

Holographic solutions from 5D $SO(2) \times ISO(3)$ $N = 4$ gauged supergravity

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Abstract

We study various types of holographic solutions from five-dimensional $N = 4$ gauged supergravity coupled to three vector multiplets with $SO(2) \times ISO(3)$ gauge group. This gauged supergravity can be obtained from the maximal gauged supergravity in seven dimensions on a Riemann surface. For a negatively curved Riemann surface H^2 , the resulting five-dimensional gauged supergravity admits a supersymmetric $N = 4$ AdS_5 critical point. This AdS_5 vacuum is dual to an $N = 2$ superconformal field theory (SCFT) arising from M5-branes wrapped on H^2 . We study holographic RG flow solutions describing deformations of this SCFT by turning on relevant, marginal and irrelevant operators to $N = 2$ non-conformal phases in the IR. Solutions describing conformal interfaces between these non-conformal phases and singular boundaries are also given. We finally study a number of supersymmetric AdS_5 black string and black hole solutions holographically dual to RG flows across dimensions from the $N = 2$ SCFT to two-dimensional SCFTs and superconformal quantum mechanics in the IR. A number of solutions describing black strings and black holes in asymptotically domain wall space-time is also found. All of the solutions can be uplifted to M-theory by a consistent truncation on $H^2 \times S^4$.

1 Introduction

The AdS/CFT correspondence [1, 2, 3] provides a very useful tool for investigating many aspects of strongly-coupled conformal field theories. The study of this holographic duality between a $(d + 1)$ -dimensional gravity theory and the dual field theory in d dimensions has been effectively achieved via different types of solutions to gauged supergravities in various dimensions. These holographic solutions include domain walls with Minkowski and AdS slices describing RG flows and conformal interfaces within the dual superconformal field theories (SCFTs). Another class of supergravity solutions interpolates between AdS spaces of different dimensionalities. In the dual field theories, these solutions describe twisted compactifications of higher-dimensional conformal field theories on a compact manifold to other conformal field theories in lower dimensions.

In this paper, we are interested in holographic solutions from half-maximal $N = 4$ gauged supergravity in five dimensions with $SO(2) \times ISO(3)$ gauge group. This gauged supergravity is obtained by coupling the pure $N = 4$ supergravity to three vector multiplets resulting in $SO(1, 1) \times SO(5, 3)$ global symmetry. By embedding the $SO(2) \times ISO(3)$ gauge group in $SO(5, 3)$, we obtain the $N = 4$ gauged supergravity with a supersymmetric $N = 4$ AdS_5 vacuum at the origin of the scalar manifold. This gauged supergravity has been shown to arise from a compactification of the $SO(5)$ maximal gauged supergravity in seven dimensions on a Riemann surface with genus greater than one, H^2 , in [4]. Using the consistent truncation of eleven-dimensional supergravity on S^4 to the $N = 4$ seven-dimensional gauged supergravity [5, 6], the $SO(2) \times ISO(3)$ gauged supergravity in five dimensions can be embedded in eleven dimensions via a consistent truncation on $H^2 \times S^4$. Furthermore, with the formulation of exceptional field theory (EFT), it has also been shown that this is the only consistent truncation of M-theory on an S^4 fibration over a Riemann surface to $N = 4$ gauged supergravity in five dimensions [4]. We will study holographic solutions from this gauged supergravity that describe various deformations of the $N = 2$ SCFT dual to the aforementioned supersymmetric AdS_5 vacuum.

We will first study holographic RG flows from the $N = 4$ AdS_5 vacuum to non-conformal phases of the $N = 2$ SCFT, see [8] to [19] for holographic RG flow solutions in other five-dimensional gauged supergravities. Since there is no other supersymmetric AdS_5 vacua in this $SO(2) \times ISO(3)$ $N = 4$ gauged supergravity, all supersymmetric RG flows are essentially break conformal symmetry leading to non-conformal or super Yang-Mills (SYM) phases corresponding to singular geometries in the IR. These solutions describe deformations of the $N = 2$ SCFT to non-conformal SYM theories. In addition, we will also study supersymmetric Janus solutions in the form of AdS_4 -sliced domain walls in contrast to the flat or Poincare domain walls in the case of RG flows. These solutions are dual to three-dimensional conformal interfaces within four-dimensional field theories, see [20] to [27] for Janus solutions in five-dimensional gauged supergravities.

The final class of solutions considered in this paper is supersymmetric black strings and black holes in asymptotically AdS_5 space. These solutions interpolate between the AdS_5 vacuum and $AdS_3 \times \Sigma$ and $AdS_2 \times \mathcal{M}_3$ geometries in the IR. Σ is a Riemann surface, and $\mathcal{M}_3$ is a 3-manifold with constant curvature. Holographically, the solutions describe RG flows across dimensions from $N = 2$ SCFT in four dimensions to two-dimensional SCFTs and superconformal quantum mechanics in the IR. In order for these solutions to preserve some amount of supersymmetry, it is necessary to perform a topological twist by turning on certain gauge fields to cancel the spin connections on Σ and $\mathcal{M}_3$ [28, 29]. Accordingly, the IR theories arise from twisted compactification of the $N = 2$ SCFT on Σ or $\mathcal{M}_3$. Similar solutions in other five-dimensional gauged supergravities can be found in [30] to [43].

The paper is organized as follows. In section 2, we review five-dimensional $N = 4$ gauged supergravity coupled to three vector multiplets with $SO(2) \times ISO(3)$ gauge group. Holographic RG flow solutions will be considered in section 3. In section 4, we look for supersymmetric Janus solutions describing three-dimensional conformal interfaces within four-dimensional field theories. We also give a number of numerical Janus solutions. In sections 5 and 6, we find supersymmetric AdS_5 black strings and black holes with near horizon geometries $AdS_3 \times \Sigma^2$ and $AdS_2 \times \mathcal{M}_3$, respectively. We give some conclusions and comments in section 7. In the appendix, we have collected some formulae for obtaining up-lifted 00-component of the eleven-dimensional metric. This is a useful tool to determine whether a given IR singularity is physical or not.

2 Five-dimensional $N = 4$ gauged supergravity with $SO(2) \times ISO(3)$ gauge group

In this section, we give a brief review of $N = 4$ gauged supergravity constructed in [44, 45]. We mainly focus on bosonic Lagrangian and supersymmetry transformations of fermionic fields which are relevant for finding supersymmetric solutions. The complete construction can be found in [44, 45] to which we refer for more detail.

2.1 Five-dimensional $N = 4$ gauged supergravity

The $N = 4$ supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini $\psi_{\mu i}$, six vectors (A_{μ}^0, A_{μ}^m) , four spin- $\frac{1}{2}$ fields χ_i and one real scalar Σ , the dilaton. Space-time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3, 4$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3, 4$. The fundamental representation of $SO(5)_R \sim USp(4)_R$ R-symmetry is described by $m, n = 1, \dots, 5$ for $SO(5)_R$ and $i, j = 1, 2, 3, 4$ for $USp(4)_R$. A vector multiplet contains a vector field A_{μ} , four gaugini λ_i and five scalars ϕ^m . For $N = 4$ supergravity coupled to n vector multiplets,

we will use indices $a, b = 1, \dots, n$ to label these multiplets $(A_\mu^a, \lambda_i^a, \phi^{ma})$.

In supergravity coupled to n vector multiplets, there are $6 + n$ vector fields denoted collectively by $A_\mu^{\mathcal{M}} = (A_\mu^0, A_\mu^m, A_\mu^a)$ and $5n + 1$ scalars in the $SO(1, 1) \times SO(5, n)/SO(5) \times SO(n)$ coset manifold. For later convenience, we have introduced a collective index $\mathcal{M} = (0, M)$ as in [44]. The $5n$ scalars parametrizing the $SO(5, n)/SO(5) \times SO(n)$ coset can be described by a coset representative $\mathcal{V}_M^A$ transforming under the global $G = SO(5, n)$ and the local $H = SO(5) \times SO(n)$ by left and right multiplications, respectively. We use the global $SO(5, n)$ indices $M, N, \dots = 1, 2, \dots, 5 + n$ while the local H indices $A, B, \dots$ can be split as $A = (m, a)$. The coset representative can then be written as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \quad (1)$$

It is also useful to define a symmetric and $SO(5) \times SO(n)$ invariant matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (2)$$

All fermionic fields are symplectic Majorana spinors subject to the condition

$$\xi_i = \Omega_{ij} C (\bar{\xi}^j)^T \quad (3)$$

with C and Ω_{ij} being the charge conjugation matrix and $USp(4)$ symplectic matrix, respectively.

As in other dimensions, gaugings of $N = 4$ supergravity in five dimensions are efficiently obtained by using the embedding tensor formalism. In the present case, the corresponding embedding tensor has the components ξ^M , $\xi^{MN} = \xi^{[MN]}$ and $f_{MNP} = f_{[MNP]}$. These components determine the embedding of a gauge group G_0 in the global symmetry group $SO(1, 1) \times SO(5, n)$. In this paper, we will consider only gaugings with $\xi^M = 0$ which admit supersymmetric AdS_5 vacua as shown in [46]. We will then set $\xi^M = 0$ from now on. This also leads to considerable simplification in various expressions. In particular, the quadratic constraints on the embedding tensor simply reduce to

$$f_{R[MN} f_{PQ]}^R = 0 \quad \text{and} \quad \xi_M^Q f_{QNP} = 0. \quad (4)$$

Furthermore, for $\xi^M = 0$, the gauge group is embedded entirely in $SO(5, n)$ with the corresponding gauge generators in $SO(5, n)$ fundamental representation given by

$$(X_M)_N^P = -f_M^{QR} (t_{QR})_N^P = f_{MN}^P \quad \text{and} \quad (X_0)_N^P = -\xi^{QR} (t_{QR})_N^P = \xi_N^P. \quad (5)$$

We have chosen $SO(5, n)$ generators of the form $(t_{MN})_P^Q = \delta_{[M}^Q \eta_{N]P}$ with $\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, 1, 1, \dots, 1)$ being the $SO(5, n)$ invariant tensor. The gauge covariant derivative reads

$$D_\mu = \nabla_\mu + A_\mu^M X_M + A_\mu^0 X_0 = \nabla_\mu + A^\mathcal{M} X_\mathcal{M} \quad (6)$$

with ∇_μ being a space-time covariant derivative including $SO(5) \times SO(n)$ composite connection.

The bosonic Lagrangian of a general gauged $N = 4$ supergravity can be written as

$$\begin{aligned} e^{-1}\mathcal{L} = & \frac{1}{2}R - \frac{3}{2}\Sigma^{-2}D_\mu\Sigma D^\mu\Sigma + \frac{1}{16}D_\mu M_{MN}D^\mu M^{MN} - V \\ & - \frac{1}{4}\Sigma^2 M_{MN}\mathcal{H}_{\mu\nu}^M\mathcal{H}^{N\mu\nu} - \frac{1}{4}\Sigma^{-4}\mathcal{H}_{\mu\nu}^0\mathcal{H}^{0\mu\nu} + e^{-1}\mathcal{L}_{\text{top}} \end{aligned} \quad (7)$$

where e is the vielbein determinant.

The covariant gauge field strength tensors read

$$\mathcal{H}_{\mu\nu}^M = 2\partial_{[\mu}A_{\nu]}^M + X_{\mathcal{NP}}{}^M A_\mu^N A_\nu^P + Z^{\mathcal{MN}} B_{\mu\nu\mathcal{N}} \quad (8)$$

with

$$Z^{MN} = \frac{1}{2}\xi^{MN} \quad \text{and} \quad Z^{0M} = -Z^{M0} = \frac{1}{2}\xi^M = 0. \quad (9)$$

In the embedding tensor formalism, the two-form fields $B_{\mu\nu\mathcal{M}}$ are introduced off-shell. These fields do not have kinetic terms and couple to vector fields via the topological term $\mathcal{L}_{\text{top}}$. It is useful to note the first-order field equations for these two-form fields

$$Z^{\mathcal{MN}} \left[\frac{1}{6\sqrt{2}}\epsilon_{\mu\nu\rho\lambda\sigma}\mathcal{H}_{\mathcal{N}}^{(3)\rho\lambda\sigma} - \mathcal{M}_{\mathcal{NP}}\mathcal{H}_{\mu\nu}^P \right] = 0 \quad (10)$$

with $\mathcal{M}_{00} = \Sigma^{-4}$, $\mathcal{M}_{0M} = 0$ and $\mathcal{M}_{MN} = \Sigma^2 M_{MN}$. This gives a duality relation between vectors and two-form fields. The field strength $\mathcal{H}_{\mathcal{M}}^{(3)}$ is defined by

$$Z^{\mathcal{MN}}\mathcal{H}_{\mu\nu\rho\mathcal{N}}^{(3)} = Z^{\mathcal{MN}} \left[3D_{[\mu}B_{\nu\rho]\mathcal{N}} + 6d_{\mathcal{NPQ}}A_{[\mu}^P \left(\partial_{\nu}A_{\rho]}^Q + \frac{1}{3}X_{\mathcal{RS}}{}^Q A_\nu^R A_{\rho]}^S \right) \right] \quad (11)$$

for $d_{0MN} = d_{MN0} = d_{M0N} = \eta_{MN}$ and

$$X_{MN}{}^P = f_{MN}{}^P, \quad X_{M0}{}^0 = 0, \quad X_{0M}{}^N = \xi_M{}^N. \quad (12)$$

The scalar potential is given by

$$\begin{aligned} V = & -\frac{1}{4} \left[f_{MNP}f_{QRS}\Sigma^{-2} \left(\frac{1}{12}M^{MQ}M^{NR}M^{PS} - \frac{1}{4}M^{MQ}\eta^{NR}\eta^{PS} \right. \right. \\ & \left. \left. + \frac{1}{6}\eta^{MQ}\eta^{NR}\eta^{PS} \right) + \frac{1}{4}\xi_{MN}\xi_{PQ}\Sigma^4(M^{MP}M^{NQ} - \eta^{MP}\eta^{NQ}) \right. \\ & \left. + \frac{\sqrt{2}}{3}f_{MNP}\xi_{QR}\Sigma M^{MNPQR} \right] \end{aligned} \quad (13)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQR} is obtained from

$$M_{MNPQR} = \epsilon_{mnpqr}\mathcal{V}_M{}^m\mathcal{V}_N{}^n\mathcal{V}_P{}^p\mathcal{V}_Q{}^q\mathcal{V}_R{}^r \quad (14)$$

by raising the indices with η^{MN} .

As mentioned above, $\mathcal{L}_{\text{top}}$ is the topological term describing the kinetic terms for two-form fields and the coupling between two-form and gauge fields. Since all solutions given in this paper have vanishing two-form fields, we will not give the explicit form of $\mathcal{L}_{\text{top}}$ here. This can be found in [44].

Supersymmetry transformations of fermionic fields are given by

$$\begin{aligned}\delta\psi_{\mu i} &= D_\mu\epsilon_i + \frac{i}{\sqrt{6}}\Omega_{ij}A_1^{jk}\gamma_\mu\epsilon_k \\ &\quad - \frac{i}{6}\left(\Omega_{ij}\Sigma\mathcal{V}_M^{jk}\mathcal{H}_{\nu\rho}^M - \frac{\sqrt{2}}{4}\delta_i^k\Sigma^{-2}\mathcal{H}_{\nu\rho}^0\right)(\gamma_\mu^{\nu\rho} - 4\delta_\mu^\nu\gamma^\rho)\epsilon_k,\end{aligned}\quad (15)$$

$$\begin{aligned}\delta\chi_i &= -\frac{\sqrt{3}}{2}i\Sigma^{-1}D_\mu\Sigma\gamma^\mu\epsilon_i + \sqrt{2}\Omega_{ij}A_2^{kj}\epsilon_k \\ &\quad - \frac{1}{2\sqrt{3}}\left(\Sigma\Omega_{ij}\mathcal{V}_M^{jk}\mathcal{H}_{\mu\nu}^M + \frac{1}{\sqrt{2}}\Sigma^{-2}\delta_i^k\mathcal{H}_{\mu\nu}^0\right)\gamma^{\mu\nu}\epsilon_k,\end{aligned}\quad (16)$$

$$\delta\lambda_i^a = i\Omega^{jk}(\mathcal{V}_M^a D_\mu\mathcal{V}_{ij}^M)\gamma^\mu\epsilon_k + \sqrt{2}\Omega_{ij}A_2^{akj}\epsilon_k - \frac{1}{4}\Sigma\mathcal{V}_M^a\mathcal{H}_{\mu\nu}^M\gamma^{\mu\nu}\epsilon_i \quad (17)$$

in which the fermion shift matrices are defined by

$$\begin{aligned}A_1^{ij} &= -\frac{1}{\sqrt{6}}\left(\sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\xi^{MN} + \frac{4}{3}\Sigma^{-1}\mathcal{V}^{ik}_M\mathcal{V}^{jl}_N\mathcal{V}^P_{kl}f^{MN}_P\right), \\ A_2^{ij} &= \frac{1}{\sqrt{6}}\left(\sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\xi^{MN} - \frac{2}{3}\Sigma^{-1}\mathcal{V}^{ik}_M\mathcal{V}^{jl}_N\mathcal{V}^P_{kl}f^{MN}_P\right), \\ A_2^{aij} &= -\frac{1}{2}\left(\Sigma^2\mathcal{V}_M^{ij}\mathcal{V}_N^a\xi^{MN} - \sqrt{2}\Sigma^{-1}\Omega_{kl}\mathcal{V}_M^a\mathcal{V}_N^{ik}\mathcal{V}_P^{jl}f^{MNP}\right).\end{aligned}\quad (18)$$

$\mathcal{V}_M^{ij}$ is defined in terms of $\mathcal{V}_M^m$ and $SO(5)$ gamma matrices Γ_{mi}^j as

$$\mathcal{V}_M^{ij} = \frac{1}{2}\mathcal{V}_M^m\Gamma_m^{ij} \quad (19)$$

with $\Gamma_m^{ij} = \Omega^{ik}\Gamma_{mk}^j$. Similarly, the inverse $\mathcal{V}_{ij}^M$ can be written as

$$\mathcal{V}_{ij}^M = \frac{1}{2}\mathcal{V}_m^M(\Gamma_m^{ij})^* = \frac{1}{2}\mathcal{V}_m^M\Gamma_m^{kl}\Omega_{ki}\Omega_{lj}. \quad (20)$$

We will use the following representation of $SO(5)$ gamma matrices

$$\begin{aligned}\Gamma_1 &= -\sigma_2 \otimes \sigma_2, & \Gamma_2 &= \mathbb{I}_2 \otimes \sigma_1, & \Gamma_3 &= \mathbb{I}_2 \otimes \sigma_3, \\ \Gamma_4 &= \sigma_1 \otimes \sigma_2, & \Gamma_5 &= \sigma_3 \otimes \sigma_2\end{aligned}\quad (21)$$

with σ_i , $i = 1, 2, 3$, being the Pauli matrices.

The covariant derivative on ϵ_i is given by

$$D_\mu\epsilon_i = \partial_\mu\epsilon_i + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon_i + Q_{\mu i}^j\epsilon_j \quad (22)$$

with the composite connection defined by

$$Q_{\mu i}^j = \mathcal{V}_{ik}^M\partial_\mu\mathcal{V}_M^{kj} - A_\mu^0\xi^{MN}\mathcal{V}_{Mik}\mathcal{V}_N^{kj} - A_\mu^M\mathcal{V}_{ik}^N\mathcal{V}^{kjP}f_{MNP}. \quad (23)$$

2.2 $N = 4$ gauged supergravity with $SO(2) \times ISO(3)$ gauge group

In this paper, we will consider $N = 4$ gauged supergravity coupled to $n = 3$ vector multiplets with $SO(2) \times ISO(3) \sim SO(2) \times (SO(3) \ltimes \mathbb{R}^3)$ gauge group. This gauged supergravity arises from a consistent truncation of eleven-dimensional supergravity on $H^2 \times S^4$ [4]. The corresponding embedding tensor is given by

$$\begin{aligned}\xi^{\hat{m}\hat{n}} &= g_1 \epsilon_{\hat{m}\hat{n}}, & \hat{m}, \hat{n} &= 1, 2, \\ f_{\tilde{m}\tilde{n}\tilde{p}} &= g \epsilon_{\tilde{m}\tilde{n}\tilde{p}}, & \tilde{m}, \tilde{n}, \tilde{p} &= 3, 4, 5, \\ f_{a+5, b+5, c+5} &= -2g \epsilon_{abc}, & f_{a+2, b+5, c+5} &= -g \epsilon_{abc}, & a, b, c &= 1, 2, 3\end{aligned}\quad (24)$$

with the gauge coupling constants g_1 and g . We have split the indices $m, n = 1, 2, \dots, 5$ as $m = (\hat{m}, \tilde{m})$ with $\hat{m} = 1, 2$ and $\tilde{m} = 3, 4, 5$. From the embedding tensor, we find that the $SO(2)$ factor is generated by ξ_{12} while the compact $SO(3) \subset ISO(3)$ is diagonally embedded in $SO(2) \times SO(3) \times SO(3) \subset SO(5, 3)$ with gauge generators $X_{\tilde{m}} = (X_3, X_4, X_5)$. The three-dimensional translation group $\mathbb{R}^3$ is generated by $X_{\tilde{m}} - X_{a+5} = (X_3 - X_6, X_4 - X_7, X_5 - X_8)$.

To give an explicit parametrization of the scalar coset $SO(5, 3)/SO(5) \times SO(3)$, we take the $SO(5, 3)$ non-compact generators to be

$$Y_{ma} = t_{m, a+5}, \quad m = 1, 2, \dots, 5, \quad a = 1, 2, 3. \quad (25)$$

Accordingly, the coset representative can be written as

$$\mathcal{V} = e^{\phi^{ma} Y_{ma}}. \quad (26)$$

As shown in [4], at the origin of $SO(5, 3)/SO(5) \times SO(3)$ with $\phi^{ma} = 0$, the $SO(2) \times ISO(3)$ gauged supergravity admits a supersymmetric AdS_5 vacuum with

$$\Sigma = - \left(\frac{g}{\sqrt{2}g_1} \right)^{\frac{1}{3}}, \quad V_0 = -3 \left(\frac{g^2 g_1}{2} \right)^{\frac{2}{3}}, \quad L = \left(\frac{4\sqrt{2}}{g^2 g_1} \right)^{\frac{1}{3}}. \quad (27)$$

The AdS_5 radius L is related to the cosmological constant V_0 via

$$L = \sqrt{-\frac{6}{V_0}}. \quad (28)$$

By choosing $g = -\sqrt{2}g_1$ or equivalently scaling Σ to $\Sigma = 1$, we find

$$V_0 = -\frac{3}{2}g^2 \quad \text{and} \quad L = \frac{2}{g} \quad (29)$$

in which we have chosen $g > 0$. The AdS_5 vacuum preserves $N = 4$ supersymmetry and $SO(2) \times SO(3) \subset SO(2) \times ISO(3)$ symmetry. This vacuum can be

identified as an $AdS_5 \times H^2 \times S^4$ solution of eleven-dimensional supergravity.

It is also useful to note all scalar masses at this $N = 4$ vacuum given in [4]. These are shown in table 1. We denote scalars by the representations under the residual symmetry $SO(2) \times SO(3)$ at the vacuum. The dilaton Σ is the singlet $(\mathbf{1}, \mathbf{1})_\Sigma$. The other representations are obtained by considering the embedding of $SO(2) \times SO(3)$ in $SO(5) \times SO(3) \subset SO(5, 3)$. Under $SO(5) \times SO(3)$, the 15 scalars transform as $(\mathbf{5}, \mathbf{3})$. By branching $SO(5) \rightarrow SO(2) \times SO(3)$ with $\mathbf{5} \rightarrow (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$, we find that

$$(\mathbf{5}, \mathbf{3}) \rightarrow (\mathbf{2}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \mathbf{3}) \quad (30)$$

under $SO(2) \times SO(3) \times SO(3)$. Finally, by taking the diagonal subgroup of the two $SO(3)$ factors, we end up with

$$(\mathbf{5}, \mathbf{3}) \rightarrow (\mathbf{2}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{5}). \quad (31)$$

In the table, we have also given the dimensions of the operators dual to these scalars given by the relation $m^2 L^2 = \Delta(\Delta - 4)$. The three massless scalars in $(\mathbf{1}, \mathbf{3})$ are Goldstone bosons corresponding to the symmetry breaking $ISO(3) \rightarrow SO(3)$ at the vacuum.

Scalars	$m^2 L^2$	Δ
$(\mathbf{1}, \mathbf{1})_\Sigma$	-4	2
$(\mathbf{1}, \mathbf{1})$	12	6
$(\mathbf{1}, \mathbf{3})$	0	4
$(\mathbf{1}, \mathbf{5})$	0	4
$(\mathbf{2}, \mathbf{3})$	5	5

Table 1: Scalar masses at the $N = 4$ supersymmetric AdS_5 vacuum with $SO(2) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators.

Furthermore, it has also been pointed out in [4] that there are no other supersymmetric AdS_5 vacua.

3 Holographic RG flows

We begin with the simplest holographic solutions describing RG flows from the $N = 2$ SCFT dual to the supersymmetric AdS_5 vacuum. To simplify the computation, we consider a truncation to $SO(2)_{\text{diag}}$ singlet sector. This $SO(2)_{\text{diag}}$ is a diagonal subgroup of $SO(2) \times SO(2) \subset SO(2) \times SO(3) \subset SO(2) \times ISO(3)$ generated by $\xi_{12} + X_3$. There are five singlets under $SO(2)_{\text{diag}}$ symmetry with the corresponding coset representative given by

$$\mathcal{V} = e^{\phi_1(Y_{12}+Y_{23})} e^{\phi_2(Y_{13}-Y_{22})} e^{\phi_3(Y_{42}+Y_{53})} e^{\phi_4(Y_{43}-Y_{52})} e^{\phi_5 Y_{31}}. \quad (32)$$

In this section, we are interested in holographic RG flow solutions with the metric ansatz given by

$$ds^2 = e^{2A(r)} dx_{1,3}^2 + dr^2. \quad (33)$$

$dx_{1,3}^2$ is the flat metric on the Minkowski space in four dimensions with the warp factor A depending only on the radial coordinate r . To preserve four-dimensional Poincare symmetry of $dx_{1,3}^2$, we take the non-vanishing scalars to depend only on r and set all the other fields to zero.

It turns out that in order to consistently truncate out all the vector fields, we need to set $\phi_2 = \phi_4 = 0$. The latter lead to non-vanishing Yang-Mills currents that become the sources for the gauge fields. With $\phi_2 = \phi_4 = 0$, we find the scalar potential

$$\begin{aligned} V = & \frac{1}{8} g_1^2 \cosh^2 \phi_3 \sinh^2 \phi_1 \Sigma^4 (3 + \cosh 2\phi_1 + 2 \cosh 2\phi_3 \sinh^2 \phi_1) \\ & + \sqrt{2} g_1 g \Sigma (\cosh^2 \phi_1 \cosh^2 \phi_3 \cosh \phi_5 + 2 \sinh^2 \phi_3 \sinh \phi_5) \\ & + \frac{1}{256} g^2 \Sigma^{-2} [42 - 48 \cosh 2\phi_1 + 6 \cosh 4\phi_1 + 4 \cosh(4\phi_1 - 2\phi_3) \\ & - 24 \cosh(2\phi_1 - 2\phi_3) + \cosh(4\phi_1 - 4\phi_3) - 152 \cosh 2\phi_3 - 2 \cosh 4\phi_3 \\ & - 24 \cosh(2\phi_1 + 2\phi_3) + \cosh(4\phi_1 + 4\phi_3) + 4 \cosh(4\phi_1 + 2\phi_3) \\ & + 128 \cosh^2 \phi_1 \sinh^2 2\phi_3 \sinh 2\phi_5 + \{2(5 + 4 \cosh 2\phi_1 + \cosh 4\phi_1) \times \\ & \times \cosh 4\phi_3 + 6 \cosh 4\phi_1 + 8 \cosh 2\phi_3 \sinh^2 \phi_1 (5 + 4 \cosh 2\phi_1) \\ & - 6 - 4 \cosh 2\phi_1\} 4 \cosh 2\phi_5]. \end{aligned} \quad (34)$$

The A_1^{ij} tensor takes a diagonal form

$$A_1^{ij} = \text{diag}(\alpha, \beta, \alpha^*, \beta) \quad (35)$$

with

$$\begin{aligned} \alpha = & \frac{2}{\sqrt{2}} \Sigma^{-1} (\cosh \phi_3 + i \sinh \phi_1 - \cosh \phi_1 \sinh \phi_3) [g_1 \Sigma^3 (\cosh \phi_3 - i \sinh \phi_1 \\ & + \cosh \phi_1 \sinh \phi_3) - \sqrt{2} g \{ \cosh \phi_3 \cosh \phi_5 + (\cosh \phi_5 - 2 \sinh \phi_5) \times \\ & \times (\cosh \phi_1 \sinh \phi_3 - i \sinh \phi_1) \}], \end{aligned} \quad (36)$$

$$\begin{aligned} \beta = & \frac{1}{16\sqrt{3}} \Sigma^{-1} \left[\sqrt{2} g e^{-\phi_5} \{ 11 + 4 e^{2\phi_5} (\sinh \phi_3 - \cosh \phi_1 \cosh \phi_3)^2 \right. \\ & - 6 \cosh 2\phi_1 \cosh^2 \phi_3 - \cosh 2\phi_3 + 4 \cosh \phi_1 \sinh 2\phi_3 \} \\ & \left. - 2 g_1 \Sigma^3 (3 + \cosh 2\phi_1 + 2 \cosh 2\phi_3 \sinh^2 \phi_1) \right]. \end{aligned} \quad (37)$$

The real eigenvalue β gives rise to the superpotential W in terms of which the scalar potential can be written as

$$V = \frac{3}{2} \Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right)^2 + \frac{9}{4} \text{sech}^2 \phi_3 \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{9}{4} \left(\frac{\partial W}{\partial \phi_3} \right)^2 + \frac{9}{2} \left(\frac{\partial W}{\partial \phi_5} \right)^2 - 6W^2 \quad (38)$$

with $W = \sqrt{\frac{2}{3}}\beta$. It should be noted that for $\phi_1 = 0$, we have $\alpha = -\beta$. In this case, the solutions preserve $N = 4$ supersymmetry. In general, the solutions preserve only $N = 2$ supersymmetry corresponding to the Killing spinors ϵ^2 and ϵ^4 .

Setting $\epsilon^1 = \epsilon^3 = 0$ and imposing the projector

$$\gamma_r \epsilon_2 = \mp i \epsilon_4 \quad \text{and} \quad \gamma_r \epsilon_4 = \pm i \epsilon_2, \quad (39)$$

we find the BPS equations from the conditions $\delta\psi_{\hat{\mu}}^i = 0$ with $\hat{\mu} = 0, 1, 2, 3$, $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ of the form

$$\begin{aligned} \phi_1' &= \mp \frac{3}{2} \text{sech}^2 \phi_3 \frac{\partial W}{\partial \phi_1}, & \phi_3' &= \mp \frac{3}{2} \frac{\partial W}{\partial \phi_3}, \\ \phi_5' &= \mp 3 \frac{\partial W}{\partial \phi_5}, & \Sigma' &= \mp \Sigma^2 \frac{\partial W}{\partial \Sigma}, & A' &= \pm W. \end{aligned} \quad (40)$$

Throughout this paper, we use $'$ to denote r -derivatives. The condition $\delta\psi_r^i = 0$ leads to the usual Killing spinors of the domain wall of the form

$$\epsilon^{2,4} = e^{\frac{A}{2}} \epsilon_0^{2,4} \quad (41)$$

with $\epsilon_0^{2,4}$ being constant spinors satisfying the projector (39).

3.1 Holographic RG flows with $SO(2) \times SO(3)$ symmetry

We begin with a simple solution with $SO(2) \times SO(3)$ symmetry. In this case, we set

$$\phi_1 = 0 \quad \text{and} \quad \phi_5 = \phi_3. \quad (42)$$

With $\phi_1 = 0$, all the eigenvalues of A_1^{ij} are degenerate up to an overall sign. The solutions then preserve $N = 4$ supersymmetry corresponding to ϵ^i with $i = 1, 2, 3, 4$. However, the γ_r projector, which in this case takes the form of

$$\gamma_r \epsilon_i = \mp (\sigma_2 \otimes \sigma_3)_i^j \epsilon_j, \quad (43)$$

will reduce the number of supercharges from 16 to 8. We also note that setting $\epsilon_1 = \epsilon_3 = 0$ in this projector, we recover the projector given in (39). Therefore, the solutions preserve $N = 2$ Poincare supersymmetry in four dimensions. These solutions would describe holographic RG flows from the $N = 2$ SCFT to non-conformal $N = 2$ field theories.

The explicit form of the relevant BPS equations are given by

$$\begin{aligned} \phi_3' &= g e^{-2\phi_3} \sinh \phi_3, \\ \Sigma' &= -\frac{1}{6} \left[g e^{-3\phi_3} (1 - 3e^{2\phi_3}) - 2\sqrt{2} g_1 \Sigma^3 \right], \\ A' &= -\frac{1}{6} \Sigma^{-1} \left[g (e^{-3\phi_3} - 3e^{-\phi_3}) + \sqrt{2} g_1 \Sigma^3 \right]. \end{aligned} \quad (44)$$

We have chosen a specific choice of sign in (43) such that the UV $N = 2$ SCFT appears in the limit $r \rightarrow \infty$. From table 1, we know that the dilaton Σ and the $SO(2) \times SO(3)$ singlet scalar given by $\phi_5 = \phi_3$ are dual to operators of dimensions $\Delta = 2$ and $\Delta = 6$, respectively. This is also confirmed by linearizing the BPS equations given above which results in

$$\phi_3 \sim e^{gr} \sim e^{\frac{2r}{L}} \quad \text{and} \quad \Sigma \sim - \left(\frac{g}{\sqrt{2}g_1} \right)^{\frac{1}{3}} + e^{-\frac{2r}{L}} \quad (45)$$

in which we have used the relations $g_1 = -\frac{g}{\sqrt{2}}$ and $L = \frac{2}{g}$.

We now explicitly solve the BPS equations given in (44). By combining Σ' and ϕ_3' equations, we can solve for Σ as a function of ϕ_3 . The result is given by

$$\Sigma^3 = - \frac{2ge^{\phi_3}(e^{2\phi_3} - 1)}{\sqrt{2}g_1e^{4\phi_3} + 2gC_0} \quad (46)$$

with an integration constant C_0 . In order to make the solution approach the $N = 4$ AdS_5 vacuum, we need to choose the constant C_0 to be

$$C_0 = - \frac{g_1}{\sqrt{2}g}. \quad (47)$$

The solution for Σ then becomes

$$\Sigma^3 = - \frac{\sqrt{2}ge^{\phi_3}}{g_1(1 + e^{2\phi_3})}. \quad (48)$$

Similarly, by combining A' and ϕ_3' equations and using the solution for Σ , we can solve for A as a function of ϕ_3

$$A = \frac{1}{3}\phi_3 + \frac{1}{3}\ln(1 - e^{2\phi_3}) + \frac{1}{6}\ln \left[\sqrt{2}g_1(e^{4\phi_3} - 1) \right] \quad (49)$$

in which we have neglected an additive integration constant that can be absorbed in rescaling of $dx_{1,3}^2$ coordinates. Finally, by using (48) in ϕ_3' equation and defining a new radial coordinate ρ via $\frac{d\rho}{dr} = \Sigma$, we find

$$g(\rho - \rho_0) = -\ln(1 + e^{\phi_3}) + \ln(1 - e^{\phi_3}) + 2e^{\phi_3} \quad (50)$$

with ρ_0 being another integration constant. ρ_0 can also be set to zero by shifting the coordinate ρ .

The solution is singular at $\rho = \rho_0$. Near this singularity, we find

$$\phi_3 \sim \ln(\rho - \rho_0), \quad \Sigma \sim (\rho - \rho_0)^{\frac{1}{3}}, \quad A \sim \frac{1}{3}\ln(\rho - \rho_0). \quad (51)$$

In this limit, the scalar potential is unbounded from above $V \rightarrow \infty$, so the solution is unphysical by the criterion of [47]. Since the gauged supergravity under

consideration here is a consistent truncation of eleven dimensional supergravity on $H^2 \times S^4$, we can also determine whether the singularity in the uplifted solution is physical or not by the criterion of [29]. We will choose the S^4 coordinates $\mu^{\hat{a}}$, $\hat{a} = 1, 2, \dots, 5$, to be

$$\begin{aligned}\mu^1 &= \cos \vartheta \cos \theta, & \mu^2 &= \cos \vartheta \sin \theta, & \mu^3 &= \sin \vartheta \sin \beta \cos \xi, \\ \mu^4 &= \sin \vartheta \sin \beta \sin \xi, & \mu^5 &= \sin \vartheta \cos \beta\end{aligned}\quad (52)$$

which satisfy $\mu^{\hat{a}}\mu^{\hat{a}} = 1$. Using the relations given in the appendix, we find

$$\hat{g}_{00} = (e^{-6\lambda} \cos^2 \vartheta + e^{4\lambda} \sin^2 \vartheta)^{\frac{1}{3}} e^{2A-4\varphi} \sim (\rho - \rho_0)^{-\frac{2}{3}} \rightarrow \infty \quad (53)$$

which also implies that the singularity is unphysical.

3.2 Holographic RG flows with $SO(2) \times SO(2)$ symmetry

We now consider a slightly more complicated solution with a smaller residual symmetry $SO(2) \times SO(2)$. In this case, we still have $\phi_1 = 0$ but unlike in the previous case $\phi_5 \neq \phi_3$. The solutions preserve $N = 4$ supersymmetry as in the previous case due to vanishing ϕ_1 . The explicit form of the corresponding BPS equations is given by

$$\phi'_3 = \frac{1}{2} g \Sigma^{-1} e^{-2\phi_3 - \phi_5} (e^{2\phi_5} - 1), \quad (54)$$

$$\phi'_5 = -\frac{1}{2} g \Sigma^{-1} e^{-2\phi_3 - \phi_5} (1 + 2e^{2\phi_3} + e^{2\phi_5}), \quad (55)$$

$$\Sigma' = \frac{1}{6} \left[2\sqrt{2} g_1 \Sigma^3 + g e^{-2\phi_3 - \phi_5} (e^{2\phi_5} + 2e^{2\phi_3} - 1) \right], \quad (56)$$

$$A' = \frac{1}{6} \Sigma^{-1} \left[g e^{-2\phi_3 - \phi_5} (e^{2\phi_5} + 2e^{2\phi_3} - 1) - \sqrt{2} g_1 \Sigma^3 \right]. \quad (57)$$

Near the supersymmetric AdS_5 vacuum, we find

$$\Sigma \sim e^{-\frac{2r}{L}}, \quad \phi_3 + \phi_5 \sim e^{\frac{2r}{L}}, \quad \phi_5 - 2\phi_3 \sim e^{-\frac{4r}{L}} \quad (58)$$

which implies that ϕ_3 and ϕ_5 are dual to two different linear combinations of operators of dimensions $\Delta = 6$ and $\Delta = 4$.

Combing (54) and (55), we find

$$\frac{d\phi_5}{d\phi_3} = \frac{1 - 2e^{2\phi_3} + e^{2\phi_5}}{1 - e^{2\phi_5}} \quad (59)$$

which can be solved by

$$e^{\phi_3 + \phi_5} \sqrt{1 - e^{2\phi_3 - 2\phi_5}} + \sin^{-1} e^{\phi_3 - \phi_5} = C \quad (60)$$

with an integration constant C . We can further simplify this expression by defining

$$\varphi_1 = \phi_3 + \phi_5 \quad \text{and} \quad \varphi_2 = \phi_3 - \phi_5 \quad (61)$$

which results in

$$e^{\varphi_1} = \frac{C - \sin^{-1} e^{\varphi_2}}{\sqrt{1 - e^{2\varphi_2}}}. \quad (62)$$

To make the solution approach the AdS_5 vacuum with $\varphi_1 = \varphi_2 = 0$, we need to choose

$$C = \frac{\pi}{2}. \quad (63)$$

We can now find the solutions for A and Σ as functions of φ_2 . It is then useful to note the BPS equation for φ_2 which takes the form

$$\varphi_2' = g\Sigma^{-1}e^{-\frac{1}{2}(\varphi_1 - \varphi_2)}(e^{-2\varphi_2} - 1). \quad (64)$$

Using (61) and (62), we can combine this equation with (56) and (57) to obtain

$$\Sigma^{-3} = \frac{g_1 e^{-\frac{\varphi_2}{2}} [\sin^{-1} e^{\varphi_2} - C - e^{\varphi_2} \sqrt{1 - e^{2\varphi_2}}]}{\sqrt{2}g(1 - e^{2\varphi_2})^{\frac{1}{4}} \sqrt{C - \sin^{-1} e^{\varphi_2}}} + \frac{\Sigma_0 e^{-\frac{\varphi_2}{2}} (1 - e^{2\varphi_2})^{\frac{3}{4}}}{C - \sin^{-1} e^{\varphi_2}}, \quad (65)$$

$$A = \frac{1}{4}\varphi_2 - \frac{1}{2}\ln \Sigma - \frac{3}{8}\ln(1 - e^{2\varphi_2}) + \frac{1}{4}\ln(\sin^{-1} e^{\varphi_2} - C) \quad (66)$$

with Σ_0 being another integration constant. As in the previous case, we have neglected an additive integration constant for A . In order to make the solution for Σ becomes $\Sigma^{-3} = -\frac{\sqrt{2}g_1}{g}$ for $\varphi_2 = 0$ at the AdS_5 vacuum, we need to set $\Sigma_0 = 0$.

Finally, using all these results, we can solve for φ_2 as

$$2g(\rho - \rho_0) = \ln(1 + e^{\frac{\varphi_2}{2}}) - \ln(1 - e^{\frac{\varphi_2}{2}}) - 2\tan^{-1} e^{\frac{\varphi_2}{2}} \quad (67)$$

with the new radial coordinate ρ defined by $\frac{d\rho}{dr} = \frac{e^{-\frac{\varphi_1}{2}}}{\Sigma}$ and ρ_0 being an integration constant. Similar to the previous case, the solution is singular at $\rho = \rho_0$ with

$$\varphi_2 \sim 2\ln(\rho - \rho_0), \quad \varphi_1 \sim \text{constant}, \quad \Sigma \sim (\rho - \rho_0)^{\frac{1}{3}}, \quad A \sim \frac{1}{3}\ln(\rho - \rho_0). \quad (68)$$

Near this singularity, we find that the scalar potential is unbounded from above

$$V \sim \frac{1}{(\rho - \rho_0)^{\frac{20}{3}}} \rightarrow \infty. \quad (69)$$

The singularity is then unphysical by the criterion of [47].

To look for the behavior of 00-component of the eleven-dimensional metric $\hat{g}_{00}$ near this singularity, in this case, we choose the S^4 coordinates to be

$$\begin{aligned} \mu^1 &= \cos \vartheta \cos \beta \cos \theta, & \mu^2 &= \cos \vartheta \cos \beta \sin \theta, & \mu^3 &= \cos \vartheta \sin \beta \sin \xi, \\ \mu^4 &= \cos \vartheta \sin \beta \cos \xi, & \mu^5 &= \sin \vartheta. \end{aligned} \quad (70)$$

Using the formulae given in the appendix, we find

$$\hat{g}_{00} = [e^{-6\lambda} \cos^2 \vartheta \cos^2 \beta + e^{4\lambda+w} \cos^2 \vartheta \sin^2 \beta + e^{4\lambda-2w} \sin^2 \vartheta]^{\frac{1}{3}} e^{2A-4\phi}. \quad (71)$$

Near the singularity, we have

$$\phi \sim \frac{1}{15} \ln(\rho - \rho_0), \quad \lambda \sim -\frac{2}{15} \ln(\rho - \rho_0), \quad w \sim \frac{4}{3} \ln(\rho - \rho_0) \quad (72)$$

which leads to

$$\hat{g}_{00} \sim \sin^{\frac{2}{3}} \vartheta (\rho - \rho_0)^{-\frac{2}{3}} \rightarrow \infty. \quad (73)$$

Therefore, the singularity is also unphysical by the criterion of [29].

3.3 Holographic RG flows with $SO(2)_{\text{diag}}$ symmetry

For non-vanishing ϕ_1 , the solutions will preserve only $N = 2$ supersymmetry and $SO(2)_{\text{diag}}$ symmetry. In this case, the BPS equations read

$$\phi'_1 = \frac{\sinh \phi_1}{2\Sigma} \left[2g \sinh \phi_5 \tanh \phi_3 + \cosh \phi_1 \{ 2g(\cosh \phi_5 - 2 \sinh \phi_5) + \sqrt{2} g_1 \Sigma^3 \} \right], \quad (74)$$

$$\begin{aligned} \phi'_3 = & \frac{1}{8} \Sigma^{-1} e^{-\phi_5} \left[\sinh 2\phi_3 \{ g + 3g \cosh 2\phi_1 - g e^{2\phi_5} (3 + \cosh 2\phi_1) \right. \\ & \left. + 2\sqrt{2} g_1 \Sigma^3 e^{\phi_5} \sinh^2 \phi_1 \} + 4g(e^{2\phi_5} - 1) \cosh \phi_1 \cosh 2\phi_3 \right], \end{aligned} \quad (75)$$

$$\begin{aligned} \phi'_5 = & \frac{1}{8} g \Sigma^{-1} e^{-\phi_5} \left[11 - \cosh 2\phi_3 - 4e^{2\phi_5} (\sinh \phi_3 - \cosh \phi_1 \cosh \phi_3)^2 \right. \\ & \left. - 6 \cosh 2\phi_1 \cosh^2 \phi_3 + 4 \cosh \phi_1 \sinh 2\phi_3 \right], \end{aligned} \quad (76)$$

$$\begin{aligned} \Sigma' = & \frac{1}{24} \left[g e^{-\phi_5} \{ 11 - 6 \cosh 2\phi_1 \cosh^2 \phi_3 + 4 \cosh \phi_1 \sinh 2\phi_3 - \cosh 2\phi_3 \right. \\ & \left. + 4e^{2\phi_5} (\sinh \phi_3 - \cosh \phi_1 \cosh \phi_3)^2 \} + 2\sqrt{2} g_1 \Sigma^3 (3 + \cosh 2\phi_1 \right. \\ & \left. + 2 \cosh 2\phi_3 \sinh^2 \phi_1) \right], \end{aligned} \quad (77)$$

$$\begin{aligned} A' = & \frac{1}{24} \Sigma^{-1} \left[g e^{-\phi_5} \{ 11 - \cosh 2\phi_3 + 4e^{2\phi_5} (\sinh \phi_3 - \cosh \phi_1 \cosh \phi_3)^2 \right. \\ & \left. - 6 \cosh 2\phi_1 \cosh^2 \phi_3 + 4 \cosh \phi_1 \sinh 2\phi_3 \} - \sqrt{2} g_1 \Sigma^3 (3 + \cosh 2\phi_1 \right. \\ & \left. + 2 \cosh 2\phi_3 \sinh^2 \phi_1) \right]. \end{aligned} \quad (78)$$

We are not able to find analytic solutions to these equations, so we will look for numerical solutions. We first consider the asymptotic behavior near the $N = 4$ AdS_5 vacuum given by

$$\Sigma \sim e^{-\frac{2r}{L}}, \quad \phi_1 \sim e^{\frac{r}{L}}, \quad \phi_3 + \phi_5 \sim e^{\frac{2r}{L}}, \quad \phi_5 - 2\phi_3 \sim e^{-\frac{4r}{L}}. \quad (79)$$

In addition to the dual operators of dimensions 2, 4 and 6 appearing in the previous case, there is a source term for an irrelevant operator of dimension

$\Delta = 5$ dual to ϕ_1 .

Due to non-vanishing ϕ_1 , the solutions preserve only four supercharges or $N = 1$ supersymmetry in four dimensions. An example of numerical solutions with $g = 2$ is shown in figure 1. In the figure, we have also given the behaviors of the scalar potential and the uplifted eleven-dimensional metric component $\hat{g}_{00}$. From these behaviors, we see that near the singularity, the scalar potential is bounded from above with $V \rightarrow 0$, and $\hat{g}_{00}$ vanishes. Therefore, the singularity is physical by both the criteria of [47] and [29]. The solution then describes an $N = 1$ supersymmetric RG flow from the $N = 2$ SCFT to $N = 2$ SYM in the IR. We also note that although the RG flow preserves only four supercharges due to non-vanishing ϕ_1 , the non-conformal phase in the IR preserves eight supercharges since $\phi_1 = 0$ near the IR singularity.

From the numerical solution, we see that near the IR singularity, $\phi_1 \rightarrow 0$ and $\phi_3 \sim \phi_5 \rightarrow \infty$. Using this asymptotic behavior in the BPS equations, we find for $r < 0$,

$$\Sigma \sim \sqrt{-\frac{3}{2gr}}, \quad \phi_3 \sim \phi_5 \sim \frac{3}{2} \ln(-gr), \quad A \sim -\frac{1}{4} \ln(-gr). \quad (80)$$

4 Supersymmetric Janus solutions

In this section, we look for supersymmetric Janus solutions describing three-dimensional conformal interfaces within $N = 2$ field theories in four dimensions. To preserve $SO(2, 3)$ conformal symmetry in three dimensions, we take the metric ansatz to be an AdS_4 -sliced domain wall

$$ds^2 = e^{2A(r)} ds_{AdS_4}^2 + dr^2 \quad (81)$$

with $ds_{AdS_4}^2$ being the metric on AdS_4 with radius ℓ . To find the relevant BPS equations for Janus solutions, we will closely follow the recent analysis in [27]. We first note that the structure of A_1^{ij} tensor given in (35) is very similar to that of [27]. In particular, there are two real and two complex eigenvalues.

As pointed out in [27], the real eigenvalues cannot lead to Janus solutions in the form of curved domain walls given above. Equivalently, the real eigenvalues can only support the flat domain walls describing holographic RG flows studied in the previous section. Accordingly, in this section, we will consider the complex eigenvalue α and take the Killing spinors of the unbroken supersymmetry to be ϵ_1 and ϵ_3 . We also point out that as in [27], α does not give rise to a viable superpotential in terms of which the scalar potential can be written.

We begin the analysis of the BPS equations by considering the variations $\delta\chi_i$ which give

$$\Sigma' \gamma_{\hat{r}} \epsilon_1 = \mathcal{A} \epsilon_3 \quad \text{and} \quad \Sigma' \gamma_{\hat{r}} \epsilon_3 = \mathcal{A}^* \epsilon_1 \quad (82)$$

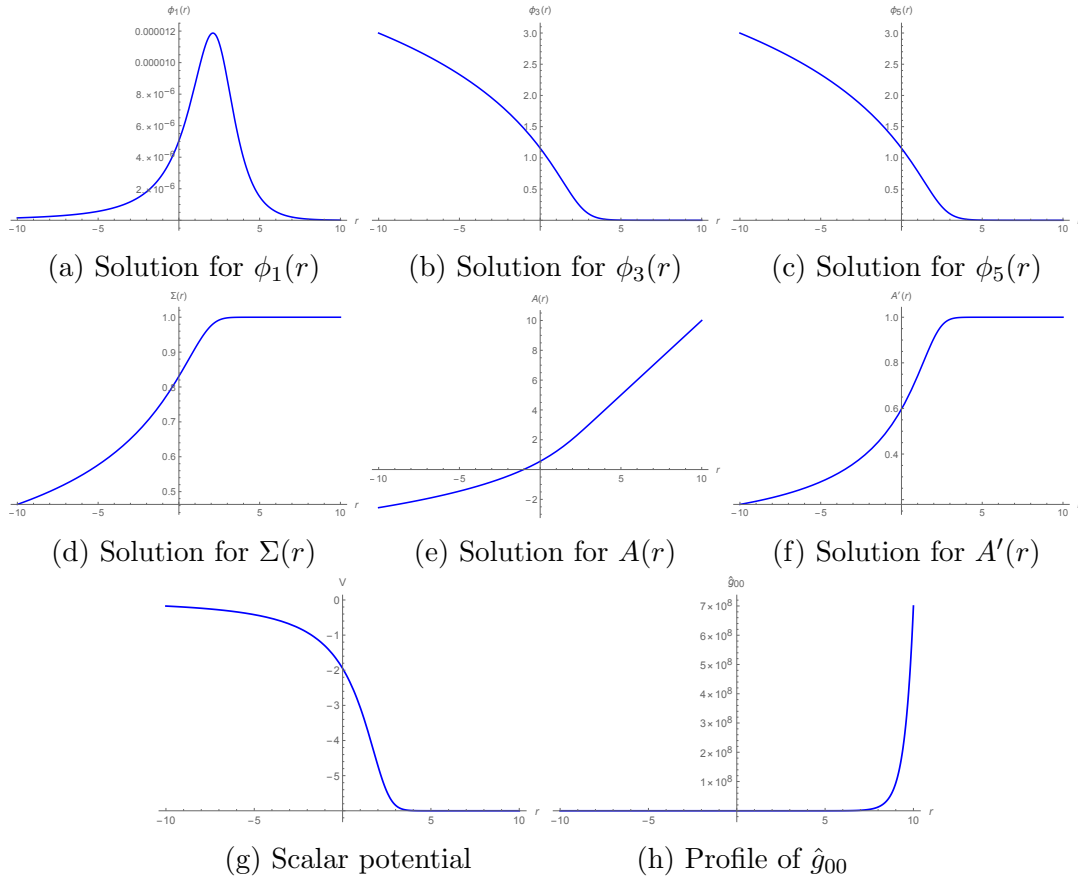


Figure 1: An $N = 1$ supersymmetric RG flow from the $N = 2$ SCFT dual to the $N = 4$ AdS_5 vacuum to $N = 2$ SYM in the IR for $g = 2$.

with

$$\begin{aligned} \mathcal{A} = & \frac{1}{3} [2g \sinh \phi_5 (\cosh \phi_1 \sinh \phi_3 + i \sinh \phi_1 - \cosh \phi_3) (i \cosh \phi_1 \sinh \phi_3 \\ & - \sinh \phi_1) + i (\cosh \phi_1 - i \sinh \phi_1 \sinh \phi_3)^2 (g \cosh \phi_5 + \sqrt{2} g_1 \Sigma^3)] . \end{aligned} \quad (83)$$

Following [27], we find that the two equations in (82) lead to the BPS equation for Σ of the form

$$\Sigma' = \eta |\mathcal{A}| \quad (84)$$

and a projector

$$\gamma_{\hat{r}} \epsilon_1 = \eta \frac{\mathcal{A}}{|\mathcal{A}|} \epsilon_3 \quad \text{and} \quad \gamma_{\hat{r}} \epsilon_3 = \eta \frac{\mathcal{A}^*}{|\mathcal{A}|} \epsilon_1 . \quad (85)$$

In these equations, we have introduced a sign factor $\eta = \pm 1$.

From $\delta \lambda_i^a$, we find two sets of equations of the form

$$\phi'_5 \gamma_{\hat{r}} \epsilon_3 = \mathcal{B}^* \epsilon_1 \quad \text{and} \quad \phi'_5 \gamma_{\hat{r}} \epsilon_1 = \mathcal{B} \epsilon_3 \quad (86)$$

and

$$(\phi'_3 - i \cosh \phi_3 \phi'_1) \gamma_{\hat{r}} \epsilon_1 = \mathcal{C}^* \epsilon_3 \quad \text{and} \quad (\phi'_3 + i \cosh \phi_3 \phi'_1) \gamma_{\hat{r}} \epsilon_3 = \mathcal{C} \epsilon_1 . \quad (87)$$

In these equations, the functions $\mathcal{B}$ and $\mathcal{C}$ are given by

$$\begin{aligned} \mathcal{B} = & g \Sigma^{-1} (\cosh \phi_3 - \cosh \phi_1 \sinh \phi_3 - i \sinh \phi_1) [\sinh \phi_5 (\sinh \phi_1 - i \cosh \phi_3 \\ & - i \cosh \phi_1 \sinh \phi_3) - 2 \cosh \phi_5 (\sinh \phi_1 - i \cosh \phi_1 \sinh \phi_3)] , \end{aligned} \quad (88)$$

$$\begin{aligned} \mathcal{C} = & -\frac{1}{4} \Sigma^{-1} [2g \cosh \phi_3 \sinh 2\phi_1 (\cosh \phi_5 - 2 \sinh \phi_5) + 2gi \cosh \phi_5 \sinh^2 \phi_1 \times \\ & \times \sinh 2\phi_3 + 4g (\sinh \phi_1 \sinh \phi_3 + i \cosh \phi_1 \cosh 2\phi_3 - i \cosh^2 \phi_1 \sinh 2\phi_3) \times \\ & \times \sinh \phi_5 - \sqrt{2} g_1 \Sigma^3 (\cosh \phi_3 \sinh 2\phi_1 + i \sinh^2 \phi_1 \sinh 2\phi_3)] . \end{aligned} \quad (89)$$

Using the $\gamma_{\hat{r}}$ projection given in (85), we find that equation (86) leads to BPS equations for ϕ_5

$$\phi'_5 = \eta \frac{\mathcal{A}^* \mathcal{B}}{|\mathcal{A}|} = \eta \frac{\mathcal{A} \mathcal{B}^*}{|\mathcal{A}|} \quad (90)$$

giving rise to an algebraic constraint for consistency of these two equations

$$\mathcal{A}^* \mathcal{B} = \mathcal{A} \mathcal{B}^* . \quad (91)$$

The explicit form of this constraint is remarkably simple

$$\Sigma^3 = -\frac{g}{\sqrt{2} g_1} \text{sech} \phi_5 = \text{sech} \phi_5 \quad (92)$$

in which we have used $g = -\sqrt{2}g_1$ in the last equality.

Repeating the same procedure in (87), we find additional two BPS equations for ϕ_1 and ϕ_3 of the form

$$\phi'_3 = \frac{\eta}{|\mathcal{A}|} \text{Re}(\mathcal{CA}) \quad \text{and} \quad \cosh \phi_3 \phi'_1 = \frac{\eta}{|\mathcal{A}|} \text{Im}(\mathcal{CA}). \quad (93)$$

We now consider the gravitino variations along AdS_4 directions with coordinates x^α for $\alpha = 0, 1, 2, 3$. The five-dimensional coordinates will be split as $x^\mu = (x^\alpha, r)$. As in [26], using the Killing spinor equations for AdS_4 of the form

$$\tilde{\nabla}_\alpha \epsilon_i = \frac{i}{2\ell} \kappa_i \gamma_r \gamma_\alpha \epsilon_i \quad (94)$$

with $\kappa_i = \pm 1$, we find

$$\left(A' - \frac{i}{\ell} \kappa_1 e^{-A} \right) \gamma_{\hat{r}} \epsilon_1 = \mathcal{W} \epsilon_3 \quad \text{and} \quad \left(A' - \frac{i}{\ell} \kappa_3 e^{-A} \right) \gamma_{\hat{r}} \epsilon_3 = \mathcal{W}^* \epsilon_1 \quad (95)$$

with

$$\begin{aligned} \mathcal{W} = & -\frac{1}{6} \Sigma^{-1} \left[i(2g \cosh \phi_5 - \sqrt{2}g_1 \Sigma^3)(\cosh \phi_1 \sinh \phi_3 + \cosh \phi_3 + i \sinh \phi_1) \right. \\ & + 4g \sinh \phi_5 (\sinh \phi_1 - i \cosh \phi_1 \sinh \phi_3) \left. \right] (\cosh \phi_1 \sinh \phi_3 \\ & - \cosh \phi_3 + i \sinh \phi_1). \end{aligned} \quad (96)$$

In obtaining the two equations in (95), we have rewritten the covariant derivative in terms of the covariant derivative $\tilde{\nabla}_\alpha$ on AdS_4 according to the relation

$$D_\alpha \epsilon_i = \tilde{\nabla}_\alpha \epsilon_i - \frac{1}{2} A' \gamma_r \gamma_\alpha \epsilon_i. \quad (97)$$

with the chirality matrix on AdS_4 given by $\gamma_r = i\gamma_{\hat{0}}\gamma_{\hat{1}}\gamma_{\hat{2}}\gamma_{\hat{3}}$. Consistency between the two equations in (95) implies $\kappa_3 = -\kappa_1$.

Using the $\gamma_{\hat{r}}$ projector given in (85) and writing $\kappa = \kappa_1 = -\kappa_3$, we find the BPS equation for A and another algebraic constraint

$$A' = \eta \frac{\text{Re}(i\mathcal{W}\mathcal{A}^*)}{|\mathcal{A}|} \quad \text{and} \quad \frac{\kappa}{\ell} e^{-A} = -\eta \frac{\text{Im}(\mathcal{W}\mathcal{A}^*)}{|\mathcal{A}|}. \quad (98)$$

It can also be verified that the two algebraic constraints in (92) and (98) are compatible with all the remaining BPS equations. Furthermore, all the BPS equations and these constraints also imply the second-ordered field equations. We also note that the two equations in (95) also imply the relation

$$A'^2 + \frac{1}{\ell^2} e^{-2A} = |\mathcal{W}|^2. \quad (99)$$

Finally, the remaining condition $\delta\psi_{\hat{r}i}$ determines the radial dependence of the Killing spinors.

The algebraic constraint given in (98) takes the form

$$\frac{\kappa}{\ell}e^{-A} = \eta \frac{\sqrt{2}gg_1}{3|\mathcal{A}|} \cosh^3 \phi_3 \sinh \phi_1 \sinh \phi_5 \Sigma^2 (\cosh \phi_1 - \tanh \phi_3)^2. \quad (100)$$

We readily see that for either $\phi_1 = 0$ or $\phi_5 = 0$, the constraint implies that the AdS_4 radius $\ell \rightarrow \infty$ resulting in a flat domain wall. From this constraint, it might appear that a further simplification with $\phi_3 = 0$ could still give curved domain wall solutions. However, this is not compatible with the BPS equation for ϕ_3 since $\phi'_3 \neq 0$ for $\phi_3 = 0$ unless $\phi_5 = 0$.

The above BPS equations can not be analytically solved. Therefore, we will look for numerical Janus solutions. Since the Killing spinors are given by only ϵ_1 and ϵ_3 subject to the projector (85), the solutions preserve only four supercharges. For regular Janus solutions, the solutions are asymptotically AdS_5 geometry on both sides of the interfaces. In particular, this implies that the metric function $A(r)$ has a turning point at a particular value of $r = r_0$ namely $A'(r_0) = 0$. As $r \rightarrow \pm\infty$, the asymptotic behavior of $A(r)$ is given by $A \sim \frac{r}{L}$ with L being the AdS_5 radius. In this case, $A(r)$ has a minimum at r_0 .

From the BPS equation for Σ given in (84), we have

$$\Sigma' = \eta \sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2} \quad (101)$$

with $\mathcal{A}_1$ and $\mathcal{A}_2$ being real and imaginary parts of $\mathcal{A}$. Follow the smoothness analysis in [48], we need to smoothly sewn the two branches of the solution with $\eta = 1$ and $\eta = -1$ at r_0 in order to obtain regular Janus solutions. In particular, this requires $\Sigma'(r_0) = 0$ or equivalently $\mathcal{A}_1 = \mathcal{A}_2 = 0$ at $r = r_0$. This also implies that Σ attains a minimum or a maximum at $r = r_0$. However, the possible choice of having $\Sigma(r_0)$ minimum leads to $A(r_0)$ being a maximum. So, we will require $\Sigma(r_0)$ to be a maximum.

For $\mathcal{A}_1 = 0$ condition, we have

$$\sinh \phi_1 \left[g \cosh \phi_1 \sinh \phi_3 (\cosh \phi_5 - 2 \sinh \phi_5) + g \cosh \phi_3 \sinh \phi_5 + \sqrt{2}g_1 \cosh \phi_1 \sinh \phi_3 \Sigma^3 \right] = 0. \quad (102)$$

To satisfy this condition, the simplest possibility is to set

$$\phi_1(r_0) = 0. \quad (103)$$

Using this result in $\mathcal{A}_2 = 0$ condition together with (92), we find

$$\begin{aligned} \Sigma(r_0)^3 &= -\frac{g}{\sqrt{2}g_1} \text{sech} \phi_5(r_0) \\ \text{and} \quad \phi_3(r_0) &= \frac{1}{2} \ln [\cosh \phi_5(r_0) (\cosh \phi_5(r_0) + \sinh \phi_5(r_0))]. \end{aligned} \quad (104)$$

With these results, the second algebraic constraint given in (100) can be used to determine the value of $A(r_0)$. Therefore, we can determine the values of all the fields at the turning point in terms of a free parameter $\phi_5(r_0)$. It turns out that for any value of $\phi_5(r_0)$,

$$A'(r_0) = \Sigma'(r_0) = \phi_1'(r_0) = \phi_3'(r_0) = \phi_5'(r_0) = 0. \quad (105)$$

However, from the constraint (92), we find that the maximal value of Σ is 1 at $\phi_5 = 0$. All these results would imply that $\Sigma = 1$ identically. This also leads to $\phi_5 = 0$ identically resulting in a flat domain wall solution.

Another possibility of setting the bracket in (102) to zero leads to either

$$\phi_1(r_0) = \cosh^{-1} \tanh \phi_3(r_0) \quad \text{and} \quad \phi_5(r_0) = \frac{1}{2} \ln[\cosh 2\phi_3(r_0) - 2] \quad (106)$$

or

$$\phi_5(r_0) = 0 \quad \text{and} \quad \phi_1(r_0) = \cosh^{-1} \left[\frac{1}{2} \coth \phi_3(r_0) \right]. \quad (107)$$

The former has no real solutions while the latter leads to $\Sigma(r) = 1$ identically as in the previous case. Therefore, there do not seem to exist any supersymmetric regular Janus solutions interpolating between the supersymmetric AdS_5 vacuum on each side of the interface.

However, a numerical search shows that there exist Janus solutions interpolating between non-conformal phases of $N = 2$ SCFT dual to the $N = 4$ AdS_5 vacuum. An example of these solutions is given in figure 2. Both sides of the interface correspond to a non-conformal phase of $N = 2$ SCFT or $N = 2$ super Yang-Mills theory in four dimensions. We note that on both sides of the interface, we have $\phi_1 = 0$ implying the enhancement of supersymmetry to eight supercharges. This $N = 2$ SYM theory is the non-conformal phase of the $N = 2$ SCFT appearing in the RG flow solution shown in figure 1. Therefore, we expect the Janus solution in figure 2 to describe conformal interfaces within $N = 2$ SYM theory. This solution is similar to those given in [49] and [50] in which supersymmetric Janus solutions in $ISO(7)$ maximal gauged supergravity in four dimensions have been found. In that case, the solutions are also attracted to the non-conformal phases rather than the conformal fixed points.

To find the numerical solution in figure 2, we have chosen the turning point $r_0 = 0$ and used (103) and (104) with $\phi_5(0) = 0.1$. For larger values of $\phi_5(0)$, one side of the solutions becomes singular. An example of these solutions with $\phi_5(0) = 1$ is shown in figure 3. This should describe a conformal boundary within the $N = 2$ SYM as pointed out in [51]. Depending on the boundary conditions, there are also solutions that are singular on both sides of the interfaces. An example of these solutions is shown in figure 4. A similar solution has also been obtained in four-dimensional $N = 4$ gauged supergravity arising from a truncation of eleven-dimensional supergravity on a tri-sasakian manifold [52].

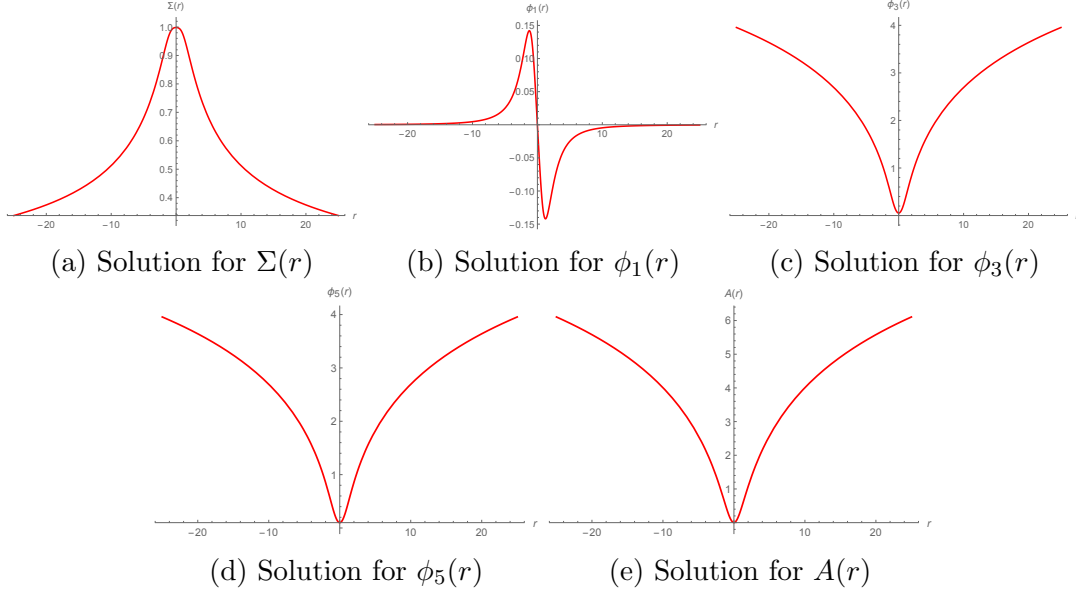


Figure 2: An example of Janus solutions interpolating between $N = 2$ SYM phases with $\ell = 1$, $\kappa = -1$ and $g = 2$.

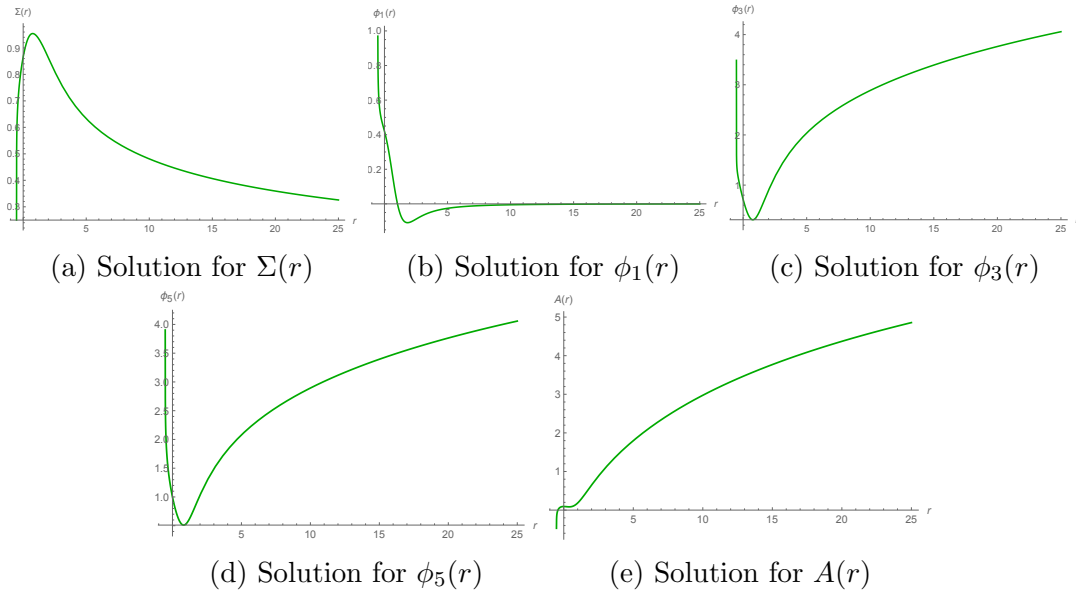


Figure 3: An example of Janus solutions interpolating between $N = 2$ SYM and a singularity with $\ell = 1$, $\kappa = -1$ and $g = 2$.

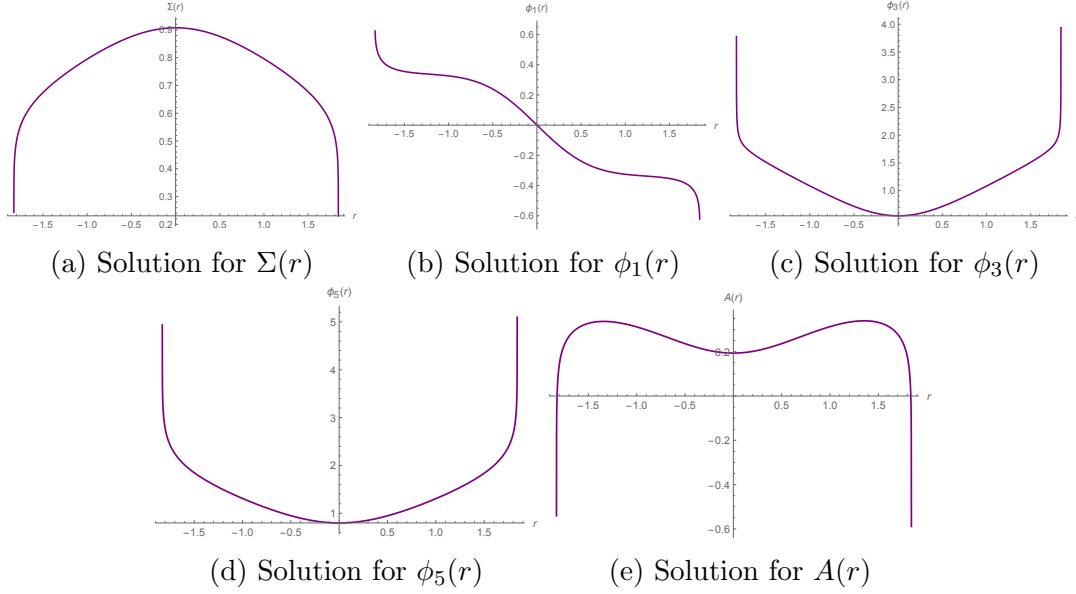


Figure 4: An example of Janus solutions interpolating between singularities on both sides with $\ell = 1$, $\kappa = -1$ and $g = 2$.

5 Supersymmetric AdS_5 black strings

In this section, we consider solutions interpolating between the $N = 4$ supersymmetric AdS_5 vacuum and an $AdS_3 \times \Sigma$ geometry with Σ being a Riemann surface. These solutions describe supersymmetric black strings in asymptotically AdS_5 space. Holographically, the solutions describe RG flows across dimensions from the $N = 2$ SCFT in four dimensions to two-dimensional SCFTs in the IR. The latter arises from twisted compactifications of the former on Σ .

The ansatz for the metric is given by

$$ds^2 = e^{2f(r)} dx_{1,1}^2 + dr^2 + e^{2h(r)} (d\theta^2 + f_\kappa^2(\theta) d\phi^2) \quad (108)$$

with

$$f_\kappa(\theta) = \begin{cases} \sin \theta, & \kappa = 1 & \text{for } \Sigma^2 = S^2 \\ \theta, & \kappa = 0 & \text{for } \Sigma^2 = T^2 \\ \sinh \theta, & \kappa = -1 & \text{for } \Sigma^2 = H^2 \end{cases} \quad (109)$$

We will split the five-dimensional coordinates as $x^\mu = (x^\alpha, r, \theta, \phi)$ with $\alpha = 0, 1$. With an obvious choice of vielbein

$$e^{\hat{\alpha}} = e^f dx^\alpha, \quad e^{\hat{r}} = dr, \quad e^{\hat{\theta}} = e^h d\theta, \quad e^{\hat{\phi}} = e^h f_\kappa(\theta) d\phi, \quad (110)$$

non-vanishing components of the spin connection are given by

$$\omega^{\hat{\alpha}}_{\hat{r}} = f' e^{\hat{\alpha}}, \quad \omega^{\hat{\theta}}_{\hat{r}} = h' e^{\hat{\theta}}, \quad \omega^{\hat{\phi}}_{\hat{r}} = h' e^{\hat{\phi}}, \quad \omega^{\hat{\phi}}_{\hat{\theta}} = \frac{f'_\kappa(\theta)}{f_\kappa(\theta)} e^{-h} e^{\hat{\phi}}. \quad (111)$$

with $f'_\kappa(\theta) = \frac{df_\kappa(\theta)}{d\theta}$.

In order to preserve some amount of supersymmetry, we will follow the standard procedure of performing a topological twist by turning on some gauge fields to cancel $\omega^{\hat{\phi}}_{\hat{\theta}}$ component of the spin connection. We will consider a twist achieved by turning on $SO(2) \times SO(2)$ gauge fields A^0 and A^3 . There are four $SO(2) \times SO(2)$ singlet scalars consisting of the dilaton and other three scalars from $SO(5,3)/SO(5) \times SO(3)$. The coset representative for the latter can be obtained by setting $\phi_1 = \phi_2 = 0$ in (32). Relevant components of the composite connection are given by

$$Q_i^j = \frac{i}{2} \left[g_1 A^0 \delta_i^k - g A^3 (\mathbf{I}_2 \otimes \sigma_3)_i^k \right] (\sigma_2 \otimes \sigma_3)_k^j. \quad (112)$$

We then turn on $SO(2) \times SO(2)$ gauge fields of the form

$$A^0 = a_0 f'_\kappa(\theta) d\phi \quad \text{and} \quad A^3 = a_3 f'_\kappa(\theta) d\phi. \quad (113)$$

The corresponding field strength tensors are given by

$$F^0 = dA^0 = -\kappa a_0 e^{-2h} e^{\hat{\theta}} \wedge e^{\hat{\phi}} \quad \text{and} \quad F^3 = dA^3 = -\kappa a_3 e^{-2h} e^{\hat{\theta}} \wedge e^{\hat{\phi}} \quad (114)$$

in which we have used the relation $f''_\kappa(\theta) = -\kappa f_\kappa(\theta)$. Finally, for solutions with r -dependent scalar fields, we need to impose the $\gamma_{\hat{r}}$ projector of the form

$$\gamma_{\hat{r}} \epsilon_i = -(\sigma_2 \otimes \sigma_3)_i^j \epsilon_j \quad (115)$$

in which we have chosen a definite sign choice in order to make the AdS_5 vacuum appear in the limit $r \rightarrow \infty$.

5.1 AdS_5 black strings preserving four supercharges

We begin with supersymmetric AdS_5 black strings preserving four supercharges. These solutions can be obtained by performing a twist using an $SO(2)$ gauge field. From the composite connection given in (112), we find that

$$\delta\psi_{i\hat{\phi}} = \frac{1}{2} \frac{f'_\kappa(\theta)}{f_\kappa(\theta)} e^{-h} \gamma_{\hat{\phi}\hat{\theta}} \epsilon_i + \frac{i}{2} [g_1 a_0 - g a_3 (\mathbf{I}_2 \otimes \sigma_3)] \frac{f'_\kappa(\theta)}{f_\kappa(\theta)} e^{-h} (\sigma_2 \otimes \sigma_3)_i^j \epsilon_j + \dots \quad (116)$$

with $\dots$ denoting other terms in the variation of $\delta\psi_{i\hat{\phi}}$. The topological twist amounts to the cancellation between the two terms appearing in (116). There are two possibilities to achieve this by turning on only one $SO(2)$ gauge field.

- A^0 -twist:

We can set $A^3 = 0$ and turn on A^0 to cancel the spin connection on Σ . This is achieved by imposing the following projector

$$\gamma_{\hat{\phi}\hat{\theta}} \epsilon_i = -i(\sigma_2 \otimes \sigma_3)_i^j \epsilon_j \quad (117)$$

together with a twist condition

$$g_1 a_0 = 1. \quad (118)$$

- A^3 -twist:

In this case, we set $A^0 = 0$ and imposing the projector

$$\gamma_{\hat{\phi}\hat{\theta}}\epsilon_i = i(\sigma_2 \otimes \mathbf{I}_2)_i^j \epsilon_j \quad (119)$$

as well as a twist condition

$$ga_3 = 1. \quad (120)$$

The A^0 -twist leads to the following BPS equations

$$\phi'_3 = g\Sigma^{-1}e^{-2\phi_3} \sinh \phi_5, \quad (121)$$

$$\phi'_5 = -\frac{1}{2}\Sigma^{-1}ge^{-2\phi_3-\phi_5}(1-2e^{2\phi_3}+e^{2\phi_5}), \quad (122)$$

$$\Sigma' = \frac{1}{6}\left[ge^{-2\phi_3-\phi_5}(2e^{2\phi_3}+e^{2\phi_5}-1)+2\sqrt{2}g_1\Sigma^3-2\sqrt{2}\kappa a_0\Sigma^{-1}e^{-2h}\right], \quad (123)$$

$$h' = -\frac{1}{6}\Sigma^{-2}\left[\sqrt{2}g_1\Sigma^4-g\Sigma e^{-2\phi_3-\phi_5}(2e^{2\phi_3}+e^{2\phi_5}-1)+2\sqrt{2}\kappa a_0e^{-2h}\right], \quad (124)$$

$$f' = -\frac{1}{6}\Sigma^{-2}\left[\sqrt{2}g_1\Sigma^4-g\Sigma e^{-2\phi_3-\phi_5}(2e^{2\phi_3}+e^{2\phi_5}-1)-\sqrt{2}\kappa a_0e^{-2h}\right]. \quad (125)$$

We also note that compatibility between the BPS equations and the field equations requires $\phi_4 = 0$. In addition, it can be verified that the two-form fields can be consistently set to zero. We now look for $AdS_3 \times \Sigma$ fixed point at which $\phi'_3 = \phi'_5 = \Sigma' = h' = 0$ and $f' = \frac{1}{L_3}$ with L_3 being the AdS_3 radius. It turns out that the BPS equations do not admit any $AdS_3 \times \Sigma$ fixed point solutions. The IR geometry is singular with

$$\phi_3 = \phi_5 \sim 0, \quad \Sigma \sim \frac{1}{\sqrt{r-r_0}}, \quad h \sim \ln(r-r_0), \quad f \sim \frac{1}{4}\ln(r-r_0) \quad (126)$$

for a finite value of $r = r_0$. Uplifting to eleven dimensions, we find that the metric component

$$\hat{g}_{00} \sim \text{constant}, \quad (127)$$

so the singularity is physically acceptable. We then expect the solution to describe an RG flow from four-dimensional $N = 2$ SCFT to non-conformal field theory in two dimensions.

For the A^3 -twist, a similar analysis leads to the following BPS equations

$$\phi'_3 = g\Sigma^{-1}e^{-2\phi_3} \sinh \phi_5, \quad (128)$$

$$\phi'_5 = -\frac{1}{2}\Sigma^{-1}e^{-\phi_5}\left[ge^{-2\phi_3}(1-2e^{2\phi_3}+e^{2\phi_5})+\kappa a_3\Sigma^2e^{-2h}(e^{2\phi_5}-1)\right], \quad (129)$$

$$\Sigma' = \frac{1}{6}\left[ge^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)+2\sqrt{2}g_1\Sigma^3-2\kappa a_3e^{-2h}\Sigma^2 \cosh \phi_5\right], \quad (130)$$

$$h' = \frac{1}{6}\Sigma^{-1}\left[ge^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)-\sqrt{2}g_1\Sigma^3+4\kappa a_3\Sigma^2e^{-2h} \cosh \phi_5\right], \quad (131)$$

$$f' = \frac{1}{6}\Sigma^{-1}\left[ge^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)-\sqrt{2}g_1\Sigma^3-2\kappa a_3\Sigma^2e^{-2h} \cosh \phi_5\right]. \quad (132)$$

These equations admit one supersymmetric $AdS_3 \times \Sigma$ fixed point given by

$$\begin{aligned} \phi_3 = \phi_5 = 0, \quad \Sigma = - \left(\frac{\sqrt{2}g}{g_1} \right)^{\frac{1}{3}}, \\ h = \frac{1}{2} \ln \left[-\kappa a_3 \left(\frac{2}{gg_1^2} \right)^{\frac{1}{3}} \right], \quad L_3 = - \left(\frac{\sqrt{2}}{g_1 g^2} \right)^{\frac{1}{3}}. \end{aligned} \quad (133)$$

This solution gives a real warp factor h only for $\kappa = -1$, so in this case, there is only an $AdS_3 \times H^2$ fixed point.

We also note that the fixed point preserves eight supercharges due to the projector (119). Recall that the supersymmetry parameters ϵ_i transforming under $SO(1, 3) \times SO(5)_R$ as $(\mathbf{4}, \mathbf{4})$. Following the analysis in [29], we decompose this representation under the subgroup $SO(1, 1) \times SO(2)_\Sigma \times SO(2) \times SO(2)_R$ in which $SO(2) \times SO(2)_R \subset SO(2) \times SO(3)_R \subset SO(5)_R$ and $SO(1, 1) \times SO(2)_\Sigma \subset SO(1, 3)$. The $SO(2)_R \subset SO(3)_R \sim SO(3) \subset ISO(3)$ corresponds to the A^3 gauge field that participates in the twist. Since the twist is performed by identifying $SO(2)_\Sigma$ with $SO(2)_R$, the unbroken supersymmetry corresponds to the twisted Killing spinors in the representations with opposite charges under $SO(2)_\Sigma$ and $SO(2)_R$; $(+, \pm, +\mp), (+, \pm, -\mp), (-, \pm, +\mp)$ and $(-, \pm, -\mp)$. This leads to $N = (2, 2)$ superconformal symmetry in two dimensions. However, due to an extra $\gamma_{\hat{r}}$ -projector given in (115), the flow solutions interpolating between the AdS_5 vacuum and this $AdS_3 \times H^2$ geometry preserve only four supercharges corresponding to $N = (2, 2)$ Poincare supersymmetry in two dimensions. This can also be explicitly seen as follows. Recall that in five dimensions, $i\gamma^{\hat{0}}\gamma^{\hat{1}}\gamma^{\hat{r}}\gamma^{\hat{\theta}}\gamma^{\hat{\phi}} = \pm \mathbf{I}_4$, the two projectors given in (115) and (119) imply that

$$\gamma^{\hat{0}}\gamma^{\hat{1}}\epsilon_i = \pm (\mathbf{I}_2 \otimes \sigma_3)_i^j \epsilon_j. \quad (134)$$

This indicates that half of the unbroken supersymmetry has opposite two-dimensional chirality to the other half leading to $N = (2, 2)$ supersymmetry.

It should be noted that since all scalars from vector multiplets vanish, this $AdS_3 \times H^2$ fixed point is the same as that found in pure $N = 4$ gauged supergravity with $U(1) \times SU(2)$ gauge group [53]. The uplifted eleven-dimensional solution has been discussed in [54]. An example of numerical solutions for these interpolating solutions is shown by the orange line in figure 5. In this solution, we have set $\phi_3 = \phi_5 = 0$ and $g = 2$ corresponding to a unit AdS_5 radius. The solution describes a black string in asymptotically AdS_5 space with a near horizon geometry given by $AdS_3 \times H^2$. Upon uplifted to eleven dimensions, this leads to a supersymmetric $AdS_3 \times H^2 \times H^2 \times S^4$ geometry preserving eight supercharges [54]. Holographically, this solution describes an RG flow from $N = 2$ SCFT in four dimensions to $N = (2, 2)$ two-dimensional SCFT in the IR.

We can also compute the central charge of the dual two-dimensional

$N = (2, 2)$ SCFT in the IR by the standard formula

$$c = \frac{3L_3}{2G_N^{(3)}}. \quad (135)$$

The Newton's constant in three dimensions is related to that in five dimensions by

$$\frac{1}{G_N^{(3)}} = \frac{e^{2h_0} \text{vol}(\Sigma)}{G_N^{(5)}} \quad (136)$$

with h_0 being the value of $h(r)$ at the $AdS_3 \times \Sigma$ fixed point. $G_N^{(5)}$ is in turn obtained by a truncation of eleven-dimensional supergravity on $\tilde{H}^2 \times S^4$ as

$$\frac{1}{G_N^{(5)}} = \frac{\Sigma_0^{-\frac{3}{5}} \text{vol}(\tilde{H}^2) \text{vol}(S^4) R_{S^4}^4}{G_N^{(11)}} \quad (137)$$

in which $G_N^{(11)} = 16\pi^7 \ell_p^9$ with ℓ_p being eleven-dimensional Plank's length. We also recall that the S^4 truncation of eleven-dimensional supergravity leads to an $AdS_7 \times S^4$ geometry with

$$L_7^2 = 4(\pi N)^{\frac{2}{3}} \ell_p^3 = \frac{4}{m^2} \quad \text{and} \quad R_{S^4} = \frac{1}{m} \quad (138)$$

with m being the gauge coupling constant in seven-dimensional gauged supergravity.

Following [29], we will work with a unit such that the AdS_7 radius $L_7 = 1$ or $m = 2$. This leads to

$$G_N^{(11)} = \frac{\pi^5}{4N^2} \quad \text{and} \quad R_{S^4} = \frac{1}{2}. \quad (139)$$

With all these, we eventually find $G_N^{(5)}$ of the form

$$\begin{aligned} \frac{1}{G_N^{(5)}} &= \frac{\Sigma_0^{-\frac{3}{5}} \text{vol}(\tilde{H}^2) \text{vol}(S^4) N^2}{4\pi^5} \\ &= \frac{\Sigma_0^{-\frac{3}{5}} |\tilde{g} - 1| N^2}{2\pi^2}. \end{aligned} \quad (140)$$

In the second line, we have used the volume of a unit S^4 , $\text{vol}(S^4) = \frac{\pi^2}{2}$ and the volume of a genus $g \neq 1$ Riemann surface

$$\text{vol}(\Sigma) = 4\pi|g - 1|. \quad (141)$$

Finally, we can determine the central charge of the dual two-dimensional SCFT

$$c = \frac{3}{\pi} L_3 N^2 \Sigma_0^{-\frac{3}{5}} e^{2h_0} |\tilde{g} - 1| |\hat{g} - 1| \quad (142)$$

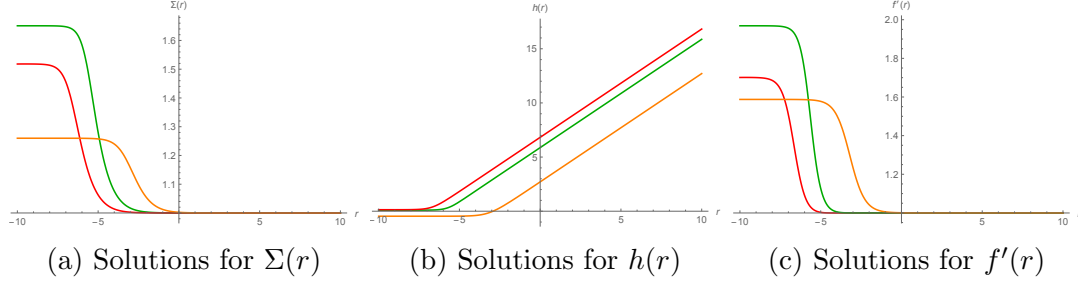


Figure 5: Examples of supersymmetric AdS_5 black string solutions. The orange line represents the solution interpolating between $N = 4$ AdS_5 vacuum and $AdS_3 \times H^2$ geometry preserving 8 supercharges. The red (green) line corresponds to a solution interpolating between AdS_5 vacuum and $AdS_3 \times H^2$ ($AdS_3 \times S^2$) preserving 4 supercharges.

with $\tilde{g}$ and $\hat{g}$ denoting the genera of $\tilde{H}^2$ and Σ , respectively. For the $AdS_3 \times H^2$ fixed point given above, we find

$$c = \frac{3(2^{\frac{4}{5}})N^2 a_3 |\tilde{g} - 1| |\hat{g} - 1|}{\pi g^2}. \quad (143)$$

By turning on scalar fields from vector multiplets, we obtain black string solutions that are not solutions of pure $N = 4$ gauged supergravity although the $AdS_3 \times H^2$ fixed point is the solution of both pure and matter-coupled $N = 4$ gauged supergravities. The numerical analysis shows that there exist solutions describing holographic RG flows from the non-conformal $N = 2$ SYM phase of the $N = 2$ SCFT to the $N = (2, 2)$ SCFT in two dimensions. An example of these solutions is shown by the green line in figure 6. By tuning the boundary condition very close to the $AdS_3 \times H^2$ fixed point, we also find solutions interpolating between domain wall and $AdS_3 \times H^2$ fixed point with an intermediate AdS_5 geometry. These solutions describe RG flows from $N = 2$ SYM to $N = 2$ SCFT in four dimensions and then to $N = (2, 2)$ SCFT in two dimensions. An example of these solutions is shown by the blue line in figure 6. Both of types of these solutions shown in figure 6 also describe black strings in asymptotically domain wall space-time.

5.2 AdS_5 black strings preserving two supercharges

We now consider a twist on Σ by $SO(2) \times SO(2)$ gauge fields. To cancel the spin connection on Σ as shown in (116), we impose the following projectors

$$(\mathbf{I}_2 \otimes \sigma_3)_i^j \epsilon_j = -\epsilon_i \quad \text{and} \quad \gamma_{\hat{\phi}\hat{\theta}} \epsilon_i = -i(\sigma_2 \otimes \sigma_3)_i^j \epsilon_j \quad (144)$$

together with a twist condition

$$g_1 a_0 + g a_3 = 1. \quad (145)$$

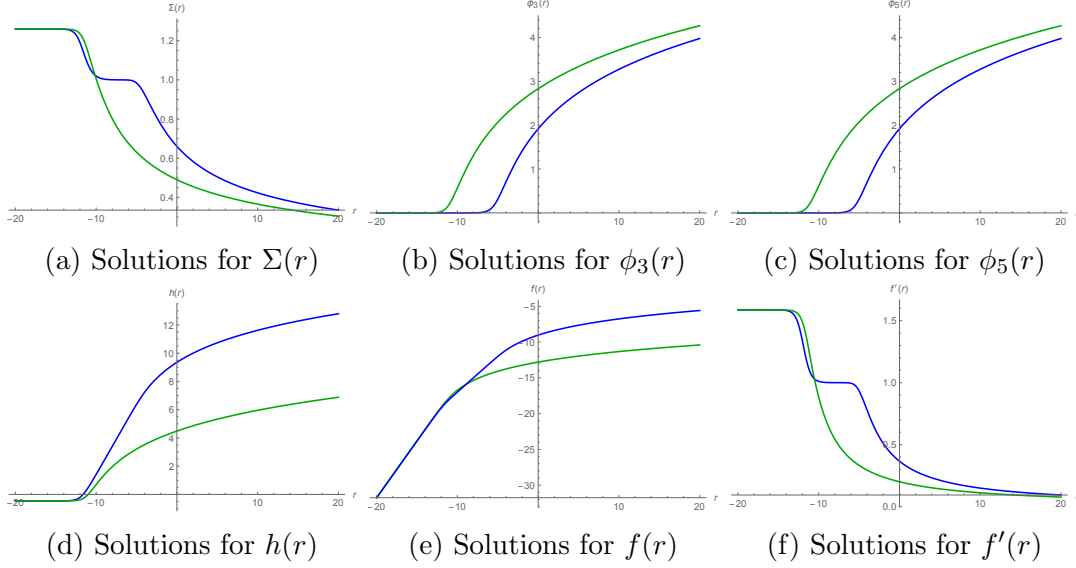


Figure 6: Examples of RG flows from $N = 2$ SYM to $N = (2, 2)$ SCFT in two dimensions (green) and from $N = 2$ SYM to four-dimensional $N = 2$ SCFT and $N = (2, 2)$ SCFT in two dimensions (blue) with $g = 2$.

Using the $\gamma_{\hat{r}}$ -projector given in (115), we find the following BPS equations

$$\phi'_3 = g\Sigma^{-1}e^{-2\phi_3}\sinh\phi_5, \quad (146)$$

$$\phi'_5 = -\frac{1}{2}\Sigma^{-1}e^{-\phi_5}\left[ge^{-2\phi_3}(1-2e^{2\phi_3}+e^{2\phi_5})+\kappa a_3\Sigma^2e^{-2h}(e^{2\phi_5}-1)\right], \quad (147)$$

$$\Sigma' = \frac{1}{6}\left[ge^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)-2\kappa\Sigma^{-1}(\sqrt{2}a_0+a_3\Sigma^3\cosh\phi_5)+2\sqrt{2}g_1\Sigma^3\right], \quad (148)$$

$$h' = \frac{1}{6}\Sigma^{-2}\left[g\Sigma e^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)-\sqrt{2}g_1\Sigma^4-2\sqrt{2}\kappa a_0e^{-2h}+4\kappa a_3\Sigma^3e^{-2h}\cosh\phi_5\right], \quad (149)$$

$$h' = \frac{1}{6}\Sigma^{-2}\left[g\Sigma e^{-2\phi_3-\phi_5}(e^{2\phi_5}+2e^{2\phi_3}-1)-\sqrt{2}g_1\Sigma^4+\sqrt{2}\kappa a_0e^{-2h}-2\kappa a_3\Sigma^3e^{-2h}\cosh\phi_5\right]. \quad (150)$$

As in the previous case, consistency with all the field equations requires $\phi_4 = 0$. In this case, due to an extra projector in (144), $AdS_3 \times \Sigma$ fixed points preserve four supercharges while the full interpolating RG flow solutions preserve only two supercharges. In this case, the first projector in (144) implies that $\epsilon_1 = \epsilon_3 = 0$. Using the relation $i\gamma^{\hat{0}}\gamma^{\hat{1}}\gamma^{\hat{r}}\gamma^{\hat{\theta}}\gamma^{\hat{\phi}} = \pm\mathbf{I}_4$ again, we find

$$\gamma^{\hat{0}}\gamma^{\hat{1}}\epsilon_{2,4} = \mp\epsilon_{2,4} \quad (151)$$

which implies that the unbroken supercharges have a definite two-dimensional chirality corresponding to $N = (0, 2)$ or $N = (2, 0)$ supersymmetry.

From the BPS equations, we find an $AdS_3 \times \Sigma$ fixed point given by

$$\Sigma = \left[\frac{\sqrt{2}(a_0 g_1 - a_3 g)}{g_1 a_3} \right]^{\frac{1}{3}}, \quad h = \frac{1}{6} \ln \left[\frac{2\kappa a_3^4}{g_1^2(g_1 a_0 - g a_3)} \right],$$

$$\phi_5 = \phi_3 = 0, \quad L_3 = \frac{2}{(a_0 g_1 - 2a_3 g)} \left[\frac{\sqrt{2}a_3^2(a_0 g_1 - a_3 g)}{g_1} \right]^{\frac{1}{3}}. \quad (152)$$

Unlike the previous case of $SO(2)$ twist, there can be both $AdS_3 \times H^2$ and $AdS_3 \times S^2$ solutions depending on the values of a_3 and g . We again find that this is the same as the $AdS_3 \times \Sigma$ solutions obtained in pure $N = 4$ gauged supergravity. An example of numerical solutions interpolating between the AdS_5 vacuum and an $AdS_3 \times H^2$ geometry is shown by the red line in figure 5. In this solution, we have chosen the following numerical values of various parameters

$$g = 2, \quad \kappa = -1, \quad a_3 = 2. \quad (153)$$

For solutions interpolating between the AdS_5 vacuum and an $AdS_3 \times S^2$ geometry, a numerical solution is shown by the green line in figure 5 with

$$g = 2, \quad \kappa = 1, \quad a_3 = -2. \quad (154)$$

Both of these solutions should describe holographic RG flows from $N = 2$ SCFT to two-dimensional $N = (0, 2)$ SCFTs. The latter arise from twisted compactifications of the former on H^2 and S^2 , respectively.

Equivalently, these solutions correspond to black strings in asymptotically AdS_5 space with near horizon geometries given by $AdS_3 \times H^2$ and $AdS_3 \times S^2$. These geometries give rise to $AdS_3 \times H^2 \times H^2 \times S^4$ and $AdS_3 \times S^2 \times H^2 \times S^4$ solutions in eleven-dimensional supergravity via a consistent truncation on $H^2 \times S^4$ [54]. As in the previous case, using (142), we can compute the central charge of the dual two-dimensional $N = (0, 2)$ SCFT

$$c = -\frac{24}{\pi} \frac{\kappa N^2 a_3^2 |\tilde{g} - 1| |\hat{g} - 1|}{g^2 (\sqrt{2}a_0 + 4a_3)} \left(\frac{a_3}{\sqrt{2}a_0 + 2a_3} \right)^{\frac{1}{5}}. \quad (155)$$

As in the previous case, by turning on scalars from vector multiplets, we find RG flow solutions interpolating between $AdS_3 \times \Sigma$ fixed points and $N = 2$ SYM and $N = 2$ SCFT in four dimensions as shown in figures 7 and 8. These solutions describe RG flows from $N = 2$ SYM in four dimensions to two-dimensional $N = (0, 2)$ SCFTs dual to $AdS_3 \times H^2$ or $AdS_3 \times S^2$ geometries in the IR. The solutions represented by red and pink lines correspond to RG flows directly from $N = 2$ SYM to two-dimensional $N = (0, 2)$ SCFTs while solutions shown by

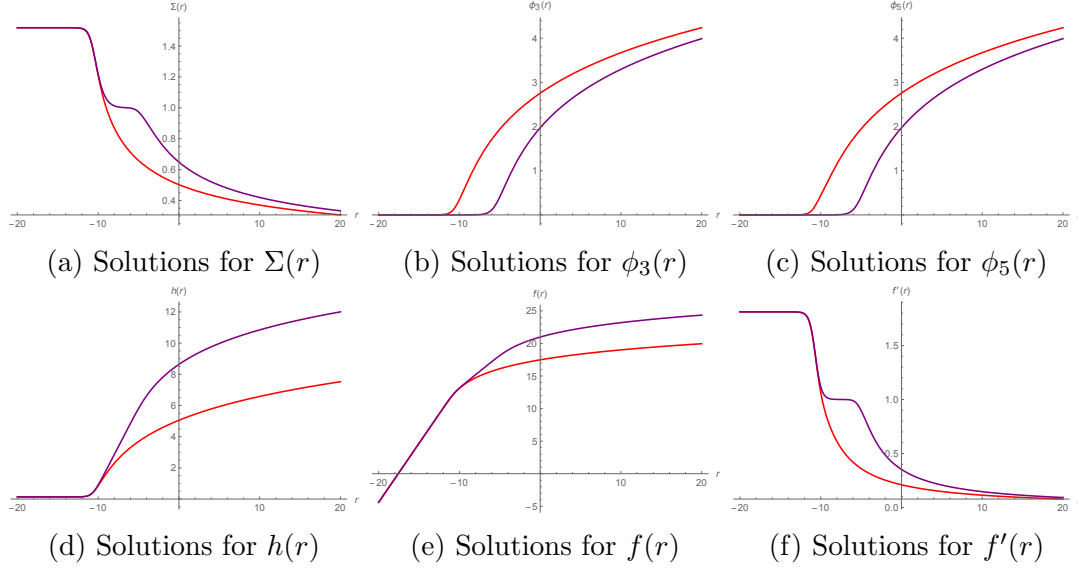


Figure 7: Examples of RG flows from $N = 2$ SYM to $N = (0, 2)$ SCFT in two dimensions dual to $AdS_3 \times H^2$ geometry (red) and from $N = 2$ SYM to four-dimensional $N = 2$ SCFT and $N = (0, 2)$ SCFT in two dimensions (purple) with $g = 2$.

purple and cyan lines describe RG flows from $N = 2$ SYM to two-dimensional $N = (0, 2)$ SCFTs via $N = 2$ SCFT in four dimensions. As in the previous case, these solutions correspond to black strings in asymptotically domain wall space-time.

We end this section by some comments on the solutions with $SO(2)_{\text{diag}}$ twist. In this case, the scalar coset representative is given by (32). The two $SO(2)$ gauge fields A^0 and A^3 are related by $g_1 A^0 = g A^3$. As in the previous section, non-vanishing ϕ_1 scalar breaks supersymmetry corresponding to the Killing spinors ϵ_1 and ϵ_3 . The topological twist is achieved by imposing the projectors

$$\gamma_{\hat{\phi}\hat{\theta}}\epsilon_2 = \epsilon_4 \quad \text{and} \quad \gamma_{\hat{\phi}\hat{\theta}}\epsilon_4 = -\epsilon_2 \quad (156)$$

and a twist condition $ga_3 = \frac{1}{2}$. However, in this case, there are no $AdS_3 \times \Sigma$ fixed points from the resulting BPS equations. We refrain from giving the detail on this analysis here.

6 Supersymmetric AdS_5 black holes

In this section, we perform a similar analysis in the case of supersymmetric AdS_5 black hole solutions with a near horizon geometry given by $AdS_2 \times \mathcal{M}_3$. We will consider the case of $\mathcal{M}_3$ being a constant curvature 3-manifold in the form of H^3

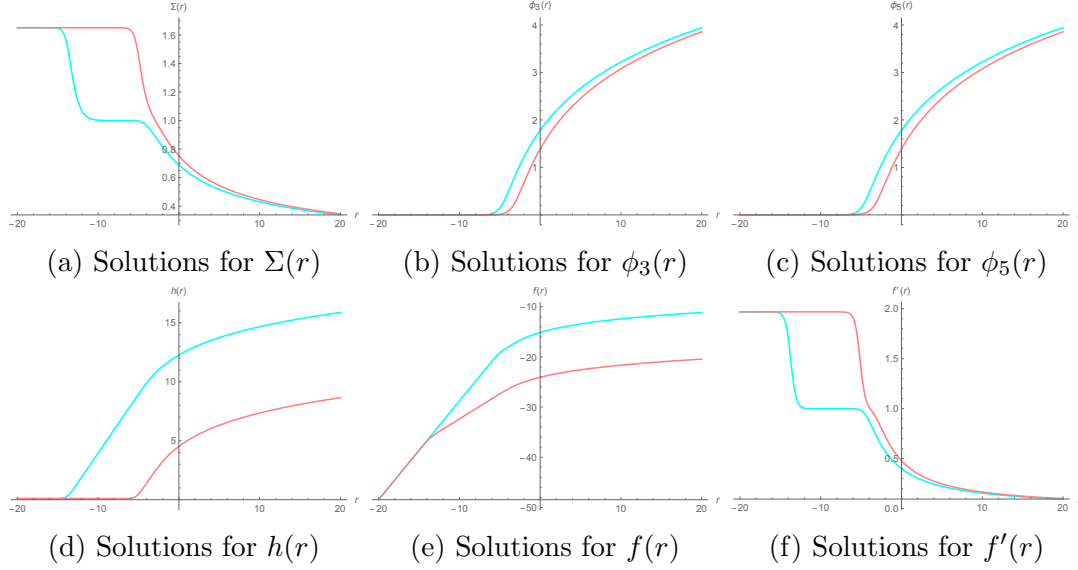


Figure 8: Examples of RG flows from $N = 2$ SYM to $N = (0, 2)$ SCFT in two dimensions dual to $AdS_3 \times S^2$ geometry (pink) and from $N = 2$ SYM to four-dimensional $N = 2$ SCFT and $N = (0, 2)$ SCFT in two dimensions (cyan) with $g = 2$.

or S^3 . The metric ansatz is given by

$$ds^2 = -e^{2f(r)} dt^2 + dr^2 + e^{2h(r)} [d\psi^2 + f_\kappa^2(\psi)(d\theta^2 + \sin^2 \theta d\phi^2)] \quad (157)$$

with $f_\kappa(\psi)$ defined in (109). For convenience, we note the vielbein and spin connection of this metric as follows

$$\begin{aligned} e^{\hat{0}} &= e^f dt, & e^{\hat{r}} &= dr, & e^{\hat{\psi}} &= e^h d\psi, \\ e^{\hat{\theta}} &= e^h f_\kappa(\theta) d\theta, & e^{\hat{\phi}} &= e^h f_\kappa(\theta) \sin \theta d\phi \end{aligned} \quad (158)$$

and

$$\begin{aligned} \omega^{\hat{0}}_{\hat{r}} &= f' e^{\hat{0}}, & \omega^{\hat{\psi}}_{\hat{r}} &= h' e^{\hat{\psi}}, & \omega^{\hat{\theta}}_{\hat{r}} &= h' e^{\hat{\theta}}, & \omega^{\hat{\phi}}_{\hat{r}} &= h' e^{\hat{\phi}}, \\ \omega^{\hat{\phi}}_{\hat{\psi}} &= \frac{f'_\kappa(\psi)}{f_\kappa(\psi)} e^{-h} e^{\hat{\phi}}, & \omega^{\hat{\phi}}_{\hat{\theta}} &= \frac{e^{-h}}{f_\kappa(\psi)} \cot \theta e^{\hat{\phi}}, & \omega^{\hat{\theta}}_{\hat{\psi}} &= \frac{f'_\kappa(\psi)}{f_\kappa(\psi)} e^{-h} e^{\hat{\theta}}. \end{aligned} \quad (159)$$

To perform a topological twist, we turn on $SO(3)$ gauge fields corresponding to A^3 , A^4 and A^5 . We then consider $SO(3)$ invariant scalars. There is only one $SO(3)$ singlet scalar from $SO(5, 3)/SO(5) \times SO(3)$ corresponding to the non-compact generator

$$Y_s = Y_{31} + Y_{42} + Y_{53}. \quad (160)$$

The coset representative is given by

$$\mathcal{V} = e^{\varphi Y_s}. \quad (161)$$

The relevant terms in the composite connection are given by

$$Q_i^j = -\frac{i}{2}gA^3(\sigma_2 \otimes \mathbf{I}_2)_i^j + \frac{i}{2}gA^4(\sigma_3 \otimes \sigma_1)_i^j - \frac{i}{2}gA^5(\sigma_1 \otimes \sigma_1)_i^j. \quad (162)$$

To cancel the components of the spin connection along $\mathcal{M}_3$, given by the second line of (159), we take the ansatz for the gauge fields of the form

$$A^3 = -a_3 \cos \theta d\phi, \quad A^4 = -a_4 f'_\kappa(\psi) \sin \theta d\phi, \quad A^5 = -a_5 f'_\kappa(\psi) d\theta. \quad (163)$$

We perform the twist by imposing the following projectors

$$\begin{aligned} \gamma_{\hat{\theta}\hat{\psi}}\epsilon_i &= -i(\sigma_1 \otimes \sigma_1)_i^j \epsilon_j, \\ \gamma_{\hat{\phi}\hat{\psi}}\epsilon_i &= -i(\sigma_3 \otimes \sigma_1)_i^j \epsilon_j, \\ \gamma_{\hat{\phi}\hat{\theta}}\epsilon_i &= -i(\sigma_2 \otimes \mathbf{I}_2)_i^j \epsilon_j \end{aligned} \quad (164)$$

and twist conditions

$$a_3 g = 1, \quad a_4 g = -1, \quad a_5 g = 1. \quad (165)$$

We also note that only two projectors in (164) are independent. Accordingly, the near horizon geometry $AdS_2 \times \mathcal{M}_3$ preserves four supercharges. As in the case of black string solutions, all the two-form fields can be consistently set to zero. It is useful to note the field strength tensors for the gauge fields

$$F^3 = \kappa a e^{-2h} e^{\hat{\theta}} \wedge e^{\hat{\phi}}, \quad F^4 = -\kappa a e^{-2h} e^{\hat{\psi}} \wedge e^{\hat{\phi}}, \quad F^5 = \kappa a e^{-2h} e^{\hat{\psi}} \wedge e^{\hat{\theta}} \quad (166)$$

in which we have written $a_5 = a_3 = -a_4 = a$ and used the relations $f''_\kappa(\psi) = -\kappa f_\kappa(\psi)$ and $1 - f'_\kappa(\psi)^2 = \kappa f_\kappa^2(\psi)$.

Using the projector (115), we find the following BPS equations

$$\varphi' = \frac{1}{2}\Sigma^{-1}e^{-2h-3\varphi}(e^{2\varphi} - 1)(ge^{2h} - a\kappa\Sigma^2e^{2\varphi}), \quad (167)$$

$$\Sigma' = \frac{1}{3}\left[ge^{-2\varphi}(\cosh \varphi + 2 \sinh \varphi) + \sqrt{2}g_1\Sigma^3 - 3\kappa a\Sigma^2e^{-2h}\cosh \varphi\right], \quad (168)$$

$$h' = -\frac{1}{6}\Sigma^{-1}\left[ge^{-3\varphi}(1 - 3e^{2\varphi}) + \sqrt{2}g_1\Sigma^3 - 6\kappa a\Sigma^2e^{-2h}\cosh \varphi\right], \quad (169)$$

$$f' = -\frac{1}{6}\Sigma^{-1}\left[ge^{-3\varphi}(1 - 3e^{2\varphi}) + \sqrt{2}g_1\Sigma^3 + 6\kappa a\Sigma^2e^{-2h}\cosh \varphi\right]. \quad (170)$$

These equations admit one $AdS_2 \times \mathcal{M}_3$ fixed point solution given by

$$\varphi = 0, \quad \Sigma = -\sqrt{2}\left(\frac{g}{g_1}\right)^{\frac{1}{3}}, \quad h = \frac{1}{2}\ln\left[-\frac{2a\kappa}{(gg_1^2)^{\frac{1}{3}}}\right]. \quad (171)$$

The solution exists only for $\kappa = -1$ giving rise to $AdS_2 \times H^3$ geometry. This solution is also the same as that of pure $N = 4$ gauged supergravity [55]. By

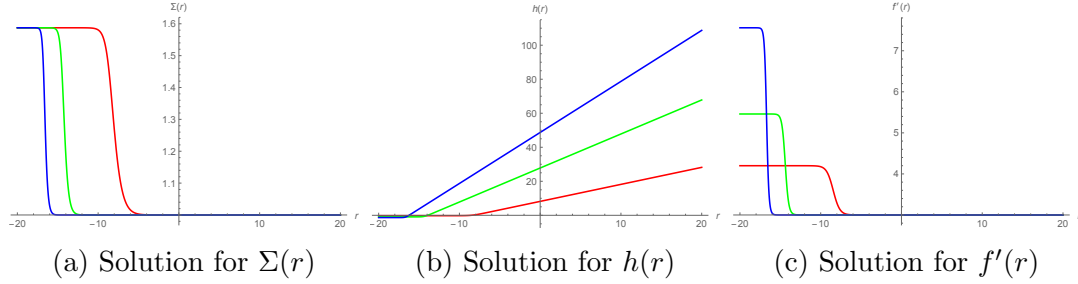


Figure 9: Supersymmetric AdS_5 black hole solutions with $AdS_2 \times H^3$ near horizon geometry for $g = 2$ (red), $g = 4$ (green) and $g = 6$ (blue).

setting $\varphi = 0$, we find examples of numerical solutions interpolating between the AdS_5 vacuum and this $AdS_2 \times H^3$ geometry as shown in figure 9 with three different values of $g = 2, 4, 6$. These solutions describe supersymmetric black holes in asymptotically AdS_5 space with $AdS_2 \times H^3$ near horizon geometry. Holographically, the solutions correspond to RG flows across dimensions from $N = 2$ SCFT in four dimensions to superconformal quantum mechanics via twisted compactifications on H^3 . Upon uplifted to eleven dimensions, these solutions lead to $AdS_2 \times H^3 \times H^2 \times S^4$ geometry in M-theory [54]. Similar to the black string solutions, turning on the scalar φ from vector multiplets leads to solutions describing RG flows from $N = 2$ SYM or $N = 2$ SYM and $N = 2$ SCFT in four dimensions to superconformal quantum mechanics as shown by the green and orange lines in figure 10, respectively. These solutions describe black holes in asymptotically domain wall space-time.

We end this section by giving the entropy of the black hole using the formulae

$$S_{\text{BH}} = \frac{A}{4G_N^{(5)}}. \quad (172)$$

Using $G_N^{(5)}$ given in (140) and $A = \text{vol}(H^3)e^{3h_0}$, we find the entropy of the black hole

$$S_{\text{BH}} = \frac{N^2 |\tilde{g} - 1| \text{vol}(H^3)}{2^{\frac{7}{5}} \pi^2 g^3}. \quad (173)$$

7 Conclusions and discussions

In this paper, we have studied various holographic solutions from five-dimensional $N = 4$ gauged supergravity with $SO(2) \times ISO(3)$ gauge group. The gauged supergravity admits a unique $N = 4$ supersymmetric AdS_5 vacuum dual to an $N = 2$ SCFT in four dimensions. The $N = 2$ SCFT arises from M5-branes wrapped on a Riemann surface with genus higher than one, H^2 . We have found solutions describing holographic RG flows preserving eight supercharges from this $N = 2$

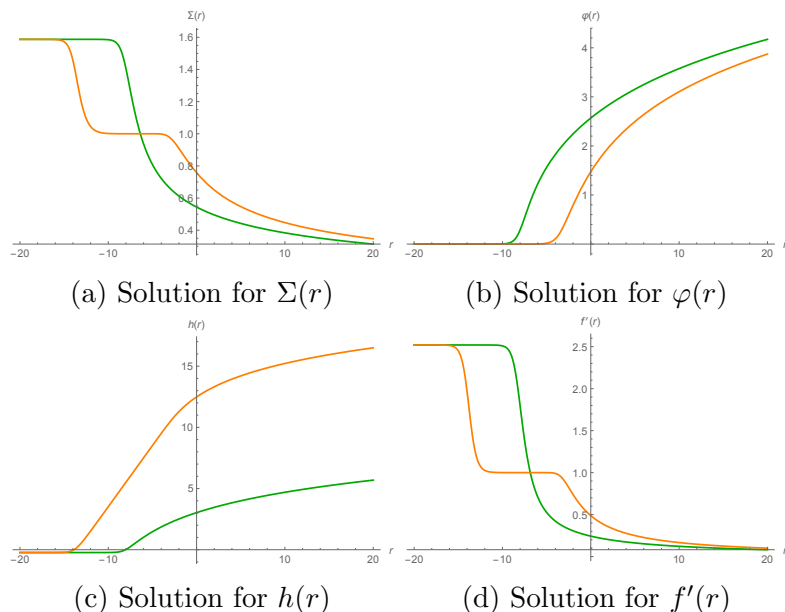


Figure 10: Examples of RG flows from $N = 2$ SYM to superconformal quantum mechanics dual to $AdS_2 \times H^3$ geometry (green) and from $N = 2$ SYM to four-dimensional $N = 2$ SCFT and superconformal quantum mechanics (orange) with $g = 2$.

SCFT to non-conformal phases with $SO(2) \times SO(3)$ and $SO(2) \times SO(2)$ symmetries. However, the five-dimensional solutions and the uplifted eleven-dimensional solutions contain singularities that are of unphysical types according to the criteria of [47] and [29]. It could be interesting to see whether these singularities can be resolved in the context of M-theory. We have also found an RG flow solution preserving four supercharges and $SO(2)_{\text{diag}}$ symmetry from $N = 2$ SCFT to $N = 2$ SYM in the IR. In this case, unlike the previous two solutions, the IR singularity turns out to be physical, but the solution can only be obtained numerically.

Another class of solutions are Janus interfaces described by AdS_4 -sliced domain walls. We have studied these solutions within the $SO(2)_{\text{diag}}$ truncation. There do not exist regular Janus solutions interpolating between AdS_5 vacua on both sides of the interfaces at least within the truncation considered here. However, we have found solutions interpolating between non-conformal or $N = 2$ SYM phases. These solutions would describe conformal interfaces within $N = 2$ SYM theories. The solutions preserve four supercharges while the SYM phases preserve eight supercharges. This solution provides the first example of non-conformal Janus solutions in five-dimensional gauged supergravities. We have also given examples of solutions interpolating between the SYM phase and a singularity as well as between singularities. We expect these solutions to describe boundary CFTs as pointed out in [51].

As a final class of solutions, we have considered supersymmetric AdS_5 black string and black hole solutions. By performing an $SO(2)$ twist on a Riemann surface, we have found an AdS_5 black string preserving four supercharges with $AdS_3 \times H^2$ near horizon geometry. On the other hand, by turning on $SO(2) \times SO(2)$ gauge fields to implement the topological twist, we have found supersymmetric AdS_5 black strings preserving two supercharges with both $AdS_3 \times H^2$ and $AdS_3 \times S^2$ near horizon geometries. These solutions holographically describe RG flows across dimensions from $N = 2$ SCFT in four dimensions to two-dimensional $N = (2, 2)$ and $N = (0, 2)$ SCFTs in the IR. We also note that the near horizon geometries enhance supersymmetry to eight and four supercharges, respectively.

It turns out that the existence of $AdS_3 \times \Sigma$ solutions requires vanishing of scalar fields from vector multiplets. Therefore, the near horizon geometries and the entire flow solutions with only the dilaton non-vanishing are the same as those of pure $N = 4$ $SU(2) \times U(1)$ gauged supergravity. Upon uplifted to eleven dimensions, these geometries would lead to $AdS_3 \times H^2 \times H^2 \times S^4$ and $AdS_3 \times S^2 \times H^2 \times S^4$ solutions of eleven-dimensional supergravity [54]. By turning on scalars from vector multiplets dual to irrelevant and marginal operators, we have found solutions interpolating between $AdS_3 \times \Sigma$ and $N = 2$ SYM in four dimensions. We interpret these solutions as holographic RG flows from $N = 2$ SYM to $N = (2, 2)$ and $N = (0, 2)$ SCFTs in two dimensions. For particular values of the boundary condition, the solutions flow from $N = 2$ SYM to $N = 2$ SCFT and then to $N = (2, 2)$ and $N = (0, 2)$ SCFTs in two dimensions. Similarly, the only $AdS_2 \times H^3$ near horizon geometry of black holes is that of pure $N = 4$ gauged supergravity. As in the case of black strings, when scalars from vector multiplets participate in the flows, we have found solutions interpolating between the $AdS_2 \times H^3$ near horizon geometry and $N = 2$ SYM with and without intermediate $N = 4$ AdS_5 geometry. These solutions are expected to describe holographic RG flows from $N = 2$ SYM in four dimensions to $N = 2$ SCFT and superconformal quantum mechanics in the IR.

It would be interesting to extend the present study to other types of holographic solutions such as line defects within $N = 2$ SCFT considered recently in [56] and solutions describing strings and black holes with the near horizon geometries involving spindles or topological disks as in [57, 58]. It could also be of particular interest to identify the field theory duals of the gravity solutions given in this paper within the $N = 2$ SCFT. In particular, it could be interesting to recover the black hole entropy given in (173) by using the twisted index of the $N = 2$ SCFT on H^3 as in [59, 60, 61]. We hope to come back to some of these issues in future works.

Acknowledgement

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A Eleven-dimensional metric components

In this appendix, we give explicit forms of the uplifted eleven-dimensional metric component $\hat{g}_{00}$ used in the main text. The procedure is to uplift the metric $g_{00} = -e^{2A}$ in five dimensions to seven dimensions by using the results of [4] and then further uplift the resulting seven-dimensional metric to eleven dimensions using the S^4 truncation of eleven-dimensional supergravity given in [6].

According to the result of [4], the seven-dimensional metric is given by

$$g_{00}^{(7)} = e^{-4\phi} g_{00} \quad (174)$$

which leads to the eleven-dimensional metric of the form

$$\hat{g}_{00} = \Delta^{\frac{1}{3}} g_{00}^{(7)} = -\Delta^{\frac{1}{3}} e^{2A-4\phi}. \quad (175)$$

The warp factor Δ is defined by

$$\Delta = T_{\hat{a}\hat{b}} \mu^{\hat{a}} \mu^{\hat{b}}, \quad \hat{a}, \hat{b} = 1, 2, \dots, 5 \quad (176)$$

with $T_{\hat{a}\hat{b}}$ being a symmetric $SL(5)$ matrix and $\mu^{\hat{a}}$ are constrained coordinates on S^4 satisfying $\mu^{\hat{a}} \mu^{\hat{a}} = 1$.

In the truncation considered in [4], the matrix $T_{\hat{a}\hat{b}}$ decomposes as

$$T_{\hat{a}\hat{b}} = \begin{pmatrix} e^{-6\lambda} & & \\ & e^{-6\lambda} & \\ & & e^{4\lambda} \mathcal{T}_{\alpha\beta} \end{pmatrix} \quad (177)$$

with $\mathcal{T}_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3$ is a symmetric matrix parametrizing $SL(3)/SO(3) \subset SL(5)/SO(5)$ submanifold. In terms of $SL(3)/SO(3)$ coset representative V , we have $\mathcal{T}_{\alpha\beta} = (VV^t)_{\alpha\beta}$.

We also note that the $SO(5, 3)$ invariant tensor used in [4] is off-diagonal of the form

$$\tilde{\eta}_{MN} = \begin{pmatrix} \mathbf{I}_2 & 0 & 0 \\ 0 & 0 & \mathbf{I}_3 \\ 0 & \mathbf{I}_3 & 0 \end{pmatrix} \quad (178)$$

in which we have rearranged some rows and columns to match the present convention. $\tilde{\eta}_{MN}$ is related to the diagonal η_{MN} used in this paper via a transformation matrix of the form

$$\mathcal{U} = \begin{pmatrix} \mathbf{I}_2 & 0 & 0 \\ 0 & -U & U \\ 0 & U & U \end{pmatrix} \quad (179)$$

with

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (180)$$

We also note some useful relations $\mathcal{U} = \mathcal{U}^{-1} = \mathcal{U}^t$.

A.1 $SO(2) \times SO(3)$ invariant scalars

In this case, we have $\mathcal{T}_{\alpha\beta} = \delta_{\alpha\beta}$, and the $SO(5, 3)/SO(5) \times SO(3)$ coset representative given in [4] becomes

$$\tilde{\mathcal{V}} = \begin{pmatrix} \mathbf{I}_2 & & \\ & e^{-\varphi_3} \mathbf{I}_3 & \\ & & e^{\varphi_3} \mathbf{I}_3 \end{pmatrix}. \quad (181)$$

By transforming to the basis with diagonal η_{MN} using $\mathcal{U}$, we precisely recover the coset representative (32) with only ϕ_3 and $\phi_5 = \phi_3$ non-vanishing. We then identify φ_3 with ϕ_3 . Using the relations given in [4]

$$\varphi_3 = 3\phi - \lambda \quad \text{and} \quad \Sigma = e^{-\phi-3\lambda}, \quad (182)$$

we can determine ϕ and λ in terms of the scalars Σ and ϕ_3 in section 3 as

$$\phi = \frac{1}{10}(3\phi_3 - \ln \Sigma) \quad \text{and} \quad \lambda = -\frac{1}{10}(\phi_3 + 3 \ln \Sigma). \quad (183)$$

A.2 $SO(2) \times SO(2)$ invariant scalars

In this case, we have

$$\mathcal{T}_{\alpha\beta} = \text{diag}(e^w, e^w, e^{-2w}). \quad (184)$$

After transforming to the basis with diagonal η_{MN} , we find the following identification

$$\phi_3 = \varphi_3 + \frac{w}{2} \quad \text{and} \quad \phi_5 = \varphi_3 - w \quad (185)$$

or

$$w = \frac{2}{3}(\phi_3 - \phi_5) = \frac{2}{3}\varphi_2 \quad \text{and} \quad \varphi_3 = \frac{1}{3}(\phi_5 + 2\phi_3) = \frac{1}{6}(3\varphi_1 + \varphi_2). \quad (186)$$

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