}_{-0.012}$	$0.308^{+0.017}_{-0.013}$	0.298 ± 0.008
	CC + DESI DR2 + DES-SN5Y	0.309 ± 0.008	0.294 ± 0.012	0.298 ± 0.009	0.298 ± 0.012	$0.321^{+0.017}_{-0.012}$	$0.323^{+0.016}_{-0.012}$	$0.320^{+0.016}_{-0.012}$	$0.315^{+0.014}_{-0.012}$	$0.321^{+0.016}_{-0.012}$	0.300 ± 0.008
	CC + DESI DR2 + Union3	0.303 ± 0.008	0.294 ± 0.011	0.298 ± 0.009	0.298 ± 0.012	$0.330^{+0.019}_{-0.014}$	$0.331^{+0.020}_{-0.015}$	$0.331^{+0.020}_{-0.015}$	$0.320^{+0.015}_{-0.013}$	$0.330^{+0.018}_{-0.015}$	0.300 ± 0.008
Ω_k	CC + DESI DR2	—	0.027 ± 0.041	—	$0.049^{+0.066}_{-0.078}$	—	—	—	—	—	—
	CC + DESI DR2 + Pantheon ⁺	—	0.051 ± 0.036	—	$0.019^{+0.049}_{-0.056}$	—	—	—	—	—	—
	CC + DESI DR2 + DES-SN5Y	—	0.066 ± 0.038	—	$-0.002^{+0.045}_{-0.056}$	—	—	—	—	—	—
	CC + DESI DR2 + Union3	—	0.050 ± 0.039	—	$-0.002^{+0.045}_{-0.058}$	—	—	—	—	—	—
ω_0	CC + DESI DR2	—	—	$-0.951^{+0.11}_{-0.091}$	$-1.09^{+0.28}_{-0.13}$	$-0.61^{+0.37}_{-0.32}$	-0.52 ± 0.29	-0.77 ± 0.14	$-0.72^{+0.33}_{-0.20}$	$-0.46^{+0.29}_{-0.34}$	—
	CC + DESI DR2 + Pantheon ⁺	—	—	-0.923 ± 0.044	$-0.937^{+0.064}_{-0.056}$	$-0.889^{+0.059}_{-0.068}$	$-0.893^{+0.052}_{-0.059}$	-0.955 ± 0.061	-0.888 ± 0.087	-0.898 ± 0.056	—
	CC + DESI DR2 + DES-SN5Y	—	—	-0.879 ± 0.041	$-0.878^{+0.057}_{-0.047}$	$-0.786^{+0.068}_{-0.081}$	$-0.801^{+0.059}_{-0.071}$	-0.954 ± 0.058	-0.738 ± 0.094	-0.817 ± 0.061	—
	CC + DESI DR2 + Union3	—	—	-0.867 ± 0.058	$-0.865^{+0.076}_{-0.062}$	-0.70 ± 0.11	$-0.729^{+0.091}_{-0.11}$	-0.938 ± 0.063	$-0.66^{+0.14}_{-0.11}$	-0.740 ± 0.095	—
ω_a	CC + DESI DR2	—	—	—	—	$-1.2^{+1.0}_{-1.3}$	$-1.29^{+0.78}_{-0.88}$	-1.07 ± 0.63	$-1.66^{+0.74}_{-1.6}$	$-0.99^{+0.64}_{-0.54}$	—
	CC + DESI DR2 + Pantheon ⁺	—	—	—	—	$-0.31^{+0.51}_{-0.43}$	$-0.27^{+0.39}_{-0.31}$	$-0.13^{+0.20}_{-0.18}$	$-0.36^{+0.78}_{-0.69}$	$-0.19^{+0.26}_{-0.21}$	—
	CC + DESI DR2 + DES-SN5Y	—	—	—	—	$-0.80^{+0.55}_{-0.48}$	$-0.65^{+0.42}_{-0.36}$	-0.35 ± 0.21	-1.31 ± 0.77	$-0.41^{+0.28}_{-0.23}$	—
	CC + DESI DR2 + Union3	—	—	—	—	-1.07 ± 0.58	$-0.80^{+0.48}_{-0.43}$	$-0.51^{+0.29}_{-0.26}$	$-1.56^{+0.63}_{-1.0}$	$-0.54^{+0.32}_{-0.28}$	—
Δ	CC + DESI DR2	—	—	—	—	—	—	—	—	—	$-0.20^{+0.60}_{-0.86}$
	CC + DESI DR2 + Pantheon ⁺	—	—	—	—	—	—	—	—	—	$-0.46^{+0.24}_{-0.37}$
	CC + DESI DR2 + DES-SN5Y	—	—	—	—	—	—	—	—	—	$-0.74^{+0.20}_{-0.23}$
	CC + DESI DR2 + Union3	—	—	—	—	—	—	—	—	—	$-0.84^{+0.28}_{-0.34}$
$ \Delta \ln \mathcal{Z}_{\Lambda\text{CDM,Model}} $	CC + DESI DR2	0	2.14	1.17	2.70	0.94	0.96	0.51	0.77	0.54	0.70
	CC + DESI DR2 + Pantheon ⁺	0	1.53	0.47	2.48	0.25	1.03	1.03	0.18	0.70	1.28
	CC + DESI DR2 + DES-SN5Y	0	0.76	2.17	0.11	3.63	2.80	2.80	4.25	3.03	4.02
	CC + DESI DR2 + Union3	0	1.51	0.96	1.05	3.02	2.92	1.85	3.41	2.62	2.80
Deviation from Λ CDM	CC + DESI DR2	0	0	0.49	0.44	1.13	1.66	1.64	1.06	1.71	0.27
	CC + DESI DR2 + Pantheon ⁺	0	0	1.75	1.05	1.75	1.93	0.74	1.29	1.82	1.80
	CC + DESI DR2 + DES-SN5Y	0	0	2.95	2.35	2.87	3.06	0.79	2.79	3.00	3.44
	CC + DESI DR2 + Union3	0	0	2.29	1.96	2.73	2.70	0.98	2.72	2.74	2.71

TABLE III: This table presents the numerical values obtained for the ω CDM, ω CDM, $\omega\omega$ CDM, $\omega_a\omega_0$ CDM, Logarithmic, Exponential, JBP, BA, and GEDE models at the 68% (1σ) confidence level, using different combinations of DESI DR2 BAO datasets with the CMB and various SNe Ia samples.

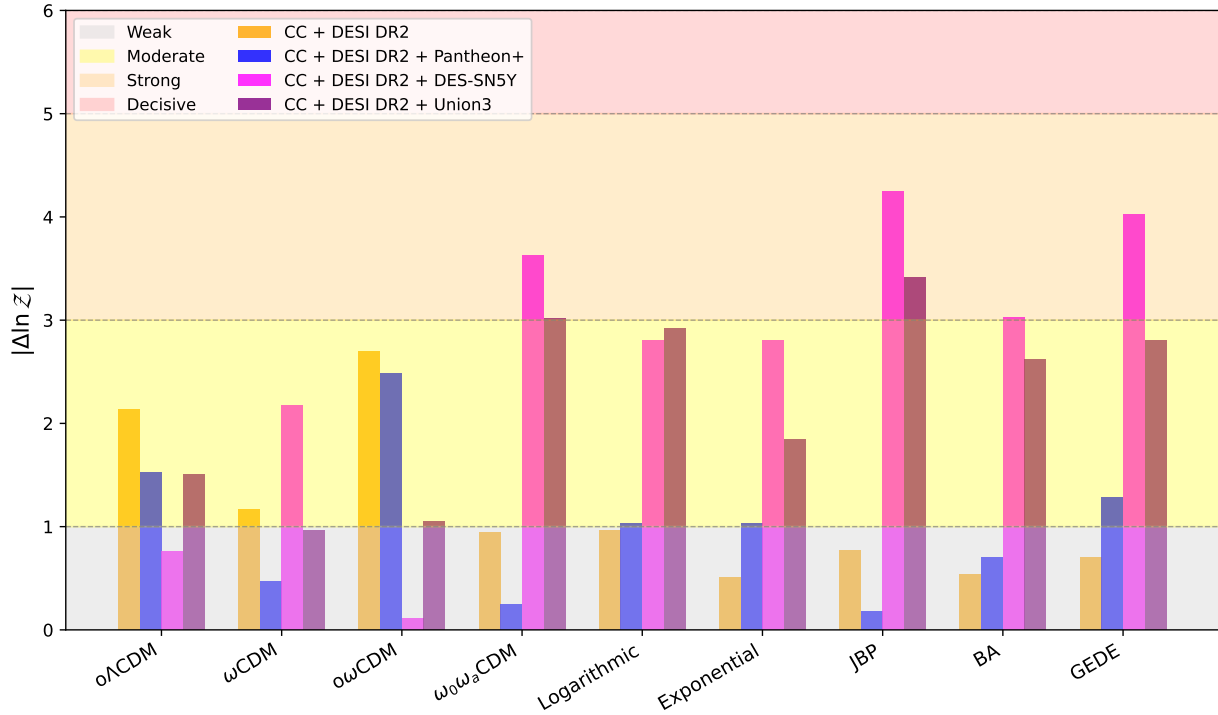


FIG. 4: The figure shows a comparison of various models relative to Λ CDM across different cosmological datasets, using the Bayesian evidence differences ($|\Delta \ln \mathcal{Z}|$).

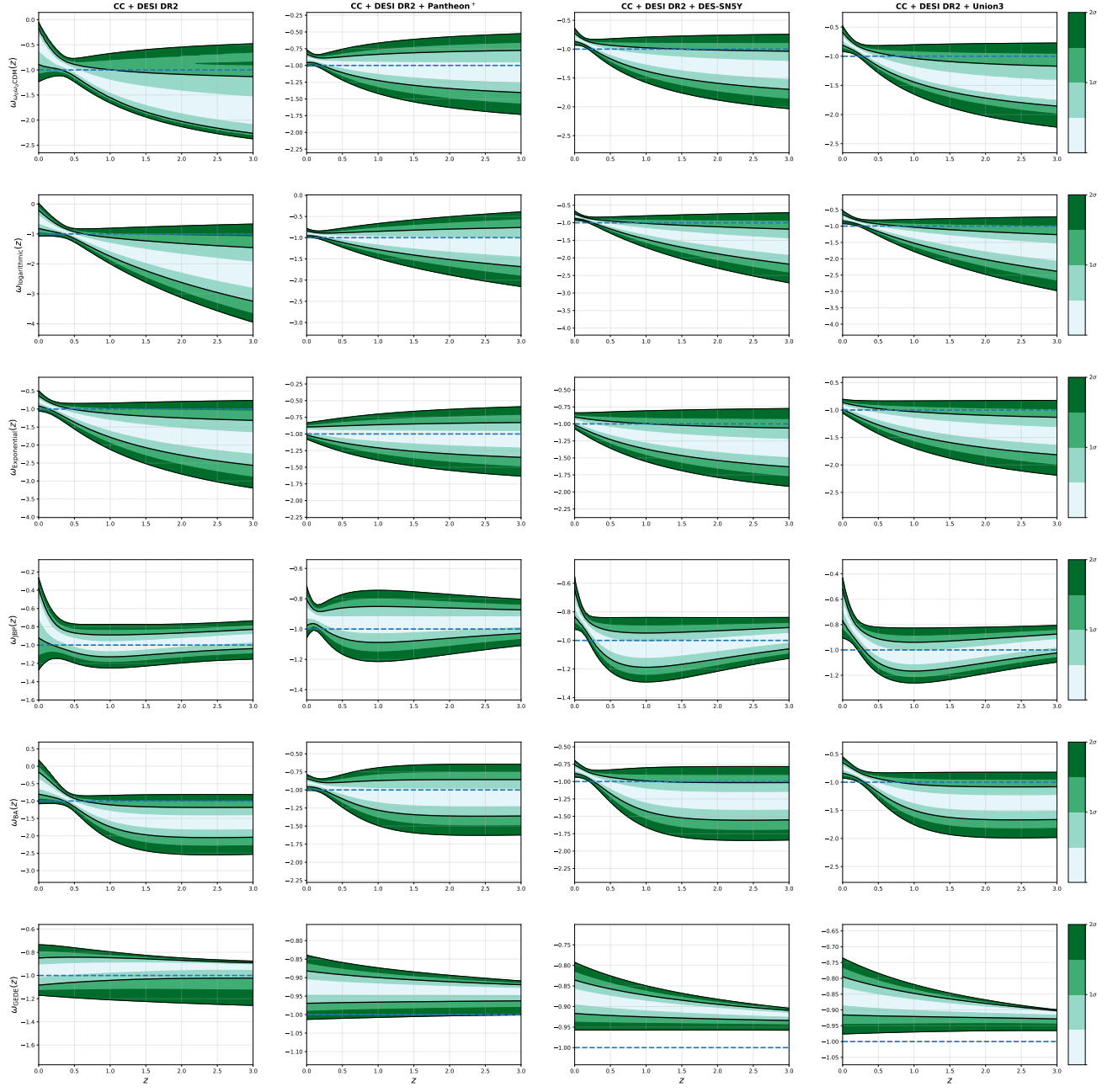


FIG. 5: This figure shows the evolution of $\omega(z)$ as a function of redshift, using DESI DR2 measurements with different combinations of SNe Ia compilations (Pantheon⁺, DESY5, Union3) and CMB, providing compelling evidence for dynamical DE.

of SNe Ia, showing the deviation from the predictions Λ CDM characterized by $(\omega_0 = -1, \omega_a = 0)$. The preference values of ω_0 and ω_a show evidence of a dynamical dark energy scenario characterized by $\omega_0 > -1$, $\omega_a < 0$ and $\omega_0 + \omega_a < -1$, corresponding to a Quintom-B-type behavior [74, 75]. Fig. 2i shows the $\Delta - \Omega_m$ plane for the GEDE model. It is crucial to note that, in each case, the GEDE model predicts a negative value of Δ , indicating an injection of dark energy at high redshifts. In-

deed, these results deviate from those reported by [76–78], since they consider the CMB in their analysis. Thus, one can also see the effect of late-time measurements on the predicted value of Δ .

Fig. 3 shows the deviation of each dark energy model from the Λ CDM model. The combination of the (CC + DESI DR2 and CC + DESI DR2 + Pantheon⁺) datasets shows only mild deviations ($\lesssim 2\sigma$). When we include DES-SN5Y or Union3 supernova samples, the devia-

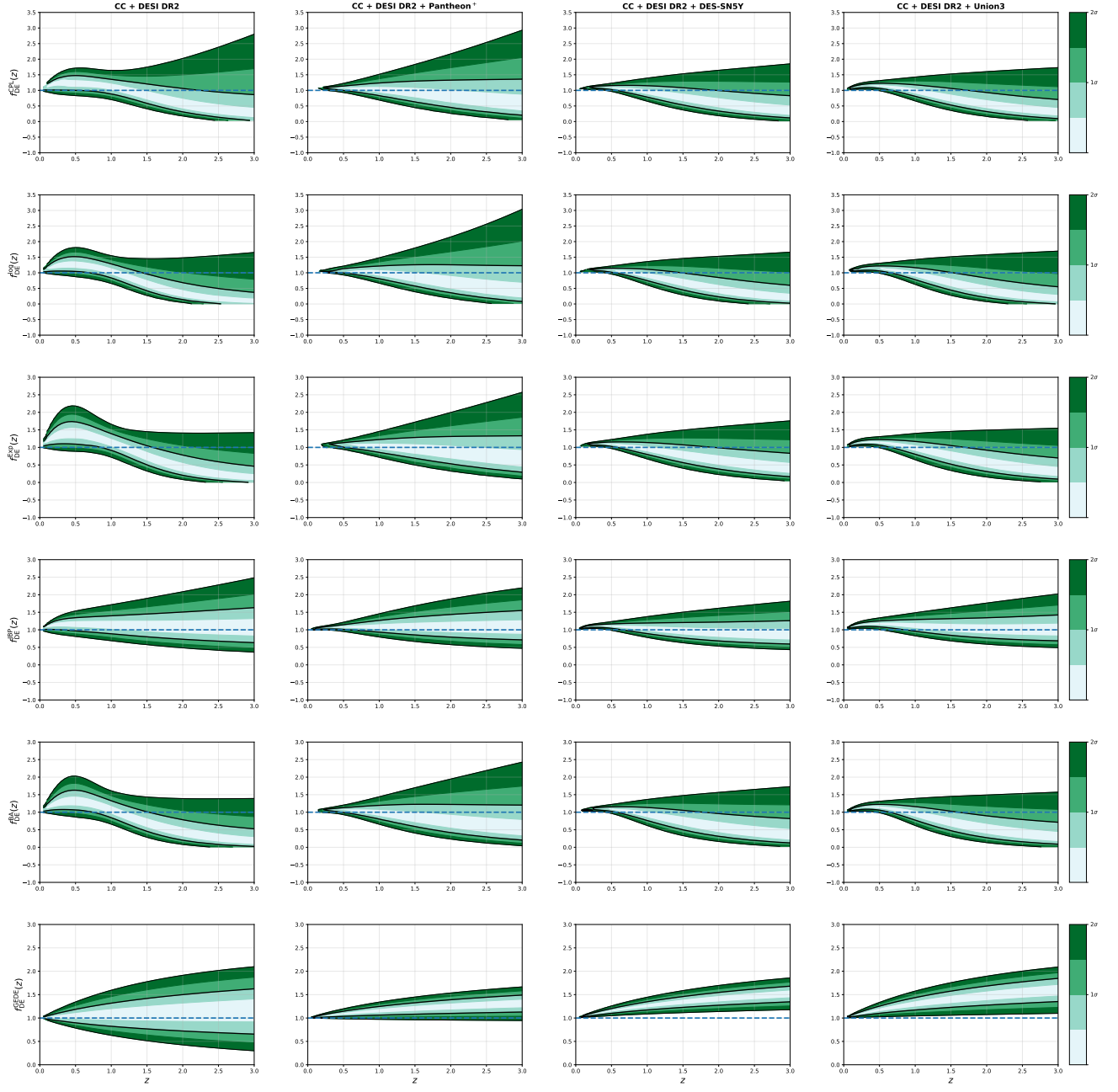


FIG. 6: This figure shows the evolution of $f_{DE}(z)$ as a function of redshift, using DESI DR2 measurements with different combinations of SNe Ia compilations (Pantheon⁺, DESY5, Union3) and CMB distance priors, providing compelling evidence for dynamical DE.

tion increases accordingly, with models such as BA, JBP, and Logarithmic reaching 3σ – 3.5σ deviations. Indeed, for most of the dark energy models, the CC + DESI DR2 combined with DES-SN5Y shows the most deviation. Indeed, it is crucial to note that DES-SN5Y also considers 194 externally sourced low- z SNe from CfA/CSP, which show this deviation from the Λ CDM model. When the low- z SN data are excluded, dynamical dark energy is not required, fully restoring Λ CDM

concordance, indeed showing a full understanding of the systematics of our local Universe within a radius of about $300, h^{-1}, \text{Mpc}$. [78, 79]. Fig. 4 shows a comparative analysis of each dark energy model relative to the Λ CDM model. It is evident that Λ CDM remains the preferred model for most combinations of data sets; however, certain dark energy models show significant support depending on the choice of datasets. For the CC + DESI DR2 combination, $o\Lambda$ CDM ($|\Delta \ln \mathcal{Z}| = 2.14$)

and $\text{o}\omega\text{CDM}$ ($|\Delta \ln \mathcal{Z}| = 2.70$) show moderate evidence, while others show weak evidence ($|\Delta \ln \mathcal{Z}| < 1$) according to the Jeffreys' scale. The inclusion of Pantheon⁺ data shows moderate evidence for $\text{o}\Lambda\text{CDM}$ (1.53) and $\text{o}\omega\text{CDM}$ (2.48), while all others show weak evidence. The inclusion of DES-SN5Y data shows strong evidence ($|\Delta \ln \mathcal{Z}| > 3$) for several dark energy models: $\omega_0\omega_a\text{CDM}$ (3.63), JBP (4.25), BA (3.03) and GEDE (4.02), indicative of notable deviations from ΛCDM . For the CC + DESI DR2 + Union3 combination, there is moderate to strong evidence for $\omega_0\omega_a\text{CDM}$ (3.02), JBP (3.41), BA (2.62) and GEDE (2.80). These results highlight the critical influence of the selection of the data set on the preference of the model, with the DES-SN5Y and Union3 datasets providing the most substantial statistical support for dynamical dark energy models over ΛCDM .

The Fig 5 shows the redshift evolution of the dark energy equation of state parameter $\omega(z)$ for various parameterizations, constrained by different combinations of datasets (CC, DESI DR2, Pantheon+, DES-SN5Y, and Union3). At low redshifts ($z \lesssim 0.5$), most models cluster around $\omega(z) \approx -1$, consistent with a cosmological constant like behavior. At intermediate redshifts ($0.5 \lesssim z \lesssim 2$), some parameterizations (e.g., JBP, BA) show mild deviations from -1 , while exponential forms allow stronger evolution. At higher redshifts ($z \gtrsim 2$), the uncertainties increase significantly and the models remain less constrained. The shaded bands represent the 68% and 95% confidence intervals from the observational data, illustrating that the present constraints allow for both nearly constant $\omega(z)$ (close to ΛCDM) and mildly dynamical dark energy scenarios.

Whereas, Fig 6 shows the redshift evolution of the normalized dark energy density function $f_{DE}(z) \equiv \tilde{\Omega}_{DE}(z)/\Omega_{DE,0}$ for various parameterizations, constrained with different dataset combinations (CC, DESI DR2, Pantheon+, DES-SN5Y, and Union3). At low redshift ($z \lesssim 0.5$), the models remain close to $f_{DE}(z) = 1$, indicating consistency with a cosmological constant. As the redshift increases ($0.5 \lesssim z \lesssim 2$), different parameterizations begin to diverge, with JBP and BA forms allowing moderate deviations, while exponential parameterizations permit a stronger evolution. At higher redshifts ($z \gtrsim 2$), the constraints weaken, leading to larger uncertainties and broader confidence regions. The shaded bands correspond to the 68% and 95% confidence intervals, showing that current observations are compatible with both a nearly constant $f_{DE}(z)$ and a mildly dynamical dark energy scenario.

V. DISCUSSION AND CONCLUSIONS

In this work, we have carried out a comprehensive Bayesian MCMC analysis of several cosmological models including ΛCDM , $\text{o}\Lambda\text{CDM}$, ωCDM , $\omega_0\omega_a\text{CDM}$, Logarithmic, Exponential, JBP, BA, and GEDE using the most recent BAO from more than 14 million galaxies and quasars drawn from the DESI Data Release 2 in combination with CC and multiple Type Ia supernova (SNe Ia) compilations (Pantheon⁺, DES-SN5Y, and Union3). We constrain the key cosmological parameters (h , Ω_m , Ω_k , ω_0 , ω_a , and Δ) to show the deviation of each dark energy model from the ΛCDM model.

Our results show that, using the CC data alone, the ΛCDM model produces $h = 0.662^{+0.053}_{-0.048}$ as a reference value. Other dark energy models predict slightly higher values h , with deviations below 1.5σ , which show no significant tension. When combining CC data with DESI DR2 and various SNe Ia samples (Pantheon⁺, DES-SN5Y, Union3), the inferred values h show dataset dependent tensions, negligible to moderate relative to ΛCDM below 2σ . Models such as $\text{o}\Lambda\text{CDM}$, $\text{o}\omega\text{CDM}$, and ωCDM show mild deviations, while the GEDE model exhibits tensions of up to $\sim 2\sigma$. The matter density parameter Ω_m shows an even smaller deviation, with most models showing a negligible to mild tension ($< 1.5\sigma$). In general, all dark energy models are statistically consistent with the prediction of ΛCDM for Ω_m .

Our results show that the $\text{o}\Lambda\text{CDM}$ and $\text{o}\omega\text{CDM}$ models yield $\Omega_k \approx 0$, supporting a spatially flat geometry consistent with WMAP, BOOMERanG, and Planck observations. Furthermore, when constrained with the CC dataset alone, all models remain statistically consistent with ΛCDM , exhibiting tensions below 1.5σ . This indicates that late-time measurements alone do not provide strong evidence for any significant deviation from the cosmological constant Λ . The inclusion of DESI DR2 and SNe Ia data improves the parameter constraints and introduces mild to moderate tensions ($\lesssim 3.5\sigma$) between ΛCDM and dynamical dark energy models. Indeed, models such as BA, JBP, and Logarithmic show the highest tensions, suggesting possible hints of dynamical dark energy. The joint posterior analyses of (ω_0, ω_a) parameter planes reveal that most dynamical DE parameterizations favor $\omega_0 > -1$ and $\omega_a < 0$, indicating that such dynamical dark energy is characterized by a Quintom-B-type behavior.

Bayesian evidence shows that, while ΛCDM remains the preferred model, the choice of data set strongly affects the model support. CC + DESI DR2 favors $\text{o}\Lambda\text{CDM}$ and $\text{o}\omega\text{CDM}$ moderately; Pantheon⁺ strengthens this evidence slightly, and DES-SN5Y strongly supports dy-

namical dark energy models such as $\omega_0\omega_a$ CDM, JBP, BA, and GEDE. The combination CC + DESI DR2 + Union3 also favors these models, highlighting that DES-SN5Y and Union3 provide the most significant evidence for deviations from Λ CDM.

The reconstruction of the EoS parameter $\omega(z)$ and the dark energy growth function $f_{DE}(z)$, based on the DESI DR2 BAO, CMB, and various combinations of SNe Ia datasets, reveals notable deviations from the Λ CDM paradigm, which supports the notion of evolving dark energy. In particular, all models considered exhibit a crossing of the phantom divide ($\omega = -1$) within the redshift interval $0.5 < z < 1.5$, indicative of quintom-like behavior. This is further substantiated by the redshift evolution of $\omega(z)$, which shows that dark energy enters a phantom phase ($\omega < -1$) at $z > 0.5$ and transitions across the phantom divide near $z \sim 0.5$. Similarly, the growth function $f_{DE}(z)$ converges to unity at low redshift, consistent with the expected behavior of a cosmological constant in the late-time Universe.

In conclusion, our analysis indicates that the standard Λ CDM cosmology remains a statistically robust description of the current Universe, although late-time observational datasets, particularly DESI DR2 and recent SNe Ia samples, provide moderate evidence supporting the possibility of dynamical dark energy models. Future Stage IV surveys and next generation observatories are poised to significantly advance our understanding of the dark sector. DESI will deliver refined

constraints from DR2 probes such as full-shape fitting, bispectrum, gravitational lensing, and peculiar velocities in 2025–2026, with DR3 results expected in 2027, which may shed light on the possible phantom crossing [80, 81] and the extent of any deviations from the Λ CDM model. The Nancy Grace Roman Space Telescope, launching in 2026, and the proposed DESI-II in the 2030s will probe the $z > 1$ Universe, opening a new frontier for dark energy studies [81, 82]. If Stage IV surveys break Λ CDM model, developing new observational methods, cross-survey analyses, and searches for specific model signatures will be crucial, and a dedicated post Λ CDM dark energy task force could provide guidance for exploring the complex physics of the dark sector.

ACKNOWLEDGEMENTS

SC acknowledges the Istituto Nazionale di Fisica Nucleare (INFN) Sez. di Napoli, Iniziative Specifiche QGSKY and MoonLight-2 and the Istituto Nazionale di Alta Matematica (INdAM), gruppo GNFM, for the support. This paper is based upon work from COST Action CA21136 – Addressing observational tensions in cosmology with systematics and fundamental physics (CosmoVerse), supported by COST (European Cooperation in Science and Technology). VKS gratefully acknowledges the facilities and institutional support provided by the Indian Institute of Astrophysics (IIA), India, during his tenure as a postdoctoral fellow.

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